5-2015

Voluntary Disclosure and the Cost of Capital

Greg Clinch

Robert E. Verrecchia
University of Pennsylvania

Recommended Citation

Clinch, G., & Verrecchia, R. E. (2015). Voluntary Disclosure and the Cost of Capital. *Australian Journal of Management, 40* (2), 201-223. http://dx.doi.org/10.1177/0312896214529441

Voluntary Disclosure and the Cost of Capital

Abstract

We investigate the association between voluntary disclosure and the risk-related discount investors apply to price. First, we study the association between (endogenous) disclosure choice and the discount in price induced by changes in the underlying model parameters: this informs empirical research that ignores endogeneity of disclosure by, for example, omitting the exogenous determinants of disclosure and the discount in price from the regressions employed. Second, we investigate the incremental effect of disclosure on the discount in price: this informs empirical research that controls for the direct effect of exogenous factors on the discount in price by, for example, including the exogenous variables in regression models employed. Finally, we examine the incremental effect of disclosure on the discount in price when changes in disclosure are not induced by changes in underlying exogenous parameters: this further informs empirical research that controls for the effect of exogenous factors on both the discount in price and disclosure, and focuses on the association between 'unexplained disclosure' and the discount.

Keywords

cost of capital, voluntary disclosure

Disciplines

Accounting | Business Administration, Management, and Operations

Voluntary Disclosure and the Cost of Capital

Greg Clinch*
Department of Accounting
University of Melbourne
clinchg@unimelb.edu.au

Robert E. Verrecchia
The Wharton School
University of Pennsylvania
verrecchia@wharton.upenn.edu

July 2011 (First Draft)

December 2011 (Current Draft)

Abstract

We investigate the association between voluntary disclosure and the risk-related discount investors apply to price. Prior research indicates that when the analysis is based on a commitment to disclose the association is negative (i.e., more disclosure is associated with a lower discount). Our results suggest that with voluntary, or endogenous, disclosure the association is not necessarily negative, and in most cases the association is positive. This implies that in studies of either the discount or cost of capital, some care should be taken to distinguish endogenous disclosure choice (i.e., voluntary disclosure) from disclosure commitment.

JEL classification: G12, G14, G31, M41

Key Words: *Cost of capital, voluntary disclosure*

* Corresponding author

We gratefully acknowledge comments and suggestions from Karthik Balakrishnan, Philip Berger, Mary Billings, Luzi Hail, Christian Leuz, Catherine Schrand, Doug Skinner, Jessica Tarica, Sorabh Tomar, David Tsui, and workshop participants at Griffith University and the University of Melbourne.

1 Introduction

The purpose of this paper is to offer results about the association between voluntary disclosure and the discount that investors apply to a firm's expected cash flow when they price the firm. In a setting where a firm's expected cash flow is fixed, the discount that investors apply to price is equivalent to a firm's cost of capital as the latter is commonly defined (e.g., Lambert, Leuz, and Verrecchia, 2007). Existing theory posits that an increased commitment to disclose information about a firm's future prospects or terminal cash flow, irrespective of the nature of the information disclosed subsequent to the commitment, results in a reduction in the firm's cost of capital.[1] In contrast, we investigate the association between *voluntary* disclosure choice and the discount investors apply to price (or cost of capital). To our knowledge, there is no theory-based literature that addresses this question.

By voluntary disclosure, we mean a policy to disclose information that depends both on features of the economy and the ex post realization of the information prior to its disclosure. Because it depends on the ex post realization of information, voluntary disclosure is an example of endogenous disclosure choice and its analysis is more nuanced than an analysis based on a commitment to disclose. The latter examines how a *ceteris paribus* change in the precision or level of disclosure as an exogenous parameter affects the discount as an endogenous variable. The former examines how a *ceteris paribus* change in an exogenous feature of the economy affects simultaneously the level of voluntary disclosure and the discount as endogenous variables, and then determines the association between disclosure and the discount. As we discuss below, the association between voluntary disclosure and the discount that results is not necessarily negative (i.e., more voluntary disclosure is not necessarily associated with

[1] See, for example, Corollary 3 in Diamond and Verrecchia (1991), Corollary 1 in Baiman and Verrecchia (1996), Proposition 3 of Easley and O'Hara (2004), Proposition 2 in Lambert, et al. (2007), Theorem 1 in Christensen, de la Rosa and Feltham (2010), Proposition 1 in Gao (2010), and Proposition 5 in Bloomfield and Fischer (2011). Note that these papers typically do not couch their results in the context of a commitment to disclose, but instead show that an increase in disclosure precision, or a reduction in the variance of measurement error, results in lower cost of capital. For all intents and purposes, an increase in disclosure precision and/or a reduction in the variance of measurement error are tantamount to an increase in the commitment to disclose.

a lower discount) and in most cases the association is positive. This implies that in studies of either the discount or cost of capital, some care should be taken to distinguish voluntary disclosure from a disclosure commitment.

Casual inspection of the extant literature on disclosure and cost of capital suggests that researchers have tended to blur the distinction between these two concepts. For example, in an early and very influential paper, Botosan (1997, p.326) hypothesizes and tests for a negative association between the level of voluntary disclosure and the cost of capital based, in part, on an appeal to the disclosure commitment models of Amihud and Mendelson (1986) and Diamond and Verrecchia (1991). More recently, Hail (2011) in his discussion of Serafeim (2011) similarly suggests that, as is the case with a commitment, voluntary disclosure might reasonably be expected to yield a negative, though weaker, association between disclosure and cost of capital. Consistent with the thesis of a negative association, Beyer, Cohen, Lys, and Walther (2010, p.308) remarks that "Much of the empirical literature to date in this second category [cross-sectional association between voluntary disclosures and the cost of capital] seeks to provide evidence that firms that disclose more have a lower cost of capital."[2] Our analysis suggests that when disclosure is voluntary/endogenous, the opposite effect is more likely to be observed.

We study the association between voluntary disclosure and the discount that investors apply to a firm's expected cash flow by extending the voluntary disclosure setting of Jung and Kwon (1988; JK) to an economy where investors are risk averse. We choose JK because while the setting itself is parsimonious, nonetheless it offers a well established framework for examining the role of voluntary disclosure.[3] Because JK is based on the assumption that investors are risk neutral, in pricing a firm's shares investors apply no discount to the firm's expected cash flow on average (we show this in our analysis below). Our assumption that

[2] See, for example, Botosan (1997), Botosan and Plumlee (2002), Easley, Hvidkjaer and O'Hara (2002), Leuz and Verrecchia (2000), Cohen (2008), and Serafeim (2011) amongst others. See Leuz and Wysocki (2008), Beyer, et al. (2010), and Berger (2011) for recent comprehensive reviews of the disclosure research.

[3] For example, using this setting: Dye and Sridhar (1995) study herding behavior in disclosures; Pae (2002) examines the allocation of resources; Shin (2003) predicts return variance; and Shin (2006) considers future disclosure risks.

investors are risk averse extends JK's setting to an economy where investors discount the firm's expected cash flow because the cash flow is uncertain. In our analysis, we focus on the expected, or average, discount in price across possible disclosure outcomes. However, we also discuss the nature of the discount in price conditional on whether or not the firm discloses.

Our setting incorporates four broad features that characterize an economy: 1) the level of investors' risk aversion; 2) the probability that the firm is uninformed; 3) the number of investors in the economy, and 4) the distribution of the firm's cash flow. We interpret a decrease or deterioration in each of these features as an exacerbation in the adverse selection environment between the firm and investors. For example, more risk aversion on the part of investors serves to exacerbate the adverse selection environment. In our main analysis we investigate the impact of changes in each of these four features on the firm's voluntary disclosure decision and on the resulting discount in price. In a majority of circumstances that involve a *ceteris paribus* change in one of the four features, we show that an exacerbation of adverse selection results in more disclosure in conjunction with a higher discount. The economic intuition that explains this is straightforward. In standard voluntary disclosure settings, a firm chooses to disclose private information by taking into account the penalty investors apply to price in the absence of disclosure as protection from the adverse selection environment they face (e.g., Verrecchia, 1983). When features of the economy change that exacerbate adverse selection, there are two effects. In valuing the firm investors apply a higher price penalty to the firm's unconditional expected cash flow as a result of heightened adverse selection, motivating the firm to increase disclosure to counteract the higher penalty. This increased disclosure works to decrease the discount in price. But the changed economic features also generally are associated with a deterioration in the general risk environment facing investors, for example through increased uncertainty or increased risk aversion. This deterioration works to increase the discount in price. The problem here is that the second-order effect of more disclosure generally does not overcome the first-order effect of deterioration in the risk environment, and hence the discount increases.[4]

It is important to emphasize that our model is compatible with the prior literature that establishes a negative relation between an improvement in the commitment to disclosure and the cost of capital. In particular, two features of our model align closely with that literature. First, we show in the context of our model that an increase in disclosure precision that decreases investors' uncertainty prior to the firm's voluntary disclosure decision is associated negatively with the discount in price. Second, holding all exogenous parameters constant in our model, the (conditional) discount manifest in price is lower when a firm discloses versus withholds its information. Despite these similarities, we show that in a voluntary disclosure setting if exogenous parameters change in a manner that results in an increased likelihood of disclosure by the firm, in general the discount also increases: greater disclosure is associated with a higher discount. The reason for this is that the exogenous factors that cause an increase in the likelihood of disclosure also cause an increase in the discount experienced by the firm if it does not disclose. And as we show below, the second effect is, in the majority of circumstances, dominant.

Perhaps the chief empirical implication of our paper is that the *contemporaneous* relation between a change in the level of disclosure and the discount in price as a result of a change in the risk environment is positive. However, to the extent to which increased disclosure is *subsequently* perceived as a commitment, then the relation between a change in the level of disclosure and the discount will be negative. A variety of empirical results already exist in the literature that accord with this implication. For example, Leuz and Schrand (2009; LS) reports evidence that the increased perceived (exogenous) uncertainty for U.S. firms that accompanied the Enron shock resulted in both an increased estimated cost of capital and increased voluntary disclosure by firms in order to mitigate transparency concerns. Moreover,

[4] Larcker and Rusticus (2010, pp.198-199) anticipates this possibility: "...firms with high risk and uncertainty in their business environment (and thus a high cost of capital) may try to increase their disclosure quality in order to reduce cost of capital. To the extent that they are only partially successful, this causes a positive relation between disclosure quality and cost of capital." One could interpret our analysis as "testing" this intuition in the context of a model of voluntary disclosure, and confirming it in the majority of circumstances (i.e., relating to variation in most of the exogneous model factors).

the association between changed cost of capital and disclosure is positive across the sample of firms in LS, despite additional evidence that the increased disclosure was successful in mitigating some of the increased perceived uncertainty. Similarly, Balakrishnan, Billings, Kelly, and Ljungqvist (2011) reports evidence of a contemporaneous relation between increased voluntary disclosure and greater illiquidity in a setting wherein exogenous terminations of analyst coverages increases adverse selection.[5] However, Balakrishnan et al. (2011) also finds that the liquidity for the firms that increased disclosure increased in subsequent periods. In a similar vein, LS finds a subsequent reduction in cost of capital following increased disclosure due to the Enron crises. These findings are consistent with subsequent, and persistent, disclosure being perceived as a commitment that results in greater liquidity (Balakrishnan et al., 2011) and a decline in cost of capital (LS).

Although voluntary disclosure and the discount are associated positively in a majority of circumstances, there are circumstances where the association is negative. For example, consider the effect of a *ceteris paribus* change in the probability that the firm is uninformed. Below we represent this probability as p. Here, in the context of JK and our extension of their model, a firm's failure to report voluntarily could either be the result of the firm's unwillingness to provide "bad news" or the fact that the firm is genuinely uninformed. As we prove below, an increase in p increases the threshold beyond which the firm discloses voluntarily. Thus an increase in p decreases the likelihood of disclosure in two ways: directly through the increase in p itself, and indirectly through the increased threshold. The result is a greater decrease in disclosure compared with changes in other model parameters where only the second, indirect effect is present. At the same time, an increase in p has a smaller effect on the discount in price conditional on non-disclosure because it does not affect the *ex ante* distribution of the firm's cash flow or investors' risk preferences. We show below that the combination of these effects implies that there is no monotonic relation between

[5] A related finding is reported by Skinner (1997) and Field, Lowry and Shu (2005) in the disclosure/legal cost setting. The observed association between disclosure and legal costs is positive (against most expectations), but negative once one controls for endogeneity (i.e., the fact that other underlying factors cause both to increase concurrently).

p and the discount. That said, we provide examples where the countervailing effect that dominates depends on the number of investors who compete for the firm's shares. Specifically, we show that when the number of investors is low the discount investors apply to price monotonically increases with p, whereas when the number is high the discount first increases and then decreases with p.[6] In both circumstances, disclosure and the discount are associated negatively for low values of p: in other words, when the likelihood of disclosure is high. Thus our examples suggest that the generally expected negative association between (voluntary) disclosure and cost of capital might be most likely to be observed when disclosure levels are already high, but not when they are low.

More generally, as we discuss further below, our results suggest that in voluntary disclosure settings such as ours the association between disclosure and the discount in price will differ depending on the nature of the exogenous factors that underlie variation in disclosure and the discount. For factors directly related to the riskiness of the firm's cash flow and/or the appetite for risk of investors, our analysis suggests the association will be positive, contrary to typically expressed expectations. For other factors, however, the association can be positive and/or negative, but will be negative when exogenous factors are such that disclosure levels and the likelihood of disclosure are high. These insights could prove useful in the design of empirical experiments about the relation between voluntary disclosure and cost of capital.

Finally, one caveat to applying the results of our analysis on the positive association between voluntary disclosure and the discount in price to empirical studies on cost of capital is that the discount is equivalent to cost of capital in a circumstance where a firm's expected cash flow is fixed. However, to the extent to which a firm's expected cash flow is not fixed because, for example, investment decisions are endogenous, then the discount that investors apply to expected cash flow is only one factor in cost of capital (albeit an important factor).

[6] These examples comport with other recent evidence that the number of investors who compete for a firm's shares may be an important conditioning variable in assessing cost of capital. See Akins, Ng, and Verdi (2011); Armstrong, Core, Taylor, and Verrecchia (2011); and Lambert, Leuz, and Verrecchia (2012).

Thus, studies that posit a negative association between voluntary disclosure and cost of capital could still be correct to the extent to which the focus is on some phenomenon other than the discount.

The remainder of the paper is organized as follows. In Section 2 we introduce our adaptation of JK's model that allows for risk aversion on the part of investors, and show that, as in JK, our adaptation implies the existence of a unique threshold level of disclosure above which the firm discloses information and below which it withholds information. In Section 3 we provide a variety of comparative static results on measures of increased disclosure, as manifest in the threshold level of disclosure and the likelihood that the firm discloses, and the extent to which investors discount the firm's expected cash flow. We summarize our results in Section 4.

2 A model of voluntary disclosure

To start, we consider a firm whose cash flow is uncertain in period 1, but becomes realized in period 2. Let $\tilde{V}$ represent the firm's (uncertain) cash flow in period 1, and let $\tilde{V} = V$ represent the realization of the firm's cash flow in period 2. We summarize the notation used in the paper in Table 1**.** [**Insert Table 1 here.**] In period 1, before the firm's cash flow is realized, the firm sells shares to investors who bid to hold claims in the firm's realized cash flow in period 2.

Also in period 1, we assume that the firm may learn in advance its realized cash flow (i.e., $\tilde{V} = V$): with probability p the firm has no knowledge of its cash flow until it is realized in period 2, and with probability $1 - p$ the firm learns its cash flow in period 1.[7] In period 1 if the firm learns $\tilde{V} = V$, then following JK we assume that the firm decides to either disclose or withhold this information based on which action maximizes the value of the firm in period

[7] While we assume that the item over which the firm may be uninformed is the firm's realized cash flow in period 2, it should be clear that p could represent the probability that the firm is uninformed about any one of a variety of economic phenomena that could be of interest to investors, such as fair value measures of assets and liabilities, future investment decisions, pending litigation, etc.

1.[8] Let $P(V)$ represent the price of the firm conditional on the firm's decision to disclose publicly $\tilde{V} = V$, and $P(ND)$ the price of the firm conditional on no disclosure (where we use ND to represent "no disclosure"). Conditional on the firm disclosing $\tilde{V} = V$ in period 1, we show below that competition among investors to buy shares in the firm results in $P(V)$ being equal to an amount that leaves investors indifferent between holding shares in the firm versus purchasing no stake in the firm. Similarly, conditional on no disclosure, competition among investors to buy shares in the firm results in $P(ND)$ being equal to an amount that leaves investors indifferent between holding shares in the firm versus purchasing no stake in the firm. In period 2 the firm's cash flow is realized, the firm liquidates, and $\tilde{V} = V$ is distributed to shareholders based on the claims to the firm that they established in period 1.

As for the distribution of the firm's cash flow, as in JK we assume that cash flow realizations $\tilde{V} = V$ are distributed over the range $[L, H]$, where "L" and "H" are mnemonics for "low" and "high," respectively. We make no assumptions about L and H other than the fact that they are real-valued numbers with the feature that $L \leq H$. Let $F(V)$ represent the cumulative probability distribution of $\tilde{V}$, and $f(V)$ the density function of $\tilde{V}$. Because $\tilde{V} \in [L, H]$,

$$F(V) = 0 \text{ for all } V \leq L \text{ and } F(V) = 1 \text{ for all } V \geq H.$$

Finally, let μ represent the firm's expected (or mean) cash flow: that is, $\mu = E\left[\tilde{V}\right] = \int_L^H V dF(V)$.

The focus of our study is on the expected price or value of the firm in period 1 based on all possible events that may transpire in period 1. Let P represent the firm's expected price or value based on all possible events, let Π represent the probability that the firm discloses in period 1 (which implies that $1 - \Pi$ represents the probability that it does not disclose), and let t represent the threshold above which the firm discloses if it knows $\tilde{V} = V$ in period

[8] For example, as JK states on p. 148: "...we assume that the firm's shareholders unanimously agree to a disclosure policy which maximizes firm value..."

1 (where below we establish the existence of such a t). We compute P as

$$P = E\left[P\left(\tilde{V}\right)|\text{Disclosure of } \tilde{V} = V \geq t\right] \times \Pi + P\left(ND\right) \times \left(1 - \Pi\right).$$

Here we study the extent to which the price or value of the firm reflects a discount to the firm's expected cash flow, which is μ. For example, let Δ represent the extent to which price discounts the firm's expected cash flow. We define and measure Δ as

$$\Delta \equiv \mu - P.$$

The chief motivation for our paper is to understand whether measures of increased disclosure, as manifest in the threshold above which the firm discloses (t) and the likelihood that the firm discloses (Π), are associated positively or negatively with the extent to which investors discount the firm's expected cash flow, as measured by Δ.

With regard to investors' utility for wealth, we assume that investors are risk averse. Let $U\left(w\right)$ represent investors' utility for wealth w. We represent risk aversion through a simple, piecewise-linear function: specifically, $U\left(w\right) = w$ for $w \leq \alpha$, and $U\left(w\right) = \alpha + \beta(w - \alpha)$ for $w > \alpha$ where $0 \leq \alpha$ and $0 \leq \beta \leq 1$. We refer to α as the "switching point" in investors' utility function for wealth, and to β as the slope in investors' utility function for wealth when an investor's wealth exceeds α. We assume $0 \leq \alpha$ to ensure that investors associate a utility of 0 to wealth of 0 (i.e., $U\left(0\right) = 0$); allowing for the possibility that $\alpha < 0$ does not qualitatively affect the results of our analysis. Using Jensen's Inequality, it is a straightforward exercise to show that $U\left(w\right)$ is a concave function and thus manifests risk aversion (we leave the proof to the interested reader). There are two advantages to representing utility as a piecewise-linear function. First, it facilitates the derivation of the P, $P\left(V\right)$, and $P\left(ND\right)$. Second, the utility function reverts seamlessly to risk neutrality by either setting $\beta = 1$ or allowing $\alpha \to \infty$, and this makes comparisons between our results and those of JK very transparent.[9]

[9] Many of our results hold with more general utility functions. For example, uniqueness of the equilibrium

Finally, we assume that investors have no endowed wealth, but can borrow funds at no cost. An implication of the assumption that investors have no endowed wealth, along with the fact that $0 \leq \alpha$, is that an investor associates a utility of $U(0) = 0$ to purchasing no stake in the firm and holding no claim to the firm's cash flow in period 1.

Recall that Δ represents the extent to which investors discount the firm's expected cash flow. When investors are risk neutral (which results from either setting $\beta = 1$ or allowing $\alpha \to \infty$), we show below that there is no discount. In other words, when investors are risk neutral it is always the case that $P = \mu$ (and thus there is no discount) irrespective of whether the firm commits to: 1) a policy of full disclosure *ex ante*; 2) a policy of no disclosure *ex ante*; or 3) behaves strategically in period 1 if it learns $\tilde{V} = V$. This result comports with the analysis in JK, which assumes that investors are risk neutral. Thus, an ancillary benefit of our analysis is that it extends the analysis in JK to a setting where investors are risk averse and a discount arises.

Note that the economy we describe has four categories of exogenous features: 1) investors' risk aversion as manifest in the switching point in investors' utility function for wealth, α, and the slope in investors' utility function for wealth, β; 2) the probability that the firm has no knowledge of the firm's realized cash flow in period 1, p; 3) the number of investors in the economy (which we represent below by N); and 4) the distribution of the firm's cash flow, $F(V)$. The goal of our analysis is to understand how a change in an exogenous feature of the economy affects simultaneously measures of increased disclosure, as manifest in the threshold level of disclosure (t) and the likelihood of disclosure (Π), and the discount investors apply to the firm's expected cash flow (Δ). When an exogenous change results simultaneously in increased disclosure and a lower (higher) discount, we say that disclosure and the discount are associated negatively (positively) through the exogenous change. Of course, an exogenous

disclosure threshold, and the fact that the threshold increases in the probability firms do not have private information, holds for all increasing, concave utility functions. Also, with general utility functions shifting the distribution of $\tilde{V}$ to the right and/or shrinking the distribution towards its mean yield the same effects as our results relating to first- and second-order stochastic dominance changes to $F(\cdot)$. Also, our results hold if we assume that investors have constant absolute risk aversion (i.e., CARA utility) and $F(\cdot)$ has a normal distribution. Details to these claims are available from the authors.

change may result in no monotonic association in general. In this circumstance we say that increased disclosure and the discount are unrelated through the exogenous change.

2.1 Price formation in period 1

We assume that in period 1 investors bid for shares in the firm's cash flow by playing a Nash game that eliminates any surplus to investors. Here we describe the market mechanism in period 1 that determines the price or value of the firm conditional on the firm's decision to disclose publicly $\tilde{V} = V$, $P(V)$, versus the price of the firm conditional on no disclosure, $P(ND)$. The role of the market mechanism is to make formal a valuation process that is very intuitive. Namely, if in period 1 the firm reveals that the value of the firm is $\tilde{V} = V$, then the market assesses the price (value) of the firm to be $P(V) = V$. Similarly, if in period 1 the firm reveals nothing, then the market assesses the price (value) of the firm to be $P(ND)$, where $P(ND)$ equals the expected value of $\tilde{V}$ conditional on no disclosure less a discount for the uncertainty investors associate with not knowing the exact realization of $\tilde{V} = V$.

To describe the market mechanism, we begin by assuming that N investors compete to hold shares of the firm in period 1. In addition, we assume that investors' demand orders are handled by a non-strategic market maker who chooses which investors will hold shares. The market maker's rule is to ask each of the N investors for a quote to purchase a fraction $1/N^{-th}$ of the firm.[10] Let $q(V)$ represent an investor's quote conditional on the firm revealing that the firm has cash flow of $\tilde{V} = V$, and let $q(ND)$ represent an investor's quote conditional on no disclosure. The market maker allocates an equal number of shares to each investor if each investor quotes the same price. If $M \leq N$ investors are tied for the highest price for holding $1/N^{-th}$ of the firm, each investor who quotes the highest price receives a fraction $1/M^{-th}$ of the firm and this is a binding commitment.

Given this commitment, a symmetric Nash equilibrium is for each investor to quote the

[10] This market mechanism is adapted from Diamond and Verrecchia (1991).

same price. The price is determined such that a deviation to a higher price for holding a fraction $1/N^{-th}$ of the firm would lead to: 1) an investor holding a larger fraction because the fraction one holds in the firm is an increasing function of price; and 2) an investor associating a negative utility to holding the larger fraction. Alternatively, a deviation to a lower price would result in an investor having no stake in the firm, and associating a utility of $U(0) = 0$ to this outcome.

Now consider the derivation of $P(V)$, the price of the firm conditional on the decision to disclose publicly $\tilde{V} = V$. The expected utility an investor associates with holding a fraction $1/N^{-th}$ of the firm at a quote of $q(V)$ is

$$
\begin{aligned}
E\left[U\left(\frac{\tilde{V}}{N} - q\left(\tilde{V}\right)\right)|\tilde{V} = V\right] &= U\left(\frac{V}{N} - q(V)\right) \\
&= \begin{cases} \alpha + \beta\left(\frac{V}{N} - q(V) - \alpha\right) & \text{if } \frac{V}{N} - q(V) > \alpha \\ \frac{V}{N} - q(V) & \text{if } \frac{V}{N} - q(V) \leq \alpha \end{cases}.
\end{aligned}
$$

This implies that if an investor quotes a price higher than $\frac{V}{N}$ then he will associate a negative utility for any fraction of the firm he holds, and if he quotes a price lower than $\frac{V}{N}$ then he will end up with no stake in the firm because other investors will quote a price $\frac{V}{N}$. Thus, each investor quotes $q(V) = \frac{V}{N}$ and ends up holding a fraction $1/N^{-th}$ of the firm. In other words, when the firm reveals in period 1 that the firm's realized cash flow is $\tilde{V} = V$, an investor is indifferent between purchasing a fraction $1/N^{-th}$ of the firm at a price quote of $q(V) = \frac{V}{N}$ versus having no stake in the firm, because in either case an investor's utility is $U(0) = 0$. Finally, when each of N investors quotes a price $q(V) = \frac{V}{N}$, then the market as a whole values the firm at $P(V) = N \times q(V) = V$.

Now we consider $P(ND)$, the price of the firm conditional on no disclosure. Because $P(V)$ is increasing in V when the firm discloses $\tilde{V} = V$ and the firm behaves strategically to maximize firm value, a disclosure/withholding region must consist of a threshold t above which the firm discloses and below which it does not. If the firm elects not to disclose (denoted by ND), this implies that either the firm did not observe $\tilde{V}$ or observed $\tilde{V} = V \leq t$. Thus, the expected utility an investor associates with holding a fraction $1/N^{-th}$ of the firm at a quote of $q(ND)$ is

$$\begin{aligned}
E\left[U\left(\frac{\tilde{V}}{N}-q(ND)\right)|ND\right] &= \frac{p}{p+(1-p)F(t)}E\left[U\left(\frac{\tilde{V}}{N}-q(ND)\right)\right] \\
&+\frac{(1-p)F(t)}{p+(1-p)F(t)}E\left[U\left(\frac{\tilde{V}}{N}-q(ND)\right)|\tilde{V}=V\leq t\right] \\
&= \frac{p}{p+(1-p)F(t)}\left[\int_{L}^{N\alpha+Nq(ND)}\left(\frac{V}{N}-q(ND)\right)dF(V)\right. \\
&+\left.\int_{N\alpha+Nq(ND)}^{H}\left(\alpha+\beta(\frac{V}{N}-q(ND)-\alpha)\right)dF(V)\right] \\
&+\frac{(1-p)F(t)}{p+(1-p)F(t)}\left[\frac{1}{F(t)}\int_{L}^{N\alpha+Nq(ND)}\left(\frac{V}{N}-q(ND)\right)dF(V)\right. \\
&+\left.\frac{1}{F(t)}\int_{N\alpha+Nq(ND)}^{t}\left(\alpha+\beta\left(\frac{V}{N}-q(ND)-\alpha\right)\right)dF(V)\right]. \quad (1)
\end{aligned}$$

Because the firm behaves strategically to maximize firm value, it must be the case that the price (value) of the firm in the absence of disclosure, $P(ND)$, equals the threshold value above which the firm discloses and below which it withholds: that is, $P(ND) = t$. But if $P(ND) = t$ and $P(ND) = N \times q(ND)$, then it must be the case that $q(ND) = \frac{t}{N}$ in equilibrium, and thus $t \leq N\alpha + Nq(ND) = N\alpha + t$ because $0 \leq \alpha$. But this implies that the interval $[N\alpha + Nq(ND), t]$ is null, and thus, eqn. (1) reduces to

$$\begin{aligned}
E\left[U\left(\frac{\tilde{V}}{N}-q(ND)\right)|ND\right] &= \frac{p}{p+(1-p)F(t)}\left[\int_{L}^{N\alpha+Nq(ND)}\left(\frac{V}{N}-q(ND)\right)dF(V)\right. \\
&+\left.\int_{N\alpha+Nq(ND)}^{H}\left(\alpha+\beta(\frac{V}{N}-q(ND)-\alpha)\right)dF(V)\right] \\
&+\frac{(1-p)F(t)}{p+(1-p)F(t)}\left(\frac{1}{F(t)}\int_{L}^{t}\left(\frac{V}{N}-q(ND)\right)dF(V)\right) \\
&= \frac{1}{N}E[\tilde{V}|ND]-q(ND)-\frac{p}{1-\Pi}(1-\beta) \\
&\times\int_{N\alpha+Nq(ND)}^{H}(\frac{V}{N}-\alpha-q(ND))dF(V), \quad (2)
\end{aligned}$$

where $\Pi = (1-p)(1-F(t))$ is the probability of disclosure, $1-\Pi = p + (1-p)F(t)$ is the probability of no disclosure, and $E[\tilde{V}|ND] = \frac{1}{1-\Pi}\left(p\mu + (1-p)\int_L^t V dF(V)\right)$ is the expected value of $\tilde{V}$ given the absence of disclosure.

Consider the value for $q(ND)$ that reduces the right-hand-side of eqn. (2) to 0; that is,

$$q(ND) = \frac{1}{N}E[\tilde{V}|ND] - \frac{p}{1-\Pi}(1-\beta)\int_{N\alpha+Nq(ND)}^{H}(\frac{V}{N} - \alpha - q(ND))dF(V). \qquad (3)$$

If an investor quotes a price higher than $q(ND)$ as determined in eqn. (3), then he will associate a negative utility for any fraction of the firm he holds, and if he quotes a price lower than $q(ND)$ then he will end up with no stake in the firm because other investors will quote $q(ND)$. Thus, each investor quotes $q(ND)$ as determined in eqn. (3) and ends up holding a fraction $1/N^{-th}$ of the firm. Conditional on no disclosure, this leaves an investor indifferent between purchasing a fraction $1/N^{-th}$ of the firm at a price quote of $q(ND)$ as determined in eqn. (3) versus having no stake in the firm, because in either case an investor's utility is $U(0) = 0$.

The only problem with eqn. (3) is that it determines $q(ND)$ implicitly because $q(ND)$ appears on both sides of the equation. Thus, our next task is to show that there exists a unique $q(ND)$ that solves eqn. (3).

2.2 A unique threshold level of disclosure

Recall that $q(ND) = \frac{t}{N}$ in equilibrium. This allows us to re-express eqn. (3) as

$$\frac{t}{N} = \frac{1}{N}E[\tilde{V}|ND] - \frac{p}{1-\Pi}(1-\beta)\int_{N\alpha+t}^{H}(\frac{V}{N} - \alpha - \frac{t}{N})dF(V),$$

or

$$t = E[\tilde{V}|ND] - \frac{p}{1-\Pi}(1-\beta)\int_{N\alpha+t}^{H}(V - N\alpha - t)dF(V). \qquad (4)$$

An immediate implication of eqn. (4) is that $t < E[\tilde{V}|ND] < \mu$. The first inequality follows because the integral in eqn. (4) is positive. The second inequality is true from the definition of $E[\tilde{V}|ND] = \frac{1}{1-\Pi}\left(p\mu + (1-p)\int_L^t V dF(V)\right)$. Thus the disclosure threshold t (and price of the firm in the absence of disclosure) is less than both the unconditional expected value of the final payoff (μ), and the conditional expected value in the event of non-disclosure ($E[\tilde{V}|ND]$).

The solution to eqn. (4) is equivalent to the existence of a t that solves $T(t) = 0$, where $T(t)$ is defined as

$$\begin{aligned} T(t) &= p\left(\beta(\mu - t) + (1-\beta)N\alpha + (1-\beta)\left(\int_L^{N\alpha+t}(V - N\alpha - t)\,dF(V)\right)\right) \\ &\quad + (1-p)\int_L^t (V-t)\,dF(V). \end{aligned} \tag{5}$$

Eqn. (5) is obtained by multiplying eqn. (4) throughout by $1-\Pi$ and rearranging. Note that $T(t)$ can also be expressed as

$$T(t) = p\left(\beta(\mu - t) + (1-\beta)N\alpha - (1-\beta)\int_L^{N\alpha+t} F(V)\,dV\right) - (1-p)\int_L^t F(V)\,dV. \tag{6}$$

Both expressions will prove useful in our analysis below.[11]

To show the existence of a unique t that solves $T(t) = 0$, it is sufficient to show that $T(t = L) > 0$, $T(t = H) < 0$, and $T(t)$ is monotonically decreasing in t. We show this in the next proposition. (All proofs are provided in the appendix.)

Proposition 1. *There exists a unique t that solves $T(t) = 0$, and thus a unique threshold above which the firm discloses $\tilde{V} = V$ and below which it withholds this information.*

Consider the following illustration of Proposition 1. Let $N = 10$, $\alpha = 0.1$, $\beta = 0.9$, and $p = 0.5$. In addition, assume that $\tilde{V}$ has a uniform distribution between 0 and 20, which

[11] Note also that when $\beta = 1$, which is equivalent to assuming that investors are risk neutral, the equilibrium condition $T(t) = 0$ using eqn. (6) is identical to the equilibrium condition in JK.

implies $\mu = 10$. Here, the threshold level of disclosure computes to $t = 8.0724$. Alternatively, when investors are risk neutral then eqn. (6) reduces to

$$T(t) = p(\mu - t) - (1-p)\int_L^t F(V)\,dV, \tag{7}$$

and t rises to 8.2843. This implies that risk aversion on the part of investors leads to a firm establishing a lower threshold level of disclosure, and thus a higher likelihood of disclosure, because risk aversion exacerbates adverse selection. We explore this, and other comparative static results, in the next section.

3 Comparative statics

Our next goal is to understand the relation between the exogenous features of the economy we describe and the two measures of increased disclosure, the threshold level of disclosure, t, and the probability of disclosure, Π. Our economy has four categories of exogenous features: 1) investors' risk aversion as manifest in the switching point in investors' utility function for wealth, α, and the slope in investors' utility function for wealth, β; 2) the probability that the firm has no knowledge of the firm's realized cash flow in period 1, p; 3) the number of investors in the economy, N; and 4) the distribution of the firm's cash flow, $F(V)$. Broadly stated, the threshold level of disclosure typically rises (falls) and the probability of disclosure typically falls (rises) in response to a change in an exogenous feature of the economy that ameliorates (exacerbates) adverse selection between the firm and investors.

For example, a reduction in the likelihood that the firm is informed in period 1 (i.e., an increase in p) ameliorates adverse selection and thus the firm raises the threshold level of disclosure in response (i.e., t rises). Similarly, a decrease in investors' risk aversion (i.e., an increase in α and/or β) or an increase in the number of investors who share the risk of holding shares in the firm in period 1 (i.e., an increase in N) ameliorates adverse selection, and here as well the firm raises the threshold level of disclosure in response. Finally, an increase in "more favorable" cash flow outcomes in the distribution of $F(V)$ in the sense of 1^{st}-order stochastic dominance (FOSD) or 2^{nd}-order stochastic dominance (SOSD) ameliorates adverse selection and also results in the firm raising the threshold level of disclosure.[12] We codify these observations in the next result.

Proposition 2. *The threshold level of disclosure, t, increases as either: 1) the switching point in investors' utility function for wealth, α, increases; 2) the slope in investors' utility function for wealth, β, increases; 3) the probability that the firm has no knowledge of the firm's realized cash flow, p, increases; 4) the number of investors who compete for firm shares, N, increases; or 5) the distribution of the firm's cash flow, $F(V)$, improves in the sense of FOSD or SOSD.*

We achieve similar results for the probability that the firm discloses, Π. Indeed, the only difference between Proposition 2 and our next result is that there is no monotonic relation between the probability of disclosure and a change in the distribution of the firm's cash flow in the sense of FOSD and SOSD.

Proposition 3. *The probability of disclosure, Π, decreases as either: 1) the switching point in investors' utility function for wealth, α, increases; 2) the slope in investors' utility function for wealth, β, increases; 3) the probability that the firm has no knowledge of the firm's realized cash flow, p, increases; or 4) the number of investors who compete for firm shares, N, increases.*

For example, consider the illustration that followed Proposition 1. In that illustration we assumed $N = 10$, $\alpha = 0.1$, $\beta = 0.9$, and $p = 0.5$, and also assumed that $\tilde{V}$ has a uniform distribution between 0 and 20 (which implies $\mu = 10$). There the threshold level of disclosure computed to $t = 8.0724$. In addition, one can show that the probability of disclosure, Π, computes to 0.2982. When the probability that the firm is uninformed in period 1 increases

[12] We refer the reader to Hanoch and Levy (1969) for a discussion of FOSD and SOSD.

from $p = 0.5$ to $p = 0.6$, then consistent with Proposition 2 t rises to 8.5164, and consistent with Proposition 3 the probability of disclosure falls to 0.2297.

To digress briefly, the reason why there is no monotonic relation between the probability of disclosure and a change in the distribution of the firm's cash flow in the sense of FOSD and SOSD is that $\Pi = (1-p)(1-F(t))$. Here, consider an improvement in the distribution of the firm's cash flow in the sense of FOSD: let the cumulative probability distribution $G(V)$ represent this improvement. FOSD implies that $1-G(V) \geq 1-F(V)$ for all V, and this would seem to suggest that the probability of disclosure increases when the distribution of cash flow improves to $G(V)$. The problem here, however, is that from Proposition 2 the threshold level of disclosure when cash flow has a distribution $G(V)$ is higher than the threshold level of disclosure when cash flow has a distribution $F(V)$ because an improvement in cash flow in the sense of FOSD ameliorates adverse selection. For example, if we represent the former threshold level by t_G and the latter by t_F, then from Proposition 2 $t_G \geq t_F$. This being the case, it is no longer clear that $1-G(t_G)$ is greater than $1-F(t_F)$, because $1-G(V)$ and $1-F(V)$ are decreasing in V and $t_G \geq t_F$.

3.1 The discount in price

Our next task is to compute P and determine the discount investors apply to the firm's expected cash flow in period 1. Recall that P represents the firm's expected price or value in period 1, where we compute P as

$$P = E\left[P\left(\tilde{V}\right) | \text{Disclosure of } \tilde{V} = V \geq t\right] \times \Pi + P(ND) \times (1-\Pi).$$

Proposition 4. *The expected value or price of the firm in period 1 is*

$$P = \mu - p(\mu - t) + (1-p)\int_L^t (t-V)\,dF(V). \tag{8}$$

As an aside, note that the three parameters that measure the risk-bearing capacity of the market (i.e., α, β, and N), do not appear explicitly in the expression for P. These parameters are implicit, however, in the calculation of t, and thus will manifest in our analysis below of the discount.

The chief implication of Proposition 4 is that the discount that investors apply to the firm relative to the firm's expected cash flow, μ, is

$$\Delta = p(\mu - t) + (1-p)\int_L^t (V - t)\, dF(V). \tag{9}$$

Alternatively, using the equilibrium condition as expressed in eqn. (5), Δ can also be expressed as

$$\Delta = p(1-\beta)\left(\int_{N\alpha+t}^H (V - N\alpha - t)\, dF(V)\right). \tag{10}$$

These expressions can be employed to establish two facts about Δ. First, it is clear from eqn. (10) that $\Delta \geq 0$ for all t. Second, although it might not seem immediately obvious from eqn. (9), $\Delta = 0$ if either $\beta = 1$ or $\alpha \to \infty$. This follows from the fact that when $\beta = 1$ or $\alpha \to \infty$, eqn. (5) reduces to

$$T(t) = p(\mu - t) + (1-p)\int_L^t (V - t)\, dF(V),$$

and thus a t such that $T(t) = 0$ in eqn. (5) implies that $\Delta = 0$ in eqn. (9). This proves our earlier claim that investors apply no discount to the firm's expected cash flow in JK, which is based on investors being risk neutral.

Next we study the behavior of the discount. The following proposition indicates that changes in the three features of the economy relating to the ability of the market to absorb risk - the two risk aversion parameters α and β, and the number of investors, N - have an unambiguous association with the discount applied to the firm's cash flow. Changes in these parameters ameliorate the severity of the adverse selection problem between the firm and investors, which results in a lower discount.

Proposition 5. *The discount investors apply to the firm's cash flow, Δ, decreases as either: 1) the switching point in investors' utility function for wealth, α, increases; 2) the slope in investors' utility function for wealth, β, increases; or 3) the number of investors who compete for firm shares, N, increases.*

Unlike Proposition 2, Proposition 5 makes no reference to the effect on Δ of a change in the distribution of the firm's cash flow, or the probability that the firm is uninformed, p. We consider the first of these issues next while we discuss the second issue in the following subsection.

With regard to the effect of a change in the distribution of the firm's cash flow, $F(V)$, in the sense of FOSD or SOSD, such a change potentially affects both the firm's mean cash flow, μ, and the discount. So as to keep the focus of the analysis on the discount, we consider instead the effect of an improvement in the distribution of the firm's cash flow in the sense of SOSD on the discount, but in a circumstance where the mean cash flow stays fixed (i.e., μ stays fixed). An improvement of this nature is referred to as a mean-preserving contraction (MPC).[13] The following proposition establishes that a mean-preserving contraction in the distribution, $F(V)$, results in an unambiguous decrease in the discount applied by investors.

Proposition 6. *The discount investors apply to the firm's cash flow, Δ, (weakly) decreases as the distribution in the firm's cash flow improves in the sense of a mean-preserving contraction (MPC): that is, a circumstance where the distribution of the firm's cash flow improves in the sense of SOSD but the mean cash flow stays fixed.*

To illustrate Proposition 6, consider again the illustration that followed Proposition 1. In that illustration we assumed $N = 10$, $\alpha = 0.1$, $\beta = 0.9$, and $p = 0.5$, and also assumed that $\tilde{V}$ has a uniform distribution between 0 and 20 (which implies $\mu = 10$). There the threshold level of disclosure computed to $t = 8.0724$. Using this value for the threshold, the

[13] Note that this approach is not feasible for FOSD because a necessary condition for FOSD is that the mean cash flow increases (i.e., μ increases). See, for example, the discussion on p. 338 of Hanoch and Levy (1969).

discount computes to $\Delta = 0.1493$. Now suppose that in this illustration everything else is held constant (*ceteris paribus*) except for the distribution of the firm's cash flow, which is now uniform over the interval $[1, 19]$. This represents a mean-preserving contraction of the distribution. In this case the equilibrium threshold increases to $t = 8.2692$, consistent with Proposition 2, while the discount decreases to $\Delta = 0.1315$, consistent with Proposition 6.[14]

Proposition 6 has additional importance because its implications are consistent with prior research that shows that improving the commitment to disclose will lead to a lower cost of capital (e.g., Lambert, et al., 2007). That is, a MPC in $F(V)$ is tantamount to a decrease in *ex ante* uncertainty about the firm's cash flow. One potential source of decreased ex ante uncertainty is an increase, or improvement in, disclosures by the firm *prior to* the firm's voluntary disclosure decision. In our model, Proposition 6 indicates that such improvements will result in an unambiguous decrease in the discount in price. Thus there is nothing in our model that is incompatible with prior research that establishes a negative association between disclosure commitments and the cost of capital.

3.2 Additional analysis and discussion

Propositions 5 and 6 relate to changes in exogenous factors concerning investors' appetite for risk (Proposition 5) or the riskiness of the firm's cash flow (Proposition 6). In both cases, changes that improve the adverse selection environment facing investors, and thus result in decreased disclosure (Propositions 2 and 3), also result in a lower discount, Δ. To gain further insight into these results, recall that Δ is the difference between the firm's expected cash flow, μ, and the expected price of its shares in period 1, P. One can express Δ equivalently as the weighted average of the discount in price if the firm discloses its information, Δ_D, and the discount in price if the firm withholds its information, Δ_{ND}: $\Delta = \Pi\Delta_D + (1 - \Pi)\Delta_{ND}$. Because by construction in our model $\Delta_D = 0$, this reduces to $\Delta = (1-\Pi)\Delta_{ND}$. That is, the

[14] Although in general a MPC does not imply an unambiguous change in the probability of disclosure, in this illustration the MPC reduces the probability of disclosure from 0.2982 to 0.2981. In other words, consistent with a higher threshold level of disclosure, the likelihood of disclosure is lower.

discount in price is the product of the probability that the firm withholds information and the (conditional) discount manifest in price in this event. This indicates that the reduction in Δ that occurs in Propositions 5 and 6 arises from a reduction in the conditional discount in price in the event of non-disclosure. That is, improvements in the risk appetite of investors and/or a decline in the riskiness of the firm's cash flow cause the probability of non-disclosure to increase, but at the same time cause the non-disclosure discount in price to decrease. In Propositions 5 and 6 the second effect dominates.

The impact on the non-disclosure discount in price itself reflects two effects. First, an improvement in investors' appetite for risk and/or a decline in the riskiness of the firm's cash flow will reduce the discount in price absent any disclosure effects. Second, the rise in the disclosure threshold will reinforce this effect indirectly. Specifically, when the disclosure threshold rises the conditional distribution of payoffs perceived by investors is less skewed to the right; the conditional distribution places less weight on values at the lower end of the distribution.[15] And because investors are risk averse, the decline in skewness is equivalent to a decline in the perceived riskiness of the firm. This second, indirect, effect also causes the discount in price to be lower. Propositions 5 and 6 indicate that the combination of these effects on the discount in the event of non-disclosure outweighs the effect on the likelihood of disclosure.

In contrast, the effect of a change in p has an ambiguous impact on Δ. The reasons why p is different are two fold. First, an increase in p increases the probability of non-disclosure both directly and indirectly through the fact that the disclosure threshold rises. Recall that the probability of non-disclosure is $1 - \Pi = p + (1 - p)F(t)$. Thus an increase in p directly increases $1 - \Pi$, as well as indirectly increasing $1 - \Pi$ through $F(t)$. In contrast, improvements in investors' appetite for risk and a decline in the riskiness of the firm's cash

[15] To see this, note that the conditional distribution of the firm's payoff given non-disclosure is a weighted mix of the unconditional distribution, $F(\cdot)$, and the truncated lower range of $F(\cdot)$ (below the disclosure threshold). Thus given non-disclosure investors perceive the distribution of the firm's payoff as 'overweighting' low values relative to the unconditional distribution. With a higher disclosure threshold, the extent of overweighting of low values is reduced.

flow, as in Propositions 5 and 6, only increase $1 - \Pi$ indirectly through $F(t)$. As a result, p has a greater impact on the probability of non-disclosure. At the same time, changes in p do not affect the underlying riskiness of the firm's cash flow (as reflected in $F(\cdot)$). This reduces the impact of p on the discount in price if the firm withholds its information, Δ_{ND}. Compared with Propositions 5 and 6, both of these effects reduce the dominance of the non-disclosure discount over the probability of non-disclosure in Δ, and imply that there is no monotonic association between p and Δ in general.

To illustrate this, consider again the illustration that followed Proposition 1 where we assumed $N = 10$, $\alpha = 0.1$, $\beta = 0.9$, and $p = 0.5$, and also assumed that $\tilde{V}$ has a uniform distribution between 0 and 20 (which implies $\mu = 10$). Figure 1 plots the discount, Δ, against p for two cases: $N = 10$ and $N = 90$. **[Insert Figure 1 here.]** Consistent with Proposition 5, the discount when $N = 10$ is greater than the discount when $N = 90$, for all values of p. However, when $N = 10$ the discount monotonically increases with p whereas when $N = 90$ the discount first increases and then decreases with p. In the former case, when the risk bearing capacity of the economy is low, the effect of the increase in p and the threshold on the likelihood of non-disclosure is sufficient to outweigh the impact on the discount in the non-disclosure price for all values of p. However when $N = 90$ and the market's risk-bearing capacity is greater, when p is sufficiently high the latter effect becomes more important; this results in a non-monotonic association between disclosure and the average discount.

Finally, we note that Figure 1 suggests a potentially interesting implication for empirical studies of voluntary disclosure and cost of capital. Figure 1 evidences a negative association between disclosure and the discount in price when p is low. Because a low p corresponds to a low $1 - \Pi$ (probability of non-disclosure), this suggests that in a voluntary disclosure setting one is more likely to observe the generally expected negative association between disclosure and cost of capital when disclosure is high, rather than when disclosure is low.

4 Summary

In this section we summarize our analysis of the association between voluntary disclosure and the discount in price. To provide the reader with a quick reference to our summary, see Table 2. [**Insert Table 2 here.**]

In our analysis we considered seven exogenous features of an economy: the switching point in investors' utility for wealth, α; the slope in investors' utility for wealth, β; the probability that the firm has no knowledge of the cash flow realization in period 2, p; the number of investors in the economy who compete for the firm's shares, N; and a change in the distribution of the firm's cash flow in the sense of FOSD, SOSD, and a MPC (mean-preserving contraction). We interpret an increase or improvement in each of these seven features as an amelioration in the adverse selection environment between the firm and investors. For example, an increase in the switching point and/or slope in investors' utility for wealth implies that investors are less risk averse, which serves to ameliorate adverse selection. Similarly, an improvement in the distribution of the firm's cash flow in the sense of a MPC implies that risk-averse investors perceive that the firm's cash flow is less uncertain, which also serves to ameliorate the adverse selection environment.

Propositions 2, 3, 5, and 6 have the following implications about the association between measures of increased disclosure, as manifest in the threshold above which the firm discloses (t) and the likelihood that the firm discloses (Π), and the extent to which investors discount the firm's expected cash flow, as measured by Δ. Proposition 2 implies that as each of these seven exogenous features of the economy increase or improve, the threshold level of disclosure increases.[16] Similarly, Proposition 3 implies that as four of these exogenous features increase or improve, the probability of disclosure declines: only for changes in the distribution of the firm's cash flow is there no monotonic relation. Finally, Propositions 5 and 6 imply that as four of these exogenous features increase or improve, the discount that investors apply to

[16] Note that while Proposition 2 does not specifically refer to a MPC, a MPC is a special case of SOSD among distributions with equal means.

the firm's expected cash flow falls. Taken altogether, these results imply that an increase or improvement in four exogenous features (α, β, N, and MPC) results in less disclosure, as measured by either an increase in the threshold level of disclosure (t) and/or a decrease in of the probability of disclosure (Π), and a lower discount. In other words, increased disclosure and the discount are associated positively through α, β, N, and MPC.

The economic intuition that explains why increased voluntary disclosure and the discount are associated positively in these circumstances is straightforward. A change in an exogenous feature of the economy that results in investors applying a higher discount to the firm's expected cash flow will also motivate the firm to increase its disclosure to counteract the higher discount. For example, investors who manifest greater risk aversion exacerbate the adverse selection environment; this results, as a first-order effect, in an increase in the discount. But as a consequence of heightened adverse selection, the firm, as a second-order effect, will increase disclosure. The problem is that the second-order effect of more disclosure will never dominate or reverse the first-order effect of a higher discount.

That said, we discuss a circumstance where the association between voluntary disclosure and discount can be negative. Specifically, we provide an example that shows that when the number of investors who compete for a firm's shares is low the discount investors apply to price increases monotonically through p, whereas when the number is high the discount eventually decreases with high p (see Figure 1). Because an increase in p increases the threshold beyond which the firm discloses voluntarily and thus results in less disclosure, this example might provide the motivation for an empirical experiment that conditions over the number of investors and attempts to associate more voluntary disclosure with a lower discount when the number is low, and more disclosure with a higher discount when the number is high.

The fact that in a majority of circumstances measures of increased disclosure and the discount are associated positively may have implications for an empirical research design that investigates the contemporaneous association between voluntary disclosure and cost of capital. According to our results, such a design would encounter at best equivocal results, and at worst a positive association due to variation across the sample in the exogenous features we study that drive both disclosure choices and the discount investors apply to firms' expected cash flows.[17] While empiricists are no doubt aware of this problem and likely attempt to control for it in their studies, nonetheless it is a point worth emphasizing.

[17] Empirical research that employs constructed disclosure indices (e.g., Botosan, 1997, amongst others) embodies this approach because a disclosure index can be thought of as an aggregation across multiple voluntary disclosure decisions made by the same firm.

Table 1

$\tilde{V}$	the firm's (uncertain) cash flow
p	probability the firm is uninformed about its cash flow in period 1
$P(V)$	price of the firm conditional on disclosing V
$P(ND)$	price of the firm conditional on no disclosure
P	expected price of the firm in period 1
$L,\ H$	lowest and highest values of $\tilde{V}$, respectively
μ	the firm's expected cash flow
Δ	the discount in price relative to μ
α	switching point in investors' utility for wealth
β	slope in investors' utility for wealth when wealth exceeds α
N	number of investors
$q\,(V)$	an investors' quote for holding the firm's shares conditional on disclosing V
$q\,(ND)$	an investors' quote for holding the firm's shares conditional on no disclosure
t	threshold level of disclosure
Π	probability of disclosure

Table of notation.

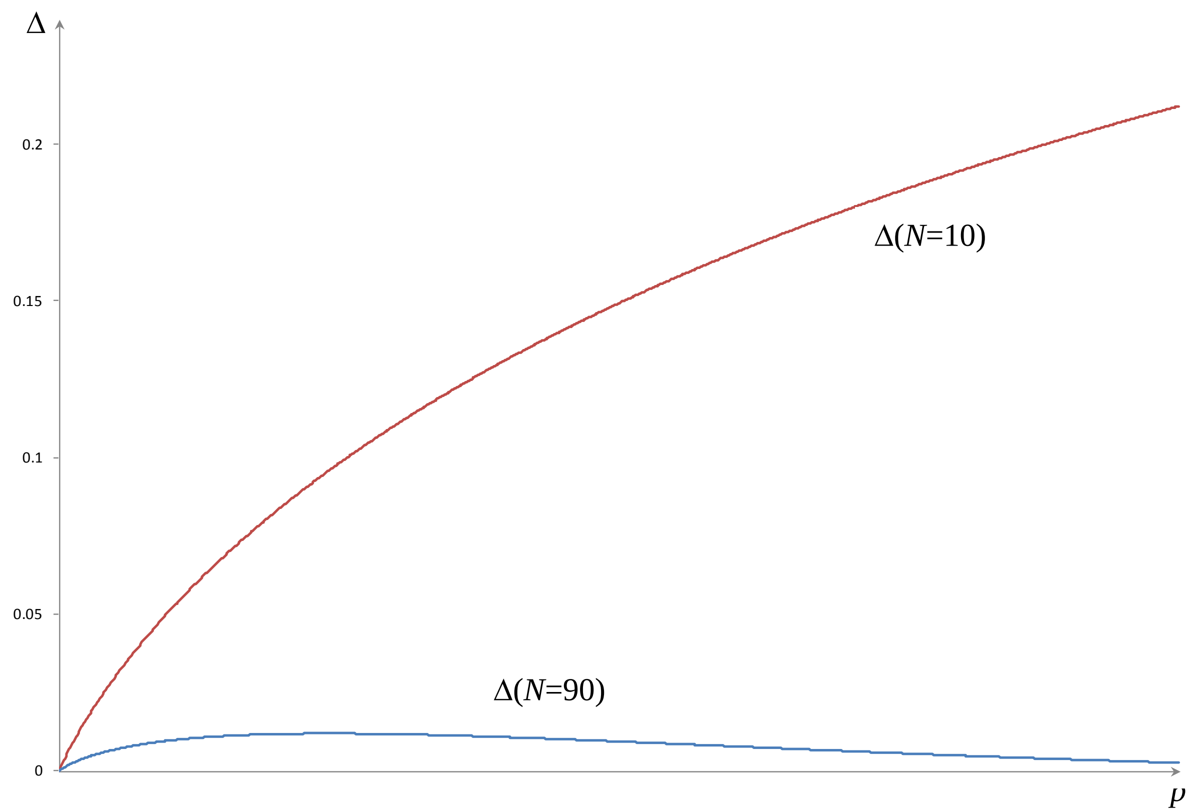
Table 2

Exogenous increase (improvement) in:	**Effect on the threshold level of disclosure, *t***	**Effect on the probability of disclosure, Π**	**Effect on the discount in price, Δ**
Switching point in utility, α	+	-	-
Slope in utility, β	+	-	-
Number of investors, *N*	+	-	-
Probability firm is uninformed, *p*	+	-	NMR
MPC (mean preserving contraction)	+	NMR	-
FOSD (1^{st}-order stochastic dominance)	+	NMR	NA
SOSD (2^{nd}-order stochastic dominance)	+	NMR	NA

NMR – Non-Monotonic Relation
NA – Not Applicable

The effect of an exogenous increase or improvement in various features of the economy on the threshold level of disclosure, *t*, the probability of disclosure, Π, and the discount in price, Δ, where "+" represents an increase, "-" represents a decrease, and NMR and NA are abbreviations for "Non-Monotonic Relation (in general)" and "Not Applicable," respectively.

Figure 1



The effect of an increase in the probability that the firm is uninformed about its cash flow in period 1, p, on the discount in price, Δ, based on the assumptions that $\alpha = 0.1$, $\beta = 0.9$, and $\tilde{V}$ is distributed uniformly between 0 and 20.

Appendix - Proofs

Proof of Proposition 1. When $T(t)$ is expressed as in eqn. (6), then $\frac{\partial}{\partial\alpha}T(t) = p(1-\beta)N(1-F(N\alpha+t)) > 0$. This implies that for all $0 \le \alpha$,

$$T(t) > T(t)|_{\alpha=0} = p\left(\beta(\mu-t) - (1-\beta)\int_L^t F(V)\,dV\right) - (1-p)\int_L^t F(V)\,dV,$$

and thus at $t = L$

$$T(t=L) > T(t=L)|_{\alpha=0} = p\beta(\mu - L) > 0.$$

Thus, $T(t=L) > 0$. When $T(t)$ is expressed as in eqn. (5), $T(t=H)$ reduces to

$$T(t=H) = \mu - H < 0.$$

Thus, $T(t=H) < 0$. Finally, note that when $T(t)$ is expressed as in eqn. (6),

$$\frac{\partial}{\partial t}T(t) = -\left(p\beta + p(1-\beta)F(N\alpha+t) + (1-p)F(t)\right) < 0,$$

which establishes that $T(t)$ is monotonically decreasing in t. Q.E.D.

Proof of Proposition 2. When $T(t)$ is expressed as in eqn. (6), consider first $\frac{\partial}{\partial\alpha}t$:

$$\frac{\partial}{\partial\alpha}t = -\frac{\frac{\partial}{\partial\alpha}T(t)}{\frac{\partial}{\partial t}T(t)} = \frac{p(1-\beta)N(1-F(N\alpha+t))}{p\beta + p(1-\beta)F(N\alpha+t) + (1-p)F(t)} > 0.$$

When $T(t)$ is expressed as in eqn. (5), consider $\frac{\partial}{\partial\beta}t$:

$$\frac{\partial}{\partial\beta}t = -\frac{\frac{\partial}{\partial\beta}T(t)}{\frac{\partial}{\partial t}T(t)} = \frac{p\int_{N\alpha+t}^{H}(V - N\alpha - t)dF(V)}{p\beta + p(1-\beta)F(N\alpha+t) + (1-p)F(t)} > 0.$$

When $T(t)$ is expressed as in eqn. (6), consider $\frac{\partial}{\partial p}T(t)$:

$$\frac{\partial}{\partial p}T(t) = \beta(\mu-t) + (1-\beta)N\alpha - (1-\beta)\int_L^{N\alpha+t} F(V)\,dV + \int_L^t F(V)\,dV.$$

But the equilibrium condition as expressed in eqn. (6) implies

$$\beta(\mu - t) + (1-\beta) N\alpha - (1-\beta)\int_L^{N\alpha+t} F(V)\,dV + \int_L^t F(V)\,dV = \frac{1}{p}\int_L^t F(V)\,dV,$$

and thus

$$\frac{\partial}{\partial p}t = -\frac{\frac{\partial}{\partial p}T(t)}{\frac{\partial}{\partial t}T(t)} = \frac{\frac{1}{p}\int_L^t F(V)\,dV}{p\beta + p(1-\beta)F(N\alpha+t) + (1-p)F(t)} \geq 0.$$

When $T(t)$ is expressed as in eqn. (6), consider $\frac{\partial}{\partial N}t$:

$$\frac{\partial}{\partial N}t = -\frac{\frac{\partial}{\partial N}T(t)}{\frac{\partial}{\partial t}T(t)} = \frac{p(1-\beta)N(1-F(N\alpha+t))}{p\beta + p(1-\beta)F(N\alpha+t) + (1-p)F(t)} > 0.$$

Finally, with regard to FOSD and SOSD, when the equilibrium condition for the existence of a threshold is expressed as in eqn. (6),

$$T(t) = p\left(\beta(\mu - t) + (1-\beta)N\alpha - (1-\beta)\int_L^{N\alpha+t} F(V)\,dV\right) - (1-p)\int_L^t F(V)\,dV,$$

the proof of this result is sufficiently similar to the proof of Proposition 3 in JK such that we refer the reader to that result. Q.E.D.

Proof of Proposition 3. Recall that Π is defined as $\Pi = (1-p)(1-F(t))$. The derivative of Π with respect to α, β, and N, has the opposite sign of the derivative of the threshold, t, with respect to α, β, and N. For example, consider $\frac{\partial}{\partial\alpha}\Pi$:

$$\frac{\partial}{\partial\alpha}\Pi = -(1-p)f(t)\frac{\partial}{\partial\alpha}t < 0.$$

A similar relation is true for β and N. In addition, we know from Proposition 2 that the derivative of the threshold with respect to α, β, and N is always positive: that is, $\frac{\partial}{\partial\alpha}t$, $\frac{\partial}{\partial\beta}t$,

$\frac{\partial}{\partial N}t$ are all positive. This proves (1), (2), and (4). In addition, consider $\frac{\partial}{\partial p}\Pi$:

$$\frac{\partial}{\partial p}\Pi = -(1-F(t)) - (1-p) f(t) \frac{\partial}{\partial p}t < 0$$

because $\frac{\partial}{\partial p}t > 0$. This proves (3). Q.E.D.

Proof of Proposition 4. To sketch the derivation of eqn. (8), note that

$$\begin{aligned} P &= E\left[P\left(\tilde{V}\right)|\text{Disclosure of } \tilde{V} = V \geq t\right] \times \Pi + P(ND) \times (1-\Pi) \\ &= \left(\frac{1}{1-F(t)}\int_t^H VdF(V)\right) \times (1-p)(1-F(t)) + t \times (p + (1-p)F(t)) \\ &= (1-p)\int_t^H VdF(V) + t(p+(1-p)F(t)) \\ &= \mu - p(\mu - t) + (1-p)\int_L^t (t-V)\,dF(V), \end{aligned}$$

where the last equality follows from the fact that $\int_L^H VdF(V) = \mu$. Q.E.D.

Proof of Proposition 5. When Δ is expressed as in eqn. (9), the derivative of the discount with respect to α, β, and N, has the opposite sign of the derivative of the threshold, t, with respect to α, β, and N. For example, consider $\frac{\partial}{\partial \alpha}\Delta$:

$$\frac{\partial}{\partial \alpha}\Delta = -p\frac{\partial}{\partial \alpha}t - (1-p)F(t)\frac{\partial}{\partial \alpha}t = -(1-\Pi)\frac{\partial}{\partial \alpha}t.$$

In addition, we know from Proposition 2 that the derivative of the threshold with respect to α, β, and N is always positive: that is, $\frac{\partial}{\partial \alpha}t$, $\frac{\partial}{\partial \beta}t$, $\frac{\partial}{\partial N}t$ are all positive. This proves our claim. Q.E.D.

Proof of Proposition 6. Let Δ_F and Δ_G represent the discounts investors apply to the firm's expected cash flow when cash flow has a distribution $F(V)$ and $G(V)$, respectively, where $G(V)$ represents a MPC of $F(V)$. Re-express (10) as

$$\Delta = p(1-\beta)\left(\mu - N\alpha - t - \int_L^{N\alpha+t} F(V)\,dF(V)\right),$$

and then express Δ_F as

$$\Delta_F = p(1-\beta)\left(\mu - N\alpha - t_F + \int_L^{N\alpha+t_F} F(V)\,dV\right), \tag{11}$$

and Δ_G

$$\Delta_G = p(1-\beta)\left(\mu - N\alpha - t_G + \int_L^{N\alpha+t_G} G(V)\,dV\right), \tag{12}$$

where t_F and t_G represent the threshold levels of disclosure when cash flow has a distribution $F(V)$ and $G(V)$, respectively. The difference between Δ_F and Δ_G is proportional (by a factor of $p(1-\beta)$) to

$$\Delta_F - \Delta_G \propto t_G - t_F + \int_L^{N\alpha+t_F} F(V)\,dV - \int_L^{N\alpha+t_G} G(V)\,dV. \tag{13}$$

From Proposition 2 we know that $t_G \geq t_F$ because a MPC implies that $G(V)$ dominates $F(V)$ in the sense of SOSD. Thus, define δ such that $\delta = t_G - t_F \geq 0$. This allows us to rewrite eqn. (13) as

$$\Delta_F - \Delta_G \propto \delta + \int_L^{N\alpha+t_F} (F(V) - G(V))\,dV - \int_{N\alpha+t_F}^{N\alpha+t_F+\delta} G(V)\,dV. \tag{14}$$

Finally, note that if $G(V)$ is a MPC of $F(V)$, then $G(V)$ dominates $F(V)$ in the sense of SOSD and thus $\int_{-\infty}^{x} (F(V) - G(V))\,dV \geq 0$ for all x. This implies that the right-hand-side of eqn. (14) is greater than or equal to

$$\delta - \int_{N\alpha+t_F}^{N\alpha+t_F+\delta} G(V)\,dV.$$

But if we define $K(\delta)$ such that $K(\delta) = \delta - \int_{N\alpha+t_F}^{N\alpha+t_F+\delta} G(V)\,dV$, then note that $K(0) = 0$ and $\frac{\partial}{\partial\delta}K(\delta) = 1 - G(N\alpha + t_F + \delta) \geq 0$; thus, $K(\delta) \geq 0$ for all $\delta \geq 0$. Consequently, the right-hand-side of eqn. (14) is greater than or equal to 0, and hence $\Delta_F \geq \Delta_G$. Q.E.D.

References

Akins, B., J. Ng, and R. Verdi, 2011, Investor competition over information and the pricing of information asymmetry, *The Accounting Review* (forthcoming).

Amihud, Y., and H. Mendelson, 1986, Asset pricing and the bid-ask spread, *Journal of Financial Economics*, 17, 223-249.

Armstrong, C., D, Taylor, J. Core, R. and Verrecchia, 2011, When does information asymmetry affect the cost of capital?, *Journal of Accounting Research* 49, 1-40.

Baiman, S., and R. Verrecchia, 1996, The Relation among capital markets, financial disclosure, production efficiency, and insider trading, *Journal of Accounting Research* 34, 1-22.

Balakrishnan, K., M. Billings, B. Kelly, and A. Ljungqvist, 2011, Shaping liquidity: On the causal effects of voluntary disclosure, unpublished manuscript, available at SSRN: http://ssrn.com/abstract

Berger, P., 2011, Challenges and opportunities in disclosure research - A discussion of 'the financial reporting environment: Review of the recent literature', Journal of Accounting and Economics, 51, 204-218.

Beyer, A., D. Cohen, T. Lys, and B. Walther, 2010, The financial reporting environment: Review of the recent literature, *Journal of Accounting and Economics*, 50, 296-343.

Bloomfield, R., and P. Fischer, 2011, Disagreement and cost of capital, *Journal of Accounting Research* 49, 41-68.

Botosan, C., 1997, Disclosure level and the cost of equity capital, The Accounting Review, 72, 323-349.

Botosan, C., and M. Plumlee, 2002, A re-examination of disclosure level and the expected cost of equity capital, Journal of Accounting Research, 40, 21-40.

Christensen, P., L. de la Rosa, and G. Feltham, 2010, Information and the cost of capital: An ex ante perspective, *The Accounting Review*, 85, 817-848.

Cohen, D., 2008, Does information risk really matter? An analysis of the determinants and economic consequences of financial reporting quality, Asia Pacific Journal of Accounting and Economics, 15, 69-90.

Diamond, D., and R. Verrecchia, 1991, Disclosure, liquidity, and the cost of capital, *Journal of Finance*, 46, 1325-1359.

Dye, R., and S. Sridhar, 1995, Industry-wide disclosure dynamics, *Journal of Accounting Research*, 33, 157-174.

Easley, D., S. Hvidkjaer, and M. O'Hara, 2002, Is information risk a determinant of asset returns?, *Journal of Finance*, 57, 2185-2221.

Easley, D., and M. O'Hara, 2004, Information and the cost of capital, *Journal of Finance*, 59, 1553-1583.

Field, L., M. Lowry, and S. Shu, 2005, "Does disclosure deter or trigger litigation?," *Journal of Accounting and Economics*, 39, 487-507.

Gao, P., 2010, Disclosure quality, cost of capital, and investors' welfare, *The Accounting Review*, 85, 1-29.

Hail, L., 2011, Discussion of Consequences and institutional determinants of unregulated corporate financial statements: Evidence from embedded value reporting, *Journal of Accounting Research*, 49, 573-594.

Hanoch, G., and H. Levy, The efficiency analysis of choices involving risk, *Review of Economic Studies*, 36, 335-346.

Jung, W., and Y. Kwon, 1988, Disclosure when the market is unsure of information endowment of managers, *Journal of Accounting Research*, 26, 146-153.

Lambert, R., C. Leuz, and R. Verrecchia, 2007, Accounting information, disclosure, and the cost of capital, *Journal of Accounting Research*, 45, 385-420.

Lambert, R., C. Leuz, and R. Verrecchia, 2012, Information precision, information asymmetry, and the cost of capital, *Review of Finance*, 16, 1-29.

Larcker, D., and T. Rusticus, 2010, On the use of instrumental variables in accounting research, *Journal of Accounting and Economics*, 49, 186-205.

Leuz, C., and C. Schrand, 2009, Disclosure and the cost of capital: Evidence from firms' responses to the Enron shock, NBER working paper No. 14897.

Leuz, C., and R. Verrecchia, 2000, The economic consequences of increased disclosure, Journal of Accounting Research, 38, 91-124.

Leuz, C., and P. Wysocki, 2008, Economic consequences of financial reporting and disclosure regulation: a review and suggestions for future research, Universities of Chicago and Miami working paper.

Pae, S., 2002, Discretionary disclosure, efficiency, and signal informativeness, *Journal of Accounting and Economics*, 33, 279-311.

Serafeim, G., 2011, Consequences and institutional determinants of unregulated corporate financial statements: Evidence from embedded value reporting, *Journal of Accounting Research*, 49, 529-571.

Shin, S-S., 2003, Disclosures and asset returns, *Econometrica*, 71, 105-133.

Shin, S-S., 2006, Disclosure risk and price drift, *Journal of Accounting Research*, 44, 351-379.

Skinner, D., 1997, Earnings disclosures and stockholder lawsuits, *Journal of Accounting and Economics*, 23, 249-282.

Verrecchia, R., 1983, Discretionary disclosure, *Journal of Accounting and Economics*, 5, 179-194.