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Robot Assembly: Another Source of Nonholonomic Control Problems

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Abstract

Assembly problems require that a robot with a few actuated degrees of freedom manipulate an environment with a greater number of unactuated degrees of freedom. Since the dynamical coupling between degrees of freedom in this setting is a function of their relative configuration, the motion of such systems is subject to constraints that preclude smooth feedback stabilization. This paper explores the extent to which assembly planning and control may be effected by recourse to some other methodical means of generating stabilizing feedback controllers. A partial solution is offered for a very simple assembly problem involving an intermittent dynamical environment.

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Robot Assembly: Another Source of Nonholonomic Control Problems

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Abstract

Assembly problems require that a robot with a few actuated degrees of freedom manipulate an environment with a greater number of unactuated degrees of freedom. Since the dynamical coupling between degrees of freedom in this setting is a function of their relative configuration, the motion of such systems is subject to constraints that preclude smooth feedback stabilization. This paper explores the extent to which assembly planning and control may be effected by recourse to some other methodical means of generating stabilizing feedback controllers. A partial solution is offered for a very simple assembly problem involving an intermittent dynamical environment.

1 Introduction

In the course of our robot juggling research [5] we have recently reported experimental success in transferring a two degree of freedom body falling freely in the earth's gravitational field from an arbitrarily chosen initial state to a specified zero velocity state via intermittent impacts with a one degree of freedom robot [4]. Our control strategy commands first a (provably correct) juggling algorithm [3] designed to bring the body to some known periodic trajectory and then switches to a catching algorithm designed (but not yet formally proven) to bring the body to rest from the known intermediate oscillatory conditions. Empirical success notwithstanding, one would prefer, in general, some methodically chosen controller. One might inquire, for instance, why this apparently ad hoc strategy should be employed, and

whether other intuitively devised schemes might succeed equally well. In particular, it seems undesirable to rely upon a switched combination of two feedback laws when a single smooth controller might do the job.

Transferring a system of bodies possessing a greater number of degrees of freedom than the number of actuated degrees of freedom available from one configuration to another is an assembly problem. Note that the possibility of solving such problems depends on the fact that the dynamical coupling between degrees of freedom is not fixed. More precisely, the allowable infinitesimal motions of the body system differ from configuration to configuration. Thus, since the motion of the bodies is subject to constraints that are not integrable, assembly problems give rise to nonholonomically constrained control systems. These seem to differ from the more commonly studied nonholonomic robotics problems involving steering wheels [11] or coupled vehicles [9] in that here the tangent bundle constraints are (at best) C^∞ rather than analytic. They seem to be very similar in most other respects, and there is good reason to hope that a principled treatment of one class should shed light on the other. One significant feature of such problems is that even while the control systems that arise are completely controllable, Brockett's [2] stabilizability condition is not met [1], and there is no possibility of constructing a single smooth feedback controller to stabilize an isolated equilibrium state.

These preliminary remarks now place the juggling problem in a clearer context. While our empirically successful "materials handling" strategy has still little pretension to systematic origins, it is at least clear that some kind of "switched" control policy will be unavoidable. In a recent paper introducing the assembly problem [8], the author has studied a very simple ex-

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ample — the *One Degree of Freedom Unit Assembly Problem* — in a quasi-static environment. That is to say, the Newtonian dynamics of a free body are replaced with Whitney’s “generalized damper” dynamics [12] as is common in the assembly planning literature [10], the system is shown to be completely controllable and not stabilizable, and a switched controller derived (relatively) methodically is shown to solve a partial version of the problem.

This paper represents a first effort to come to grips (so to speak) with the possibility for justification (or improvement) of our experimental success in “materials handling” through juggling and catching via a formal examination of the attendant assembly problem. Section 2 places the One Degree of Freedom Unit Assembly Problem back in a dynamical setting — the free body obeys Newton’s laws — and demonstrates that any physically reasonable model of contact between the robot and body gives rise to a dynamical model which is again completely controllable but not feedback stabilizable. To simplify the synthesis problem, it is convenient to pass to an impulse model of contact, thereby fixing the formal representation of the problem as occurring within an intermittent dynamical environment in conformity with our ongoing juggling work [5]. A partial solution to this assembly problem is offered in Section 3. The paper concludes with a brief speculation about where these ideas may lead.

2 The One Degree of Freedom Unit Assembly Problem

Consider a point unit mass body restricted to a single horizontal axis of motion, $b \in \mathbb{R}$, governed, in isolation, by Newton’s law, $\ddot{b} = 0$. Consider as well a point unit mass robot restricted to a single axis of motion parallel¹ to that of the body, $r \in \mathbb{R}$. The robot is controlled by a force we are free to command, ϕ , and its motion is governed by Newton’s laws, $\ddot{r} = \phi$. Let the robot have a position controlled “gripper”, $\text{GRIP}(t)$, which, when extended, ($\text{GRIP} = 1$) is capable of engaging the body when the two point masses are aligned at the same relative positions on their respective axes.

¹It would be more appropriate to place the robot and body in the same configuration space. This would merely complicate the gripper model in the present circumstances since a model of pulling would necessarily be required in addition to the simple model of pushing — impacts — introduced below. In higher degree of freedom problems this complication will not arise and the robot and bodies may be located in the same space.

2.1 The Task

It is desired to place the body at a specified goal location. The problem at hand is to devise an autonomous strategy for the robot that will enable it to move toward the body, “grab it”, and place it in the arbitrarily designated new location, and proceed to its nest. More formally, we seek a feedback controller, $(\phi, \text{GRIP})(t) = G(r, \dot{r}, b, \dot{b}, \dots)(t)$ which causes the robot to bring the body to the specified goal assuming arbitrary initial conditions. There are two versions of the problem. In the first instance, the robot’s state is left unspecified after the body has reached the goal. In the second instance, the robot is commanded to return to some specified nest state as well. This paper provides a smooth feedback controller, G that solves the first version — “half” — of the unit assembly problem. It will be seen that the second version cannot be effected via a smooth feedback controller because of the intrinsic nonholonomic constraints that arise. This is the problem solved by empirical demonstration in [4].

2.2 Contact Models

This section addresses the issue of how to represent the contact event between body and robot gripper. Dynamical assembly interactions connote in their simplest instance contact conditions between body and robot that are of extremely short duration and involve very large forces. In the limit, such impact conditions lead to the notion of impulses introduced to maintain inequality constraints in configuration [7]. In the present setting it will prove useful to work with both the continuous and impulsive models of contact force. To maintain some generality in the later controllability/stabilizability discussion a general force model of contact is first presented that makes the minimal assumptions necessary to realize the required physical constraints. For purposes of controller synthesis it proves convenient to replace this general force law with an impulse model of instantaneous collision.

2.2.1 Force Model

Contact forces, $c(\delta, \dot{\delta})$, depend upon the relative configuration, $\delta = b - r$. Roughly speaking, they obtain when the body and robot’s gripper approach each other in close proximity. That is, given some small “contact region”, $\epsilon > 0$, we will require

$$c(\delta, v) \begin{cases} > 0 & 0 < \delta < \epsilon, v < 0 \\ < 0 & -\epsilon < \delta < 0, v > 0 \\ = 0 & \text{otherwise} \end{cases} \quad (1)$$

according to the arguments of [7, §2.3.B.ii.c]. Clearly such a function cannot be analytic, but it is expedi-

ent to assume smoothness in order to avoid technical issues of existence and uniqueness in the sequel.²

As matters stand, this crude model does not prohibit the possibility that sufficiently large and opposing velocities might give rise to trajectories whose relative configuration changes sign. Such trajectories would violate the physical principle that two bodies cannot occupy the same space at the same time. This principle may be enforced by letting c become unbounded as $\delta \rightarrow 0$, or, effectively the same, assuming that the initial kinetic energy of the joint system is limited (and that c does not add energy to the system) as a function of the maximal value of c .

2.2.2 Impulse Model

In the limit, as $|c| \rightarrow \infty$, the time over which the contact forces are engaged becomes vanishingly small, and the integral of work done remains a finite value called the “impulse” [7, §2.5]. Energy is generally lost (the contact forces are not conservative) during the impact, and this leads to the notion of a coefficient of restitution [7, §4.2],

$$\dot{b}_F - \dot{r}_F = -\alpha (\dot{b}_I - \dot{r}_I),$$

where the subscripts I, F , denote initial and final velocities respectively. Generally speaking, one knows the initial velocities and desires to predict the final velocities. To do so, introduce one last assumption, $\dot{r}_I = \dot{r}_F$, indicating the ability of the robot’s controlled source of external forces to effectively cancel out any impact effects. This implies

$$\dot{b}_F = -\alpha \dot{b}_I + (1 + \alpha) \dot{r}. \quad (2)$$

2.3 The Control Problem

This section addresses the systems theoretic properties of the coupled robot-body system that arise from the models introduced above. The general force model is used to show that while this system is completely controllable, it is not stabilizable — that is, no isolated equilibrium state may be made asymptotically stable via a smooth feedback law, G . The impulse model is then used to derive an abstracted representation of the robot-body interaction that we have previously termed the *Environmental Control Problem* [5].

2.3.1 Qualitative Properties of the Force Model

The robot is controlled by a perfect (but bounded) force, ϕ . Dynamical coupling occurs through contact

²The function cannot even be differentiable at the origin, but it will prove necessary to avoid this regime anyway through the methods below.

forces (1) arising from the robot’s gripper engaging the body: if the robot exerts a force c on the body then the body exerts a force, $-c$, on the robot. But this contact occurs only if the robot’s gripper is engaged. The simplest representation of this situation may be given by multiplying the force law, c , by the multiplicative decision variable, GRIP , which, when zero, precludes any dynamical coupling at all. The coupled system may be modeled as

$$\begin{aligned} \ddot{r} &= -\text{GRIP} \cdot c + \phi \\ \ddot{b} &= \text{GRIP} \cdot c. \end{aligned}$$

Denoting the joint state vector as $x = [b, \dot{b}, r, \dot{r}]^T$, this dynamical system may be rewritten as

$$\dot{x} = \begin{bmatrix} x_2 \\ -\text{GRIP} \cdot c(x_1 - x_3, x_2 - x_4) \\ x_4 \\ \text{GRIP} \cdot c(x_1 - x_3, x_2 - x_4) + \phi \end{bmatrix} \quad (3)$$

It is easy to see that this system is completely controllable. Given initial and final states, x_I, x_F , respectively, set $\text{GRIP} = 0$, and transfer the decoupled robot (double integrator) system to match the state of the body at some future time, t_1 , where matching means bringing the robot to the boundary of the ϵ region of contact at a matching velocity so that the robot and the body’s goal point form an interval containing the body’s present location, $x_{I1} + t_1 x_{I2} \in [x_{F1}, x_3(t_1)]$. Now engage the gripper (set $\text{GRIP} = 1$), cancel the effects of contact on the robot (set $\phi = -c + \hat{\phi}$), and drive the body’s state until time t_2 in such a manner that it will “drift” to the desired final state, (x_{F1}, x_{F2}) at time t_3 . This may be done by using $\hat{\phi}$ to control the relative position and velocity of the robot and body, thereby employing c as a control input to the body. Disengage the gripper at time t_2 and bring the robot to its desired final state at t_3 via the input ϕ .

Lemma 1 *The coupled system (3) is not smoothly stabilizable. to any isolated equilibrium state, x^* for which $|x_1^* - x_3^*| > \epsilon$.*

Proof: According to Brockett’s observation [2], the vector field must be onto some neighborhood of the origin. But $c \equiv 0$ in a neighborhood every hypothesized x^* , thus the second component of the vector field is always zero on that neighborhood, and it cannot be onto any open ball about 0.

□

2.3.2 Impulse Model

Letting $\dot{r}(n)$ denote the velocity of the robot at impact n and $\Delta t_n \triangleq t_{n+1} - t_n$ denote the time the robot

allows to elapse between impact n and impact $n + 1$, we have

$$\begin{aligned} b(n+1) &= b(n) + \Delta t_n \left(-\alpha \dot{b}(n) + (1 + \alpha) \dot{r}(n) \right) \\ \dot{b}(n+1) &= -\alpha \dot{b}(n) + (1 + \alpha) \dot{r}(n). \end{aligned} \quad (4)$$

While both Δt_n and $\dot{r}(n)$ are formally control inputs, it will suffice for the present purposes to concentrate on the second. Thus, denoting

$$w(n) = \begin{bmatrix} b(t_n) \\ \dot{b}(t_n) \end{bmatrix}; \quad u(n) = \dot{r}(t_n) - \dot{b}(t_n),$$

the system may be equivalently rewritten in the form

$$\begin{aligned} w(n+1) &= f(w(n), u(n)) \\ f(w, u) &= Aw + hu \\ A &= \begin{bmatrix} 1 & \Delta t_n \\ 0 & 1 \end{bmatrix} \\ h &= \begin{bmatrix} \Delta t_n \\ 1 \end{bmatrix} (1 + \alpha) \text{ GRIP}. \end{aligned}$$

This system is analogous to what we have termed the environmental control system in our juggling work [5]. It's dynamics express all the logically possible outcomes of robot-body collisions abstracting away any consideration of the robot's intrinsic dynamics.

We now seek an "impact schedule"—that is, a function, $g : \mathcal{W} \rightarrow \mathcal{U}$ such that the closed loop dynamics $f_g(w) = f(w, g(w))$ has a globally asymptotically stable goal state, w^* . Assuming a constant choice of t_n , this problem is readily solved via an appeal to linear systems theory. For example, it is easy to see that

$$w^* = Aw^* + hu$$

has a solution, u , if and only if $w_2^* = 0$. Thus, not surprisingly, only zero velocity goal points may be made asymptotically stable fixed points by an appropriate choice of g . Notice that f is invariant under all zero-velocity translations, that is, if $T(w) \triangleq w - (w_1^*, 0)$ then $T \circ f(T^{-1}(w), u) = f(w, u)$, so there is no loss of generality in assuming $w_1^* = 0$ as well. Finally, since this system is completely controllable,

$$\text{Det}[h, Ah] = (1 + \alpha)^2 t_n,$$

it is now clear that any fixed point stabilization problem may be solved by an appropriate choice of gains,

$$g(w) = k^T w; \quad k = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}. \quad (5)$$

3 Solving the Partial Assembly Problem

In our juggling problem [5] the environmental control system (4) was nonlinear in state as well as input, and

linearized feedback strategies produced closed loop systems, f_g with unacceptably small domains of attraction [3]. It was necessary to pass to a nonlinear strategy. In the present case, there can be no objection to a linear feedback law since an asymptotically stable equilibrium state attracts the entire state space. However the difficulties in implementing the abstract impact schedule — in measuring the body state just at time of impact; in controlling the robot's velocity accurately enough at the exactly right time and place required — and the absolute reliance upon the abstract dynamical model implied by the map g turned out to degrade juggling performance considerably.

In contrast, extremely reliable juggling behavior with very strong robustness properties emerged from forcing the robot to track a reference trajectory obtained by passing the body's state trajectory through a carefully chosen memoryless nonlinearity [5, 3]. Thus, in setting up the present problem in Section 2, we have explicitly required that the robot's feedback controller, G be a function of the continuous time robot-body state.

3.1 Augmented Reference Dynamics

Forcing the robot to track the output of a memoryless nonlinear transformation of the body's state,

$$r(t) = \mu(b, \dot{b})(t),$$

achieved juggling in an intermittent dynamical environment forced by the earth's gravitational field [5]. In this setting, the same strategy will not work. For example, if $\dot{b}(0) = 0$ but $b \neq 0$ then $\dot{r}(0) = 0$, and the first (and all succeeding) impacts to the environmental control system (4) are zero, and no motion occurs.

This problem will be circumvented by adding memory to the controller. Consider the second order dynamical system

$$\dot{a} = \begin{bmatrix} 0 & 1 \\ -\omega^2(1 + \beta^2) & -2\omega\beta \end{bmatrix} a + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d$$

where both ω, β are positive so that the system has complex conjugate poles and oscillatory solutions obtain. Forcing this system with the output of the body,

$$d \triangleq [\omega(1 + \beta^2), 2\omega\beta] \begin{bmatrix} b \\ \dot{b} \end{bmatrix},$$

and defining error coordinates as $(e_1, e_2) = (b, \dot{b}) - (a_1, a_2)$ implies

$$\dot{e} = \begin{bmatrix} 0 & 1 \\ -\omega^2(1 + \beta^2) & -2\omega\beta \end{bmatrix} e.$$

It is now convenient to introduce a change of basis,

$$W \triangleq \begin{bmatrix} \omega\sqrt{1+\beta^2} & \frac{\beta}{\sqrt{1+\beta^2}} \\ 0 & \frac{1}{\sqrt{1+\beta^2}} \end{bmatrix},$$

to the complex normal form [6],

$$\bar{A} \triangleq WAW^{-1} = \omega \begin{bmatrix} -\beta & 1 \\ -1 & -\beta \end{bmatrix},$$

since the resulting polar coordinate,

$$\theta(e) \triangleq \arctan \left(\frac{[0, 1]We}{[1, 0]We} \right),$$

displays the oscillatory dynamics in a particularly simple form,

$$\dot{\theta} = -\omega.$$

Assuming that the augmented dynamics are initially relaxed, $a(0) = (0, 0)$, relative motion is guaranteed in the error system as long as the body is not placed at the goal, $(b(0), \dot{b}(0)) \neq 0$. It follows that $\dot{\theta}$ is a nonzero constant, hence, θ has a zero crossing every $t_I = \pi/\omega$ seconds.

3.2 A Mirror Law for the Augmented Reference Dynamics

Defining the “mirror strategy”

$$\begin{aligned} \mu((b, \dot{b}), e)(t) &\triangleq \kappa_1(e)b + \kappa_2(e)\dot{b} \\ \kappa_1(e) &\triangleq (k_1/\omega)\theta(e) + 1 \\ \kappa_2(e) &\triangleq (k_2/\omega)\theta(e) \end{aligned}$$

now implies that the robot will “overtake” the body, $r(t) = \mu((b, \dot{b}), e)(t) = b(t)$, if and only if

$$k^T \begin{bmatrix} b \\ \dot{b} \end{bmatrix} (t) = 0 \text{ or } \theta(e)(t) = 0.$$

Now assign the gripper the strategy

$$\text{GRIP}(t) = \begin{cases} 0 & : \quad k^T \begin{bmatrix} b \\ \dot{b} \end{bmatrix} = 0 \\ 1 & : \quad k^T \begin{bmatrix} b \\ \dot{b} \end{bmatrix} \neq 0 \end{cases}$$

so that the robot cannot engage the body unless $\theta = 0$, and this occurs at time $t_n = nt_I, n = 0, 1, 2, \dots$. Thus, Δt_n in (4), is effectively set uniformly to t_I . At the moment of collision, the robot has the velocity

$$\dot{r}(t_n) = k_1 b(t_n) + (k_2 + 1)\dot{b}(t_n),$$

and this implies

$$\begin{aligned} u(n) &= \dot{r}(t_n) - \dot{b}(t_n) \\ &= k_1 b(t_n) + k_2 \dot{b}(t_n) \\ &= k^T w(n), \end{aligned}$$

realizing³ any desired linear impact schedule (5).

³Note that the gripper is disengaged, $\text{GRIP} = 0$, at time t_n , if and only if the desired impact schedule called for a zero impulse to be applied anyway.

4 Conclusion

The mirror law proposed above solves the partial one degree of freedom unit assembly problem. Any (standard) asymptotically exact tracking method that forces the robot to track this μ results in a trajectory delivering the necessary impact schedule to guarantee that every initial error in actual body state be asymptotically reduced to zero through repeated impacts. It must be emphatically stated that this solution *does not* address the more fundamental problem of stabilizing a nonholonomically constrained system. The partial problem requires stabilization to a subspace (i.e. bring the body-robot system to the body’s goal) to which there are no smoothness obstructions. The full problem (i.e., bring the body-robot system to the body’s goal and the robot’s nest) remains.

A solution to the full problem should presumably cause the robot to nest if and only if the body is at its goal point. How this should be accomplished is not entirely clear at present. For one would wish to generalize such strategies to encompass both more bodies and higher degrees of freedom. The latter generalization seems reasonably achievable within the present context. The former generalization seems to require a more methodical means of point stabilization in non-holonomically constrained systems.

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