Responsive Math Teaching: Planning and Coaching Protocol

Responsive Math Teaching Project

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Responsive Math Teaching: Planning and Coaching Protocol

Abstract
The Responsive Math Teaching Project’s (RMT) Planning and Coaching Protocol is an 18-page booklet that includes the RMT Instructional Model as well as a planning and coaching guide for each phase of the RTM instructional cycle. These guides provide questions and facilitation moves to support educators in the planning, implementing, coaching, and/or reflecting on an RMT lesson.

Keywords
education, teaching, math, teacher learning, professional development, math education, mathematics, instructional model, math classroom, instruction, planning, coaching

Disciplines
Education

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PLANNING AND COACHING PROTOCOL

Mathematical Goal

Launch

Facilitate Productive Struggle

Individual/Small Groups

Whole Group

Return to Mathematical Goal

Discuss Learner Thinking

Whole Group

Reflect

Learner

Plan

Teacher Preparation

Community

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• What is your mathematical goal?

• Do you need to make adjustments based on evidence from prior lessons?

• What task will you use? Does it need revisions?
  • Is it low-floor, high-ceiling? Does it allow for multiple approaches? If not, how can you open the problem up?
  • Will it help get to your mathematical goal?
  • Is it accessible to your students? Possible revisions include:
    • changing the numbers
    • changing the context
    • changing the conditions

• What do you anticipate students will think?
  • What strategies/ideas will students use/have?
  • What stumbling blocks are they likely to encounter?
  • What representations could you use to help make
• How will you help learners begin to **make sense of the task individually**?

• How will you **establish collective understanding** of the task and context (without telling them how to do it)? Think about:
  • eliciting prior knowledge, clarifying context and vocabulary, and representing important information

• How will you **build a bridge to solving**? Think about:
  • communicating expectations for work, products, and tools
  • assessing individual and class readiness to engage in productive struggle
<table>
<thead>
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<th>LAUNCH</th>
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<td><strong>How will you help learners begin to make sense of the task individually?</strong></td>
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| Give multiple ways to **process the task** (e.g. hearing it read out loud, reading silently, choral reading, different voices) | • Listen carefully. I’m going to read it twice.  
• Your only job right now is to read the problem and try to make sense of it yourself.  
• Feel free to write on the problem to help you make sense of it.  
• Lizzy, will you read the problem out loud? |
| When developmentally appropriate, encourage learners to **write notes** about their thinking |  |
| Have learners describe task or problem in their own words | • Who wants to try summarizing the problem in their own words?  
• What are we being asked to figure out? |
| Ask learners to **visualize the problem** and describe what is happening in the situation | • What is happening in this situation? What’s going on here? |
| **How will you establish collective understanding** of the task and context (without telling them how to do it)? |
| Elicit and/or make connections to **prior knowledge and experience** and fill gaps as needed | • Who here has cooked with a family member before? Did you use measuring cups? Can you share a personal story about your experience?  
• This problem uses the term regular polygon. We learned about them last month when we did the ____ activity. Talk to your partner about what it means to be a regular polygon. |
| Provide opportunity for **clarification** of task and vocabulary (scaffolded or open-ended questions) | • What do you think it means when it says a ‘fair share’?  
• Is this the kind of problem that could have more than one solution, or are we looking for a single correct solution? |
| **Record** group sense-making of problem | • List important information/constraints the group agrees to on the board. |
| If needed, ask **explicit questions** without specifying an equation, strategy, or operation for solving | • It asks us to find the largest rectangle. Could the answer be a square? Why or why not? |
| **How will you build a bridge to solving?** |
| **Set expectations** for working (individuals, partners, groups, timing), products (solution AND justification), and available tools | • Take a few minutes to start solving the problem yourself. In a few minutes I’ll ask you to start talking with someone else about your ideas.  
• Think about using the counters or the base ten blocks up here if they might help you.  
• So you have two jobs. First—to find out how much money he started with. Second—to have some kind of work that shows your thinking and proves your solution.  
• In fifteen minutes, I’m going to ask each group to share what they are finding. |
| **Assess** individual and class readiness to engage in productive struggle on the problem | • Consider: Is there anyone who seems like they may need extra support understanding the problem or getting started? |
FACILITATE PRODUCTIVE STRUGGLE

• What opportunities will learners have for collaboration?
  • Be specific—what will this look like?

• How will you support learner thinking without lowering the cognitive demand?
  • Consider getting learners started
  • Consider getting learners “unstuck”
  • Consider pushing learners’ ideas further
  • Consider tools/supplies/resources
  • Consider alternate versions of the task

• How will you make decisions about what to do in the next parts of the lesson?
  • Keep track of student strategies and misconceptions
  • Remind yourself to look for opportunities to assign competence
### FACILITATE PRODUCTIVE STRUGGLE

**What opportunities will students have for collaboration?**

| Provide opportunities for students to work collaboratively, in pairs or small groups | • Talk with your partner about your ideas and their ideas. Are they thinking about the problem in a similar or different way?  
  • You two have similar strategies. Why don’t you see if you can help each other to find a solution? |
| --- | --- |
| **Scaffold collaboration** between students | • What do you think about what Keira just said?  
  • That’s a great question. Why don’t you ask your partner that question and see what she thinks. |

**How will you support student thinking without lowering the cognitive demand?**

| Help students understand the problem by rereading the problem, focusing on understanding the situation and/or clarifying terms or contexts | • Read the problem out loud to me.  
  • What do we know?  
  • Try explaining what’s going on in your own words. |
| --- | --- |
| Help students get started or unstuck by:  
  • using models, diagrams or acting out the problem  
  • thinking about a similar, simpler scenario  
  • building on their informal and/or incomplete strategies  
  • asking questions and/or providing just enough information (scaffolding)  
  • pointing out an approach that has helped another group get started | • Let’s act it out  
  • Can you draw a picture of what is happening?  
  • What are you thinking about?  
  • What are you trying to find out?  
  • What’s hard or standing in your way?  
  • Does this remind you of anything we’ve done before?  
  • Let’s all come back together for a moment. Jamal has noticed an important pattern. |
| Provide appropriate tools, resources, and supplies for students to engage with task | • Remember there are counters if you would like to use them.  
  • What are some materials you could use to help you solve this? |
| For students that need it, provide alternate versions of the task (simpler/more complex) or follow-up tasks | • Vary the numbers, the problem structure, or the context  
  • Provide graphic organizers or sentence frames  
  • Provide supports “just in time”—when a student demonstrates the need because the struggle is no longer productive |
| Coach mathematical participation | • What would happen if you used a different number? Let’s try!  
  • See if you can convince your groupmates that it works |

**How will you make decisions about what to do in the next parts of the lesson?**

| Circulate to visit, listen in on, and monitor | • Try to check in with each group, asking questions to gauge understanding  
  • Monitor time, progress, and group dynamics |
| --- | --- |
| Determine what strategies students are using to solve the task and keep a mental or written record | • Look for a range of student strategies you could share with the class (perhaps take a picture)  
  • Look for common misconceptions that need addressing |
| Look for opportunities to assign competence, particularly to low-status students, by giving praise that is public, specific, and meaningful to the task. | • I noticed Eric was doing something really important—he was looking for patterns.  
  • I love how Jada is trying out different combinations! |
DISCUSS LEARNER THINKING

• How will you decide **what will be shared, by whom and in what order?**

• How will you **facilitate and support the sharing and discussion of work and strategies?**
**DISCUSS LEARNER THINKING**

**How will you decide what will be shared, by whom and in what order?**

| Consider the **type of mathematical goal** you have identified | ● Build or reinforce a new conceptual understanding  
● Connect a conceptual understanding to a procedure  
● Develop procedural fluency (with understanding) |
|---------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| Using this knowledge, strategically select solution approaches to make visible to the class | ● Related and/or contrasting strategies  
● Common misconceptions  
● Useful notation or organization |
| Intentionally sequence or organize solutions to lead to the mathematical goal. | ● Accessible/concrete to efficient/abstract  
● Common to unconventional  
● Exhibition protocols |
| Attend to **equitable participation, identities, and status.** Consider who will represent work: the facilitator, a student or multiple students. | ● Whose work will be represented and why?  
● Whose work and/or voices are not represented?  
● Allow everyone’s work to be shown? (e.g., gallery walks) |

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**How will you facilitate and support the sharing and discussion of work and strategies?**

| Coach students on how to present their ideas and solutions to peers | ● Turn and face the class so that we can hear and see you.  
● Could you stand to the side so we can see your drawing?  
● Could you say that again so everyone can hear your ideas? |
|-------------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| Represent learner strategies visually by projecting or recording learner solutions using pictures, equations, and/or words and orienting learners to parts of solutions | ● Use visual models (number lines and area models) to make strategies accessible.  
● Use colors, circling, arrows, etc. to highlight parts of solutions. |
| Cultivate rich learner explanations by providing wait time, probing, and pressing learner thinking  
● follow up with “why” and “how”  
● ask learners to unpack a strategy, provide evidence or counterexamples, and/or explain why a strategy works | ● Can you say more about that?  
● Why do you think Tiffany’s method works?  
● Tell us how your strategy works.  
● I’m wondering if this method will always work?  
● I want to hear from someone else who hasn’t spoken yet |
| Engage learners in making sense of each other’s thinking by:  
● revoicing for clarity  
● prompting and scaffolding participation from learners  
● asking learners to rephrase, agree or disagree, or add on to another learner’s response | ● I think what you’re saying is ______. Did I get your idea right?  
● Do you agree with Joy’s idea? Or have a question for her?  
● Who can try to explain her idea in your own words?  
● What do you think he means when he says the bigger one?  
● Compliment or question |
| Intentionally make space for and assign competence to marginalized and/or low-status learner contributions by:  
● acknowledging mathematical practices  
● attributing an idea to a specific learner  
● treating errors as an opportunity for learning | ● That mistake helped us all to better understand this!  
● Rakeem’s method is really important for us to think about.  
● Lila, you are trying different approaches like a mathematician!  
● Let’s see if we can build on Lila’s great idea. |
| Introduce standard language, notation, and/or models in response to learner ideas and questions | ● Here is one way to visualize what Anna is doing. [Teacher draws a model.] How do you see Anna’s strategy in this picture?  
● How could we write that with equations?  
● Did you notice how in John’s pattern we keep multiplying numbers by themselves? We call these square numbers. Mathematicians have a special notation for this. |
• What are the connections you need learners to make to **explicitly bridge their thinking and strategies to the important mathematical ideas**? What is your plan for helping learners to make these **connections**?

• How will you **help learners to solidify connections and consolidate their understanding**? How will you formatively assess learners’ understanding?
| Facilitate a **whole class debrief** | Try to surface and highlight:  
- an understanding of the main mathematical ideas of the lesson  
- any remaining questions or uncertainties  
- connections to other concepts  
- real-world applications |
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<tr>
<td>Help learners to <strong>formalize</strong> their ideas by generalizing patterns and relationships, articulating theories and/or proving or disproving conjectures</td>
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- Do you see any patterns?  
- Will that strategy always work? How do you know?  
- Can you think of an example where that wouldn’t be true?  
- Why do you think that makes sense?  
- Let’s look at the table we made. What did we discover about the relationship between the value of a digit when it is in the tens place, and the value of that same digit when it is in the hundreds place? |
| Connect **procedures to concepts** |  
- Let’s take a look at Chris’ drawing and at Jamir’s equation. How are they connected to each other?  
- Is there a more efficient way to record those steps?  
- Now I’m going to show you how we can write that a different way. |
| **How will you help learners to solidify connections and consolidate their understanding?** How will you formatively assess learners’ understanding? Choose one or more of the options below. |  
| Give learners the opportunity to **revise their thinking** on the task | Revisions to rough draft thinking might include:  
- Making the work clear and understandable  
- Making a convincing argument  
- Changing the answer, or adding additional solutions  
- Trying a different strategy to get to the solution  
- Looking for patterns or generalizing a rule |
| Engage learners in **written reflection** |  
- **General Feedback**: What is something you learned today? What do you still have questions about?  
- **Focused on the mathematical goal**: In your own words, what makes a relationship proportional?  
- **Focused on identity**: How do you feel about being called on to share your thinking? What’s it like to listen to other people’s strategies?  
- **Focused on group process**: How well did your group work together? |
| Collaboratively **create a chart** to anchor problem solving | Clearly record things like:  
- steps to a procedure discovered through learner’s strategy (e.g. Martha’s Method of Factoring)  
- definitions, theorems or mathematical truths discovered and clarified through the course of the lesson  
- a list of different kinds of strategies that could be used to solve a certain kind of problem based on the class's work for the day |
| **Select and administer an exit slip** | **Characteristics of a good exit slip:**  
- can be solved quickly, in about 5 minutes  
- provides an opportunity for learners to use different strategies  
- requires mathematical justification or explanation  
- is not exactly the same kind of problem they just solved |
REFLECT

• How will you use the evidence you collected to make decisions about instructional next steps?

• What options do you have for follow-up work to build from and expand learners’ current understanding?
  • A mini lesson to the whole class
  • A small group lesson on an identified area of need
  • Practice problems or activity
  • Adding in a new math routine
  • A new task
  • Something else?
**REFLECT**

**How will you use the evidence you collected to make decisions about instructional next steps?**

| Look for evidence of developing understanding or underlying issues in your students’ work | • What is the good news that I can build upon?  
• What issues or concepts do I want to make sure to address before moving forward? |
|---|---|
| Analyze evidence of learner participation, identity, and/or group processes | • Is there evidence that my students are engaging in problem solving and discussion?  
• Are all students developing positive mathematical identities?  
• How could I help Amira develop more confidence in her own thinking? |
| Review video and/or observation notes in relation to professional learning goals | • Identify a moment that went well – what facilitation moves did you use and how did they support your learners?  
• Identify a moment that didn’t go as well as you hoped – what could you have done differently and what can you learn from this?  
• Set goals for your next lesson to build upon |

**What options do you have for follow-up work to build from and expand learners’ current understanding?**

| Does the whole group need more instruction? | • If you notice that learners are not labeling their answers, you might do a mini-lesson on identifying the meanings of the quantities in word problems  
• A mini-lesson on equipartitioning area models might follow a task where learners had difficulty visually representing a whole with equally sized fractional units (i.e. sixths) |
|---|---|
| Teach or plan to teach a **mini-lesson** on an identified need to the whole class | • Work with a small group on an identified area of need, using a different approach, model, or activity  
• Intentionally pair learners based on identified strengths/needs |
| Does a small group have a specific need? | • Introduce a mathematical routine (e.g. number talks, Which One Doesn’t Belong, Quick Images)  
• Have students play a math game to reinforce and practice a specific skill  
• Give students a small number of problems to work on with a partner/small group |
| Provide **differentiated support or instruction to a small group** | --- |
| Would students benefit from more practice? | --- |
| Choose **additional problems or an activity** to give students more opportunities to utilize a new math skill or concept | --- |

**RETURN TO MODEL**

![Responsive Math Teaching (RMT) model](image-url)