Essays In Financial Fragility And Regulations

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Essays In Financial Fragility And Regulations

Abstract
This dissertation studies financial fragility caused by coordination failure and discusses plausible regulations to alleviate coordination problems and enhance social welfare. It consists of two chapters.

In the first chapter, "Capital Flows in the Financial System and Supply of Credit," I study how capital flows in the financial system affect the coordination problem among banks in supplying credit to the real economy. When credit contraction raises concerns about an economic recession, the economy can end up in a self-fulfilling credit freeze that banks abstain from lending for fear that others would withhold lending. I show that capital flows across banks can alleviate the self-fulfilling credit freeze problem because banks that are prone to supply credit can borrow from other banks to extend more credit to the real economy. However, when the interest rates for interbank capital flows are low, they signal dim economic prospects and deter credit supply by banks. As a result, the economy can get stuck in an equilibrium with low interest rates. In such equilibrium, freezing interbank capital flows and real credit crunch co-exist and reinforce each other through a vicious feedback loop. This is consistent with the observations in the shallow post-crisis recovery—low real interest rates, contraction in wholesale funding markets, low credit growth, and sluggish economic growth. My model suggests that regulations addressing counter-party risks can be a remedy to prevent capital flows in the financial system from freezing, which breaks the vicious feedback loop and stabilizes the real credit market. This paper develops a model to study how capital flows in the financial system affect the coordination problem among banks in supplying credit to the real economy.

In the second chapter, "Intervention with Screening in Global Games," joint with Junyuan Zou, we propose a novel intervention program to reduce coordination failure. Compared with conventional government-guarantee type programs, such as demand deposit insurance, ours incur a lower cost of implementation and suffers less from moral hazard problems. The proposed program effectively screens agents based on their heterogeneous beliefs of the coordination results. In equilibrium, only a small mass of "pivotal agents" self-select to participate in the program. However, the effect is amplified by strategic complementarities, and coordination failure can be significantly reduced. We demonstrate the generality of the proposed program with applications in panic-based bank runs, debt rollover problems, self-fulfilling market freezes, and underinvestment problems in the real economy.

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ABSTRACT

ESSAYS IN FINANCIAL FRAGILITY AND REGULATIONS

Lin Shen

Itay Goldstein

This dissertation studies financial fragility caused by coordination failure and discusses plausible regulations to alleviate coordination problems and enhance social welfare. It consists of two chapters.

In the first chapter, “Capital Flows in the Financial System and Supply of Credit,” I study how capital flows in the financial system affect the coordination problem among banks in supplying credit to the real economy. When credit contraction raises concerns about an economic recession, the economy can end up in a self-fulfilling credit freeze that banks abstain from lending for fear that others would withhold lending. I show that capital flows across banks can alleviate the self-fulfilling credit freeze problem because banks that are prone to supply credit can borrow from other banks to extend more credit to the real economy. However, when the interest rates for interbank capital flows are low, they signal dim economic prospects and deter credit supply by banks. As a result, the economy can get stuck in an equilibrium with low interest rates. In such equilibrium, freezing interbank capital flows and real credit crunch co-exist and reinforce each other through a vicious feedback loop. This is consistent with the observations in the shallow post-crisis recovery low real interest rates, contraction in wholesale funding markets, low credit growth, and sluggish economic growth. My model suggests that regulations addressing counter-party risks can be a remedy to prevent capital flows in the financial system from freezing, which breaks the vicious feedback loop and stabilizes the real credit market. This paper develops a model to study how capital flows in the financial system affect the coordination problem among banks in supplying credit to the real economy.
In the second chapter, “Intervention with Screening in Global Games,” joint with Junyuan Zou, we propose a novel intervention program to reduce coordination failure. Compared with conventional government-guarantee type programs, such as demand deposit insurance, ours incur a lower cost of implementation and suffers less from moral hazard problems. The proposed program effectively screens agents based on their heterogeneous beliefs of the coordination results. In equilibrium, only a small mass of “pivotal agents” self-select to participate in the program. However, the effect is amplified by strategic complementarities, and coordination failure can be significantly reduced. We demonstrate the generality of the proposed program with applications in panic-based bank runs, debt rollover problems, self-fulfilling market freezes, and underinvestment problems in the real economy.
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CHAPTER 1 : Capital Flows in the Financial System and the Supply of Credit

Lin Shen

1.1. Introduction

In the aftermath of the 2007–2008 financial crisis, financial institutions worldwide, especially those in Europe, have displayed considerable reluctance to supply credit to the real economy, which is perceived as a key contributing factor to the sluggish economic recovery. Indeed, such credit contraction can slow firms’ investment, households’ consumption, and economic activities, resulting in a sluggish economy that, in turn, reduces the profits from lending and deters financial institutions (referred to as banks hereafter) from extending credit to the real economy. Therefore, banks may abstain from lending in fear that other banks would withhold lending, and the ensuing aggregate credit contraction impedes economic growth. This gives rise to a so-called self-fulfilling credit freeze, during which banks withdraw real credit supply en masse (Bebchuk and Goldstein, 2011).

The reasoning behind the self-fulfilling credit freeze highlights the strategic interactions among banks in supplying credit to the real economy. However, it overlooks the interconnectedness of banks in the financial system. In the modern market-based financial system, capital actively flows across banks in various wholesale funding markets, such as interbank loan markets and securitized asset markets. The capital flows in the financial system may

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2In a 2016 policy address (Praet, 2016), Peter Praet, a member of the Executive Board of the European Central Bank (ECB), claimed that the “episode inevitably set back the recovery: banks in large parts of the eurozone became less willing and capable of keeping credit flowing to the real economy.” In a 2014 speech (Cœuré, 2014), his colleague, Benoît Cœuré, also emphasized the “vicious circle of low growth, low investment and low credit.”

3Capital flows in this paper broadly refer to wholesale funding markets that facilitate resource reallocation within the financial system, including long-term interbank loans (Craig and Ma, 2017), term Fed Funds (Kuo et al., 2014), asset-backed securities (ABS) (Loutskina and Strahan, 2009; Stein, 2010; Loutskina, 2011), and
affect the real credit supply process via two opposing channels. On the one hand, they can potentially alleviate the credit freeze problem, because banks willing to extend credit can borrow from others and thus supply more credit to the real economy. On the other hand, the process itself also reveals information. In particular, interest rates for interbank capital flows can aggregate information possessed by individual banks and reflect the aggregate economic prospects. Low interest rates may discourage banks from supplying credit, because low interest rates “signal that the economy’s long-run growth prospects are dim,” as Stanley Fischer, the former vice chairman of the Federal Reserve, stated in a 2016 speech (Fischer, 2016).

In view of the two opposing forces, in aggregate, do capital flows in the financial system dampen or improve the supply of credit to the real economy? To answer this question, I build a model featuring resource reallocation and information revelation and analyze their interactions in channeling capital from “Wall Street” to “Main Street.” A key result of the model is that the possibility of interbank capital flows can destabilize the financial system and hamper the aggregate real credit supply. Specifically, when interest rates are low, negative information about economic prospects raises banks concerns about counterparty risks and deters banks from lending to one another. In essence, negative information blocks flow of capital and undermines resource reallocation in the financial system. As a result, the economy can get stuck in an equilibrium characterized by low interest rates. In such equilibrium, freezing interbank capital flows and real credit crunches coexist and reinforce each other in a vicious feedback loop.

This result has major policy implications. Specifically, in the wake of the latest financial crisis, policy makers have implemented stress tests and tighter regulations in general, to address counterparty risks and secure the soundness of the financial system in adverse economic conditions. However, opponents argue that regulatory overreach may restrict so forth. The notion is broader than overnight interbank lending, which serves the role of liquidity insurance (Bhattacharya et al., 1985; Allen and Gale, 2000; Freixas et al., 2000).

4For example, in the United States, the Federal Reserve conducts annual Dodd-Frank Act stress testing (DFAST), and, in Europe, the European Central Bank (ECB) conducts European-wide stress tests.
banks from extending credit and eventually hinder economic growth. The paper contributes to the policy debate by highlighting a new channel through which financial regulations addressing counterparty risks help stabilize the credit market. That is, such regulations can reduce coordination failure in the credit supply process, i.e., self-fulfilling credit freezes. In particular, in times of low interest rates, such regulations maintain active capital flows in the financial system, which also relieves banks concerns about aggregate credit contraction and incentivizes them to supply credit to the real economy.

Following Bebchuk and Goldstein (2011), I begin with a benchmark model without interbank capital flows using the global-games methodology, a canonical framework for analyzing coordination problems. In the benchmark model, banks receive heterogeneous private signals about macroeconomic fundamentals and independently decide how much credit to supply to the real economy. When economic fundamentals are weak, a massive credit contraction occurs, because banks abstain from lending for fear that insufficient credit supply by other banks impedes economic growth. Although heterogeneous private information naturally motivates trading among agents (banks, in this paper), most existing papers on coordination problems abstract from the possibility of trading and assume that agents make isolated decisions.

A distinct feature in my model is that I introduce a market for banks to borrow or lend capital (referred to as the interbank capital market, hereafter). In particular, the interbank capital market plays two roles in the real credit supply process. First, optimistic banks with positive private signals about the macroeconomic fundamentals can borrow from pessimistic banks with negative private signals and extend more credit to the real economy. Hence, the interbank capital market performs an allocative role in the financial system by channeling the capital of the pessimistic banks to the optimistic banks and eventually to the real economy. Second, the market-clearing rate (referred to as the interest rate, hereafter)

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5The structure of the interbank capital market is similar to that in Geanakoplos (2010). Following the same structure, Che and Sethi (2014) analyze CDS market for corporate bonds. This paper endogenizes the belief heterogeneity in Geanakoplos (2010) based on the banks’ trading behaviors in the interbank capital market.
reveals the average borrowing demand in the interbank capital market. Observing the interest rate, banks can infer other banks’ private information, their willingness to supply credit, and overall economic prospects. In this sense, the interbank capital market also plays an informative role in shaping banks’ beliefs about economic prospects and affecting banks’ willingness to supply credit.

I then analyze the aggregate effect of the allocative role and the informative role of the interbank capital market in two cases: one with loose regulations and severe concerns of counterparty risks and the other with tight regulations easing the concerns of counterparty risks.

In the former case, the interbank capital market is underregulated such that borrowing banks may take on excessive leverage and default in economic recessions. In this case, capital flows in the financial system dampen the real credit supply, because the economy can stay trapped in a vicious feedback loop between an interbank capital market freeze and a real credit freeze. The intuition of the feedback loop is as follows. When few banks demand borrowing from others, the interbank capital market freezes with a low market-clearing interest rate. Because banks borrow from others in order to extend more credit to the real economy, in the financial system, a low interest rate reflects a low average willingness to supply credit. A low willingness to supply credit, on the one hand, indicates aggregate credit contraction, which impedes economic growth; on the other hand, it reveals that banks receive negative private signals about economic fundamentals. Hence, observing the low interest rate, banks become pessimistic about economic prospects, and many banks withhold real credit supply. To make matters worse, because of concerns of counterparty risks, these banks also refuse to lend to the remaining optimistic banks, thereby inhibiting the allocative role of the interbank capital market. With many banks being pessimistic and holding onto their capital, the real credit supply also freezes. Finally, the real credit freeze leads to an economic recession, in which banks earn low profits on their credit supply. Low profitability, in turn, justifies the low borrowing demand in the interbank capital market.
and the low interest rate.

In the latter case, when tight financial regulations, such as stress tests, prevent excessive leverage-taking by banks and ease concerns about counterparty risks in the interbank capital market, the vicious feedback loop can be prevented. This occurs via the following process. In the absence of counterparty risks, pessimistic banks always have the incentive to lend to other banks even when the interest rate is low and economic prospects are dim. In other words, such regulations maintain an active allocative role for the interbank capital market in channeling capital to the real economy. Moreover, compared with the former case, banks become less pessimistic when they observe a low interest rate, because they know that the interbank capital market keeps channeling capital to the optimistic banks and eventually to the real economy. In other words, the negative informative role of the interbank capital market is also limited. In aggregate, more banks demand borrowing from others, rejuvenating the interbank capital market. At the same time, these banks actively supply credit, defrosting the aggregate real credit crunch.

Overall, this paper not only makes a theoretical contribution but also derives empirical and policy implications. Theoretically, it contributes to the global-games framework by analyzing how agents’ trading behaviors affect the coordination outcomes through the interaction of resource reallocation and information revelation. In addition to the theoretical contribution, my model provides empirical implications consistent with the observations in the shallow post-crisis recovery: low interest rates, contraction in the volume of interbank capital flows, low credit growth, and sluggish economic growth. Moreover, it provides a general framework for studying how regulations in financial markets can mitigate coordination problems among market participants.

Besides the regulations that address the counterparty risks in the interbank capital market, I provide two other applications of the model. First, I demonstrate that short-selling constraints can help stabilize asset markets vulnerable to fire sales. Second, I illustrate that the ban on naked credit default swaps (CDS) on sovereign debt improves the sovereign
stability when self-fulfilling debt runs are possible. These two applications convey a common message: even when economic fundamentals are relatively strong, regulations should prevent aggressive bets against the market (short selling and speculation with naked CDS) and maintain market participants’ confidence, especially in adverse scenarios, because crises can be self-fulfilling for two reasons. First, aggressive bets against the market directly impose downward pressures on the financial markets. Second, the price of trading among market participants reveals the severity of the negative bets and overall pessimism, further encouraging market participants to short the market.

**Literature Review**  This paper relates to several strands of literature. First, it speaks to the literature on financial fragility associated with interbank capital markets and highlights a new source of fragility posed by the interbank capital market in the context of self-fulfilling credit freezes. Previous literature has shown that the interbank capital market can intensify the fragility of the financial system through network effects (Allen and Gale, 2000), the moral hazard problems of bank managers (Brusco and Castiglionesi, 2007), liquidity hoarding (Acharya and Skeie, 2011; Heider et al., 2015), and resource misallocation (Boissay, 2011; Boissay et al., 2016; Boissay and Cooper, 2016). Through a different lens, this paper incorporates the coordination problem in banks’ credit supply decisions and illustrates how interbank capital markets can destabilize the real credit market through a vicious feedback loop. The paper most closely related to mine is Liu (2016), who examines the feedback between interbank capital markets with low liquidity and panic-based bank runs on financial institutions. However, my paper considers the coordination issue among financial institutions rather than depositors. More importantly, Liu (2016) assumes away counterparty risks in the interbank capital markets, whereas my paper emphasizes that addressing counterparty risks can be a remedy to the interbank capital market freeze.

Second, this paper builds on the literature that studies the positive externalities of bank

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6Since December 1, 2011, the European Parliament has banned naked CDS on debt for sovereign nations to fight the eurozone crises. See European Commission (2011) for details.

lending on the aggregate credit supply to the real economy. I incorporate the interconnected feature of the financial system and analyze the effect of interbank capital flows on banks’ real lending decisions. Extant papers indicate that the positive externalities of bank lending can stem from two channels. First, bank lending stimulates real economic activities and increases the profits of lending for the whole banking sector. Firms’ investment, households’ consumption, and economic activities in general are interdependent; therefore, economic underdevelopment can result from a coordination failure among economic agents (Diamond, 1982; Bulow et al., 1985; Kiyotaki, 1988; Cooper and John, 1988; Schaal and Taschereau-Dumouchel, 2015). Because economic activities depend on access to credit, Bebchuk and Goldstein (2011) argue that banks have more incentive to lend if they believe others extend credit to support the growth of the economy. Second, bank lending generates information externalities (Nakamura et al., 1993; Pagano and Jappelli, 1993). Specifically, bank lending helps develop borrowers’ credit history, reduces information asymmetry between banks and borrowers, and, therefore, eases borrowers’ access to credit. My paper shows that interbank capital flows can also freeze and aggravate banks’ coordination problems caused by the externalities in bank lending.

Finally, this paper follows the literature on global games pioneered by Carlsson and Van Damme (1993) in terms of modeling techniques and analyzes how allowing agents to trade may affect the coordination results. The research closest to mine in this respect was conducted by Angeletos and Werning (2006) and Hellwig et al. (2006), among others. These papers demonstrate that financial markets can restore equilibrium multiplicity in global games, because they reveal public information and synchronize agents’ beliefs. The interbank capital market in my paper also plays a similar informative role. However, in addition to that,

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8See Hertzberg et al. (2011) and Giannetti and Saidi (2017) for empirical evidence.
9See Brown et al. (2009) and Doblas-Madrid and Minetti (2013) for empirical evidence.
10See Morris and Shin (2003) for a review of global games. Applications of global games include, but are not limited to, bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005), currency attacks (Morris and Shin, 1998), credit freezes (Bebchuk and Goldstein, 2011), debt rollovers (Morris and Shin, 2004; He and Xiong, 2012), and political revolutions (Edmond, 2013).
11Angeletos et al. (2006) show that if a government has information about economy fundamentals, government intervention itself serves as a public signal for weak fundamentals and dampens the effectiveness of the intervention.
my paper highlights an allocative role of the interbank capital market and analyzes the interactions of the two roles in affecting the coordination outcome. More importantly, my paper suggests that when tight regulations secure an active allocative role of the interbank capital market, they can also boost banks’ confidence and mitigate the negative informative role.

Outline of the paper The rest of the paper is arranged as follows. Section 2 describes the benchmark model of the credit market and demonstrates the possibility of a self-fulfilling credit market freeze. Section 3 introduces an interbank capital market to the baseline model. Section 4 discusses the case in which the interbank capital market is underregulated and risky. Section 5 illustrates how regulations removing counterparty risks can stabilize the credit market. Section 6 discusses two other applications of the model. Finally, section 7 concludes.

1.2. Self-Fulfilling Credit Market Freeze

In this section, following Bebchuk and Goldstein (2011), I introduce a benchmark model of a credit market and illustrate how the strategic complementarities in banks’ lending decisions can lead to a self-fulfilling credit market freeze.

It has been widely recognized in the macro literature that an underdeveloped economy can result from a coordination failure of interdependent economic agents (Diamond, 1982; Kiyotaki, 1988; Cooper and John, 1988; Schaal and Taschereau-Dumouchel, 2015). This interdependence can stem from various channels. A firm’s success directly depends on the success of other firms in the same supply chain. For firms without direct business relationships, a firm’s success can generate more income for its employees and raise the demand for other firms’ products. Access to credit is crucial for firms’ investment and success, so the positive spillover effect among the operating firms leads to strategic complementarities.

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12 Chodorow-Reich (2013) find that credit contraction accounts for between one-third and one-half of the employment decline at small and medium-sized firms in the year following the Lehman bankruptcy. Huber (2018) provides evidence that the lending cut by a large German bank affected firms, through lower aggregate demand and agglomeration spillovers in counties exposed to the lending cut.
in banks’ lending decisions. Specifically, one bank is more willing to lend if it expects that other banks are supplying credit to support the success of other firms. In general, economic activities are interdependent, and the prosperity of an economy relies on access to credit for all economic participants. Therefore, bank lending in this paper is not limited to firm loans but also includes consumer credit, mortgage, etc. In the benchmark model, self-fulfilling credit freezes can result from banks abstaining from lending for fear that other banks are withdrawing lending.

1.2.1. The Model

A mass one of ex-ante identical banks are indexed by $i \in [0, 1]$. The banks are risk neutral, and each has 1 unit of capital. They simultaneously make lending decisions $a_i \geq 0$ to maximize their expected profits. In the benchmark model without interbank capital markets, the budget constraint of bank $i$ is simply $a_i \leq 1$; that is, the maximum credit that bank $i$ can supply is its endowed capital. For each unit of capital received, an operating firm makes an investment that generates a return of $R_h$ if successful and $R_l$ if not, where $R_h > 1 > R_l$. To focus on the supply of credit in the economy, I assume the mass of operating firms is greater than 1, and, therefore, banks can extract all the surplus from extending credit. Hence, banks’ profits from lending represent the productivity of the economy. Besides risky lending to the real sector, banks have access to a safe storage technology that is unproductive and generates a return normalized to 1. The safe storage technology can be interpreted as the excess reserves held by banks.

To incorporate the strategic complementarities of banks’ lending decisions, I assume the operating firms’ investments are successful when a sufficient amount of firms receive credit and invest. As a result, banks’ lending is profitable when a sufficient amount of credit is supplied to the real economy. Specifically, the payoff of a bank that lends out $a_i$ units of

$^{13}$Gilchrist et al. (2018) show that a reduction in the supply of home mortgage loans has detrimental real effects on economic activities.

$^{14}$As long as banks’ profits from real credit supply are positively correlated with the productivity of the operating firms, all results in this paper remain qualitatively robust.
capital and puts $1 - a_i$ into storage is as follows:

$$
u_i(a_i) = \begin{cases} 
R_h a_i + 1 - a_i & \text{if } A \geq 1 - \theta \text{ (Expansion)}, \\
R_l a_i + 1 - a_i & \text{if } A < 1 - \theta \text{ (Recession)},
\end{cases}$$

where $A = \int_0^1 a_i \, di$ is the aggregate amount of lending in the economy, and $\theta$ stands for the fundamentals of the economy. To facilitate discussion, I say the economy is in an expansion if $A \geq 1 - \theta$ and in a recession if $A < 1 - \theta$. In an economic expansion, the productivity of the real economy is high, and banks’ credit supply is profitable. The strategic complementarities in banks’ lending decisions come from the fact that when other banks supply more credit resulting in a high $A$, the economy is more likely to be in an expansion, which incentivizes each bank to lend. When the fundamentals $\theta$ are high, a few firms receiving credit is sufficient for the economy to be in an expansion.

Without information friction, when $\theta \in [0, 1)$, all banks lending to full capacity ($a_i = 1$) and all banks holding on to their capital ($a_i = 0$) are both equilibria. However, all banks lending to full capacity is strictly more efficient than the other equilibrium. Therefore, as shown in the upper panel of Figure 1, the first-best outcome is that all banks coordinate to lend when $\theta \geq 0$ and abstain from lending when $\theta < 0$.

The global-games framework is useful for connecting the coordination outcome to the underlying fundamentals of the economy. More importantly, the framework realistically characterizes the information and belief structure that no agent (bank in my model) has perfect information about the fundamentals of the economy. Instead, each agent observes a noisy private signal about the fundamentals of the economy. Therefore, the global-games framework highlights the strategic interactions of agents with heterogeneous private information.

The information structure of the model follows the standard global-games setup. The economic fundamentals $\theta \in \mathbb{R}$ are drawn from an improper prior and are not directly observable to the banks when they make lending decisions. Instead, each bank receives a
noisy signal about the fundamentals \( s_i = \theta + \sigma s \varepsilon_i \), where \( \varepsilon_i \overset{i.i.d.}{\sim} \mathcal{N}(0, 1) \). Note that two dominance regions always exist: \((-\infty, \bar{s})\) and \((\bar{s}, \infty)\), defined as

\[
\Pr(\theta \geq 1|s_i = \bar{s}) = \Pr(\theta \geq 0|s_i = \bar{s}) = \frac{1 - R_l}{R_h - R_l}.
\]

With the most pessimistic belief that \( A = 0 \), a bank is indifferent between lending or not when it receives signal \( \bar{s} \). Therefore, a bank’s dominant strategy when \( s_i > \bar{s} \) is \( a_i = 1 \). Similarly, with the most optimistic belief that \( A = 1 \), a bank is indifferent between lending or not when it observes signal \( \bar{s} \). Hence, when \( s_i < \bar{s} \), \( a_i = 0 \) is the dominant strategy.

1.2.2. Equilibrium Analysis

Proposition 1 characterizes the equilibrium. Following the standard results in global games, there exists a unique equilibrium in which all agents follow a threshold strategy. Banks are risk neutral, so they either supply zero credit or the maximum credit bounded by their endowed capital.

**Proposition 1** There is a unique Bayesian Nash Equilibrium in which all banks follow the same strategy that

\[
a_i(s_i) = \begin{cases} 
1, & \text{if } s_i \geq s_0^*, \\
0, & \text{if } s_i < s_0^*, 
\end{cases}
\]

where \( s_0^* = \frac{1 - R_l}{R_h - R_l} + \sigma_s \Phi^{-1} \left( \frac{1 - R_l}{R_h - R_l} \right) \).

With the continuum of banks, based on the realization of fundamentals \( \theta \), we can determine the distribution of banks’ private signals and apply the law of large numbers to calculate the aggregate lending \( A \) and predict the coordination outcomes. As \( \theta \) increases, more banks receive high signals that lie above the threshold and result in a higher aggregate lending \( A \). Therefore, there also exists a fundamental threshold above which abundant credit supply supports an economic expansion and below which a shortage of credit leads to an economic
recession. In particular, the fundamental threshold is given by

\[ \theta_0^* = \frac{1 - R_l}{R_h - R_l}. \]

Figure 1 compares the coordination outcome in the ideal first-best case and in the equilibrium of the benchmark model. In the middle region where \( \theta \in [0, \theta_0^*] \), if all banks were to lend to the maximum capacity, there would be sufficient credit supplied to support the growth of the real economy as shown in the first-best case. However, a lot of banks receive low private signals and form self-fulfilling beliefs that the economy is likely to be in a recession with insufficient credit supply. As a result, they rationally choose not to extend credit, which indeed leads to an economic recession caused by the shortage of credit supply.

1.3. Interbank Capital Market

The heterogeneous private signals of banks naturally generate heterogeneity in beliefs and the incentive to trade among banks. In this section, I introduce the structure of the interbank capital market that allows banks to trade based on their heterogeneous beliefs. As mentioned in the introduction, the interbank capital market not only plays an allocative role in channeling capital within the financial system but also an informative role in revealing public information and shaping banks’ beliefs through the market-clearing interest rate.

1.3.1. Capital Flows in the Financial System

In the interbank capital market, banks can borrow from or lend to one another. Let \( x_i > 0 \) denote the amount of capital that bank \( i \) borrows. If \( x_i < 0 \), it means that bank \( i \) lends
out $|x_i|$ units of capital. For the rest of the paper I will refer to bank $i$’s lending decision to operating firms $a_i$ as the real lending decision, and $x_i$ as the interbank borrowing decision.

For each unit of capital borrowed in the interbank capital market, borrowers promise to repay $r$ after the realization of the coordination outcomes, that is, whether the economy is in an expansion or in a recession. Conditional on the nominal interest rate $r$, banks submit their demand schedules. In equilibrium, the market-clearing condition of the interbank capital market endogenously determines the interest rate $r$. Because banks’ profit from the real credit supply represents the productivity of the real economy and their demand for borrowing is driven by the incentive to extend more credit to the real economy, the interest rate $r$ indicates banks’ average expectations about the productivity of the real economy. Therefore, the market-clearing interest rate $r$ is effectively the “equilibrium” real interest rate, which reflects the marginal productivity of the real economy.\footnote{See Barsky et al. (2014), Laubach and Williams (2016), and Holston et al. (2017) for estimates of the “equilibrium” real interest rate.}

When the economy is in a recession, banks suffer losses from supplying credit to the real economy and may default on their interbank liabilities. In this model, banks are protected by limited liabilities, and there is no deadweight loss associated with bankruptcy.\footnote{I make this assumption to isolate the benefit of strict regulations in reducing coordination failure in the financial system. Otherwise, strict regulations would generate an additional benefit in preventing the deadweight loss of bankruptcy.} If a bank does not have enough capital to repay its interbank liabilities, it defaults and depletes all available capital to its lender(s). As a result, lending banks in the interbank capital market may suffer from counterparty credit risks.

To address the counterparty risks, banks are subject to the following regulatory constraint:

$$R_l a_i + 1 + x_i - a_i \geq cr x_i.$$ 

Specifically, the regulatory constraint can be interpreted as the stress test on banks to make sure that they can withstand recessions. The left-hand side of the inequality represents bank
i’s total capital in a recession. In a recession, banks earn a return of $R_l$ for their risky lending to the real economy and a return of 1 for their investment in the safe storage. The right-hand side represents the capital required to pass the stress test, which is proportional to its interbank liability $rx_i$ by a factor of $c$. In other words, the regulatory constraint ensures that in recessions, borrowing banks maintain enough capital to cover at least $c$ fraction of their liabilities.\textsuperscript{17} Hence, parameter $c$ represents the regulatory stringency. The regulatory constraint effectively puts an upper limit on the leverage that each bank can take. As stated in Lemma 1 below, borrowing banks always take on the maximum leverage.

**Lemma 1** The regulatory constraint binds for all borrowing banks in the interbank capital market, and lending banks expect to receive promised return $r$ in expansions ($A \geq 1 - \theta$) and $cr$ in recessions ($A < 1 - \theta$).

Intuitively, if a bank borrows in the interbank capital market, the bank must believe that it is profitable to extend credit at the interest rate. In this sense, the market-clearing interest rate reflects the average belief of banks about the productivity of the real economy. What is unique about this model is that in equilibrium the interest rate itself affects banks’ incentive to engage in credit supply activities and the real productivity. Banks are risk neutral, and their investment technologies have a constant return to scale, so borrowing banks always borrow to the maximum. When $c = 1$, the interbank capital market lending is completely risk-free, and no bank defaults, even in economic recessions.\textsuperscript{18} If $c < 1$, borrowing banks take on excessive leverage and default in recessions. In this case, the interbank capital market is underregulated and risky.

The regulatory constraint may appear restrictive, because it does not allow banks to trade at different interest rates conditional on the leverage of the borrowing banks, which represents the riskiness of interbank lending. Appendix A.1.1 addresses this concern by extending the

\textsuperscript{17}In the main text, I restrict attention to $c \in [c, 1]$. Appendix A.1.4 analyzes the case in which $c > 1$, that is, that case with excessive regulations.

\textsuperscript{18} Appendix A.1.4 analyzes the case with excessive regulations, $c > 1$. Compared with the case in which $c = 1$, although each bank borrows less, in terms of aggregate credit supply, they are equivalent.
model to allow banks to post collaterals to reduce the riskiness of interbank lending and charge different interest rates conditional on the riskiness. I show that when interest rates are low, banks bilaterally agree not to post collateral and solely rely on the regulations to contain the counterparty risks.

To summarize the payoff structure, if bank $i$ borrows $x_i$ units of capital in the interbank capital market, makes $a_i$ units of loans to the operating firms, and stores the rest of its capital $1 + x_i - a_i$ in the safe storage, its payoff is

$$u_i(a_i, x_i) = \begin{cases} R_h a_i - r x_i + 1 + x_i - a_i & \text{if } A \geq 1 - \theta \text{ (Expansion)}, \\ R_l a_i - cr x_i + 1 + x_i - a_i & \text{if } A < 1 - \theta \text{ (Recession)}. \end{cases}$$

1.3.2. Information Structure

Like in the benchmark model without interbank capital market, the fundamentals $\theta$ follow an improper prior, and each bank receives a private signal $s_i = \theta + \sigma_s \epsilon_i$, where $\epsilon_i \overset{i.i.d.}{\sim} \mathcal{N}(0, 1)$. In addition, banks can make inferences based on the market-clearing interest rate $r$. Specifically, bank $i$ forms its belief about the probability of an expansion $\pi(s_i, r) = Pr(A \geq 1 - \theta | s_i, r)$ based on its private signal $s_i$ and the market-clearing interest rate $r$. Given its belief $\pi$, bank $i$ then optimally chooses real lending $a(\pi, r)$ and interbank borrowing $x(\pi, r)$ to maximize its expected payoff as follows,

$$\max \{a_i, x_i\} \mathbb{E}(u_i(a_i, x_i) | \pi) = \pi_i(R_h a_i - r x_i) + (1 - \pi)(R_l a_i - cr x_i) + 1 + x_i - a_i$$

s.t. $a_i \leq 1 + x_i$ \hspace{1cm} (Budget Constraint)

$a_i \geq 0$ \hspace{1cm} (Short-Selling Constraint)

$R_l a_i + 1 + x_i - a_i \geq cr x_i$ \hspace{1cm} (Regulatory Constraint).

Without any frictions, the Law of Large Numbers implies that the interbank capital market rate $r$ perfectly aggregates banks’ private information and fully reveals the current funda-
mentals $\theta$. In particular, I denote the aggregate demand in the interbank capital market by $D(\theta, r)$. The functional form of $D(\theta, r)$ will be characterized later in the equilibrium analysis, and a unique solution $\theta = z(r)$ to $D(z(r), r) = 0$ will be solved. Therefore, observing the interest rate $r$, banks can back out the fundamentals $\theta = z(r)$ from the market-clearing condition. To preserve the belief heterogeneity and provide banks with an incentive to trade, I introduce noise to banks’ interpretation of the public information contained in $r$ in the following form:

$$\theta = z(r) + \sigma_p \epsilon_{pi},$$

where $\epsilon_{pi} \sim N(0, 1)$ is the interpretation noise of bank $i$ and is independent of $\epsilon_i$’s. That is, absent other information, observing an interest rate $r$, banks believe fundamentals $\theta \sim N(z(r), \sigma_p^2)$. Banks are aware of this interpretation noise and therefore don’t solely rely on the public information when forming beliefs. Hence, they put nonzero weight on their private information and form heterogeneous beliefs in equilibrium. In fact, Appendix A.1.2 demonstrates that the interpretation noise can be micro-founded by the noisy demand of capital in the financial system. Although they are equivalent in terms of equilibrium implications, the noisy demand of capital is mathematically more complicated. Therefore, I proceed with the interpretation noise in the main text. Moreover, for the rest of the paper, I will focus on the limit when the interpretation noise vanishes, that is, $\sigma_p \to 0$. This is when banks extract precise information from the interest rate $r$, or the noisy demand of capital in the financial system goes to zero.

1.3.3. Time line

The time line of the model with capital flows in the financial system is as follows. After the realization of fundamentals $\theta$, bank $i$ receives its private signal $s_i$. Conditional on the interest rate $r$ and its private signal $s_i$, bank $i$ forms its belief about the probability of an economic expansion $\pi(s_i, r)$. Based on its belief, bank $i$ submits a demand schedule $x_i(\pi, r)$ to indicate its demand of capital when the interest rate is $r$. Given the demand schedule of all banks, interest rate $r^*(\theta)$ is determined to clear the interbank capital market.

Following
that, bank $i$ makes real lending $a_i(\pi, r)$ to the real economy, and the aggregate credit supply $A$ is realized. Depending on the relationship between the aggregate credit supply $A$ and the threshold $1 - \theta$, the coordination outcome is realized, that is, whether the economy is in an expansion or in a recession. Finally, the return from real credit supply is realized, and payoffs from trading across banks are settled.

1.4. Risky Interbank Capital Market

This section analyzes the case in which the financial system is underregulated $(c < 1)$. As a result, borrowing banks take on excessive leverage and default in economic recessions, and, hence, the interbank capital market is risky. I solve the model in two steps. First, I analyze banks’ optimal portfolio choice given their beliefs and discuss the allocative role of the interbank capital market. Second, given banks’ portfolio choice, I write the market-clearing condition of the interbank capital market and illustrate the informative role by analyzing the information content of the interest rate $r$, which endogenously shapes banks’ beliefs.

1.4.1. The Allocative Role

In this subsection, given banks’ beliefs, I analyze their optimal real lending $a(\pi, r)$ and their optimal demand for borrowing from other banks $x(\pi, r)$ and discuss the allocative role of the interbank capital market in the credit supply process.

If the interest rate $r \leq r = \frac{R_h - R_l}{c(R_h - 1) + 1 - R_l}$, no banks find it profitable to lend to other banks. Specifically, compared with the safe storage, the pessimistic banks find the interest rate $r$ too low to justify for the counterparty risks. If $r \geq R_h$, it’s too costly for banks to borrow, resulting in zero demand in the interbank capital market. In both cases, the interbank capital market cannot clear. Therefore, I restrict attention to $r \in (r, R_h)$.

Figure 2 summarizes the optimal portfolio choices. The figure shows that the portfolio choice analysis can be further decomposed into two cases.
If the interest rate is high, when \( r \in \left[ \frac{1}{c}, R_h \right] \) (subfigure a), \(^{19}\) banks’ optimal portfolio choices given their beliefs are

\[
\begin{align*}
  a(\pi, r) &= \frac{cr}{cr - R_l} \quad \text{and} \quad x(\pi, r) = \frac{R_l}{cr - R_l}, \quad \text{if} \quad \pi > \pi^*_2(r), \\
  a(\pi, r) &= 0 \quad \text{and} \quad x(\pi, r) = -1, \quad \text{if} \quad \pi < \pi^*_2(r),
\end{align*}
\]

where the belief cutoff \( \pi^*_2(r) = \frac{cr - R_l}{R_h - R_l - r(1-c)} \). In this case, even in economic recessions when the borrowing banks default on their interbank obligations, the lending banks still earn a return of \( cr \geq 1 \) for the interbank lending. Hence, lending in the interbank capital market always generates positive returns and strictly dominates the storage technology. I say the interbank lending is guaranteed in this case. As a result, pessimistic banks with \( \pi < \pi^*(r) \) optimally lend out all their endowed capital in the interbank capital market. Meanwhile, optimistic banks with \( \pi > \pi^*(r) \) find it optimal to take on maximum leverage and supply all available capital to the real economy, including their endowed capital and the capital borrowed in the interbank capital market. Therefore, the interbank capital market is playing an active allocative role in the sense that it channels the capital of the pessimistic banks to the optimistic banks, who then supply all capital in the financial system to the real economy.

\(^{19}\)Note that I restrict the regulatory stringency, \( c \in \left( \frac{1}{R_h}, 1 \right) \), to analyze the general case with a guaranteed region and a nonguaranteed region. In fact, if \( c \in \left( \frac{1}{R_h}, \frac{1}{R_h} \right) \), the guaranteed region disappears; however, the equilibrium implications in Proposition 2 still hold. If \( c < \frac{1}{R_h} \), the counterparty risks are so prominent that the interbank capital market melts completely.

---

**Figure 2: Portfolio Choice (Risky Interbank Capital Market)**

<table>
<thead>
<tr>
<th>Lending banks</th>
<th>Borrowing banks</th>
<th>π</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^*_1(r) = 0 )</td>
<td>( \pi^*_2(r) )</td>
<td>π</td>
</tr>
</tbody>
</table>

(a) \( r \in \left[ \frac{1}{c}, R_h \right] \)

<table>
<thead>
<tr>
<th>Nonparticipating banks</th>
<th>Lending banks</th>
<th>Borrowing banks</th>
<th>π</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^*_1(r) )</td>
<td>( \pi^*_2(r) )</td>
<td>π</td>
<td></td>
</tr>
</tbody>
</table>

(b) \( r \in \left( \frac{R_h}{c}, \frac{1}{c} \right) \)
In fact, the interbank capital market improves not only the capital allocation but also the risk allocation within the financial system. Specifically, optimistic banks that are more willing to bear the risks from real credit supply take on leverage and retain more risks compared with pessimistic banks. In other words, the interbank capital market effectively transfers risks from pessimistic banks to optimistic banks. The key mechanism for the risk transfer is the fact that the risky real credit supply guarantees a safe return of $R_l$ regardless of the state of the economy. This enables optimistic banks to pledge the safe return from their endowed capital to borrow from pessimistic banks.

If the interest rate is low, when $r \in \left(\frac{R_t}{c}, \frac{1}{c}\right)$ (subfigure b), banks’ optimal portfolio choices given their beliefs are

$$a(\pi, r) = \frac{cr}{cr - R_t} \quad \text{and} \quad x(\pi, r) = \frac{R_t}{cr - R_t}, \quad \text{if } \pi > \pi_2^*(r), \quad (1.4)$$

$$a(\pi, r) \in [0, \frac{cr}{cr - R_t}] \quad \text{and} \quad x(\pi, r) = a(\pi, r) - 1, \quad \text{if } \pi = \pi_2^*(r), \quad (1.5)$$

$$a(\pi, r) = 0 \quad \text{and} \quad x(\pi, r) = -1, \quad \text{if } \pi \in (\pi_1^*(r), \pi_2^*(r)), \quad (1.6)$$

$$a(\pi, r) = 0 \quad \text{and} \quad x(\pi, r) \in [-1, 0], \quad \text{if } \pi = \pi_1^*(r), \quad (1.7)$$

$$a(\pi, r) = 0 \quad \text{and} \quad x(\pi, r) = 0, \quad \text{if } \pi < \pi_1^*(r), \quad (1.8)$$

where $\pi_1^*(r) = \frac{r - 1 - c}{1 - c}$ and $\pi_2^*(r) = \frac{cr - R_t}{R_h - R_t - r(1-c)}$. In contrast with the previous case with a high interest rate, in recessions when the borrowing banks default, the lending banks can only recover $cr < 1$. In this case, interbank lending is not guaranteed, and the counterparty risks significantly undermine the allocative role of the interbank capital market in the credit supply process. Concerns about counterparty risks are subjective to banks’ beliefs of the probability of an economic recession. To the most pessimistic banks that believe in a high probability of an economic recession, the low interest rate does not compensate for counterparty risks. As a result, they optimally choose to use the safe storage technology and not to participate in the interbank capital market. In aggregate, only the capital endowed to the banks with medium beliefs is channeled through interbank capital market
to the real economy. Moreover, as the interest rate $r$ decreases, the lending banks expect to recover less in an economic recession, and concerns about counterparty risks becomes more salient. When the interest rate $r$ reaches its lower bound $\underline{r}$, the two belief cutoffs converge, that is, $\pi_1^*(\underline{r}) = \pi_2^*(\underline{r})$, and no banks are willing to lend in the interbank capital market.

1.4.2. The Informative Role

In this subsection, I analyze the information conveyed by the market-clearing interest rate $r$ and discuss the informative role of the interbank capital market in the credit supply process. Given the optimal portfolio choices by banks, we can write the market-clearing condition as follows:

$$D(\theta, r) = \int_{-\infty}^{+\infty} x(\pi(\theta + \sigma_s \epsilon_s, r))\phi(\epsilon_s)d\epsilon_s = 0.$$ 

In more detail, throughout the paper, I restrict attention to monotone equilibrium in which banks’ belief function $\pi(s_i, r)$ strictly increases in the private signal $s_i$. The formal equilibrium definition will be introduced in Section 1.4.3. Therefore, substituting in the optimal portfolio choice by banks, the market-clearing condition can be expressed as

$$D(\theta, r) = \frac{R_l}{r_c - R_l} \left(1 - \Phi \left( \frac{s_2^*(r) - \theta}{\sigma_s} \right) \right) - \left( \Phi \left( \frac{s_2^*(r) - \theta}{\sigma_s} \right) - \Phi \left( \frac{s_1^*(r) - \theta}{\sigma_s} \right) \right) = 0,$$

where $s_1^*(r)$ and $s_2^*(r)$ are the signal cutoffs such that banks form the cutoff beliefs, that is, $\pi(s_1^*(r), r) = \pi_1^*(r)$ and $\pi(s_2^*(r), r) = \pi_2^*(r)$, and $\Phi$ is the cumulative distribution function of a standard normal variable. When $r \in [\frac{1}{c}, R_h)$, all pessimistic banks lend in the interbank capital market, and $\pi_1^*(r) = 0$, which implies $s_1^*(r) = -\infty$.

The aggregate demand $D(\theta, r)$ strictly increases in $\theta$. Therefore, banks can extract the perfect information about the realization of the fundamentals $\theta = z(r)$, which is the unique solution to $D(z(r), r) = 0$. As mentioned before, to preserve the belief heterogeneity, I assume that banks make noisy inferences and receive the following noisy signal:

$$\theta = z(r) + \sigma_p \epsilon_{pi}.$$
Appendix A.1.2 shows that the interpretation noise can be micro-founded by the noisy demand of capital in the interbank capital market.

To understand the intuition behind the informative role of the interbank capital market, one can go through a bank’s inference process when the bank observes a drop in the market-clearing interest rate $r$. First, the regulatory constraint relaxes, and each bank can borrow more in the interbank capital market, that is, $\frac{R_i}{r_c - R_i}$ increases. Meanwhile, each lending bank still lends 1 unit of capital. Therefore, for the market to clear, the mass of banks demand borrowing relative to lending must decrease. In addition, as $r$ drops, it becomes cheaper to borrow and less profitable to lend in the interbank capital market, reflected by a decrease in $\pi_2^*(r)$. However, even when borrowing becomes more attractive relative to lending, fewer banks demand borrowing relative to lending in the interbank capital market. Hence, banks must be holding, on average, more pessimistic beliefs about economic prospects.

To summarize, when the interest rate $r$ is low, even with the cheap financing opportunities, very few banks demand borrowing, which indicates that banks, on average, are less willing to extend credit to the real economy. Because of the strategic complementarities, all banks observing a low interest rate $r$ become more pessimistic, and it can be verified in equilibrium that the belief function $\pi(s_i, r)$ indeed decreases in $r$.

Below, I sketch the steps necessary to characterize the belief function $\pi(s_i, r)$. After bank $i$ observes its private signal $s_i$ and the interest rate $r$, it updates its belief about the posterior distribution of fundamentals $\theta$ according to Bayes’ rule, which gives a posterior p.d.f. as follows:

$$f(\theta|s_i, r) \propto \phi \left( \frac{s_i - \theta}{\sigma_s} \right) \phi \left( \frac{z(r) - \theta}{\sigma_p} \right) \quad \forall r \in (r_c, R_h).$$

Therefore, the posterior distribution of $\theta$ is a normal distribution with mean $\mu(s_i, r) = \delta s_i + (1 - \delta)z(r)$ and variance $\sigma^2 = \delta \sigma_s^2$, where $\delta = \frac{\sigma_p^2}{\sigma_s^2 + \sigma_p^2}$ is the relative informativeness of the private signal.

Given the realization of fundamentals $\theta$, banks know the distribution of private signals $s_i$'s.
Hence, given the interest rate $r$, banks can infer the belief distributions and the aggregate credit supply to the real sector as follows:

$$A(\theta, r) = \int_{-\infty}^{\infty} a(\pi(\theta + \sigma_s \epsilon_s, r)) \phi(\epsilon) d\epsilon.$$  

Because $A(\theta, r)$ increases in $\theta$, there exists a unique fundamental threshold $\theta^*(r)$ above which there is sufficient credit to support the economic growth, that is, $A(\theta^*(r), r) = 1 - \theta^*(r)$. Finally, banks form their own beliefs by estimating the probability of economic recessions, $\pi(s_i, r) = \int_{\theta^*(r)}^{\infty} f(\theta|s_i, r) d\theta$. The belief function $\pi(s_i, r)$ and the fundamental threshold $\theta^*(r)$ should be consistent and therefore are simultaneously determined in equilibrium.

1.4.3. Interbank Capital Market Freeze and Fragility

In this section, I characterize the equilibrium and illustrate how the two roles of the interbank capital market interact and their interaction increases the fragility of the real credit market. Using the elements from previous analyses, here I introduce the formal definition of monotone equilibrium in the model:

**Definition 1** A monotone equilibrium is characterized by banks’ strategy $\{a(\pi, r), x(\pi, r)\}$, a belief function $\pi(s_i, r)$, a fundamental threshold function $\theta^*(r)$, an information revelation function $z(r)$, and a market-clearing interest rate function $r^*(\theta)$ such that

1. (Profit maximizing) $\{a(\pi, r), x(\pi, r)\}$ solves the portfolio choice problem;

2. (Bayesian updating) $\pi(s_i, r)$ satisfies Bayes’ rule such that

$$\pi(s_i, r) = \int_{\theta^*(r)}^{\infty} f(\theta|s_i, r) d\theta \quad \forall r \in (r, R_{h});$$
3. (Fundamental cutoff) the aggregate credit supply at $\theta^*(r)$ satisfies

$$A(\theta^*(r), r) = \int_{-\infty}^{+\infty} a(\pi(\theta^*(r) + \sigma_s \epsilon_s, r), r) \phi(\epsilon_s) d\epsilon = 1 - \theta^*(r) \quad \forall r \in (\underline{r}, \overline{R}_h);$$

4. (Information revelation) $z(r)$ satisfies

$$D(z(r), r) = \int_{-\infty}^{+\infty} x(\pi(z(r) + \sigma_s \epsilon_s, r), r) \phi(\epsilon_s) d\epsilon = 0 \quad \forall r \in (\underline{r}, \overline{R}_h);$$

5. (Market clearing) $r^*(\theta)$ is selected from the correspondence $\hat{R}(\theta) \equiv \{ r : z(r) = \theta \}$.

As discussed in the previous analyses, banks’ strategy is uniquely characterized in equations 1.1–1.8. Moreover, because the information revelation function $z(r)$ is the unique solution to the market-clearing condition, following the analyses in section 1.4.2, we can also uniquely characterize the belief function $\pi(s_i, r)$ and the fundamental threshold $\theta^*(r)$. Lemma 2 below summarizes the uniqueness of these elements in the equilibrium. However, the interest rate function $r^*(\theta)$ is not unique, and the multiplicity of $r^*(\theta)$ also gives rise to the equilibrium multiplicity.

**Lemma 2** Banks’ strategy $\{ a(\pi, r), x(\pi, r) \}$, the belief function $\pi(s_i, r)$, the fundamental threshold function $\theta^*(r)$, and the information revelation function $z(r)$ are unique.

In the proof of Lemma 2, I provide the unique solution to $\pi(s_i, r)$, $\theta^*(r)$, and $z(r)$. Given the uniqueness of other elements in the equilibrium, each monotone equilibrium can be uniquely characterized by the interest rate function $r^*(\theta)$.

As mentioned before, I focus on the limiting case in which the interpretation noise vanishes, that is, $\sigma_p \to 0$. In the limit, banks’ inference about the public information is precise or, equivalently, the demand noise in the interbank capital market vanishes. Appendix A.1.3 briefly discusses the equilibrium implications for when the noise in the public signal is nonnegligible ($\sigma_p > 0$) and demonstrates that the model is smooth in $\sigma_p$ such that the
Proposition 2. In the limit of vanishing interpretation noise, that is, \( \sigma_p \to 0 \),

1. for any \( \theta \geq 1 \), \( \hat{r}(\theta) = \mathcal{R}_h \), and the economy is in an expansion;

2. for any \( \theta \in (0,1) \), \( \hat{r}(\theta) \in \{ \mathcal{R}_h, \mathcal{R}_l \} \), the economy is in an expansion if \( \hat{r}(\theta) = \mathcal{R}_h \), and the economy is in a recession if \( \hat{r}(\theta) = \mathcal{R}_l \);

3. for any \( \theta \leq 0 \), \( \hat{r}(\theta) = \mathcal{R}_l \), and the economy is in a recession.

Appendix A.1.7 provides the proof. Figure 3 visualizes the equilibrium coordination outcome and compares it with the benchmark case. When the macroeconomic fundamentals \( \theta \geq 1 \), there is no coordination concern, because the dominating strategy for banks is to supply all the endowed capital to the real economy. In the limit when \( \sigma_p \to 0 \), banks receive precise public signals about the fundamentals of the economy. Hence, there exists a unique equilibrium in which banks actively extend credit and the economy is in an expansion. Because banks hold optimistic beliefs about the economic outlook, high borrowing demand in the interbank capital market drives up the interest rate. Similarly, when the fundamentals \( \theta \leq 0 \), the dominating strategy for banks is to invest the endowed capital in the safe storage. With precise information about the fundamentals, in the unique equilibrium, banks abstain from lending, and the economy enters a recession. Meanwhile, that banks do not have an incentive to borrow from other banks results in a low interest rate in equilibrium.

The most interesting case occurs when the macroeconomic fundamentals \( \theta \in (0,1) \). The
comparison of the two panels in Figure 3 indicates that the risky interbank capital market introduces fragility to the real credit supply and economic growth. Specifically, in the shaded area, when $\theta \in (\theta^*_0, 1)$, the risky interbank capital market opens up the possibility of an economic recession caused by a shortage of credit supply. The main mechanism behind the fragility and the equilibrium multiplicity when $\theta \in (0, 1)$ is the feedback loop between the interbank capital market and the real credit market.

In particular, if the demand of capital within the financial system is low, the interbank capital market freezes, resulting in a low market-clearing interest rate. As explained in the previous section on the informative role of the interbank capital market, a low interest rate implies overall pessimism about the productivity in the real economy and low willingness to extend credit. Therefore, the low interest rate depresses banks’ beliefs and discourages them from extending credit to the real economy. In addition, concerns about counterparty risks prevent the most pessimistic banks from lending in the interbank capital market, which dampens the allocative role of the interbank capital market. When the interest rate is low, very few banks find it sufficient to compensate for the counterparty risks associated with lending to other banks. Hence, the interbank capital market only channels very limited capital endowed to these lending banks to the real economy. Together, the two effects explain how a freezing interbank capital market causes the real credit supply to also freeze.

Next, consider the other direction in the feedback loop. If the real credit market freezes, which restrains the investment by firms, consumption by households and economic activities in general, the economy results in a recession with low return on banks’ credit supply. The low return on bank lending justifies banks’ incentive to invest their capital in the safe storage and not participate in the interbank capital market. Therefore, a freeze in the real credit market reinforces the interbank capital market freeze. As a result, the economy is trapped in an equilibrium with almost all banks holding onto their capital. No capital flows within the financial system, and no capital flows out to the real economy.

Similarly, when the interest rate is high, there exists a “good” feedback loop. Intuitively,
a high borrowing demand of capital within the financial system generates a high interest rate. In terms of the informative role of the interbank capital market, the high interest rate implies overall optimism about aggregate credit supply and economic outlook, which directly incentivizes banks to extend credit to the real economy. Moreover, the high interest rate also encourages banks to lend to other banks in spite of the counterparty risks. Hence, the interbank capital market plays an active allocative role in channeling funds to the real economy. Together, the two effects invigorate the real credit market. At the same time, the active real credit market promotes economic growth, which generates a high return on banks’ real credit supply and justifies banks’ soaring demand in the interbank capital market. Hence, in this equilibrium, the economy expands with capital actively flowing both within the financial system and out to the real economy.

1.5. Risk-Free Interbank Capital Market

This section analyzes the case in which financial regulations are strict enough \((c = 1)\) to ensure that interbank lending is risk-free.\(^{20}\) By comparing it to the case with risky interbank capital market, I first discuss how financial regulations separately affect the allocative role and the informative role of the interbank capital market. Following this discussion, I evaluate the aggregate effect of strengthening financial regulations and illustrate how they can be a remedy to stabilize the real credit market.

1.5.1. The Allocative Role

Based on their heterogeneous beliefs about the probability of an economic expansion, banks make their optimal portfolio choices. If the interest rate \(r < 1\), lending in the interbank capital market generates negative returns and is strictly dominated by the safe storage technology. Therefore, banks are not willing to lend to others within the financial system. If \(r \geq R_h\), it’s too costly for banks to lever up, and no banks demand borrowing from other banks. In both cases, the interbank capital market cannot clear. Hence, I restrict attention

\(^{20}\)Appendix A.1.4 analyzes the case with excessive regulations \((c > 1)\) and demonstrates that the equilibrium implications are the same in the limit when the noise on the public signal vanishes \((\sigma_p \to 0)\).
to the two nontrivial cases, $r \in (1, R_h)$ and $r = 1$, summarized in Figure 4.

\[
\begin{align*}
\pi^*(r) &= \frac{r - R_l}{R_h - R_l} & \pi^*(1) &= \frac{1 - R_l}{R_h - R_l} \\
(a) & \ r \in (1, R_h) & (b) & \ r = 1
\end{align*}
\]

\[
\begin{align*}
a = 0, x = 1 & & a = \frac{1}{r - R_l}, x = \frac{R_l}{r - R_l} \\
\pi^*(r) &= \frac{r - R_l}{R_h - R_l} & \pi^*(1) &= \frac{1 - R_l}{R_h - R_l}
\end{align*}
\]

Figure 4: Portfolio Choice (Risk-Free Interbank Capital Market)

If the interest rate is high, when $r \in (1, R_h)$, the optimal portfolio choices are essentially the same as those in the risky interbank capital market case in which interbank lending is guaranteed. Pessimistic banks lend their endowed capital to other banks, because interbank lending is safe and generates a higher return compared with the safe storage. Optimistic banks absorb capital from pessimistic banks and supply all available capital to the real economy. Therefore, in aggregate, the interbank capital market plays an active allocative role in channeling the capital held by pessimistic banks to optimistic banks. Moreover, the risk-free interbank capital market maximizes the risk transfer between the two groups of banks. Optimistic banks effectively hold the risky portion of real lending and pass on the risk-free portion, that is, the guaranteed return of firm loans, $R_l$, to pessimistic banks.

The sharp contrast to the risky interbank capital market case arises when the interest rate is low. Without worrying about counterparty risks, even the most pessimistic banks are willing to lend to other banks. Specifically, if $r = 1$, pessimistic banks are indifferent between lending to other banks and investing in the safe storage. Similarly, because borrowing from other banks is costless, optimistic banks can borrow to infinity, as long as they invest their excessive borrowing in the safe storage to fulfill the regulatory constraint. In aggregate, the interbank capital market still performs a positive allocative role in reallocating capital to optimistic banks willing to provide credit to the real economy, although it is possible that not all capital endowed to pessimistic banks is channeled to optimistic banks. This is because
strict financial regulations maximize the risk transfer between the two groups of banks regardless of the interest rate. In comparison, when the financial system is underregulated, the risk transfer diminishes as the interest rate falls, as more banks find the interest rate too low to compensate for the counterparty risks. In the extreme, when interest rate gets close to its lower bound, \( r \), the interbank capital market freezes completely.

1.5.2. The Informative Role

Given the optimal portfolio choice by banks, we can write the market-clearing condition of the interbank capital market and discuss the information content of the market-clearing interest rate \( r \). The aggregate demand in the interbank capital market is given by

\[
D(\theta, r) = \frac{R_l}{r - R_l} \left( 1 - \Phi \left( \frac{s^*(r) - \theta}{\sigma_s} \right) \right) - \Phi \left( \frac{s^*(r) - \theta}{\sigma_s} \right) \quad \text{if } r \in (1, R_h),
\]

\[
D(\theta, r) \geq \frac{R_l}{r - R_l} \left( 1 - \Phi \left( \frac{s^*(r) - \theta}{\sigma_s} \right) \right) - \Phi \left( \frac{s^*(r) - \theta}{\sigma_s} \right) \quad \text{if } r = 1,
\]

where \( s^*(r) \) is the signal cutoff endogenously defined by \( \pi(s^*(r), r) = \pi^*(r) \). Note that when the interest rate \( r \in (1, R_h) \), because \( D(\theta, r) \) strictly increases in \( \theta \), there exists a unique solution \( \theta = z(r) \) such that \( D(z(r), r) = 0 \). However, when \( r = 1 \), a continuum of fundamentals \( \theta \) solves the market-clearing condition, \( D(\theta, 1) = 0 \). With a small abuse of notation, I denote \( z(1) = \max\{\theta : D(\theta, 1) = 0\} \) or, equivalently,

\[
\frac{R_l}{r - R_l} \left( 1 - \Phi \left( \frac{s^*(r) - z(1)}{\sigma_s} \right) \right) - \Phi \left( \frac{s^*(r) - z(1)}{\sigma_s} \right) = 0.
\]

For all \( \theta < z(1) \), the market can clear at \( r = 1 \). As mentioned before, banks make noisy interpretations of the public information contained in \( r \) as follows:

\[
\theta = z(r) + \sigma_p \epsilon_p \quad \text{if } r \in (1, R_h), \tag{1.9}
\]

\[
\theta \leq z(1) + \sigma_p \epsilon_p \quad \text{if } r = 1, \tag{1.10}
\]
where
\[ z(r) = s^*(r) - \sigma_s \Phi^{-1} \left( \frac{R_l}{r} \right). \]

\( r = 1 \) conveys a vague signal that clusters the bad fundamentals \( \theta \) with the good ones because of the indeterminacy in banks portfolio choices. Specifically, when economic fundamentals \( \theta \) decline, more banks receive low private signals, and fewer banks receive high private signals. Hence, more banks are willing to lend to others in the financial system, whereas fewer banks are willing to borrow. However, the interbank capital market can still clear at \( r = 1 \), because each lending bank is willing to lend less, and each borrowing bank is willing to borrow more in the interbank capital market.

According to Bayes’ rule, we can determine the posterior distribution of fundamentals \( \theta \) given a bank’s private signal \( s_i \) and the public signal \( r \). When \( r \in (1, R_h) \), like in the risky interbank capital market case, the posterior distribution of \( \theta \) is \( \mathcal{N}(\mu(s_i, r), \sigma^2) \), a normal distribution with mean \( \mu(s_i, r) = \delta s_i + (1 - \delta)z(r) \) and variance \( \sigma^2 = \delta \sigma_s^2 \). Because a low interest rate reflects a meager demand of interbank borrowing and a greater reluctance to extend credit to the real economy, in equilibrium, a lower interest rate \( r \) depresses banks’ beliefs \( \pi(s_i, r) \).

In stark contrast to the case with risky interbank capital market, when the interest rate declines to its lower bound \( r = 1 \), the posterior p.d.f. of fundamentals \( \theta \) is
\[ f(\theta | s_i, r = 1) \propto \phi \left( \frac{s_i - \theta}{\sigma_s} \right) \Phi \left( \frac{z(1) - \theta}{\sigma_p} \right), \]
which is strictly first-order stochastic dominated by \( \mathcal{N}(\mu(s_i, 1), \sigma^2) \). When \( \sigma_p \to 0 \), the posterior distribution of \( \theta \) converges to a truncated normal distribution, \( \mathcal{N}(s_i, \sigma_s) \), truncated from above at \( z(1) \). As a result, banks’ belief function is discontinuous at \( r = 1 \), that is, \( \lim_{r \to 1^+} \pi(s_i, r) > \pi(s_i, 1) \). In other words, observing the vague signal \( r = 1 \), banks form not only inaccurate beliefs but also pessimistic ones about economic prospects.

Although \( r = 1 \) conveys worse information than a higher interest rate, the discontinuity
in the information content works in favor of aggregate credit supply and economic growth compared with the risky interbank capital market case. Intuitively, in the risky interbank capital market case, the low interest rate \( r = r \) precisely informs banks that the current interbank capital market freezes completely. Banks are not optimistic enough to borrow in the interbank capital market even with the cheap financing opportunity, and banks are not lending because of the concern of counterparty risks. Because banks are not optimistic enough to supply credit to the real economy, the low interest rate also precisely indicates a freezing real credit market. In comparison, when the interbank capital market is risk-free, the imprecise information conveyed by \( r = 1 \) only indicates the possibility that interbank capital flows are inactive, and not all capital endowed to the pessimistic banks is channeled through interbank capital market to the real economy. As a result, banks’ beliefs are not as pessimistic as those in the risky interbank capital market case, thanks to strict financial regulations.

1.5.3. Financial Regulations and Stabilities

Definition 1 of the monotone equilibrium still applies but with slight modifications. In particular, as explained in section 1.5.2, the definition of the information revelation function \( z(r) \) is modified to accommodate the discontinuity at \( r = 1 \) and the possibility that a continuum of fundamentals \( \theta \) can clear at \( r = 1 \). Hence, in definition 1, the fourth condition that governs \( z(r) \) is modified such that \( z(r) = \max\{\theta : D(\theta, r) = 0\} \). Accordingly, the interest rate correspondence is modified as \( \hat{R}(\theta) \equiv \{r : z(r) = \theta \text{ if } r \in (1, R_h), \text{ and } \theta < z(r) \text{ if } r = 1\} \).

Like in the risky interbank capital market case, although other elements in the equilibrium can be uniquely characterized, there exists a continuum of market-clearing interest rate functions \( r^*(\theta) \) that gives rise to the multiplicity of equilibrium. Proposition 3 characterizes the fundamental threshold function in the limit:

**Proposition 3** In the limit of vanishing interpretation noise, the fundamental threshold function...
function $\theta^*(r)$ is given by

$$
\lim_{\sigma_p \to 0} \theta^*(r) = \begin{cases} 
0 & \text{if } r \in (1, R_h), \\
\frac{1-R_l}{R_h-R_l} & \text{if } r = 1.
\end{cases}
$$

When $r \in (1, R_h)$, all banks’ budget constraints hold. Hence, pessimistic banks lend all available capital to optimistic banks, which then lend their endowed capital and that borrowed in the interbank capital market to the real economy. Because the aggregate lending $A = 1$, there is sufficient lending to support economic expansions as long as $\theta \geq 0$.

When $r = 1$, two forces counterplay on each other. On one hand, the interbank capital market channels capital to optimistic banks and uniformly scales up the real credit supply by each optimistic bank. On the other hand, as analyzed in section 1.5.2, $r = 1$ uniformly truncates the distribution of fundamentals $\theta$ from above, which depresses banks’ confidence and reduces their incentive to lend to the real sector. In aggregate, the two forces cancel out, and the fundamental threshold is the same as that in the benchmark model $\theta^*_0$. The interbank capital market has no effect on the aggregate coordination results when $r = 1$.

Technically, Appendix A.1.8 shows that the Laplacian property still holds with an interest rate of $r = 1$.

Proposition 4 summarizes the interest rate and the coordination outcome for all realizations of fundamentals $\theta$. Like in the risky interbank capital market case, I focus on the limit of vanishing interpretation noise and denote $\hat{r}(\theta) = \lim_{\sigma_p \to 0} r^*(\theta)$. Appendix A.1.3 discusses the nonlimiting case and demonstrates the generality of the equilibrium results.

**Proposition 4** In the limit of vanishing noise on the public signal, that is, $\sigma_p \to 0$

1. for any $\theta > \bar{\theta}$, $\hat{r}(\theta) = R_h$, and the economy is in an expansion;

2. for any $\theta \in [\theta^*_0, \bar{\theta}]$, $\hat{r}(\theta) \in \{R_h, 1\}$, and the economy is in an expansion regardless of the interest rate;
3. for any \( \theta \in (0, \theta_0^*) \), \( \hat{r}(\theta) \in \{R_h, 1\} \), the economy is in an expansion if \( \hat{r}(\theta) = R_h \), and the economy is in a recession if \( \hat{r}(\theta) = 1 \);

4. for any \( \theta \leq 0 \), \( \hat{r}(\theta) = 1 \), and the economy is in a recession;

where \( \theta_0^* = \frac{1-R_l}{R_h-R_l} \) and \( \tilde{\theta} = \theta_0^* + \sigma_s [\Phi^{-1}(R_l + (1-R_l)\theta_0^*) - \Phi^{-1}(R_l)] > \theta_0^* \).

Appendix A.1.7 provides the proof. Figure 5 visualizes the comparison of the coordination outcome in the risky interbank capital market case and in the risk-free interbank capital market case. Recall that a risky interbank capital market introduces fragilities to the credit market in the sense that when \( \theta \in (\theta_0^*, 1) \), an economic recession caused by a shortage of credit becomes possible. By comparing the two panels in Figure 5, one can clearly see that imposing strict financial regulations stabilizes the real credit market, and when \( \theta \in (\theta_0^*, 1) \), the credit supply always abundantly supports an economic expansion, regardless of the interest rate. The main mechanism for why financial regulation can stabilize the credit market works through the feedback loop between the interbank capital market and the real credit market. By removing the counterparty risks of interbank lending, financial regulations prevent the interbank capital market from freezing, which directly improves the credit supply through the allocative role of the interbank capital market. Meanwhile, the functioning allocative role of the interbank capital market boosts banks’ confidence in the economy and limits its negative informative role. Moreover, banks’ confidence in the economy, in turn, encourages banks to borrow from others and supply more credit to the real economy, thereby further enhancing the allocative role of the interbank capital market. These two effects reinforce each other and jointly improve the real credit supply.
and promote economic growth. Finally, the prosperous economy generates a high return for the real credit supply, which reinforces banks’ incentive to participate in the interbank capital market.

Financial regulations that eliminate counterparty risks enhance the stability of the credit market in terms of aggregate coordination outcomes, that is, whether the economy is in an expansion or in a recession. In addition to that, they also ensure that the interbank capital market improves the efficiency of the credit market for any fundamentals $\theta$ and any market-clearing interest rate $r$ as summarized in the proposition below.

**Proposition 5** *In the limit of vanishing interpretation noise, that is, $\sigma_p \to 0$, the risk-free interbank capital market increases the efficient credit supply in economic expansions and decreases the inefficient credit supply in economic recessions.*

Figure 6 visualizes the results by plotting the aggregate credit supply of the financial system. The black line represents the benchmark case without interbank capital market. The red (blue) line represents the risk-free interbank capital market case in which the interest rate is high (low). When the interest rate is high, that is, $\hat{r} = R_h$, all capital is channeled through interbank capital market to the real economy, resulting in an aggregate credit supply $A = 1$.

Supply of credit is efficient in economic expansions. The interbank capital market improves
the efficiency of the credit market by increasing aggregate credit supply.

When the interest rate is low, that is, $\hat{r} = 1$, although the interest rate conveys vague information, it still serves as a coordination device that synchronizes banks’ lending decisions. Therefore, in aggregate, the total credit supplied by the financial system $A$ is more responsive to changes in fundamentals. At the fundamental threshold $\theta_0^*$, the aggregate credit supply $A = 1 - \theta_0^*$ in both cases. Hence, if the fundamentals are above the threshold, the interbank capital market helps create a more efficient credit supply. Similarly, in economic recessions when the fundamentals fall below the threshold, the interbank capital market helps reduce an inefficient credit supply.

1.5.4. Empirical and Policy Implications

**Empirical Implications** That this paper highlights the feedback between capital flows in the financial system and the credit supply to the real economy offers important empirical implications.

First, the equilibrium interest rate positively correlates with the aggregate credit supply and real economic growth. Before discussing this implication, I will first clarify the interpretation of the interest rate in the model. Banks’ returns from credit supply reflect the productivity of the real economy, which is constructive ($R_h > 1$) in expansions and destructive ($R_l < 1$) in recessions. In the financial system, each bank has unbiased—but noisy—private information and a heterogeneous estimation of real productivity. Through trading with one another, banks learn about the information possessed by other banks. In other words, the interest rate for interbank capital flows aggregates all the private information in the financial system and serves as a precise indicator of real productivity. Hence, mapping into the real world, one can think of the interest rate for interbank capital flows as the equilibrium real rate that reflects the marginal productivity of capital in the real economy.\(^{21}\)

Regardless of the counterparty risks for the interbank capital market, a low interest rate in

\(^{21}\)For simplicity, the paper assumes linear production technology, and the marginal productivity equals the average productivity in the real economy.
the model signals bad economic prospects, which discourages banks from supplying credit to the real economy. With limited access to credit, operating firms and consumers cut production and consumption, respectively, thereby depressing real economic growth. The low real economic growth, in turn, generates low returns on bank loans, which justifies the low borrowing demand for interbank capital and the low equilibrium interest rate.

Consistent with the models implications, during the 2007–2008 financial crisis, the real interest rate significantly dropped and remained low in the post-crisis recovery. Holston et al. (2017) estimates equilibrium interest rates at a quarterly frequency from the first quarter of 1961 to the third quarter of 2016. According to their estimations, in the United States, the equilibrium interest rate dropped from a pre-crisis level of over 2% to less than 1% and stayed below 1% afterward.\footnote{Other estimates have reached similar conclusions. See Barsky et al. (2014), Laubach and Williams (2016), and Holston et al. (2017) for examples.} Moreover, estimates of equilibrium interest rates for the Euro Area, the United Kingdom, and Canada all demonstrate the same pattern. In the meantime, the real economy experienced a credit crunch and sluggish growth worldwide. Regulators around the world also have expressed concerns that the post-crisis slow credit growth can encumber real investment and consumption, which have contributed to the shallow recovery after the crisis.\footnote{See the 2015 speech by James McAndrews (McAndrews, 2015), the former Executive Vice President and Director of research of the New York Fed, and the evidence of concerns in the ECB mentioned in the introduction.}

Second, trading volumes across financial institutions positively correlate with aggregate credit supply and real economic growth. As demonstrated by the model, when banks stop trading with one another, capital also gets stuck within the financial system and stops flowing to the real economy. This halted capital impedes economic growth. Meanwhile, the sluggish economic growth justifies banks’ reluctance to supply credit to the real economy and the low trading volumes across financial institutions.

Consistent with the models implications, many wholesale funding markets experienced turbulence and contraction in volume during the 2007–2008 financial crisis, and some have
yet to recover.\textsuperscript{24} For example, the interbank market dependence ratio, which is the total amount owed to credit institutions over total assets for domestic banks in the EU, has been declining from 15.5\% in 2007 to 6.2\% in 2017.\textsuperscript{25} In the United States, the total interbank loans held by all commercial banks plummeted from its pre-crisis peak of close to $500 billion to around $100 billion after the crisis and remain dim afterward.\textsuperscript{26} In addition to interbank loans, turmoil in the securitized asset markets also dampen the real credit supplied by the financial system (Loutskina and Strahan, 2009; Stein, 2010; Loutskina, 2011; di Patti and Sette, 2016). In fact, the lack of a revival of securitization is perceived by the policy makers in Europe as a major constraint for credit growth in the post-crisis recovery (Bank of England and European Central Bank, 2014a,b).

\textbf{Policy Implications} This paper emphasizes the benefits of strengthening financial regulations to address counterparty risks among financial institutions. In particular, this paper conveys two messages that regulators should bear in mind when formulating regulations to enhance financial stability.

First, even when economic fundamentals appear to be strong, it is important for the policy makers to ensure that banks can survive under severe economic conditions. The reason is that concerns about counterparty risks among financial institutions can lead to freezing interbank transactions and create real crisis in a self-fulfilling manner. Appendix A.1.1 shows that banks cannot solve the problem on their own, because they don’t fully endogenize the benefit of their own safe transactions on boosting others’ confidence in the economy. In consonance with the models implications, following the systemic fallouts of financial institutions during the financial crisis, regulators around the world have implemented stress tests to mandate sufficient capital buffers to absorb losses during adverse economic conditions. Generally speaking, the regulatory constraint in my model speaks to all prudential regula-

\textsuperscript{24}Examples include the mortgage-backed securities market (Gorton, 2009), the repo market (Gorton and Metrick, 2012), the short-term interbank lending market (Iyer and Peydro, 2011; Iyer et al., 2014; Cingano et al., 2016), and the interbank term loan market (Craig and Ma, 2017; Kuo et al., 2014).

\textsuperscript{25}Source: ECB Statistical Data Warehouse (series key CBD2.A.B0.W0.11.Z.Z.A.F.I3004.Z.Z.Z.Z.Z.Z.PC).

\textsuperscript{26}Source: Federal Reserve Bank of St. Louis (FRED) database (series IBLACBW027NBOG).
tions that require banks to maintain a healthy balance sheet and mitigate the counterparty risks for interbank transactions, such as capital requirements and minimum leverage ratio requirements.

Second, focusing on overall regulatory stringency is not enough. Recognizing the heterogeneities across different asset classes is equally important. To be more specific, I revisit the regulatory constraint,

\[ R_l a_i + 1 + x_i - a_i \geq c r x_i. \]

On the left-hand side of the inequality, the risky real lending \(a_i\) is evaluated at a discount of \(1 - R_l\) relative to the safe storage investment \(1 + x_i - a_i\). For simplicity, so far, I have focused on the overall regulatory stringency \(c\). In fact, an insufficient discount for the risky real lending \(a_i\) can also result in excessive leverage for the borrowing banks and counterparty risks. Therefore, in the design of stress tests, regulators should evaluate risky real lending at its default payoff \(R_l\) when there is a systemic shortage of credit to the real economy. In other words, in the spirit of a “macro-prudential” approach, regulators should endogenize coordination externalities among banks in stress tests. Following the same reasoning, in the design of risk weights in the capital requirements to limit excessive leverage taking, regulators should account for possibilities of systemic default on bank loans due to credit contraction and economic recession. In line with the models implications, in the 2017 reforms of the Basel III framework, the Basel Committee focused on enhancing the granularity and risk sensitivity in the calculation of risk-weighted assets (BIS, 2017).

Moreover, the discount for the risky lending \(a_i\) also speaks to the haircuts for risky collateral posted for interbank transactions. To limit the build-up of excessive leverage outside the traditional banking system, the Financial Stability Board (FSB) initiated a regulatory framework for haircut floors on securities-financing transactions (FSB, 2015). This model provides theoretical justification for mandating haircuts and guidelines for the level of haircuts. The bottom line is that even when the economic fundamentals appear to be strong, regulators should mandate haircuts such that the collateral value provided by risky assets
are evaluated by their payoffs during recessions.

Note that a direct capital injection to interbank capital market is not efficient in boosting credit supply when the economy is stuck in an equilibrium with low interest rates. In this equilibrium, banks are pessimistic about economic prospects; therefore, they are not willing to extend credit to the real economy or to demand borrowing from other banks. Frozen interbank capital flows and frozen real credit supply are a result of the overall pessimism instead of financing difficulties given the low cost of borrowing from other banks. Aimed at enhancing credit supply, the ECB conducted two long-term refinancing operations (LTROs) in December 2011 and February 2012. The LTROs provide funding to banks at a 3-year maturity and with low interest rates. Despite the large scale of 1 trillion euro lending to banks, evidence shows that the transmission to real credit supply was limited. Many banks took advantage of the cheap financing opportunities to invest in high-yield government bonds instead of supplying credit to support economic recovery. Carpinelli and Crosignani (2017) estimated that of the €181.5 billion borrowed, Italian banks in their sample invested €22.6 billion in credit to firms and €82.7 billion in government bonds. Consistent with my model, when interest rates are low, the efficiency of this kind of capital injection is hindered by the overall pessimism of economic prospects.

One final remark about the policy implications is that this paper focuses on the aggregate credit supply by the whole financial system and hence doesn’t account for frictions, such as relationship lending. In particular, if bank-specific lending relationships exist, that is, if banks have heterogeneous lending opportunities, stringent regulations can limit individual banks' lending capacity and harm the efficiency of capital allocation.

1.6. Other Applications

Beyond the context of regulations that address counterparty risks in the financial system, my model also provides a general framework to study how financial market regulations can mitigate coordination problems among market participants. In this section, I analyze
two applications and provide a theoretical justification for short-selling restrictions and the ban of naked CDS on sovereign debt. The common message conveyed by these two exercises is that regulations against aggressive speculation stabilize the financial markets. These regulations not only directly ease downward pressures imposed by speculation but also boost market confidence.

1.6.1. Short Selling Restrictions

Although regulators around the world have introduced various restrictions on short selling to prevent the damage of unfair speculation, most existing literature argues against short-selling constraints, because they are detrimental for liquidity, slow the price discovery process, and may create pricing bubbles. My model can be applied to financial markets vulnerable to fire-sale activities and provides a theoretical justification for short-selling constraints in these financial markets. Appendix A.1.5 provides a formal and detailed analysis. Below, I will briefly discuss the key results and the intuition.

The coordination problem in the real credit supply process can be reinterpreted as follows. The action $a_i$ represents bank $i$’s investment in a financial asset. $A = \int_0^1 a_i \, di$ is the total holding the financial asset, and $1 - A$ measures the fire-sale pressure. If $A$ decreases, it imposes a higher fire-sale pressure. As a consequence, holding onto the investment in the financial asset becomes more costly. When $A < 1 - \theta$, the market for the financial asset melts because of low fundamentals $\theta$ or high fire-sale pressures $1 - A$, and investments in the asset generate a return of $R_l < 1$. Otherwise, the investment is profitable with a return of $R_h > 1$.

The interbank capital market setup follows the main model naturally. To isolate the effect of short-selling restrictions, I restrict attention to the case in which financial regulations are tight ($c = 1$), and banks are not concerned about counterparty risks. Therefore, if banks

\footnote{During the 2007–2008 financial crisis, the SEC temporarily blocked short selling of financial stocks to protect investors and markets. Europe and Australia also enacted similar bans on short-selling activities. \footnote{See Diamond and Verrecchia (1987), Saffi and Sigurdsson (2010), and Boehmer and Wu (2012) for examples. Goldstein and Guembel (2008) exceptionally argues that if firms’ market prices feed back to their real decisions, short sellers can manipulate the market prices by short selling and make speculative profits.}}
are restricted from short selling, like in the main model, the equilibrium implications follow the risk-free interbank capital market case (see the upper panel of Figure 7).

![Figure 7: Coordination Outcome (Short Selling)](image)

However, as shown in the lower panel of Figure 7, if short selling is allowed (see the shaded area), when the fundamentals \( \theta \in (\theta^*_0, \theta^*_1) \), there is a possibility of a financial market meltdown because of fire-sale activities. Intuitively, in terms of resource reallocation, pessimistic banks with low private signals optimally choose to speculate by short selling financial assets. Short-selling activities directly aggravate the fire-sale pressure by reducing the aggregate investment \( A \). On top of this, aware of the aggravated fire-sale pressure, banks become more pessimistic about the health of the financial market. Therefore, the mass of pessimistic banks that short sell the financial asset increases, which further intensifies the fire-sale pressure. These two effects reinforce each other and jointly lead to a meltdown of the financial market for higher fundamentals \( \theta \). Hence, short-selling restrictions not only directly reduce fire-sale pressure but also maintain the confidence of banks investing in the financial asset, which, in aggregate, stabilizes the financial market.

### 1.6.2. Credit Default Swaps

Since December 1, 2011, the European Parliament has banned naked CDS on sovereign debts to fight the eurozone crises (European Commission, 2011). The International Monetary Fund (IMF), however, has warned that the ban might raise hedging costs and reduce investors’ interest in the underlying debt market (IMF, 2013). In this section, I apply the model of the interbank capital market and credit market to the CDS market and the underlying sovereign debt market. By comparing the two cases—one with naked CDS banned and the other with naked CDS allowed—I show that the ban of naked CDS enhances the
underlying sovereign stability. Appendix A.1.6 provides a formal and detailed analysis. Below, I will briefly discuss the key results and the intuition.

The baseline coordination problem can be reinterpreted as follows. Instead of banks, the problem comprises a unit mass of investors, each endowed with one unit of capital. \( a'_i \) denotes investor \( i \)'s investment in the sovereign debt. Thus, \( A' = \int_0^1 a'_i di \) represents the aggregate investment in sovereign debt. Sovereign debt is exposed to self-fulfilling debt rollover problems. In particular, the government is solvent if \( A' \geq 1 - \theta \), where \( 1 - \theta \) stands for the government budget deficit. In words, the government is solvent if enough investors keep lending to the government, such that the government can maintain its normal operations. In this case, the government repays its sovereign debt obligations in full, and investors earn a return of \( R_h > 1 \). Otherwise, the government defaults on the sovereign debt, and investors recover a return of \( R_l < 1 \).

In addition to the sovereign debt market, investors can trade on the CDS market based on their heterogeneous private information. Let \( p \) denote the price for the CDS contracts, which is determined by market-clearing condition in equilibrium. If investor \( i \) issues \( x'_i > 0 \) units of CDS contracts, she receives \( px'_i \) and pays the buyers \( x'_i \) if the sovereign debt defaults. If investor \( i \) is a net buyer of CDS contracts, \( x'_i < 0 \). To isolate the effect of the ban on the naked CDS, I impose short-selling constraints and strict regulatory constraints \((c = 1)\) to eliminate counterparty risks.

**Naked CDS Banned**

When naked CDS is banned, investors can only buy CDS contracts for hedging purpose. It turns out that the covered CDS market is equivalent to a risk-free interbank capital market in terms of coordination outcomes. Specifically, pessimistic investors invest in sovereign debt and purchase CDS contracts in proportion to hedge the sovereign default risk. Doing so effectively creates a risk-free portfolio. The optimistic investors, on the other hand, sell CDS contracts, and then invest their endowed capital and the proceeds from selling CDS contracts, and then invest their endowed capital and the proceeds from selling CDS contracts.
contracts in the sovereign debt. Therefore, optimistic investors are effectively borrowing from pessimistic investors, so that they can invest more in the sovereign debt. A high CDS price $p$ implies a higher hedging cost and, therefore, corresponds to a low interest rate $r$ in the risk-free interbank capital market. In this case in which naked CDS are banned, the price is bounded above at $\bar{p}$, which corresponds to an interest rate of $r = 1$.

The upper panel in Figure 8 summarizes the equilibrium coordination outcome. Like in the risk-free interbank market case, when $\theta \in (0, \theta_0^*)$, two different market-clearing CDS prices predict different coordination outcomes. The intuition is that if the investors observe a high CDS price indicating high default risks of the sovereign debt, they become pessimistic and refuse to invest in the sovereign debt. The aggregate reduction in the sovereign debt investment accelerates the shortage of funding for the government, causing a default on the sovereign debt. Similarly, when the CDS price is high, investors are optimistic and actively invest in sovereign debt. Doing so prevents sovereign default. However, as long as $\theta \geq \theta_0^*$, the government has an abundant budget, so that it always stays solvent, regardless of the price of the CDS contracts.

Naked CDS Allowed

If naked CDS is allowed, pessimistic investors optimally choose to speculate against the solvency of the government by purchasing CDS contracts without investing in sovereign debt. In stark contrast to the previous case, in this case the price of CDS contracts can rise above $\bar{p}$. At this high CDS price, optimistic investors also find it optimal to speculate by
selling CDS contracts without investing in the risky sovereign debt. As a result, all investors spend all their endowment on speculation based on their private beliefs, resulting in zero aggregate investment in sovereign debt. In other words, speculation activities completely crowd out sovereign debt investment.

As shown in the lower panel of Figure 8, without the ban on naked CDS contracts, when fundamentals $\theta \in (\theta^*_0, 1)$, naked CDS trading harms sovereign stability in the sense that it creates the possibility that all investors—both pessimistic and optimistic—speculate based on their beliefs, which crowds out the sovereign debt market. Observing a high CDS price, all investors know that even optimistic agents speculate instead of investing in sovereign debt. Knowing this exaggerates their concerns about the solvency of the government and reduces their incentives to invest in the sovereign debt. Therefore, the ban on abusive speculations with naked CDS enhances sovereign stability.

1.7. Conclusion

In this paper, I study the effect of capital flows in the financial system on the aggregate credit supply. I show that capital flows within the financial system play a positive allocative role in channeling capital to banks prone to supply credit to the real economy. However, without strict financial regulations to address counterparty risks, interbank lending can destabilize the credit market. When low interest rates reveal dim economic prospects and depress banks’ incentive to supply credit, the economy can get stuck in an equilibrium with freezing capital flows, both within the financial system and out to the real economy. Above all, this paper emphasizes the benefits of imposing strict financial regulations to ease the concern of counterparty risks and maintain the optimal functioning of the interbank capital market and the real credit market.
CHAPTER 2 : Intervention with Screening in Global Games

Lin Shen  Junyuan Zou

2.1. Introduction

In many economic environments, strategic complementarities among agents can give rise to coordination failure.\(^3\) To reduce the welfare loss from coordination failure, policy makers may intervene by providing incentives for agents to play the socially desirable equilibrium. For instance, during the recent financial crisis, governments around the world provided explicit and implicit guarantees on debt obligations of financial institutions to prevent “runs” on the financial systems. While these policies proved to be effective in restoring financial stability, some drawbacks also emerged. First, implementing guarantee programs at such large scale exposes the policy maker to large costs, which jeopardized sovereign debt sustainability and led to the sovereign debt crisis in many European countries (Acharya et al., 2014; Farhi and Tirole, 2016). Second, the policies were criticized for their vulnerability to moral hazard problems (Kareken and Wallace, 1978; Keeley, 1990; Cooper and Ross, 2002).\(^4\)

Given that such large-scale interventions are costly, a natural question is whether it is possible to reduce the size of intervention programs without compromising the effectiveness. To answer this question in a general context, we consider a coordination game with incomplete information as in standard global games (Morris and Shin, 2003). When agents receive pri-
vate signals, they form interim beliefs regarding the expected payoffs from taking different actions. We propose a group of programs with voluntary participation that screen agents based on their interim beliefs. Compared with conventional government-guarantee type of programs, it has two main advantages. First, in equilibrium, only a small group of marginal investors self-select into the program, which reduces the implementation costs. Also, our proposed programs have the advantage that moral hazard problems are limited to the small group of participating agents.

This paper provides novel insights for the design of intervention policies to reduce coordination failure in various economic contexts. Some existing literature (Sakovics and Steiner, 2012; Choi, 2014) has studied policies that target ex-ante important agents based on their payoff functions. We contribute to this literature by highlighting the role of agents’ interim beliefs of the economic fundamental and other agents’ actions. If an ex-ante important agent is very optimistic about the coordination result, there’s no need to provide extra incentives for her to take the socially desirable action. It is more cost-efficient if the resources are allocated to agents who have medium beliefs and are at the margin of taking the socially desirable action.

In our benchmark model, we explore a canonical binary-action coordination game under the global games framework. Global games are useful for linking coordination outcome to the underlying fundamental and determining the unique equilibrium. More importantly, they highlight the strategic interactions of agents with heterogeneous private information. In the model, a continuum of agents are each endowed with an investment opportunity. Their investments feature strategic complementarities. Specifically, the investments are successful if and only if the mass of agents investing exceeds a threshold which decreases in the fundamental of the economy. In addition, each agent receives a noisy private signal of the fundamental and makes inferences about the other agents’ investment decisions. The game has a unique equilibrium where all agents follow the same threshold strategy. In terms of welfare, there exists a region of weak fundamentals in which agents do not invest,
however, the investments would have been successful if all agents were to invest. Therefore, social welfare will be improved if the policy maker can lower the investment threshold and reduce the coordination failure region. The setup of the model is fairly general such that it can be applied to various economic contexts. In section 6, we discuss three coordination problems and make policy recommendations based on the proposed intervention policy.

Next, we allow the policy maker to offer a subsidy-tax program with voluntary participation to all agents who invest. If an investor accepts the offer, she receives a direct subsidy. In return, she is required to pay tax when the investment is successful. We classified the intervention programs into three categories based on the subsidy-to-tax ratio. If a program is too austere, i.e. has low subsidy-to-tax ratio, no agents will participate. We call this type of programs the zero-participation programs. If a program is too generous such that all investors participate, we call it a full-participation program. Many existing intervention policies, including government guarantees and direct subsidies, benefit all agents uniformly and therefore fall into this category. We show that full-participation programs can effectively reduce coordination failure however are costly to implement. To reduce the costs of implementation, we propose partial-participation programs with medium subsidy-to-tax ratios. A partial participation program is equivalent to a costly insurance policy and screens agents based on their interim belief of success. The most optimistic investors who believe in a high probability of paying the tax do not take the offer. At the same time, the most pessimistic agents who believe in a high probability of coordination failure do not find it worthwhile to invest solely to take advantage of the offer. Only agents with intermediate beliefs will participate in the program since it provides protection against coordination failure and investment loss. We show that with a partial-participation program there is a unique Bayesian Nash equilibrium, in which all agents follow the same threshold strategy with two thresholds. An agent will invest and reject the offer if she receives a high signal; she will invest and accept the offer if her signal is medium and between the two thresholds; she will not invest if she receives a low signal. When the information friction goes to zero, the two thresholds converge, and the expected mass of agents who accept the offer goes to
zero, which implies zero expected cost of implementation for the policy maker. Furthermore, with proper choice of subsidy and tax, coordination failures can be eliminated, and the first-best investment threshold can be achieved.

To understand intuitively how partial-participation programs can improve coordination results at a minimal cost, let us start with the original threshold equilibrium without intervention programs. For agents receiving signals right below the investment threshold, without any intervention policy, they will not invest in fear of the coordination failure and investment loss. The partial-participation programs provide protection against investment loss and give them extra incentive to make the investment. Therefore, with the partial-participation programs, all agents rationally expect the mass of agents who invest to increase and the strategic complementarities strengthen all agents’ incentive to invest. Hence, agents receiving even lower signals would be willing to accept the offer and invest, which is also expected by all agents in the economy and gives them more incentive to invest. Repeating the thought process, the extra incentive to invest provided by the partial-participation programs is amplified by higher-order beliefs, and the investment threshold can be reduced significantly in equilibrium. Given that all agents are more optimistic and less worried about coordination failure, the downside protection of the partial-participation programs becomes less appealing, and the mass of investors who accept the offer in equilibrium is actually small.

We then compare government guarantee programs with partial-participation programs in the presence of moral hazard problems. Government guarantee programs are a special case of full-participation programs and have been widely used to reduce coordination failure. We extend the benchmark model by assuming that after investment, an investor can earn private benefit by shirking, which will reduce the success probability of her own investment. Both types of intervention programs reduce investors’ “skin in the game” hence induce shirking at the expense of the policy maker and social welfare. For example, in the context of credit freeze when banks abstain from lending, government guarantees reduce banks’
incentive to screen and monitor borrowers. Moral hazard problem critically limits the scale of the government-guarantee type of programs. Specifically, if a government guarantees a large amount of investment losses, all investors, including the most optimistic ones, would participate and shirk. In contrast, for partial-participation programs, the moral hazard problem is limited to the program participants. For the optimistic agents, rejecting the offer and exerting effort gives higher payoff than participating and shirking. Hence, the social welfare loss only incurs for medium-belief agents, the mass of whom goes to zero in the limit of vanishing information frictions. As a result, in the limit, there exist partial-participation programs that can restore the first best, yet no government-guarantee type of programs can restore the first best.

Besides the benchmark model, we also show that the results could be generalized to allow unobservable ex-ante heterogeneity in agents’ payoff and information structure. Regarding ex-ante agent heterogeneity, a closely related paper is Sakovics and Steiner (2012). The difference is that they only allow the policy maker to provide direct subsidies conditional on agents’ observable heterogeneities. Under their setup, the most cost-efficient subsidies should target the important agents with specific ex-ante characteristics. However, their policy space falls into the category of full-participation programs in our model, and the policy maker can save costs and limit moral hazard problems by switching to a partial-participation program. In other words, we show that subsidization should target the interim rather than ex-ante “pivotal” types. Moreover, since the “pivotal” agents self-select to participate in partial-participation programs, the policy maker does not need to observe agents’ ex-ante characteristics. We also show that the binary payoff structure in the baseline model can be generalized to a continuous monotonic payoff function.

Our paper is related to two lines of literature. First, our model is built on the literature of global games which was pioneered by Carlsson and Van Damme (1993). Researchers have applied the global games techniques to analyze coordination failures in different contexts, to name a few, bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005), currency
attack (Morris and Shin, 1998), credit freeze (Bebchuk and Goldstein, 2011), debt rollovers
(Morris and Shin, 2004; He and Xiong, 2012), and political revolutions (Edmond, 2013).

We take a general approach and propose intervention programs that can be applied to
reduce coordination failure in different contexts. Morris and Shin (2003) reviews the most
commonly applied setup and applications of global games. Our main model in section 2 is
a special case with binary payoffs. In section 5, we discuss a generalized payoff structure
as in Morris and Shin (2003). In both cases, we show that there exists costless intervention
to reduce the coordination threshold and eliminate coordination failures in the limit of zero
information friction.

Second, our mechanism shares similar ideas found in the literature that explores policies
targeting a specific group of agents to reduce coordination failures. For example, within
the contracting literature, Segal (2003) and Bernstein and Winter (2012) show that the
optimal policy is to divide and conquer, i.e. subsidize a subset of players so that they invest
even if no one else invests, then the surplus of players in the no-subsidy set can be fully
extracted. Sakovics and Steiner (2012) and Choi (2014) analyzed a coordination game with
ex-ante heterogeneous agents and showed that different types should be subsidized in a
certain order. These papers all demonstrate that subsidizing a subset of agents to ensure
their participation can efficiently encourage the participation of the rest of the agents and
reduce coordination failure. Our proposed intervention program is different in terms of
implementation. The policy maker offers the same option to all agents, and a subset of
agents self-select to participate in the program. In the generalization of unobservable ex-
ante heterogeneity, we show that our proposed intervention program is more cost-efficient
and does not require information about agents’ heterogeneity. Cong et al. (2017) and Basak
and Zhou (2017) analyze intervention policies under dynamic settings. In both papers, the
policy maker target a subset of agents in each period. The coordination result of the current
period serves as a public signal of the fundamental of the economy. They emphasize the
effect of the public signal on agents’ beliefs and behaviors in the subsequent period(s).
Another closely related paper is Morris and Shadmehr (2017), which analyzes the reward
schemes for a revolutionary leader to elicit effort from citizens. The optimal reward scheme also screens citizens for their optimism. However, they consider bounded reward schemes imposed on a continuous and unbounded effort choice set, while we focus on subsidy-tax programs that agents can voluntarily choose to participate in. More importantly, while they assume zero cost for implementing any reward scheme, we target minimizing the cost of intervention.

The rest of the paper is organized as follows. In section 2, we present a benchmark model of a binary-action investment game and introduce intervention policies that can reduce coordination failures. Section 3 and 4 compare the proposed program with government-guarantee type of programs in terms of implementation cost and robustness to moral hazard problems. Two extensions of the benchmark model are discussed in section 5. Section 6 presents several applications of the benchmark model and discusses policy recommendations in each context. Finally, section 7 concludes.

2.2. The Benchmark Model

In this section, we analyze a binary-action investment game in which each agent’s investment outcome depends on the aggregate investment in the economy. In such an environment, inefficient coordination failure can arise in which agents abstain from investment because of their self-fulfilling expectation that other agents will not invest. Then we introduce intervention policies and show how they can encourage investment and reduce coordination failure.

2.2.1. Setups

There is a unit mass of ex-ante identical infinitesimal agents, indexed by $i \in [0,1]$. These agents are endowed with the same investment opportunity, and they simultaneously make investment decisions $a_i \in \{0,1\}$. $a_i = 1$ if agent $i$ invests, and $a_i = 0$ if agent $i$ does not invest. Not investing results in zero payoffs, while investing incurs a fixed cost $c > 0$ and generates a profit of $b > c$ if agent $i$’s project is successful and 0 if it fails. We assume all
agents’ investment payoffs are perfectly correlated. The investments would be successful when the fundamentals of the economy are strong enough or a sufficient number of agents invest. Specifically, the payoff from an investment project is

\[ \pi(\theta, l) = \begin{cases} b - c, & \text{if } l \geq 1 - \theta, \\ -c, & \text{if } l < 1 - \theta. \end{cases} \]

where \( l = \int_0^1 a_i d\bar{i} \) represents the fraction of investors or the aggregate investment level, and \( \theta \) stands for the fundamentals of the economy. Note that agents’ investment decisions feature strategic complementarities, because each project is more likely to succeed when more agents choose to invest. When the fundamentals are higher, it requires less aggregate investment to make the projects successful. Without information friction, when \( \theta \in [0, 1) \), all agents investing \((l = 1)\) and all agents not investing \((l = 0)\) are both Nash equilibria. However, all agents investing is strictly more efficient than the other equilibrium. Therefore, the first-best outcome is that all agents coordinate to invest when \( \theta \geq 0 \) and not to invest when \( \theta < 0 \).

We follow the standard global games setup and assume the following information structure. The fundamental \( \theta \) is drawn from a uniform distribution with support \([\underline{\theta}, \overline{\theta}]\) and it is not directly observable to the agents when they make investment decisions.\(^5\) Instead, each agent receives a noisy signal about the fundamental \( x_i = \theta + \sigma \varepsilon_i \), where \( \varepsilon_i \) is identically and independently distributed with a continuous and strictly increasing c.d.f. \( F(\varepsilon) \), the support of which is \([-\frac{1}{2}, \frac{1}{2}]\). Furthermore, we assume that \( \underline{\theta} < -\sigma \) and \( \overline{\theta} > 1 + \sigma \). Under this assumption, there exist two dominance regions of signals, \([-\theta - \frac{1}{2} \sigma, \bar{x})\) and \((\bar{x}, \theta + \frac{1}{2} \sigma]\), with \( \bar{x} \) and \( \bar{\bar{x}} \) defined as

\[ \Pr(\theta \geq 1| x = \bar{x}) = \frac{c}{b}, \]
\[ \Pr(\theta \geq 0| x = \bar{x}) = \frac{c}{b}. \]

\(^5\)We assume a uniform prior to obtain an analytical solution to the coordination game. This is without loss of generality since it can be viewed as a limiting case as the size of the information friction goes to zero.
Intuitively, with the lowest aggregate investment level \( l = 0 \), an agent is indifferent between the two actions when she receives signal \( \bar{x} \). Therefore, her dominant strategy when signal \( x > \bar{x} \) is to invest. Similarly, with the highest aggregate investment level \( l = 1 \), an agent is indifferent between the two actions if she observes signal \( x \). Hence, when \( x < \bar{x} \), not investing is the dominant strategy.

### 2.2.2. Equilibrium without Intervention

In this subsection, we analyze the equilibrium without intervention and identify the inefficiencies due to coordination failure. Proposition 6 characterizes the equilibrium.

**Proposition 6** Without intervention, there is a unique equilibrium in which all agents follow the same strategy

\[
a_i(x_i) = \begin{cases} 
1, & \text{if } x_i \geq \xi^*_0, \\
0, & \text{if } x_i < \xi^*_0.
\end{cases}
\]

where \( \xi^*_0 = \frac{\xi}{\theta} + \sigma F^{-1}\left(\frac{\xi}{\theta}\right) \).

Since there is a continuum of agents, given the realization of fundamentals \( \theta \), we can apply the law of large numbers to calculate the aggregate investment \( l \) and predict the coordination outcomes. In equilibrium, all agents follow the same threshold strategy. Therefore, the coordination outcome also has a threshold above which the investment projects are successful. Let \( \theta^*(\xi) \) denote the fundamental threshold when all agents follow the threshold strategy \( \xi \), then it is defined by

\[
F\left(\frac{\theta^*(\xi) - \xi}{\sigma}\right) = 1 - \theta^*(\xi).
\]

In words, at the fundamental threshold, the fraction of investors \( l \) equals the cutoff \( 1 - \theta \).
Then the fundamental threshold in equilibrium is given by

\[ \theta^*(\xi_0^*) = \frac{c}{b}. \]

The fundamental realizations can be divided into three regions as shown below. In the

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<th>Inefficient Coordination Failure</th>
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<td>[ \theta^<em>(\xi_0^</em>) = \frac{c}{b} ]</td>
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middle region \( \theta \in [0, \frac{c}{b}) \), if all agents coordinate to invest, the investment projects would have been successful. However, the agents have self-fulfilling beliefs that other agents do not invest. As a result, they rationally choose not to invest. Since a unit of successful investment generates a positive surplus of \( b - c \), in the middle region, coordination failure leads to social welfare loss of \( b - c \). Hence, the first-best scenario has a fundamental threshold \( \theta^* \) equal to zero. And in the next section, we will show how our proposed intervention program can lower this cutoff and reduce inefficiencies caused by coordination failure.

### 2.2.3. Intervention Program

Having characterized the equilibrium in the game without intervention, we now describe the subsidy-tax intervention program that the policy maker can use to boost investment and reduce coordination failure. The intervention program consists of two parts, a direct subsidy \( s \in [0, c] \) and a contingent tax \( t \in [0, b] \). Specifically, if an investor decides to accept the offer, she receives an upfront subsidy \( s \) regardless of the investment outcome and pays a lump-sum tax \( t \) only if the investment succeeds.\(^6\) The program is only available to the investors and they voluntarily decide whether to participate in the program. Note that there is an implicit assumption that the actions taken by the agents are observable to the policy maker and can be contracted on. We make this assumption because, as shown in Bond

\(^6\)Since in the benchmark model there’s only two possible payoffs from investing, we only need to specify a contingent lump-sum tax. In section 2.5.2, we analyze a more general setup where there’s a continuum of investment outcomes and we allow tax to be proportional to the investment revenue.
and Pande (2007), if the policy maker cannot observe individual actions, its ability to use subsidy-tax schemes as a coordination device is greatly limited. This assumption imposes certain limitations on the application of our proposed intervention mechanism. For example, in the context of currency attack, it is hard to trace agents’ action and tax conditional on agents’ investment behavior. Therefore, the intervention program discussed in this paper cannot be applied to solving currency deflation caused by coordination failure (Morris and Shin, 1998). Despite this limitation, there is a wide range of real-world applications. In section 6, we discuss three representative examples.

Mathematically, if an investor accepts the offer, her payoff is modified to

\[
\tilde{\pi}(\theta, l) = \begin{cases} 
  b - t - (c - s), & \text{if } l \geq 1 - \theta, \\
  -(c - s), & \text{if } l < 1 - \theta.
\end{cases}
\]

The upfront subsidy \(s\) reduces the cost of investment and encourages agents to invest. The contingent tax \(t\) directly helps the policy maker recover the cost of providing subsidies. More importantly, it will become clear later that the contingent tax \(t\) indirectly saves cost by deterring participation of optimistic agents. The timeline of the coordination game with the intervention program is modified as follows. At the beginning of the game, the policy maker announces the intervention program \((s, t)\). Note that since the subsidy \(s\) and tax \(t\) are both state independent, the announcement of the intervention program does not convey any information possessed by the policy maker. Angeletos et al. (2003) demonstrates that the informational role of state contingent policy can lead to multiple equilibria in global games. Therefore, the intervention programs analyzed in this paper are free from the signaling concern of state contingent policies and do not require the policy maker to have superior information about the fundamentals of the economy. Then the fundamental \(\theta\) is realized, and each agent receives a noisy signal of the fundamental. After observing the signal, agents simultaneously make their decisions on whether to invest and if so, whether to participate in the intervention program. As soon as the decisions are made, active investors pay the cost \(c\), and the policy maker transfers the subsidy \(s\) to all investors participating in the intervention.
program. Then the aggregate investment \( l \) and the investment returns are realized. Finally, the policy maker collects tax \( t \) from the investors participating in the intervention program if the investments are successful. The timeline is summarized in Figure 10 below.

![Figure 10: Timeline of the Investment Game](image)

Although the intervention program is specified as a subsidy-tax program, it can be interpreted as other forms of intervention with transfers between the policy maker and the investors, contingent on the coordination result. For example, a government-guarantee type program that promises to cover the loss of failed investment up to \( s^g \leq c \) is equivalent to a subsidy-tax program with \( s = t = s^g \). To see this, under both programs, the net transfer from the government to any participating investor is 0 in the case of successful investments and \( s^g \) in the case of failed investments. Similarly, an asset purchase program in which the policy maker buys \( \frac{l}{b} \) fraction of the project with price \( s \) is equivalent to a subsidy-tax program \((s, t)\).

### 2.2.4. Equilibrium with Intervention

We now analyze the equilibrium with intervention and demonstrate how the intervention program works to reduce coordination failure. With the intervention program, an agent has three choices: \( \{a = 1, \text{Reject}\} \), \( \{a = 1, \text{Accept}\} \), and \( \{a = 0\} \). Note that although agents make two decisions, whether to invest and conditional on investing, whether to accept the offer, only their investment decisions affect the coordination results. Therefore, an agent only cares about the investment decisions of the others but not their participation in the intervention program. As a result, to analyze the best response and equilibrium strategies, it is sufficient to condition on other agents’ investment strategies. Let \( \hat{p}_i = \)
Pr[l ≥ 1 − θ|x_i] denote the interim belief of success of agent i given her private signal x_i and other agents’ investment strategies a_{-i}(x). The expected payoffs from \{a = 1, Reject\} and \{a = 1, Accept\} are

\[
E[\pi(\theta, l)|x_i] = \hat{p}_i b - c, \quad (2.1)
\]
\[
E[\tilde{\pi}(\theta, l)|x_i] = \hat{p}_i (b - t) - (c - s) \quad (2.2)
\]

respectively. And the expected payoff from \{a = 0\} is zero. Figure 11 depicts the expected payoff as a function of the interim belief \(\hat{p}\). It can be divided into three cases according to the subsidy-tax ratio \(\frac{s}{t}\).

In the first case when \(\frac{s}{t} \geq 1\), accepting the offer dominates rejecting the offer. This is because investors always receive a higher subsidy s than their tax payment required by the intervention program. We call this type of programs the full-participation programs. Without intervention, the belief threshold for investment is the cost-benefit ratio \(\frac{c}{b}\). With
a full-participation program, the threshold is lowered to \( \frac{c + s}{b + t} \). In the third case with \( \frac{c}{t} < \frac{s}{t} \), rejecting dominates accepting the offer. We call this type of programs the zero-participation programs. Thus, the threshold belief under the zero-participation program is the same as the original cost-benefit ratio \( \frac{c}{t} \). The second case is the most interesting. When \( \frac{c}{b} \leq \frac{s}{t} < 1 \) (figure 11.b), an agent would only accept the offer and invest when she has an intermediate belief \( \hat{p} \in \left[ \frac{c-s}{b-t} \right] \). We call this type of programs the partial-participation programs. Notice in both case 1 and case 2, the provision of the intervention program lowers the threshold belief to \( \frac{c-s}{b-t} \). The difference is that, in case 2, the most optimistic agents do not participate in the intervention program, which is cost saving especially when the information friction is small. We will analyze the cost of the programs in detail in section 3.

Next we sketch the analyses of equilibrium with intervention. It will become clear later that iterated deletion of dominated strategies allows us to focus on cutoff investment strategies. We say an agent follows a cutoff investment strategy with threshold \( k \), if her investment strategy is

\[
a_i(x; k) = \begin{cases} 
1, & \text{if } x \geq k, \\
0, & \text{if } x < k.
\end{cases}
\]

(2.3)

Let \( p(x; k) \) denote the interim belief of success when an agent receives private signal \( x \) and all other agents follow a cutoff investment strategy \( k \),

\[
p(x; k) = Pr(\theta > \theta^*(k) | x) = F\left( \frac{x - \theta^*(k)}{\sigma} \right),
\]

(2.4)

where \( \theta^*(k) \) is the fundamental threshold for successful investment and satisfies \( F\left( \frac{k - \theta^*(k)}{\sigma} \right) = \theta^*(k) \). An agent’s interim belief of success \( p(x; k) \) increases in \( x \) and decreases in \( k \), because a high private signal \( x \) indicates a high realization of fundamentals \( \theta \), and a low investment threshold \( k \) implies a high aggregate investment \( l \). Both imply a high probability of success.

In all three cases depicted in figure 11, the optimal investment strategy is that an agent
invests if and only if her belief $p(x, k)$ exceeds a threshold. Since $p(x, k)$ is monotonic in both $x$ and $k$, an agent’s best response to other agents’ cutoff strategy $k$ is also a cutoff investment strategy based on her own signal. The two dominance regions form two extreme cutoff investment strategies. Starting there, by iterated deletion of dominated strategies, we are able to prove the uniqueness of the equilibrium with intervention. The details of the analyses can be found in the proof of proposition 7 below. The following proposition characterizes the equilibrium with a subsidy-tax intervention program $(s, t)$.

**Proposition 7** When the policy maker offers a subsidy-tax intervention program $(s, t) \gg 0$, the game has a unique equilibrium. There are three different cases,

1. When $\frac{s}{t} \geq 1$, the equilibrium is for any agent $i$,

\[
\begin{align*}
a_i &= 1, \text{Accept, if } x_i \geq \xi^*(s, t), \\
a_i &= 0, \text{if } x_i < \xi^*(s, t).
\end{align*}
\]

where

\[
\xi^*(s, t) = \frac{c - s}{b - t} + \sigma F^{-1} \left( \frac{c - s}{b - t} \right),
\]

2. When $\frac{s}{t} \leq \frac{c}{b} < 1$, the equilibrium is for any agent $i$,

\[
\begin{align*}
a_i &= 1, \text{Reject, if } x_i \geq \eta^*(s, t), \\
a_i &= 1, \text{Accept, if } \xi^*(s, t) \leq x_i < \eta^*(s, t), \\
a_i &= 0, \text{if } x_i < \xi^*(s, t),
\end{align*}
\]

Frankel et al. (2003) prove existence, uniqueness and monotonicity in multi-action global games. However, our setup does not satisfy the continuity assumption. Therefore, we provide our own proof in the Appendix.
where

\[ \xi^*(s,t) = \frac{c-s}{b-t} + \sigma F^{-1}\left(\frac{c-s}{b-t}\right), \]
\[ \eta^*(s,t) = \frac{c-s}{b-t} + \sigma F^{-1}\left(\frac{s}{t}\right). \]

3. When \( \frac{s}{t} < \xi \), the equilibrium is for any agent \( i \),

\[ a_i = 1, \text{Reject, if } x_i \geq \xi^*(s,t), \]
\[ a_i = 0, \text{if } x_i < \xi^*(s,t), \]

where

\[ \xi^*(s,t) = \frac{c}{b} + \sigma F^{-1}\left(\frac{c}{b}\right). \]

The ratio of the upfront subsidy \( s \) and the ex-post tax \( t \) can be interpreted as the generosity of the program. If the offer is generous (case 1), all investors find it profitable to accept the offer and the equilibrium investment cutoff depends on the modified cost \( c' = c - s \) and benefit \( b' = b - t \). If the offer is austere (case 3), all investors will not be interested in the offer. Therefore the equilibrium investment cutoff is the same as the original cutoff without the intervention program. The most interesting case is case 2, in which the generosity of the offer is medium. Investors with high private signals have strong beliefs in the success of the project, so they will reject the subsidy offer since they believe in a high probability of paying a net tax in the future. However, even without subsidies, these optimistic agents would invest anyway. Agents with low private signals have strong beliefs in the failure of the project, so even with the subsidy \( s \), they still suffer a loss of \( c - s \) from investing. Therefore, these agents would not invest regardless of the intervention program. In contrast, investors receiving signals around the threshold do not have strong beliefs about the coordination results. Without the intervention program, some of these agents would not invest. The
intervention program provides insurance against losses in case of failed investment and gives these agents extra incentive to invest. With the extra incentive, these agents’ decisions are effectively altered and the aggregate action \( l \) therefore increases. The increase in \( l \), in turn, strengthens all agents’ incentive to invest. Agents with even lower signals would participate in the program and change their decisions to invest. Through iterations of higher-order beliefs, the action cutoff is significantly lowered. Moreover, agents with signals around the old cutoff are significantly more optimistic, and therefore the intervention program is no longer appealing to them. In equilibrium, the mass of investors accepting the offer is rather small. We call these investors the “pivotal” investors, since the equilibrium investment cutoff is determined by their modified cost and benefit.

In case 1 and 2, the fundamental cutoff above which the investment projects are successful is

\[
\theta^*(\xi^*(s, t)) = \frac{c - s}{b - t}. \quad (2.5)
\]

Note that the new fundamental cutoff is lower than that without government intervention. Therefore, the provision of the intervention program successfully reduces the inefficient coordination failure region. If the government picks \( s = c \) and \( t \in [s, b) \), the fundamental cutoff can be reduced to 0, eliminating the whole region of inefficient coordination failure as demonstrated in Figure 12.

<table>
<thead>
<tr>
<th>Efficient No Investment</th>
<th>Inefficient Coordination Failure</th>
<th>Efficient Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \theta^* = \frac{c - s}{b - t} )</td>
<td>( \theta_0^* = \frac{c}{b} )</td>
</tr>
</tbody>
</table>

Figure 12: Coordination Results after Intervention

2.3. Cost of Implementation

In this section, we compare the implementation cost of partial-participation and full-participation intervention programs in two cases, one with negligible information frictions and one with
non-negligible information frictions. We then discuss the intuitions why partial-participation is more cost-efficient than full-participation programs.

2.3.1. Cost of the Intervention Programs

We compare the expected cost of the partial-participation and full-participation programs conditional on the same target fundamental threshold \( \theta^* \) of successful investment. To allow for the possibility that the policy maker values tax and subsidy differently, the value of tax for the policy maker is normalized to 1 and the cost of providing subsidy is assumed to be \( \tau \). The ex-post cost of providing the intervention program to an individual investor is

\[
\hat{c}(\theta, s, t) = \begin{cases} 
\tau s - t, & \text{if } l \geq 1 - \theta, \\
\tau s, & \text{if } l < 1 - \theta.
\end{cases}
\] (2.6)

When \( \hat{c}(\theta, s, t) \) is negative, the policy maker profits from providing this intervention program.

For the rest of the analyses, we focus on \( \tau \geq 1 \) for two reasons. First, we believe it is a realistic characterization. If subsidy is provided before tax collection, \( \tau > 1 \) reflects the funding cost of the policy maker due to the opportunity cost of other welfare-improving programs. Alternatively, if the program is government guarantee, \( \tau > 1 \) reflects the cost of commitment, such as setting aside funds specifically for the program. Moreover, any administrative cost incurred by providing subsidy or collecting tax can raise \( \tau \). Secondly, if \( \tau < 1 \), given negligible information frictions, the policy maker can easily restore the first best and profit at the same time by offering \( t = s = c \). \(^8\) The coordination problem then becomes trivial. Therefore, for the rest of the paper, we assume \( \tau \geq 1 \).

Let \( C(\theta, s, t; \sigma) \) denote the ex-post total cost of providing a subsidy-tax intervention program \( (s, t) \) given the realized fundamental \( \theta \) and the information friction \( \sigma \). For full-participation programs, i.e., \( \frac{s}{t} \geq 1 \), all investors participate in the intervention program. The ex-post  

\(^8\)To be precise, the policy maker should set \( t = s = c - \varepsilon \) with a very small \( \varepsilon \) to avoid over-investment when \( \theta < 0 \) and keep the left dominance region.
cost of implementation is

\[ C(\theta, s, t; \sigma) = \begin{cases} 
\tau s - t \left[ 1 - F\left( \frac{s^*(s,t) - \theta}{\sigma} \right) \right], & \text{if } \theta \geq \frac{c-s}{b-t}, \\
\tau s \left[ 1 - F\left( \frac{s^*(s,t) - \theta}{\sigma} \right) \right], & \text{if } \theta < \frac{c-s}{b-t}. 
\end{cases} \]  

(2.7)

For partial-participation programs \( \frac{s}{b} \leq \frac{t}{b} < 1 \), only pivotal investors participate in the intervention program. In this case,

\[ C(\theta, s, t; \sigma) = \begin{cases} 
\tau s - t \left[ F\left( \frac{s^*(s,t) - \theta}{\sigma} \right) - F\left( \frac{x^*(s,t) - \theta}{\sigma} \right) \right], & \theta \geq \frac{c-s}{b-t}, \\
\tau s \left[ F\left( \frac{s^*(s,t) - \theta}{\sigma} \right) - F\left( \frac{x^*(s,t) - \theta}{\sigma} \right) \right], & \theta < \frac{c-s}{b-t}. 
\end{cases} \]  

(2.8)

If \( \frac{s}{t} < \frac{c}{b} \), no agents will find it profitable to opt in to the intervention program, therefore \( C(\theta, s, t; \sigma) = 0 \).

Proposition 8 below compares the ex-post and ex-ante expected cost of partial-participation programs and full-participation programs, which restore first best in the limit of vanishing information frictions.

**Proposition 8** With strictly costly subsidy \( \tau > 1 \), when the information friction \( \sigma \) goes to 0, there exists a continuum of full-participation programs \((s, t)\) and a continuum of partial-participation programs \((s', t')\) achieving the first-best outcome, where \( s = s' = c \) and \( t \leq c < t' \leq b \).

For any such \((s, t)\) and \((s', t')\), given \( \theta \), the full-participation program \((s, t)\) is ex-post more costly than the partial-participation program \((s', t')\). Specifically,

\[ \lim_{\sigma \to 0} C(\theta, s, t; \sigma) = \tau s - t > \lim_{\sigma \to 0} C(\theta, s', t'; \sigma) = 0, \quad \text{if } \theta > 0; \]

\[ \lim_{\sigma \to 0} C(\theta, s, t; \sigma) = \tau s - t > \lim_{\sigma \to 0} C(\theta, s', t'; \sigma) = \frac{s'}{\tau}(\tau s' - t'), \quad \text{if } \theta = 0; \]

\[ \lim_{\sigma \to 0} C(\theta, s, t; \sigma) = \lim_{\sigma \to 0} C(\theta, s', t'; \sigma) = 0, \quad \text{if } \theta < 0. \]

Moreover, the full-participation program \((s, t)\) is ex-ante strictly more costly than the partial-
participation program \((s', t')\). Specifically,  
\[
\lim_{\sigma \to 0} \mathbb{E}_\theta C(\theta, s, t; \sigma) > \lim_{\sigma \to 0} \mathbb{E}_\theta C(\theta, s', t'; \sigma) = 0.
\]

The proof is in the Appendices. When the information friction is small, although both full-participation programs and partial-participation programs can effectively reduce coordination failures and restore the first-best outcome, the partial-participation programs are ex-post weakly less costly than the full-participation programs in all states. Intuitively, compared with full-participation programs, partial-participation programs have less participants since the optimistic investors are deterred from participating. This subsequently reduces the cost of implementation. If the policy maker evaluates the ex-ante expected cost of the programs, in the limit of negligible information frictions, the partial-participation programs incur zero cost and strictly dominate the full-participation programs.

Now we extend the analysis to the case of non-negligible information frictions \(\sigma > 0\). To facilitate the comparison of the cost of different programs given the same fundamental \(\theta^*\), we introduce an alternative parameterization of the intervention programs. Specifically, an intervention program \((s, t)\) can be equivalently parameterized by \((\theta^*, \lambda)\) as follows,

\[
\begin{align*}
    s &= \frac{c - \theta^* b}{1 - \theta^*} + \theta^* \lambda, \\
    t &= \frac{c - \theta^* b}{1 - \theta^*} + \lambda.
\end{align*}
\]

\(\theta^* = \frac{c - s}{b - t}\) is the target fundamental threshold, and \(\lambda = \frac{t - s}{1 - \theta^*}\) is proportional to the net tax charged by the program when the project succeeds. \(\lambda\) can also be interpreted as the scale of the program because given the same target \(\theta^*\), both tax and subsidy are strictly increasing in \(\lambda\). Intuitively, when \(\lambda\) increases, the intervention program charges a higher net tax and becomes less attractive. To achieve the same target, the government needs to increase the direct subsidy \(s\) (and the tax \(t\) at the same time) to provide more downside protection to the investors. When \(\lambda \in \left[\frac{c - \theta^* b}{b - t}, 0\right]\), the subsidy-to-tax ratio \(\frac{s}{t} \geq 1\), and the program is a full-participation program. When \(\lambda \in \left(0, \frac{b - c}{1 - \theta^*}\right)\), \(\frac{c}{b} < \frac{s}{t} < 1\), and the program is a partial-participation program. Note that to achieve the same target \(\theta^*\), the partial-participation
programs are larger in scale $\lambda$ than the full-participation programs. The reason is that partial-participation programs are less generous, i.e. have lower subsidy-to-tax ratio than full-participation programs, the magnitude of partial-participation programs need to be larger to provide more downside protection as compensation.

Suppose the policy maker is considering switching from a full-participation program $(\theta^*, \lambda)$ to a partial-participation program $(\theta^*, \lambda')$. The change in the expected cost of implementation comes from both the extensive and the intensive margin. On the extensive margin, the most optimistic investors will no longer enter the program. Hence, this effect always reduces the expected cost of intervention. However, on the intensive margin, the cost of providing the program to an individual investor could increase or decrease. Formally, the difference in unit cost is

$$\hat{c}(\theta, s', t') - \hat{c}(\theta, s, t) = \begin{cases} 
(\tau \theta^* - 1)(\lambda' - \lambda), & \text{if } \theta \geq \theta^*, \\
\tau \theta^*(\lambda' - \lambda), & \text{if } \theta < \theta^*.
\end{cases}$$

With vanishing information frictions, the effect on the intensive margin is negligible because the mass of participants in partial-participation programs goes to zero except for the knife-edge case of $\theta = \theta^*$. Therefore, switching to any partial-participation program will always reduce the cost of implementation. This is no longer true with non-negligible information frictions. In proposition 9, we provide two sufficient conditions such that switching to a partial-participation program reduces the expected cost of implementation.

**Proposition 9** For any $\sigma > 0$, if $1 \leq \tau < G(\theta^*, 1)$ or $\theta^*(1 + \sigma) < 1$, there exists a partial-participation program $(\theta^*, \lambda)$ which achieves $\theta^*$ at lower expected cost than any full-participation program targeting $\theta^*$, where $G(\alpha, \beta)$ is defined for any $0 \leq \alpha \leq \beta \leq 1$ as

$$G(\alpha, \beta) = \int_{\alpha}^{\beta} \frac{F^{-1}(\beta) - F^{-1}(\alpha)}{\alpha(F^{-1}(\beta) - F^{-1}(\alpha))} F(x) dx.$$
The proof involves technical details and is included in the Appendix. Here we provide some intuitions. Since partial-participation programs provide more subsidy and charges more tax to each participants, the effect on the intensive margin depends on the ratio of expected mass of taxpayers to the expected mass of subsidy receivers. This ratio is equal to $G(\theta^*, 1)$ in a partial-participation program $(\theta^*, \lambda)$ when $\lambda$ approaches 0. If $\tau < G(\theta^*, 1)$, the ratio is large enough such that the increase in expected tax revenue is greater than the increase in expected subsidy provision. Hence, the effect on the intensive margin also works in favor of the partial-participation programs, and switching to a partial-participation program with small $\lambda$ reduces the expected cost. Notice for any given $\theta^* < 1$, $G(\theta^*, 1) > 1$.

Therefore, the special case of $\tau = 1$ always satisfies the first condition. The second condition governs the relative importance of the two margins. If $\theta^*$ and $\sigma$ are jointly small, the participation threshold $\eta^*$ for partial-participation programs is also small, therefore the mass of participants is significantly reduced. In particular, if the second condition holds, the effect on the extensive margin dominates that on the intensive margin, making the proposed partial-participation program less costly than any full-participation programs. In summary, Proposition 9 gives three circumstances in which the most cost-efficient subsidy-tax program is a partial-participation program: ambitious target (small $\theta^*$), small information frictions (small $\sigma$), or small cost of subsidy $\tau$. Note that as a special case, if the policy maker targets at the first-best $\theta^* = 0$, there always exists a partial-participation program that dominates all full-participation programs.

We use a numerical example to demonstrates how switching to a partial-participation program from a full-participation program can reduce the expected cost of the intervention. Suppose the prior on $\theta$ is uniformly distributed on $[\underline{\theta}, \bar{\theta}] = [-0.2, 1.2]$. The private noise $\varepsilon$ follows a uniform distribution over $[-\frac{1}{2}, \frac{1}{2}]$ and $\sigma = 0.2$. $c$ and $b$ are set to 1 and 1.25, so the benchmark success threshold is $\bar{c} = 0.8$. The policy maker has a cost parameter $\tau = 1.05$ and targets a success threshold $\theta^* = 0.2$. The least costly full-participation program to achieve the equilibrium threshold is $s = t = 0.9375$. The ex-post cost as a function of the realized fundamental is represented by the solid blue line in Figure 13. The cost is positive.
for all $\theta > \theta^*$ because all investing agents sign up for the program and there's a positive cost $\tau s - t$ of providing this program to each agent. When $\theta$ falls below $\theta^*$, the cost surges because the investment projects fail and the policy maker can’t recover $t$. Now the policy maker switches to a partial-participation program. There’s a continuum of partial-participation programs that targets the same threshold $\theta^*$. We take $(s', t') = (0.97, 1.1)$ for an example. The red dashed line in the top panel of Figure 13 represents the ex-post cost function of program $(s', t')$. It has a similar shape as the cost function of the full-participation program. However, it converges to 0 when $\theta$ is large enough so that all agents receive signals higher than $\eta^*$ and no agents participate in the intervention program. The difference between the two cost functions is plotted in the bottom panel. Compared to the full-participation program, the partial-participation program incurs lower cost when $\theta > \theta^*$ because of the higher tax charge and the lower participation rate. When $\theta < \theta^*$, since the partial-participation
program provides higher subsidy, it incurs higher cost than the full-participation program. On average, the partial-participation program incurs lower expected cost.

2.3.2. Discussions

From previous analyses, we show that partial-participation intervention programs can improve the coordination results to the first-best outcome in the investment game, yet has zero cost when the information friction vanishes. This result seems striking at first glance. The most important reason why the partial-participation intervention program works effectively at a minimal cost is that it targets precisely the marginal agents who are on the investment threshold and can be incentivized to invest relatively easily. These agents are also the “pivotal” investors whose investment decisions are crucial in the determination of the investment threshold. The figure below demonstrates how through higher-order beliefs, our proposal effectively reduces coordination failure.

In each iteration, the lower axis denotes the signal received by an agent, and the upper axis denotes the corresponding belief. Start from the cutoff strategy \( \xi_0^* \), which is the original cutoff without intervention. The partial intervention program incentivizes agents to lower the investment threshold to \( \xi_1^* \). Since all agents understand that more agents are willing to
invest, given the same private signals, they all believe in a higher aggregate action $l$ and a higher probability of successful investment $p(x; \xi_1)$. Therefore, they are willing to lower their investment threshold further to $\xi_2^*$. Similarly, with the additional mass of agents receiving signals between $\xi_1^*$ and $\xi_2^*$ investing, all agents are more optimistic about the success of the investment and therefore further lower their investment threshold to $\xi_3^*$. At the same time, as the agents become more optimistic about their investments, the intervention program becomes less attractive, which implies a decreasing sequence of participation thresholds $\eta_n^*$. With an infinite number of iterations, both the investment threshold and the participation threshold are significantly lowered. As the information friction decreases, investors become more certain about the coordination results, so the mass of “pivotal” investors shrinks to zero. However, as long as there exist a few pivotal investors, the intervention program will have a significant effect on the investment threshold due to higher-order beliefs.

Our partial-participation programs share similar spirit to the targeted intervention programs. Sakovics and Steiner (2012) analyze coordination games with heterogeneous agents and argue that the optimal subsidy schedule is to target a certain type of agent. In section 5, we examine an extension with heterogeneous agents and show that there exist partial-participation programs that incur zero cost to restore first-best outcome in the limit of negligible information frictions. Similar to the main model, in equilibrium, only a small mass of “pivotal” agents self-select to accept the policy maker’s offer. The only difference is that different agent types have different thresholds, and the “pivotal” agents are the ones receiving signals around their own thresholds. The result conveys one message contrasting Sakovics and Steiner (2012) that policy makers should target interim rather than ex-ante important types. Also, one common problem with targeted intervention programs is that information acquisition to identify the targeted type(s) can be costly. The policy maker needs to correctly identify each agent’s type to implement the targeted intervention programs. In contrast, our proposed intervention programs incentivize the “pivotal” agents to self-reveal their types, therefore the implementation only requires information on the payoff structure of different types. As a result, our proposed program is superior to the targeted
intervention programs in terms of reducing the costs of collecting information.

2.4. Interventions in the Presence of Moral Hazard

In this section, we address the concern of moral hazard problem of government guarantees and demonstrate our proposal’s robustness to moral hazard problems. For example, in the context of self-fulfilling credit freeze (Bebchuk and Goldstein, 2011), banks may abstain from lending in fear that the other banks will withdraw lending, which results in a coordination failure of credit crunch. If the government provides guarantees on bank losses, the banks may have the incentive to shirk in screening and monitoring the borrowers, since the losses caused by shirking is guaranteed by the government. In order to do incorporate moral hazard problems in the model, we modify the game into two stages. The first stage is the same as the benchmark model with an intervention program, except that the payoffs are not realized until the second stage. If the realized fundamental \( \theta < 1 - l \), we say the aggregate state is \textit{Bad}. In this case, the investment project fails and the game ends immediately. If the realized fundamental \( \theta \geq 1 - l \), we say the aggregate state is \textit{Good}. In this case, the game enters the second stage, in which investors make their effort choices. If an investor exerts effort, the investor pays a cost of effort \( c^e \), and her project succeeds with probability \( 1 - \gamma \). As in the benchmark model, the project generates \( b \) in case of success and 0 in case of failure. And for the participants in the intervention program, they are required to pay tax \( t \) if their investments are successful. We make the following assumption on the parameters.

\textbf{Assumption 1} The investment opportunity has the following properties,

\begin{enumerate}
\item shirking is inefficient, \( c^e < \gamma b \);
\item the investment projects are ex-ante efficient, \( b > c + c^e \).
\end{enumerate}

\(^9\)The results hold as long as the success probability when exerting effort is between \( 1 - \gamma \) and 1, which prevents the policy maker from inferring effort choice based on ex-post investment outcome. Otherwise, the moral hazard problem can potentially be solved by imposing ex-post punishment when the policy maker observes failed investment.
Given the assumptions above, the first-best scenario is that all agents invest and exert effort if the fundamental $\theta \geq 0$, and all agents do not invest otherwise.

The equilibrium with moral hazard problem can be solved backward. In the second stage, an investor would exert effort if and only if

$$b - t - c^e \geq (1 - \gamma)(b - t). \quad (2.9)$$

This condition can be interpreted as a constraint on the size of the tax $t$,

$$t \leq b - \frac{c^e}{\gamma}. \quad (2.10)$$

When the tax is above the threshold, participating investors has too little “skin in the game” to exert effort, resulting in inefficient outcomes. Intuitively, with a higher cost of effort $c^e$ or lower losses caused by shirking $\gamma$, the incentive problem is more severe, imposing a tighter constraint on the size of tax $t$.

Next, we will analyze the equilibrium under different programs and examine whether a full-participation program like government guarantee or a partial-participation program can achieve first best when there is moral hazard problem in the private investment project. In the context of our model, we interpret the government guarantee program as a subsidy-tax program $(s, t)$ with $s = t$, which is the full-participation programs with least cost. Since participating in the government guarantee program weakly dominates investing alone, every investor will take advantage of this program.

**Government Guarantee.** The moral hazard problem in the second stage imposes an upper limit on the scale of the government guarantee program if the policy maker wants to enforce effort.
The expected payoff from investing with the government guarantee program is

$$\mathbb{E}[\tilde{\pi}(\theta, l)|x_i] = \begin{cases} \hat{p}_i(b - t - c^e) - (c - s), & \text{if } t \leq b - \frac{c^e}{\gamma}, \\ \hat{p}_i(1 - \gamma)(b - t) - (c - s), & \text{if } t > b - \frac{c^e}{\gamma}. \end{cases}$$  \hspace{1cm} (2.11)$$

From the analysis of the benchmark model, we know that in the unique Bayesian Nash equilibrium, the fundamental threshold above which the aggregate state is good is equal to the belief of the marginal investor. Given a program with $t \leq b - \frac{c^e}{\gamma}$ that prevents shirking, the fundamental threshold in equilibrium is

$$\theta^* = \frac{c - s}{b - t - c^e}.$$  \hspace{1cm} (2.12)$$

Given a program with $t > b - \frac{c^e}{\gamma}$ that tolerates shirking, the fundamental threshold in equilibrium is

$$\theta^* = \frac{c - s}{(1 - \gamma)(b - t)}.$$  \hspace{1cm} (2.13)$$

In both cases, reducing the fundamental threshold to the first best $\theta^* = 0$ requires the subsidy $s$ to be as close to $c$ as possible. However, by the nature of the intervention program, this also requires the contingent tax $t = s$ to be as close to $c$ as possible. The scale of the intervention program is constrained by the incentive constraint as shown in (2.10), and whether the constraint is binding depends on the severity of the moral hazard problem.

**Assumption 2** The moral hazard problem is severe, $\frac{c^e}{\gamma} > b - c$.

Given Assumption 2 above, the maximum program size $t$ that prevents shirking in the second period is strictly less than $c$, the cost of the investment project. Therefore, the government guarantee program cannot achieve efficient fundamental threshold in the first stage and prevent shirking in the second period at the same time. The result is summarized
in Proposition 10 below. When Assumption 2 does not hold, the government guarantee program with \( t = c \) achieves the first-best outcome.

**Proposition 10** Given Assumption 1 and 2, no government guarantee program can restore the first-best outcome when \( \sigma \to 0 \).

**Partial-participation Programs.** Now let us consider a subsidy-tax program with \( s \in \mathbb{[}c, b, 1) \). Given that the tax is higher than the subsidy, whether to participate in the program depends on investors’ idiosyncratic beliefs of the probability that the aggregate state is good. As in the benchmark model, the program is the most attractive to agents with intermediate beliefs. What complicates the analyses is that agents will take into account their effort decisions in the second period when they compare the cost and benefit of participating in the program. When the moral hazard problem in the second period is not severe, i.e., Assumption 2 does not hold, the policy maker can choose \( s = c \) and \( t \in \mathbb{[}c, b) \) to implement the first-best outcome, which is the same as government guarantee programs. In the following analyses, we focus on the case when the moral hazard problem is severe, i.e., Assumption 2 holds, and full-participation government guarantee programs cannot achieve the first best.

Given that Assumption 2 holds and \( t > c \), the optimistic agents will reject the intervention offer and exert effort, the agents with medium beliefs will accept the intervention offer and shirk. Intuitively, the intervention offer reduces participant’s investment risk as well as “skin in the game”. The most optimistic agents who strongly believe in the success of investment do not want to share the profits with the policy maker. Therefore, they will reject the offer and fully endogenize the payoff from investment which incentivizes them to make the first-best effort choice. In contrast, the agents with medium beliefs are willing to invest only if the policy maker bears part of the investment risk. However, the intervention program also reduces their “skin in the game” because they need to share the investment profits with the policy maker but bear the full cost of effort. As a result, these agents will participate in the intervention program and shirk.
Formally, given the optimal effort choices in the second stage, the expected payoffs from \(\{a = 1, \text{Reject}\}\) and \(\{a = 1, \text{Accept}\}\) for an agent who receives signal \(x_i\) and forms belief \(\tilde{p}_i\) are

\[
E[\pi(\theta, l)|x_i] = \tilde{p}_i(b - c^e) - c, \quad (2.14)
\]

\[
E[\tilde{\pi}(\theta, l)|x_i] = \tilde{p}_i(1 - \gamma)(b - t) - (c - s). \quad (2.15)
\]

The expected payoffs are linear and increasing in the belief \(\tilde{p}_i\), and the slopes are different. The difference in the slopes of \(E\pi(\theta, l)\) and \(E\tilde{\pi}(\theta, l)\),

\[
(b - c^e) - (1 - \gamma)(b - t) = \gamma b + t(1 - \gamma) - c^e > 0 \quad (2.16)
\]

is strictly positive given Assumption 1a. Investing alone is the optimal choice if and only if the belief \(\tilde{p}_i\) exceeds the critical participation belief

\[
p_2^*(s, t) \equiv \frac{s}{\gamma b + t(1 - \gamma) - c^e}. \quad (2.17)
\]

Not investing is the optimal action choice if and only if the belief of the agent is worse than the critical investment belief

\[
p_1^*(s, t) \equiv \frac{c - s}{(1 - \gamma)(b - t)}. \quad (2.18)
\]

The optimal action choice if the belief of success probability is between \(p_1^*(s, t)\) and \(p_2^*(s, t)\) is to invest and accept the offer.

Similar to those in the benchmark model, the critical beliefs determine the equilibrium thresholds of investment and participation regarding the private signal \(x\). Investment efficiency in the first stage requires the critical investment belief \(p_1^*(s, t)\) to be as close to 0 as possible, which implies that the policy maker should choose subsidy \(s = c\). On the other hand, if \(t\) can be selected properly such that the critical participation belief \(p_2^*(s, t) < 1\),

\[ \]
the investors who are very optimistic about the aggregate state would choose to invest and reject the offer. The exclusion of optimistic investors from the program improves efficiency in the second stage game and reduces the policy maker’s cost from inefficient failures due to shirking. As the information friction goes to zero, the mass of “pivotal” investors who participate in the program goes to zero. The following proposition summarizes the result.

**Proposition 11** Given Assumption 1 and 2, the equilibrium outcome given a subsidy-tax program \((s, t)\) with \(s = c\) and \(\frac{c + c^* - \gamma b}{1 - \gamma} < t < b\) converges to the first best when \(\sigma \to 0\). The ex-ante cost of providing such program also converges to 0 when \(\sigma \to 0\).

The above proposition demonstrates the advantage of the partial-participation programs compared with full-participation programs like government guarantee when the moral hazard problem is relatively severe. In the benchmark model, both types of programs can achieve the first-best outcome at zero cost with diminishing information friction if \(\tau = 1\). They are different in terms of the program size: full-participation programs invite all investors, while partial-participation programs only target the “pivotal” investors. Absent other frictions, the size of a program does not alter the efficiency or the cost of implementing the program. However, the moral hazard problem causes welfare losses in proportion to the size of a program. When using a government guarantee program, the policy maker faces a trade-off between the first-stage investment efficiency and the second-stage effort efficiency. A program with high subsidy over tax ratio \((\frac{s}{t})\) encourages investment in the first stage but deters effort input in the second stage. This trade-off limits the role of the government guarantee program in improving social efficiency. On the contrary, despite the moral hazard problem, a partial-participation program still achieves the first-best outcome at zero cost. The advantage of partial-participation programs in dealing with moral hazard is that they only involve a small mass of investors. Although these participating investors shirk in the second stage, it will have a limited impact on the social welfare since the mass of these participating investors goes to zero as the information friction vanishes. In general, the partial-participation program proposed in this paper is superior to the full-participation
programs such as government guarantee in the presence of any size-related inefficiency.

2.5. Extensions

2.5.1. Unobservable Ex-ante Heterogeneity

In this part, we study whether the existence of ex-ante heterogeneity in agents’ payoff structure and information structure changes our results. The assumptions on the heterogeneity resemble those in Sakovics and Steiner (2012). Our analyses differ from their paper in two dimensions. First, they studied the optimal intervention when the policy maker can only provide a lump-sum subsidy, while we consider subsidy-tax programs. Second, they assume the types of agents are observable, while we allow for hidden types.

There are \( N \) groups of infinitesimal agents indexed by \( g \), each group with mass \( m^g \). There are three folds of heterogeneity. First, the agents differ in their profitability. They pay the same investment cost \( c \) yet earn different revenue \( b^g \) from successful investment. Assume there is no inefficient project, so \( b^g > c \) for all \( g \). Second, the agents impose different levels of externalities for the coordination results. Specifically, the aggregate action \( l = \sum_{g=1}^{N} \int_0^{m^g} w^g a^g_i \, di \). Same as in the benchmark model, the condition that investment is successful is \( l \geq 1 - \theta \). The weights are normalized such that \( \sum_{g=1}^{N} w^g m^g = 1 \). Lastly, each agent receives a private signal \( x^g_i = \theta + \sigma \epsilon^g_i \), where \( \epsilon^g_i \) is independent across agents and follows a group-specific distribution with c.d.f. \( F^g(\epsilon) \), the support of which is \([-\frac{1}{2}, \frac{1}{2}]\). We assume an agent’s group is not observable to the policy maker. However, the policy maker knows the composition of agents.

The equilibrium without intervention is summarized by the following proposition.

**Proposition 12** Without intervention, there is a unique equilibrium in which an agent in group \( g \) invests if and only if her private signal is greater or equal to \( \xi_0^g \), which is given by

\[
\xi_0^g = \sum_{g=1}^{N} m^g w^g \frac{c}{b^g} + \sigma F^{-1}_g \left( \frac{c}{b^g} \right). \tag{2.19}
\]
From the above proposition, we can calculate the fundamental threshold \( \theta^* \) above which the investments are successful. The expression for the fundamental threshold is given by

\[
\theta^* = \sum_{g=1}^{N} m^g w^g \frac{c_{gb}}{b_g},
\]

which is a weighted average of the cost-benefit ratio of different types of agents. Let \( b_{min} = \min \{b^g\}_{g=1}^{N} \). The following proposition shows our previous results still hold when there is unobservable heterogeneity among agents.

**Proposition 13** Given a subsidy-tax program with \( s < c \) and \( s < t < b_{min} \), there exists a unique equilibrium in which a type \( j \) agent follows the strategy below,

\[
a = 1, \text{Reject, if } x \geq \eta^*_g(s, t), \\
a = 1, \text{Accept, if } \xi^*_g(s, t) \leq x < \eta^*_g(s, t), \\
a = 0, \text{if } x < \xi^*_g(s, t),
\]

where

\[
\xi^*_g(s, t) = \sum_{g=1}^{N} m^g w^g \frac{c_{gb}}{b_g} - s + \sigma F^{-1}_g \left( \frac{c_{gb} - s}{b_g - t} \right),
\]

\[
\eta^*_g(s, t) = \sum_{g=1}^{N} m^g w^g \frac{c_{gb}}{b_g} - t + \sigma F^{-1}_g \left( \frac{s}{t} \right).
\]

When \( s = c \) and \( c < t < b_{min} \), the equilibrium outcome converges to the first-best outcome and the expected cost of the program converges to 0 when \( \sigma \to 0 \).

If agents also differ in the cost of investment, i.e., \( c^g \) can be different across groups, we need to relax the assumption that type are unobservable to the government. Instead, we assume the government can observe \( c_i \) for each individual agent. If \( c_i = c^g_1 = c^g_2 \), the government
The intuition for how our proposed intervention program works in the case with ex-ante heterogeneous agents is essentially the same as in the benchmark model. The intervention program incentivizes “pivotal” agents who originally choose not to invest to change their decisions. All agents knowing that there is an increase in the aggregate action \( l \) all believe in a higher probability of success. Amplified by higher-order beliefs, the intervention program can efficiently restore the first-best coordination results. Note that the notion of “pivotal” agents refers to the interim type of agents. Since different groups earn different profitabilities from successful investments, they require a different success probability to agree to invest. Our intervention program identifies and targets agents with beliefs right below the cutoffs of their own group. In Sakovics and Steiner (2012), they only look at direct subsidy programs and argue that an efficient program should target the ex-ante “pivotal” group, the group with low \( b^g \) and high \( w^g \) in our setup. Our results above demonstrate that by allowing an additional intervention tool, the contingent tax \( t \), we are able to reduce coordination failure at a much lower cost. Moreover, the implementation of our proposed program does not require information on an agent’s group, therefore our proposed program could save the potential cost of information acquisition.

2.5.2. General Payoff Structure

In this section, we follow the setups of the symmetric binary-action global games in Morris and Shin (2003) and allow for general monotonic payoff functions.

As in the benchmark model in section 2, an agent’s payoff from not investing \((a_i = 0)\) is normalized to zero. An agent’s payoff from investing \((a_i = 1)\) is modified to be a continuous function \( \pi(x, l) \), which weakly increases in both the private signal \( x \) and the aggregate action \( l = \int_0^1 a_i di \).\(^{10}\) The fundamental \( \theta \) follows a uniform distribution on \([\underline{\theta}, \bar{\theta}]\). The private signal

\(^{10}\)We assume the payoff is a function of the private signal instead of the fundamental for simplicity of demonstration. Our results still hold under the alternative setup. See Morris and Shin (2003) for the
received by agent $i$ is $x_i = \theta + \sigma \epsilon_i$, where $\epsilon_i$ are i.i.d. and has a density function $f(\epsilon)$ and a distribution function $F(\epsilon)$ with support $[-\frac{1}{2}, \frac{1}{2}]$.

For simplicity, we only consider the family of linear intervention programs. In general, we could allow transfer as a non-linear function of the agents’ payoff. The intervention program $(s, t)$ consists of two parts, a direct subsidy $s \geq 0$ and a proportional tax $t \in [0, 1]$. If an agent accepts the offer, she receives the direct upfront subsidy $s$ and pays the proportional tax after the realization of the investment outcome. Her payoff from accepting the offer is\textsuperscript{11}

$$\tilde{\pi}(x, l) = (1 - t)\pi(x, l) + s.$$  \hspace{1cm} (2.21)

Agents who receive low private signals believe in low realization of the fundamental $\theta$ and low aggregate action $l$, so they are pessimistic about their payoffs from investments. Therefore, they expect to pay low tax and are more willing to accept the offer than optimistic agents.

Recall the partial-participation programs in the benchmark model. These programs do not appeal to the optimistic agents who do not need extra incentive to invest, which efficiently saves resources and reduces the cost of the program. The proportional tax $t$ captures this feature and helps to target agents receiving medium signals.

We adopt the standard assumptions on the payoff function in the literature.

\textbf{Assumption 3} The payoff function $\pi(x, l)$ satisfy the following properties:

1. (Monotonicity) The payoff function $\pi(x, l)$ is weakly increasing in both arguments.

2. (Strict Laplacian State Monotonicity) $\int_0^1 \pi(x, l) dl$ is strictly increasing in $x$.

\textsuperscript{11} One might notice that when $\pi(x, l) < 0$, investors end up paying a negative “tax”. In fact, let $\pi = \pi(\theta - \frac{1}{2} \sigma, 0)$ be the lower bound of the payoff. The intervention program can be implemented by providing a positive subsidy $s - t\pi$ and imposing a proportional tax $t$ on the positive tax base $\pi(x, l) - \pi$. 

\hspace{1cm} 78
3. *(Limit Dominance)* There exists $\theta_0, \theta_1 \in (\bar{\theta} + \frac{1}{2}\sigma, \bar{\theta} - \frac{1}{2}\sigma)$ such that

$$
\pi(x, 1) < 0, \text{ for all } x < \theta_0, \\
\pi(x, 0) > 0, \text{ for all } x > \theta_1,
$$

(2.22) (2.23)

4. *(Continuity)* $\int_0^1 g(l)\pi(x, l)dl$ is continuous in $x$ for any density function $g$.

The first assumption states the strategic complementarities among the investment choices of different agents. The individual payoff of investing increases when more agents invest. Also, a higher fundamental increases everyone’s incentive to invest, given the same aggregate investment. Note that the payoff function need not be strictly increasing or continuous. For example, the payoff function in our benchmark model in Section 2 is a step function. The role of the second assumption is to make sure the equilibrium is unique when it exists, with or without the intervention program. The third assumption ensures the existence of two dominance regions so that we can adopt the iterated deletion of dominated strategies from both sides. The last assumption regulates integration of the payoff function so the equilibrium always exists.

The equilibrium without intervention is characterized in the proposition below. The “natural outcome” serves as a benchmark to analyze the effect of intervention programs.

**Proposition 14** Without intervention ($s = t = 0$), when the information friction $\sigma$ is small enough, there is a unique equilibrium in which each agent invests if and only if her private signal $x \geq \xi_0^*$ given by

$$
\int_0^1 \pi(\xi_0^*, l)dl = 0.
$$

Compare the coordination results characterized in the above proposition with the first-best outcome. In the first-best scenario, if all agents investing can generate positive surplus, the social optimal outcome is for all agents to invest. In other words, the first-best scenario is that all agents follow the same cutoff strategy $\theta_0$, the upper bound for the left dominance
region. By Assumption 3, unless $\pi(\theta_0, l) = 0$ for any $l \in [0, 1]$, the natural coordination outcome $\xi^* > \theta_0$. Therefore if the realized fundamental $\theta \in (\xi^*, \theta_0)$, there would be a coordination failure. And the goal of intervention is to reduce the coordination threshold from $\xi^*$ to as close to $\theta_0$ as possible.

Next we analyze the equilibrium with an intervention program $(s, t)$. We focus on the partial-participation programs and demonstrate its zero cost of implementation in the limiting case. Proposition 15 summarizes the conditions for such partial-participation programs. For the convenience of discussion, we first define partial-participation programs with targeted threshold for coordination failure as follows.

**Definition 2** A intervention program $(s, t)$ is a partial-participation program with target $\xi^*$ if and only if it satisfy the following three conditions,

1. (Intervention Target) $\int_0^1 \pi(\xi^*, l)dl = -\frac{s}{1-t}$.
2. (Optimism Exclusion) $\pi(\xi^*, 1) > \frac{s}{t}$,
3. (Left Dominance Region) $\pi(\theta, 1) < -\frac{s}{1-t}$,

Denote a coordination game with information friction $\sigma$ and intervention program $(s, t)$ by $G(\sigma; s, t)$, we can prove the following proposition.

**Proposition 15** Given a partial-participation program $(s, t)$ with target $\xi^*$, the following two properties must be satisfied in any Bayesian Nash equilibrium of the coordination game $G(\sigma; s, t)$,

1. Agents invests if and only if their private signal $x > \xi^*$;
2. There exists a threshold $\eta^*(\sigma)$ such that investing agents strictly prefer not to participate in the intervention program if and only if their private signal $x > \eta^*(\sigma)$.

When $\sigma \to 0$, $\eta^*(\sigma)$ converge to $\xi^*$. 

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The above proposition provides conditions under which there exist partial-participation programs to reduce the investment threshold to $\xi^*$. Same as in the benchmark regime-change model, in the limit, ex ante expected mass of participants goes to zero, which implies zero cost of implementation. The question remaining is whether there exist such programs to costlessly restore the first-best scenario, i.e. $\xi^* = \theta_0$. The proposition below answers this question.

Proposition 16 If $\int_0^1 \pi(\theta_0, l) dl \geq \pi(\theta, 1)$, for any $\xi^* \in (\theta_0, \xi^*_0)$, there exists a partial-participation program with target $\xi^*$.

With the intervention program, all agents become more optimistic about their investment payoff. Therefore, the left dominance region, where agents prefer not to invest even if $l = 1$, shrinks. The condition specified in Proposition 16 guarantees that the left dominance region still exists with the intervention program. If the condition is violated, there might be multiple equilibria when targeting $\xi^*$ close to $\theta_0$. However, if we follow the equilibrium refinements proposed in Goldstein and Pauzner (2005), we can select the equilibrium described in Proposition 15 even without the left dominance region. Therefore, following the refinements, there always exists a partial-participation program that restores the first-best scenario. Moreover, the left dominance region may disappear because we limit our attention to programs with linear transfers. Linear transfer schedules generally gives a lot of subsidies in case of low fundamental. If the policy maker lowers subsidies in the case of very low fundamental realizations (when $\theta < \theta_0$) or adds convexity to the tax schedule properly, the left dominance region as well as the uniqueness of the equilibrium can be recovered. Either way, there always exists an intervention program that can restore the first-best scenario.

2.6. Selected Applications

The partial-participation programs can be applied to various contexts with coordination problems. In this section, we discuss three representative applications.
2.6.1. Debt Rollover

It has been widely recognized in the literature that panic-based debt run can leads to inefficient firm default. Specifically, consider a firm with many small debt-holders. The firm is more likely to survive if more debt-holders roll over their debts. Therefore, debt-holders’ rollover decisions features strategic complementarities. When the fundamental of the firm is weak, debt-holders might stop rolling over their debts because they worry the others would also stop, which can leads to self-fulfilling debt run. Our analyses suggest tranching can be a cost-efficient way to reduce such coordination failure. Instead of one standard debt contract, the firm can issue two types of debts with different seniorities. The senior debt promises lower return yet provides higher payment than the junior debt when the firm defaults. Without tranching, debt-holders who have medium beliefs and coordination concerns would not roll over their standard debts. With the safer option of senior debt, they are willing to lend to the firm which eases the liquidity concern of the firm and boosts all debt-holders’ beliefs in the firm’s survival. This effect can be amplified by higher order beliefs. In equilibrium, only the pivotal debt-holders choose the senior option. However, the availability of the safer senior debt improves all debt-holders’ belief in that the firm can raise enough funds to survive.

Bank run is another similar application. To implement the partial-participation programs, the government can offer optional but costly deposit insurance. In fact, Alipay, the largest online payment platform in China, offers all users an option to purchase insurance against losses on their associated financial accounts. The insurance is costly if their accounts are safe yet provides protection when the platform fails. Therefore, it would work in a similar way as the senior debt option to reduce coordination failure. It is less costly than the mandatory deposit insurance because it screens for the “pivotal depositors” and leaves out the optimistic depositors who would not run even without insurance protection.
2.6.2. Market Freeze

During the 2008 financial crisis, many financial institutions and investors significantly reduced their leverage. This process pushed down the market prices of Commercial Mortgage-Backed Securities (CMBS) and Residual Mortgage-Backed Securities (RMBS). The markets for RMBS and CMBS froze, and prices were well below their fundamentals. Among others, coordination failure can prevent the market from thawing. If only a few investors participate in the market for Mortgage-Backed Securities (MBS), the liquidity in the market is not enough to drive the prices back to the fundamental and the participating investors suffer losses on their investments. However, if a significant amount of liquidity is injected in the market, the prices are more likely to be driven back to reflect the fundamental and investors who bought at a discount can profit from the investment.

In March of 2009, the US Treasury announced the Legacy Securities Public-Private Investment Program (PPIP). Under the program, private equity was matched by government equity and debt to form Public-Private Investment Funds (PPIFs) and purchase highly rated legacy MBS from financial institutions. Private investors in the PPIFs effectively receive investment subsidies from the government and are levered up for their investment. They earn higher investment return in good times and are protected by limited liabilities in bad times. Hence, PPIP is uniformly beneficial to all qualifying private investors and can be interpreted as full-participation programs in our model. PPIP is not efficient in resource allocation in the sense that part of the government funding is provided to the optimistic investors who would have invested in MBS market without PPIP. According to our analyses, the government can reduce the cost of rejuvenating the market by offering a partial-participation program instead. Mapping into the context of PPIP, the government could offer to inject equity into PPIFs in proportion to debt holdings by private investors. This option of debt investment reduces the losses from freezing the MBS market. As a return, the government shares the profit of investment if the market for MBS is successfully rejuvenated. This offer incentivizes the pivotal investors to invest in the MBS market. Since
all investors are aware of the offer, they know that the aggregate investment will increase and hence also have more incentive to invest.

2.6.3. Shopping Mall Investment

We analyze a real investment problem in this section. Pashigian and Gould (1998) documents the strategic complementarities among department stores in the same shopping mall. Specifically, department stores with reputations can bring in mall traffic and increase the sales of less-known stores. As discussed in Sakovics and Steiner (2012), the difference in reputation maps into $w_g$, the importance in coordination outcome of different groups in section 5.1.

Consider a newly opened shopping mall inviting different brands to open new stores. Since all stores benefit from customers’ visit to the shopping mall, all stores’ investment return increases in the occupancy ratio of the shopping mall. Therefore, coordination failure could lead to low occupancy ratio and failure of the shopping mall. In order to boost investment, according to our analyses, the shopping mall manager could offer an equity injection option. Specifically, if a brand accepts the equity injection offer and opens a new store in the shopping mall, the shopping mall manager pays part of the investment cost and receives proportional profit made by the store as a return. This offer is not appealing to the optimistic brands because they do not want to share the profits with the shopping mall. For brands that are around investment threshold, the equity injection offer reduces their investment risk and increases their expected payoff from the investment. Amplified by higher-order beliefs, all brands significantly lower their investment threshold. Moreover, in equilibrium, only the “pivotal” brands accept the offer. Therefore the resources to finance the intervention program are efficiently allocated.

It is reasonable to assume different brands have different profit functions. We have shown in section 5.2 that the interim critical agents who are around their own investment thresholds self-select to accept our offer. The result that the equity injection offer effectively reduces
coordination failure and incurs low financing cost for the shopping mall owner still holds.

2.7. Conclusions

In this paper, we analyze a canonical coordination game under global games framework and propose a novel intervention program for a policy maker to reduce coordination failures. The intervention program screens for the marginal agents who receive medium signals, which reduces the cost of implementation for the program. At the same time, correctly incentivizing the marginal agents have a significant impact on all agents due to strategic complementarities and the amplification through higher order beliefs. In the limit of zero noise in agents’ private signals, our proposed program eliminates all coordination failures at zero cost since the expected mass of marginal investors goes to zero. Compared with conventional government guarantee type of programs, our proposed program not only incurs lower cost of implementation but also is shown to be more robust to moral hazard problems.

We demonstrate with three examples that our proposed program has a wide range of applications in improving coordination failures. As a concluding remark, we would like to point out some limitations of the proposed program. First, the program requires the policy maker to observe and condition the provision of the program on agents’ action choices, which might not be feasible. For example, in the context of panic-based currency attack, it is hard to trace the identities of the currency holders and give them an optional offer. Second, the effectiveness of the proposed program relies on agents’ rationality. If agents possess bounded rationality, the amplification effect through higher order beliefs will be limited.
A.1. Appendix to Chapter 1

A.1.1. Endogenizing Interbank Lending Contracts

Recall that the borrowing banks in the interbank capital market are subject to the regulatory constraint below.

\[ R_l a_i + 1 + x_i - a_i \geq c r x_i \] (A.1)

When there is insufficient regulatory requirement \((c < 1)\), banks might agree bilaterally to post additional collateral (i.e. increase \(c\)) to mitigate the counter-party risks,. To address this concern, in this section, I check the pairwise stability and see whether both the lending bank and the borrowing bank can be better-off with safer interbank contracts. I first show that without any friction, banks would voluntarily choose to use risk-free contracts for interbank lendings. However, with minimum cost of inspection associated with posting additional collateral, in the “bad equilibrium” with low interest rate, banks would stick to the risky contracts.

Below I consider the case without any friction and show that all banks choose to trade with risk-free contracts. Let \(c_{\text{min}}\) denote the exogenous minimum regulatory requirement. Banks can contract on additional collateral and achieve any \(c \in [c_{\text{min}}, 1]\). Given interest rate function \(r(c)\), for a bank with belief \(\pi\), let \(\{a^*(\pi, c), x^*(\pi, c)\}\) denote the bank’s portfolio choice if it trades with collateral level \(c\) and \(c^*(\pi)\) denote the bank’s choice of collateral level. In equilibrium, given interest rate function \(r(c)\), banks’ portfolio strategy \(\{a^*(\pi, c), x^*(\pi, c)\}\) and contract strategy \(c^*(\pi)\) must satisfy

1. (Individual Rationality) \(a^*(\pi, c)\) and \(x^*(\pi, c)\) solves the portfolio choice problem for a bank with belief \(\pi\) on interbank capital market \(c\).

2. (Pairwise Stability) For any two banks \(i \neq j \in [0, 1]\), there is no \(c\) such that
(a) \( x^*(\pi,i,c)x^*(\pi,j,c) < 0; \)

(b) \( U(a^*(\pi,i,c),x^*(\pi,i,c)|\pi,i,c) \geq U(a^*(\pi,i,c^*(\pi_i)),x^*(\pi_i,c^*(\pi_i))|\pi,i,c^*(\pi_i)); \)

(c) \( U(a^*(\pi,j,c),x^*(\pi,j,c)|\pi,j,c) \geq U(a^*(\pi,j,c^*(\pi_j)),x^*(\pi_j,c^*(\pi_j))|\pi,j,c^*(\pi_j)); \)

(d) at least one of (b) and (c) holds with strict inequality.

where \( U(a,x|\pi,c) = \pi(R_ha - rx) + (1 - \pi)(R_la - rx) + 1 + x - a \) is the expected payoff function for a bank with belief \( \pi \) and portfolio choice \( \{a,x\} \) on interbank capital market \( c \).

It turns out that the only market with positive trading volume is the risk-free interbank capital market with collateral requirement \( c = 1 \), which is summarized in Proposition 17 below.

**Proposition 17** In equilibrium, all risky interbank capital markets \( (c < 1) \) have zero trading volume.

The figure below plots a bank’s expected payoff as a function of its belief, and the red line highlights its optimal choice. For the optimistic banks with high \( \pi \), they borrow in the interbank capital markets and are indifferent between risky and safe contracts. However, for the lending banks with low \( \pi \), they strictly prefer to lend with the safe contract which optimizes risk allocation in the financial system. Intuitively, trading on interbank capital markets improves risk allocation. Since the lending banks believe in a high probability of economic recession, it is more costly for them to bear risks than the borrowing banks with optimistic beliefs. Through interbank transactions, the borrowing banks effectively lever up their risk-taking by promising the safer cash flows backed by collaterals to the lending banks. When the collateral requirement \( c \) decreases, each borrowing bank is able to take a higher leverage and earn higher expected payoff, the interest rate representing the cost of borrowing surges at the same time because the lending banks demand higher promised return to compensate for the risk. The risk-free interbank capital market with
\( c = 1 \) allows the borrowing banks to bear all the risks and the lending banks to be free from risks, which optimizes the risk allocation and maximizes the total surplus. Therefore, the risk-free interbank capital market is the only active market in equilibrium.

In a frictionless world, all banks choose to use risk-free contracts backed by sufficient collateral, and hence no bank defaults in equilibrium. However, in reality, unsecured interbank lending does exist. Moreover, even for collateralized wholesale funding market like asset-backed securities (ABS) and mortgage-backed securities (MBS), defaults do occur indicating that these securities are undercollateralized and not risk-free. Below I introduce the cost of inspection to the lenders in the interbank capital market and demonstrate that in the bad equilibrium with interbank capital market freeze, banks optimally choose to trade with risky contracts. Note that as long as writing safe contracts is more costly than writing risky contracts, the result goes through. Since for each unit of interbank lending, safe contract requires more collateral, it incurs a higher cost associated with inspecting the quality of the collateral. Especially for opaque and nonstandard assets, in order to make sure the value of the collateral is sufficient in recessions, the cost of inspection and monitoring is not negligible.

Specifically, lenders in the interbank capital market need to pay a cost of inspection \( I(c) \) for each unit lent when they require additional collateral, where \( I(c) > 0 \ \forall c > c_{\text{min}} \) and \( I(c_{\text{min}}) = 0 \). In the bad equilibrium with interbank capital market freeze, all banks are
relatively pessimistic about the coordination outcome, and very few banks borrow in the interbank capital market resulting in a low interest rate. Therefore, in the bad equilibrium, borrowing banks extract almost all surplus from the interbank trading. As long as the safe interbank contract incurs higher cost of inspection than risky interbank contract, the total surplus from trading is lower for safe interbank lending. Hence, in the bad equilibrium with low cost of borrowing in the interbank capital markets, borrowing banks can extract more surplus from risky interbank lending than safe interbank lending. Recall from section 5 that without inspection cost, for interbank capital market with collateral level $c$, the lower bound for interest rate is $r(c) = \frac{R_h - R_l}{c(R-1) + 1 - R_l}$. Proposition 18 below states that when the interest rate for interbank $c_{min}$ is low enough, all other interbank capital markets with additional collateral requirement are crowded out.

**Proposition 18** When $r(c_{min}) \rightarrow r(c_{min})$, banks lend with risky contracts with collateral level $c_{min}$.

The figure below plots the banks’ expected payoffs as a function of their beliefs. The dashed line represents the expected payoff of a lender on interbank capital market $c > c_{min}$ when there is no inspection cost. Without inspection cost, additional collateral improves risk allocation between the lending banks and the borrowing banks, and banks always prefer to lend with additional collateral. However, in the bad equilibrium with low interest rates, the surplus extracted by the lending banks goes to zero. As long as the inspection cost is
positive, lending on interbank capital markets with additional collateral requirements are dominated by safe storage or lending without additional collateral.

A.1.2. Demand Noise on the Interbank Capital Market

In this appendix, I show how the noise on the public signal can be micro-founded by demand noise in the interbank capital market.

With the demand noise in the interbank capital market \( \epsilon_D(r, \epsilon_p) \), the aggregate demand depends on the realization of the demand shock \( \sigma_p \), and the market-clearing condition is re-written as follow,

\[
D_N(\theta, r, \epsilon_p) = D(\theta, r) + \epsilon_D(r, \epsilon_p) = 0,
\]

where \( D(\theta, r) = \int_{-\infty}^{\infty} x(\pi(\theta + \sigma_s \epsilon_s, r))\phi(\epsilon_s)d\epsilon_s \) is the aggregate rational demand by the banks. I follow Angeletos and Werning (2006) and Hellwig et al. (2006), and assume the following functional form of the demand noise \( \epsilon_D(r, \epsilon_p) \),

**Assumption 4** The demand noise in the interbank capital market takes the following functional form,

\[
\epsilon_D(r, \epsilon_p) = \Phi\left(h(d, r) + \frac{\sigma_p \epsilon_p}{\sigma_s} + d\right) - \frac{rc}{r_c - R_l} \Phi\left(h(d, r) + \frac{\sigma_p \epsilon_p}{\sigma_s}\right)
\]

where \( d = \frac{\Phi^{-1}(\pi_{\epsilon}^2) - \Phi^{-1}(\pi_{\epsilon}^1)}{\sqrt{\delta}} \), \( \epsilon_p \sim N(0, 1) \) and \( h(d, r) \) is an implicit function such that

\[
\frac{\Phi(h(d, r))}{\Phi(h(d, r) + d)} = 1 - \frac{R_l}{rc}.
\]

Below I demonstrate how the noisy demand \( \epsilon_D(r, \epsilon_p) \) introduces noise on the public information revealed by the market-clearing interest rate \( r \). Recall that without frictions, the market-clearing interest rate conveys perfect information about the realization of the
fundamentals $\theta = z(r)$, which is the unique solution to
\[
D(z(r), r) = \frac{R_l}{rc - R_l} \left( 1 - \Phi \left( \frac{s_2^*(r) - \theta}{\sigma_s} \right) \right) - \left( \Phi \left( \frac{s_2^*(r) - \theta}{\sigma_s} \right) - \Phi \left( \frac{s_1^*(r) - \theta}{\sigma_s} \right) \right) = 0.
\]

With the implicit function $h(d, r)$ defined in Assumption 4, we can express the perfect information $z(r)$ as follows,
\[
z(r) = s_2^*(r) + \sigma_s h \left( \frac{s_2^*(r) - s_1^*(r)}{\sigma_s}, r \right).
\]

Then it can be easily verified by substituting $\theta = z(r) + \sigma_p \epsilon_p$ in equation A.2 that
\[
D_N(z(r) + \sigma_p \epsilon_p, r, \epsilon_p) = 0.
\]

In other words, observing interest rate $r$, banks infer that the fundamentals $\theta = z(r) + \sigma_p \epsilon_p$.

As mentioned in section 1.4.2, when the interest rate is high such that interbank lending is guaranteed, it corresponds to a special case that the lower belief cutoff to become a lending bank $\pi_1^*(r) = 0$, and the lower signal cutoff $s_1^*(r) = -\infty$. Substituting in $\pi_1^*(r) = 0$, the expression for the demand noise $\epsilon_D(r, \epsilon_p)$ in Assumption 4 can be simplified as
\[
\epsilon_D(r, \epsilon_p) = 1 - \frac{rc}{rc - R_l} \Phi \left( \Phi^{-1} \left( 1 - \frac{R_l}{rc} \right) + \frac{\sigma_p}{\sigma_s} \epsilon_p \right).
\]

Similarly, when strict regulations ($c = 1$) make sure that interbank capital market is risk-free, the demand noise follows the above simplified form.

One final remark for introducing demand noise in the interbank capital market is that the coordination outcome depends on the realization of the demand shock. Therefore, given realized fundamentals $\theta$, we can only predict the probability of an economic expansion. Even so, it can be easily verified that the probability of an economic expansion increases in $\theta$. Nonetheless, in this paper, I focus on the limiting case when $\sigma_p \to 0$, which gives
\[
\lim_{\sigma_p \to 0} \epsilon_D(\epsilon_p, r) = 0 \quad \forall r \in [1, R).
\]

Therefore, in the limit, the coordination result becomes
A.1.3. Non-Negligible Public Noise

In this appendix, I briefly discuss the equilibrium in the non-limiting case when the noise on the public signal $\sigma_p > 0$, and demonstrate that the model is smooth in $\sigma_p$ such that results hold in general in the non-limiting case.

I first consider the risky interbank capital market case in section 1.4. Figure 15 plots the fundamental threshold $\theta^*(r)$ (red line) and the market-clearing interest rate $r^*(\theta)$ (blue). The general expressions for the two functions can be found in the proof of Proposition 2. Panel (a) is the non-limiting case when the noise on the public signal $\sigma_p > 0$, while panel (b) is the limiting case discussed in the main text. Given any interest rate $r$, we can find the corresponding fundamentals that clears at this rate from the blue line representing $r^*(\theta)$, and the fundamental threshold from the red line representing $\theta^*(r)$. Therefore, if the blue line lies above the red line at fundamentals $\theta$, the economy is in an expansion with profitable credit supply to the real sector.

![Figure 15: Equilibrium (Risky Interbank Capital Market)](image)

Note that when the public noise is non negligible (panel a), if $\theta$ is in the middle region, there
can be three different market-clearing interest rates. Figure 16 plots the demand and supply curve for a given realization of $\theta$ in the multiple equilibria region. Although there are three possible market-clearing interest rate, point B is unstable in the sense that if interest rate increases, there are more demand and less supply pushing the rate even higher. Moreover, it converges to C in the limit. In panel (a) of Figure 15, the “hump” on the left gets narrower as the the noise $\sigma_p$ vanishes and disappears in the limit. Therefore, I focus on the two stable equilibria, i.e. point A and C. Point A corresponds to the efficient equilibrium in which interbank capital flows are active and the interest rate is high, resulting in high aggregate credit supply and economic expansion. In contrast, point C corresponds to the inefficient equilibrium in which interbank capital market freezes and the interest rate is low, resulting in low aggregate credit supply and economic recession.

Figure 16: Demand/Supply Curve (Risky Interbank Capital Market)

Figure 17 plots the case when interbank lending is risk-free. Panel (a) displays the general results when the demand noise is non-negligible, while panel (b) corresponds to the limiting case.

Parameter values: $\theta = 0.2$, $c = 0.8$, $R_l = 1.3$, $\delta = 0.01$, $\sigma_s = 0.5$
In the general case, as long as $\theta > 0$, there exists an equilibrium in which the interest rate $r \in (1, R_h)$, and abundant credit supply supports an economic expansion. While for $\theta \leq \theta^*(1)$, there exists another equilibrium with interest rate $r = 1$, and the shortage of credit supply leads to an economic recession.

![Figure 17: Equilibrium (Risk-Free Interbank Capital Market)](image)

A.1.4. Extra Financial Regulations: $c > 1$

This section analyzes the case in which banks face extra financial regulations. Figure 18 below presents the optimal portfolio choice of a bank given its belief $\pi(s_i, r)$. Compared with the risk-free interbank capital market case ($c = 1$), a bank faces tighter leverage constraint and therefore is able to borrow less. In other words, the extra regulations effectively increase the cost of borrowing in the interbank capital markets. In particular, when the interest rate $r = 1$, the optimistic banks will not borrow to invest in the safe storage, and their portfolio choice is deterministic. However, the pessimistic banks are still indifferent between interbank lending and the safe storage. As a result, similar to the case with $c = 1$, an interest rate $r = 1$ conveys a vague public signal about the fundamental.
Market clearing condition in the interbank capital market, the interbank capital market rate \( r \) serves as a public signal with normally distributed noise, i.e.

\[
z(r) = \theta - \sigma_p \epsilon_p \quad \text{if} \quad r \in (1, R_h),
\]

\[
z(1) \geq \theta - \sigma_p \epsilon_p \quad \text{if} \quad r = 1,
\]

where \( z(r) = s^*(r) - \sigma_s \Phi^{-1} \left( \frac{R_l}{\sigma_r} \right) \).

When the interest rate \( r \in (1, R_h) \), all banks’ budget constraints are binding. In other words, although each optimistic bank borrows less with stringent regulatory requirement, in aggregate, all capital available in the financial system are lent out to the real sectors through interbank trading. Hence, in the limit of vanishing demand noise in the interbank capital market, the aggregate lending \( A = 1 \) and therefore the fundamental threshold \( \theta^* = 0 \) which is the first-best outcome.

When the interest rate \( r = 1 \), pessimistic banks who receive negative signals about the fundamentals \( \theta \) are indifferent between lending in the interbank capital market and the safe storage technology. Similar to the case with just sufficient regulatory requirement, fundamentals \( \theta \in (-\infty, \bar{\theta}) \) can clear at \( r = 1 \) in equilibrium, while \( \bar{\theta} \) is higher under extra regulatory requirement. The reason is that when fundamentals \( \theta \) are high, a large mass of banks receive positive signals and are willing to borrow in the interbank capital market generating an upward pressure on the interest rate \( r \). Under extra regulatory requirement, each
optimistic bank is allowed to borrow less which limits the upward pressure on the interest rate, and hence the interbank capital markets can clear at \( r = 1 \) for higher realizations of fundamentals \( \theta \). As a result, an interest rate \( r = 1 \) conveys more positive information about the fundamentals \( \theta \). In summary, although the extra regulatory requirement limit the borrowing capacity of each optimistic bank, and hence the allocative role of interbank capital markets, the information conveyed by the interest rate \( r = 1 \) becomes more positive which enhances all banks beliefs through the informative role. These two effects cancel out giving the same fundamental threshold \( \theta^* \). The proposition below summarizes the equilibrium in the limit of vanishing noise on the public signal.

**Proposition 19** In the limit of vanishing noise on the public signal, i.e. \( \sigma_p \to 0 \),

1. for any \( \theta > \bar{\theta}(c) \), \( \hat{r}(\theta) = R_h \), and the economy is in an expansion;

2. for any \( \theta \in [\theta_0^*, \bar{\theta}(c)] \), \( \hat{r}(\theta) \in \{R_h, 1\} \), and the economy is in an expansion regardless of the interest rate;

3. for any \( \theta \in (0, \theta_0^*) \), \( \hat{r}(\theta) \in \{R_h, 1\} \), the economy is in an expansion if \( \hat{r}(\theta) = R_h \), and the economy is in a recession if \( \hat{r}(\theta) = 1 \);

4. for any \( \theta \leq 0 \), \( \hat{r}(\theta) = 1 \), and the economy is in a recession;

where \( \theta_0^* = \frac{1 - R_l}{R_h - R_l} \) and \( \bar{\theta}(c) = \frac{1 - R_l}{R_h - R_l} + \sigma_s \left[ \Phi^{-1} \left( \frac{R_l}{c} \right) + \left( 1 - \frac{R_l}{R_h - R_l} \right) \Phi^{-1} \left( \frac{R_l}{c} \right) \right] \) which increases monotonically in \( c \geq 1 \). The fundamental threshold function \( \lim_{\sigma_p \to 0} \theta^*(r) \) is the same for any \( c \geq 1 \).

Under extra regulatory requirement \((c > 1)\), the fundamental threshold function \( \lim_{\sigma_p \to 0} \theta^*(r) \) is the same as in the case when \( c = 1 \). Therefore, in terms of aggregate coordination outcome or the return of bank lending to the real sectors, extra regulations seems costless. However, they reduce efficient lending and raise inefficient lending for all realizations of fundamentals \( \theta \). When the interest rate \( \hat{r}(\theta) = R_h \), all capital held by the pessimistic banks are channeled through interbank capital market to the optimistic banks, and hence the aggregate lending

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by the banking system \( A = 1 \). The proposition below summarizes the welfare implication when \( \hat{r} = 1 \) under extra regulatory requirement \((c > 1)\).

**Proposition 20** In the limit of vanishing noise trading, i.e. \( \sigma_p \to 0 \), in the equilibrium with interest rate \( \hat{r}(\theta) = 1 \),

- when \( \theta \in (-\infty, \theta^*_0) \), the economy is in a recession, and the inefficient lending \( A \) strictly increases in \( c \) for all \( c \geq 1 \);
- when \( \theta \in (\theta^*_0, \bar{\theta}(1)) \), the economy is in an expansion, and the efficient lending \( A \) strictly decreases in \( c \) for all \( c \geq 1 \).

To summarize, although the aggregate state of the economy is the same, extra financial regulations imposes welfare loss in the equilibrium with \( r = 1 \) for all realizations of fundamentals \( \theta \).

**A.1.5. Analysis of Short Selling Restrictions**

As mentioned in Section 1.6.1, if banks are restricted from short selling as in the main model, the equilibrium implications follow Proposition 4 in the risk-free interbank capital market case.

Below I relax the short selling constraint by allowing \( a_i \) to be negative and compare the coordination outcomes with the case when short selling is restricted. In particular, given its belief \( \pi \), bank \( i \) solves the portfolio choice problem below,

\[
\max_{\{a_i, x_i\}} \pi(R_h a_i - r x_i) + (1 - \pi)(R_l a_i - r x_i) + 1 + x_i - a_i
\]

s.t. \( a_i \leq 1 + x_i \) \hspace{1cm} (Budget Constraint)

\[ R_l a_i + 1 + x_i - a_i \geq r x_i \] \hspace{1cm} (Regulatory Constraint - Borrower)

\[ R_h a_i + 1 + x_i - a_i \geq r x_i \] \hspace{1cm} (Margin Constraint - Short Seller)
Note that when short selling is allowed, the short sellers may default if the asset lives through the fire sales and generates a high return $R_h$. To eliminate the counter-party risks and isolate the effect of short selling, I impose a margin constraint on the short sellers to make sure they don’t default when they make a wrong bet.

Figure 19 below characterizes the optimal portfolio choice of a bank given its belief $\pi$. In comparison to the case when short selling is restricted, as plotted in Figure 4, the optimistic banks don’t short sell, and their portfolio choices remain the same. Specifically, they borrow in the interbank capital market and invest all available capital in the financial asset. However, the pessimistic banks with low private signals choose to speculate by short selling. They lend in the interbank capital market and use their interbank lending as margin to short sell the financial assets.

When the interest rate $r \in (1, R_h)$, short selling activities have no aggregate effect on the financial market. The reason is that the short sellers also lend in the interbank capital market, and their capital is eventually invested in the financial asset by the optimistic banks, which cancels out their short selling practices. To see it more clearly, since all banks budget constraint binds, i.e., $a_i = 1 + x_i$, the market-clearing condition in the interbank capital market implies that the aggregate investment in the financial asset $A = 1$ in equilibrium.

However, if the interest rate $r = 1$, the pessimistic banks, which short sell the financial asset, may invest in the safe storage technology. In this case, short selling can have real
impacts on the financial market. Proposition 21 below summarizes the destabilizing effect of short selling activities.

**Proposition 21** When short selling is allowed, in the limit of vanishing interpretation noise, there exists an equilibrium in which for any \( \theta \in (\theta_0^*, \theta_1^*) \), the interest rate \( \hat{r}(\theta) = 1 \) and the financial market melts down, where \( \theta_1^* = \frac{R_h(1-R_l)}{(R_h-1)(R_h-R_l)} > \theta_0^* \).

Recall that if short selling is restricted, as shown in the lower panel of Figure 5, as long as fundamentals \( \theta > \theta_0^* \), the economy is in an expansion, which corresponds to a healthy financial market in this section. However, if short selling is allowed, when the fundamentals \( \theta \in (\theta_0^*, \theta_1^*) \), there is a possibility of financial market meltdown due to fire sale activities. Intuitively, short selling activities directly aggravate the fire sale pressure by reducing the aggregate investment \( A \). On top of that, aware of this direct effect, banks observing a low interest rate \( r = 1 \) become more pessimistic about the health of the financial market. Therefore, the mass of pessimistic banks that short sell the financial asset increases, which further intensifies the fire sale pressure. These two effects reinforce each other and jointly lead to a meltdown of the financial market for higher fundamentals \( \theta \). Hence, short selling restrictions not only directly reduce fire sales but also maintain the confidence of banks investing in the financial asset, which in aggregate stabilizes the financial market.

A.1.6. Analysis of Credit Default Swaps

**Naked CDS Banned**

When naked CDS on sovereign debt is banned, I call the market a covered CDS market. In this subsection, I establish the equivalence between the covered CDS market and the risk-free interbank capital market analyzed in section 1.5. Below is the portfolio choice
problem for investor $i$ given its belief $\pi$.

$$\max_{\{a,x\}} \pi R_h a'_i + (1 - \pi)(R_l a'_i - x'_i) + 1 + px'_i - a'_i$$

s.t. $a'_i \leq 1 + px'_i$ \hspace{1cm} (Budget Constraint)

$$1 + (R_l - 1)a'_i + px'_i \geq x'_i$$ \hspace{1cm} (Regulatory Constraint)

$$a'_i(R_h - R_l) + x' \geq 0$$ \hspace{1cm} (Covered Constraint)

$$a'_i \geq 0$$ \hspace{1cm} (Short-Sell Constraint)

It turns out that there is a one-to-one mapping between investors’ portfolio choice and banks’ portfolio choice in the risk-free interbank capital market case. Specifically, a risk-free covered portfolio can be constructed by investing in sovereign debt and purchasing CDS contracts in proportion to hedge the risk. The upper bound for the price of CDS contract, $\bar{p} = \frac{R_h - 1}{R_h - R_l}$, is such that the covered portfolio generates a return of one, such that investors are indifferent between the covered portfolio and the safe storage. As shown in Figure 20, if the price of CDS contract is low $p \in (0, \bar{p})$, the covered portfolio generates higher returns than the safe storage, and the pessimistic investors invest their endowed capital in the covered portfolio. The optimistic investors, on the other hand, sell CDS contracts, then invest their endowed capital and the proceeds from selling CDS contracts in the sovereign debt. Effectively, the optimistic investors are borrowing from the pessimistic investors to relax their budget constraint and invest more in the sovereign debt. In this sense, the CDS market is playing a similar allocative role as the interbank capital market in channeling funds from the pessimistic investors to the optimistic investors. When the price of CDS contract is high $p = \bar{p}$, the pessimistic investors are indifferent between the safe storage and the risk-free covered portfolio. From the optimistic investors’ perspective, the profit from selling CDS contract is so high that they are indifferent between investing directly in the sovereign debt and selling CDS contract. In fact, if an investor makes the portfolio choice $\{a', x'\} = \{a - \frac{r}{R_h}x, \frac{(R_h - R_l)r}{R_h}x\}$ given the price of a CDS contract $p = \frac{R_h - r}{(R_h - R_l)r}$, she receives
the same payoff as investing $a$ and borrows $x$ at interest rate $r$ in the bank’s problem. Hence, the covered CDS market is equivalent to a risk-free interbank capital market in terms of portfolio choice and the allocative role.

The information structure also follows the interbank capital market model. Besides its private signal $s_i$, investor $i$ also observes the market clearing CDS price $p$, which clears the CDS market and reveals public information. Since the optimal portfolio choice is equivalent to that in the interbank capital market model, there is also a one-to-one mapping for the market-clearing condition. Following that, with the same interpretation noise when investors extract information from the market-clearing CDS price, we can figure out the one-to-one mapping for the belief-updating process of the investors. Therefore, the covered CDS market is also equivalent to a risk-free interbank capital market in the informative role. With the equivalence in both resource allocation and information revelation, the equilibrium implications described in Proposition 3-5 hold for the covered CDS market. The upper panel in Figure 8 summarizes the equilibrium coordination outcome.

**Naked CDS Allowed**

If naked CDS is allowed, that is, if the covered constraint is removed from investors’ portfolio choice problem, the CDS market is no longer equivalent to an interbank capital market. In particular, the pessimistic investors can speculate against the solvency of the government by investing in naked CDS contracts.
Investors’ optimal portfolio choice is summarized in Figure 21. In contrast to the previous case, the pessimistic investors now choose to speculate by purchasing CDS contracts without investing in the sovereign debt. For the optimistic investors, when the price of CDS contracts is low, such that \( p \in (0, \bar{p}) \), they make the same portfolio choice as in the covered CDS market case displayed in Figure 20. Specifically, they sell CDS contracts to relax their budget constraints, then invest their endowed capital and the proceeds from selling CDS contracts in sovereign debt. Therefore, the CDS market plays an active allocative role in channeling all capital from the pessimistic investors to the optimistic investors and boosting the investment in the sovereign debt. Hence, speculations by the pessimistic investors do not threaten the solvency of the government when the price of CDS contracts is low.

\[
\begin{align*}
\pi^*(p) &= \frac{1}{1 + (R_h - R_l)p} \\
\pi'(p) &= \frac{1 - R_l}{R_h - R_l} \\
\pi'(p) &= 1 - p
\end{align*}
\]

(a) \( p \in (0, \bar{p}) \)

(b) \( p = \bar{p} \)

(c) \( p \in (\bar{p}, 1) \)

Figure 21: Portfolio Choice (Naked CDS Allowed)

The starkest contrast to the previous case is that the price of CDS contracts can rise above \( \bar{p} \). At such high CDS price, the optimistic investors find it more profitable to speculate by selling CDS contracts without investing in the risky sovereign debt. As a result, all investors spend all their endowment on speculation based on their private beliefs, resulting in zero aggregate investment in the sovereign debt. In other words, the speculation activities completely crowd out the sovereign debt investment. Proposition 22 below summarizes the destabilizing effect of naked CDS trading on the solvency of the government.
Proposition 22  If naked positions are allowed, in the limit of vanishing interpretation noise, there exists an equilibrium in which for any $\theta \in (\theta_0^*, 1)$, the CDS price $p = 1$ and the sovereign debt defaults.

As shown in the lower panel of Figure 8, without the ban on naked CDS contracts, when fundamentals $\theta \in (\theta_0^*, 1)$, naked CDS trading harms the sovereign stability in the sense that it creates the possibility that all investors, pessimistic and optimistic, speculate based on their beliefs, which crowds out the sovereign debt market. Therefore, the ban of such abusive speculations with naked CDS enhances the sovereign stability.

A.1.7. Proofs

Proof of Proposition 1.

Given a bank’s belief of the probability that bank lending to the real sector is profitable $\pi$, the bank solves the portfolio choice problem below.

$$\max_{a \in [0,1]} \pi (R_h a + 1 - a) + (1 - \pi) (R_l a + 1 - a)$$

Therefore, a bank will optimally chooses $a = 1$ if its belief $\pi \geq \frac{1 - R_l}{R_h - R_l}$ and $a = 0$ otherwise.

Next, I will prove the uniqueness of equilibrium by iterated deletion of dominated strategies. Strategies survive $n$ rounds of iterated deletion of dominated strategies if and only if

$$a(s) = 0, \text{ if } s < \underline{s}_n,$$

$$\text{and } a(s) = 1, \text{ if } s \geq \overline{s}_n. \quad (A.3)$$

where $\{(\underline{s}_n, \overline{s}_n)\}_{n=0}^{\infty}$ satisfies

$$-\infty = \underline{s}_0 < \underline{s}_1 \leq \cdots \leq \underline{s}_n \leq \cdots \leq \overline{s}_n \leq \cdots \leq \overline{s}_1 < \overline{s}_0 = +\infty. \quad (A.5)$$

This result can be proven by induction. Let $\underline{s}_0 = -\infty$ and $\overline{s}_0 = +\infty$. Therefore, the first
round of deletion starts with the full set of strategies. Let \( \pi(s; k) \) denote the belief of a bank with private signal \( s \) when all other banks follow a cutoff strategy \( k \),

\[
\pi(s; k) = Pr(\theta > \theta^*_k | s) = \Phi \left( \frac{s - \theta^*_k}{\sigma_s} \right)
\]

where \( \theta^*_k \) is the fundamental threshold when all banks follow the cutoff strategy \( k \) and satisfies \( \Phi \left( \frac{k - \theta^*_k}{\sigma_s} \right) = \theta^*_k \). Suppose round \( n \in \mathbb{N} \) of deletion has been completed. In round \( n+1 \), the best scenario for a bank to lend is that all other banks follow a cutoff strategy with threshold \( s_n \). Therefore, for any \( s \) such that \( \pi(s; s_n) < 1 - R_l \), \( a(s) = 0 \) is the dominate strategy. Similarly, the worst scenario for a bank to choose \( a(s) = 1 \) is that all other banks follow a cutoff strategy with threshold \( \bar{s}_n \). As a result, for \( s \) such that \( \pi(s; \bar{s}_n) > \frac{1-R_l}{R_h-R_l} \), \( a(s) = 1 \) is the dominant strategy.

Given that \( \pi(s; k) \) is non-decreasing in \( s \), the strategy profiles that survives deletion of dominated strategies can be summarized in the form of (A.19)(A.20), with \( (s_{n+1}, \bar{s}_{n+1}) \) defined inductively as

\[
s_{n+1} = \inf \left\{ s : \pi(s; s_n) \geq \frac{1-R_l}{R_h-R_l} \right\} \tag{A.6}
\]

and

\[
\bar{s}_{n+1} = \sup \left\{ x : \pi(x; \bar{s}_n) \leq \frac{1-R_l}{R_h-R_l} \right\} \tag{A.7}
\]

The monotonicity of \( \pi(s; k) \) guarantees that \( s_{n+1} \leq \bar{s}_{n+1} \) given \( s_n \leq \bar{s}_n \). Note that the dominance region assumption implies that \( s_1 > -\infty \) and \( \bar{s}_1 < +\infty \). Therefore, \( \{(s_n, \bar{s}_n)\}_{n=0}^{\infty} \) is a well-defined sequence of real couple which satisfies (A.21).

Now we’ve proved that \( \{s_n\}_{n=1}^{\infty} \) and \( \{\bar{s}_n\}_{n=1}^{\infty} \) are both monotonic and bounded sequences. Thus, they converges to two finite numbers \( \underline{s} \) and \( \bar{s} \) respectively when \( n \to \infty \). And the two
limits satisfy
\[ \underline{s} \leq \bar{s}. \]  \hspace{1cm} (A.8)

The definition (A.22)-(A.23) implies that \( \pi(s; \underline{s}) \geq 1 - R_l R_h - R_l \) and \( \pi(s; \bar{s}) \leq \frac{1-R_l}{R_h - R_l} \). Note that
\[ \pi(s; s) = \Phi \left( \frac{s - \theta^* s}{\sigma s} \right) = \theta^* s, \]  \hspace{1cm} (A.9)
is strictly increasing in \( s \). Therefore \( \underline{s} = \bar{s} = s_0^* \) must be the unique solution to \( \theta^*(s) = \frac{1-R_l}{R_h - R_l} \), which is
\[ s_0^* = \frac{1-R_l}{R_h - R_l} + \sigma s \Phi^{-1} \left( \frac{1-R_l}{R_h - R_l} \right). \]  \hspace{1cm} (A.10)

Since there’s only one strategy that survives the iterated deletion of dominated strategies, the equilibrium of the game is unique and the associated equilibrium strategy is the cutoff strategy with threshold \( s_0^* \). \( \blacksquare \)

**Proof of Lemma 1.** If bank \( i \) chooses to become a borrower \( (x_i \geq 0) \), its portfolio choice problem given its belief \( \pi_i \) is
\[
\max_{\{a,x\}} \mathbb{E}(u) = \pi_i \max \{0, R_h a_i + 1 + x_i - a_i - r x_i \} + (1 - \pi_i) \max \{0, R_l a_i + 1 + x_i - a_i - r x_i \}
\]
\[
s.t. \quad a_i \leq 1 + x_i \quad \text{(Budget Constraint)}
\]
\[
R_l a_i + 1 + x_i - a_i \geq c r x \quad \text{(Regulatory Constraint)}
\]
\[
a_i \geq 0 \quad \text{(Short-Sell Constraint)}
\]
\[
x_i \geq 0 \quad \text{(Borrowing Constraint)}
\]

Borrower’s portfolio choices can be broken down into three cases depending on the default outcome. The first case is when \( R_h a_i + 1 + x_i - a_i - r x_i < 0 \), i.e. borrower \( i \) defaults in both
expansions and recessions. In this case, the expected payoff of a borrower is zero, which is strictly dominated by \( \{a, x\} = \{0, 0\} \) with an expected payoff of one. Hence, a borrower will never choose a portfolio that always defaults.

The second case is when \( R_l a_i + 1 + x_i - a_i - rx_i \geq 0 \), i.e. borrower \( i \) pays back in full in both expansions and recessions. Under this extra constraint, the expected payoff function can be written as \( \mathbb{E}(u) = (\pi_i R_h + (1 - \pi_i) R_l) a_i + 1 + x_i - a_i - rx_i \), and the optimal portfolio choice is

\[
\begin{align*}
    a_i &= 0 \text{ and } x_i = 0, \text{ if } \pi_i < \frac{1 - R_l}{R_h - R_l}, \\
    a_i &= 1 \text{ and } x_i = 0, \text{ if } \pi_i \in \left( \frac{1 - R_l}{R_h - R_l}, \frac{r - R_l}{R_h - R_l} \right), \\
    a_i &= \frac{r}{r - R_l} \text{ and } x_i = \frac{R_l}{r - R_l}, \text{ if } \pi_i > \frac{r - R_l}{R_h - R_l}.
\end{align*}
\]

Hence, if bank \( i \) never defaults, it will borrow only when \( \pi_i \geq \frac{r - R_l}{R_h - R_l} \), and its expected payoff is \( \mathbb{E}(u) = \pi_i \frac{r(R_h - R_l)}{r - R_l} \).

The last case is when \( R_l a_i + 1 + x_i - a_i - rx_i < 0 \leq R_h a_i + 1 + x_i - a_i - rx_i \), i.e. borrower \( i \) defaults in recessions and pays back in full in expansions. In this case, the expected payoff function can be written as \( \mathbb{E}(u) = \pi_i(R_h a_i + 1 + x_i - a_i - rx_i) \). Note that in this case, the optimal portfolio choice is independent of bank \( i \)'s belief. Bank \( i \) optimally chooses \( \{a_i, x_i\} = \left\{ \frac{cr}{cR_l - R_l}, \frac{R_l}{cR_l - R_l} \right\} \) and achieves expected payoff \( \mathbb{E}(u) = \pi_i \frac{r(R_h c - R_l)}{cR_l - R_l} > \pi_i \frac{r(R_h - R_l)}{r - R_l} \), i.e. higher than that achieved when bank \( i \) never defaults.

Hence, in equilibrium, all borrowing banks choose \( \{a_i, x_i\} = \left\{ \frac{cr}{cR_l - R_l}, \frac{R_l}{cR_l - R_l} \right\} \) and default only in recessions. When borrowing bank \( i \) defaults, it pays back the lender in the interbank capital market \( \frac{R_l a_i + 1 + x_i - a_i}{x_i} = cr \) for each dollar borrowed. Therefore, all lending banks expect to receive full promised return \( r \) when firm loans are profitable and \( cr \) otherwise.  

**Proof of Lemma 2.** Banks’ strategy functions are uniquely characterized by equation 1.1-1.8. Below I characterize the other elements of the equilibrium. The information contained
from market-clearing interest rate $r$ is $z(r)$, which is characterized by

$$D(z(r), r) = \frac{R_l}{rc - R_l} \left( 1 - \Phi \left( \frac{s_2^*(r) - \theta}{\sigma_s} \right) \right) - \left( \Phi \left( \frac{s_2^*(r) - \theta}{\sigma_s} \right) - \Phi \left( \frac{s_1^*(r) - \theta}{\sigma_s} \right) \right) = 0.$$ 

We can express the information revelation function $z(r)$ as follows,

$$z(r) = s_2^*(r) + \sigma_s h \left( \frac{s_2^*(r) - s_1^*(r)}{\sigma_s}, r \right), \tag{A.11}$$

where $h(d, r)$ is an implicit function such that

$$\frac{\Phi(h(d, r))}{\Phi(h(d, r) + d)} = 1 - \frac{R_l}{rc}.$$ 

Given bank $i$’s private signal $s_i$ and the public signal $z(r)+\sigma_p\epsilon_p$, $\theta$ follows a normal posterior distribution with mean $\mu(s_i, r) = \delta s_i + (1 - \delta)z(r)$ and standard deviation $\sigma = \sqrt{\delta} \sigma_s$. Therefore, the beliefs of the marginal banks who receives signal $s_2^*(r)$ and $s_1^*(r)$ are

$$\pi(s_2^*(r), r) = \Phi \left( \frac{\mu(s_2^*(r), r) - \theta^*(r)}{\sigma} \right) = \pi_2^*(r)$$

$$\pi(s_1^*(r), r) = \Phi \left( \frac{\mu(s_1^*(r), r) - \theta^*(r)}{\sigma} \right) = \pi_1^*(r)$$

which implies

$$s_2^*(r) - \theta^*(r) = \sqrt{\delta} \Phi^{-1}(\pi_2^*(r)) - (1 - \delta)h \left( \frac{s_2^*(r) - s_1^*(r)}{\sigma_s}, r \right), \tag{A.12}$$

and

$$s_2^*(r) - s_1^*(r) = \frac{\Phi^{-1}(\pi_2^*) - \Phi^{-1}(\pi_1^*)}{\sqrt{\delta}}. \tag{A.13}$$

At the fundamental threshold $\theta^*(r)$, the aggregate investment

$$A(\theta^*(r), r) = \frac{cr}{cr - R_l} \left( 1 - \Phi \left( \frac{s_2^*(r) - \theta^*(r)}{\sigma_s} \right) \right) = 1 - \theta^*(r).$$
Plugging in equation A.12, we have

\[
\theta^*(r) = \frac{cr}{cr - R_l} \Phi \left[ \sqrt{\delta} \Phi^{-1} \left( \pi_2^*(r) \right) - (1 - \delta) h \left( \frac{\Phi^{-1} \left( \pi_2^*(r) \right) - \Phi^{-1} \left( \pi_1^* \right)}{\sqrt{\delta}}, r \right) \right] - \frac{R_l}{cr - R_l}.
\]

Hence, we can get the expression of signal threshold from equation A.12

\[
s_2^*(r) = \theta^*(r) + \sigma_s \left[ \sqrt{\delta} \Phi^{-1} \left( \pi_2^*(r) \right) - (1 - \delta) h \left( \frac{\Phi^{-1} \left( \pi_2^*(r) \right) - \Phi^{-1} \left( \pi_1^* \right)}{\sqrt{\delta}}, r \right) \right].
\]

From equation A.13, we can then get the expression for \( s_1^*(r) \)

\[
s_1^*(r) = s_2^*(r) - \sigma_s \frac{\Phi^{-1} \left( \pi_2^* \right) - \Phi^{-1} \left( \pi_1^* \right)}{\sqrt{\delta}}.
\]

Substituting in the expressions for \( s_1^*(r) \) and \( s_2^*(r) \), we can characterize \( z(r) \) that is uniquely defined in equation A.11. Therefore, substituting in the expression for \( z(r) \) and \( \theta^*(r) \), we can characterize the belief function as follows,

\[
\pi(s_i, r) = \Phi \left( \frac{\delta s_i + (1 - \delta) z(r) - \theta^*(r)}{\sigma} \right).
\]

\[\blacksquare\]

**Proof of Proposition 2.**

**Large interest rate** When \( r \in [c^{-1}, R_h] \), \( \pi_1^*(r) = 0 \) and \( s_1^*(r) = -\infty \). Hence, substituting in the expression of \( s_2^*(r) \), we can rewrite the market-clearing condition as follows,

\[
\theta = z(r^*(\theta)) = \theta^*(r^*(\theta)) + \sigma_s \left( \sqrt{\delta} \Phi^{-1} \left( \pi_2^* (r^*(\theta)) \right) - \delta \Phi^{-1} \left( \frac{R_l}{cr} \right) \right)
\]

where

\[
\theta^*(r) = \frac{cr}{cr - R_l} \Phi \left[ \sqrt{\delta} \Phi^{-1} \left( \pi_2^* (r) \right) + (1 - \delta) \Phi^{-1} \left( \frac{R_l}{cr} \right) \right] - \frac{R_l}{cr - R_l}.
\]
Note that $\lim_{\sigma_p \to 0} z(r) = 0 \forall r \in (r, R_h)$ and $\lim_{r \to R_h} z(r) = \infty$. In other words, for $\theta > 0$, there exists at least one market clearing rate $r^*(\theta) \in [c^{-1}, R_h)$. Next, I prove that the limit of the interest rate $\hat{r}(\theta) = \lim_{\sigma_p \to 0} r^*(\theta) = R_h$. That is for any $\epsilon > 0$, there exists a $\bar{\sigma}_p > 0$ such that $\forall \sigma_p < \bar{\sigma}_p$, $|R_h - r^*(\theta)| < \epsilon$, i.e. $r^*(\theta) > R_h - \epsilon$.

- $\forall r \in [1, R_h - \epsilon]$, $\lim_{\sigma_p \to 0} \theta^*(r) = 0$. Therefore, there exists a $\bar{\sigma}_1$ such that $\forall \sigma_p < \bar{\sigma}_1$, $|\theta^*(r)| < \frac{\theta}{2}$.

- Similarly, $\forall r \in [1, R_h - \epsilon]$, $\lim_{\sigma_p \to 0} \sigma_s \left( \sqrt{3} \Phi^{-1} (\pi_2^*(r)) - \delta \Phi^{-1} \left( \frac{R_l}{c_r} \right) \right) = 0$. Therefore, there exists a $\bar{\sigma}_2$ such that $\forall \sigma_p < \bar{\sigma}_2$, $\left| \sigma_s \left( \sqrt{3} \Phi^{-1} (\pi_2^*(r)) - \delta \Phi^{-1} \left( \frac{R_l}{c_r} \right) \right) \right| < \frac{\theta}{2}$.

Therefore, for any $\sigma_p < \bar{\sigma}_p = \min\{\bar{\sigma}_1, \bar{\sigma}_2\}$ and any $r \in [1, R_h - \epsilon]$, we have $z(r) < \theta$.

Moreover, since $z(r)$ is continuous and $\lim_{r \to R_h} z(r) = \infty$, the solution to market-clearing condition $r^*(\theta) > R_h - \epsilon$. Therefore, $\hat{r}(\theta) = \lim_{\sigma_p \to 0} r^*(\theta, \epsilon_p) = R_h$. Moreover, since $\theta^*(r) \geq 0$ if and only if $\sqrt{3} \Phi^{-1} (\pi_2^*(r^*(\theta))) \geq \delta \Phi^{-1} \left( \frac{R_l}{c_r^*(\theta)} \right)$. Therefore, $\theta^*(r^*(\theta))$ and $\theta - \theta^*(r^*(\theta))$ have the same sign. As a result, if $\theta > 0$, it must be $\theta^*(r^*(\theta)) > 0$ and $\theta - \theta^*(r^*(\theta)) > 0$.

Therefore, for any $\theta > 0$, there exists a market clearing interest rate $\hat{r} = R_h$, and the economy is in an expansion.

**Small interest rate** When the interest rate $r^*(\theta) \in (\bar{r}, c^{-1})$, substituting in the expression of $s_2^*(r)$ and $s_1^*(r)$, we can rewrite the market-clearing condition as follows,

$$\theta = z(r^*(\theta)) = \theta^*(r^*(\theta)) + \Theta(r^*(\theta)).$$

where

$$\theta^*(r) = \frac{c_r}{c_r - R_l} \Phi \left[ \sqrt{3} \Phi^{-1} (\pi_2^*(r)) - (1 - \delta) h \left( \frac{\Phi^{-1}(\pi_2^*(r)) - \Phi^{-1}(\pi_1^*(r))}{\sqrt{\delta}}, \right) r \right] - \frac{R_l}{c_r - R_l},$$

and

$$\Theta(r) = \sigma_s \left( \sqrt{3} \Phi^{-1} (\pi_2^*(r)) + \delta h \left( \frac{\Phi^{-1}(\pi_2^*(r)) - \Phi^{-1}(\pi_1^*(r))}{\sqrt{\delta}}, \right) r \right).$$

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Note that as \( r \) decreases to \( \bar{r} \), \( h \left( \frac{\Phi^{-1}(\pi_0^2) - \Phi^{-1}(\pi_1^1)}{\sqrt{\delta}}, r \right) \rightarrow -\infty. \) Therefore, \( \theta^*(r) \) increases to 1 at the speed of \( h \) and \( \Theta(r) \) decreases to \( -\infty \) at a slower speed of \( \delta h \) as \( r \) decreases to \( \bar{r}. \) Moreover, both functions \( \lim_{\sigma_p \rightarrow 0} \theta^*(r) = 0 \) and \( \lim_{\sigma_p \rightarrow 0} \Theta(r) = 0 \) \( \forall r \in (\bar{r}, c^{-1}). \) Hence, when \( \sigma_p \rightarrow 0, \) as \( r \) decreases to \( \bar{r}, \) \( \theta^*(r) \) first dominates \( \Theta(r) \) resulting in the sum close to 1 then decreases to \( -\infty \) as \( \Theta(r) \) later dominates. In other words, for \( \theta \in (-\infty, 1), \) there exists at least one market clearing rate \( r \in (\bar{r}, c^{-1}). \) Next I will show that \( \hat{r}(\theta) = \lim_{\sigma_p \rightarrow 0} r^*(\theta) = \bar{r} \) for all \( \theta \) that has a market clearing rate \( r \in (\bar{r}, c^{-1}). \) That is for any \( \epsilon > 0, \) there exists a \( \bar{\sigma}_p > 0 \) such that \( \forall \sigma_p < \bar{\sigma}_p, |\bar{r} - r^*(\theta)| < \epsilon, \) i.e. \( r^*(\theta) < \bar{r} + \epsilon. \)

- \( \forall r \in [\bar{r} + \epsilon, c^{-1}], \lim_{\sigma_p \rightarrow 0} \theta^*(r) = 0. \) Therefore, there exists a \( \bar{\sigma}_1 \) such that \( \forall \sigma_p < \bar{\sigma}_1, |\theta^*(r)| < \left| \frac{\theta}{2} \right|. \)

- Similarly, \( \forall r \in [\bar{r} + \epsilon, c^{-1}], \lim_{\sigma_p \rightarrow 0} \Theta(r) = 0. \) Therefore, there exists a \( \bar{\sigma}_2 \) such that \( \forall \sigma_p < \bar{\sigma}_2, |\Theta(r)| < \left| \frac{\theta}{2} \right|. \)

Therefore, for any \( \sigma < \bar{\sigma} = \min\{\bar{\sigma}_1, \bar{\sigma}_2\} \) and any \( \forall r \in [\bar{r} + \epsilon, c^{-1}], \) we have \(-|\theta| < \theta^*(r) + \Theta^*(r) < |\theta|\). Moreover, since \( \theta^*(r) + \Theta^*(r) \) is continuous and \( \lim_{r \rightarrow \bar{r}} (\theta^*(r) + \Theta^*(r)) = -\infty, \) the solution to market-clearing condition \( r^*(\theta) < \bar{r} + \epsilon. \) Therefore, \( \hat{r}(\theta) = \lim_{\sigma_p \rightarrow 0} r^*(\theta, \epsilon_p) = \bar{r}. \) Moreover, \( \Theta(r) \) increases in \( r, \) and as \( r \) decreases to \( \bar{r} \lim_{r \rightarrow \bar{r}^-} \Theta(r) = -\infty. \) Therefore, for any \( \theta < 1, \) there exists an interest rate \( \hat{r}(\theta) = \bar{r}, \) and \( \theta - \theta^*(\bar{r}) = \Theta(r) < 0, \) i.e. the economy is in a recession. \( \blacksquare \)

**Proof of Proposition 3.** When \( r \in (1, R_h), \) the posterior belief of marginal bank with private signal \( s^*(r) \) is normally distributed with mean \( \mu(s^*(r), r) = s^*(r) - (1 - \delta) \sigma_s \Phi^{-1} \left( \frac{R_l}{r} \right) \) and standard deviation \( \sigma = \sqrt{\delta} \sigma_s. \) Hence the belief of the marginal bank who receives signal \( s^*(r) \) is

\[
\pi(s^*(r), r) = \Phi \left( \frac{\mu(s^*(r), r) - \theta^*(r)}{\sigma} \right) = \frac{r - R_l}{R_h - R_l}
\]

which implies

\[
\frac{s^*(r) - \theta^*(r)}{\sigma_s} = (1 - \delta) \Phi^{-1} \left( \frac{R_l}{r} \right) + \sqrt{\delta} \Phi^{-1} \left( \frac{r - R_l}{R_h - R_l} \right)
\]

(A.14)
At the fundamental threshold $\theta^*(r)$, the aggregate investment

$$A(\theta^*(r), r) = \frac{r}{r - R_l} \left(1 - \Phi \left( \frac{s^*(r) - \theta^*(r)}{\sigma_s} \right) \right) = 1 - \theta^*(r).$$

Plugging in equation A.14, we have

$$\theta^*(r) = \frac{r}{r - R_l} \Phi \left[ \sqrt{\delta} \Phi^{-1} \left( \frac{r - R_l}{R_h - R_l} \right) + (1 - \delta) \Phi^{-1} \left( \frac{R_l}{r} \right) \right] - \frac{R_l}{r - R_l}. \quad (A.15)$$

Hence, we can get the expression of signal threshold from equation A.14

$$s^*(r) = \theta^*(r) + \sigma_s \left[ \sqrt{\delta} \Phi^{-1} \left( \frac{r - R_l}{R_h - R_l} \right) + (1 - \delta) \Phi^{-1} \left( \frac{R_l}{r} \right) \right]. \quad (A.16)$$

In the limit,

$$\lim_{\sigma_p \to 0} \theta^*(r) = \frac{r}{r - R_l} \Phi \left[ \Phi^{-1} \left( \frac{R_l}{r} \right) \right] - \frac{R_l}{r - R_l} = 0.$$

Below I analyze the equilibrium thresholds when $r = 1$. For the marginal bank who receive private signal $s^*(1)$, the posterior p.d.f. of $\theta$ is

$$f(\theta|s^*(1), r = 1) \propto \phi \left( \frac{s_i - \theta}{\sigma_s} \right) \Phi \left( \frac{z(1) - \theta}{\sigma_p} \right).$$

In the limit of vanishing noise on the public signal, it converges to a truncated normal distribution $\mathcal{N}(s^*(1), \sigma_s)$ truncated from above at $z(1) = s^*(1) - \sigma_s \Phi^{-1}(R_l)$. Therefore, in the limit, the belief of the marginal bank is

$$\lim_{\sigma_{p} \to 0} \pi(s^*(1), 1) = \frac{1}{1 - R_l} \left( \Phi \left( \frac{\lim_{\sigma_{p} \to 0} s^*(1) - \lim_{\sigma_{p} \to 0} \theta^*(1)}{\sigma_s} \right) - R_l \right) = \frac{1 - R_l}{R_h - R_l}.$$

The aggregate lending at the fundamental threshold $\theta^*(1)$ can be expressed as

$$\lim_{\sigma_{p} \to 0} A(\theta^*(1), 1) = \frac{1}{1 - R_l} \left( 1 - \Phi \left( \frac{\lim_{\sigma_{p} \to 0} s^*(1) - \lim_{\sigma_{p} \to 0} \theta^*(1)}{\sigma_s} \right) \right).$$
Substituting in the expression \( \lim_{\sigma_r \to 0} \pi(s^*(1), 1) \), we have

\[
\lim_{\sigma_p \to 0} A(\theta^*(1), 1) = 1 - \lim_{\sigma_r \to 0} \pi(s^*(1), 1) = \frac{R_h - 1}{R_h - R_l}.
\]

Therefore, the fundamental threshold \( \lim_{\sigma_p \to 0} \theta^*(1) = 1 - \lim_{\sigma_p \to 0} A(\theta^*(1), 1) = \frac{1 - R_l}{R_h - R_l} \). Hence, the signal threshold in the limit is

\[
\lim_{\sigma_p \to 0} s^*(1) = \frac{1 - R_l}{R_h - R_l} + \sigma_s \Phi^{-1} \left( \frac{R_l}{R_h - R_l} - \frac{1 - R_l}{R_h - R_l} \right).
\]  

(A.17)

**Proof of Proposition 4.** If the market-clearing interest rate \( r^*(\theta) \in (1, R_h) \), the market-clearing condition implies that

\[
\theta = z(r^*(\theta)) = s^*(r^*(\theta)) - \sigma_s \Phi^{-1} \left( \frac{R_l}{r^*(\theta)} \right)
\]

Substituting in the expression of \( s^*(r) \) in equation A.16,

\[
\theta = z(r^*(\theta)) = \theta^*(r^*(\theta)) + \sigma_s \left( \sqrt{\delta} \Phi^{-1} \left( \frac{r^*(\theta) - R_l}{R_h - R_l} \right) - \delta^{-1} \left( \frac{R_l}{r^*(\theta)} \right) \right)
\]

Given lemma 2 below, it can be easily verified that \( z(r) \) monotonically increases in \( r \) in the limit. The proof of lemma 2 can be found at the end.

**Lemma 3** In the limit when \( \sigma_p \to 0 \), \( \theta^*(r) \) monotonically increases in \( r \).

Note that \( \lim_{\sigma_p \to 0} z(r) = 0 \) \( \forall r \in (1, R_h) \) and \( \lim_{r \to R_h} z(r) = \infty \). Since \( z(r) \) increases in \( r \) monotonically, for any \( \theta > 0 \), there exists a market clearing interest rate \( r^*(\theta) \in (1, R_h) \).

Next, I prove that the limit of the interest rate \( \hat{r}(\theta) = \lim_{\sigma_p \to 0} r^*(\theta) = R_h \). That is for any \( \epsilon > 0 \), there exists a \( \bar{\sigma}_p > 0 \) such that \( \forall \sigma_p < \bar{\sigma}_p, |R_h - r^*(\theta)| < \epsilon \), i.e. \( r^*(\theta) > R_h - \epsilon \).
• \( \forall r \in [1, R_h - \epsilon], \lim_{\sigma_p \to 0} \theta^*(r) = 0. \) Therefore, there exists a \( \bar{\sigma}_1 \) such that \( \forall \sigma_p < \bar{\sigma}_1, |\theta^*(r)| < \frac{\theta}{2}. \)

• Similarly, \( \forall r \in [1, R_h - \epsilon], \lim_{\sigma_p \to 0} \sigma_s \left( \sqrt{\delta} \Phi^{-1} \left( \frac{r - R_l}{R_h - R_l} \right) - \delta \Phi^{-1} \left( \frac{R_l}{r} \right) \right) = 0. \) Therefore, there exists a \( \bar{\sigma}_2 \) such that \( \forall \sigma_p < \bar{\sigma}_2, |\sigma_s \left( \sqrt{\delta} \Phi^{-1} \left( \frac{r - R_l}{R_h - R_l} \right) - \delta \Phi^{-1} \left( \frac{R_l}{r} \right) \right)| < \frac{\theta}{2}. \)

Therefore, for any \( \sigma_p < \bar{\sigma}_p = \min\{\bar{\sigma}_1, \bar{\sigma}_2\} \) and any \( r \in [1, R_h - \epsilon] \), we have \( z(r) < \theta. \)

Moreover, since \( z(r) \) is continuous and \( \lim_{r \to R_h} z(r) = \infty \), the solution to market-clearing condition \( r^*(\theta) > R_h - \epsilon. \) Therefore, \( \hat{r}(\theta) = \lim_{\sigma_p \to 0} r^*(\theta, \epsilon_p) = R_h. \)

Next, I show that the economy is in an expansion. It suffices to show that \( \theta > \theta^*(r^*(\theta)) \).

The interbank capital market clearing implies that

\[
\theta - \theta^*(r^*(\theta)) = \sqrt{\delta} \Phi^{-1} \left( \frac{r^*(\theta) - R_l}{R_h - R_l} \right) - \delta \Phi^{-1} \left( \frac{R_l}{r^*(\theta)} \right). 
\]

Recall the expression of fundamental threshold \( \theta^*(r) \) in equation A.15. It can be easily verified that \( \theta^*(r) \geq 0 \) if and only if \( \sqrt{\delta} \Phi^{-1} \left( \frac{r^*(\theta) - R_l}{R_h - R_l} \right) \geq \delta \Phi^{-1} \left( \frac{R_l}{r^*(\theta)} \right). \) Therefore, \( \theta^*(r^*(\theta)) \) and \( \theta - \theta^*(r^*(\theta)) \) have the same sign. As a result, if \( \theta > 0 \), it must be \( \theta^*(r^*(\theta)) > 0 \) and \( \theta - \theta^*(r^*(\theta)) > 0. \) Therefore, for any \( \theta > 0 \), there exists a market clearing interest rate \( \hat{r} = R_h \), and the economy.

When the equilibrium interest rate \( r^*(\theta) = 1 \), the interbank capital market-clearing condition implies

\[
\theta \leq z(1) = s^*(1) - \sigma_s \Phi^{-1}(R_l). 
\]

Let \( \bar{\theta} = \lim_{\sigma_p \to 0} z(1). \) Substituting in the expression for \( s^*(1) \) in equation A.17, we have

\[
\theta \leq \bar{\theta} = \frac{1 - R_l}{R_h - R_l} + \sigma_s \left[ \Phi^{-1} \left( R_l + (1 - R_l) \frac{1 - R_l}{R_h - R_l} \right) - \Phi^{-1}(R_l) \right]. 
\]

Therefore, for \( \theta \leq \bar{\theta} \), there exists a market clearing interest rate \( \hat{r}(\theta) = 1. \) As proved in Proposition 3, \( \lim_{\sigma_p \to 0} \theta^*(1) = \theta^*(0). \) Therefore, for any \( \theta \in [\theta^*(0), \bar{\theta}] \), the economy is in an
Proof of Lemma 3. By definition, \( \lim_{\sigma_p \to 0} \sqrt{\delta} = \lim_{\sigma_p \to 0} \sqrt{\frac{\sigma_p^2}{\sigma_s^2 + \sigma_p^2}} = 0 \). Therefore, it is equivalent to look at the limit of \( \sqrt{\delta} \to 0 \). The Taylor expansion of \( \theta^*(r) \) with respect to \( \sqrt{\delta} \) is

\[
\theta^*(r) = \frac{r}{r - R_l} \phi \left( \Phi^{-1} \left( \frac{R_l}{r} \right) \right) \Phi^{-1} \left( \frac{r - R_l}{R_h - R_l} \right) \sqrt{\delta} + O(\delta).
\]

Let \( \alpha_1(r) = \phi \left( \Phi^{-1} \left( \frac{R_l}{r} \right) \right) \) and \( \alpha_2(r) = \Phi^{-1} \left( \frac{r - R_l}{R_h - R_l} \right) \). Then the term with order of \( \sqrt{\delta} \), i.e. \( \alpha(r) = \frac{r - R_l}{r} \alpha_1(r) \alpha_2(r) \sqrt{\delta} \).

\[
\alpha'(r) = \sqrt{\delta} \left( -\frac{R_l}{(r - R_l)^2} \alpha_1 \alpha_2 + \frac{R_l}{r - R_l} \Phi^{-1} \left( \frac{R_l}{r} \right) \alpha_2 + \frac{r}{(r - R_l)(R_h - R_l)} \Phi^{-1}(\alpha_2) \right)
\]

\[
> \sqrt{\delta} \left( -\frac{R_l}{(r - R_l)^2} \alpha_1 \alpha_2 - \frac{1}{(r - R_l)} \alpha_1 \alpha_2 + \frac{r}{(r - R_l)(R_h - R_l)} \Phi^{-1}(\alpha_2) \right)
\]

\[
= \frac{\sqrt{\delta} r \alpha_1}{(r - R_l)^2 \Phi(\alpha_2)} \left( -\alpha_2 \phi(\alpha_2) + \frac{r - R_l}{R_h - R_l} > \sqrt{\delta} r \alpha_1 \left( -\Phi(\alpha_2) + \frac{r - R_l}{R_h - R_l} \right) = 0 \right)
\]

Therefore, in the limit when \( \sigma_p \to 0 \), \( \theta^*(r) \) monotonically increases in \( r \).

Proof of Proposition 5.

As specified in Proposition 1, in the benchmark without an market, banks follow the threshold strategy \( s_0^* = \theta_0^* - \sigma_s \Phi^{-1} \left( 1 - \theta_0^* \right) \), where \( \theta_0^* = \frac{1 - R_l}{R_h - R_l} \) is the fundamental threshold. Hence, we can write the equilibrium aggregate lending as a function of the realized fundamentals as follow,

\[
A_0(\theta) = \Phi \left( \frac{\theta - s_0^*}{\sigma_s} \right) = \Phi \left( \frac{\theta - \theta_0^*}{\sigma_s} + \Phi^{-1} \left( 1 - \theta_0^* \right) \right).
\]

When the interest rate \( \hat{r}(\theta) = R_h \), all banks’ budget constraint binds, i.e. \( a_i = 1 + x_i \forall i \). Therefore, in aggregate, \( A(\theta, \hat{r}(\theta)) = D(\theta, \hat{r}(\theta)) + 1 \). The market-clearing condition, \( D(\theta, \hat{r}(\theta)) = 0 \) implies that the aggregate lending in equilibrium is \( A(\theta, \hat{r}(\theta)) = 1 > \)
\( A_0(\theta) \ \forall \theta > 0 \). That is the interbank capital market increases the efficient credit supply in economic expansions when the interest rate \( \hat{r}(\theta) = R_h \).

When the interest rate \( \hat{r}(\theta) = 1 \), the equilibrium aggregate lending is

\[
\lim_{\sigma_p \to \theta} A(\theta^*(1), 1) = \frac{1}{1 - R_l} \Phi \left( \frac{\theta - s^*(1)}{\sigma_s} \right)
\]

Substituting in the expression for \( s^*(1) \) in equation A.17,

\[
\lim_{\sigma_p \to \theta} A(\theta, 1) = \frac{1}{1 - R_l} \Phi \left( \frac{\theta - \theta_0^*}{\sigma_s} + \Phi^{-1}(1 - R_l)(1 - \theta_0^*) \right)
\]

Next, I will compare the aggregate lending with that in the benchmark. Define the difference between the aggregate lending as follow

\[
\Delta(\theta) = \lim_{\sigma_p \to \theta} A(\theta, 1) - A_0(\theta).
\]

Note that \( \Delta(\theta_0^*) = 0 \), i.e. at the fundamental threshold, the welfare achieved by introducing the interbank capital market is the same as in the benchmark. Moreover, \( \lim_{\theta \to -\infty} \Delta(\theta) = 0 \), because the aggregate lending goes to zero as the fundamentals decrease in both cases. Take the first order derivative with respect to \( \theta \),

\[
\frac{d\Delta(\theta)}{d\theta} = \beta \left( e^{\gamma(\theta - \theta_0^*)} - (1 - R_l) \right)
\]

where \( \beta \) and \( \gamma \) are two positive constant. There exists a unique \( \theta = \theta_0^* + \gamma^{-1}ln(1 - R_l) < \theta_0^* \) such that \( \frac{d\Delta(\theta)}{d\theta} \geq 0 \) if and only if \( \theta \geq \theta_0^* \). Therefore, when the realized fundamentals \( \theta > \theta_0^* \), \( \Delta(\theta) > \Delta(\theta_0^*) = 0 \), i.e. the interbank capital market increases the efficient credit supply. When the realized fundamentals \( \theta < \theta_0^* \), \( \Delta(\theta) < \max \{\Delta(\theta_0^*), \lim_{\theta \to -\infty} \Delta(\theta)\} = 0 \), i.e. the interbank capital market decreases the inefficient credit supply. 

**Proof of Proposition 17.** Lemma 1 specifies the optimal portfolio choice for borrowers in the interbank capital market with collateral requirement \( c \). Therefore, for a borrower
who has belief $\pi_i$ and borrows on interbank capital market $c$, its expected payoff can be written as

$$\pi_i \cdot \frac{r(c)(R_hc - R_l)}{cr(c) - R_l}.$$ 

Since borrowers always default when they suffer losses from their bank loans to the operating firms, to maximize their expected payoffs, they can simply maximize their payoffs when their bank loans are profitable. Therefore, every borrower regardless of its belief solves the same maximization problem, i.e.

$$\max_c \pi_i \cdot \frac{r(c)(R_hc - R_l)}{cr(c) - R_l} = \pi_i \cdot \frac{r(c)(R_hc - R_l)}{cr(c) - R_l}.$$ 

Let $\bar{u} = \max_c \frac{r(c)(R_hc - c - r)}{cr(c) - r}$ denote a borrower’s maximum payoff when bank loans are profitable. In equilibrium, if an interbank capital market $c$ has positive trading volume, its interest rate $r(c)$ must satisfy

$$r(c) = \frac{\bar{u}R_l}{(\bar{u} - R_h)c + R_l}.$$ 

Hence, we can restrict attention to interbank capital markets with above interest rate.

Below I analyze formally the portfolio choice problem from a lender’s perspective. For lender $j$ with belief $\pi_j$, it achieves expected payoff specified below by lending on interbank capital market $c$.

$$\pi_j r(c) + (1 - \pi_j)cr(c) = \frac{\bar{u}R_l(\pi_j + (1 - \pi_j)c)}{c(\bar{u} - R_h) + R_l}.$$ 

It must be higher than $\pi_j \bar{u}$ which is the expected payoff for being a borrower. Therefore, lenders in the interbank capital markets must have beliefs $\pi \leq \frac{R_l}{\bar{u} - R_h + R_l}$. In equilibrium, if lender $j$ lends on a risky interbank capital market with $c < 1$, it must earn a higher return than lending on the risk-free interbank capital market with $c = 1$, i.e.

$$\frac{\bar{u}R_l(\pi_j + (1 - \pi_j)c)}{c(\bar{u} - R_h) + R_l} \geq \frac{\bar{u}R_l}{\bar{u} - R_h + R_l}.$$
which gives \( \pi_j \geq \frac{R_l}{u - R_h + R_l} \). Therefore, a lender who lends on interbank capital market with \( c < 1 \) must have belief \( \pi = \frac{R_l}{u - R_h + R_l} \). Because of the continuous signal structure, the mass of banks with belief \( \pi = \frac{R_l}{u - R_h + R_l} \) is zero. Hence, in equilibrium, risky interbank capital markets with \( c < 1 \) have zero trading volume. ■

**Proof of Proposition 18.** Since the cost of contracting is imposed on lenders in the interbank capital market, borrowers’ decision on which interbank capital market to enter is the same as in the main model. As shown in the proof of Proposition 17, in equilibrium, all borrowers receive the same payoff in expansions and solve the same maximization problem below.

\[
\max_c \pi_i r(c) (R_h c - R_l) - I(c) = \pi_i \max_c r(c) (R_h c - R_l) - I(c).
\]

Note that on any interbank capital market \( c \), the borrowers’ expected payoff decreases in \( r(c) \). Therefore, the maximum expected payoff a borrower can achieve on interbank \( c \) is when \( r(c) = \bar{r}(c) = \frac{R_l - R_l}{c(R_h - 1) + 1 - R_l} \). Substitute in the express for \( \bar{r}(c) \), we can calculate the maximum payoff for a borrower is \( \pi_i \frac{R_l - R_l}{1 - R_l} \) regardless of the collateral level \( c \). Therefore, when \( r(c_{\text{min}}) \to \bar{r}(c_{\text{min}}) \), the maximum payoff that borrowing banks receive in expansion is

\[
\bar{u} = \max_c \frac{r(c) (R_h c - R_l)}{cr(c) - R_l} = \frac{R_h - R_l}{1 - R_l}.
\]

And all interbank capital markets with positive trading volume should have \( r(c) = \bar{r}(c) \).

However, from lending bank j’s perspective, its expected payoff from lending on interbank \( c \) is

\[
\pi_j \bar{r}(c) + (1 - \pi_j) c \bar{r}(c) - I(c) = \frac{(R_h - R_l) (\pi_j + (1 - \pi_j) c)}{c(R_h - 1) + 1 - R_l} - I(c).
\]

It must be greater than the expected payoff from being a borrower or investing in the safe storage, i.e.

\[
\frac{(R_h - R_l) (\pi_j + (1 - \pi_j) c)}{c(R_h - 1) + 1 - R_l} - I(c) \geq \max\{\pi \bar{u}, 1\}.
\]

There is solution to the above inequality when \( c = c_{\text{min}} \) and no solution for any \( c > c_{\text{min}} \).
Proof of Proposition 19. Below I prove the monotonicity of $\bar{\theta}(c)$. It is equivalent to show that $f(t) = \Phi^{-1}\left(t + (1 - t) \frac{1 - R_l}{R_h - R_l}\right) - \Phi^{-1}(t)$ decreases monotonically in $t$. Take the first order derivative with respect to $t$,

$$\frac{df}{dt}(t) = \left(1 - \frac{1 - R_l}{R_h - R_l}\right) \left(\frac{1}{\Phi'\left(\Phi^{-1}\left(t + (1 - t) \frac{1 - R_l}{R_h - R_l}\right)\right)}\right)^{-1} - \left(\Phi^{-1}(t)\right)^{-1}.$$ 

Therefore it suffices to prove that

$$g(t) = \left(1 - \frac{1 - R_l}{R_h - R_l}\right) \phi\left(\Phi^{-1}(t)\right) - \phi\left(\Phi^{-1}\left(t + (1 - t) \frac{1 - R_l}{R_h - R_l}\right)\right)$$

is non-positive for any $t \in [0, R_l]$. Note that

$$\frac{dg}{dt}(t) = \left(1 - \frac{1 - R_l}{R_h - R_l}\right) \left[\Phi^{-1}\left(t + (1 - t) \frac{1 - R_l}{R_h - R_l}\right)\right] - \Phi^{-1}(t)$$

is non-negative for any $t \in [0, 1]$. Therefore, $g(t) \leq g(1) = 0$ for any $t \in [0, R_l]$.

The proof of the rest of the proposition follows the proof of proposition 3 and 4. ■

Proof of Proposition 20. The aggregate lending under regulatory requirement $c$ is

$$A(\theta, c) = \frac{1}{1 - R_l/c} \Phi\left(\frac{\theta - \theta_0^*}{\sigma_s} + \Phi^{-1}\left(\left(1 - \frac{R_l}{c}\right)(1 - \theta_0^*)\right)\right).$$

Let $t(c) = 1 - \frac{R_l}{c}$, which increases in $c$ monotonically. Hence, the aggregate lending function can be expressed as

$$A(\theta, t) = \frac{1}{t} \Phi\left(a(\theta) + \Phi^{-1}(t(1 - \theta_0^*))\right),$$

where $a(\theta) = \frac{\theta - \theta_0^*}{\sigma_s}$. Take the first order derivative with respect to $t$,

$$\frac{d}{dt} A(\theta, t) = \frac{1}{t^2} \left[\frac{t(1 - \theta_0^*)}{\phi(\Phi^{-1}(t(1 - \theta_0^*)))} - \frac{\Phi(a(\theta) + \Phi^{-1}(t(1 - \theta_0^*)))}{\phi(a(\theta) + \Phi^{-1}(t(1 - \theta_0^*)))}\right].$$
Note that $\frac{\Phi(x)}{\phi(x)}$ strictly increases in $x$.\(^1\) Hence, the above first order derivative $\frac{d}{dt} A(\theta, t) \geq 0$ when $a(\theta) \geq 0$. As a result, $\frac{d}{dt} A(\theta, t) \geq 0$ when $\theta \leq \theta^*_0$, and $\frac{d}{dc} A(\theta, c) \geq 0$ when $\theta \leq \theta^*_0$. \(\blacksquare\)

**Proof of Proposition 21.**

If the market-clearing interest rate $r^*(\theta) = 1$, the fundamentals

$$\theta \leq z(1) = s^*(1) - \sigma_s \Phi^{-1} \left( \frac{R_l(R_h-1)}{R_h - R_l} \right).$$

For the marginal banks that receive private signal $s^*(1)$, the posterior p.d.f. of $\theta$ is

$$f(\theta|s^*(1), r = 1) \propto \phi \left( \frac{s_i - \theta}{\sigma_s} \right) \Phi \left( \frac{z(1) - \theta}{\sigma_p} \right).$$

In the limit of vanishing noise on the public signal, it converges to a truncated normal distribution $N(s^*(1), \sigma_s)$ truncated from above at $z(1) = s^*(1) - \sigma_s \Phi^{-1} \left( \frac{R_l(R_h-1)}{R_h - R_l} \right)$. Therefore, in the limit, the belief of the marginal bank is

$$\lim_{\sigma_p \to 0} \pi(s^*(1), 1) = \frac{\lim_{\sigma_p \to 0} \Phi \left( \frac{s^*(1) - \theta^*(1)}{\sigma_s} \right) - \frac{R_l(R_h-1)}{R_h - R_l}}{1 - \frac{R_l(R_h-1)}{R_h - R_l}} = \frac{1 - R_l}{R_h - R_l}. \tag{1}$$

The aggregate lending at the fundamental threshold $\theta^*(1)$ can be expressed as

$$\lim_{\sigma_p \to 0} A(\theta^*(1), 1) = \frac{1}{1 - R_l} \left( 1 - \frac{1}{\sigma_p \to 0} \Phi \left( \frac{s^*(1) - \theta^*(1)}{\sigma_s} \right) \right) - \frac{1}{R_h - 1} \lim_{\sigma_p \to 0} \Phi \left( \frac{s^*(1) - \theta^*(1)}{\sigma_s} \right).$$

Substituting in the expression $\lim_{\sigma_p \to 0} \pi(s^*(1), 1)$, we have

$$\lim_{\sigma_p \to 0} A(\theta^*(1), 1) = 1 - \frac{R_h}{R_h - 1} \lim_{\sigma_p \to 0} \pi(s^*(1), 1) = 1 - \frac{R_h(1 - R_l)}{(R_h - 1)(R_h - 1)}. \tag{2} \text{\footnote{See Gordon (1941) for details.}}$$

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Therefore, the fundamental threshold is
\[
\lim_{\sigma_p \to 0} \theta^*(1) = 1 - \lim_{\sigma_p \to 0} A(\theta^*(1), 1) = \frac{R_h(1 - R_t)}{(R_h - 1)(R_h - R_t)}.
\]

Hence, the signal threshold in the limit is
\[
\lim_{\sigma_p \to 0} s^*(1) = \lim_{\sigma_p \to 0} \theta^*(1) + \sigma_s \Phi^{-1} \left( \lim_{\sigma_p \to 0} \Phi \left( \frac{s^*(1) - \theta^*(1)}{\sigma_s} \right) \right).
\] (A.18)

The interbank capital market-clearing condition when \( r = 1 \) implies that
\[
\theta \leq z(1) = s^*(1) - \sigma_s \Phi^{-1} \left( \frac{R_t(R_h - 1)}{R_h - R_t} \right).
\]

Substituting in the expression for \( s^*(1) \) in equation A.18, we have
\[
z(1) = \lim_{\sigma_p \to 0} \theta^*(1) + \sigma_s \left[ \Phi^{-1} \left( \lim_{\sigma_p \to 0} \Phi \left( \frac{s^*(1) - \theta^*(1)}{\sigma_s} \right) \right) - \Phi^{-1} \left( \frac{R_t(R_h - 1)}{R_h - R_t} \right) \right].
\]

Since
\[
\lim_{\sigma_p \to 0} \pi(s^*(1), 1) = \lim_{\sigma_p \to 0} \Phi \left( \frac{s^*(1) - \theta^*(1)}{\sigma_s} \right) - \frac{R_t(R_h - 1)}{R_h - R_t} = \frac{1 - R_t}{R_h - R_t},
\]
the positive numerator implies that \( \lim_{\sigma_p \to 0} \Phi \left( \frac{s^*(1) - \theta^*(1)}{\sigma_s} \right) > \frac{R_t(R_h - 1)}{R_h - R_t} \). Therefore, \( z(1) > \lim_{\sigma_p \to 0} \theta^*(1) \). Let \( \theta^*_1 = \lim_{\sigma_p \to 0} \theta^*(1) \), then for any \( \theta \in (\theta^*_0, \theta^*_1) \), there exists a market clearing interest rate \( \hat{r} = \theta^*(1) \). Moreover, since \( \theta \) is less than the fundamental threshold \( \theta^*_1 \), the economy is in a recession. ■

**Proof of Proposition 22.** When CDS price \( p \in (\bar{p}, 1) \), the aggregate investment in sovereign debt \( A = 0 \). Therefore, the fundamental threshold \( \theta^*(p) = 1 \ \forall p \in (\bar{p}, 1) \). The market-clearing condition on the CDS market
\[
\int_{-\infty}^{\infty} x(\theta + \sigma_s \epsilon, p)\phi(\epsilon)d\epsilon = 0
\]
implies that the CDS price contains perfect information about the fundamentals \( \theta \), i.e.

\[
\theta = s^*(p) - \sigma_s \Phi^{-1}(p).
\]

To preserve the belief heterogeneity, I assume that banks make noisy inference based on the information and update their beliefs based on the noisy public signal

\[
\theta = s^*(p) - \sigma_s \Phi^{-1}(p) + \sigma_p \epsilon_p.
\]

Hence, for a marginal investor with private signal \( s^*(p) \), the posterior distribution of fundamentals \( \theta \) given her private signal \( s^*(p) \) and the public signal \( p \) is \( \mathcal{N}(s^*(p) - \sigma_s(1 - \delta)\Phi^{-1}(p), \delta \sigma_s^2) \), and her belief

\[
\pi(s^*(p), p) = 1 - \Phi\left( \frac{\theta^*(p) - s^*(p) + \sigma_s(1 - \delta)\Phi^{-1}(p)}{\sqrt{\delta}\sigma} \right) = 1 - p.
\]

Substituting in \( \theta^*(p) = 1 \), we can solve for the signal cutoff

\[
s^*(p) = 1 + \sigma_s(1 - \delta - \sqrt{\delta})\phi^{-1}(p).
\]

From the market-clearing condition, we know that for fundamentals

\[
\theta = 1 - \sigma_s(\delta + \sqrt{\delta})\phi^{-1}(p),
\]

there exists an equilibrium with CDS price \( p \in (\bar{p}, 1) \). Note that the right-hand side of the above equation decreases monotonically in \( p \), \( \lim_{\sigma_p \to 0} 1 - \sigma_s(\delta + \sqrt{\delta})\phi^{-1}(p) = 1 \ \forall p \in (\bar{p}, 1) \) and \( \lim_{p \to 1} 1 - \sigma_s(\delta + \sqrt{\delta})\phi^{-1}(p) = -\infty \ \forall \sigma_p > 0 \). Therefore, for any \( \theta \in (-\infty, 1) \), there exists an equilibrium with CDS price \( p^*(\theta) \in (\bar{p}, 1) \) and the sovereign debt defaults. Below I show that in the limit \( \hat{p}(\theta) = \lim_{\sigma_p \to 0} p^*(\theta) = 1 \ \forall \theta \in (-\infty, 1) \). That is to show \( \forall \epsilon > 0 \), there exists a \( \bar{\sigma}_p > 0 \) such that \( \forall \sigma_p < \bar{\sigma}_p, |1 - p^*(\theta)| < \epsilon \), i.e. \( p^*(\theta) > 1 - \epsilon \).
$\forall p \in (\bar{p}, 1 - \epsilon)$, \(\lim_{\sigma_p \to 0} \sigma_s(\delta + \sqrt{\delta})\phi^{-1}(p) = 0\). Therefore, there exists a \(\bar{\sigma}_1 > 0\) such that $\forall \sigma_p < \bar{\sigma}_1$, $\sigma_s(\delta + \sqrt{\delta})\phi^{-1}(p) < \frac{1-\theta}{2}$.

$\forall \sigma_p < \bar{\sigma}_2 = \frac{1-\theta}{2\epsilon_p}$, $\sigma_p \epsilon_p > \frac{1-\theta}{2}$.

Hence, for any $\sigma_p < \bar{\sigma}_p = \min\{\bar{\sigma}_1, \bar{\sigma}_2\}$ and any $p \in (\bar{p}, 1 - \epsilon)$, we have $1 - \sigma_s(\delta + \sqrt{\delta})\phi^{-1}(p) + \sigma_p \epsilon_p > \theta$. Moreover, since $1 - \sigma_s(\delta + \sqrt{\delta})\phi^{-1}(p)$ is continuous and \(\lim_{p \to 1} 1 - \sigma_s(\delta + \sqrt{\delta})\phi^{-1}(p) = -\infty\), the market clearing CDS price \(p^*(\theta) > 1 - \epsilon\).

Therefore, for any $\theta \in (\theta^*_0, 1)$, there exists a market clearing CDS price $\hat{p}(\theta) = 1$. Since the fundamental threshold $\theta^*(p) = 1 \forall p \in (\bar{p}, 1)$, in this region, the economy is in a recession.

A.1.8. Laplacian Property

For a bank with private signal at the threshold, when she decides whether to extend credit to the real sector, she makes inferences on the aggregate lending and hence the rank of her signal among all banks. Her private signal is entirely uninformative about her rank and consequently about the aggregate lending. Therefore, she is uncertain about the realized proportions of types above and below the threshold and is indifferent between extending credit and not.

**Lemma 4 (Laplacian Property)** In the limit of vanishing demand noise, the aggregate lending is, conditional on interest rate $r = 1$ and the threshold signal $s^*(1)$, uniformly distributed on $[0,1]$.

**Proof of Lemma 4.**

In the limit of vanishing demand noise, for a bank observing private signal $s^*(1)$ and interest rate $r = 1$, fundamentals $\theta$ follow a truncated normal distribution with cumulative
distribution function as follows,

\[ F(\theta | s_i = s^*(1), r = 1) = \frac{\Phi \left( \frac{\theta - s^*(1)}{\sigma_s} \right)}{\Phi \left( \frac{\theta - s^*(1)}{\sigma_s} \right)} \]

where by the market-clearing condition \( \tilde{\theta} = z(1) = s^*(1) - \sigma_s \Phi^{-1}(r) \). Therefore, the expression can be simplified as

\[ F(\theta | s_i = s^*(1), r = 1) = \frac{1}{1 - R_l} \Phi \left( \frac{\theta - s^*(1)}{\sigma_s} \right) \]

The aggregate lending at fundamentals \( \theta \) when the interest rate \( r = 1 \) is

\[ A(\theta, 1) = \frac{1}{1 - R_l} \Phi \left( \frac{\theta - s^*(1)}{\sigma_s} \right) \]

Hence, the c.d.f. of aggregate lending conditional on interest rate \( r = 1 \) and the threshold signal \( s^*(1) \) is

\[ Pr(A(\theta, 1) < a | s_i = s^*(1), r = 1) = F(|s^*(1) + \sigma_s \Phi^{-1}((1 - R_l)a)) = a. \]

Therefore, the aggregate lending \( A(\theta, 1) \) is uniformly distributed on \([0, 1]\).

A.2. Appendix to Chapter 2

A.2.1. Proofs

**Proof of Proposition 6.** It can be proved by iterated deletion of dominated strategies. Let \( p(x; k) \) denote the interim belief of success when an agent receives private signal \( x \) and all other agents follow a cutoff investment strategy \( k \) as defined by Equation (2.4). First, we want to show that strategies survive \( n \) rounds of iterated deletion of dominated strategies.
if and only if

\[ a(x) = 0, \text{ if } x < \xi_n, \quad \text{(A.19)} \]

and \( a(x) = 1, \text{ if } x \geq \bar{\xi}_n. \) \( \text{(A.20)} \)

where \( \{ (\xi_n, \bar{\xi}_n) \}_{n=0}^{\infty} \) satisfies

\[ -\infty = \xi_0 < \xi_1 \leq \cdots \leq \xi_n \leq \cdots \leq \xi_1 < \xi_0 = +\infty. \] \( \text{(A.21)} \)

This result can be proved by induction. Let \( \xi_0 = -\infty \) and \( \bar{\xi}_0 = +\infty \), so the first round of deletion starts with the full set of strategies. Suppose round \( n \in \mathbb{N} \) of deletion has been completed. In round \( n + 1 \), the best scenario for an agent to invest is that all other agents follow a cutoff strategy with threshold \( \xi_n \). Therefore, for any \( x \) such that \( p(x; \xi_n) < \frac{c}{b} \), \( a(x) = 1 \) is strictly worse than \( a(x) = 0 \). Similarly, the best scenario for an agent to choose \( a_i = 1 \) is that all other agents follow a cutoff strategy with threshold \( \xi_n \). As a result, for \( x \) such that \( p(x; \bar{\xi}_n) > \frac{c}{b} \), any strategy profile with \( a(x) = 1 \) is strictly better than \( a(x) = 0 \).

Given \( p(x; k) \) is non-decreasing in \( x \), the strategy profiles that survives deletion of dominated strategies can be summarized in the form of (A.19)(A.20), with \( (\xi_{n+1}, \bar{\xi}_{n+1}) \) defined inductively as

\[ \xi_{n+1} = \inf \left\{ x : p(x; \xi_n) \geq \frac{c}{b} \right\} \] \( \text{(A.22)} \)

and

\[ \bar{\xi}_{n+1} = \sup \left\{ x : p(x; \bar{\xi}_n) \leq \frac{c}{b} \right\} \] \( \text{(A.23)} \)

The monotonicity of \( p(x; k) \) guarantees that \( \xi_{n+1} \leq \bar{\xi}_{n+1} \) given \( \xi_n \leq \bar{\xi}_n \). Note the dominance region assumption implies that \( \xi_1 > -\infty \) and \( \bar{\xi}_1 < +\infty \) when \( \sigma \) is small enough. Therefore,
\( \left\{ (\xi_n, \bar{\xi}_n) \right\}_{n=0}^{\infty} \) is a well-defined sequence of real couple which satisfies (A.21).

Now we’ve proved that \( \{\xi_n\}_{n=1}^{\infty} \) and \( \{\bar{\xi}_n\}_{n=1}^{\infty} \) are both monotonic and bounded sequences. Thus, they converges to two finite numbers \( \bar{\xi} \) and \( \bar{\bar{\xi}} \) respectively when \( n \to \infty \). And the two limits satisfy

\[
\bar{\xi} \leq \bar{\bar{\xi}}. \tag{A.24}
\]

The definition (A.22)(A.23) implies that \( p(\xi; \bar{\xi}) \geq \bar{\xi} \) and \( p(\bar{\xi}; \bar{\bar{\xi}}) \leq \bar{\bar{\xi}} \). Note that

\[
p(\xi; \bar{\bar{\xi}}) = F \left( \frac{\xi - \theta^*(\xi)}{\sigma} \right) = \theta^*(\xi), \tag{A.25}
\]

is strictly increasing in \( \xi \). Therefore \( \bar{\xi} = \bar{\bar{\xi}} \) must be the unique solution to \( \theta^*(\xi) = \bar{\bar{\xi}} \), which is

\[
\xi^*_0 = \frac{c}{b} + \sigma F^{-1} \left( \frac{c}{b} \right). \tag{A.26}
\]

Since there’s only one strategy that survives the iterated deletion of dominated strategies, the equilibrium of the game is unique and the associated equilibrium strategy is the cutoff investment strategy with threshold \( \xi^*_0 \).

**Lemma 5** Suppose the optimal strategy of an agent as a function of her interim belief of success \( \hat{p}_i \) can be characterized as

\[
a_i = 1, \text{ Reject, if } \hat{p}_i > p^*_2,
\]

\[
a_i = 1, \text{ Accept, if } p^*_1 < \hat{p}_i \leq p^*_2,
\]

\[
a_i = 0, \text{ if } \hat{p}_i \leq p^*_1,
\]

where \( p^*_1 \) and \( p^*_2 \) are two threshold beliefs that satisfy \( 0 \leq p^*_1 < p^*_2 \leq 1 \). There is a unique
Bayesian Nash equilibrium and the equilibrium strategy of any agent is

\[ a_i = 1, \text{Reject, if } x_i \geq \eta^*, \]
\[ a_i = 1, \text{Accept, if } \xi^* \leq x_i < \eta^*, \]
\[ a_i = 0, \text{if } x_i < \xi^*, \]

where \( \xi^* = p_1^* + \sigma F^{-1}(p_1^*) \) and \( \eta^* = p_1^* + \sigma F^{-1}(p_2^*) \)

**Proof of Lemma 5.** We want to find a sequence \( \{(\xi_n, \bar{\xi}_n)\}_{n=0}^{\infty} \) such that strategies survive \( n \) rounds of iterated deletion of dominated strategies only if

\[ a(x) = 0, \text{ if } x < \xi_n, \]  \hspace{1cm} (A.27)
\[ \text{and } a(x) = 1, \text{ if } x \geq \bar{\xi}_n. \]  \hspace{1cm} (A.28)

The reason that we can only iterate on the investment cutoff without keeping track of the participation decisions is that an agent’s investment decision is independent of other agents’ participation decisions. The recursive expression for \( \{(\xi_n, \bar{\xi}_n)\}_{n=0}^{\infty} \) is

\[ \xi_{n+1} = \inf \{ x : p(x; \xi_n) \geq p_1^* \}, \]  \hspace{1cm} (A.29)
\[ \bar{\xi}_{n+1} = \sup \{ x : p(x; \xi_n) \leq p_1^* \}. \]  \hspace{1cm} (A.30)

Applying the same techniques in the proof of Proposition 6, it becomes clear that the limit of the two cutoff sequences converges to

\[ \xi^*(s, t) = p_1^* + \sigma F^{-1}(p_1^*), \]  \hspace{1cm} (A.31)

which is the investment cutoff in the unique Bayesian Nash equilibrium of the global game.
The associated participation cutoff $\eta$ is the solution to

$$p(\eta; \xi^*(s,t)) = p^*_2.$$  \hfill (A.32)

Solving the above equation yields

$$\eta^*(s,t) = p^*_1 + \sigma F^{-1}(p^*_2).$$  \hfill (A.33)

**Proof of Proposition 7.** In case 1, *invest-and-reject* is dominated by *invest-and-accept*. Therefore, we can rewrite the investment payoff by letting $b' = b - t$ and $c' = c - s$ and directly apply Proposition 6. Similarly, *invest-and-accept* is jointly dominated by *invest-and-accept* and *not-invest* in case 3. Since the intervention program is never going to be accepted, the equilibrium is the same as that described in Proposition 6. Case 2 is a direct implication of Lemma 5. \hfill $\blacksquare$

**Proof of Proposition 8.**

As specified in Equation 2.5, with program $(s, t)$, the fundamental cutoff is $\frac{c - s}{b - t}$. Therefore, the programs targeting at the first-best fundamental cutoff 0 should satisfy $s = c$. Hence, the subsidy to tax ratio of a program targeting at the first best is $\frac{s}{t} = \frac{c}{t}$. If the ratio is greater than 1, the program is a full-participation program. Otherwise, it is a partial-participation program.

As a result, if $(s, t)$ satisfies the following two conditions, it is a full-participation program targeting the first best.

1. $0 \leq t \leq c$,
2. \( s = c \).

If \((s', t')\) satisfies the following two conditions, it is a partial-participation program targeting the first best.

1. \( c < t' \leq b \),
2. \( s' = c \).

Lastly, we calculate the limit of the cost functions as specified in Equation 2.7 and 2.8. For any \( \theta > 0 \),

\[
\lim_{\sigma \to 0} C(\theta, s, t) = \lim_{\sigma \to 0} (\tau s - t) \left[ 1 - F \left( \frac{0 - \theta}{\sigma} + F^{-1}(0) \right) \right] = (\tau s - t) \left[ 1 - F(-\infty) \right] = \tau s - t
\]

\[
\lim_{\sigma \to 0} \frac{s'}{v'} (\tau s' - t') = \lim_{\sigma \to 0} \left( \tau s' - t' \right) \left[ F \left( \frac{0 - \theta}{\sigma} + F^{-1} \left( \frac{s'}{v'} \right) \right) - F \left( \frac{0 - \theta}{\sigma} + F^{-1}(0) \right) \right]
\]

\[
= (\tau s' - t') \left[ F(-\infty) - F(-\infty) \right] = 0
\]

If \( \theta = 0 \),

\[
\lim_{\sigma \to 0} C(\theta, s, t) = \lim_{\sigma \to 0} (\tau s - t) \left[ 1 - F \left( F^{-1}(0) \right) \right] = \tau s - t
\]

\[
\lim_{\sigma \to 0} \frac{s'}{v'} (\tau s' - t') = \lim_{\sigma \to 0} \left( \tau s' - t' \right) \left[ F \left( \frac{F^{-1} \left( \frac{s'}{v'} \right)}{F^{-1}(0)} \right) - F \left( F^{-1}(0) \right) \right] = \frac{s'}{v'} (\tau s' - t')
\]

The cost of a partial-participation program is strictly less than that of a full-participation program.

\[
\frac{s'}{v'} (\tau s' - t') = \tau c \frac{c}{v'} - c < \tau c - c \leq \tau s - t
\]

For any \( \theta < 0 \),

\[
\lim_{\sigma \to 0} C(\theta, s, t) = \lim_{\sigma \to 0} \tau s \left[ 1 - F \left( \frac{0 - \theta}{\sigma} + F^{-1}(0) \right) \right] = \tau s \left[ 1 - F(\infty) \right] = 0
\]
\[
\lim_{\sigma \to 0} C(\theta, s', t') = \lim_{\sigma \to 0} \tau s' \left[ F \left( \frac{0 - \theta}{\sigma} + F^{-1} \left( \frac{s'}{\theta} \right) \right) - F \left( \frac{0 - \theta}{\sigma} + F^{-1}(0) \right) \right] = \tau s' \left[ F(\infty) - F(\infty) \right] = 0
\]

**Proof of Proposition 9.** We compare the expected cost of a full-participation program with \((s, t)\) a partial-participation program \((s', t')\) with small enough \(\lambda > 0\).

The expected cost of the full-participation program is

\[
E_\theta[C(\theta, s, t)] = \frac{\tau s}{\bar{\theta} - \theta} \int_0^{\bar{\theta}} \left[ 1 - F \left( \frac{\xi^* - \theta}{\sigma} \right) \right] d\theta - \frac{t}{\bar{\theta} - \theta} \int_{\theta'}^{\bar{\theta}} \left[ 1 - F \left( \frac{\xi^* - \theta}{\sigma} \right) \right] d\theta,
\]

and that of the partial-participation program \((s', t')\),

\[
E_\theta[C(\theta, s', t')] = \frac{\tau s'}{\bar{\theta} - \theta} \int_0^{\bar{\theta}} \left[ F \left( \frac{\eta^* - \theta}{\sigma} \right) - F \left( \frac{\xi^* - \theta}{\sigma} \right) \right] d\theta - \frac{t'}{\bar{\theta} - \theta} \int_{\theta'}^{\bar{\theta}} \left[ F \left( \frac{\eta^* - \theta}{\sigma} \right) - F \left( \frac{\xi^* - \theta}{\sigma} \right) \right] d\theta,
\]

where \(\xi^*\) and \(\eta^*\) are the investment threshold and participation threshold defined as in Proposition 7, \(\xi^* = \theta^* + \sigma F^{-1}(\theta^*), \eta^* = \theta^* + \sigma F^{-1} \left( \frac{s'}{\sigma} \right) \). To suppress notations, we omit the dependence of \(\eta^*\) on \((s', t')\). The difference between the cost of full-participation program \((s, t)\) and that of partial-participation program \((s', t')\) can be decomposed into two parts, \(E_\theta[C(\theta, s, t)] - E_\theta[C(\theta, s', t')] = \Delta_1 + \Delta_2\), where

\[
\Delta_1 = \frac{\tau s - t}{\bar{\theta} - \theta} \int_{\theta'}^{\bar{\theta}} \left[ 1 - F \left( \frac{\eta^* - \theta}{\sigma} \right) \right] d\theta + \frac{\tau s}{\bar{\theta} - \theta} \int_{\theta'}^{\bar{\theta}} \left[ 1 - F \left( \frac{\eta^* - \theta}{\sigma} \right) \right] d\theta,
\]

\[
\Delta_2 = -\frac{\tau \theta^*(t' - t)}{\bar{\theta} - \theta} \int_0^{\bar{\theta}} \left[ F \left( \frac{\eta^* - \theta}{\sigma} \right) - F \left( \frac{\xi^* - \theta}{\sigma} \right) \right] d\theta + \frac{t'}{\bar{\theta} - \theta} \int_{\theta'}^{\bar{\theta}} \left[ F \left( \frac{\eta^* - \theta}{\sigma} \right) - F \left( \frac{\xi^* - \theta}{\sigma} \right) \right] d\theta.
\]

\(\Delta_1\) and \(\Delta_2\) are the cost difference on the extensive margin and intensive margin respectively.

Notice \(E[C(\theta, s, t)]\) is linear in \(s\) and \(t\). Therefore, the expected cost of any full-participation program lies between the cost of the guarantee program \(\lambda_1 = 0\) with \((s, t) = \left( \frac{\theta^* - b_1}{1 - \theta^*}, \frac{c - \theta^* b_1}{1 - \theta^*} \right),\)

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and the pure subsidy program \( \lambda_2 = -\frac{c - \theta^* b}{1 - \theta^*}, \) with \((s, t) = (c - \theta^* b, 0)\). In the remaining part of the proof, we show that if either of the two conditions is satisfied, the proposed partial-participation program \((s', t') = (\frac{c - \theta^* b}{1 - \theta^*} + \theta^* \lambda, \frac{c - \theta^* b}{1 - \theta^*} + \lambda)\) with small positive \(\lambda\) has lower cost than both the guarantee program and the pure subsidy program.

Consider the pure subsidy program \((s, t) = (c - \theta^* b, 0)\). Plugging \((s, t)\) into the expression of \(\Delta_1\), we have

\[
\Delta_1 = \frac{\tau(c - \theta^* b)}{\theta - \bar{\theta}} \int_{\frac{1}{2}}^{\bar{\theta}} \left[ 1 - F\left(\frac{\eta^* - \theta}{\sigma}\right) \right] d\theta,
\]

\[
= \frac{\tau(c - \theta^* b)}{\theta - \bar{\theta}} \int_{\frac{1}{2}}^{\bar{\theta}} \int_{\frac{1}{2}}^{\theta + \frac{1}{2} \sigma} \frac{1}{\sigma} f\left(\frac{x - \theta}{\sigma}\right) dx d\theta,
\]

\[
= \frac{\tau(c - \theta^* b)}{\theta - \bar{\theta}} \int_{\frac{1}{2}}^{\theta + \frac{1}{2} \sigma} \int_{\frac{1}{2}}^{\theta} \frac{1}{\sigma} f\left(\frac{x - \theta}{\sigma}\right) d\theta dx,
\]

\[
= \frac{\tau(c - \theta^* b)}{\theta - \bar{\theta}} \int_{\frac{1}{2}}^{\theta + \frac{1}{2} \sigma} \left[ 1 - F\left(\frac{x - \theta}{\sigma}\right) \right] dx,
\]

\[
= \frac{\tau(c - \theta^* b)}{\theta - \bar{\theta}} \left[ \bar{\theta} + \frac{1}{2} \sigma - \eta^* - \sigma \int_{-\frac{1}{2}}^{\frac{1}{2}} F(y) dy \right] > \frac{\tau(c - \theta^* b)}{\theta - \bar{\theta}} (1 - \theta^*),
\]

which is strictly positive.

For \(\Delta_2\), notice

\[
\int_{\frac{1}{2}}^{\beta} \left[ F\left(\frac{\eta^* - \theta}{\sigma}\right) - F\left(\frac{\xi^* - \theta}{\sigma}\right) \right] d\theta = \int_{\frac{1}{2}}^{\beta} \int_{\frac{1}{2}}^{\eta^*} \frac{1}{\sigma} f\left(\frac{x - \theta}{\sigma}\right) dx d\theta,
\]

\[
= \int_{\frac{1}{2}}^{\eta^*} \int_{\frac{1}{2}}^{\beta} \frac{1}{\sigma} f\left(\frac{x - \theta}{\sigma}\right) d\theta dx,
\]

\[
= \int_{\frac{1}{2}}^{\eta^*} \left[ F\left(\frac{x - \alpha}{\sigma}\right) - F\left(\frac{x - \beta}{\sigma}\right) \right] dx,
\]
therefore

\[ \Delta_2 = \left( \frac{c - \theta^* b}{1 - \theta^*} + \varepsilon \right) \frac{1}{\theta - \overline{\theta}} \left[ \int_{\xi^*}^{\eta^*} F \left( \frac{x - \theta^*}{\sigma} \right) dx - \tau \theta^* (\eta^* - \xi^*) \right] , \]

\[ = \left( c - \theta^* b \right) \frac{\theta^* (\eta^* - \xi^*)}{\theta - \overline{\theta}} \left[ \int_{F^{-1}(\theta^*)}^{F^{-1}(\frac{s^*}{\sigma})} \frac{F(y)}{\theta^* (F^{-1}(\frac{s^*}{\sigma}) - F^{-1}(\theta^*))} dy - \tau \right] , \]

\[ = \left( c - \theta^* b \right) \frac{\theta^* (\eta^* - \xi^*)}{\theta - \overline{\theta}} \left[ G \left( \theta^*, \frac{s^*}{\sigma} \right) - \tau \right] . \]

Taking \( \lambda \) to 0, we have

\[ \lim_{\lambda \to 0^+} \Delta_2 = \left( c - \theta^* b \right) \frac{\theta^* (\eta^* - \xi^*)}{\theta - \overline{\theta}} \left[ G(\theta^*, 1) - \tau \right] = \frac{c - \theta^* b}{\theta - \overline{\theta}} \theta^* [G(\theta^*, 1) - \tau]. \]

If the first condition holds, \( \tau < G(\theta^*, 1) \), \( \lim_{\varepsilon \to 0^+} \Delta_2 > 0 \), \( \Delta_1 + \Delta_2 \) is strictly positive for small enough \( \lambda \). Also, if the second condition holds, \( \theta^* + \sigma < 1 \),

\[ \lim_{\lambda \to 0^+} \Delta_1 + \Delta_2 > \tau \left( \frac{c - \theta^* b}{\theta - \overline{\theta}} \right) (1 - \theta^* - \theta^* \sigma) > 0. \]

Now let’s turn to the guarantee program with \( s = t = \frac{c - \theta^* b}{1 - \theta^*} \). For \( \Delta_1 \), since \( \eta^* = \theta^* + \sigma F^{-1}(\frac{s^*}{\sigma}) < \theta^* + \frac{1}{2} \sigma \), we have

\[ \Delta_1 > \frac{(\tau - 1) s}{\theta - \overline{\theta}} \int_{\theta^*}^\overline{\theta} \left[ 1 - F \left( \frac{\theta^* + \frac{1}{2} \sigma - \theta}{\sigma} \right) \right] d\theta + \frac{\tau s}{\theta - \overline{\theta}} \int_{\theta^*}^\overline{\theta} \left[ 1 - F \left( \frac{\theta^* + \frac{1}{2} \sigma - \theta}{\sigma} \right) \right] d\theta \geq 0. \]

The last inequality is strict when \( \tau > 1 \). For \( \Delta_2 \), we have

\[ \Delta_2 = \frac{\sigma \theta^* (F^{-1}(\frac{s^*}{\sigma}) - F^{-1}(\theta^*))}{\theta - \overline{\theta}} \left[ G \left( \theta^*, \frac{s^*}{\sigma} \right) - \tau \right] . \]

If \( \tau > 1 \), \( \lim_{\lambda \to 0^+} \Delta_1 > 0 \), \( \lim_{\lambda \to 0^+} \Delta_2 = 0 \). Thus, \( \mathbb{E}_\theta[C(\theta, s, t)] - \mathbb{E}_\theta[C(\theta, s', t')] = \Delta_1 + \Delta_2 > 0 \) for small enough \( \lambda \).
If \( \tau = 1 \), since \( \frac{s'}{t'} > \frac{c}{b} > \theta^* \), \( G(\theta^*, \frac{s'}{t'}) > 1 = \tau \), \( \Delta_2 > 0 \) for any positive \( \lambda \). Combining with \( \Delta_1 \geq 0 \), we have \( \mathbb{E}_\theta[C(\theta, s, t)] - \mathbb{E}_\theta[C(\theta, s', t')] = \Delta_1 + \Delta_2 > 0 \) for any positive \( \lambda \).

To sum up, in either case, when \( \lambda \) being positive and small enough, the partial participation program \( (s', t') = (c - \theta^* b, \frac{c - \theta^* b}{1 - \theta^*} + \lambda) \) has lower expected cost than any full-participation program targeting \( \theta^* \).

**Proof of Proposition 11.** If we can choose \((s, t)\) properly such that \(0 < p^*_1(s, t) < p^*_2(s, t) < 1\), Lemma 5 implies in the unique Bayesian Nash equilibrium, agents follow a threshold strategy

\[
\begin{align*}
a_i &= 1, \text{Reject, if } x_i \geq \eta^*(s, t), \\
a_i &= 1, \text{Accept, if } \xi^*(s, t) \leq x_i < \eta^*(s, t), \\
a_i &= 0, \text{if } x_i < \xi^*(s, t),
\end{align*}
\]

where

\[
\begin{align*}
\xi^*(s, t) &= p^*_1(s, t) + \sigma F^{-1}(p^*_1(s, t)), \\
\eta^*(s, t) &= p^*_2(s, t) + \sigma F^{-1}(p^*_2(s, t)).
\end{align*}
\]

Moreover, \( \xi^*(s, t) \) and \( \eta^*(s, t) \) both converges to \( p^*_1(s, t) \) when \( \sigma \to 0 \). Thus, for any continuous belief of the fundamental held by the government, the ex-ante cost of the program converges to 0 when \( \sigma \to 0 \).

Now we want to show that it is possible to choose \((s, t)\) such that \(0 < p^*_1(s, t) < p^*_2(s, t) < 1\) and \( p^*_1(s, t) \) can be arbitrarily close to 0. Let \( s = c - \varepsilon \) and \( \frac{c + c^* - \gamma b}{1 - \gamma} < t < b \). The choice of
$t$ is feasible since Assumption 1b implies $\frac{c+e^{-\gamma_b}}{1-\gamma} < b$. Note $\frac{c+e^{-\gamma_b}}{1-\gamma} < t$ implies

$$p_2^*(s, t) = \frac{s}{\gamma_b + t(1-\gamma)} - e < \frac{c - \varepsilon}{c},$$

$$p_1^*(s, t) = \frac{c - s}{(1-\gamma)(b-t)} = \frac{\varepsilon}{(1-\gamma)(b-t)}.$$

Therefore, for any fixed $t$, when $\varepsilon \to 0$, $p_1^*(s, t)$ converges to 0 and $p_2^*(s, t)$ converges to a positive number which is strictly less than 1. □

**Proof of Proposition 12.** The proof is similar to the proof of Lemma 5. We want to find a sequence $\{(\xi_{g0}^n, \bar{\xi}_{g0}^n)_{g=1}^N\}_{n=0}^\infty$ such that the strategies of group $g$ agents survive $n$ rounds of iterated deletion of dominated strategies only if

$$a^g(x) = 0, \text{ if } x < \xi_{g0}^n, \quad (A.34)$$

and

$$a^g(x) = 1, \text{ if } x \geq \bar{\xi}_{g0}^n. \quad (A.35)$$

To simplify notations, let $\xi_n = (\xi_{g0}^n)_{g=1}^N$ and $\bar{\xi}_n = (\bar{\xi}_{g0}^n)_{g=1}^N$ be the vectors of threshold signals. The recursive expression for $\{(\xi_{g0}^n, \bar{\xi}_{g0}^n)_{g=1}^N\}_{n=0}^\infty$ is

$$\xi_{g0}^{n+1} = \inf_x \{x : p^g(x; \xi_n) \geq \frac{c}{b^g}\}, \quad (A.36)$$

$$\bar{\xi}_{g0}^{n+1} = \sup_x \{x : p^g(x; \bar{\xi}_n) \leq \frac{c}{b^g}\}. \quad (A.37)$$

We can prove by induction that

$$-\infty = \xi_0 < \xi_1 \leq \cdots \leq \xi_n \leq \cdots \leq \bar{\xi}_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \quad (A.38)$$

Since any bounded monotonic sequence has a finite limit, take $n$ to $\infty$, we have

$$\bar{\xi} \geq \xi. \quad (A.39)$$

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Now we want to show $\bar{\xi} = \xi$. It can be proved by contradiction. Suppose $\bar{\xi} > \xi$. Let $h$ be the group such that $\bar{\xi}^g - \xi^g = \max_g \{\bar{\xi}^g - \xi^g\} > 0$. Note that $\theta^*(\bar{\xi})$ is the solution to

$$\sum_{g=1}^N w^g m^g F^g \left( \frac{\bar{\xi}^g - \theta}{\sigma} \right) = \theta. \quad (A.40)$$

Therefore, $\theta^*(\bar{\xi}) - (\bar{\xi}^h - \xi^h)$ is the solution to

$$\sum_{g=1}^N w^g m^g F^g \left( \frac{\bar{\xi}^g - (\bar{\xi}^h - \xi^h) - \theta}{\sigma} \right) - \theta - (\bar{\xi}^h - \xi^h) = 0. \quad (A.41)$$

Also notice $\theta^*(\bar{\xi})$ is the solution to

$$\sum_{g=1}^N w^g m^g F^g \left( \frac{\xi^g - \theta}{\sigma} \right) - \theta = 0. \quad (A.42)$$

Let’s compare (A.41) and (A.42). Since $\bar{\xi}^g > \xi^g - (\bar{\xi}^h - \xi^h)$ and $\bar{\xi}^h - \xi^h > 0$, the left hand side of (A.42) is strictly larger than the left hand side of (A.41) for any given $\theta$. Given the left hand side of (A.42) is strictly decreasing in $\theta$, we must have $\theta^*(\bar{\xi}) - (\bar{\xi}^h - \xi^h) < \theta^*(\bar{\xi})$.

Therefore,

$$p^h(\bar{\xi}^h; \bar{\xi}) = P_{r^h}[\theta > \theta^*(\bar{\xi})|\xi^h],$$

$$= F^h \left( \frac{\bar{\xi}^h - \theta^*(\bar{\xi})}{\sigma} \right),$$

$$= F^h \left( \frac{\bar{\xi}^h - [\theta^*(\bar{\xi}) - (\bar{\xi}^h - \xi^h)]}{\sigma} \right),$$

$$> F^h \left( \frac{\xi^h - \theta^*(\theta^*(\bar{\xi}))}{\sigma} \right),$$

$$= p^h(\bar{\xi}^h; \bar{\xi}).$$

However, (A.36) and (A.37) implies $p^h(\bar{\xi}^h; \bar{\xi}) = p^h(\xi^h; \bar{\xi}) = \frac{c^h}{b^h}$. Contradiction. This implies $\bar{\xi} = \xi = \xi_0$.  

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To solve for \( \xi_0 \), note \( \xi_0 \) and \( \theta_0 \) are the solutions to

\[
\sum_{g=1}^{N} w^g m^g F^g \left( \frac{\xi^g - \theta}{\sigma} \right) = \theta, \quad (A.43)
\]

\[
F^g \left( \frac{\xi^g - \theta}{\sigma} \right) = \frac{c}{b^g}, \quad \text{for any } g = 1, \ldots, N. \quad (A.44)
\]

Plugging (A.44) into (A.43) we have

\[
\theta_0 = \sum_{g=1}^{N} m^g w^g \frac{c}{b^g}, \quad (A.45)
\]

\[
\xi^g_0 = \sum_{g=1}^{N} m^g w^g \frac{c}{b^g} + \sigma F^{-1}_g \left( \frac{c}{b^g} \right), \quad \text{for any } g = 1, \ldots, N. \quad (A.46)
\]

\[\blacksquare\]

**Proof of Proposition 13.** The optimal response of an agent in group \( g \) is

\[
a_i = 1, \text{Reject, if } \hat{p}_i \geq \frac{s}{t},
\]

\[
a_i = 1, \text{Accept, if } \frac{c - s}{b^g - t} \leq \hat{p}_i < \frac{s}{t},
\]

\[
a_i = 0, \text{if } \hat{p}_i < \frac{c - s}{b^g - t};
\]

We can apply the same method in the proof of Proposition 12 and show that in any equilibrium, agents of group \( g \) invest if and only if their private signal is greater or equal to

\[
\xi^*_g(s, t) = \sum_{g=1}^{N} m^g w^g \frac{c - s}{b^g - t} + \sigma F^{-1}_g \left( \frac{c - s}{b^g} \right). \quad (A.47)
\]

Given the investment thresholds, we know the fundamental threshold above which there
will be successful investment is

$$\theta^*(s, t) = \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g-t}.$$  
(A.48)

Therefore, the signal $\eta^*(s, t)$ that makes an agent from group $g$ indifferent between accepting and rejecting the intervention program is

$$\eta^*(s, t) = \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g-t} + \sigma F^{-1}_{g}(\frac{s}{t}).$$  
(A.49)

Proof of Proposition 14. Consider an agent who receives private signal $x$ and knows that all other agents invest if and only if observing private signal $k$. The expected payoff from investing is

$$U(k, x) = \int_{\theta} f \left( \frac{x-\theta}{\sigma} \right) \pi \left( \theta, 1 - F\left( \frac{k-\theta}{\sigma} \right) \right) d\theta$$

Note that $U(k, x)$ weakly decreases in $k$ and weakly increases in $x$. Intuitively, an agent has higher expected payoff if everyone else is more willing to invest or the agent receives a high signal indicating a high fundamental $\theta$. Also note that $U(-\infty, x) < 0$ for $x < \theta_0$ and $U(+\infty, x) > 0$ for $x > \theta_1$.

Next we prove the uniqueness of equilibrium by iterated deletion of dominated strategies. The strategy profile of an agent is the action as a function of the private signal received. We denote it by $a(x) : \mathbb{R} \to \{0, 1\}$. We will prove that strategy survives $n$ rounds of iterated
deletion of dominated strategies if and only if

\[ a(x) = 0, \text{ if } x < \xi_n, \quad \text{(A.50)} \]

and

\[ a(x) = 1, \text{ if } x \geq \bar{\xi}_n. \quad \text{(A.51)} \]

where \( \{(\xi_n, \bar{\xi}_n)\}_{n=0}^{\infty} \) satisfies

\[ -\infty = \xi_0 < \xi_1 \leq \cdots \leq \xi_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \quad \text{(A.52)} \]

This result can be proved by induction. Let the starting node be \( \xi_0 = -\infty \) and \( \bar{\xi}_0 = +\infty \), meaning that there is no restrictions on agents’ strategy. Suppose round \( n \in \mathbb{N} \) of deletion has been completed. In round \( n + 1 \), the most optimistic belief for an agent is that all other agents follow a cutoff strategy with threshold \( \xi_n \). Therefore, for any \( x \) such that \( U(\xi_n, x) < 0 \), \( a(x) = 1 \) is strictly dominated by \( a(x) = 0 \). Similarly, the most pessimistic belief for an agent is that all other agents follow a cutoff strategy with threshold \( \bar{\xi}_n \). As a result, for \( x \) such that \( U(\bar{\xi}_n, x) > 0 \), any strategy profile with \( a(x) = 0 \) is strictly dominated by \( a(x) = 1 \).

Given \( U(k, x) \) is non-decreasing in \( x \), the strategy profiles that survives deletion of dominated strategies must satisfy the restrictions in (A.50) and (A.51), with \( (\xi_{n+1}, \bar{\xi}_{n+1}) \) defined inductively as

\[ \xi_{n+1} = \inf \{ x : U(\xi_n, x) \geq 0 \} \quad \text{(A.53)} \]

and

\[ \bar{\xi}_{n+1} = \sup \{ x : U(\bar{\xi}_n, x) \leq 0 \} \quad \text{(A.54)} \]

The monotonicity of \( U(k, x) \) guarantees that \( \xi_{n+1} \leq \bar{\xi}_{n+1} \). Note that the dominance region
assumption implies that $\xi_1 > -\infty$ and $\bar{\xi}_1 < +\infty$. Therefore, $\{(\xi_n, \bar{\xi}_n)\}_{n=0}^{\infty}$ is a well-defined sequence of real couples which satisfies (A.52).

Now we’ve proved that $\{\xi_n\}_{n=1}^{\infty}$ and $\{\bar{\xi}_n\}_{n=1}^{\infty}$ are both monotonic and bounded sequences. Thus, they converges to two finite numbers $\underline{\xi}$ and $\bar{\xi}$ respectively when $n \to \infty$. The definition (A.53) and (A.54) imply that $U(\xi, \xi) \geq 0$ and $U(\bar{\xi}, \bar{\xi}) \leq 0$. Notice for $y \in [\theta_0, \theta_1]$,

$$U(y, y) = \int_{\theta_0}^{\theta_1} \frac{1}{\sigma} f \left( \frac{y - \theta}{\sigma} \right) \pi \left( y, 1 - F \left( \frac{y - \theta}{\sigma} \right) \right) d\theta = \int_{0}^{1} \pi(y, l) dl,$$

strictly increases in $y$ and $\underline{\xi} \leq \bar{\xi}$, it must be the case that $U(\underline{\xi}, \underline{\xi}) = U(\bar{\xi}, \bar{\xi}) = 0$.

Since $U(y, y)$ is continuous in $y$, $U(\theta, \bar{\theta}) \leq 0$, $U(\bar{\theta}, \bar{\theta}) \geq 0$, there is a unique solution to $U(y, y) = \int_{0}^{1} \pi(y, l) dl = 0$. Denote the solution by $\xi_0^*$, and we have $\underline{\xi} = \bar{\xi} = \xi_0^*$. Therefore, the only strategy that survives the iterated deletion of dominated strategies is the cutoff investment strategy with cutoff $\xi_0^*$.

**Proof of Proposition 15.** Consider an agent who receives private signal $x$ and knows that all other agents invest if and only if their signal is above $k$. The expected payoff from investing and rejecting the intervention offer is

$$U^R(k, x) = \int_{\theta_0}^{\theta_1} \frac{1}{\sigma} f \left( \frac{x - \theta}{\sigma} \right) \pi \left( \theta, 1 - F \left( \frac{x - \theta}{\sigma} \right) \right) d\theta.$$

The expected payoff from investing and accepting the offer is

$$U^A(k, x) = (1 - t)U^R(k, x) + s$$

Therefore, the maximum expected payoff from investing is

$$U(k, x) = \max\{U^R(k, x), U^A(k, x)\}$$

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We prove a lemma that will be useful later.

**Lemma 6** Given that all other agents invest if and only if their signal is above \( k \), there exist two functions \( k^*_1(k) \) and \( k^*_2(k) \) such that an agent strictly prefers not investing if her private signal \( x < k^*_1(k) \) and strictly prefers investing if \( x > k^*_2(k) \). \( k^*_1(k) \) and \( k^*_2(k) \) are given by

\[
\begin{align*}
k^*_1(k) &= \inf \left\{ k^* : UR(k, k^*) \geq -\frac{s}{1-t} \right\}, \\
k^*_2(k) &= \sup \left\{ k^* : UR(k, k^*) \leq -\frac{s}{1-t} \right\},
\end{align*}
\]

Both \( k^*_1(k) \) and \( k^*_2(k) \) are weakly increasing in \( k \).

**Proof of Lemma 6.** The Left Dominance Region assumption in Definition 2 and Limit Dominance in Assumption 3 make sure that the two function \( k^*_1(k) \) and \( k^*_2(k) \) are well defined. By continuity of \( UR(k, x) \) in \( x \), we have

\[
UR(k, k^*_1(k)) = UR(k, k^*_2(k)) = -\frac{s}{1-t}.
\]

For any \( x < k^*_1(k) \), \( UR(k, x) < -\frac{s}{1-t}, \) \( UA(k, x) = (1-t)UR(k, x) + s < 0 \). Therefore, \( U(k, x) = \max\{UR(k, x), UA(k, x)\} < 0 \), the agent will not invest if observing \( x < k^*_1(k) \).

On the other hand, for any \( x > k^*_2(k) \), \( UR(k, x) > -\frac{s}{1-t}, \) \( UA(k, x) = (1-t)UR(k, x) + s > 0 \). Therefore, \( U(k, x) = \max\{UR(k, x), UA(k, x)\} > 0 \), the agent will invest after observing signal \( x > k^*_2(k) \).

Since \( UR(k, x) \) is weakly decreasing in \( k \), we can easily show that both \( k^*_1(k) \) and \( k^*_2(k) \) are weakly increasing in \( k \). ■

With Lemma 6, we can prove the uniqueness of equilibrium by iterated deletion of dominated strategies. Denote the investment strategy by \( a(x) \). We want to show a strategy survives \( n \)
rounds of iterated deletion of dominated strategies if and only if

\[ a(x) = \begin{cases} 
0, & \text{if } x < \xi_n, \\
1, & \text{if } x > \bar{\xi}_n, 
\end{cases} \]

where \( \xi_0 = -\infty \), \( \bar{\xi}_0 = \infty \). \( \xi_n \) and \( \bar{\xi}_n \) are defined inductively by \( \xi_{n+1} = k_1^*(\xi_n) \), \( \bar{\xi}_{n+1} = k_2^*(\bar{\xi}_n) \).

Since \( k^*(\xi) \) increases in \( \xi \), \( \xi_n \) and \( \bar{\xi}_n \) are increasing and decreasing sequences, respectively. As \( n \to \infty \), \( \xi_n \to \xi \) and \( \bar{\xi}_n \to \bar{\xi} \). Therefore, \( \xi = k_1^*(\xi) \) and \( \bar{\xi} = k_2^*(\bar{\xi}) \). \( \xi \) and \( \bar{\xi} \) must both be the solution to

\[ U_R(\xi, \xi) = -s \frac{s}{1 - t}. \]

Let \( l = 1 - F\left(\frac{\xi - \theta}{\sigma}\right) \), the equation can be written as

\[ \int_0^1 \pi(\xi, l) \, dl = -s \frac{s}{1 - t}. \quad (A.55) \]

By Strict Laplacian State Monotonicity in Assumption 3, the left hand side is continuous and strictly increasing in \( \xi \). Also, \( \int_0^1 \pi(\theta, l) \, dl < -s \frac{s}{1 - t}, \int_0^1 \pi(\bar{\theta}, l) \, dl > 0 \geq -s \frac{s}{1 - t} \), there is a unique solution to the equation above, \( \xi = \xi = \xi^* \). Notice \( \xi^* \) is independent of \( \sigma \). Then by iterated deletion of dominated strategies, it is the unique investment cutoff in equilibrium.

Given the investment cutoff, we can solve for the private signal \( x \) such that \( U^A(\xi^*, x) = U^R(\xi^*, x) \), or equivalently \( U^R(\xi^*, \eta^*(\sigma)) = \frac{s}{t} \). Let \( \eta^*(\sigma) \) be the maximum value that satisfies

\[ U^R(\xi^*, \eta^*) = \int_{\xi}^{\eta^*} \frac{1}{\sigma} f\left(\frac{\eta^* - \theta}{\sigma}\right) \pi \left(\eta^*, 1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right) \, d\theta = \frac{s}{t}. \quad (A.56) \]

For any signal \( x > \eta^*(\sigma) \), an agent strictly prefers investing and not participating in the intervention program. Notice when \( x > \xi^* + \sigma, U^R(\xi^*, x) = \pi(x, 1) > \frac{s}{t} \), therefore, \( \eta^*(\sigma) \) is
well-defined.

Since $U^R(k, x)$ increases in $x$, and $U^R(\xi^*, \xi^*) = -\frac{s}{1-t} \leq \frac{s}{t} = U^R(\xi^*, \eta^*(\sigma))$, therefore, $\eta^*(\sigma) \geq \xi^*$. It immediately follows that $\lim_{\sigma \to 0} \eta^*(\sigma) = \eta \geq \xi^*$. Next, we prove $\eta = \xi^*$ by contradiction. Suppose $\eta > \xi^*$, take $\sigma \to 0$ in the left hand side of (A.56), we have

$$
\lim_{\sigma \to 0} \int_0^\theta \frac{1}{\sigma} f \left( \frac{\eta^*(\sigma) - \theta}{\sigma} \right) \pi \left( \eta^*(\sigma), 1 - F \left( \frac{\xi^* - \theta}{\sigma} \right) \right) d\theta = \pi \left( \eta, 1 \right) \geq \pi \left( \xi^*, 1 \right) \geq \frac{s}{t}.
$$

Contradiction to (A.56). Therefore, $\lim_{\sigma \to 0} \eta^*(\sigma) = \eta = \xi^*$.

**Proof of Proposition 16.** According to Definition 2, a partial-participation program with target $\xi^*$ should satisfy the following conditions

1. $\pi(\theta, 1) < -\frac{s}{1-t}$
2. $\pi(\xi^*, 1) > \frac{s}{t}$
3. $\int_0^1 \pi(\xi^*, l) dl = -\frac{s}{1-t}$

As long as the government offers $(s, t)$ given by

$$
\left( -\frac{\pi(\xi^*, 1)}{\int_0^1 \pi(\xi^*, l) dl} + 1 \right)^{-1} < t < 1,
$$

(A.57)

and

$$
s = -(1 - t) \int_0^1 \pi(\xi^*, l) dl,
$$

(A.58)

the three conditions listed above are satisfied. First, by assumption, $\pi(\theta, 1) \leq \int_0^1 \pi(\theta_0, l) dl < \int_0^1 \pi(\xi^*, l) dl = -\frac{s}{1-t}$. Second, (A.57) can be written as $\pi(\xi^*, 1) > -\frac{1-t}{s} \int_0^1 \pi(\xi^*, l) dl = \frac{s}{t}$. Finally, the third condition directly follows equation (A.58).


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