PHYSICAL REVIEW C **84**, 024327 (2011)

The puzzle of ^{32}Mg

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(Received 2 April 2011; revised manuscript received 2 August 2011; published 29 August 2011)

An analysis of results of the ^{30}Mg(t,p) ^{32}Mg reaction demonstrates that the ground state is the normal state and the excited 0^+ state is the intruder, contrary to popular belief. Additional experiments are suggested.

DOI: 10.1103/PhysRevC.84.024327 PACS number(s): 21.10.Hw, 27.30.+t, 21.10.Jx, 25.55.Hp

I. INTRODUCTION

In the so-called island of inversion, for very neutron-rich nuclei, the $N = 20$ shell gap narrows appreciably, allowing the lowest $(fp)^2$ configuration to compete with the normal *sd*-shell structure at low excitation. It is thought that this intruder could even dominate the ground state (gs) in some nuclei. Of course, this lowering of the *fp* shell into the low-excitation region is already well known for $N \sim Z$ nuclei, but perhaps not quite to the same extent. For example, in ^{38}Ar, which has $N = 20$, the presence of three low-lying 0^+ states in an excitation region where only one *sd*-shell 0^+ state exists has long been taken as evidence of excitations into the *fp* shell (see Ref. [1] and references therein). An important nucleus in this region is ^{32}Mg, whose gs has been reported to possess several puzzling features. Here, we briefly review the history as it relates to ^{32}Mg.

II. HISTORY CONCERNING ^{32}Mg

Lowering of the 2_1^+ state in ^{32}Mg [2] was taken [3], along with their measurement [3] (the first) of $B(E2)$, as evidence of strong deformation and hence of core excitation. The 2^+ energy remains low for ^{34}Mg [4].

In a neutron-knockout experiment [5] on ^{32}Mg, excited $3/2^-$ and $7/2^-$ states were strongly populated, leading to deduced occupancies of 0.59 and 1.19, respectively, consistent with a dominant $(fp)^2$ structure for ^{32}Mg, with a ratio of $p^2/f^2 = 0.50(18)$. (Throughout, we will frequently use f and p for $1f_{7/2}$ and $2p_{3/2}$, respectively.) These were considerably larger than in ^{30}Mg, as can be seen in Table I.

In two-proton knockout [6] from ^{32}Mg to ^{30}Ne, the neutron overlap was found to be very small, ∼0.4, despite the expectation that ^{30}Ne(gs) was dominated by $(fp)^2$. The authors interpreted this small overlap as requiring about 50% four-particle–four-hole (4p-4h) in ^{30}Ne(gs)—a very unlikely scenario.

Reference [7], with a magnetic-moment measurement, concluded that the $1/2^+$ gs and $(3/2^+)$ first-excited state at 50 keV in ^{31}Mg were primarily of the structure $(fp)^2(sd)^{-3}$ [i.e., an *sd* hole in a predominant $(fp)^2(sd)^{-2}$ gs of ^{32}Mg]. The ^{31}Mg structure would consist of two neutrons in the deformed Nilsson orbital $1/2^-$ [330], none in $3/2^+$ [202], and one in $1/2^+$ [200].

In ^{33}Mg, beta-decay results [8,9] provide support for a gs J^π of $3/2^+$, while a negative magnetic moment [10] favors $3/2^-$. It appears [11] that this issue remains to be settled.

Several workers [3,4,12–15] have investigated the $B(E2$; gs $\rightarrow 2_1^+)$ in ^{32}Mg and neighboring nuclei. In ^{32}Mg, if a correction for feeding from above is applied, results are in general agreement that $B(E2\uparrow)$ is about 328(48) e^{2}fm^4, or slightly larger. This value has been suggested to require dominant $(fp)^2$ structure in ^{32}Mg(gs) and 2_1^+, but it is only slightly larger than in ^{30}Mg and ^{28}Ne and approximately equal within the uncertainties. Several $B(E2)$'s are listed in Table II. Reference [16] has stated "^{30}Mg has a rather small $B(E2)$ for gs $\rightarrow 2_1^+$" and "the large $B(E2$: gs $\rightarrow 2_1^+)$ value for ^{32}Mg has clearly established its strongly deformed ground state." The numbers in Table II for ^{30,32}Mg would not support that view. For ^{34}Mg, which should be $(fp)^2$, the $B(E2)$ value is significantly larger.

The history of the $B(E2)$ measurements in ^{32}Mg is summarized in Table III. With the exception of a large value reported by Chiste *et al.* [15], the measurements agree, but the analysis of the data causes the results to fall into two separate groups—depending on the magnitude of the correction for feeding from above. Takeuchi *et al.* [17] state that their assignment of 4^+ to the state at 2321 keV should lead to a small feeding correction, and they prefer $B(E2) = 454(78)$ e^{2}fm^4. Chiste *et al.* [15] report a large value for ^{32}Mg, but their $B(E2)$ for ^{30}Mg is also very large—larger than the accepted value by more than 2σ.

Now comes a ^{30}Mg(t,p) experiment, in inverse kinematics, at an incident center-of-mass energy of 4.9 MeV. With a beam of ^{30}Mg incident on a target of tritium in titanium, Wimmer *et al.* [16] used the ^{30}Mg(t,p) reaction to populate the gs of ^{32}Mg and an excited 0^+ state at 1.058 MeV. Cross sections were 10.5(7) and 6.5(5) mb, respectively. Despite the analysis in that paper, the results of that experiment appear to be in conflict with the previous picture of ^{32}Mg, as we now discuss.

III. THE ^{30}Mg(t,p) ^{32}Mg REACTION

The authors [16] claim the excited 0^+ state is dominantly of *sd*-shell character. They use a two-state model to interpret their results. They assume the gs is pure $(fp)^2$ and they vary the p^2/f^2 ratio in order to fit the observed absolute cross section. Their ratio of $p^2/f^2 = 1.02$ is quite different from the value of 0.50 in Ref. [4]. To fit the excited 0^+-state cross section they use a mixture of *sd* shell and p^2 with constructive interference between the two components. This choice of relative phase is incorrect on quite fundamental grounds. In fact, their two-state picture cannot be even close to the real situation, as we discuss later.

TABLE I. Results of neutron knockout from ^{30,32}Mg from Ref. [5].

	C^2S			
Nucleus	$\ell = 1$	$\ell = 3$	Sum	Ratio
^{30}Mg	0.19(7)	0.41(10)	0.60(12)	0.46(20)
^{32}Mg	0.59(11)	1.19(36)	1.78(38)	0.50(18)
Ratio	0.32(13)	0.34(13)		

But, first we digress in order to discuss general features of configuration mixing and the (*t*,*p*) reaction near a major shell boundary, with special emphasis on $N = 20$. We also mention results from another such boundary. We are concerned here with two-neutron excitations across a major neutron shell. For the case of ^{32}Mg, which has $N = 20$, the relevant physics is contained in just two basis states, zero-particle–zero-hole (0p-0h) and two-particle–two-hole (2p-2h), where the particles are in the *fp* shell and holes are in the *sd* shell. In the *sd*-shell space of ^{32}Mg, there are no other states that arise from neutron excitations within the *sd* shell. And the 0^+ states from proton excitations will lie much higher in excitation. For the 2p-2h excitation, the 2h are undoubtedly well approximated by the *sd*-shell wave function of the gs of ^{30}Mg. (We leave for later the possibility of excitations out of the ^{30}Mg core.) In a deformed picture of ^{32}Mg (supposedly the source of the gap narrowing), only one $(fp)^2$ 0^+ excitation will lie low. Others will exist at much higher excitation energies. Also for this reason (only one low-lying *fp* deformed orbital), we expect 4p-4h excitations to be minimal. These 0p-0h and 2p-2h basis states are each eigenfunctions of realistic Hamiltonians; they are normalized and orthogonal. The two-body residual interaction can (will) mix them, providing two physical states that are again orthonormal. (We mention briefly that, near $N = Z$, three 0^+ states are needed [1] at low excitation: 0p-0h, 2p-2h, and 4p-4h. But that 4p-4h state is an alpha-particle excitation with two protons and two neutrons in the deformed *fp* orbital. That type of excitation is, of course, absent here because two like nucleons fill a deformed orbital.)

Frequently, many configurations are necessary to account for the nuclear structure, but it is often the case that they can

TABLE II. $B(E2\text{:gs} \to 2_1^+)$ in e^{2}fm^4 for ^{28}Ne and ^{30,32,34}Mg.

Nucleus	$B(E2)$	Ref.
^{28}Ne	269(136)	[14]
^{30}Mg	295(26)	[14]
^{32}Mg	333(70)[a]	[14]
^{32}Mg	>328(48)[a]	[12]
^{34}Mg	<670	[14]
^{34}Mg	631(126)	[13]
^{34}Mg	541(102)[b]	[12]
^{34}Mg	>438(83)[b]	[12]

[a]After correcting for feeding from above (but see Table III).
[b]No evidence of feeding from above but, if present, the lower limit applies.

TABLE III. History of $B(E2)$ measurements in ^{32}Mg.

Year	$B(E2;\text{ gs} \to 2_1^+)$ (e^{2}fm^4)	Ref.
1995	454(78)	[3]
1999	440(55)	[14]
1999	330(70)[a]	
2001	662(90)[b]	[15][b]
2001	449(63)	[13]
2005	447(57)	[12]
2005	>328(48)[a]	

[a]After correcting for feeding from above.
[b]Reference [15] also reports a very large value for ^{30}Mg, larger than the accepted value by more than 2σ.

be "reconfigured" into only one or two simpler structures. For example, as mentioned above, we would expect only one $(fp)^2$ 0^+ configuration to come low in excitation, even though it may be a complicated admixture of $(1f_{7/2})^2$, $(2p_{3/2})^2$, $(1f_{5/2})^2$, and $(2p_{1/2})^2$. It would be unwise to consider four basis states here because clearly one is enough, with four components, where the amplitudes of the four configurations in the Nilsson orbital $1/2^-[330]$ are determined by the value of the deformation. And, although the latter two configurations are certain to be present at some level, their contribution is not important [amplitudes are small, and their contribution to (*t*,*p*) is even smaller] and we consider only the first two, frequently abbreviated *f* and *p*, respectively. The same reasoning applies to the lowest $(sd)^{-2}$ excitation; one basis state involving a linear combination of $(1d_{5/2})^{-2}$, $(2s_{1/2})^{-2}$, and $(1d_{3/2})^{-2}$. It is often true that, at a major shell boundary, only one core-excited component (for each J^π) is needed to explain the "intruder" properties and the mixing between the intruder and the normal state(s) [18,19]. We thus have only two relevant basis states; namely, the closed *sd* shell ^{32}Mg(gs) and a two-neutron excitation into the lowest *fp*-shell Nilsson orbital.

When dealing with 2*n* transfer reactions in the presence of configuration mixing, it is often convenient to use direct-reaction phases, in which positive relative signs in the wave functions correspond to constructive interference in the 2*n* transfer amplitude [20]. Of course, any phase convention is acceptable, as long as it is consistently applied. Here, we use shell-model phases. In the mixing of the two basis states referred to above, with normal shell-model phases, the 0^+ state that is lowest after mixing (the gs) will have a relative negative sign between 2p-2h and 0p-0h components. For the (*t*,*p*) reaction on a 2h target, this state will have constructive interference between the $(fp)^2$ transfer amplitude into the 2p-2h component and the $(sd)^2$ transfer into the 0p-0h component. This result arises because the $(fp)^2$ two-neutron wave function has one more radial node than does the one for $(sd)^2$. Thus, negative phase in the nuclear structure leads to positive phase in the (*t*,*p*) reaction and vice versa. Because the only important basis states at low excitation are these two, then near to the gs there will be another 0^+ physical state having the orthonormal configuration admixture—and hence the opposite (i.e., positive) wave-function phase but with destructive interference in the (*t*,*p*) reaction.

We turn now to another neutron major shell boundary; namely, $N = 8$ Here, ^{12}Be(gs) is dominated by the $(sd)^2$ excitation, accounting for 60% to70% of the wave function [21–23]. This fact was clearly established with the ^{10}Be(t,p) reaction [21], and later confirmed by other work [22,23]. In the (t,p) reaction, the experimental gs cross section is about five times as large as that expected for the pure p-shell state. This is also a general result that transfer into the next major shell will have a larger cross section than transfer into the lower one, for the following two reasons: (1) the configurations that make a good two-nucleon cluster are the ones that lie lowest in the next major shell and (2) the larger number of quanta of excitation in the next-higher shell causes a greater radial extent to the radial wave function. In ^{12}Be, the "other" physical 0^+ state has been identified [24] and is extremely weak in the (t,p) reaction.

Now consider ^{14}C, which also has $N = 8$. Here, the gs is primarily a p-shell structure with only about 12% excitation of two neutrons into the sd shell [18]. And here the excited 0^+ and gs are observed to have about the same cross section in ^{12}C(t,p), [16,25]. Of course, more than two 0^+ states exist in these nuclei but, in both ^{12}Be and ^{14}C a simple two-state model with mixing explains the data. The nonparticipation of the next (third) 0^+ state in ^{14}C can be seen clearly [25] in the fact that it [the second $(sd)^2$ 0^+ state] behaves nearly identically to the second 0^+ state in ^{16}C, which has no p-shell state.

These ^{12}Be and ^{14}C features are quite general. At the boundary of a major shell, if the first-excited 0^+ state has about the same $2n$ transfer cross section as the gs, then the gs is predominantly the normal state, and the excited state is the intruder. Conversely, when the excited 0^+ state is very weak, then the gs is dominantly the intruder.

Now we return to the results of the ^{30}Mg(t,p) reaction [16], in which the cross sections are 10.5(7) for the gs and 6.5(5) for the excited 0^+ state. (For convenience I sometimes use the abbreviation es for the latter.) The authors of Ref. [16] used a two-state model to explain their data, but some of what they did is incorrect. First, they allowed the configuration amplitudes of each state to vary independently to fit their data. For the gs they had a linear combination of f^2 and p^2, with the p^2/f^2 ratio adjusted to fit the cross section. For the excited 0^+ state, they used a combination of $(sd)^2$ and p^2, with the amount of p^2 adjusted to fit the cross section. There are several things wrong with this approach. Their excited 0^+ wave function cannot arise from any Hamiltonian diagonalization; the relative phase they took for it [constructive in (t,p)] is wrong; and their two states could never have emerged from any mixing. And their two states are not even close to being orthogonal. It is impossible for their two states to arise from any realistic model of nuclear structure.

However, it is possible to understand the (t,p) data in a simple, self-consistent, rigorous model, as we now demonstrate. Consider

$$\text{gs} = \alpha\,(fp)^2 - \beta\,(sd\,\text{shell})\,, \quad 0_2^+ = \beta\,(fp)^2 + \alpha\,(sd\,\text{shell})\,,$$

with α, β positive and $\alpha^2 + \beta^2 = 1$. Reference [16] has $\alpha \gg \beta$. Let $R^2 = \sigma(fp)^2/\sigma(sd)^2$, where the σ are calculated cross sections, and $\sigma(sd)^2$ is that for the pure sd-shell gs for $N = 18$ to $N = 20$; $\sigma(fp)^2$ is that for transfer from an empty shell into the lowest $(fp)^2$ 0^+ state. We expect $R^2 = 3$ to 10, depending mostly on the amount of $(p_{3/2})^2$ in the $(fp)^2$ wave function. For $N = 18$ to $N = 20$, the $(sd)^2$ transfer (listed in Ref. [16]) is mostly $(d_{3/2})^2$, which produces the smallest cross section of the three 0^+ $(sd)^2$ amplitudes. The larger $(fp)^2$ cross section will depend sensitively on the amount of $(p_{3/2})^2$ in the $(fp)^2$ wave function. For any specific $(fp)^2$ wave function, the value of R^2 can be computed. If the p^2/f^2 ratio is not accurately known, then the values of R and β can be determined from the sum of the two cross sections and their ratio. From values given in Ref. [16], their $(fp)^2$ wave function would have an R^2 of about 3.2. We can estimate it from the summed cross section: With no core excitation in ^{30}Mg(gs),

$$\sigma(\text{gs}) + \sigma(0_2^+) = \sigma(fp)^2 + \sigma(sd)^2.$$

Reference [16] gives $\sigma(sd)^2 = 3.2$ mb, so that with the measured values of σ, we would have $R^2 = 4.3(3)$—consistent with the expectation above. With no mixing the es/gs ratio would be $1/R^2$, and mixing will only decrease the ratio, because mixing increases the gs cross section and decreases it for the es. Now, let $r^2 = \sigma(0^+{}_2)/\sigma(\text{gs})$, where these σs are the measured ones. Then

$$(-xR + 1)/(R + x) = \pm r, \quad \text{where} \quad x = \beta/\alpha.$$

For $R^2 > 1.6$, we must take the negative value. Then, with $R^2 = 4.3(3)$ and $r^2 = 0.62(6)$, we get $x = 2.06(13)$ [i.e., $\beta^2 = 0.81(2)$, $\alpha^2 = 0.19(2)$]. Of course, the total uncertainties are larger than these because of the simplicity of the model. Nevertheless, this analysis demonstrates that the gs is mostly the sd-shell state and the excited 0^+ state is mostly the $(fp)^2$ state. This conclusion is inconsistent with much of the previous understanding of ^{32}Mg. Even with my gs, an appreciable $(fp)^2$ component in the 2^+ state will easily lead to a $B(E2)$ that is larger than the sd-shell value.

My results are stable with respect to small changes. For example, the admixture within the $(fp)^2$ wave function may not be exactly identical in the gs and excited state, and likewise for the sd-shell component. But those differences should produce only small changes in the $2n$ transfer amplitudes. Furthermore, both Ref. [16] and I assume the ^{30}Mg(gs) is pure sd shell. Allowing core excitation in ^{30}Mg(gs) does not qualitatively change the arguments but gives slightly different numerical results. For example, a 30% admixture (almost certainly an overestimate) of $(fp)^2$ $(sd)^{-4}$ into ^{30}Mg(gs) complicates the arithmetic somewhat, but changes β^2 only from 0.81 to 0.74 and R^2 from 4.3 to 3.5.

It is true that a neutron-knockout experiment on ^{32}Mg [5] found substantial population of $3/2^-$ and $7/2^-$ states in ^{31}Mg, leading to the suggestion of a dominant $(fp)^2$ configuration for ^{32}Mg. If the gs of ^{32}Mg is predominantly of $(fp)^2$ character, I see only two possibilities:

(i) The absolute cross-section scale of Ref. [16] is too large by about a factor of two. Then the data can be fitted with R^2 in the range 1.0 to 1.6, with very little mixing. However, such a small value of R^2 implies a very small amount of $(p_{3/2})^2$ in the $(fp)^2$ wave function—in apparent conflict with the results of Ref. [5] and in conflict with the idea of large deformation. Or,

(ii) The new state is in another nucleus—probably the gs of that nucleus, if even-even. (If odd-odd or odd-*A*, it could be a low-lying state.)

If the absolute cross-section scale is correct and both 0^+ states are in ^{32}Mg, I see only one other option:

(iii) Is it possible that, in the knockout experiments, the ^{32}Mg nuclei were primarily in the excited 0^+ state, rather than in the gs? By comparison with ^{30}Mg [26], the lifetime of the excited 0^+ state could easily be in the range of 50 to100 ns. This explanation could also account for the very small overlap (~0.4) encountered in the two-proton-removal experiment [6]. It is a simple matter to compute the overlap between my excited 0^+-state wave function and the published ^{30}Ne(gs) [6]. The result is 0.46, very close to the published value of ~0.4.

IV. SUMMARY

Several experiments have suggested that the gs of ^{32}Mg has a neutron configuration that is predominantly $(fp)^2(sd)^{-2}$, even though much of the data [e.g., the $B(E2)$] do not seem to require that conclusion. A straightforward analysis of the ^{30}Mg(t,p) reaction results leading to the gs and the excited 0^+ state demonstrates that the gs is predominantly 0p-0h and the excited state is 2p-2h. If nothing is wrong with the (t,p) experiment, the only reasonable resolution that I see to this dilemma is that the ^{32}Mg nuclei in the knockout experiments were primarily in the excited 0^+ state rather than in the gs. It would be interesting to measure the lifetime of the excited 0^+ state, which is estimated here to be in the range of 50 to 100 ns. Finally, if the neutrons in ^{30}Ne(gs) are predominantly $(fp)^2$ $(sd)^{-2}$, then I expect the excited 0^+ state to be very weak in the reaction ^{28}Ne(t,p)^{30}Ne.

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