

Mass of $^{18}\text{Mg}(\text{g.s.})$

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We use a potential model, together with spectroscopic factors from a combination of weak coupling and a shell-model calculation, to compute the mass of the ground state of ^{18}Mg , considered as a mirror of ^{18}C . The result is $E_{2p} = 3.87(10)$ MeV.

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I. INTRODUCTION

Energies of ground states (g.s.'s) of proton-rich nuclei are notoriously difficult to predict. In nuclei around $A = 16$, a large complicating factor is the competition between $2s_{1/2}$ and $1d_{5/2}$ occupancy, coupled with the fact that the two behave very differently in mirror nuclei. Compared to $1d_{5/2}$, a $2s_{1/2}$ state is much lower in excitation in the proton-excess member of a mirror pair. This effect is well understood, and is sometimes called the Thomas-Ehrman effect. Getting an accurate prediction for the energy of a proton-rich nucleus in this mass region depends sensitively on an accurate treatment of this $s_{1/2}$ - $d_{5/2}$ competition.

After several years in which the mass-excess prediction for ^{19}Mg covered a wide range [1,2], we presented results of a calculation [3] that gave $E_{2p} = 0.87(7)$ MeV. A later experiment [4] found 0.75(5) MeV, just at the limit of the combined uncertainties. Another proton-rich nucleus for which predictions have been made is ^{17}Na [5,6], but no experimental results have yet appeared. Here we report our expectation for ^{18}Mg .

Nothing is known experimentally about the properties of ^{18}Mg , but two calculations of its energy have appeared. Ebata *et al.* [7] used time-dependent Hartree-Fock-Bogoliubov with the Skyrme functional SkM* [8] to compute the energy of $^{18}\text{Mg}(\text{g.s.})$. They concluded that the last proton was bound by 200 keV. Patra *et al.* [9] used relativistic mean field theory to calculate the g.s. properties of many nuclei from Ne to Ca. For ^{18}Mg , their total binding energy was 93.286 MeV, implying $E_{2p} = 4.03$ MeV for $^{18}\text{Mg} \rightarrow ^{16}\text{Ne} + 2p$. However, they missed ^{18}Ne by 2.8 MeV and ^{20}Mg by 1.5 MeV. Those may be an indication of the uncertainties in their predictions.

II. METHOD

Our approach relies on the completeness of a one-nucleon expansion in basic quantum mechanics. For any state in nucleus A , its wave function can be written as a sum of terms, each of which is the product of an expansion coefficient, the wave function of a state in nucleus $A - 1$, and a nucleon wave function of relative motion: The sum is over all terms consistent with isospin, parity, and angular-momentum coupling. A shell-model calculation provides numerical values for the expansion

coefficients, which are in one-to-one correspondence with the spectroscopic factors. This procedure is exact, but it is convenient only when the total number of important core states is small. The present calculation uses spectroscopic factors for $^{20}\text{O} \rightarrow ^{19}\text{O}$ from a full $(sd)^4$ shell model calculation, with the universal sd-shell (USD) interaction [10].

In our procedure, we assume that the expansion coefficients are equal for a given state and its mirror, and that the effects of the Coulomb interaction are limited to changes in the radial wave function. We have frequently used this technique to compute mass excesses of proton-rich nuclei and/or energies of excited states in those nuclei. Our procedure employs a Woods-Saxon nuclear potential for the calculation of a single-particle wave function. Geometric parameters are $r_0 = 1.26$, $a = 0.60$, and $r_{0C} = 1.40$ fm. For each term in the one-nucleon expansion of the total core + neutron wave function mentioned above, we vary the potential well depth to fit the known energy of the state. We then use this potential plus the Coulomb potential of a uniformly charged sphere to calculate the energy of the core + proton sp state. After this has been done for all the components being included, we weight these energies with the relevant spectroscopic factors to obtain our predicted energy for the mirror state. In the present case, the first step involves $^{17}\text{C} + n$, and the second has $^{17}\text{Na} + p$. As for ^{19}Mg , the needed core-state energies in ^{17}Na are not known, but theoretical estimates exist.

In more normal cases, in which the nuclear structure and excitation energies of the core states are reliably known, our method has been shown to produce results that agreed with experimental values to within about 30 to 40 keV. For the case of ^{19}Mg , where we had to compute the energies of the ^{18}Na core states, we had assigned an uncertainty of ± 70 keV to our calculations. Here the uncertainty could be even larger, perhaps as large as 100 keV.

III. RESULTS

In a recent calculation [11] of the ^{20}O - ^{20}Mg energy difference, we found that a severe truncation of the number of core states worked extremely well. For the g.s., the use of only three states in $^{19}\text{O}/^{19}\text{Na}$ gave very nearly identical results as using all the core states in a full $(sd)^4$ shell-model

TABLE I. Core states and results for ^{18}Mg (energies in MeV).

J^π	$E_x(^{17}\text{C})^a$	$E_p(^{17}\text{Na})^b$	S^c	$E_p(^{18}\text{Mg})^d$	E_{2p}^e
$3/2^+$	0	3.39	0.0687	0.125	3.52
$1/2^+$	0.21	3.05	0.3217	-0.079	2.97
$5/2^+$	0.32	3.72	3.4014	0.235	3.96
Total			3.7918		3.87

^aReference [12].^bReference [6].^cReference [11] + weak coupling.^dPresent.^e $E_{2p} = E_p(^{17}\text{Na}) + E_p(^{18}\text{Mg})$.

calculation. This is perhaps not surprising, because these three states exhaust 95% of the total sum-rule strength.

For ^{18}C - ^{18}Mg we have also truncated the calculations to the same three states. In ^{17}C , these are the three lowest states [12], all closely spaced. The ordering is different from ^{19}O because the $s_{1/2}$ orbital comes lower with decreasing A . But, it is already nearly a pure single-particle state in ^{19}O , so it should have a similar structure in ^{17}C . The $3/2^+$ and $5/2^+$ states are primarily of $(d_{5/2})^3$ structure, but the $3/2^+$ does have a small $d_{3/2}$ spectroscopic factor.

With each of these three ^{17}C states as cores, we varied the well depth of our potential well to reproduce the neutron binding energy of $^{18}\text{C}(\text{g.s.})$. We then used this same potential, plus the Coulomb potential of a uniformly charged sphere, with $r_0 = 1.40$ fm, to calculate the proton energy of $^{18}\text{Mg}(\text{g.s.})$ as $^{17}\text{Na} + p$. For the energies of the core states in ^{17}Na , we used the shell-model results of Ref. [6]. They differ considerably from results of the microscopic cluster calculations of Ref. [5]. We then weighted these single-particle separation energies with their respective spectroscopic factors to get a final prediction for the energy of $^{18}\text{Mg}(\text{g.s.})$. The result is $E_{2p} = 3.87(10)\text{MeV}$ (Table I). This is only slightly larger than the single-proton unbound energy of ^{17}Na , but it *is* larger. The primary decay of $^{18}\text{Mg}(\text{g.s.})$ is thus expected to be single-proton decay to one or more of the three low-lying states of ^{17}Na . Decay to the $1/2^+$ state has a much smaller spectroscopic factor than decay to the $5/2^+$, but the $1/2^+$ state wins on energy and penetrability grounds. So, it will probably dominate the decay of $^{18}\text{Mg}(\text{g.s.})$. Our result is depicted in Fig. 1.

Because all the states of ^{17}Na are unbound, ^{18}Mg will be a $2p$ emitter, dominated by sequential decay through one or

	3.87
3.72	$5/2^+$
3.39	$3/2^+$
3.05	$1/2^+$

^{16}Ne ^{17}Na ^{18}Mg

FIG. 1. Previous results for ^{17}Na from Ref. [6] and present result for ^{18}Mg . Energies are given relative to $^{16}\text{Ne}(\text{g.s.})$.

more of the first three resonances of ^{17}Na . The width of ^{18}Mg will depend sensitively on the energies and widths of these ^{17}Na resonances—none of which are known. Estimation of the expected ^{18}Mg width must await information for ^{17}Na . We can achieve a rough estimate by ignoring the (unknown) widths of the ^{17}Na states. If the ^{18}Mg and ^{17}Na energies of Fig. 1 are even remotely correct, the dominant ^{18}Mg decay will be to the $1/2^+$ state of ^{17}Na . For a proton decay energy of 0.82 MeV, the single-particle decay width is about 27 keV. If we ignore the natural width of the $1/2^+$ state, the ^{18}Mg width is just the spectroscopic factor (0.32) times the single-particle width, leading to an expected width of about 9 keV.

IV. SUMMARY

We have computed the ^{18}C - ^{18}Mg energy difference using shell-model (plus weak coupling) spectroscopic factors, and the three lowest core states of $^{17}\text{C}/^{17}\text{Na}$. For ^{17}Na only calculated energies exist. For this reason, we assign an uncertainty of 100 keV to our final ^{18}Mg energy: $E_{2p} = 3.87(10)\text{MeV}$. If predictions for ^{17}Na should change, or if any of the levels are observed experimentally, our prediction for ^{18}Mg could change. The most robust result of the present work is an energy of about 150 keV for ^{18}Mg decay to $^{17}\text{Na}(5/2^+)$.

Despite its very large negative Q value, a possible reaction to populate ^{18}Mg is $^{12}\text{C}(^{24}\text{Mg}, ^{18}\text{C})^{18}\text{Mg}$. Because of the Barshay-Temmer theorem [13], the angular distribution of each ejectile should be symmetric about 90° in the center-of-mass system, so the ^{18}C could be detected at forward angles without a loss of count rate. We eagerly await an experimental search for the g.s. of ^{18}Mg .

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