

Widths and spectroscopic factors in ^{21}O

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A recent $^{20}\text{O}(d,p)^{21}\text{O}$ experiment, in reverse kinematics, discovered two new states in ^{21}O at 4.77(10) and 6.17(11) MeV, with J^π assignments of $3/2^+$ and of $3/2^+$ or $7/2^-$, respectively. Both widths and spectroscopic factors were reported, along with the branching ratio for the upper state to decay to the 2^+ state of ^{20}O . We have computed single-particle widths for all the relevant decays and have used them to extract additional information for these two states, including the spectroscopic factors for 2^+ decay of the upper state with the two possible J^π values. Our analysis prefers $7/2^-$ for J^π .

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Neutron-rich nuclei have turned out to be a good laboratory for finding new magic numbers and eliminating others, and for fine tuning the shell-model interaction to be used in calculations of nuclei far from stability. Excitation energies, J^π values, and spectroscopic factors are important for structure models as they evolve to account for properties of such nuclei. One recent example is an investigation [1] of the $^{20}\text{O}(d,p)^{21}\text{O}$ reaction, in reverse kinematics. They measured angular distributions for four states, at excitation energies of 0, 1.213(7), 4.77(10), and 6.17(11) MeV. For the ground state (g.s.), their results were consistent with an earlier assignment [2] of $J^\pi = 5/2^+$. They assigned $J^\pi = 1/2^+$ for the state at 1.21 MeV, based on its $\ell = 0$ angular distribution. They identify this state as the one previously reported at 1.33(9) [3] and 1.218(4) [4] MeV. They did not observe a probable $3/2^+$ state reported [3,4] at 2.13 MeV.

The unbound state at 4.77 MeV has a clear $\ell = 2$ angular distribution. The authors assign $J^\pi = 3/2^+$ to this state, on the grounds that its spectroscopic factor of 0.58(14) is too large for a $5/2^+$ excited state. The angular distribution for the 6.17-MeV state is consistent with either $\ell = 2$ or 3, leading to a suggestion of $J^\pi = (3/2^+ \text{ or } 7/2^-)$. From coincident gamma rays connecting the 2^+ and g.s. of ^{20}O , they report a branching ratio (BR) for this state of $\Gamma_{2^+} / \Gamma_{\text{tot}} = 0.71(22)$. Quantities measured in Ref. [1] for these two unbound states are listed in Table I.

In the present report, for the two unbound states, we exploit the well-known relationship between width and spectroscopic factor ($\Gamma = S\Gamma_{\text{sp}}$, where Γ_{sp} is the single-particle (sp) width computed in a potential model) to determine whether additional information can be extracted for these states. Comparison of the spectroscopic factor computed from the width to the one obtained from a transfer reaction has long been [5] a powerful tool for determining ℓ values. Our sp widths are also listed in Table I. They were calculated in a Woods-Saxon well, with $r_0 = 1.25$ fm and $a = 0.65$ fm. We look first at the 6.17-MeV state, which has $S = 0.30(7)$ or $0.20(5)$ for $\ell = 2$ or 3 respectively, and a BR to the 2^+ state of $\Gamma_{2^+} / \Gamma_{\text{tot}} = 0.71(22)$. As there are only two open channels,

the rest of the decay width is for the g.s. These spectroscopic factors, together with sp widths from Table I, give g.s. widths of 330(80) or 19(5) keV for $\ell = 2$ or 3 respectively—to be compared with the experimental width of 320(260) keV. As this width is only 1.2σ from zero, it might be thought that it contains very little information. However, let us see what transpires.

Because the $\ell = 3$ sp width for decay to the 2^+ state is only 1.6 keV, if the state is $7/2^-$ the decay to 2^+ is almost certainly not $\ell = 3$, but $\ell = 1$. The $\ell = 1$ sp width is 860 keV. We then ask, what value of $\ell = 1$ spectroscopic factor S_1 for $7/2^- \rightarrow 2^+$ is needed to fit the observed BR? And then, what does this S_1 imply about the total width? From Table I, the g.s. width, if $7/2^-$, is 19(5) keV. The 2^+ width is $860S_1$ keV. We plot the BR computed from these widths as a function of assumed S_1 in Fig. 1. In our figures, we make use of the total width, the BR, and the spectroscopic factors from the (d,p) reaction, through the constraint $\Gamma_{\text{g.s.}} = S\Gamma_{\text{sp}}$, at the $\pm 1\sigma$ level. We note agreement with the experimental BR for $0.02 < S_1 < 0.29$. We then plot the total width versus assumed S_1 in Fig. 2. (Note the semilogarithmic scale.) Again, we have agreement, this time for $0.05 < S_1 < 0.65$. Thus, if the 6.17-MeV state is $7/2^-$, then an $\ell = 1$ spectroscopic factor of $0.05 < S_1 < 0.29$ for p -wave decay to the 2^+ state fits both the BR and the total width. Reference [1] also mentioned that decay of a $7/2^-$ state to the 2^+ could be by $p3/2$.

For the $3/2^+$ possibility, decay to the 2^+ state could be via $\ell = 0$ or 2. Unbound states with $\ell = 0$ are odd entities, and an $\ell = 0$ partial decay branch might be expected to distort the peak shape. As no such distortion is apparent, we will first assume the decay to 2^+ is via $\ell = 2$. But, this assumption remains to be tested. We return to this point below. For $3/2^+$, from the reported spectroscopic factor, the g.s. width is 330(80) keV. The 2^+ width is $62S_2$ keV. Again, we plot the computed BR versus assumed S_2 in Fig. 3. First, we briefly discuss the allowed range of S_2 . In the simplest shell-model calculation, for any state in ^{21}O the sum of S 's to all positive-parity states of ^{20}O is 5. This includes $\ell = 0$ and 2. For any 0^+ state in ^{20}O , S must be less than 1, but that limit does not hold if $J(^{20}\text{O}) \neq 0$.

TABLE I. Quantitative information for two unbound levels in ^{21}O . (Energies and widths in MeV.)

E_x	E_n	Ref. [1]				Present	
		Γ	ℓ	J^π	S	Γ_{sp}	$\Gamma_{\text{g.s.}} = S\Gamma_{\text{sp}}^{\text{b}}$
4.77(10)	0.96(10)	0.46(20)	2	$3/2^+$	0.58(14)	0.145	0.084(20)
6.17(11)	2.36(11)	0.32(26)	2	$3/2^+$	0.30(7)	1.10	0.33(8)
			3	$7/2^-$	0.20(5)	0.096	0.019(5)
	0.68 ^a		1			0.86 ^a	
			2			0.062 ^a	
			3			$1.6 \times 10^{-3\text{a}}$	

^aFor decay to the 2^+ state.

^bUsing S from column 6 and our sp widths.

Even if the $3/2^+$ state has a large amount of $d3/2$ sp strength, the value of S to the 2^+ state will also contain $d5/2$ strength. It is unlikely, however, that S to the 2^+ state is as large as, say, 3. From Fig. 3, we note that, in order to fit the BR at the 1σ level, we must have $S_2 > 3.4$, above the limit of reasonableness. The curve labeled f is for $\ell = 3$ decay to the 2^+ state. It is clear, as alluded to above, that no amount of $\ell = 3$ will fit the BR. The total width (Fig. 4, again semilogarithmic) provides little additional information because any value of S_2 produces agreement (but see Fig. 5). Again, there is no agreement for $\ell = 3$.

For the 6.17-MeV state, we conclude that the data of Ref. [1] are internally consistent for a J^π value of $7/2^-$ if the decay to the 2^+ state is p wave with S_1 in the range 0.05–0.29. For $J^\pi = 3/2^+$, and d -wave decay to the 2^+ state, the data would require $S_2 > 3.4$ to be consistent at the 1σ level. This may indicate that most of the 2^+ decay is s wave. Independently of the $\ell = 0,2$ admixture for decay to 2^+ , if the state is $3/2^+$,

in order to have agreement with the BR at the 1σ level, the width to the 2^+ state (see Fig. 5) must be >240 keV, while for agreement with the measured total width at the 1σ level this width must be <330 keV. So, for the $3/2^+$ possibility we have (at the 1σ level) $240 \text{ keV} < \Gamma_{2^+} < 330 \text{ keV}$. If this is all (or mostly) s -wave decay, we leave for others the task of converting this width into an $\ell = 0$ spectroscopic factor. In all of the above, we have used the expression $\Gamma = S\Gamma_{\text{sp}}$ to convert measured S 's into g.s. widths.

A value of 285(45) keV for decay to the 2^+ state would leave 35(265) keV for the g.s. width, to be compared with the g.s. width of 330(80) keV computed from the reported spectroscopic factor. Alternatively, the sum of $\Gamma_{\text{g.s.}}$ (from S) and Γ_{2^+} (from BR) is 615(92) keV, compared to the reported total width of 320(260) keV. Inspection of the spectrum would appear to rule out a possible total width of 615(92) keV. Consideration of all this information (including the absence of any obvious peak distortion) leads us to conclude that $7/2^-$ is more likely than $3/2^+$ for the upper state.

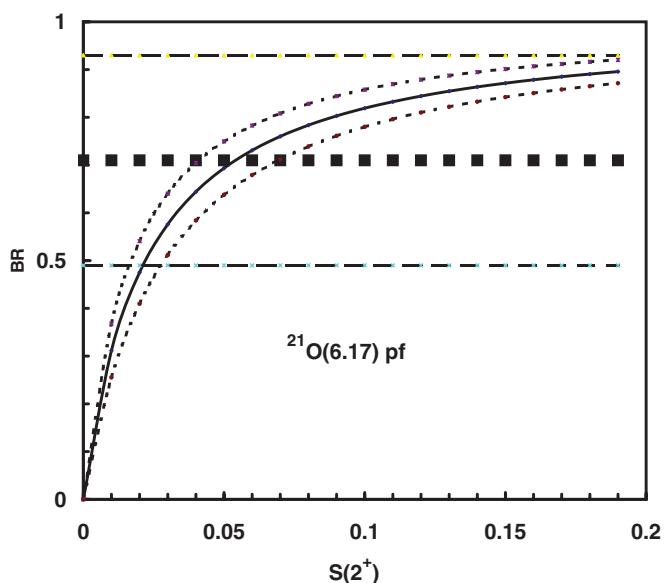


FIG. 1. (Color online) Branching ratio $\Gamma_{2^+}/\Gamma_{\text{tot}}$ for $^{21}\text{O}(6.17)$. Squares represent the experimental value, with dashed horizontal lines at the $\pm 1\sigma$ limits. Plotted vs the assumed S_1 , the solid curve is the BR for $J^\pi = 7/2^-$, assuming $\ell = 1$ for decay to 2^+ . The short-dashed curves are at $\pm 1\sigma$.

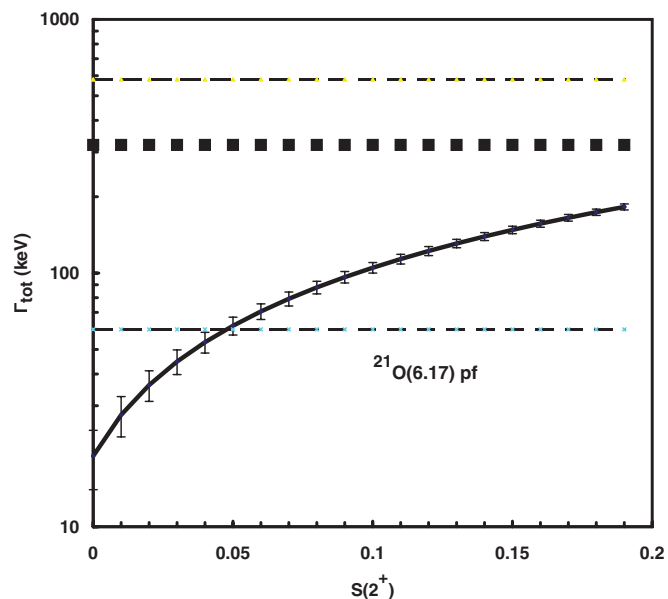


FIG. 2. (Color online) Total width of the 6.17-MeV state of ^{21}O . Squares with horizontal dashed lines are the experimental result. The solid curve is for $7/2^-$, plotted vs the assumed value of S_1 .

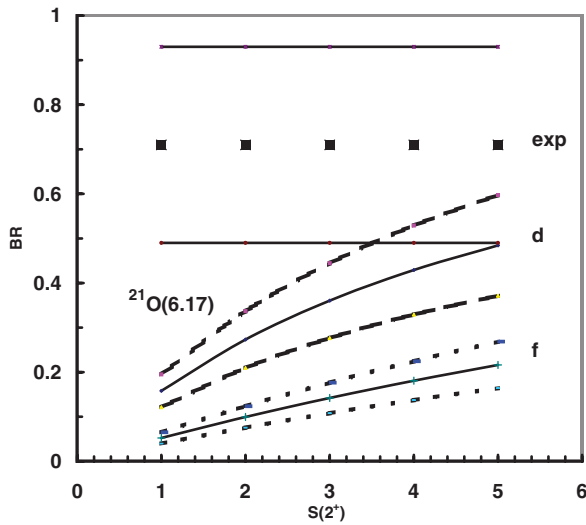


FIG. 3. (Color online) Experimental BR as in Fig. 1. Solid curves, with dashed curves at $\pm 1\sigma$, are computed BRs, assuming both branches have the same ℓ value (d or f).

We turn now to the 4.77-MeV state, whose width is 0.46(20) MeV. With a spectroscopic factor of 0.58(14), the width should have been 84(20) keV—quite a large difference from the stated width. In their data the total width of the 4.77-MeV peak appears to be about 1.44 MeV. The authors do not state the resolution width at this energy, but they do give the full width at half maximum (FWHM) at the 1.21-MeV state as 0.77(17) MeV. They state that they extracted the natural width by convoluting Gaussian and Breit-Wigner shapes. With their quoted width of 0.46(20) MeV and our estimate of a total width of 1.44 MeV in their spectrum, we deduce a FWHM of about 0.85 MeV for the resolution at 4.77 MeV. This is reasonably close to the value of 0.77(17) they quote for 1.21 MeV. The width measured directly by Ref. [1] from their spectrum differs from the width calculated from the spectroscopic factor by an

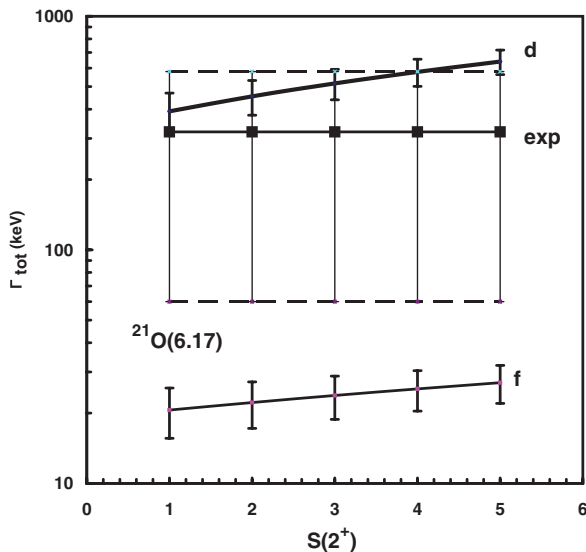


FIG. 4. (Color online) As in Fig. 3, but for total width. (Note semilogarithmic scale.)

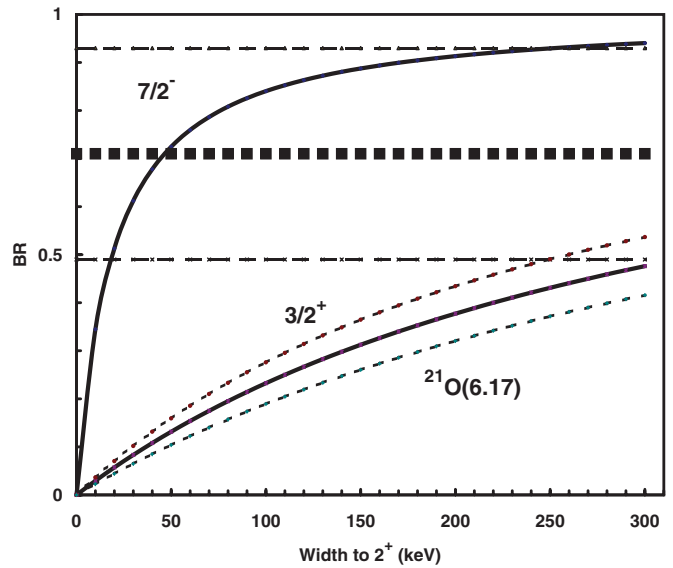


FIG. 5. (Color online) As in Fig. 1, but for both $7/2^-$ and $3/2^+$, plotted vs assumed width for decay to 2^+ .

amount 0.38(19) MeV—about a 2σ difference. Another way to state this discrepancy is to compute S from the measured width and compare it to the S extracted from comparison of measured cross section to a distorted-wave calculation. Using $S = \Gamma_{\text{exp}}/\Gamma_{\text{sp}}$ and our Γ_{sp} of 145 keV, we get $S_{\Gamma} = 3.2(1.4)$. Of course, this S must rigorously be less than unity. The value of S from the cross section is $S_{\sigma} = 0.58(14)$. The ratio of the two (which should be 1.0) is 5.5(20). As this is not a 3σ effect, it is not clear how seriously to take this discrepancy. Perhaps another look at this reaction might be worthwhile.

To summarize: If the 6.17-MeV state is $7/2^-$, the 2^+ decay is almost certainly via $\ell = 1$, with S_1 in the range 0.05–0.29. If it is $3/2^+$ and the 2^+ decay is d wave, the BR requires $S_2 > 3.4$ —which is very unlikely. These results are summarized in Table II. If the 6.17-MeV state is $3/2^+$, decaying to the 2^+ state by a combination of $\ell = 0 + 2$, the partial 2^+ width is $\Gamma_{\text{exc}} = 285(45)$ keV. The g.s. width is 330(80), obtained from the reported [1] spectroscopic factor and our sp width (Table I). As these are the only open channels, the total width is then 615(92) keV, to be compared with the measured width [1] of 320(260) keV.

TABLE II. Partial and total widths (keV) for the 6.17-MeV state for various J^{π} and ℓ combinations.

J^{π}	ℓ to 2^+	$\Gamma_{\text{exc}}^{\text{a}}$	$\Gamma_{\text{g.s.}}^{\text{b}}$	$\Gamma_{\text{tot}}^{\text{c}}$	$\Gamma_{\text{meas}}^{\text{d}}$
$3/2^+$	2		Not possible		
	$0 + 2$	285(45)	330(80)	615(92)	320(260)
$7/2^-$	3		Not possible		
	1	146(103)	19(5)	165(103)	320(260)

^aWidth for decay to 2^+ obtained from reported BR [1] and present analysis.

^bFrom Table I.

^c $\Gamma_{\text{tot}} = \Gamma_{\text{exc}} + \Gamma_{\text{g.s.}}$

^dTotal width reported in Ref. [1].

If the state is $7/2^-$, the data do not allow $\ell = 3$ for the 2^+ decay, but $\ell = 1$ is acceptable, giving an excited-state width of $\Gamma_{\text{exc}} = 146(103)$ keV. The g.s. width obtained from the reported S is $19(5)$ keV, resulting in a total width of $165(103)$ keV, to be compared with the measured width of $320(260)$ keV. Thus, our results favor $J^\pi = 7/2^-$ for this state, even though the $3/2^+$ possibility is allowed at the 1.1σ level.

For the two J^π possibilities, the total widths that result from our analysis differ by 3.2σ . So, even a modest reduction in the uncertainty of the experimental width should allow a clear choice.

For the 4.77-MeV state, the spectroscopic factor derived from the measured width is $5.5(20)$ times the S derived from the measured (d,p) cross section.

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