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Three-dimensional Motion Reconstruction and Analysis of the Right Ventricle Using Tagged MRI

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Abstract

Right ventricular (RV) dysfunction can serve as an indicator of heart and lung disease and can adversely affect the left ventricle (LV). However, normal RV function must be characterized before abnormal states can be detected. We describe a method for reconstructing the 3D motion of the RV images by fitting of a deformable model to extracted tag and contour data from multiview tagged magnetic resonance images (MRI). The deformable model is a biventricular finite element mesh built directly from the contours. Our approach accommodates the geometrically complex RV by using the entire lengths of the tags, localized degrees of freedom (DOFs), and finite elements for geometric modeling. We convert the results of the reconstruction into potentially useful motion variables, such as strains and displacements. The fitting technique is applied to synthetic data, two normal hearts, and a heart with right ventricular hypertrophy (RVH). The results in this paper are limited to the RV free wall and septum. We find noticeable differences between the motion variables calculated for the normal volunteers and the RVH patient.

1 Introduction

Abnormal motion of the RV serves as an indicator of several types of heart disease, such as RV ischemia and hypertrophy [4]. Dysfunction of the RV can also adversely affect the LV [22]. However, there currently is no in-depth knowledge about the motion of the healthy RV and its alteration due to various diseases. One reason that researchers do not agree about the exact pumping mechanism of the normal RV may be the limitations of conventional means of studying RV motion. Since studies have found that RV contraction varies with increases in pressure and volume, and pathological states such as ischemia [4], a method for accurately capturing the motion of the RV wall could both answer questions about its normal function and be used as an indicator of the presence and severity of various diseases.

The RV receives blood from the right atrium and pumps it into the pulmonary artery (Fig. 1). It appears crescent-like in a cross-sectional view and, unlike the LV, is difficult to approximate with any simple parameterized 3D shape. The RV shares a common wall with the LV (the septum), while its outer free wall is in mechanical contact with the pericardium and, through it, the lungs. The

Figure 1: Heart anatomy. Portions of LV and RV walls are cut away to reveal cavities [11]. The RV can be separated into an inflow tract, outflow tract, and apex.

RV cavity can be conceptually separated into an inflow tract, a highly trabeculated apical portion, and a relatively smooth outflow tract [13]. The normally 3-5mm-thick RV free wall is thin relative to the 9-11mm-thick LV free wall [13]. This relative thinness, complex geometry, and lack of trackable landmarks in the wall make it difficult to capture RV wall motion using conventional imaging modalities. The development of noninvasive tissue tagging methods that are used in conjunction with MRI has opened the possibility of more fully investigating RV wall motion.

The anatomy of the RV poses problems for motion reconstruction in several ways: the current image resolution results in tags spaced approximately 6mm apart, so that a limited number of tags fall on the RV wall; the RV geometry cannot be described using any simple primitive; and RV wall motion varies temporally and regionally, thereby making it difficult to describe concisely. This paper presents a new methodology that overcomes these problems and effectively captures RV shape and motion from tagged MRI data. Previous related work at our institution involved fitting a generalized primitive deformable model of the LV to MRI tag intersections [18, 25]. In the current work, we retain the basic deformable modeling framework of using Lagrange's equations of motion to deform the model under forces calculated from image data. However, in order to accommodate the complexities posed by the RV, we significantly modify the method of calculating image-derived forces, the geometrical description of the deformable model, and our model degrees of freedom.

In the deformable modeling framework, image data are used to calculate forces on the model. Our new method for calculating these image-derived forces accounts for significant RV through-plane motion and the fact that only a small number of tag point intersections fall within the normally thin-walled RV. Instead of working with tag stripes as in [12], we reconstruct the tag surfaces from the extracted stripes and apply forces between these surfaces and corresponding surfaces registered to the deformable model. Also, the new contour force formulation also accounts for the sparsity of contour data near the RV free wall which results from its asymmetric shape.

In this paper, we also describe a unique method for modeling the complex RV geometry: instead of fitting a deformable geometric primitive to the initial contour data, we build a geometric model of the biventricular geometry *directly* from the initial contours. This approach allows us to account for the significant geometric variations which can occur in some RV diseases. The initial geometric model is a volumetric finite element mesh (FEM) which we fit to extracted contour and tag data from successive images acquired through systole. Due to the complex RV geometry and motion, we use the maximum possible number of model DOFs for the given image data. Instead of lumping the model DOFs into parameter functions as in [25], we use stiffness from finite elements for locally

smoothing the relatively sparse input data which can result from imaging the thin RV wall.

Once we reconstruct the detailed RV motion, we calculate both global and local motion variables and assess their clinical relevancy. We use computer graphics techniques to visualize the dense set of 3D derived motion data. Our quantitative results are consistent with previously documented knowledge of RV motion and provide a more detailed motion analysis of the RV, as compared to previous methods. In addition, providing a biventricular motion reconstruction opens the way for the quantitative study of inter-ventricular dependence and relative motion. However, in this paper we will focus on our analysis of the RV free wall and septal systolic contraction.

2 Related Work

The static geometry of the RV was initially described with cylinders, ellipsoids, and other geometric primitives [15]. Advances in imaging modalities have allowed for more accurate models, where, in most cases, a predefined model is fit to image data. Due to its thinness, the RV free wall has been modeled as a shell using finite elements [37], a combination of quadratic surfaces [8], and biquadratic spline surface patches [29]. Volumetric RV models have usually included the LV, but excluded the outflow tract. A Constructive Solid Geometry (CSG) approach was used for modeling the geometry from short-axis contours [26] taken from ex-vivo wall contours. Another approach has been to fit a general RV-LV finite element mesh to digitized data from excised hearts [21].

Right ventricular wall motion has previously been studied invasively by implanting markers into limited portions of the RV wall and using an imaging modality to track the motion of the markers [30, 28, 5]. One-dimensional relative motion data has been gathered by tracking pairs of radiopaque beads tracked with biplane cineradiography [30, 5]) or pairs of crystals with ultrasound [28]. By implanting several markers close together on the same ventricular surface (e.g., endocardium), researchers have been able to derive biaxial, or in-plane, deformations [17, 6, 34]. The human RV has been studied using the noninvasive MRI tissue tagging method. The tags were used as input data for 1D and 2D analyses of RV wall motion within selected image planes [9], or for a 3D motion reconstruction of the RV midwall surface (see below) [37].

The MRI tissue tagging method provides a noninvasive means for tracking RV deformation and displacement in humans. Since tagged MR images do not capture important through-plane motion, data from multiple views must be combined to capture the complete 3D motion. Motion reconstruction is also necessary when combining information from mutually orthogonal stripes (called 1D tags since they provide 1D motion data) in a particular image plane. However, almost all 3D reconstruction methods have been developed for the LV. The types of input data include tag intersections (2D tags) [35, 19, 33], 1D tags [23, 20] or a combination of 2D tags and segmented contour data [25, 36]. The 3D reconstruction approaches include: fitting a 3D displacement field composed of an analytic series to image data [23]; physics-based deformable modeling [18] applied to a parameterized superquadric [24]; curve and spline surface reconstruction of tag data to be used as input to linear optimization [20]; and nonlinear least-squares fitting of a finite element model [36]. Up to now the only MRI tagging study of the motion of the RV was an adaptation of the latter fitting method to a *surface* model of the RV free wall, using intersections of mid-wall contours with tags as input data. In contrast, this paper describes the use of the entire length of the tags for reconstructing the volumetric 3D motion of the RV.

Figure 2: Mid-wall short-axis 1D tagged MR images at (a) beginning systole (b) mid-systole (c) end-systole.

3 Image Acquisition and Data Extraction

In order to capture the motion in the ventricular walls using MRI images, we use a 1D SPAMM (SPAtial Modulation of Magnetization) tagging technique [3]. This noninvasive technique produces a family of parallel tagging planes oriented perpendicular to the image planes. Prior to imaging, a sequence of non-selective radio frequency excitation pulses are generated separated by intervals of magnetic field gradients in the direction perpendicular to the desired tag planes. The intersections of the resulting tag planes with the imaging plane appear as dark stripes in the images (Fig. 2) and represent perturbations of the magnetization of the tissue. As the heart contracts and the tagged tissue planes deform (and become nonplanar tag surfaces), the initially straight lines in the image deform to reveal within-wall motion. As a result of through-plane motion, the portion of a tag surface seen in an initial image may move out of the imaging plane and be replaced by the intersection of a different portion of the same tag surface with the imaging plane. Since the tag lines only provide information about motion in the direction perpendicular to the initial tag planes, three mutually perpendicular tag plane orientations must be combined to capture the 3D motion. Since the tag planes must be oriented perpendicular to the image planes, both short- and long-axis views must be used. For our studies, we use two sets of stacked short-axis image planes with tag planes which appear as vertical or horizontal stripes, and rotating long-axis images whose tag planes appear horizontal in the images. The schematic in Fig. 3 shows these combinations along with the motion information that each provides.

We extract tags and contours from the images using SPAMMVU, a program developed in the Department of Radiology at our institution [2]. The heart walls are segmented using active contours, or snakes [14], which the user can manipulate. As show in Fig. 5, four contours are extracted from each image. Two closed contours (labeled LV-epi and LV-endo for the epicardial and endocardial borders, respectively) define the borders of the LV free wall and septum. Two open contours define the RV endo- and epicardium and connect to the LV-epi contour at the insertion points. Tags from multiple time phases (between end-diastole and end-systole) are tracked with a tool that is an adaptation of the deformable mesh scheme of [38] for 1D stripes (Fig. 4). The mesh, which can be manipulated by the user, is a set of mutually orthogonal interlocked active contours where one orientation of contours tracks the stripes and the others maintain structural support. The extracted stripes are approximated as a series of points by sampling the tag lines at 2mm intervals while the contours are defined as a set of points separated at variable-length intervals.

Figure 3: 1st row: Schematic LV with image (light) and tag (dark) planes. 2nd row: 2D images with example of tag motion from initial (dark lines) to final (dashed lines) times. Highlighted arrows indicate for which direction we get motion information.

Figure 4: Set of active contours for tracking 1D tags. a) Initial mesh at end-diastole. b) Final mesh at end-systole. Lighter colored squares are marked active since they fall within the contours. The stripes are tracked by deforming a mesh of interlocked active contours.

Figure 5: Heart walls are segmented into 4 contours labeled LV-epi, LV-endo, RV-epi, and RV-endo.

Figure 6: Finite elements are generated in two layers, seen as white and grey. a) Short-axis view
b) Long-axis view.

4 Biventricular Model Geometry

Although the RV cannot be well approximated by any simple geometric primitive, an accurate geometric model is necessary for reconstructing the 3D motion. Since the septum plays an important role in the function of both ventricles, our approach was to create a biventricular geometric model. Instead of a more commonly-used approach of fitting a predefined model to contour data, we build a volumetric finite element mesh directly from contours extracted from end-diastolic images. This automated approach is more robust and applicable to the complex geometry we are trying to capture, particularly for disease states that may alter the ventricular geometry in ways that may not be well approximated by a predefined model. A feature of our finite element mesh is that we can control the number, size, and placement of the elements. We do not use existing finite element mesh generation packages since they would not have given us this control and may have required additional pre- and post-processing. Our approach allows us to integrate the geometry-building phase and fitting phase into one program. The finite element mesh construction is performed in two stages: 1) partition the contours into the set of model points (nodes) which set up the topology for 2) generating the finite elements which connect these nodes.

Figure 7: Short-axis contours are used for selecting points to be used as nodes in the deformable model. a) Points are sampled from endocardial contours at equally spaced intervals between the insertion points, r_i , and the points on the the LV-endo contour nearest them, e_i . b) Connections across the muscle wall are made between the sampled endocardial points and the reconstructed epicardial wall. c) Final tessellation of one (non-planar) level. A new LV-mid sampled contour is formed.

4.1 Contour Partitioning

In order to make the problem of tessellating a 3D volume more tractable, our approach is to tessellate cross-sections having the same topology and generate volumetric elements to connect these levels. Since the cross-sectional topology in the main portion of the biventricular unit remains more or less constant in short-axis, rather than long-axis views, we use the short-axis contours for the mesh generation. We also simplify the problem of subdividing the biventricular geometry by first conceptually defining two 'layers': an inner layer defines half of the LV free wall and the full septum (shown grey) and an outer layer (shown white) defines the LV free wall and continues out into the RV free wall as was done by [21] (Fig. 6).

The two layered topology is maintained when we tessellate cross-sectional slices. The series of points sampled from the original contours or defined during the contour partitioning are called sampled contours. The number of intervals used to sample the contours will later determine the number of elements made, and can be adjusted according to the desired number of model DOFs. We partition the endocardial contours extracted from each image plane as described below and

shown in Fig. 7a:

1. Label the points of insertion of the RV wall into the LV. These are the end-points of the open RV-epi (r_1, r_2) and RV-endo (r_3, r_4) contours.
2. Find the corresponding points, e_i , on the LV-endo contour which are closest to the 4 insertion points, r_i .
3. Sample points at equal intervals from the septal portion of the LV-endo (between e_3 and e_4) and LV-epi (between r_3 and r_4) contour. These points are then joined to make elements that define the septum.
4. Sample points at equal intervals from the RV-endo contour between the insertion points (r_3 and r_4).
5. Sample points at equal intervals from the free wall portion of the LV-endo contour (between e_1 and e_2).

Once we have points on the endocardial surfaces, we find the corresponding points on the epicardial walls so that later we can define elements across the muscle walls. We label this step as 'connecting' the free wall portions of the sampled endocardial contours with the epicardial points. Making these connection in the same plane as the images would result in an artificial thickening of the RV free wall due to its oblique intersection with the images. In order to get the true wall thickness, we first reconstruct the surfaces defined by the sampled RV endocardial contours and the original epicardial contours. Fig. 7b shows the surface normals from the triangulated endocardial walls which provide the direction in which to search for the intersection with the outer triangulated epicardial surface (not shown). The intersection points on the outer surface are shown along with the epicardial contour they define. A more complicated surface reconstruction can also be incorporated into this technique if desired. For simplicity, since the main portion of the LV does not generally have as much through-plane curvature, we connect the inner and outer walls radially within the image planes. We also generate points halfway between these LV connections to produce a new midwall contour (LV-mid). A cross-sectional view of the final tessellation is shown in Fig. 7c.

4.2 Finite Element Mesh Generation

Since we cannot readily represent the biventricular geometry with any simple primitive, we build a discretized volumetric model using finite elements. The geometry of a finite element is defined by the location of its vertices, or nodes, and shape functions, which determine the position (and other quantities) of points within the element. The shape functions and elements used in the mesh are shown in Appendix B. Given the complexity of the RV shape, we use a large number of smaller elements with linear shape functions rather than larger elements with higher-order shape functions. The nodes of the finite elements are points from the sampled contours and surfaces described in the previous section. Eight-noded elements are used to connect nodes across the wall and between tessellated levels (Fig. 8b). Six-noded wedge-shaped elements are used to prevent the elements from getting too skewed and to define the apex (Fig. 8c). In order to be able to visualize the endocardial or epicardial surface of either ventricle, the faces of the finite elements which lie on these surfaces

Figure 8: Finite element mesh generation. Flat shading is used to show triangulation of heart wall surfaces. a) Slice of elements connecting two tessellated levels. b) Wedge elements are used to define the apex. c) Final biventricular finite element model with shaded epicardial walls of LV (light pink) and RV (dark pink). d) Shaded endocardial walls of LV (green) and RV (yellow).

are labeled during this mesh-building step and later triangulated for shaded surface display. The FE elements will later play a role in both the fitting method and the calculation of strains. The total number of elements varies with heart size and number of intervals used in partitioning the elements. The complete mesh shown in Fig. 8d has 294 elements and 488 nodes.

5 Right Ventricular Motion Reconstruction

Once we have an accurate geometric model of the ventricles, our goal is to use the information from image-derived data to capture the full 3D motion of the ventricles. The deformable modeling approach is a commonly-used approach, in which a model deforms to fit to data using energy minimization or a classical physics-based equation of motion [16]. The specifics of how to deform the model and apply the deformation constraints vary with the particular application. Previous work in our laboratory [25] has successfully used Lagrange's equations of motion with spring-like (proportional to displacement) pseudo-forces which attract the model to image-derived data. Although we maintain this basic framework, application of the fitting technique to the RV has led to a major alteration in the force calculation and the use of local, rather than global, smoothing.

5.1 Model Dynamics

Our model is deformed using a reduced form of Lagrange's equation of motion:

$$\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}_q \quad (1)$$

where the vector $\mathbf{q}$ contains the displacement degrees of freedom and the vector $\mathbf{f}_q$ contains the image-derived forces. We neglect the inertial term since we want the model to come to rest when it reaches the position most consistent with the data and use a damping coefficient of one. In previous work, the degrees of freedom were converted to parameter functions which were chosen using previous knowledge about typical patterns of LV deformation. However, a frame of reference is necessary for describing parameters such as radial shortening and contraction. Since the RV shape is too complex to define such a global frame of reference and there are currently no simple parameters which describe RV contraction, we use local degrees of freedom: the general 3D displacement of each node of the finite element mesh. Due to the relative sparsity of the data available from the tagged images, we introduce localized spatial smoothing through the finite element stiffness matrix, $\mathbf{K}$. Stiffness can also be viewed as an internal resistance which results in a force equal to $\mathbf{K}\mathbf{q}$. Using these considerations, Lagrange's equation of motion is written for each node, i , as

$$\dot{\mathbf{q}}_i = \mathbf{f}_{i,internal} + \mathbf{f}_{i,external} \quad (2)$$

where $\mathbf{q}_i$ is the 3D nodal displacement, $\mathbf{f}_{i,internal} = [\mathbf{K}\mathbf{q}]_i$ is an internal stiffness force, and $\mathbf{f}_{i,external}$ is an external, image-derived force. It is important to note that these forces are only used to deform our model and are not meant to replicate the actual forces or material properties in the heart wall muscle.

The spring-like external forces are used to deform the model to the image-derived data. In previous work [25, 12], separate spring constants were used for the contour and tag data. The Euler integration used a constant step size so that damping effects caused the model to come to rest before it reached the data positions and the user had to increase the strength of these forces. In this work, the forces are directly proportional to the distances between the model and image-derived data: $\mathbf{f}_{data} = \mathbf{d}_{data}$, where $\mathbf{d}_{data}$ is the distance from a point on the model to the location of the appropriate data.

We integrate equation 2 using the adaptive Euler technique [27], where the step size, h , varies according to the amount of integration error. Since the step size is multiplied by the forces, the adaptive step sizing effectively modulates the strength of the forces, thereby automating the fitting. The model is fit sequentially to data from each image time phases, with usually five or more phases through systole. Displacement errors equal to the magnitude of the distance-proportional tag and contour forces, $\mathbf{f}_{data}$ are calculated during the fitting. The criterion for ending the fitting to data from a particular time phase, and ending the fitting or continuing with data from the next time phase, is that all tag and contour displacement errors are less than 0.8mm rms, which is less than the image resolution.

In the following, we describe the calculation of the internal forces and two types of external image-derived forces, which we call contour and SPAMM forces.

Figure 9: Schematic triangulated model surface and contour data. In previous work, forces (a few shown here) were applied from contour data points, resulting in forces only being applied to nodes which were nearest to the image plane locations from which the contours originated.

5.2 External forces From Contour Data

Due to the coarse tag sampling and the thinness of the RV wall, the contour data are also used to constrain the motion of the model. Each portion of a contour provides motion information in the direction of the local normal. In previous work, each contour data point was used for calculating a force between it and the model and applying it to points on the model closest that data point (Fig. 9). This method of applying forces *from the data to the model* led to the model only being constrained at locations where the image planes happened to fall [25, 12]. This was especially problematic for constraining the RV since the center of rotation of the long axis image planes is at the LV axis. Moving the center to the septum or using parallel planes does not solve the problem due to the crescent shape of the RV. Thus, our current approach is to calculate forces *from the model to the data* by calculating a force at each node by using contours from the closest image planes. The contour forces are applied in the direction normal to the model boundary, where the boundary normals are calculated from the triangulated surfaces generated during the geometry generation phase. Although the boundary normal may not initially be equal to the contour normal, the model normals are recalculated as the model is being pulled closer to the contours, causing the two normals to gradually coincide.

During the motion fitting process, we keep track of which image planes a node is nearest (Fig. 10). Except near the extremities of the heart, most nodes will have two nearby planes in which we calculate a force. We then combine these two forces according to the distance between the node and the the image planes in order to get a linear interpolation of the constraint from the two images. During the mesh generation, we save the contour that the node was sampled from (e.g., LV-epi) and fit the node to that contour in subsequent images. The following steps describe the force calculation at each iteration and in each image plane (Fig. 10).

1. At each time step, project node point $\mathbf{x}$ and its associated normal $\mathbf{n}$ onto the image plane to get $\mathbf{x}_p$ and $\mathbf{n}_p$, respectively.
2. Find the intersections between the contour and a vector pointing from $\mathbf{x}_p$ in positive or negative $\mathbf{n}_p$ direction.
3. Use the intersection nearest to $\mathbf{x}_p$, $\mathbf{r}$, to calculate the force in the plane: $\mathbf{f}_p = \mathbf{r} - \mathbf{x}_p$.

If the node lies between two image planes, the forces from each plane are combined as described above. This force is then projected onto the original node normal, $\mathbf{n}$, in order calculate the contour force, $\mathbf{f}_{contour}$.

Figure 10: Contour force calculation. Since we use mostly parallelepiped elements, model surface is shown as connected polygons. We project model node, x , and its normal, n , onto the nearest image planes. Forces are calculated in both image planes and combined to calculate the final contour force.

5.3 External Forces from SPAMM data

As was described in Section 3, the tags provide information about motion within the myocardium. In this paper, we call the tags 'tag surfaces' in the general case, and 'tag planes' when describing their initial configuration. Our approach to calculating the SPAMM forces accounts for the fact that the tag data extracted from the images provide information about the time evolution of the deformation of image tags. Thus, the initial tags are registered to the model and, during the fitting, SPAMM forces are applied to these registered points, thereby allowing each model stripe to constrain the motion of the model as it follows its corresponding tag through time. In previous work, only the tag stripes or just their intersections in the image planes were registered and followed [12, 25]. However, since the already limited number of tags which fell in the RV wall and were registered to the model were also limited to the image plane locations, the model was underconstrained. In Section 5.3.1, we describe how we provide more constraints by registering the *entire tag surface*, which we call a material surface, to the model.

Another limitation in the previous work was that once the points were registered in the image planes, the movement of these registered points out of the image plane due to through-plane motion meant that we calculated subsequent motion information from a plane in which the registered tags no longer resided [12, 25]. Since the RV can move through more than two short-axis image planes (about 15mm near the base of the ventricle), this leads to less accurate fitting results. We overcome this limitation by noting that the tag lines seen in the MRI images are the intersection of the tag surfaces with the image planes (Fig. 14). If the image plane separation is small enough (about 6mm in our case), there will be only a small variation in the shape of the tag surface between image planes. Therefore, we reconstruct the position of the entire tag surface from the tag stripe positions at each image time phase (see Sec 5.3.2). The reconstruction results in the effective existence of tag data irrespective of the image plane locations. Then, the SPAMM forces are calculated between the material surfaces of the model and the reconstructed tag surfaces so that the former are attracted to the latter. We capture the 3D motion by simultaneously fitting the deformable model to tag surfaces that provide motion information in three mutually perpendicular directions. The SPAMM force calculation requires registration of the material surfaces to the model prior to fitting, tag surface reconstruction for each time phase, and the force calculation between model material points

Figure 11: Schematic LV with 1D coordinate system for defining series of material planes.

Figure 12: Biventricular mesh with initial material planes shaded green and material points drawn black. Spacing between points shown here is 0.9 mm in the direction along the stripes and 4.2mm in the direction between stripes. Each of these points is registered to the element in which it lies.

and the appropriate tag surfaces.

5.3.1 Material Surface Registration

Prior to fitting, material surfaces are generated so that they coincide with the initial tag plane positions and then are registered to the model. This process involves the localization of the original tags, generation of material surfaces which are initially planar, and registration of the material planes to the model. First, in order to locate the tag planes, we take advantage of the fact that they are initially planar and uniformly spaced. Therefore, given the location of one of the tag planes and the tag spacing, each set of material planes can be defined by a 1D coordinate system with the axis oriented in the direction of the tag plane normals, $\mathbf{tn}$ (Fig. 11). The seed point, S , and tag plane spacing can be found by calculating the average position of the tag stripes and their spacing from the tag positions extracted from the images. To be more accurate, these quantities can also be directly measured from the images and entered by the user. Since we know that the tags to be tracked completely fall on the heart wall, we calculate a bounding box for the heart and generate material planes that start at a seed point and fill the bounding box (Fig. 11).

Figure 13: Element coordinate systems. a) The coordinates of a material point remain constant in the local (e, n, s) system b) although its global position will change with the global x, y, z position of the element.

Next, since we will calculate forces between points on the model and the tag surfaces, we define the material surfaces as a collection of equally-spaced points, called material points. As shown in Fig. 12, these points are triangulated to display the material surface. The material point spacing is chosen so that enough material points fall across the heart walls to properly constrain the model; we currently use a separation of 0.7mm in the direction along the stripes and 1.7mm in the direction between stripes (Fig. 12). We register each model material point by finding its coordinates in a non-deforming, stationary local coordinate system of the appropriate finite element (Fig. 13a). The transformation from the local position, (e, n, s) , to the global position, (x, y, z) , of a point (Fig. 13b) is written in terms of the finite element shape functions:

$$x = \sum_{j=1}^n N_j(e, n, s)x_j, \quad y = \sum_{j=1}^n N_j(e, n, s)y_j, \quad z = \sum_{j=1}^n N_j(e, n, s)z_j \quad (3)$$

where (x_j, y_j, z_j) is the position of the j^{th} node in the element numbering system. The shape functions, N_j , can be seen as weighting the global coordinate of a node according to where a point lies in the (e, n, s) system. In the work reported in this paper, we use six-noded wedge and eight-noded parallelepiped elements, whose linear shape functions are given in Appendix A. Given the positions of the nodes (x_j, y_j, z_j) and the position of a tag point on the stripe (x, y, z) , we solve the set of three equations (Eq. 3) for the local coordinates (e, n, s) using a Newton-Raphson method. Since each finite element is defined to be a 2 by 2 cube centered at the origin of its local coordinate system, the point falls within a particular element if: $-1 \leq e, n, s \leq 1$. This test is performed first for the element for which the material point lies in its bounding box and is nearest the material point. If the material point lies outside this element, neighboring elements are then tested.

5.3.2 Tag Surface Reconstruction

The registered material surface is composed of material points which are not likely to fall in the image planes where we have tag data. In earlier work [25, 12], the material points were parallel projected onto an image plane. This approach did not account for the curvature of the tag surface as it deforms with the heart muscle. We overcome this limitation by reconstructing the tag surfaces from the tracked stripe data. However, this requires a correspondence between a stripe tag seen in an image and the tag surface to which it belongs. Fig. 14 shows a shaded tagging surface and the tag stripes produced from its intersection with series of short-axis image planes. Since the tag

Figure 14: The intersection of the tag surfaces with the image planes produces dark bands in the image planes.

Figure 15: Reconstructed tag surfaces (shown shaded) along with original tag stripes (black lines). a) Stripes originated from stacked, short-axis image planes. b) Stripes originated from a set of rotating long-axis image planes, resulting in a radial pattern.

tracking procedure is performed within each image plane, there is no information about how stripes in one image plane correspond to stripes in the adjacent image plane.

Rather than aligning stripes between image planes, we take advantage of the fact that we just generated material planes at the exact locations of the initial tagging planes. Also, the stripe data (a set of points defining the stripe) that we extract from the image planes at the initial times approximate straight lines. Therefore, the average position of a stripe, $\mathbf{a}$, is found by averaging the positions of its active points. A coordinate, c , is then found for the stripe in the material plane coordinate system, $\mathbf{tn}$ (Fig. 11) by projection: $c = \mathbf{tn} \cdot (\mathbf{a} - \mathbf{s})$, where $\mathbf{s}$ is the origin of the coordinate system. This coordinate, c , is compared to the coordinates of the material planes to find the plane to which the stripe belongs. This correspondence is maintained for all image time phases. Currently, the reconstruction is performed by triangulating between stripes in adjacent images that belong to the same tag surfaces, resulting in a linear interpolation of the tag surface between image the planes (Fig. 15). Note that although the resulting triangulated surface is coarser between image planes, it is defined in detail within the image plane along the directions of its corresponding stripes.

Figure 16: Calculation of SPAMM forces. a) Project reconstructed tag surface, S_1 , in the direction of tag normal, tn , to get the planar surface, S_2 . Do the same for each material point, mp , to get the projected point, p . b) Each triangle, T from S_1 , will have a corresponding projected triangle, T_p . Once the triangle T_p in which p lies is found, the force is calculated between mp and T in the direction of tn .

5.3.3 SPAMM Force Calculation

The SPAMM forces are designed to pull the model towards the image data. Therefore, a force is calculated between each material point, mp , and the appropriate reconstructed tag planes, keeping in mind that a tag surface only provides motion information normal to the initial tag planes. The force is determined by finding the intersection between a material point and the tag surface in the direction of the tag plane normal, tn . This calculation first requires a preprocessing step for each image time phase where each triangle (T) from the reconstructed surface, S_1 , (Fig. 16a) is projected in the direction of tag plane normal, tn , to form T_p . The resulting planar surface, S_2 , will have the same topology as S_1 since we know that surfaces in the heart do not fold over. (For convenience, we project these triangles onto the same planes in which they were originally located).

The SPAMM force is calculated during each iteration as follows (Fig. 16b):

1. Project material point, mp , along tn onto the original tag plane to get p .
2. Determine which projected triangle, T_p , if any, contains p .
3. Find intersection, r , of mp with T (the original triangle corresponding to T_p) in the tn direction.
4. SPAMM force is: $f_s = r - mp$.

The SPAMM force calculated on a material point must be distributed to the nodes of the element to which the material point belongs since the nodes are the model's degrees of freedom. The force

Figure 17: Each collection of material points defining a material surface (not drawn) is attracted to the corresponding tag data surface via the SPAMM forces.

on each node is weighted by the shape functions:

$$\mathbf{f}_{j,S} = N_j \mathbf{f}_S \quad (4)$$

where the subscript j refers to the node number within an element. This is the manner in which concentrated loads are distributed to the nodes in finite element theory, so that the collection of material points registered to the model effectively attracts the model material plane to the reconstructed tag surface (Fig. 17). For a particular element and material surface orientation, the SPAMM forces at each node are normalized according to the number of material points belonging to that element. Then, if a node belongs to several elements, the contribution to the total nodal SPAMM force is summed.

5.4 Total Image-derived External Force

Nodes on the LV mid wall (LV-mid contour generated during the geometry phase) will have no contours to be used in contour force calculation. Therefore, in order to keep the forces balanced, the total external force is the average of the SPAMM and contour data for a surface node, $f_{i,external} = 0.5(\mathbf{f}_{i,contour} + \mathbf{f}_{i,SPAMM})$, and $f_{i,external} = \mathbf{f}_{i,SPAMM}$ for a node in the midwall.

5.5 Internal Forces Due To Stiffness

The degrees of freedom of our model are the 3D displacements of the finite element nodes. We calculate forces from segmented data contours and tag data, which may have inaccuracies due to

human error during segmentation or image noise. Also, the SPAMM forces are applied to model material surfaces whose spacing depend on the initial tag spacing, about 6 or 7 mm at the current image resolution. In order to account for the discrete sampling of motion data due to tags and factors which cause inaccuracies in the input data, we use localized smoothing by adding a stiffness component to the deformable model. In our formulation, we consider stiffness to be an internal force, $\mathbf{f}_{i,internal}$. We use stiffness to impose a continuity and smoothing constraint, and not to model the actual cardiac material properties (a topic of future research). Between image time phases, we consider each element to consist of an isotropic, linear, compressible material.

The element force vector, f_e , contains the forces on the element nodes. It is calculated for each element as:

$$\mathbf{f}_e = \mathbf{K}\mathbf{d}_e \quad (5)$$

where $\mathbf{d}_e$ contains the nodal displacements. The stiffness matrix, $\mathbf{K}$, incorporates the geometry and material properties of the element and is computed from

$$\mathbf{K} = \int \int \int \mathbf{B}^T \mathbf{D} \mathbf{B} dV \quad (6)$$

The matrix $\mathbf{D}$ contains the material property information and relates nodal stresses, σ , to nodal strains, ε :

$$\sigma = \mathbf{D}\varepsilon \quad (7)$$

The matrix $\mathbf{B}$ relates nodal strains to displacements:

$$\varepsilon = \mathbf{B}\mathbf{d}_e \quad (8)$$

Once the element forces are calculated, each element contributes to the total internal force for of that node, $\mathbf{f}_{i,internal}$. A detailed derivation of the finite element formulation for calculating the stiffness matrix, $\mathbf{K}$, is presented in [39]. We use a Poisson ratio of 0.05 and a Young's modulus of 0.02 for the material properties. The Young's modulus can be altered to increase the smoothing needed for increased noise due to poor image quality. In order to prevent element distortions of the linear elements, we recalculate the stiffness matrix after fitting to each time phase, thereby making the smoothing piecewise linear.

6 Deformation Analysis and Motion Quantification

The 3D motion reconstruction provides us with displacement of the model nodes. An effective dense set of displacement data within the finite elements representing the ventricles can be interpolated between the nodes with the FE shape functions. However, since simply viewing the motion of a finite element mesh is of little use to researchers and clinicians, one overall goal is to quantify the motion and visualize the results in an efficient manner. An aid in visualizing the results is to provide color plot of the motion variables on the endocardial or epicardial wall surfaces.

Previous studies of the LV [25] described the reconstructed motion in terms of variables that clinicians are already familiar with for that ventricle, for example, twisting and radial contraction. In a preliminary work for the current motion fitting method, we attempted to quantify twisting in the RV [12]. However, due to the complex shape of the RV, it was difficult to find a single axis about which to calculate twisting and radial contraction. Even when an axis was calculated, these

circular measures were not satisfactory for the crescent-shaped RV. An attempt to calculate global descriptors of RV motion is also hampered by the current lack of knowledge about RV motion. An area of future research will be to apply the 3D motion reconstruction technique described in this paper to a significant number of normal RVs and work closely with clinicians to determine variables with succinctly describe RV motion in health. We will also determine which motion variable(s) are most appropriate for indicating the presence of several types of diseases.

Similar to previous studies of RV motion, in this paper we quantify the local wall motion. We calculate both the 3D regional displacement and deformation of the biventricular model during systole. Along with a color plot of displacement, we also display the paths of model points. However, a better indicator of the degree of muscle contraction is a quantity which describes differential motion, such as strain (Appendix B) [31]. The strain can be considered to be the ratio of the change in length of an arbitrary line segment superimposed onto the material to its initial length. When looking at strain at particular point in the material, the orientation of this line segment must be specified. Therefore, we use tensors to define the strain magnitude, direction, and orientation.

Given the initial and final positions of a point in the material, $\mathbf{X}$ and $\mathbf{x}$, respectively, we start by calculating the deformation gradient tensor, $\mathbf{F}$:

$$F_{pq} = \frac{\partial x_p}{\partial X_q} \quad (9)$$

where the subscripts p and q range from 1 to 3 and denote one of the 3D Cartesian coordinates. The tensor, $\mathbf{F}$, includes both the rotation and deformation around a point in the material. It can be shown that the Lagrangian strain tensor, $\mathbf{E}$, only includes the deformation of the material and is related to $\mathbf{F}$ as follows [31]:

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T \mathbf{F} - \mathbf{I}) \quad (10)$$

where $\mathbf{I}$ is the identity matrix. We use the Lagrangian strain formulation to describe systolic deformation in a region surrounding a point in the heart relative to its initial position at end-diastole.

The finite elements are used to compute strains numerically. Since we do not have any explicit functions relating $\mathbf{X}$ and $\mathbf{x}$, we can calculate derivatives in Eq. 9 by using the chain rule:

$$F_{pq} = \frac{\partial x_p}{\partial X_q} = \frac{\partial x_p}{\partial \xi_i} \frac{\partial \xi_i}{\partial X_q} \quad (11)$$

where ξ_i is a component of the local finite element coordinate system (see Appendix B for details).

Since the Lagrangian strain is a tensor which includes orientation information, it is defined relative to a reference coordinate frame. Since the issue of defining a coordinate frame for the RV remains an open research question, we use the Lagrangian strain tensor to calculate the coordinate system independent principal strains and directions. In a 3D analysis, two of the principal directions indicate the direction of maximum extension and compression in the region around a point. These directions and their corresponding magnitudes (which are the principal strains) supply potentially useful information about the directions of wall thickening and contraction in the heart. The principal directions are computed by finding the eigenvectors of $\mathbf{E}$, where the corresponding eigenvalues are the principal strains. A positive value indicates extension while a negative value indicates

compression [31]. In this paper, we report the results of the motion reconstruction using global displacements and local principal strains.

7 Results and Discussion

7.1 Validation Using Synthetic Data

As part of the validation of the 3D motion reconstruction method, we first generated synthetic data for a known deformation and geometry as was done by [35, 34]. The deformation modes were not meant to replicate the ventricular wall motion exactly, but rather to determine whether the fitting can reconstruct a complex motion pattern of a somewhat similar geometry. Similarly, the semi-spherical phantom geometry of the phantom was chosen to model the oblique intersection of ventricular walls with image planes, and not to exactly replicate the ventricular geometry. The deformation of 4mm-thick hollow spherical model was prescribed with equations similar to those of [34]. Using spherical coordinates, the equations are:

$$r = R \quad (12)$$

$$\theta = \Theta + \Theta_{min} - \Omega(R\cos(\Phi) + R_o) - \gamma(R - R_i) \quad (13)$$

$$\phi = \Phi_{max} + \lambda(\Phi - \Phi_{max}) \quad (14)$$

where $(R, \Phi, \Theta) =$ initial position, $(r, \phi, \theta) =$ deformed position, $R_i =$ inner radius, $R_o =$ outer radius. One end of the sphere is open and Φ ranges from Φ_{min} to Φ_{max} . Twisting of the model is the superposition of a minimum twist (Θ_{min}), twisting between planes in the longitudinal direction (controlled by Ω), and twisting between the inner and outer walls (controlled by γ).

We generated parallel tag planes with a 6mm separation and with tag points sampled every 2mm along a stripe. Simulated short-axis image planes (8 total) were separated at 6mm intervals, and simulated rotating long-axis image planes (as in our actual imaging protocol) were separated at 20 degree intervals. Similar to [35], we used the above equations to solve for the positions of tags and contours in the appropriate image planes. Geometric and deformation parameters are shown in Table 1.

Spherical Phantom Parameters			
Geometric		Deformation	
R_i	30mm	Ω	$0.573^\circ/\text{mm}$
R_{outer}	34mm	γ	$0.573^\circ/\text{mm}$
Φ_{min}	$\pi/3$	λ	0.88
Φ_{max}	π	Θ_{min}	3.0°

Table 1: Geometric and deformation parameters of phantom.

The synthetic tag and contour data were generated for an initial and final time phase. The initial deformable model was not generated from contours, but was a finite element mesh with the geometric parameters of Table 1. The rms tag and contour displacement errors were reduced from an initial 2.4mm to 0.3mm and an intimal 0.46mm to 0.22mm, respectively. The rms error between the calculated and fitted nodal displacements was 1.02mm rms. We used the same smoothing parameters as in the normal case, and simulated the piecewise smoothing described

Figure 18: Spherical phantom: (a) Initial configuration, with horizontal and vertical material surfaces. (b) Deformed model, tag planes, and associated stripe tags (light bands). A color plot of the displacement is displayed on inner wall.

earlier by intermittently recalculating the element stiffness matrices 4 times during the fitting. The initial and deformed position of the phantom along with several model material planes with two different orientations are shown in Fig. 18.

7.2 Reconstructed RV Wall Motion

The 3D reconstruction technique was applied to data from 2 normal volunteers and a patient with RVH. All human studies had 5 time phases through systole. The tag plane separation was 6mm in one normal heart and 7mm in the second normal and the RVH patient. The model was fit to the data until the tag and contour errors for each time phase were decreased to less than 0.5mm rms for the normals and less than 0.7mm for the hypertrophic heart (the starting errors were approximately 2mm rms). The total fitting time for both data sets was about 40 minutes on an SGI 02 RS10000 workstation (Mountain View, CA) for the normal and abnormal cases. Since several algorithms have not yet been optimized, the fitting time can be reduced by half. The time for the automatic mesh generation was relatively small, about 15 seconds.

The best way to visualize the reconstructed motion was by viewing movie loops, as shown in the accompanying video. Note that no temporal smoothing or interpolation was used, so that each model location shown corresponds to a particular image time phase. In these segments, it can be observed that the ventricles contracted as a unit, with the right ventricle following the twisting motion of the LV at the insertion points. When looking from the base, there was an initial clockwise rotation followed by a counterclockwise twisting of the base with respect to the apex. This relative twisting of the LV portion of the model was similar to that reported in the literature [1]. There was also a significant contraction of the base towards the apex in the RV free wall.

The 3D reconstruction provided interesting information about RV contraction and we focus our quantitative analysis on it. The greatest motion in the direction perpendicular to the short-axis image planes was about 19mm and 15mm near the base of the RV in the normal hearts, which was similar to previous observations [32]. This through-plane motion of more than two image planes was an indication that the fitting accuracy would increase when calculating the image forces from the nearest image planes during the fitting.

Figure 19 and a segment in the video show the paths of points in the middle of the RV free wall

Figure 19: Normal heart displacement at end-systole. Color plot of displacement on the RV endocardial a) free wall and b) septal wall viewed from the vantage point of the LV. The paths of points located in the centers of the elements are shown from end-diastole (white) to end-systole (blue).

as it contracts. These paths are also shown on the RV free wall and septal wall in Fig. 19. The color plot of displacement with respect to the initial position revealed a nonuniform distribution of displacement, with the greatest amount occurring in the base of the RV. The magnitude of displacement was smaller and the distribution more uniform in the septal wall.

Although the RV followed along with the LV through its insertion points at the septum, it exhibited a significant contraction of its own. A segment of the video and Fig. 20 show a color plot on the endocardial surface of the RV free wall of the minimum principal strains (E_3) at each time phase. The associated principal directions (red lines) were plotted at the mid wall with lengths relative to the maximum contraction at those points. Since the corresponding strains were negative, these lines indicated the directions of maximal contraction, and were in agreement with the results of [37]. These strains were fairly uniform at end-systole, with a extremum of -0.27 occurring towards the apex. An increasing gradient in the magnitude of E_3 towards the apex was also observed by [34], who implanted a matrix of beads onto the epicardial surface. However, since the region which they studied was limited to the midwall, they found a maximum value of -0.18 [34]. The fact that the maximum contraction occurred towards the apex whereas the maximum displacement occurred near the base indicates that the base undergoes mostly translation as it is pulled down by the middle and apical portion of the free wall. We also found that the minimum principal strain directions were more circumferential in the septal wall compared to the free wall (Fig 22). Another study using 2D analysis limited to MRI image planes found greater long-axis contraction in the RV free wall and greater short-axis contraction in the septal wall [10]. The researchers explained that this observation was due to the fiber orientation being more circumferential in the septal wall than in the free wall [10].

In section 5.3.3 we described how the fitting method produces material surfaces registered to the model which follow the image data. The motion of these material surfaces is shown in a segment of the video for two different tag surface orientations. The associated endocardial walls are first shown shaded for spatial reference. The yellow bands are the tag stripes which were assigned a correspondence to these surfaces. The close agreement between these material surfaces and their

Figure 20: Normal heart deformation: color plot of minimum principal strain on the RV endocardium as biventricular model deforms from end-diastole to end systole (initial phase + 4 time intervals). Red lines are the minimum principal strain directions drawn at the centers of the elements, with lengths normalized by the strain magnitude

associated stripes can be seen. The twisting motion in the LV and RV walls can also be seen by the warping of the vertical material surfaces.

Both right ventricular volume and pressure overload may result in right ventricular hypertrophy (RVH), which can progress to heart failure. We studied one RVH patient whose RV was dilated to the extent that the RV apex was closer to the diaphragm than the LV apex and, therefore, an extreme portion of the RV apex was not defined by the finite element mesh. The results of our 3D motion reconstruction showed noticeable differences in contraction patterns during RVH. The dilation and thickening of the wall due to the increased volume and pressure, respectively, can be seen in the video. The video segment shows a decrease in twisting and rotation in both the RV and LV and less through-plane motion in the RV as was observed by [37]. The entire contraction of the RV appears to be less efficient. The hypertrophic RV produced pressures which cause the septum to collapse into the LV cavity during systole as can be seen in the video segment. This can also be seen in the 3D motion reconstruction, as the shaded LV endocardial surface can be seen to move inwards compared to the LV. Fig. 21 shows the minimum principal strains in the RV free wall. Although the maximum magnitude of contraction was approximately the same as that in the normal, the color plot shows that this maximum occurred at the apex and that the contraction was noticeably reduced towards the base, as was observed by [10]. While the minimum principal

Figure 21: Right ventricular hypertrophy: color plot of minimum principal strain on the RV endocardial free wall as model deforms from end-diastole to end systole (initial phase + 4 time intervals). Dark lines are the minimum principal strain directions drawn at the centers of the elements, with lengths normalized by the strain magnitude.

strain directions lie in planes parallel to the ventricular walls (as in the normal case), they were more circumferentially oriented during RVH. Young, et al. [37] observed similar changes in principal strain magnitudes and directions.

The MRI-SPAMM tagging method combined with the 3D reconstruction technique provided both regional and distributed global information about biventricular motion. Although we derived motion variables which promise to be clinically useful, a statistical study involving several normal and abnormal hearts is necessary to determine which variable is the best for which disease. This method opens the way for characterizing the normal RV wall motion and possibly deriving global motion parameters akin to those found for the LV, which are more intuitive to physicians. The accuracy of the technique described in this paper was limited by the image quality and tag spacing. Better image quality would have decreased the noise of the input data and more closely spaced tag data would have provided more constraints for the model. Our method included local smoothing to account for both these factors. In the future, we can use higher-order finite elements or more elements when a smaller tag spacing leads to more constraints. In addition, we did not model a portion of the RV outflow tract, which is oriented obliquely in both short and long-axis data, due to limitations in building the geometric mesh from short-axis contours only. However, once the RV outflow tract is accurately modeled, the reconstruction method described in this paper can be directly applied to a model which encompasses the outflow tract.

Figure 22: Minimum principal strains in the septum of the normal heart study at end-systole. Color plot on surface facing RV cavity, viewed from the vantage point of the LV. Minimum strain directions are plotted at the center of the wall.

8 Conclusion

We have developed a deformable modeling technique for reconstructing the 3D of the ventricles from multiview tagged MR images. We accommodate for the thinness and complex geometry of the right ventricle by using 1D tags and a detailed model. The deformable model fitting method is a significant improvement of previous work since it uses tag *surfaces* as input data and accounts for the sparse contour data. Due to the relative lack of knowledge about RV motion, we use general degrees of freedom at the finite element nodes with localized smoothing between the nodes. This method allows us to capture the 3D regional variation in strain and other motion patterns. An example of the usefulness of this method is the observation that although the maximum displacement occurs at the base, the maximum contraction is in the mid-region and apex, which could not have been observed without a quantitative analysis of tagged tissue. The methods described in this paper can be used to study a significant number of normals in order to characterize the motion of the RV. Once the normal RV motion is characterized on a global and regional scale, the fitting method can also be applied to abnormal cases in order to determine which motion variables are the best indicators of various diseases. Finally, having a biventricular model should be useful for researching ventricular interactions.

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A Finite Element Shape Functions

The shape functions are used to interpolate a motion variable (displacement, position) from the nodes to a point in the element. We used eight-noded and six-noded elements whose node num-

Figure 23: Elements used to build deformable model. a) 8-noded parallelepiped. b) 6-noded wedge.

bering and local coordinates are shown in Fig. 23 [39]. The shape functions for an eight-noded parallelepiped are:

$$N_1 = (1 + \xi)(1 + \eta)(1 + \zeta) \quad (15)$$

$$N_2 = (1 + \xi)(1 + \eta)(1 - \zeta) \quad (16)$$

$$N_3 = (1 - \xi)(1 + \eta)(1 - \zeta) \quad (17)$$

$$N_4 = (1 - \xi)(1 + \eta)(1 + \zeta) \quad (18)$$

$$N_5 = (1 + \xi)(1 - \eta)(1 + \zeta) \quad (19)$$

$$N_6 = (1 + \xi)(1 - \eta)(1 - \zeta) \quad (20)$$

$$N_7 = (1 - \xi)(1 - \eta)(1 - \zeta) \quad (21)$$

$$N_8 = (1 - \xi)(1 - \eta)(1 + \zeta) \quad (22)$$

The shape functions for a six-noded wedge-shaped element are:

$$N_1 = 0.5L_1(1 - \eta) \quad (23)$$

$$N_2 = 0.5L_2(1 - \eta) \quad (24)$$

$$N_3 = 0.5L_3(1 - \eta) \quad (25)$$

$$N_4 = 0.5L_1(1 + \eta) \quad (26)$$

$$N_5 = 0.5L_2(1 + \eta) \quad (27)$$

$$N_6 = 0.5L_3(1 + \eta) \quad (28)$$

where L_1 , L_2 , and L_3 are the area coordinates of triangle 1-2-3 and 4-5-6 in Fig. 23b [39].

B Strain Calculation Using Finite Elements

Due to significant deformation in the heart wall during systole, we must use finite deformation theory for our strain calculations as is described in the next section.

B.1 Continuum Mechanics: Finite Deformation Theory

In this section, we define concepts in continuum mechanics that we will use in the strain formulation. A more detailed description is given in [31]. Finite deformation theory begins by defining the

deformation of a point in a continuum as

$$\mathbf{x} = \mathbf{x}(\mathbf{X}, t) \quad (29)$$

where $\mathbf{X}$ and $\mathbf{x}$ are the initial and final positions, respectively, of a point in the continuum. This vector equation can be written for each coordinate direction. For example, a uniform extension is written as:

$$x = \lambda_x X, \quad y = \lambda_y Y, \quad z = \lambda_z Z \quad (30)$$

Since we are concerned with relative deformation, we calculate the 3×3 deformation gradient tensor, $\mathbf{F}$. The components of $\mathbf{F}$ are

$$F_{pq} = \frac{\partial x_p}{\partial X_q} \quad (31)$$

where x_p is one of the three components of $\mathbf{x}$. In our model, we calculate the Lagrangian strain, $\mathbf{E}$, which is a measure of the deformation of a point in the model with respect to its initial position.

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T \mathbf{F} - \mathbf{I}) \quad (32)$$

where $\mathbf{I}$ is the identity matrix.

B.2 Finite Element Strain Calculation

Since we cannot take the derivatives in Eq. 31 directly, we expand it with the chain rule:

$$F_{pq} = \frac{\partial x_p}{\partial X_q} = \frac{\partial x_p}{\partial \xi_i} \frac{\partial \xi_i}{\partial X_q} \quad (33)$$

This equation is written in matrix form as the product of 2 matrices, $\mathbf{S}$ and $\mathbf{T}$:

$$\begin{bmatrix} \frac{\partial x}{\partial X} & \frac{\partial x}{\partial Y} & \frac{\partial x}{\partial Z} \\ \frac{\partial y}{\partial X} & \frac{\partial y}{\partial Y} & \frac{\partial y}{\partial Z} \\ \frac{\partial z}{\partial X} & \frac{\partial z}{\partial Y} & \frac{\partial z}{\partial Z} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \zeta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \zeta} \end{bmatrix}}_{\mathbf{S}:3 \times 3} \underbrace{\begin{bmatrix} \frac{\partial \xi}{\partial X} & \frac{\partial \xi}{\partial Y} & \frac{\partial \xi}{\partial Z} \\ \frac{\partial \eta}{\partial X} & \frac{\partial \eta}{\partial Y} & \frac{\partial \eta}{\partial Z} \\ \frac{\partial \zeta}{\partial X} & \frac{\partial \zeta}{\partial Y} & \frac{\partial \zeta}{\partial Z} \end{bmatrix}}_{\mathbf{T}:3 \times 3} \quad (34)$$

B.2.1 Matrix: $\mathbf{S}$

This matrix is the derivative of the position of a point in the element with respect to the local coordinates of that element. We recall that positions are functions of the local coordinates through the interpolation function equations. These equations are used to interpolate the position of a point in the element from the positions of the nodes. For example, the x -coordinate of a position within the element is:

$$x = \sum_{i=1}^n N_i x_i = N_1 x_1 + N_2 x_2 + \cdots + N_n x_n \quad (35)$$

Where (x_i, y_i, z_i) is the deformed position of the i^{th} node.

The derivative with respect to a local coordinate, ξ for example, is

$$\frac{\partial x}{\partial \xi} = x_{,\xi} = \sum_{i=1}^n N_{i,\xi} x_i = N_{1,\xi} x_1 + N_{2,\xi} x_2 + \cdots + N_{n,\xi} x_n \quad (36)$$

In matrix form, the relations for all 3 displacement derivatives taken with respect to the 3 local coordinates is:

$$\begin{bmatrix} x_{,\xi} & x_{,\eta} & x_{,\zeta} \\ y_{,\xi} & y_{,\eta} & y_{,\zeta} \\ z_{,\xi} & z_{,\eta} & z_{,\zeta} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & \dots & x_n \\ y_1 & y_2 & y_3 & y_4 & \dots & y_n \\ z_1 & z_2 & z_3 & z_4 & \dots & z_n \end{bmatrix} \begin{bmatrix} N_{1,\xi} & N_{1,\eta} & N_{1,\zeta} \\ N_{2,\xi} & N_{2,\eta} & N_{2,\zeta} \\ N_{3,\xi} & N_{3,\eta} & N_{3,\zeta} \\ N_{4,\xi} & N_{4,\eta} & N_{4,\zeta} \\ \vdots & \vdots & \vdots \\ N_{n,\xi} & N_{n,\eta} & N_{n,\zeta} \end{bmatrix} \quad (37)$$

B.2.2 Matrix: $\mathbf{T}$

It is easier to calculate the inverse of $\mathbf{T}$, which we call $\mathbf{U}$:

$$\mathbf{U} = \mathbf{T}^{-1} = \begin{bmatrix} X_{,\xi} & X_{,\eta} & X_{,\zeta} \\ Y_{,\xi} & Y_{,\eta} & Y_{,\zeta} \\ Z_{,\xi} & Z_{,\eta} & Z_{,\zeta} \end{bmatrix} \quad (38)$$

The initial coordinates, (X, Y, Z) , are also interpolated with the use of the interpolation functions. For the X coordinate,

$$X = \sum_{i=1}^n N_i X_i = N_1 X_1 + N_2 X_2 + \dots + N_n X_n \quad (39)$$

Where (X_i, Y_i, Z_i) are the initial positions of the i^{th} node. The derivative with respect to a local coordinate, ξ for example, becomes

$$\frac{\partial X}{\partial \xi} = X_{,\xi} = \sum_{i=1}^n N_{i,\xi} X_i = N_{1,\xi} X_1 + N_{2,\xi} X_2 + \dots + N_{n,\xi} X_n \quad (40)$$

The nine equations (similar to the one above) for the derivatives of the 3 initial coordinates with respect to the natural coordinates can be summarized in matrix form:

$$\mathbf{U} = \begin{bmatrix} X_{,\xi} & X_{,\eta} & X_{,\zeta} \\ Y_{,\xi} & Y_{,\eta} & Y_{,\zeta} \\ Z_{,\xi} & Z_{,\eta} & Z_{,\zeta} \end{bmatrix} = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & \dots & X_n \\ Y_1 & Y_2 & Y_3 & Y_4 & \dots & Y_n \\ Z_1 & Z_2 & Z_3 & Z_4 & \dots & Z_n \end{bmatrix} \begin{bmatrix} N_{1,\xi} & N_{1,\eta} & N_{1,\zeta} \\ N_{2,\xi} & N_{2,\eta} & N_{2,\zeta} \\ N_{3,\xi} & N_{3,\eta} & N_{3,\zeta} \\ N_{4,\xi} & N_{4,\eta} & N_{4,\zeta} \\ \vdots & \vdots & \vdots \\ N_{n,\xi} & N_{n,\eta} & N_{n,\zeta} \end{bmatrix} \quad (41)$$

The inverse of $\mathbf{U}$ then gives us $\mathbf{T}$.

B.3 Location of Strain Calculation

For the case of linear elements, we calculate the strain at the center of the each element. In order to calculate the strain at the nodes, we average the strains from all the elements that contain that particular node.

References

- [1] T. Arts, S. Meerbaum, R.S. Reneman, and E. Corday. Torsion of the left ventricle during the ejection phase in the intact dog. *Cardiovascular Research*, 18(3):183–93, 1984.
- [2] L. Axel, D. Bloomgarden, C.N. Chang, D. Kraitichman, and A.A. Young. SPAMMVU: a program for the analysis of dynamic tagged MRI. In *Proceedings of the Soc. of Magnetic Resonance in Medicine 12th Annual Meeting*, page 724, 1993.
- [3] L. Axel and L. Dougherty. Heart wall motion: Improved method of spatial modulation of magnetization for MR imaging. *Radiology*, 272:349–50, 1989.
- [4] D. Barnard and J.S. Alpert. Right ventricular function in health and disease. *Curr Probl Cardiol*, 12:422–29, 1987.
- [5] K.W. Chin, G.T. Daughters, E.L. Alderman, and D.C. Miller. Asynergy of right ventricular wall motion in man. *J Thorac Cardiovasc Surg*, 97:104–9, 1989.
- [6] C.J. Chuong, M.S. Sacks, G. Templeton, F. Schwiep, and Jr. R.L. Johnson. Regional deformation and contractile function in canine right ventricular free wall. *Am J Physiol*, 260:H1224–1235, 1991.
- [7] R.D. Cook, D.S. Malkus, and M.E. Plesha. *Concepts and Applications of Finite Element Analysis*. John Wiley and Sons, New York, third edition, 1989.
- [8] F.P. Czegledy and J.Katz. A new geometric description of the right ventricle. *J Biomed Eng*, 15:387–91, September 1993.
- [9] Z.A. Fayad. *Magnetic Resonance Imaging Studies of Right Ventricular Regional Mechanical Function*. PhD thesis, University of Pennsylvania, Philadelphia, 1996.
- [10] Z.A. Fayad, Victor A. Ferrari, D.L. Kraitichman, A.A. Young, H.I. Palevsky, D.C. Bloomgarden, and L. Axel. Right ventricular regional function using MR tagging: normals versus chronic pulmonary hypertension. *Magnetic Resonance in Medicine*, 39:116–123, 1998.
- [11] H. Greer. *Structure of the Heart: Course Notes*. Harry Greer, New York, 1999.
- [12] E. Haber, D.N. Metaxas, and L. Axel. Motion analysis of the right ventricle from MRI images. In *Medical Image Computing and Computer-Assisted Intervention - MICCAI*, pages 177–188, Cambridge, MA, 1998.
- [13] J.W. Hurst. *Atlas of the Heart*. Gower Medical, New York, 1988.
- [14] M. Kass, A. Witkins, and D. Terzopoulos. Snakes: active contour models. *Int J Comput Vision*, 1:321–31, 1987.
- [15] M.A. Konstam and N. Pandian. Assessment of right ventricular function. In *The Right Ventricle*. Kluwer Academic Publishers, Boston, 1988.

- [16] T. McInerney and D. Terzopoulos. Deformable models in medical image analysis: A survey. *Medical Image Analysis*, 1(2):91–108, 1996.
- [17] G.D. Meier, A.A. Bove, W.P. Santamore, and P.R. Lynch. Contractile function in canine right ventricle. *Am J Physiol*, 239:H794–804, 1980.
- [18] D.N. Metaxas. *Physics-based deformable models: applications to computer vision, graphics, and medical imaging*. Kluwer Academic Publishers, Cambridge, 1996.
- [19] C.C. Moore, W.G. O'Dell, E.R. McVeigh, and E.A. Zerhouni. Calculation of three dimensional left ventricular strains from biplanar tagged MR images. *Journal of Magnetic Resonance Imaging*, 2:165–75, 1992.
- [20] M.J. Moulton, L.L. Cresswell, S.W. Downing, R.L. Actis, B.A. Szabo, M.W. Vannier, and M.K. Pasque. Spline surface interpolation for calculating 3-D ventricular strains from MRI tissue tagging. *Am J Physiol*, 39:H281–H297, 1996.
- [21] P.M.F. Nielsen, I.J. Le Grice, B.H. Smaill, and P.J. Hunter. Mathematical model of geometry and fibrous structure of the heart. *J Appl Physiol*, 260:H1365–H1378, 1991.
- [22] A.V. Noordegraaf, J.T. Marcus, B. Roseboom, P.E. Postmus, T.J. Faes, and P.M. de Vries. The effect of right ventricular hypertrophy on left ventricular ejection fraction in pulmonary emphysema. *Chest*, 112:640–45, 1997.
- [23] W.G. O'Dell, C.C. Moore, W.C. Hunter, E.A. Zerhouni, and E.R. McVeigh. 3-dimensional myocardial deformations: calculation with displacement field fitting to tagged MR images. *Radiology*, 195:829–35, 1996.
- [24] J. Park. *Model-based shape and motion analysis: Left ventricle of a heart*. PhD thesis, University of Pennsylvania, Philadelphia, 1996.
- [25] J. Park, D. Metaxas, and L. Axel. Analysis of left ventricular wall motion based on volumetric deformable models and MRI-SPAMM. *Med Image Analysis J*, 1(1):53–71, 1996.
- [26] J.S. Pirolo, S.J. Bresina, L.L. Creswell, K.W. Myers, B.A. Szabo, M.W. Vannier, and M.K. Pasque. Mathematical three-dimensional solid modeling of biventricular geometry. *Annals of Biomed Eng*, 21:199–219, 1993.
- [27] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery. *Numerical Recipes in C: The Art of Scientific Computing*. Cambridge University Press, Cambridge, 1992.
- [28] R.A. Raines, M.M. LeWinter, and J.W. Covell. Regional shortening patterns in canine right ventricle. *Am J Physiol*, 231:H1395–1400, 1976.
- [29] M.S. Sacks, C.J. Chuong, G.H. Templeton, and R. Peshock. *In vivo* 3-D reconstruction and geometric characterization of the right ventricular free wall. *Annals of Biomed Eng*, 21:263–275, 1993.
- [30] W.P. Santamore, G.D. Meier, and A.A. Bove. Contractile function in canine right ventricle. *Am J Physiol*, 239:H794–804, 1980.

- [31] A.J.M. Spencer. *Continuum Mechanics*. Longman, London, 1980.
- [32] M. Stuber, E. Nagel, S.E. Fischer, M.B. Scheidegger, and P. Boesiger. Systolic long axis contraction of the human myocardium. In *Proceedings of the SMR*, 1995.
- [33] Jr. T.S. Denney and J.L. Prince. Reconstruction of 3D left ventricular motion from planar tagged cardiac MR images: an estimation theoretic approach. *IEEE Trans on Med Imaging*, 4(4):625–635, December 1995.
- [34] L.K. Waldman, J.J. Allen, R.S. Pavelec, and A.D. McCulloch. Distributed mechanics of the canine right ventricle: effects of varying preload. *J Biomechanics*, 29(3):373–81, 1996.
- [35] A.A. Young and L. Axel. Non-rigid heart wall motion using MR tagging. In *Proceedings of the IEEE Computer Society Conference on computer vision and pattern recognition*, pages 399–404, Campaign, Illinois, 1992.
- [36] A.A. Young and L. Axel. Three-dimensional motion and deformation of the heart wall: Estimation with spatial modulation of magnetization - a model-based approach. *Radiology*, 185:241–47, 1992.
- [37] A.A. Young, Z.A. Fayad, and L. Axel. Right ventricular mid-wall surface motion and deformation using magnetic resonance tagging. *J Appl Physiol*, 271:2677–88, 1996.
- [38] A.A. Young, D. L. Kraitchman, L. Dougherty, and L. Axel. Tracking and finite element analysis of stripe deformation in magnetic resonance tagging. *IEEE Trans on Med Imaging*, 14(3):413–21, September 1995.
- [39] O.C. Zienkiewicz and R.L. Taylor. *The Finite Element Method*. McGraw-Hill, New York, fourth edition, 1989.