

Order Number 9015068

Imperfectly competitive financial markets

Caballe Vilella, Jorge, Ph.D.

University of Pennsylvania, 1989

U·M·I

300 N. Zeeb Rd.
Ann Arbor, MI 48106

IMPERFECTLY COMPETITIVE FINANCIAL MARKETS

Jorge Caballe Vilella

A Dissertation

in

Economics

Presented to the Faculties of the University of Pennsylvania in
Partial Fulfillment of the Requirements for the Degree of
Doctor of Philosophy.

1989



Supervisor of Dissertation



Graduate Group Chairperson

DEDICATIONS

To my parents

To Clara

ACKNOWLEDGMENTS

I am grateful to all the people who have helped me to make this dissertation.

First, I am grateful to the members of my dissertation committee: Beth Allen (advisor), Richard Kihlstrom, George Mailath and Andrew Postlewaite for their advice and encouragement. I have also benefited from very helpful comments by Patricio Arrau, Utpal Bhattacharya, James Foster, Nick Gonedes, Yukiko Hirao, Matthew Jackson, Murugappa Krishnan (who has also co-authored a part of this dissertation), Pete Kyle, Bill Novshek, Pietro Reichlin, K. Rao Kadiyala, Asani Sarkar, Tim VanZandt, Jean-Luc Vila and participants at workshops at Penn and at IESE, Spain.

Among the people that helped to make life at Penn much easier, I would like to mention Albert Ando, Carmen Ansotegui, Guillermo Calvo and Antonio Villanacci. I am specially grateful to Dave Cass for his unconditional support.

My two years of graduate studies at the Universitat Autònoma of Barcelona were also a very important period in my life that influenced my subsequent education. I thank specially professors Xavier Calsamiglia, Joan Maria Esteban, Andreu Mas-Colell and Joaquim Silvestre because with their academic guidance and encouragement persuaded me to continue my studies in the United States.

Finally I want to thank my wife Clara for, among other

invaluable things, her tireless help in typing the manuscript many times.

Financial support at different stages of my studies provided by Bank of Spain; CIRIT, Generalitat of Catalunya; and School of Arts and Sciences at the University of Pennsylvania is gratefully acknowledged.

ABSTRACT

IMPERFECTLY COMPETITIVE FINANCIAL MARKETS

Jorge Caballe Vilella

Beth Allen

The first part of this dissertation, titled "Strategic Behavior and Asymmetric Information in Financial Markets", studies the effects of changes in the precision of both public and private information in financial markets in which traders are not price-takers but act strategically. Two different mechanisms of price formation are considered. The first one is a mechanism with market orders and competitive market makers. The second one is based on limit orders and market clearing. Under both regimes, the disclosure of more public information increases the expected profits of liquidity traders at the expense of privately informed agents. These results are potentially changed in two cases: when the acquisition of private information is costly and when the disclosure requirements are not uniform across firms and we allow for discretionary liquidity traders. The implications for price volatility, trading volume, incentives to produce private information, efficiency of associations of investors and mechanism design are explored.

In the second part (a joint work with Murugappa Krishnan) titled "Insider Trading and Asset Pricing in an Imperfectly Competitive

Multi-Security Market", we study a multi-security financial market in a correlated environment with asymmetric information and imperfect competition, in which market makers learn about each return from every order flow, even as an informed trader manipulates what they can learn. Our model is a generalization of a single-security model by Kyle. In contrast to a previous analysis by Admati under perfect competition, where the effect of a correlated environment is only to generate various ambiguities, strategic behavior restores various theoretical regularities, and can "neutralize" all of the correlatedness arising from the structure of returns and liquidity noise. Even with imperfect private information, strategic behavior helps generate an equilibrium with simpler structure, which is valuable for applications, especially for justifying traditional event study procedures even when there is private information.

TABLE OF CONTENTS

PART 1: STRATEGIC BEHAVIOR AND ASYMMETRIC INFORMATION IN FINANCIAL	
MARKETS	1
1. Introduction and Related Literature	2
2. The Model with Market Orders and Competitive Market Makers	12
2.A. The Model	12
2.B. Equilibrium	19
3. The Welfare Effects of Public Information	22
3.A. Free Private Information	22
3.B. Costly Information Acquisition	26
4. Price Volatility and Trade Volume Implications	31
5. Strategic Liquidity Traders	35
6. The Effects of Private Information	42
7. Common Private Information	46
8. A Monopolistic Market for Information	50
9. Associations of Investors	57
10. A Rational Expectations Model with Imperfect Competition	60
10.A. The Model	60
10.B. Equilibrium	65
11. Comparative Statics of the Rational Expectations with Imperfect Competition Model	68
12. A Comparison Result	73

13. Conclusion	75
- Appendix	79
- References	87
PART 2: INSIDER TRADING AND ASSET PRICING IN AN IMPERFECTLY COMPETITIVE MULTI-SECURITY MARKET (with Murugappa Krishnan)	
1. Introduction	95
2. Model	99
3. Equilibrium	105
4. General Properties of Equilibrium	113
5. Perfect Private Information	127
6. A Two-Asset Example	129
7. Conclusion	139
- Appendix	141
- References	150

LIST OF TABLES

PART 1

Table 1 — Time Structure of the Market Orders Model	18
---	----

PART 2

Table 1 — Time Structure of Basic Model	105
Table 2 — Limits	132
Table 2 — Limits (continued)	133
Table 3 — Comparative Statics	133
Table 3 — Comparative Statics (continued)	134

LIST OF ILLUSTRATIONS

PART 1

Figure 1 — Production of Private Information	93
Figure 2 — Expected Demand Function	93

PART 2

Figure 1a — $a_{11}(\rho_v; \rho_z)$	152
Figure 1b — $a_{11}(\rho_z; \rho_v)$	152
Figure 2a — $a_{1j}(\rho_v; \rho_z), i \neq j$	152
Figure 2b — $a_{1j}(\rho_z; \rho_v), i \neq j$	152
Figure 3a — $b_{11}(\rho_v; \rho_z)$	153
Figure 3b — $b_{11}(\rho_z; \rho_v)$	153
Figure 4a — $b_{1j}(\rho_v; \rho_z), i \neq j$	153
Figure 4b — $b_{1j}(\rho_z; \rho_v), i \neq j$	153
Figure 5a — $E(\tilde{\pi}(\rho_v; \rho_z))$	154
Figure 5b — $E(\tilde{\pi}(\rho_z; \rho_v))$	154
Figure 6 — $E(\tilde{\pi}(\rho_v) m \text{ securities}), \rho_z = 0$	154

Part 1

STRATEGIC BEHAVIOR AND ASYMMETRIC INFORMATION
IN FINANCIAL MARKETS

1. INTRODUCTION AND RELATED LITERATURE

This paper studies the effects of public disclosure of information by firms about the return of the securities they issue. More generally, I will study the effects of changes in the precision of both public and private information in financial markets. This problem has been addressed in several papers following the path-breaking work by Hirshleifer (1971). Among others, I mention the papers by Allen (1987b), Diamond (1985), Hakansson, Kunkel and Ohlson (1982), Ross (1979), Verrecchia (1982b) and Kyle (1984).

The papers by Allen, Diamond and Verrecchia are the ones that are most closely related to the present one. The difference between their approach and mine is that they assume a perfectly competitive financial market in the tradition of Grossman and Stiglitz (1980) or Diamond and Verrecchia (1981). I will follow instead the paradigm pioneered by Kyle (1984, 1985, 1986), Glosten and Milgrom (1985) and Kihlstrom and Postlewaite (1983). These authors depart from the previous models by assuming that each agent in the market has a nonnegligible effect on prices and each agent takes into account this effect in order to formulate his optimal demand for risky asset. These models of imperfect competition seem appropriate to study the performance of thin markets with few traders who are aware of the fact that they can influence the equilibrium prices, markets with dominant

traders, or markets with price setters (market makers) who make inferences from the observed demand in order to price the traded securities.

The traditional model of rational expectations with fully revealing prices, as stated by Grossman (1976, 1978, 1981a) Grossman and Stiglitz (1976) or Kihlstrom and Mirman (1975), has been subject to several criticisms. The first problem raised by Beja (1977) and Grossman and Stiglitz (1980) refers to the incentives to use private information in such markets. If the equilibrium price becomes a sufficient statistic for all the information in the market, then there are no incentives for the agents to use their private information, especially if this information is costly. Agents can observe the equilibrium price and make all the relevant inferences from this sufficient statistic. But if all the agents disregard their private information, then prices cannot aggregate all the existing information in the economy. This paradox has been solved in two different ways.

First, Grossman (1977) and Grossman and Stiglitz (1980) eliminate the possibility of fully revealing prices by introducing a source of noise that is uncorrelated with the return of the risky asset. In this case, prices depend on the realizations of both private signal and noise. Thus, Grossman and Stiglitz show that there are still incentives to become informed because not all the private information is revealed to the uninformed agents through the price system. There have been several stories that justify the existence of noise in financial markets. We can associate this noise with a random supply of risky asset or with the random demand for asset made by agents that are

liquidity constrained or whose trading is determined exclusively by life-cycle reasons.

Another solution to the Grossman-Stiglitz paradox comes from the work by Milgrom (1981) which specifies the extensive form of a second-price auction for a single object. In this auction, prices are submitted first by the bidders and afterwards the auctioneer selects the second highest price as the one at which the transaction is carried out. The agent who gets the object is the agent who has submitted the highest bid. Given this sequence of moves, bidders are forced to use their (probably costly) private information even if the equilibrium price becomes fully revealing¹.

Another criticism to the models with fully revealing prices comes from the fact that if the number of agents is finite and agents know how prices are formed, then they should act strategically in order to manipulate the equilibrium price.

Because the price taking assumption in markets with few agents leads to the "schizophrenic behavior" problem posed by Hellwig (1980), the (noisy) rational expectations literature in financial markets has focused on large markets with a continuum of agents, each of them with negligible weight with respect to the whole market.

The solution of these problems suggested by the literature on imperfect competition that I will follow uses partially some of the

¹ The equilibrium price in Milgrom (1981) is an order statistic. Usually, order statistics are not sufficient, but Milgrom is able to provide an example of auction whose equilibrium price reveals all the private information.

ideas mentioned above. For instance, several models by Kyle (1984, 1985, 1986) have a source of noise (due to liquidity constrained traders) that prevents prices from being fully revealing so that informed agents still have incentives to participate in the market.

Another feature of Kyle's models is that, as in Milgrom (1981), the price is selected at the end of the auction. However, the strategic variables for the agents are not prices anymore but quantities (Admati and Pfleiderer (1988a) and Kyle (1985)) or demand schedules (Jackson (1988) and Kyle (1986)). Therefore, given this extensive form of the game, agents are forced to use their private information in the first stage of the game.

Finally, the most important departure from the noisy rational expectations literature is that the price taking assumption is relaxed. The number of informed agents is assumed finite and these agents behave strategically because they know the mechanism by which the prices are formed.

As I have mentioned, Allen (1987b), Diamond (1985) and Verrecchia (1982b) have studied the issue of the value of information in financial markets in the context of a noisy rational expectation economy with perfect competition, in which agents are characterized by constant absolute risk aversion utility functions and all random variables are normally distributed. Even though the computations are somewhat involved, these authors are able to give explicit solutions to the equilibrium and to sign the effects of their comparative statics experiments.

The Allen and Diamond models yield different predictions about

the effects of public information on non-liquidity traders' welfare. These results depend on the way in which noise is introduced. An additional significant factor is that the private information is common to all informed agents in Allen's paper whereas it is diverse in Diamond's case.

Verrecchia (1982b) points out the difficulty of answering the question about the value of public or private information. He argues that slightly changing the scenarios and the sequence of events of the models would change the results dramatically. An interesting analysis in Verrecchia's paper refers to the incentives to acquire private information when more public information is available. His results say that the disclosure of public information reduces the amount of information produced privately.

On the other hand, Kyle (1984) studies the effects of disclosing a public signal in an imperfectly competitive financial market. His analysis focuses on a futures market for an agricultural good where the return on the future depends on the positions taken by speculative traders and the random demand for the good made by non-speculative consumers. He closes the model by using a market clearing condition in which the price of the good is determined after the speculative round of trade is concluded and the stochastic demand is revealed. On the contrary, the market I will consider is like the one modeled by Diamond (1985) or Admati and Pfleiderer (1988): a market for an asset with a random return that is independent of the actions taken by the speculative traders during the round of trade. However, Kyle's results about the effects of public information are similar to mine.

Some of the questions addressed in previous papers are analyzed here in the context of imperfectly competitive financial markets. I will focus mainly on a market with a finite number of agents who own diverse pieces of private information. Liquidity constrained agents are the source of noise in this market. The mechanism of price formation is modeled in two different ways. I first consider a model with competitive market makers who select a price equal to the expected return of the risky asset conditional on all information available to them. The strategic variables of informed agents are quantities (market orders) of risky asset that they want to buy or sell. Secondly, I briefly study a model that resembles the traditional noisy rational expectations model with perfect competition in which prices are formed by automatic market clearing after all demand functions (limit orders) are submitted. Most of my results hold irrespective of the mechanism under consideration.

One shortcoming of my approach is that, as in Admati and Pfleiderer (1988a), Easley and O'Hara (1987), Glosten and Milgrom (1985) and Kyle (1985), for tractability I assume that participants in the market are risk neutral. Risk neutrality negates some results in the previous literature. However, this exercise has intrinsic value because provides an explicit equilibrium for a case where perfect competitive equilibrium fails to exist. If the risk neutral agents (indexed by n) are price takers and observe a signal s_n about the return \tilde{v} of an asset, their demand will be $x_n = -\infty$ if $E(\tilde{v}|s_n) < p$, $x_n = \infty$ if $E(\tilde{v}|s_n) > p$ or $x_n = [-\infty, +\infty]$ if $E(\tilde{v}|s_n) = p$. Only in the improbable case in which $s_n = s$ for all n , does equilibrium exist. In this case, $p = E(\tilde{v}|s)$ is the

equilibrium price.

With imperfect competition the "generic" non-existence result is changed because agents take into account their effects on prices. Each agent reduces his intensity of reaction to private information in order to reduce the amount of information revealed to the market maker or to other agents. The revelation of information would push the prices up when informed agents receive good news and eliminate part of the profits that the agents would obtain if prices did not react to individual actions.

Furthermore, the assumption of risk neutrality complements the imperfect competition assumption. In thin markets with few agents, usually the traders are agents who are either (wealthy) insiders or mutual funds with very high risk bearing capacity. A further advantage of the risk neutrality assumption is that permits one to concentrate exclusively on strategic interactions among participants in the financial market.

The several aspects and results of the paper are summarized as follows:

I model public information, as in Diamond (1985) or Kyle (1984), assuming that all agents are able to observe a common signal about the return of the risky asset in addition to their private signals. More precise public information makes informed agents worse off because it dissipates the informational advantage of these agents with respect to the liquidity traders, even if their relative informational position with respect to the market maker and other strategic traders remains unchanged. More public information makes the

market more liquid, i.e., prices becomes less sensitive to order flows. This in turn implies a transfer of expected profits from informed traders to liquidity traders.

My above result is potentially changed in two cases. First, if private information is costly, the number of informed agents decreases when more precise public information is disclosed. This implies that the new equilibrium will involve less competition among insiders. This new equilibrium may generate less surplus for liquidity traders. Secondly, liquidity traders in a multi-securities world are attracted to the most liquid market. This fact alters camouflage opportunities across markets.

I study the effects of more precise information release on the behavior of the price process and the expected volume of transactions done by the market maker. The volatility of prices increases in an economy where public information is more intensively released, because prices can more accurately replicate returns. The expected volume of the market maker's trade decreases as public information becomes more precise in the market orders model.

Private information exhibits decreasing returns in my models. Too much private diverse information may be harmful to informed agents. This result does not hold in the market orders model with common private information across agents. In this case, the informed agents' advantage with respect to the market maker always increases.

Another issue studied here concerns the efficiency of syndicates of investors. I consider two kinds of associations of investors: associations in which its members precommit ex ante to share

their private information and associations that submit collective demands on behalf of its members. The relative efficiency of these associations from the point of view of the informed agents is independent of the precision of public information.

I also study the performance of a very stylized monopolistic market for information. For markets with a single insider who can produce private information, the induced change of incentives to produce private information when further public information is released depends on the average cost of producing such private information.

Finally, I obtain the following comparison result: if private information is very precise compared to the public information, then informed agents are better off when the price is determined by competitive risk neutral market makers than when it is formed by automatic market clearing. The converse is true for liquidity traders. This result is a consequence of the decreasing returns associated with private information and the information sharing involved in the limit orders model.

The paper is organized as follows:

Section 2 presents a model with market orders and market makers. This model has also two kinds of agents: informed agents and liquidity traders. I obtain the Bayesian-Nash equilibrium of the proposed game when there is available a piece of public information modeled like in Diamond (1985).

Section 3 derives some welfare implications of increasing the precision of public information when the private information is free and when it is costly.

Section 4 studies some empirical implications of releasing public information on price volatility and volume of trade done by the market maker.

Section 5 modifies the previous model by introducing discretionary liquidity traders that choose optimally in which market they will submit their demands.

Section 6 analyses the effects of increasing the precision of private information.

Section 7 considers the case of common private information. This case is useful in order to analyze the performance of a monopolistic market for information (Section 8) and the efficiency of associations of investors (Section 9).

Section 10 considers a different model of price formation in the same spirit as in Jackson (1988) and Kyle (1986). The equilibrium price is defined by market clearing where agents' strategies are demand functions (limit orders). The comparative statics experiments of sections 3, 4 and 6 are performed for the new model in Section 11.

Section 12 compares both mechanisms from the point of view of the expected profits of both types of agents.

Section 13 concludes the paper.

2. THE MODEL WITH MARKET ORDERS AND COMPETITIVE MARKET MAKERS.

2.A. The Model.

We are going to study the price formation of a single asset in a market with three kinds of agents: informed traders, noise traders and market makers.

There are T liquidity (or noise) traders, indexed by t , in the market. The noise traders either buy or sell quantities of risky asset motivated by liquidity constraints. These liquidity constraints can be justified by life-cycle reasons and other needs that arise outside the financial market. The important feature of the behavior of these liquidity traders is that they trade for reasons that are not related to the payoff of financial asset. I will assume that the net demand \tilde{z}_t for shares of each liquidity trader t is normally distributed. Without loss of generality, assume that \tilde{z}_t has zero mean and variance equal to σ^2 for all t , and $\text{Cov}(\tilde{z}_m, \tilde{z}_k) = 0$ for $m \neq k$. Therefore, the total net demand $\tilde{z} = \sum_{t=1}^T \tilde{z}_t$ for shares by the liquidity traders, is normally distributed with zero mean and variance σ_z^2 , where $\sigma_z^2 = T\sigma^2 > 0$.

Alternatively, interpret the noise \tilde{z} as a random supply of risky asset (cf. Grossman and Stiglitz (1980) and Diamond and Verrecchia (1981), among others). In this case I do not need to worry about the welfare implications on the liquidity traders of our comparative statics.

Finally, I must say that the amount of noise parameterized by the variance σ_z^2 may be very small, but in any case I need it in order

to avoid having fully revealing prices.

There are N informed traders (indexed by n) in the market who trade on the basis of private information about the future payoff of the risky asset. They are not aware of the exact needs of the liquidity constrained traders. However, they know the parameters of the distribution of the total net demands \tilde{z} for shares by the noise traders.

I consider a single asset whose expected return \tilde{v} is normally distributed with mean \bar{v} and precision (the inverse of the variance) equal to $\tau_v \in (0, \infty)$. Each informed trader receives a piece of private information that takes the form of a signal \tilde{s}_n where $\tilde{s}_n = \tilde{v} + \tilde{\epsilon}_n$. The noise $\tilde{\epsilon}_n$ of the signal is also normally distributed with mean 0 and precision $\tau_\epsilon > 0$ for all n (i.e., \tilde{s}_n is informative about the return \tilde{v}).

I assume that the firm issuing the asset can be forced by law to disclose at no cost reliable information about the expected return of the asset. I disregard the direct costs of producing information and the indirect cost of disclosing reliable information using costly signals as dividends (Bhattacharya (1979), Miller and Rock (1985)) or capital structure (Ross (1977)). This assumption allows one to focus exclusively on the effects of public information on the stock market traders' welfare. Specifically, as in Diamond (1985), I assume that the firm is enforced to release a public signal \tilde{s}_0 that takes the form

$$\tilde{s}_0 = \tilde{v} + \tilde{\epsilon}_0,$$

where $\tilde{\epsilon}_0$ has a normal distribution, with zero mean and precision $\tau_0 > 0$.

We can imagine that the level of precision τ_0 can be enforced by the legislator in the following way. The firm must be audited and

the auditor has to release all the information that he is able to obtain. The precision τ_0 is controlled ex ante by establishing the different activities of the firm that should be audited. By enlarging the set of audited activities, the precision τ_0 is increased. All the participants in the market observe the realization of \tilde{s}_0 before the transactions are conducted.

I assume that $(\tilde{v}, \tilde{z}, \tilde{\epsilon}_1, \dots, \tilde{\epsilon}_N, \tilde{\epsilon}_0)$ are mutually independent random variables.

Denote the optimal demand of the informed trader n as $\tilde{x}_n = x_n(\tilde{s}_n, \tilde{s}_0)$ where $x_n(\dots)$ is a measurable function of \tilde{s}_n and \tilde{s}_0^2 . Note that the demands are not allowed to be contingent on prices; this possibility will be studied in Section 10. Note also that $x_n(\cdot, \cdot)$ denotes the demand strategy, \tilde{x}_n is the quantity demanded as a random variable, and x_n denotes the realization of this random variable.

All informed traders are assumed to be risk neutral, that is to say, they only care about maximizing the expected future payoff. This assumption is consistent with the behavior of mutual funds that have a very diversified portfolio and that are risk neutral as a group.

Our model has two periods. In period 1 each trader submits market orders to a market maker who is also assumed to be risk neutral. The market maker establishes a price p for the risky asset once he has observed the total net quantity demanded by the traders and the public signal. It is important to note that the market maker only observes

² Given the assumptions on the pricing rule below (mainly, linearity), mixed strategies are never optimal for insiders.

"total demands". Thus, he cannot know if an order comes from an informed trader or from a noise trader.

I assume competition among market makers. This competition among price setters forces them to select a price such that they earn zero expected profits. The reasons for this are exactly the same that force price to equate the marginal cost in the Bertrand model of oligopolistic competition. The market maker is prepared to buy or sell any amount of risky asset that is supplied or demanded³. Neither the market maker nor the informed agent have short-selling constraints.

Thus, a market maker must sell \tilde{w} shares where \tilde{w} is the net total order flow

$$\tilde{w} = \sum_{n=1}^N x_n (\tilde{s}_n, \tilde{s}_0) + \tilde{z}. \quad (2.1)$$

The zero profit condition (or market efficiency condition) implies that

$$\tilde{p} = p(\tilde{w}, \tilde{s}_0) = E(\tilde{v} | \tilde{w}, \tilde{s}_0). \quad (2.2)$$

The price selected equals the expected return conditional on all information available to the market maker. The market maker uses the order flow to make inferences about \tilde{v} because it contains part of

³ The assumption of competitive market makers is consistent with the institutional arrangements of the Over-the-counter (OTC) market or the Intermarket Trading System (ITS) that forces competition between the New York Stock Exchange (NYSE) monopolistic specialists and regional specialists. For other rules of dynamic market making with inventory costs, see Amihud and Mendelson (1980), Garman (1976) or O'Hara and Oldfield (1986).

the private information owned by informed agents⁴.

Prices are random variables that are measurable with respect to order flows and the public signals. $p(\dots)$, \tilde{p} and p denote the pricing rule, the equilibrium price as a random variable and the realization of this random variable respectively.

In period 2, the realization of \tilde{v} is observed and each agent receives his payoff.

This sequence of events to determine price formation looks like the one used by Admati and Pfleiderer (1988a), Diamond and Verrecchia (1987) Easley and O'Hara (1987), Glosten and Milgrom (1985), Gould and Verrecchia (1985) and Kyle (1985) and resembles the structure proposed by the already classical paper of Kreps and Wilson (1982) on sequential equilibrium. But my approach follows more closely the articles by Kyle and Admati and Pfleiderer because I assume that the market maker selects a single price after he has observed the order

⁴ A third possible justification of the random variable \tilde{z} is to assume that there is some noise in the communication channel between the informed speculators and the market maker. This means that the market maker is only able to observe a garbled order flow. Obviously \tilde{z} must be again uncorrelated to all other random variables.

flow.⁵

Gould and Verrecchia (1985) alter this sequence of events. In their paper the market maker selects the price first and afterwards the informed agents submit demands conditional on price and private information⁶.

⁵ Diamond and Verrecchia (1987), Easley and O'Hara (1987) and Glosten and Milgrom (1985) study the possibility of bid-ask spreads (i.e., a buying price and a selling one) in a dynamic setup. Glosten and Milgrom, for instance, assume that in each period of time there is only the possibility of buying or selling a unit of asset. From this observed behavior, the market maker infers part of the information contained in the order flow using updating bayesian rules. The agent who submits the market order in each period is selected randomly and can be either an informed trader or a noise trader. With this structure the market maker can select only two contingent prices (the selling price and the buying one) before the order flow is observed. Finally, the possibility of sequential learning by the market maker is not analyzed in our (essentially) one-period model.

⁶ In Gould and Verrecchia (1985) the assumption of risk aversion cannot be relaxed, because the price is fixed by the specialist first and risk neutrality would lead informed traders to take infinite positions in the risky asset held by.

The following table summarizes the sequence of events:

Table 1 — Time Structure of the Market Orders Model

<p><u>Date 1</u></p>	<p>a) - The public signal $\tilde{s}_0 = \tilde{v} + \tilde{\epsilon}_0$ is announced and observed by everybody.</p> <p>- The informed agents observe $\tilde{s}_n = \tilde{v} + \tilde{\epsilon}_n$.</p> <p>- The liquidity traders observe their financial needs \tilde{z}_t.</p> <p>b) - The informed agents submit net demands $x_n(\tilde{s}_n, \tilde{s}_0)$ conditional on public and private information to the market maker.</p> <p>- The liquidity traders submit their net demands \tilde{z}_t.</p> <p>c) - The market maker selects a price that equates the expected return conditional to the public signal and the order flow, $\tilde{p} = E(\tilde{v} \tilde{w}, \tilde{s}_0)$.</p> <p>d) - Transactions are carried out at the price selected by the market maker in c). All the demands are absorbed by the market maker.</p>
<p><u>Date 2</u></p>	<p>- The return \tilde{v} is revealed and each agent receives this return multiplied by the quantity of asset bought on Date 1.</p>

2.B. Equilibrium.

Only informed traders make strategic decisions in the model. They want to maximize expected profits conditional on their information.

The optimal demand for risky asset of informed agent n is

$$x_n(s_n, s_0) = \operatorname{argmax}_{x \in \mathbb{R}} E \left[\left[\tilde{v} - p(\tilde{w}, \tilde{s}_0) \right] x \mid s_n, s_0 \right]$$

The Nash equilibrium of the game we are studying consists of N strategies $x_n(\tilde{s}_n, \tilde{s}_0)$, $n = 1, \dots, N$, that maximize the expected profits for each informed trader, given the observed signals, and a price function $p(\tilde{w}, \tilde{s}_0)$ that makes the expected profits of the market maker equal to zero for each pair of public signal and order flow.

I restrict attention to linear and symmetric equilibria, where $x_n(\tilde{s}_n, \tilde{s}_0)$ and $p(\tilde{w}, \tilde{s}_0)$ are linear functions⁷. The question of existence of nonlinear equilibria in this setup remains unanswered but seems implausible under Gaussian assumptions.

The proofs make extensive use of the following lemma which is a version of the projection theorem for normally distributed random variables.

⁷ For nonlinear equilibria under different distributional assumptions, see Gale and Hellwig (1987) and Laffont and Maskin (1988).

LEMMA 2.1: Let $\tilde{v}, \tilde{u}_1, \dots, \tilde{u}_K$ be normally and independently distributed random variables with $E(\tilde{v}) = \bar{v}$ and $E(\tilde{u}_k) = 0$, ($k = 1, \dots, K$) and precisions $\tau_v, \tau_1, \dots, \tau_K$, respectively.

Then:

$$\left[\text{Var}(\tilde{v} | \tilde{v} + \tilde{u}_1, \dots, \tilde{v} + \tilde{u}_K) \right]^{-1} = \tau_v + \sum_{k=1}^K \tau_k,$$

and

$$E\left[\tilde{v} | \tilde{v} + \tilde{u}_1, \dots, \tilde{v} + \tilde{u}_K \right] = \frac{\tau_v \bar{v} + \sum_{k=1}^K \tau_k (\tilde{v} + \tilde{u}_k)}{\tau_v + \sum_{k=1}^K \tau_k}.$$

Proof: It follows from DeGroot (1970, p. 55). ■

Now, I can state now my first result:

PROPOSITION 2.2: There exists a unique symmetric, linear equilibrium which is given by

$$\tilde{p} = p(\tilde{w}, \tilde{s}_0) = \delta + \lambda \tilde{w} + \gamma \tilde{s}_0,$$

and

$$\tilde{x}_n = x(\tilde{s}_n, \tilde{s}_0) = \alpha + \beta \tilde{s}_n + \kappa \tilde{s}_0,$$

where

$$\alpha = - \frac{\beta \bar{v}}{\tau_v + \tau_0},$$

$$\beta = \left[\frac{\sigma_z^2}{\left(\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_c} \right) N} \right]^{\frac{1}{2}},$$

$$\kappa = - \frac{\beta \tau_0}{\tau_v + \tau_0},$$

$$\delta = \frac{\tau_v \bar{v}}{\tau_v + \tau_0},$$

$$\lambda = \left[\frac{\left(\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_c} \right) N}{\sigma_z^2} \right]^{\frac{1}{2}} \frac{1}{2 \left[\frac{\tau_v + \tau_0}{\tau_c} \right] + N + 1},$$

$$\gamma = \frac{\tau_0}{\tau_v + \tau_0}.$$

Proof: See the appendix. ■

Note that the equilibrium can be written in the following way:

$$x_n(\tilde{s}_n, \tilde{s}_0) = \beta \left[\tilde{s}_n - \frac{\tau_v \bar{v} + \tau_0 \tilde{s}_0}{\tau_v + \tau_0} \right] = \beta \left[\tilde{s}_n - E(\tilde{v} | \tilde{s}_0) \right], \quad (2.3)$$

$$p(\tilde{w}, \tilde{s}_0) = \frac{\tau_v \bar{v} + \tau_0 s_0}{\tau_v + \tau_0} + \lambda \tilde{w} = E(\tilde{v} | \tilde{s}_0) + \lambda \tilde{w}. \quad (2.4)$$

The quantities demanded by each agent depend on the deviation of the private information with respect to the posterior expectation after observing the public signal. On the other hand, the price of the asset will be equal to the posterior expectation after observing s_0 plus

a term that embodies the information contained in the order flow.

It is straightforward to prove that the unconditional expectation $E(x_n(\tilde{s}_n, \tilde{s}_0))$ of individual demands is equal to zero and, from this, that $E(p(\tilde{w}, \tilde{s}_0)) = \bar{v}$, i.e., prices are unbiased estimates of returns.

Equation (2.3) and (2.4) imply that analyzing the effects of the disclosure of public information is an equivalent problem to studying the effects of a reduction in the variance of the prior distribution of \tilde{v} .

The coefficient λ in Proposition 2.2 is the inverse of the depth of the market, according to Kyle's (1985) terminology. In other words, $1/\lambda$ measures the order flow required to change the price of the risky asset by one dollar. By inspection, λ is decreasing in τ_0 . Therefore the depth of the market is increasing in τ_0 . This reflects the fact that, when more public information is available, market maker's inferences are less dependent on the order flow. The next section gives a more detailed explanation of this important fact.

3. THE WELFARE EFFECTS OF PUBLIC INFORMATION.

3.A. Free private information.

The welfare effects of public disclosure of information by firms are parameterized by τ_0 . Higher values of τ_0 mean more stringent disclosure requirements. Our analysis requires computation of expected

profits for both types of agents before the realizations of the random variables are observed.

COROLLARY 3.1: Expected profits of informed traders are

$$E(\pi^n) = \left[\frac{\sigma_z^2 \left[\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_e} \right]}{N} \right]^{\frac{1}{2}} \frac{1}{2 \left[\frac{\tau_v + \tau_0}{\tau_e} \right] + N + 1} \quad (3.1)$$

For a liquidity constrained individual trader t (before he is able to observe his liquidity needs), expected profits are

$$\begin{aligned} E(\pi^t) &= -\lambda E(z_t^2) = \\ &= - \left[\frac{\left(\left[\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_e} \right] N \right)^{\frac{1}{2}}}{\sigma_z^2} \right]^{\frac{1}{2}} \frac{1}{2 \left[\frac{\tau_v + \tau_0}{\tau_e} \right] + N + 1} \sigma^2 \end{aligned} \quad (3.2)$$

Proof: For the informed traders, compute

$$E(\pi^n) = E\left[(\tilde{v} - \tilde{p})\tilde{x}_n\right] = E\left[(v - \delta - \lambda\tilde{w} - \gamma\tilde{s}_0)(\alpha + \beta\tilde{s}_n + \kappa\tilde{s}_0)\right]$$

where \tilde{w} is defined in (2.1) and $(\alpha, \beta, \kappa, \delta, \lambda, \gamma)$ are given in Proposition 2.2.

For the liquidity constrained traders compute

$$E\left[(\tilde{v} - \tilde{p})\tilde{z}_t\right] = E\left[\left[\tilde{v} - \delta - \lambda \left[\sum_{n=1}^N \tilde{x}_n + \sum_{t=1}^T \tilde{z}_t \right] - \gamma\tilde{s}_0\right]\tilde{z}_t\right]$$

to obtain, after some algebra, the expressions given in the Corollary.

An alternative (and easier) proof consists of computing the total profits $\sum_{t=1}^T E(\pi^t) = -\lambda\sigma_z^2$ of noise traders. This, together with the zero profit condition for the market maker, implies that total expected profits for insiders are equal to the negative of total expected profits of liquidity traders. Therefore the expected profits of each insider are equal to $\frac{\lambda\sigma_z^2}{N}$. ■

Inspection of (3.1) and (3.2) clearly shows that expected profits for liquidity constrained traders are always negative and increasing in τ_0 . The opposite is true for strategic traders.

When there is more public information available in the economy, informed agents want to keep their relative informational advantage with respect to the market maker. They can react to the arrival of more precise public information, putting more noise in the order flow in such a way that the market maker is not able to make as precise an inference as he could when the precision of the public signal was lower. In fact, the informational content of the order flow alone — without considering the public signal — is decreasing in τ_0 . To see this, compute $[\text{Var}(\tilde{v}|\tilde{w})]^{-1}$ as a measure of the precision of the order flow exclusively

$$[\text{Var}(\tilde{v}|\tilde{w})]^{-1} = \tau_v + \frac{N\tau_v^2 \tau_c}{(\tau_v + \tau_0)^2 + N\tau_0 \tau_c},$$

which is clearly decreasing in τ_0 . This shows that insiders modify their trading behavior in order to avoid revealing too much information

to the market maker.

However, τ_u as defined in the expression (A.9) of the appendix is a measure of the informational content of the order flow when the market maker observes both w and s_0 . Compute its equilibrium value, using the coefficients obtained in Proposition 2.2, to obtain

$$\tau_u(\tau_0) = \frac{N}{\frac{1}{\tau_v + \tau_0} + \frac{2}{\tau_c}}, \quad (3.3)$$

which is increasing in τ_0 . Therefore, the market maker's overall informedness is increasing in τ_0 for two reasons: he receives a more precise public signal and he is able to make more accurate predictions about \tilde{v} when he observes \tilde{w} together with s_0 .

Corollary (3.1) tells us that insiders cannot overcome the initial unfavorable shock of more precise public information. The market maker uses the public signal in a way that dissipates insiders' informative advantage.

I have shown in the previous section that when public information becomes more precise, the market maker puts less weight in the order flow in order to estimate \tilde{v} (λ is decreasing in τ_0). This means that the noise trading is not going to affect prices too much. This is precisely what liquidity traders want.

If a liquidity trader has to sell shares, he wants to get the highest price for his supply. If the market maker is very sensitive to the order flow, he will interpret this supply as "bad news" about \tilde{v} and he will lower the equilibrium price. The same argument applies for a liquidity trader who wants to buy assets. Therefore, liquidity traders

prefer to trade in deep markets (with low λ) in order to minimize their cost of trading.

The implications of Corollary 3.1 are obvious. If there are no legal requirements about public disclosure of information, the decision about how much information is disclosed is endogenously taken by the firm. If I assume that the managers of the firm are the insiders of my model, then managers will precommit ex ante to a policy of no information disclosure. This is true even if the relative informational position of managers with respect to other traders remains unchanged.

On the other hand, any legal requirement about public disclosure will imply a transfer of expected profits from insiders to liquidity traders or vice versa.

3.B. Costly information acquisition

If I assume that the acquisition of information is costly and that there is free entry in the market, then the number of informed agents will be endogenously determined in equilibrium. The number of informed agents will be the one at which if an additional agent purchases the private signal, then his net expected profits will be negative.

Note that an agent who is not liquidity constrained and observes only the public signal has no incentives to enter in this market. The market maker may subtract the demands of these uninformed agents from the order flow. The informational content of this demands

is already known by the market maker provided that uninformed agents can only formulate demands contingent on the public signal. Therefore, the equilibrium price will not depend on these agents' demands. The difference between the expected return and the equilibrium price conditional on the public signal is always zero. Note that

$$E(p(\tilde{w}_I, \tilde{s}_0) | \tilde{s}_0) = E(E(\tilde{v} | \tilde{w}_I, \tilde{s}_0) | \tilde{s}_0) = E(\tilde{v} | \tilde{s}_0)$$

where w_I is the order flow submitted by informed agents and by liquidity constrained traders exclusively. This means that uninformed agents' demands are indeterminate, since their expected conditional profits are always equal to zero. I set the demands of these uninformed agents equal to zero as a mere convention.

The model of entry that I am using, assumes that informed agents and the market maker modify their strategies appropriately when a new insider enters⁸.

COROLLARY 3.2: If the cost of purchasing the signal s_n is c , where

$$c \leq \frac{1}{2(\tau_v + \tau_0)} \left[\frac{\sigma_z^2}{\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_c}} \right]^{\frac{1}{2}}, \quad (3.4)$$

⁸ If a potential entrant cannot credibly announce his presence in the market, then I have to compute his optimal strategy and corresponding profits assuming that the equilibrium strategies of the other insiders and the market maker remain unchanged. It can be shown that the comparative statics of this alternative model of entry are the same as in the one I am considering.

then the number of informed traders, N^* , is given by

$$N^* = \operatorname{argmax} \left\{ N \in \mathbb{N} \mid E(\pi^n(N, \tau_0)) \geq c \right\},$$

where $E(\pi^n(N, \tau_0))$ are expected profits of insiders when the number of insiders is N and the precision of public information is τ_0 .

Proof: Assumption (3.4) is necessary to allow the existence of at least one informed trader. $E(\pi^n(N, \tau_0))$ is given in (3.1) and is clearly strictly decreasing in N . Informed traders will enter in the market of the risky asset until

$$E(\pi^n(N, \tau_0)) \geq c > E(\pi^n(N+1, \tau_0)) \quad \blacksquare$$

Since $E(\pi^n(N, \tau_0))$ is decreasing in N , the number of informed traders is decreasing in c (not in a continuous basis given our "discrete" framework).

Moreover, since $E(\pi^n(N, \tau_0))$ is decreasing in τ_0 , implicit differentiation proves that if the accuracy of public information released by firms increases then the number of informed traders tends to decrease. This means that legislation that forces firms to release public information reduces insider trading activity.

However, when the precision of public information increases, the effect on expected profits of liquidity traders is ambiguous for some values of τ_0 . The next Corollary shows why this is possible.

COROLLARY 3.3: The depth of the market ($1/\lambda$) is increasing in N if and only if

$$\left(\frac{N-1}{2}\right)\tau_c \geq \tau_v + \tau_o .$$

Proof: It is immediate after computing the derivative of λ with respect to N . ■

In order to interpret Corollary 3.3, it should be noted that an increase in N has two opposite effects. First, it increases the informational content of order flows. This tends to increase λ because the order flow becomes a more reliable signal about the expected return. Secondly, it increases the degree of competition among informed traders and this tends to reduce λ .

The reason for this second effect is that λ is the regression coefficient of \tilde{w} in the estimation of \tilde{v} against order flows and public signals. If all informed agents received the same signal, then \tilde{w} would have the same informational content regardless of N . Therefore, the market maker should scale down λ to compensate the increased variance of \tilde{w} due to the larger number of informed agents.

The above exercise is fictitious because if the number of informed traders increases, then the "quality" of the signal improves: a new informed trader supplies a new signal about \tilde{v} and this new signal is incorporated in the order flow. Therefore, the order flow becomes a more informative signal for the market maker and thus he will put more weight on it (λ will increase). This effect goes in the opposite

direction relative to the one of increasing competition among informed traders.

Note that Corollary 3.3 tells us that when either the precision of informed traders is low or there are few insiders, it is very difficult for the market maker to make inferences about \tilde{v} . In this case, an increase in N has an important effect on the amount of information contained in the order flow and this effect dominates the effect of increasing competition, i.e., λ increases.

Assume now that the level of public information τ_0 is such that the net expected profits $E(\pi^i(N, \tau_0)) - c$ for the insiders are equal to zero. In this case, if τ_0 increases only a little bit, then the number of informed agents in the market will decrease in one unit and this has ambiguous effects on the depth of the market and therefore on liquidity traders' welfare. The reader may feel uncomfortable because I am taking the derivative of λ with respect to N where N is a natural number. I can compare directly the expected profits of liquidity traders when there are N insiders with the profits when there are $N - 1$ of them for a given τ_0 .

It can be proved that

$$E(\pi^t(N, \tau_0)) > E(\pi^t(N - 1, \tau_0)) \text{ iff } -4(b^2 + b) + N^2 - N - 1 > 0 \quad (3.5)$$

$$\text{where } b = \frac{\tau_v + \tau_0}{\tau_c}$$

Note that expected profits of an informed trader, and the number of informed agents in the market, are not only determined by b ,

but by both b and τ_c^9 . Therefore, I have an extra degree of freedom that prevents us from signing the expression in the right hand side of (3.5).

Thus, I have shown that if the acquisition of private information is costly, then disclosure of further public information has ambiguous effects on the noise traders' welfare, provided that the change in τ_0 induces reductions in the number of informed agents participating in that market.

4. PRICE VOLATILITY AND TRADE VOLUME IMPLICATIONS.

In this section I derive some empirical implications of increasing the precision of public information. The experiment consists of comparing two markets with the same fundamentals (N , τ_c , τ_v) and different precisions of the public signal, and compare the expected behavior of prices and trade volume in these two markets. I assume that private information is observable at no cost.

The level of price volatility is given by the variance of the random variable $\tilde{p} = \delta + \lambda\tilde{w} + \gamma\tilde{s}_0$ ¹⁰. Replace \tilde{w} by its equilibrium value $(\sum_{n=1}^N x_n(\tilde{s}_n, \tilde{s}_0) + \tilde{z})$ and use the values of α , β , κ , δ , λ and γ in

⁹ For fixed b and c , the equilibrium number of informed traders is decreasing in τ_c .

¹⁰ The variance of prices as a measure of volatility is to be interpreted as the variance of the equilibrium price in (independent) repeated rounds of trade. The same argument applies for the expected volume of trade.

Proposition 2.2, to obtain

$$\text{Var}(\tilde{p}) = \frac{1}{(\tau_v + \tau_0)} \left[\frac{N}{2 \left[\frac{\tau_v + \tau_0}{\tau_\varepsilon} \right] + N + 1} + \frac{\tau_0}{\tau_v} \right], \quad (4.1)$$

where the first term of (4.1) incorporates the variance of \tilde{p} induced by the variance of the order flow and the second term incorporates the reaction of price to the arrival of public signals.

The first term is decreasing in τ_0 because, as I have argued in Section 3, the order flow becomes a relatively more imprecise signal about \tilde{v} and then the market maker reduces the elasticity of the price with respect to order flows. Moreover, the random variable \tilde{w} has less variance now (see below). When τ_0 increases the market maker puts more weight in the public signal (γ is increasing in τ_0) and, even if the variance of \tilde{s}_0 is smaller, the second term in (4.1) is increasing in τ_0 .

The reader will notice that it should appear a third term in (4.1) involving $\text{Cov}(\tilde{w}, \tilde{s}_0)$. It turns out that this covariance is equal to zero in equilibrium. The reason is that the insiders' demands and the pricing rule depend on the deviation of the random variables s_n and \tilde{w} from their expected values conditional on s_0 . The combination of the pricing rule with the insiders' optimal behavior rules out any correlation between order flows and public signals.

It can be shown that the derivative of $\text{Var}(\tilde{p})$ with respect to τ_0 is always positive. This is the logical result if we have in mind the discussion in Section 3.A. I showed there that the overall informedness of the market maker was increasing in τ_0 . This means that

the market maker will be able to predict more accurately the realization of \tilde{v} , i.e., the variance of \tilde{p} will approach the variance of \tilde{v} . Notice that the variance of \tilde{v} is the upper bound of $\text{Var}(\tilde{p})$ because

$$\text{Var}(\tilde{p}) = \text{Var}\left[E(\tilde{v}|\tilde{w}, \tilde{s}_0)\right] \leq \text{Var}(\tilde{v}) = \frac{1}{\tau_v},$$

and, from (4.1), it is obvious that $\lim_{\tau_0 \rightarrow \infty} \text{Var}(\tilde{p}) = \frac{1}{\tau_v}$.

Surprisingly, $\text{Var}(\tilde{p})$ is independent of the noise variance σ_z^2 . The reason is that when the level of noise increases, the insiders have more camouflage and they trade more actively based upon private information, so that they leave unchanged the precision of the order flow as a signal of \tilde{v} .

This result gives us another normative argument about the desirability of public disclosure of information and about the incentives to disclose information by firms. Following Fishman and Hagerty (1988), I can assume that there are Q original shareholders before date 1 who want to cancel their positions in the firm. After trading has been conducted, the original shareholder q will receive a payoff equal to $p \cdot \theta_q$ where θ_q is the number of shares that this shareholder owned¹¹. If those shareholders are risk averse, their expected utility will be obviously decreasing in $\text{Var}(\tilde{p})$. If the

¹¹ In this case the total order flow has mean equal to the total number of shares $\sum_{q=1}^Q \theta_q$ held by the original Q stockholders. According to my assumptions, only deviations from this mean will be informative for the market maker.

managers of the firm want to maximize the expected utility of current shareholders, they will commit themselves to a policy of no information disclosure because this is the policy that minimizes the variance of stock prices. In fact, a legislation that forces disclosure reduces the insurance opportunities for the original stockholders. If those stockholders were risk neutral, then they are indifferent between alternative disclosure policies.

Another testable implication I obtain refers to the intensity of the market maker's activity. As I have said, the market maker has an expected position equal to zero. However, we can use as a measure of trading volume processed by the market maker the absolute value of \tilde{w} (that is the net demand presented to the market maker).

We know that for a normally distributed random variable \tilde{x} with zero mean, $E|\tilde{x}| = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \sigma_x$, where σ_x is the standard deviation of \tilde{x} . Thus, the expected volume of trade done by the market maker is given by

$$E(V) = \left(\frac{2}{\pi}\right)^{1/2} \sqrt{\text{Var}(\tilde{w})} . \quad (4.2)$$

COROLLARY 4.1: The expected volume of trade done by the market maker is decreasing in τ_0 .

Proof: The standard deviation of the order flow is

$$\sqrt{\text{Var}(\tilde{w})} = \sqrt{\text{Var}\left[\sum_{n=1}^N x_n(\tilde{s}_n, \tilde{s}_0) + \tilde{z}\right]} = \sqrt{\text{Var}\left[\sum_{n=1}^N x_n(\tilde{s}_n, \tilde{s}_0)\right] + \sigma_z^2} ,$$

and only the first term under the root is potentially sensible to changes in τ_0 . Evaluate the variance of the informed agents' aggregate

demand in equilibrium, to obtain

$$\begin{aligned} \text{Var} \left[\sum_{n=1}^N x_n (\tilde{s}_n, s_0) \right] &= \text{Var} (N\alpha + N\beta \sum_{n=1}^N \tilde{s}_n + N\kappa \tilde{s}_0) = \\ &= \sigma_z^2 \left(\frac{\frac{N}{\tau_v + \tau_0} + \frac{1}{\tau_c}}{\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_c}} \right), \end{aligned} \quad (4.3)$$

that is obviously strictly decreasing in τ_0 when $N > 1$ and constant for $N = 1$. ■

The intuition behind this result is that the market maker has to trade to compensate for insiders' demands that are not matched by other insiders' demands. This means that the market maker matches only the demands submitted by insiders that have drawn extreme signals. When the precision of the public signal increases, informed agents react less aggressively to their private signals. This reduces the occurrence of "unmatched" demands. This means that there will be less expected trade crossed between informed agents and the market maker.

5. STRATEGIC LIQUIDITY TRADERS.

All my previous results rely crucially on the fact that noise traders' behavior is independent of the values of the parameters in the model. Specifically, I was assuming that σ_z^2 is independent of τ_0 .

I show in this section how the previous results change when I

relax the previous assumption in a two-security world example. I will also assume that disclosure requirements are not uniform across firms.

Assume that there are two securities $s = 1, 2$, traded in two different markets. The random returns \tilde{v}_1 and \tilde{v}_2 of these securities are uncorrelated. In each market there are N_s distinct informed traders who receive signals about security \tilde{v}_s of the form $\tilde{s}_{ns} = \tilde{v}_s + \tilde{\epsilon}_{ns}$, $n = 1, \dots, N_s$, $s = 1, 2$. For each security, the statistical properties of the random variables \tilde{v}_s and $\tilde{\epsilon}_{ns}$ are exactly as in section 2.

There is also a piece of public information $\tilde{s}_{0s} = \tilde{v}_s + \tilde{\epsilon}_{0s}$ about the return of each security, with again the same statistical properties as in section 2.

In each market there are T_s distinct nondiscretionary liquidity traders indexed by t . Each nondiscretionary liquidity trader in market s has to trade an amount of shares of security s equal to the random variable \tilde{z}_{st} , where $\tilde{z}_{st} \sim N(0, \sigma_s^2)$ for $t = 1, \dots, T_s$, and $s = 1, 2$.

There are also D discretionary liquidity traders indexed by d . Each discretionary liquidity trader has to trade an amount of shares equal to the realization of the random variable \tilde{y}_d where $\tilde{y}_d \sim N(0, \sigma_d^2)$ for $d = 1, \dots, D$. However, each discretionary liquidity trader is free to choose in which market he will trade. For the sake of simplicity I will assume that each liquidity trader can only trade in a single

market¹².

Finally, I assume that

$$\left[(\tilde{v}_1, \tilde{v}_2), (\tilde{\epsilon}_{11}, \dots, \tilde{\epsilon}_{1N_1}), (\tilde{\epsilon}_{21}, \dots, \tilde{\epsilon}_{2N_2}), \right. \\ \left. (\tilde{\epsilon}_{01}, \tilde{\epsilon}_{02}), (\tilde{z}_{11}, \dots, \tilde{z}_{1T_1}), (\tilde{z}_{21}, \dots, \tilde{z}_{2T_2}), (\tilde{y}_1, \dots, \tilde{y}_D) \right]$$

are normal and mutually independent random variables with means

$$\left[(\tilde{v}_1, \tilde{v}_2), (0, \dots, 0), (0, \dots, 0), (0, 0), (0, \dots, 0), (0, \dots, 0), (0, \dots, 0) \right]$$

and variances $\left[\left(\frac{1}{\tau_{v1}}, \frac{1}{\tau_{v2}} \right), \left(\frac{1}{\tau_{\epsilon1}}, \dots, \frac{1}{\tau_{\epsilon1N_1}} \right), \left(\frac{1}{\tau_{\epsilon2}}, \dots, \frac{1}{\tau_{\epsilon2N_2}} \right), \right.$

$$\left. \left(\frac{1}{\tau_{01}}, \frac{1}{\tau_{02}} \right), \left[\sigma_1^2, \dots, \sigma_1^2 \right], \left[\sigma_2^2, \dots, \sigma_2^2 \right], \left[\sigma_d^2, \dots, \sigma_d^2 \right] \right]$$

Define the random variable $\tilde{z}_s = \sum_{t=1}^{T_s} \tilde{z}_{st}$. Thus, $\tilde{z}_s \sim N(0, \sigma_{zs}^2)$

where $\sigma_{zs}^2 = T_s \sigma_s^2$.

Define $\tilde{h}_s = \sum_{d \in D_s} y_d$ where D_s is the set of discretionary

liquidity traders who choose to trade in security s .

In each market the price is selected by different risk neutral market makers who make zero expected profits.

¹² If the discretionary traders can split their net demands among several markets, it can be shown that the proportion of shares traded in each market is proportional to the depth of the market (see Bhushan (1988) or Admati and Pfleiderer (1988) in a different context).

The equilibrium pricing rule in each market is given by

$$p_s(\tilde{w}_s, \tilde{s}_{0s}) = \delta_s + \lambda_s \tilde{w}_s + \gamma_s \tilde{s}_{0s},$$

where $w_s = \sum_{n=1}^{N_s} x_n(s_{ns}, \tilde{s}_{0s}) + \tilde{z}_s + \tilde{h}_s$. The optimal demand of an informed agent is $x_n(\tilde{s}_{ns}, \tilde{s}_{0s}) = \alpha_s + \beta_s \tilde{s}_{ns} + \kappa_s \tilde{s}_{0s}$, $n = 1, \dots, N_s$.

The optimal values of α_s , β_s , κ_s , δ_s , λ_s and γ_s are as in Proposition 2.2, replacing τ_v , τ_0 , τ_c and σ_z^2 by τ_{vs} , τ_{0s} , τ_{cs} and $\left[\sigma_{zs}^2 + \text{Var}(\tilde{h}_s)\right]$ respectively.

Profits of both informed traders and nondiscretionary liquidity traders are similar to those in (3.1) and (3.2) with the appropriate relabelling (again, σ_z^2 must be replaced by $\sigma_{zs}^2 + \text{Var}(\tilde{h}_s)$).

Expected profits (before observing \tilde{y}_d) of discretionary liquidity traders who choose to trade in security s are

$$E(\pi^d) = -\lambda_s \sigma_d^2 \quad d \in D_s \quad (5.1)$$

The following lemma is an adaptation of Proposition 1 in Admati and Pfleiderer (1988a).

LEMMA 5.2: There exists generically a unique equilibrium in which all discretionary liquidity trading is concentrated in the same market.

Proof: From (5.1), expected profits of discretionary traders are maximized when λ_s is the smallest. If the market maker follows the equilibrium pricing rule, informed agents behave optimally and all the discretionary liquidity traders trade in market s , then the value of λ_s will be

$$\lambda_s(H) = \left[\frac{1}{\tau_{vs} + \tau_{0s}} + \frac{1}{\tau_{\epsilon s}} \right]^{\frac{1}{2}} \cdot \frac{1}{2 \left[\frac{\tau_{vs} + \tau_{0s}}{\tau_{\epsilon s}} \right] + N + 1},$$

where $H = D\sigma_d^2$.

Choose the security $s^* \in \{1, 2\}$ that minimizes $\lambda_s(H)$. Assume without loss of generality that $s^* = 1$. It is an equilibrium for all discretionary traders to choose security 1 to trade, because $\lambda_1(H) \leq \lambda_2(H) < \lambda_2(0)$.

To prove genericity, note that if it happens that discretionary agents want to split their demand, this is because $\lambda_1(H) = \lambda_2(H)$. Any small change on the parameters τ_{vs} , τ_{0s} , σ_{zs}^2 , σ_d^2 ($s = 1, 2$) will imply $\lambda_1(H) \neq \lambda_2(H)$. ■

The security s^* that minimizes $\lambda_s(H)$ is an endogenous variable that depends on the level of public information in each market. If, for instance, the firm that issues security s (different from s^*) is forced to make more precise disclosures of public information, then τ_{0s} increases and security s may become eventually the one that minimizes $\lambda_s(H)$. In this case, all discretionary liquidity trading is automatically concentrated in security s .

Informed agents in the market for security s^* (the market where the trading was concentrated previously) suffer a loss because there is less noise trading providing camouflage. Define $\pi_s^n(y, \tau_0)$ as the expected profits of informed agents in market s when the variance $\text{Var}(h_s)$ of discretionary trading is equal to y , and the precision of

public information is τ_0 . It is easy to see that

$$\pi_{s^*}^n(H, \tau_{0s^*}) > \pi_{s^*}^n(0, \tau_{0s^*}) .$$

Moreover, nondiscretionary liquidity traders in market s^* experiment also a loss, because now this market is less liquid. Note that $\lambda_{s^*}(H, \tau_{0s^*}) < \lambda_{s^*}(0, \tau_{0s^*})$ where $\lambda(\cdot, \cdot)$ is defined similarly to $\pi_s^n(\cdot, \cdot)$.

With respect to the market for security s , in which all discretionary trading is concentrated now, the results are the following. When the precision of public information increases from τ_{0s} to τ'_{0s} in such a way that all discretionary trading is attracted to security s , I have to compare $\lambda_s(0, \tau_{0s})$ with $\lambda_s(H, \tau'_{0s})$ in order to evaluate the effects on nondiscretionary liquidity traders' welfare. It is easy to see that $\lambda_s(y, \tau_{0s})$ is strictly decreasing in both variables, and this implies that $\lambda_s(0, \tau_{0s}) > \lambda_s(H, \tau'_{0s})$. This means that expected profits of nondiscretionary liquidity traders increase.

On the other hand, the results for informed agents in market s are ambiguous. We have to decide whether $\pi_s^n(H, \tau'_{0s})$ is greater than $\pi_s^n(0, \tau_{0s})$. $\pi_s^n(\cdot, \cdot)$ is increasing in the first term and decreasing in the second one. The comparison between expected profits will depend on the magnitude of H and on the increment of τ_{0s} necessary to induce the concentration of all discretionary trading in market s .

From the point of view of discretionary liquidity traders, the increase from τ_{0s} to τ'_{0s} makes them better off, because they are able to trade in a deeper market.

Note also that the results I obtained in Section 4 referred to

volume of trade done by the market maker in market s are modified substantially. Assume that when the precision increases from τ_{0s} to τ'_{0s} , all discretionary liquidity trading that was concentrated in market s^* shifts to market s , and use the same measure of volume of trade as in Section 4. Then, we have to compare

$$E\left[V_s(H, \tau'_{0s})\right] = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \left(\sigma_{zs}^2 + H\right)^{\frac{1}{2}} \left[\left(\frac{\frac{N}{\tau_v + \tau'_{0s}} + \frac{1}{\tau_\varepsilon}}{\frac{1}{\tau_v + \tau'_{0s}} + \frac{1}{\tau_\varepsilon}} \right)^{\frac{1}{2}} + 1 \right]$$

with

$$E\left[V_s(0, \tau_{0s})\right] = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \sigma_{zs} \left[\left(\frac{\frac{N}{\tau_v + \tau_{0s}} + \frac{1}{\tau_\varepsilon}}{\frac{1}{\tau_v + \tau_{0s}} + \frac{1}{\tau_\varepsilon}} \right)^{\frac{1}{2}} + 1 \right].$$

Note that $E\left[V_s(y, \tau_0)\right]$ is increasing in y , and decreasing in τ_0 . Therefore, the comparison is ambiguous and depends on the values of H , τ_{0s} and τ'_{0s} .

When public information becomes more precise, discretionary liquidity trading is concentrated in market s and this implies greater expected volume of trade. The effect on the expected volume of trade crossed between insiders and market maker is ambiguous. I have already shown that if the variance of noise is constant, then the expected volume of trade is decreasing in τ_0 . However, if the variance of liquidity trading increases, then there is more camouflage provided by the noise. In this case, insiders will react more sharply to private information, and so increase the expected volume of trade. The dominant

effect is ambiguous.

The results about volatility of prices obtained in Section 4 remain unchanged because $\text{Var}(\tilde{p})$ is independent of the noise variance, as can be seen from (4.1)

6. THE EFFECTS OF PRIVATE INFORMATION.

I have shown that more public information makes informed agents worse off. This may also be the case when more precise private information becomes available. We can imagine the endowments of private information as signals coming from informational leaks in the firm issuing the security. I am still assuming that each agent has access to a different source of information with independent noise ϵ_n . The case of common information will be studied in the next section.

It can be proved that for some parameters of the model, more private information (in the sense of more precision obtained from the leaks) may make informed agents worse off. More precisely, I can state the following Corollary:

COROLLARY 6.1: Expected profits of informed agents are decreasing in τ_ϵ iff $\left(\frac{N-3}{2}\right) \tau_\epsilon \geq (\tau_v + \tau_0)$.

Proof: It follows immediately from evaluating the derivative of $E(\pi^n)$ in (3.1) with respect to τ_ϵ . ■

There is an initial positive effect for the insiders of having more precise private information: their informational position with respect to the market maker improves.

The reason for the result in Corollary 6.1 is that if private information is very precise relative to public information, and there is a further increase in τ_e , then the correlation between the demands submitted by insiders increases notably. In fact, insiders use more heavily their private information that was already very precise. This means that there is less trade crossed among insiders, because private signals cannot differ too much. This in turn implies less noise in the order flow as a signal of \tilde{v} and, therefore the market maker is able to predict more accurately $\frac{\sum_{n=1}^N s_n}{N}$, that is the sufficient statistic of all private information. Thus, this negative effect on the price rule outweighs the initial positive effect of more private information.

Note that if $N < 3$, expected profits are always increasing in τ_e . When insiders do not face too much competition, they are always better off by being better informed.

It is interesting to measure the amount of private information that is publicly revealed through the price system in this economy. Kyle (1985) proposed as a measure of how much private information is revealed by prices the difference between the prior precision of \tilde{v} and the posterior precision of \tilde{v} conditional on prices. In our model with public information the right measure would be

$$R = \left[\text{Var}(\tilde{v} | \tilde{p}, \tilde{s}_0) \right]^{-1} - \left[\text{Var}(\tilde{v} | \tilde{s}_0) \right]^{-1}$$

When public signals are available, the variance of the prior is in fact $\text{Var}(\tilde{v}|\tilde{s}_0)$. To empirically interpret this measure note that, after \tilde{v} is observed, we can run two regressions. One regression would be of \tilde{v} against \tilde{s}_0 alone, and the other one of \tilde{v} against both \tilde{p} and \tilde{s}_0 . The residual variances of these regressions are $\text{Var}(\tilde{v}|s_0)$ and $\text{Var}(\tilde{v}|\tilde{p}, \tilde{s}_0)$ respectively. The difference between these magnitudes will give us the reduction on the prior variance due to the private information incorporated on prices.

It can be proved that

$$\text{Var}(\tilde{v}|\tilde{p}, \tilde{s}_0) = \frac{1}{\tau_v + \tau_0 + \frac{1}{\frac{1}{\tau_v + \tau_0} + \frac{2}{\tau_\epsilon}} N}$$

and
$$\text{Var}(\tilde{v}|\tilde{s}_0) = \frac{1}{\tau_v + \tau_0},$$

therefore,

$$R = \frac{N}{\frac{1}{\tau_v + \tau_0} + \frac{2}{\tau_\epsilon}}$$

Like $\text{Var}(\tilde{p})$, this magnitude is independent of σ_z^2 because insiders trade more aggressively when there is more liquidity trading providing camouflage. This implies that the informative content of the order flow remains constant. Note that R is increasing in both τ_ϵ and N . Either more private information or more insiders increase the

informational content of both order flows and prices¹³.

Moreover, R is increasing in τ_0 . As we have seen in Section 3, insiders reveal more of their private information through the order flow when the market maker is able to observe both \tilde{w} and \tilde{s}_0 .

An interesting exercise suggested by Kyle (1986) consists of fixing the total amount of private information, given by $M = N\tau_c$, and to increase the number of agents. This means that there are more informed agents in the market but each of them owns a more imprecise signal now. The total amount of information is split equally among the N agents. Take the limit when $N \rightarrow \infty$ and keep M constant, to obtain

$$\lim_{\substack{N \rightarrow \infty \\ N\tau_c = M}} R = \frac{M}{2}.$$

In other words, prices do not reveal more than half of the total private information. The weight of each individual becomes small with respect to the whole market and each agent knows very little about \tilde{v} (τ_c becomes small). However, informed agents use their private information in such a way that never the market maker can learn more than half of the total private information.

I will show in Section 11 how this result is also true in a model in which prices are selected according to market clearing rules.

Another interesting result is that if we keep the number of insiders constant and assume that private information becomes perfect

¹³ The fact that $\lim_{N \rightarrow \infty} R = \infty$ is similar to the results in Milgrom (1979).

($\tau_c \rightarrow \infty$), prices do not become fully revealing either,

$$\lim_{\tau_c \rightarrow \infty} R = N(\tau_v + \tau_0) .$$

This result tells us that the posterior precision is increased by N times the prior precision. In particular I recover Kyle (1985) result that says that when there is a single perfectly informed insider the posterior precision doubles the prior precision.

7. COMMON PRIVATE INFORMATION.

Until now I have assumed that the private information possessed by insiders is diverse. My results about the effects of public information do not change substantially if I assume that all informed agents receive the same signal $s = v + \epsilon$, where, as before, $\epsilon \sim N(0, 1/\tau_c)$. Keeping unchanged the structure of all other random variables and the sequence of events, I can state the following proposition:

PROPOSITION 7.1: The unique linear symmetric equilibrium with common private information is given by the following demand for each agent:

$$\tilde{x}_n = x(\tilde{s}, \tilde{s}_0) = \alpha^* + \beta^* \tilde{s} + \kappa^* \tilde{s}_0 ,$$

and the pricing rule

$$\tilde{p} = p(\tilde{w}, \tilde{s}_0) = \delta^* + \lambda^* \tilde{w} + \gamma^* \tilde{s}_0 ,$$

where

$$\alpha^* = -\beta^* \frac{\tau_v \bar{v}}{\tau_v + \tau_0},$$

$$\beta^* = \left[\frac{\sigma_z^2}{N \left[\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_\varepsilon} \right]} \right]^{\frac{1}{2}},$$

$$\kappa^* = -\beta^* \frac{\tau_0}{\tau_v + \tau_0},$$

$$\delta^* = \frac{\tau_v \bar{v}}{\tau_v + \tau_0},$$

$$\lambda^* = \frac{1}{(N+1)(\tau_v + \tau_0)} \left[\frac{N}{\sigma_z^2 \left[\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_\varepsilon} \right]} \right]^{\frac{1}{2}},$$

$$\gamma^* = \frac{\tau_0}{\tau_v + \tau_0},$$

Proof: The proof mimics the one in Proposition 2.2. The only difference is that I have to replace $E(\beta \sum_{j \neq n} s_j | s_n)$ in the proof of Proposition 2.2 by $\beta(N-1)s$. The details are left to the reader. ■

From the equilibrium values given in Proposition 7.1, compute both the informed traders' expected profits $E^*(\pi^n)$ and the liquidity traders' expected profits $E^*(\pi^l)$,

$$\begin{aligned}
E^*(\pi^n) &= E\left[(\tilde{v} - p(\tilde{w}, \tilde{s}_0)) \cdot x_n(\tilde{s}, \tilde{s}_0)\right] = \\
&= \frac{1}{(N+1)(\tau_v + \tau_0)} \cdot \left[\frac{\sigma_z^2}{N\left[\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_e}\right]} \right]^{\frac{1}{2}} \quad (7.1)
\end{aligned}$$

$$E^*(\pi^t) = -\lambda\sigma^2 = -\frac{1}{(N+1)(\tau_v + \tau_0)} \cdot \left[\frac{N}{\sigma_z^2\left[\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_e}\right]} \right]^{\frac{1}{2}} \sigma^2 \quad (7.2)$$

It is easy to see that $E^*(\pi^n)$ is decreasing in τ_0 and $E^*(\pi^t)$ is increasing in τ_0 . The reason for this is the same that for the case of diverse information: the market becomes more liquid and the insiders lose their informational advantage.

However, the result I obtained in the previous section referred to the value of private information is totally changed when private information is common. In this case, insiders' expected profits are always increasing in τ_e . More private information means that informed agents as a group improve their position with respect to the market maker. However, private information does not exhibit any externality now.

When private information was diverse, higher levels of private information, parameterized by τ_e , might result in lower expected profits earned by the insiders. The market maker might be able to predict more accurately \tilde{v} after observing the order flow \tilde{w} . The reason for obtaining that result does not apply in the common information setup. Given that all the insiders receive the same information and submit the same net

demand, it is not true now that the demands become more correlated. This would imply more precise inferences made by the price setter.

I proceed to analyze the implications for volume of trade done by the market maker and price volatility. From the discussion in Section 4, the standard deviation of the order flow is the relevant measure for the expected volume of trade done by the market maker. It can be shown that

$$E(V^T) = E\left[\left| \sum_{n=1}^N x_n(\tilde{s}, \tilde{s}_0) + \tilde{z} \right| \right] = \left[\frac{2}{\pi} \right]^{\frac{1}{2}} \sqrt{\text{Var} [N(\alpha^* + \beta^* \tilde{s} + \kappa^* \tilde{s}_0) + \sigma_z^2]} = \left[\frac{2}{\pi} \right]^{\frac{1}{2}} \sigma_z (N^{1/2} + 1) .$$

The expected volume of trade done by the market maker is independent of the level of public information. Here, more public information does not reduce the occurrence of "unmatched" demands because all insiders receive the same private information. This implies that there is no trading crossed among insiders and all their demands have to be met by the market maker.

If we allow for liquidity discretionary traders in this common private information setup, then an increase in τ_0 that attracts those discretionary traders will always mean higher expected volume of trade because there will be more noise trading and also more insider trading.

Finally I can compute the variance of the price as a measure of volatility. It can be proved that

$$\text{Var}(\tilde{p}) = \text{Var}(\delta^* + \lambda^* \tilde{w} + \gamma^* \tilde{s}_0) = \frac{1}{\tau_v + \tau_0} \left[\frac{N}{(N+1) \left[1 + \frac{\tau_v + \tau_0}{\tau_c} \right]} + \frac{\tau_0}{\tau_v} \right]$$

that is an increasing function of τ_0 as for the case with diverse information. Again, the price selected by the market maker tends to adjust the true realization of \tilde{v} when there is more public information available.

8. A MONOPOLISTIC MARKET FOR INFORMATION.

In this section I analyze the performance of a monopolistic market for information in the spirit of Admati and Pfleiderer (1986, 1988b). There is a risk neutral monopolist in this market who is able to produce information and has two options: a) To use the information by himself and trade actively in the market for the risky asset, or b) To sell the information to others.

If the monopolist chooses option b, then he has to select the number of agents to whom is going to sell the information, the precision of this information and, finally, its price. In order to simplify the analysis I assume that information garbling is not allowed. I am also going to assume that the information owned by the informed agent is verifiable by the potential buyer. Thus, I abstract from the reliability problems analyzed for instance in Allen (1987a).

I proceed to describe the technology that produces information. The monopolist may produce signals of the asset return \tilde{v} at unitary cost $c' > 0$. Each signal s_j takes the form $\tilde{s}_j = \tilde{v} + \tilde{\epsilon}_j$ where $\tilde{\epsilon}_j$ are i.i.d. normal with zero mean and variance $\frac{1}{\tau}$ for all j . Therefore, if the monopolist produces J signals, then the unbiased and most efficient estimate about \tilde{v} is

$$\tilde{s} = \frac{\sum_{j=1}^J \tilde{s}_j}{J} = \tilde{v} + \frac{\sum_{j=1}^J \tilde{\epsilon}_j}{J}.$$

Define $\tilde{c} = \frac{\sum_{j=1}^J \tilde{\epsilon}_j}{J}$, and it follows that the precision $\tau_{\tilde{c}}$ of \tilde{c} is equal to $J\tau$. Finally, it is clear that the cost of producing an estimate with precision $\tau_v + \tau_{\tilde{c}}$ is $c\tau_{\tilde{c}}$ where $c = \frac{c'}{\tau}$.

Therefore, I have shown that the problem of selecting a level of precision for the estimate of \tilde{v} is equivalent to the one of selecting the number J of observations.

As I have also said, if the monopolist chooses to sell his information he has to choose the number of buyers to whom he will sell it, the level of precision he will produce and the price of that information.

The latter variable can be determined in a straightforward way. Given our monopolistic setup, we can assume that the monopolist extracts all the surplus from the buyers. This means that the price $p^*(N, \tau_{\tilde{c}})$ of a signal with noise precision $\tau_{\tilde{c}}$ sold to N buyers is equal to the certainty equivalent of the profits per capita when there are N informed agents who trade using the same signal with noise precisions

equal to τ_e . From risk neutrality and from (7.1), we can conclude that

$$p^*(N, \tau_e) = \frac{1}{(N+1)(\tau_v + \tau_0)} \cdot \left[\frac{\sigma_z^2}{N \left(\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_e} \right)} \right]^{\frac{1}{2}} \quad (8.1)$$

Therefore, the maximization problem faced by the monopolist

is

$$\text{Max}_{\tau_e, N} \pi(N, \tau_e) = p^*(N, \tau_e) \cdot N - c\tau_e \quad \text{subject to } N \leq \bar{N}$$

where \bar{N} is the number of potential buyers. I assume that $\bar{N} \geq 1$.

LEMMA 8.1: The optimal values (N^* , τ_e^* , p^*) for the monopolist's maximization problem are

$$N^* = 1,$$

$$c = \frac{1}{4} \cdot \frac{\sigma_z}{\tau_v + \tau_0} \cdot \frac{1}{\left[\frac{(\tau_e^*)^{2/3}}{\tau_v + \tau_0} + (\tau_e^*)^{1/3} \right]^{3/2}} \quad (8.2)$$

$$p^* = \frac{1}{2(\tau_v + \tau_0)} \left[\frac{\sigma_z^2}{\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_e^*}} \right]^{\frac{1}{2}} \quad (8.3)$$

Proof: After differentiating $\pi(N, \tau_e)$ with respect to N and simplifying, we obtain

$$\frac{\partial \pi(N, \tau_c)}{\partial N} = \frac{1}{2N^{1/2}} \cdot \left[\frac{1-N}{(N+1)^2} \right] \cdot \left[\frac{\sigma_z}{\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_c}} \right]^{\frac{1}{2}} \cdot \frac{1}{\tau_v + \tau_0}$$

Then, $\pi(N, \tau_c)$ is strictly increasing (decreasing) iff $N < 1$ ($N > 1$). Therefore, for any value of τ_c , $N^* = 1$ is the value of N that maximizes $\pi(N, \tau_c)$.

Differentiate $\pi(N, \tau_c)$ with respect to τ_c and make $N = 1$, to obtain (8.2).

To see that the optimal value τ_c^* defined implicitly in (8.2) is unique and belongs to the open interval $(0, \infty)$, note that the right hand side of (8.2) is a continuous and strictly decreasing function of τ_c that I denote $F(\tau_c)$. It can be checked that $\lim_{\tau_c \rightarrow \infty} F(\tau_c) = 0$ and

$\lim_{\tau_c \rightarrow 0} F(\tau_c) = \infty$. Then, continuity of $F(\cdot)$ proves the existence of

$\tau_c^* \in (0, \infty)$. Uniqueness follows from strict monotonicity of $F(\cdot)$.

I obtain the optimal value of p^* replacing in (8.1) N and τ_c by their optimal values. ■

Lemma 8.1 tells us that if the monopolist chooses to sell information, then he wants to sell it to a single buyer. This buyer will be able to extract the maximum surplus from the financial market. This single trader will have to worry only about competing with the market maker and not with other informed traders. This result resembles the one in the theory of oligopolistic competition that says that the sum of profits obtained by oligopolistic firms operating in a market is

lower than the profits obtained by a monopolistic firm operating in the same market. Obviously, the risk neutral monopolist is indifferent between selling his information to a single agent extracting all the surplus or using the information by himself.

An obvious comparative statics result I get from (8.2), after implicitly differentiating, is that the equilibrium private precision τ_c^* is decreasing in the unitary cost c of producing it.

Now, I am in a position to analyze the effects of public information on the incentives to produce private information. Basically, I want to study how τ_c^* responds to changes in τ_0 . The answer is given in the next proposition.

PROPOSITION 8.2: There exists a $c^* > 0$ such that the optimal value of τ_c^* in the monopolist problem is increasing in τ_0 if and only if the unitary cost of producing private information is less than c^* .

Proof: Applying the Implicit Function Theorem to (8.2), it can shown that

$$\frac{\partial \tau_c^*(\tau_0)}{\partial \tau_0} = -\frac{2a^{1/3} - a^{4/3}}{4a^{1/3} + a^{2/3}} \quad \text{where } a = \frac{\tau_c^*}{\tau_v + \tau_0}. \quad (8.4)$$

It follows that $\frac{\partial \tau_c^*}{\partial \tau_0} > 0$ if and only if $a > 2$. Notice that for given τ_v and τ_0 , the optimal private precision $\tau_c^*(c)$ is strictly decreasing in c and tends to zero (infinite) when c tends to infinite (zero). Therefore, define c^* implicitly as $\tau_c^*(c^*) = 2(\tau_v + \tau_0)$ and the result follows. ■

This proposition tells us that when the production of information is very costly, the direct negative effect of increasing public information, as a consequence of Corollary 3.1, is never overcome by means of producing more private information. However, when c is low enough, the insider wants to produce still more private information in order to maintain his relative advantage with respect to the market maker.

Figure 1 illustrates the previous discussion. In this figure the information costs c_i are ranked as follows: $c_1 < c_2 < c_3$. When c_1 is lower, the optimal demand for information τ_c^* is more likely to be increasing in τ_0 .

(INSERT FIGURE 1 HERE)

This result contrasts with the results in Verrecchia (1982) and Gonedes (1980) for perfectly competitive economies with risk averse agents. These authors claimed that additional public disclosures motivates the agents to cut back the production of information. However, when the strategic relationship between the informed agent and the market maker is taken into account, the effects on the production of private information depend on the cost of produce it.

Another question posed by Gonedes (1980) is: How does public disclosure affect the total level of informedness of the trader?. The overall level $\hat{\tau}$ of a trader's informedness can be defined as the sum of precisions of prior, public and private information, i.e.,

$$\hat{\tau} = \tau_v + \tau_0 + \tau_c.$$

The following corollary gives the comparative statics result:

COROLLARY 8.3: The overall level of the trader's informedness is increasing in τ_0 .

Proof: Compute the derivative of $\hat{\tau}$ with respect to τ_0 .

$$\frac{\partial \hat{\tau}}{\partial \tau_0} = 1 + \frac{\partial \tau_c^*(\tau_0)}{\partial \tau_0} = 1 - \frac{2a^{1/3} - a^{4/3}}{4a^{1/3} + a^{2/3}} = \frac{a^{4/3} + a^{2/3} + 2a^{1/3}}{4a^{1/3} + a^{2/3}} > 0 \quad \blacksquare$$

Public information increases the trader's informedness despite the fact that it may reduce the amount of private information produced. This result is similar to the one in Verrecchia (1982).

Finally, let us observe that more public information decreases always the informed agent's welfare even if this agent can react changing the level of private information he produces. This result follows from an application of the Envelope Theorem to the optimal profits function when the level of public information is τ_0 .

$$\pi^*(\tau_0, \tau_c^*(\tau_0)) = \frac{1}{2(\tau_v + \tau_0)} \cdot \left[\frac{\sigma_z^2}{\frac{1}{\tau_v + \tau_0} + \frac{1}{\tau_c^*(\tau_0)}} \right]^{\frac{1}{2}} - c\tau_c^*(\tau_0) ,$$

and obviously

$$\frac{\partial \pi^*(\tau_0, \tau_c^*(\tau_0))}{\partial \tau_0} < 0 .$$

9. ASSOCIATIONS OF INVESTORS.

The results in Section 8 give us an immediate Corollary about the desirability of associations or syndicates of investors from the point of view of the informed traders¹⁴.

I consider two types of associations: 1) Associations in which its members precommit before receiving their signal to share their private information. Afterwards, agents will compete in the financial market using a more precise common information, and 2) Associations in which informed traders not only share their information but the association submits a collective demand to the market maker based upon all the information collected by its members. Profits will be distributed equally among members of the association.

Let us assume that each agent owns a private signal s_n ($n = 1, \dots, N$) about \tilde{v} with the statistical properties of Section 2. The total private informedness in the economy is $N\tau_\varepsilon$ (the sum of precisions of private signals). Denote $\pi^j(N, \tau_\varepsilon)$, $j = n, s$, as the expected profits of informed agents when there are N informed agents, each of them receiving a signal with precision τ_ε . The superindex j can take the values n or s depending on whether the private information is diverse (Section 2) or common (Section 7) respectively. $\pi^n(N, \tau_\varepsilon)$ is given in (3.1) and $\pi^s(N, \tau_\varepsilon)$ is given in (7.1).

The desirability of associations of type 1 depends on the

¹⁴ The implementation of associations of investors as the equilibrium of an information transmission game (as in Gal-Or (1985)) takes us beyond the scope of this paper.

relation between the magnitudes of $\pi^n = \pi^n(N, \tau_g)$ and the expected profits $\pi^s = \pi^s(N, N\tau_g)$ when the N informed agents share a more precise common information.

On the other hand, to analyze the desirability of associations of investors in which the demand is submitted collectively (type 2), we have to compute the profits "per capita" $\pi^A = \frac{1}{N} [\pi(1, N\tau_g)]$ obtained through the association. When $N = 1$ the superindex is redundant.

From Lemma 8.1, we know that $\pi^s < \pi^A$. In fact, Lemma 8.1 says that

$$N\pi^s = N\pi^s(N, N\tau_g) < \pi(1, N\tau_g) = N\pi^A \quad \text{for } N > 1,$$

and thus $\pi^s < \pi^A$. This confirms our intuition that collusive behavior delivers higher profits per capita than competition. The following corollary compare this two magnitudes with π^n .

COROLLARY 9.1: $\pi^s < \pi^n < \pi^A$

Proof: 1) $\pi^n < \pi^A$. Divide π^n by π^A and obtain

$$R(a) = \frac{\pi^n}{\pi^A} = \frac{\left[1 + (N+1)a + Na^2\right]^{\frac{1}{2}}}{1 + \frac{N+1}{2}a},$$

where a is defined in (8.4). It can be proved that the derivative of $R(\cdot)$ with respect to a is strictly negative whenever $N > 1$. It can be proved that $\lim_{a \rightarrow 0} R(a) = 1$ and $\lim_{a \rightarrow \infty} R(a) = \frac{2N^{1/2}}{N+1} < 1$, for $N > 1$.

Therefore, $\pi^n < \pi^A$ for all values of $a < \infty$ and $N > 1$.

2) $\pi^s < \pi^n$. Similarly, compute $R^*(a) = \frac{\pi^s}{\pi^n}$ and obtain

$$R^*(a) = \frac{N^{1/2}}{N+1} \cdot \left[\frac{a + (N+1)^2 a^2 + 4(N+1)a}{1 + Na^2 + (N+1)a} \right]^{1/2},$$

where $R^*(a)$ has a strictly positive derivative with respect to a ,

$\lim_{a \rightarrow \infty} R^*(a) = 1$, and $\lim_{a \rightarrow 0} R^*(a) = \frac{2N^{1/2}}{N+1} < 1$ for $N > 1$. Therefore, $\pi^s < \pi^n$

for all values of $a < \infty$ and $N > 1$. ■

To interpret Corollary 9.1, note that the configuration associated with π^A is a monopolistic one in which the association is not facing competition and has also more information than the private agents separately. Therefore $\pi^A > \pi^n$ is the logical result.

A little bit more surprising is that $\pi^n > \pi^s$, i.e., that information pooling and competing delivers lower expected profits than competing without information sharing. The reason is that when all agents make trades based on the same information, the order flow is more informationally "pure" in the sense of having less noise due to the existence of diverse signals (possibly in opposite directions). The existence of this diverse information makes difficult for the market maker to predict $\frac{\sum_{n=1}^N s_n}{N}$, the sufficient estimate of all private information available in the economy.

To see that this is the case, consider the informational content τ_u of the order flow, as defined in the expression (A.9) of the Appendix. We have to compare τ_u^s with τ_u^n , where the superindexes have

the same meaning as before¹⁵. It is easy to see that

$$\tau_u^s = \frac{1}{\tau_v + \tau_0} + \frac{2}{\tau_\varepsilon} \quad \text{and} \quad \tau_u^n = \frac{1}{\tau_v + \tau_0} + \frac{N+1}{N\tau_\varepsilon},$$

and it is clear that $\tau_u^s > \tau_u^n$ iff $N > 1$. Even if each agent owns better information, informed agents as a whole lose ground with respect to the market maker.

Finally, let us point out that the optimality properties of syndicates of investors from the point of view of informed agents are qualitatively independent of the precision of public information; in fact they are independent of a .

10. A RATIONAL EXPECTATIONS MODEL WITH IMPERFECT COMPETITION.

10. A. The Model.

I introduce a different mechanism of price formation in this section. This mechanism resembles the one used in the traditional models of noisy rational expectations. The equilibrium prices will be formed by automatic market clearing and the quantities demanded by

¹⁵ τ_u^s is the precision of the random variable \tilde{u}^* such that

$$\frac{w - N\alpha^* - N\kappa^* s_0}{N\beta^*} = \tilde{v} + \tilde{\varepsilon} + \frac{\tilde{z}}{N\beta^*} = \tilde{v} + \tilde{u}^*. \quad \text{Here, } \tilde{\varepsilon} = \frac{\sum_{n=1}^N \varepsilon_n}{N} \text{ and } \alpha^*, \beta^*, \kappa^*$$

are defined in Proposition 7.1.

informed agents will be conditional on prices (limit orders). Section 11 will perform the comparative statics of this model in order to see how robust are the results obtained in previous sections under a different mechanism of price formation.

When agents are allowed to submit limit orders, the information sharing among informed agents is increased notably. Since insiders are now able to condition their demands on prices, they are able to infer part of others' information from the prices at which the transactions are carried out.

The structure of the model I propose is based on Kyle (1986) and Jackson (1988) whose work is in turn based on Grossman (1981b) and Wilson (1979) respectively.

Each informed agent receives again a piece of private information s_n and a public signal s_0 . I assume that both private and public signals are received at no cost and I will maintain this assumption throughout my remaining analysis¹⁶. The statistical properties of these signals and all other random variables are exactly the same as in the market orders model.

After observing these signals, informed agents select a continuous demand function¹⁷, i.e., a function that specifies for each price the number of shares that they are willing to buy.

¹⁶ The model of this section can be viewed as the imperfectly competitive version of the model in Grossman (1976).

¹⁷ The space of actions can be enlarged to allow for upper hemicontinuous, convex-valued and real-valued correspondences. See Kyle (1986) for a specification of a pricing rule defined in this actions space.

Let us denote X_n as the strategy of agent n . The strategy X_n is a map from \mathbb{R}^2 (the cartesian product of private and public signals) into the set of functions from \mathbb{R} (prices) to \mathbb{R} (quantities). Therefore, $X_n(\cdot; \tilde{s}_n, \tilde{s}_0)$ is a random variable that takes values in the set of continuous functions. $X_n(\cdot; s_n, s_0)$ is a particular demand function corresponding to the particular combination (s_n, s_0) of private and public signals. $x_n = x_n(p, s_n, s_0) = X_n(p; s_n, s_0)$ denotes the quantity of asset bought by an agent, given a particular realization of \tilde{s}_n, \tilde{s}_0 and the price \tilde{p} (I will show below why prices are random variables). In other words, x_n is the realization of the random variable $\tilde{x}_n = x_n(\tilde{p}, \tilde{s}_n, \tilde{s}_0) = X_n(\tilde{p}; \tilde{s}_n, \tilde{s}_0)$.

There is also a source of noise \tilde{z} that is independent of \tilde{s}_0, \tilde{s}_n ($n = 1, \dots, N$) and \tilde{p} . This random variable \tilde{z} can be justified in the same way as in the previous model.

I proceed to define the pricing rule. Prices are formed according to an automatic "market clearing" rule. I assume that there is a computerized system that receives all the limit orders plus noise.

The observed aggregate net demand is defined by

$$D(p) = \sum_{n=1}^N x_n(p, s_n, s_0) + z.$$

Define the set of equilibrium prices

$$E = \left\{ p \in \mathbb{R} \mid D(p) = 0 \right\}.$$

If $E = \emptyset$, the computer-auctioneer shuts down the market. This means that trade is not allowed. In this case, $x_n = 0$ ($n = 1, \dots, N$) and liquidity traders cannot sell or buy any share. Strategic agents will

make zero expected profits in this case¹⁸.

If $E \neq \emptyset$, the equilibrium price p^* is selected according to

$$p^* = \operatorname{argmin}_{p \in E} |p| .$$

The previous equation defines the random variable \tilde{p}^* implicitly. However, given the restrictions I will impose in the next section, I will be able to give an explicit formula for that random variable. Note that \tilde{p}^* is $(\tilde{z}, \tilde{s}_0, \tilde{s}_1, \dots, \tilde{s}_N)$ -measurable but is not $(\tilde{s}_0, \tilde{s}_1, \dots, \tilde{s}_N)$ -measurable, i.e. \tilde{p}^* is not a sufficient estimate for all information available in the economy.

It is also important to note that p^* is a function of all the strategies used by informed agents. Therefore, I can write

$$\tilde{p}^* = p^*(X_1, \dots, X_N; \tilde{z}, \tilde{s}_0, \tilde{s}_1, \dots, \tilde{s}_N) .$$

The expected profits for the informed agent N , after observing s_n and s_0 are

$$E\left[\tilde{\pi}^N(X_1, \dots, X_N) | s_n, s_0\right] = E\left\{(\tilde{v} - \tilde{p}^*) \cdot X_N(\tilde{p}^*, \tilde{s}_n, \tilde{s}_0) | s_n, s_0\right\} .$$

The Bayesian-Nash Equilibrium of this game is defined as the set of strategies X_1^*, \dots, X_N^* such that,

¹⁸ This is a situation that will never occur in equilibrium. However, this contingency has to be characterized in order to have a well defined extensive form of the game.

$$E\left[\hat{\pi}^n(X_1^*, \dots, X_n^*, \dots, X_N^*) | s_n, s_0\right] \geq E\left[\hat{\pi}^n(X_1^*, \dots, X_n, \dots, X_N^*) | s_n, s_0\right],$$

$n = 1, \dots, N$, for all X_n , and for all s_n and s_0 .

This equilibrium is called a rational expectation equilibrium with imperfect competition (REEIC).

It is worth to highlight the difference with the traditional rational expectations equilibrium with perfect competition (REEPC) in which prices are also selected according to a market clearing rule. In the REEPC, each agent is "price taker": he considers that he cannot affect the equilibrium price. This leads in our economy with a finite number agents to "schizophrenic behavior" according to Hellwig (1980). Each agent recognizes how the price is formed and that his behavior has a nonnegligible effect on prices but he does not try to manipulate the mechanism of price formation. There is a solution to this conceptual difficulty in Hellwig (1980), for the single security case, and in Admati (1985), for the case of several securities. Both authors consider a large market in which each individual agent does not affect the equilibrium price because he has measure zero with respect to the total mass of agents in the economy.

The concept of rational expectation equilibrium with imperfect competition gives a different solution to the "schizophrenia" problem. We still have a finite number of agents who realize that they have an effect on prices and, therefore, they act strategically. Applying the Bayesian-Nash equilibrium concept to a game with demand schedules as actions, we get an equilibrium concept that collects all the strategic interactions between agents and the process of price formation.

Note also that this model of competition based on a game of demand schedules submission gives a little bit more of sense to the rational expectations equilibrium as described for instance in Grossman (1981a). There was in that description a problem of circularity because an agents' demand depends on the equilibrium price, and this price depends simultaneously on that demand. This circularity disappears in the demand schedules submission game in which the sequence of events is perfectly structured.

10. B. Equilibrium.

For reasons of tractability, I am going to restrict the space of demand schedules to the space of linear demand functions. A strategy X_n for agent n is a mapping from the space \mathbb{R}^2 of signals to the space L of linear demand functions

$$X_n: \mathbb{R}^2 \longrightarrow L$$

$$(s_n, s_0) \longmapsto A_n(s_n, s_0) + C_n(s_n, s_0) \cdot p$$

Furthermore, I assume that

$$A_n(s_n, s_0) = \hat{\alpha}_n + \hat{\beta}_n s_n + \hat{\kappa}_n s_0$$

$$\text{and } C_n(s_n, s_0) = \hat{\mu}_n, \quad \text{for all } (s_n, s_0) \in \mathbb{R}^2$$

I will restrict attention to symmetric equilibria, i.e.,

$$\hat{\alpha}_n = \hat{\alpha}, \quad \hat{\beta}_n = \hat{\beta}, \quad \hat{\kappa}_n = \hat{\kappa} \text{ and } \hat{\mu}_n = \hat{\mu} \text{ for all } n.$$

PROPOSITION 10.1: When $N > 2$, there exists a unique symmetric and linear rational expectations equilibrium with imperfect competition.

This equilibrium is given by

$$x_n = \hat{\alpha} + \hat{\beta} s_n + \hat{\kappa} s_0 - \hat{\mu} p,$$

where

$$\hat{\alpha} = \left[\frac{N(N-2)\sigma_z^2}{(N-1)\tau_e} \right]^{\frac{1}{2}} \cdot \frac{2\tau_v \bar{v}}{N^2},$$

$$\hat{\beta} = \left[\frac{(N-2)\sigma_z^2 \tau_e}{N(N-1)} \right]^{\frac{1}{2}},$$

$$\hat{\kappa} = \left[\frac{N(N-2)\sigma_z^2}{(N-1)\tau_e} \right]^{\frac{1}{2}} \cdot \frac{2\tau_0}{N^2},$$

$$\hat{\mu} = \left[\frac{N(N-2)\sigma_z^2}{(N-1)\tau_e} \right]^{\frac{1}{2}} \cdot \frac{2(\tau_v + \tau_0) + N\tau_e}{N^2}.$$

Proof: See the Appendix. ■

The requirement of $N > 2$ is similar to the one in the theory of Cournot competition. There, the equilibrium fails to exist if the demand is infinitely elastic. Here, liquidity traders have an

infinitely elastic demand and having more than two informed agents suffices to have bounded demands for the insiders.

From the equilibrium given in Proposition 10.1, and since the equilibrium price is

$$\bar{p} = \frac{N\hat{\alpha} + \hat{\beta} \sum_{n=1}^N \tilde{s}_n + M\hat{\kappa}\tilde{s}_0 + \tilde{z}}{M\hat{\mu}}, \quad (10.1)$$

it is easy to check that $E(\bar{p}) = \bar{v}$. It can also be proved that

$$E\left[x_n(\bar{p}, \tilde{s}_n, \tilde{s}_0)\right] = 0.$$

Recall that the depth of the market is the inverse of the induced change on prices when the order flow increases one unit. From (10.1), it is obvious that the depth of the market is equal to $M\hat{\mu}$. Looking at the equilibrium value of $\hat{\mu}$ in Proposition 10.1, it is also obvious that the depth of the market is increasing in τ_0 .

In this REEIC model there is no market maker who makes inferences from order flows and, therefore, the argument to explain why the price is less sensitive to quantities should be modified accordingly.

When more precise public information is available, the "expected" individual demand schedules become more elastic. Note that the expected intercept of the equilibrium demand schedules with the "quantities axis" is $I(\tau_0) = E(\hat{\alpha} + \hat{\beta}\tilde{s}_n + \hat{\kappa}\tilde{s}_0) = \hat{\alpha} + (\hat{\beta} + \hat{\kappa})\bar{v}$, that is clearly increasing in τ_0 (for $\bar{v} > 0$). This is because agents put more weight on public signals. For $\bar{v} < 0$ the argument is symmetric. Since the expected equilibrium price is equal to \bar{v} , the expected individual

equilibrium demand schedule has to shift as in figure 2 when τ_0 increases.

(INSERT FIGURE 2 HERE)

Individual demand functions will become more elastic, and this means that both $\hat{\mu}$ and $N\hat{\mu}$ (the inverse of the slope of aggregate demand in figure 2) increase. Higher values of $N\hat{\mu}$ imply in turn that if there is a shock in the "quantities axis" in figure 2 due to liquidity trading, then the "reflected shock" on the "prices axis" is more attenuated.

11. COMPARATIVE STATICS OF THE RATIONAL EXPECTATIONS WITH IMPERFECT COMPETITION MODEL.

In this section I apply the analysis of sections 3, 4 and 6 to the model developed in the previous section.

The discussion at the end of Section 10 implies our first Corollary.

COROLLARY 11.1: The expected profits of informed agents are decreasing in τ_0 and the opposite is true for the expected profits of liquidity traders.

Proof: For the strategic traders compute

$$E(\hat{\pi}^n) = E\left[(\tilde{v} - \tilde{p}) \cdot x_n(\tilde{p}_n, \tilde{s}_n, \tilde{s}_0)\right].$$

Use (10.1) and the equilibrium values of $\hat{\alpha}$, $\hat{\beta}$, $\hat{\kappa}$, and $\hat{\mu}$ in Proposition 10.1, to obtain

$$E(\hat{\pi}^n) = \left(\frac{\sigma_z^2(N-1)}{N(N-2)\tau_c} \right)^{\frac{1}{2}} \cdot \frac{1}{2 \left[\frac{\tau_v + \tau_0}{\tau_c} \right] + N} \quad (11.1)$$

which is clearly decreasing in τ_0 .

For liquidity traders, compute

$$E(\hat{\pi}^t) = E \left[(\tilde{v} - \tilde{p}) \cdot \tilde{z}_t \right] = E \left[-\frac{1}{N\hat{\mu}} z_t^2 \right]$$

that is equal to

$$E(\hat{\pi}^t) = - \left(\frac{N(N-1)}{\sigma_z^2(N-2)\tau_c} \right)^{\frac{1}{2}} \cdot \frac{1}{2 \left[\frac{\tau_v + \tau_0}{\tau_c} \right] + N} \cdot \sigma^2 \quad (11.2)$$

which is increasing in τ_0 . ■

Once I have proved that more public information implies a deeper market, the effects on expected profits are justified as in Section 3: there is a transfer of expected profits from insiders to liquidity traders.

Other implications of the model with market orders and price selected by market makers can be recovered in this model of limit orders and price selected by automatic market clearing. For instance, I can compute the variance of prices in the

REEIC model and obtain

$$\text{Var}(\tilde{p}) = \frac{(2\tau_0 + N\tau_\epsilon)^2 + \frac{N(2N-3)\tau_\epsilon \tau_v}{N-2} + 4\tau_0 \tau_v}{\tau_v [2(\tau_v + \tau_0) + N\tau_\epsilon]^2},$$

and it can be proved that $\frac{\partial \text{Var}(\tilde{p})}{\partial \tau_0} > 0$ as in the case of market orders.

When the precision of public information increases, insiders are more sensitive to the realization of the public signal. This means that there is more volatility in the intercept of the demand function with the "quantities axis" (see figure 2). This increased volatility in this axis overcomes the fact that the demand function has less slope.

The discussion and the results in Section 4 about desirability of public disclosure when there are risk-averse original stockholders that want to cancel their position in the firm, applies also in the present model.

Finally, I obtain again the following ambiguous result about the value of private information for the informed agents, whose proof is immediate.

COROLLARY 11.2: Expected profits of informed agents are decreasing in τ_ϵ iff $\frac{N}{2} \tau_\epsilon \geq \tau_v + \tau_0$. ■

Private information exhibits the same decreasing returns that we have seen in section 6. After some level of overall private information, additional private information makes insiders worse off. The intuition behind this result is that when private information is

very precise, agents put still more weight on their own private signals and then, the expected intercept on the quantities axis of the equilibrium individual demand schedules becomes increasing in τ_c . This means a flatter demand schedule ($\hat{\mu}$ increases), which implies a deeper market. As I have argued, more depth results in a transfer of expected profits from insiders to liquidity traders. On the other hand, when the precision of private signals is very low, an increase on that precision results in more willingness to share information among insiders. The way of increasing information sharing among informed agents consists of steeper demands. This implies a greater sensitivity of the equilibrium price to private information.

The amount of private information revealed exclusively by prices, parameterized by

$$R = \left[\text{Var}(\tilde{v} | \tilde{p}, \tilde{s}_0) \right]^{-1} - \left[\text{Var}(\tilde{v} | \tilde{s}_0) \right]^{-1}$$

is now

$$R = \frac{N(N-2)\tau_c}{2N-3}.$$

Note that even if the number of agents goes to infinite keeping constant the aggregate precision $M = N\tau_c$ of private information and dividing this total information equally among insiders, we get

$$\lim_{N \rightarrow \infty} R = \frac{M}{2},$$

$$N\tau_c = M$$

that is the same result that I obtained in section 6.

As Kyle (1986) argues, this limit REEIC model looks like a

monopolistic competition model. Even if agents know very little about \tilde{v} (τ_c becomes small as N increases) and the size of each agent becomes negligible, traders always face a residual supply curve with non-zero slope $\left[-\frac{1}{(N-1)\tilde{\mu}} < 0\right]$. This induces them to restrict their trade and to use their private information in such a way that no more than half of total private information is revealed by prices.

A difference with the market orders model appears when we fix the number of insiders and compute the limit of R when τ_c goes to infinite. This limit is equal to infinite. Since private information is perfect, if the insiders can submit demand schedules, then they are able in fact to observe the information received by the other agents ($s_n = \tilde{v}$, $n = 1, \dots, N$). Moreover, they observe the realization of \tilde{z} through the equilibrium price. This means that all uncertainty vanishes from the point of view of insiders and then, given risk neutrality, the aggregate demand schedule becomes flat. Depending on whether $\tilde{v} > (<) \tilde{p}$, the demand for assets is $+\infty$ ($-\infty$). This means that the only equilibrium price that clears the market is $\tilde{p} = \tilde{v}$ and prices become fully revealing.

From the previous paragraph it is obvious that, in a model with common private information, prices fully reveal all private information. In this case, the unique equilibrium price is such that $\tilde{p} = \tilde{s} = \tilde{s}_n$ for $n = 1, \dots, N$.

12. A COMPARISON RESULT

I have studied two mechanisms of price formation. These two mechanisms are the most popular in the finance literature and it seems natural to ask which mechanism is more efficient. The answer to this question is always ambiguous, because, as I have explained throughout the paper, what is good for liquidity traders is bad for insiders.

The result of the comparison follows from the discussion about the value of private information. Too much private information is harmful for informed agents in both models of price formation. As I have said, the limit orders model involves more information sharing than the market orders model because the equilibrium price reveals others' private information. Therefore, when private information is very precise, the participants in a regime with limit orders and automatic market clearing would prefer to switch to a regime with market orders in which there is no information sharing and all agents become less informed. The converse argument applies when τ_{ε} is low.

The following Proposition confirms the previous intuitive argument.

PROPOSITION 12.1: For $N > 2$, there exists a $\tau_{\varepsilon}^* \in (0, \infty)$ such that

$$E(\pi^n) > E(\hat{\pi}^n) \text{ iff } \tau_{\varepsilon} > \tau_{\varepsilon}^* .$$

Proof: Using the expressions for insiders' expected profits in both regimes given in (3.1) and (11.1), compute their ratio and simplify to obtain

$$Q(a) = \frac{E(\hat{\pi}^n)}{E(\pi^n)} = \left[\frac{(N-1)}{(N-2)(1+a)} \right]^{\frac{1}{2}} \left[1 + \frac{a}{2+Na} \right],$$

where, again, $a = \frac{\tau_e}{\tau_v + \tau_0}$. It can be proved that $\frac{\partial Q(a)}{\partial a} < 0$ when $N > 2$.

Also $\lim_{a \rightarrow 0} Q(a) = \left[\frac{N-1}{N-2} \right]^{\frac{1}{2}} > 1$ and $\lim_{a \rightarrow \infty} Q(a) = 0$. Therefore, by continuity

of $Q(\cdot)$, there exists an $a^* \in (0, \infty)$ such that $Q(a^*) = 1$. Then,

$\tau_e^* = a^*(\tau_v + \tau_0)$ is the desired threshold that equates both expected profits. ■

Note that, when public information is very precise, a tends to zero and the mechanism with limit orders and market clearing is more efficient. However, when public information becomes very imprecise a tends to τ_e/τ_v and the relative efficiency of each mechanism will depend on whether τ_e is greater or smaller than $a^* \cdot \tau_v$.

Again, the results are reversed for liquidity traders' expected profits. Thus, our results indicate that if legislators only care about liquidity traders, i.e., uninformed agents that participate in financial markets in order to make intertemporal transfers of wealth exclusively, then it is optimal to force as much public disclosure of information as possible. Once the level of public information τ_0^* is determined the optimal mechanism of trading will depend on the value of $a = \tau_e/(\tau_v + \tau_0^*)$. When \tilde{z} is not associated with liquidity traders the results indicate that public information should be forbidden ($\tau_0 = 0$) and the optimal mechanism should be selected depending on the value of $a = \tau_e/\tau_v$.

13. CONCLUSION

We have studied the effects of public information in two contexts and we have proved that more public information dissipates the informational advantage of informed agents and improves the position of liquidity traders.

Legislation on public disclosure tends to protect liquidity traders. As I have shown, more public information increases the expected profits earned by liquidity traders (in fact, what public information does is to reduce the cost of trading for these agents). This result is obtained in both regimes of price formation: a regime with competitive market makers and a regime with automatic market clearing.

If we assume that the noise \tilde{z} is not associated with liquidity trading, but with random supply of risky asset or with some noise in the communication process, then the disclosure of public information decreases the welfare of all active participants in the market.

These results are modified in the market orders model either when private information is costly or when disclosure requirements are not uniform across firms and we allow for discretionary liquidity traders who choose in which market they will trade. In the later case, both informed and liquidity traders may benefit from disclosure of public information in their own market at the expense of traders in the market in which the discretionary liquidity trading was previously

concentrated.

I have shown that in general more public information means greater price volatility (in both regimes) and lower expected volume of trade done by the market maker provided that private information is diverse. On the other hand, the relative efficiency of associations of investors is independent of the precision of public information. With respect to the incentives to produce private information, I have shown that the results are ambiguous depending on the unitary cost of producing such private information. Finally, the relative efficiency of the two mechanisms of price formation depends on the precision of both public and private information.

Our model has obvious limitations and, therefore, it has room for extensions. The most important limitation comes from the assumption of risk neutrality. This assumption allows us tractability but prevent us from studying the effects of public information on the risky position of each agent and on the risk sharing among participants in the market. It can be easily proved that, with CARA utility functions, the linear equilibrium involves the solution to a fifth-order polynomial. This implies that, if we wanted to model risk aversion, we should confine our analysis to numerical examples. However, the equilibria obtained in the two models of price formation analyzed in this paper are the ones corresponding to limit equilibria of economies with risk averse agents whose coefficient of risk aversion tends to zero. Continuity is not lost in the limit, given our strategic behavior assumption. This is in stark contrast with the traditional models of rational expectations with perfect competition. this means that our results are still applicable

to markets with mutual funds or insurance companies that hold a very diversified portfolio and that have a very large risk bearing capacity, given the large number of individuals that represent.

Given the difficulty of modeling explicitly risk aversion in this imperfectly competitive setup (at least with our distributional assumptions), I think that there are two more promising topics that should be explored in order to put the models of imperfect competition in financial markets at the same level as the ones of perfect competition. The first one refers to the extension of the model to a multi-security world as Admati (1985) did in a perfectly competitive setup. This extension would allow us to characterize the behavior of insiders in several markets who manipulate one market in order to send misleading signals to other markets.

The second line of research involves a more general analysis of the information acquisition problem. I have studied in this paper the incentives that a monopolistic insider has to produce information. The obvious extension should be to allow for several endogenously informed agents. In that model we should specify a two-stage game. In the first stage, the insiders would select the amount of information they will produce and, in the second stage, they will compete using that information. Verrecchia (1982a, 1982b) has studied this problem for the competitive case. Matthews (1984) has some results for auctions with prices as strategic variables and finite number of agents.

Finally, I should point out that our paper has several empirical implications on volume of trade, price volatility and informational content of prices. Therefore, future research should also

involve some empirical tests applied to thin financial markets in order to assess the power of the models developed in this paper.

APPENDIX

Proof of Proposition 2.2: Suppose that each informed agent makes the conjecture that the others' demand will take the form

$$\tilde{x}_j = \alpha + \beta \tilde{s}_j + \kappa \tilde{s}_0, \quad (\text{A.1})$$

and the pricing rule will be linear

$$\tilde{p} = \delta + \lambda \tilde{w} + \gamma \tilde{s}_0. \quad (\text{A.2})$$

The total order flow as conjectured by agent n will be

$$\begin{aligned} \tilde{w} &= (N-1)\alpha + \beta \sum_{j \neq n} \tilde{s}_j + (N-1)\kappa \tilde{s}_0 + x_n + \tilde{z} = \\ &= (N-1)\alpha + (N-1)\beta \tilde{v} + \beta \sum_{j \neq n} \tilde{\varepsilon}_j + (N-1)\kappa \tilde{s}_0 + x_n + \tilde{z}. \end{aligned}$$

Agent n maximizes his expected profits $E(\pi^n | s_n, s_0)$

conditional on the signals he has received. These profits are given by

$$\begin{aligned} E(\pi^n | s_n, s_0) &= E\left[(\tilde{v} - \tilde{p})x_n | s_n, s_0\right] = E\left[(\tilde{v} - \delta - \lambda \tilde{w} - \gamma \tilde{s}_0)x_n | s_n, s_0\right] = \\ &= E\left\{\left[\tilde{v} - \delta - \lambda \left[(N-1)\alpha + (N-1)\beta \tilde{v} + \beta \sum_{j \neq n} \tilde{\varepsilon}_j + (N-1)\kappa \tilde{s}_0 + x_n + \tilde{z}\right] - \gamma \tilde{s}_0\right]x_n | s_n, s_0\right\}. \end{aligned}$$

The first order condition of this maximization problem is

$$x_n = \frac{\left[1 - (N-1)\lambda\beta\right]E(\tilde{v}|s_n, s_0) - \left[(N-1)\lambda\kappa + \gamma\right]s_0^{-\delta - (N-1)\lambda\alpha}}{2\lambda}, \quad (\text{A.3})$$

and the second order condition is

$$-2\lambda < 0, \text{ i.e., } \lambda > 0. \quad (\text{A.4})$$

The expected payoff conditional on both private and public signal can be computed using Lemma 2.1.,

$$E(\tilde{v}|s_n, s_0) = \frac{\tau_v \bar{v} + \tau_\varepsilon s_n + \tau_0 s_0}{\tau_v + \tau_\varepsilon + \tau_0}. \quad (\text{A.5})$$

Plugging (A.5) into (A.3) and using (A.1) to equate coefficients, we obtain the following equations:

$$\alpha = \frac{\left[1 - (N-1)\lambda\beta\right]\tau_v \bar{v}}{\tau_v + \tau_\varepsilon + \tau_0} - \delta - (N-1)\lambda\alpha}{2\lambda}, \quad (\text{A.6})$$

$$\beta = \frac{\left[1 - (N-1)\lambda\beta\right]\tau_\varepsilon}{\tau_v + \tau_\varepsilon + \tau_0}, \quad (\text{A.7})$$

$$\kappa = \frac{\left[1 - (N-1)\lambda\beta\right]\tau_0}{\tau_v + \tau_\varepsilon + \tau_0} - (N-1)\lambda\kappa - \gamma}{2\lambda}. \quad (\text{A.8})$$

On the other hand, the zero profit condition for the market maker implies

$$\tilde{p} = E(\tilde{v} | \tilde{w}, \tilde{s}_0) .$$

Notice that to observe \tilde{s}_0 and \tilde{w} is informationally equivalent to observe \tilde{s}_0 and the following random variable:

$$\frac{\tilde{w} - N\alpha - N\kappa\tilde{s}_0}{N\beta} = \tilde{v} + \frac{\sum_{n=1}^N \tilde{\epsilon}_n}{N} + \frac{\tilde{z}}{N\beta} = \tilde{v} + \tilde{u} .$$

Thus, it can be shown that the precision of the random variable $\tilde{u} = \frac{\sum_{n=1}^N \tilde{\epsilon}_n}{N} + \frac{\tilde{z}}{N\beta}$ is

$$\tau_u = \frac{N^2 \beta^2 \tau_\epsilon}{N\beta^2 + \sigma_z^2 \tau_\epsilon} , \quad (\text{A.9})$$

Using again Lemma 2.1,

$$E(\tilde{v} | s_0, w) = \frac{\tau_v \tilde{v} + \tau_0 s_0 + \tau_u \left[\frac{w - N\alpha - N\kappa s_0}{N\beta} \right]}{\tau_v + \tau_0 + \tau_u} = \delta + \lambda w + \gamma s_0 . \quad (\text{A10})$$

Therefore, equating coefficients, after plugging (A.9) into (A.10), we

obtain the following equations:

$$\delta = \frac{\tau_v \bar{v} - \frac{N^2 \beta \alpha \tau_c}{N\beta^2 + \sigma_z^2 \tau_c}}{\tau_v + \tau_0 + \frac{N^2 \beta^2 \tau_c}{N\beta^2 + \sigma_z^2 \tau_c}}, \quad (\text{A.11})$$

$$\lambda = \frac{\frac{N\beta\tau_c}{N\beta^2 + \sigma_z^2 \tau_c}}{\tau_v + \tau_0 + \frac{N^2 \beta^2 \tau_c}{N\beta^2 + \sigma_z^2 \tau_c}}, \quad (\text{A.12})$$

$$\gamma = \frac{\tau_0 - \frac{N^2 \beta \kappa \tau_c}{N\beta^2 + \sigma_z^2 \tau_c}}{\tau_v + \tau_0 + \frac{N^2 \beta^2 \tau_c}{N\beta^2 + \sigma_z^2 \tau_c}}. \quad (\text{A.13})$$

In equilibrium, the conjectures of all agents must be fulfilled. Therefore, the Bayesian-Nash equilibrium of the game is given by the values of $(\alpha, \beta, \kappa, \delta, \lambda, \gamma)$ that solve simultaneously equations (A.6), (A.7), (A.8), (A.11), (A.12) and (A.13).

To solve this nonlinear system of equations, first solve the subsystem consisting of (A.7) and (A.12) which only contains λ and β as unknowns. In the third-order polynomial equation that appears, I select the unique root consistent with the second order condition (A.4). Finally, plug the solutions for λ and β in the remaining equations to

obtain the solution written in the statement of the proposition.

The details and the messy algebra necessary to get this solution are left to the reader. ■

Proof of Proposition 10.1: The market clearing condition,

$$\sum_{n=1}^N x_n(p, s_n, s_0) + z = 0 \quad \text{for all } (z, s_0, s_1, \dots, s_N) \in \mathbb{R}^{N+2},$$

according to our conjectured linearity, takes the form

$$N\hat{\alpha} + \hat{\beta} \sum_{n=1}^N s_n + N\hat{\kappa}s_0 - N\hat{\mu}p + z = 0.$$

This implies that the equilibrium price for each realization of \tilde{z} , \tilde{s}_0 , \tilde{s}_n ($n = 1, \dots, N$) is

$$p = \frac{N\hat{\alpha} + \hat{\beta} \sum_{n=1}^N s_n + N\hat{\kappa}s_0 + z}{N\hat{\mu}}. \quad (\text{A.14})$$

Since each informed trader considers the others' strategies as given, he is facing the following residual demand:

$$p = \frac{(N-1)\hat{\alpha} + \hat{\beta} \sum_{j \neq n} s_j + (N-1)\hat{\kappa}s_0 + z}{(N-1)\hat{\mu}} + \frac{x_n}{(N-1)\hat{\mu}}. \quad (\text{A.15})$$

Therefore, strategic agents solve the following maximization

problem

$$\begin{aligned} \max_{x_n \in \mathbb{R}} E \left[(\tilde{v} - \tilde{p}) x_n \mid s_n, s_0, p \right] = \\ = \max_{x_n \in \mathbb{R}} E \left[\left[\tilde{v} - \frac{(N-1)\hat{\alpha} + \hat{\beta} \sum_{j \neq n} \tilde{s}_j + (N-1)\hat{\kappa} \tilde{s}_0 + \tilde{z}}{(N-1)\hat{\mu}} - \frac{x_n}{(N-1)\hat{\mu}} \right] x_n \mid s_n, s_0, p \right]. \end{aligned}$$

The first order condition for this problem is

$$\begin{aligned} E(\tilde{v} \mid s_n, s_0, p) - \frac{2}{(N-1)\hat{\mu}} x_n - \\ - E \left[\frac{(N-1)\hat{\alpha} + \hat{\beta} \sum_{j \neq n} \tilde{s}_j + (N-1)\hat{\kappa} \tilde{s}_0 + \tilde{z}}{(N-1)\hat{\mu}} \mid s_n, s_0, p \right] = 0. \quad (\text{A.16}) \end{aligned}$$

Because of (A.15), (A.16) may be written as

$$E(\tilde{v} \mid s_n, s_0, p) - \frac{x_n}{(N-1)\hat{\mu}} - p = 0,$$

and this implies,

$$x_n = (N-1)\hat{\mu} \left[E(\tilde{v} \mid s_n, s_0, p) - p \right]. \quad (\text{A.17})$$

The second order sufficient condition for the maximization problem is

$$- \frac{2}{(N-1)\hat{\mu}} < 0,$$

that is to say, $\hat{\mu}$ must be strictly positive.

Note that to observe the random variables \tilde{p} , \tilde{s}_n , \tilde{s}_0 is

informationally equivalent to observe \tilde{s}_0, \tilde{s}_n and the following random variable:

$$\frac{N\hat{\mu} - N\hat{\alpha} - \hat{\beta}\tilde{s}_n - N\hat{\kappa}s_0}{(N-1)\hat{\beta}} = \tilde{v} + \tilde{y},$$

$$\text{where } \tilde{y} = \frac{\sum_{j \neq n} \tilde{\epsilon}_j}{N-1} + \frac{\tilde{z}}{(N-1)\hat{\beta}}.$$

It can be proved that the precision of \tilde{y} is

$$\tau_y = \frac{(N-1)^2 \hat{\beta}^2 \tau_\epsilon}{(N-1)\hat{\beta}^2 + \sigma_z^2 \tau_\epsilon}. \quad (\text{A.18})$$

then, applying Lemma 2.1, I can compute the following expectation:

$$E(\tilde{v} | s_n, s_0, p) = \frac{\tau_v \tilde{v} + \tau_\epsilon s_n + \tau_0 s_0 + \tau_y \left[\frac{N\hat{\mu} - N\hat{\alpha} - \hat{\beta}s_n - N\hat{\kappa}s_0}{(N-1)\hat{\beta}} \right]}{\tau_v + \tau_0 + \tau_\epsilon + \tau_y}. \quad (\text{A.19})$$

Plugging (A.19) into (A.17) and now making the conjecture that

$$x_n = x_n(p, s_n, s_0) = \hat{\alpha} + \hat{\beta}s_n + \hat{\kappa}s_0 - \hat{\mu}p,$$

and equating coefficients, we obtain the following system:

$$\hat{\alpha} = (N-1)\hat{\mu} \left[\frac{\tau_v \tilde{v} - \tau_y \left[\frac{N\hat{\alpha}}{(N-1)\hat{\beta}} \right]}{\tau_v + \tau_0 + \tau_\epsilon + \tau_y} \right], \quad (\text{A.20})$$

$$\hat{\beta} = (N-1)\hat{\mu} \left[\frac{\tau_{\varepsilon} - \frac{\tau_y}{(N-1)}}{\tau_v + \tau_0 + \tau_{\varepsilon} + \tau_y} \right], \quad (\text{A.21})$$

$$\hat{\kappa} = (N-1)\hat{\mu} \left[\frac{\tau_0 - \tau_y \left[\frac{N\hat{\kappa}}{(N-1)\hat{\beta}} \right]}{\tau_v + \tau_0 + \tau_{\varepsilon} + \tau_y} \right], \quad (\text{A.22})$$

$$\hat{\mu} = (N-1)\hat{\mu} \left[1 - \frac{\tau_y \left[\frac{N\hat{\mu}}{(N-1)\hat{\beta}} \right]}{\tau_v + \tau_0 + \tau_{\varepsilon} + \tau_y} \right]. \quad (\text{A.23})$$

Using the expression (A.18) for τ_y , and after some tricky algebra, it can be proved that the unique solution to the non-linear system (A.20)-(A.23) that satisfies the second order condition ($\hat{\mu} > 0$) is given by the expressions in the statement of the proposition provided that $N > 2$. The strategies characterized by the values of $\hat{\alpha}$, $\hat{\beta}$, $\hat{\kappa}$ and $\hat{\mu}$ that solve the above system constitute a Bayesian-Nash equilibrium of the demand schedules submission game. ■

REFERENCES

- Admati, A. R. (1985). "A Noisy Rational Expectations Equilibrium for Multi-Asset Securities Markets." Econometrica 53, 629-657.
- Admati, A. R. and P. Pfleiderer (1986). "A Monopolistic Market for Information." Journal of Economic Theory 39, 400-438.
- Admati, A. R. and P. Pfleiderer (1988a). "A Theory of Intraday Patterns: Volume and Price Variability." The Review of Financial Studies 1, 3-40.
- Admati, A. R. and P. Pfleiderer (1988b). "Selling and Trading on Information in Financial Markets." American Economic Review 78, 96-103
- Allen, F. (1987a). "Contracts to Sell Information." Working Paper. University of Pennsylvania.
- Allen, F. (1987b). "The Social Value of Asymmetric Information." Working Paper. University of Pennsylvania.
- Amihud, Y. and H. Mendelson (1980). "Dealership Market." Journal of Financial Economics 8, 31-53.
- Bhattacharya, S. (1979). "Imperfect Information, Dividend Policy and the Bird in the Hand Fallacy." Bell Journal of Economics 10, 259-270.
- Beja, A. (1977). "The Limits of Price Information in Market Processes." Working Paper. University of California at Berkeley.

- Bhushan, R. (1988). "Cross-Sectional Variations in Trading Costs and Trading Volume." Working Paper. Massachusetts Institute of Technology.
- DeGroot, M. H. (1970). Optimal Statistical Decisions. McGraw-Hill, New York, N.Y.
- Diamond, D. W. (1985). "Optimal Release of Information by Firms." The Journal of Finance 40, 1071-1094.
- Diamond, D. W. and R. E. Verrecchia (1981). "Information Aggregation in a Noisy Rational Expectations Economy." Journal of Financial Economics 9, 221-235.
- Diamond, D. W. and R. E. Verrecchia (1987). "Constraints on Short-Selling and Asset Price Adjustment to Private Information." Journal of Financial Economics 18, 277-311.
- Easley, D. and M. O'Hara (1987). "Price, Trade Size, and Information in Securities Markets." Journal of Financial Economics 19, 69-90.
- Fishman, M. J. and K. M. Hagerty (1988). "Disclosure Decisions by Firms and the Competition for Price Efficiency." Working Paper. Northwestern University.
- Gal-Or, E. (1985). "Information Sharing in Oligopoly." Econometrica 53, 329-343.
- Gale, D. and M. Hellwig (1987). "Informed Speculation in Large Markets." Working Paper. University of Pittsburgh.
- Garman, M. B. (1976). "Market Microstructure." Journal of Financial Economics 3, 257-275.

- Glosten, L. R. and P. R. Milgrom (1985). "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders." Journal of Financial Economics 14, 71-100.
- Gonedes, N. (1980). "Public Disclosure Rules, Private Information Production Decisions and Capital Market Equilibrium." Journal of Accounting Research 18, 441-475.
- Gould, J. P. and R. E. Verrecchia (1985). "The Information Content of Specialist Pricing." Journal of Political Economy 93, 66-83.
- Grossman, S. J. (1976). "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information." The Journal of Finance 31, 573-585.
- Grossman, S. J. (1977). "The Existence of Futures Markets, Noisy Rational Expectations and Informational Externalities." Review of Economic Studies 64, 431-449.
- Grossman, S. J. (1978). "Further Results on the Informational Efficiency of Competitive Stock Markets." Journal of Economic Theory 18, 81-101.
- Grossman, S. J. (1981a). "An Introduction to the Theory of Rational Expectations under Asymmetric Information." Review of Economic Studies 48, 541-559.
- Grossman, S. J. (1981b). "Nash Equilibrium and the Industrial Organization of Markets with Large Fixed Costs." Econometrica 49, 1149-1172.
- Grossman, S. J. and J. E. Stiglitz (1976). "Information and Competitive Price Systems." American Economic Review 66, 246-253.

- Grossman, S. J. and J. E. Stiglitz (1980). "On the Impossibility of Informationally Efficient Markets." American Economic Review 70, 393-408.
- Hakansson, N. H., J. G. Kunkel and J. A. Ohlson (1982). "Sufficient and Necessary Conditions for Information to have Social Value in Pure Exchange." The Journal of Finance 37, 1169-1181.
- Hellwig, M. F. (1980). "On the Aggregation of Information in Competitive Markets." Journal of Economic Theory 22, 477-498.
- Hirshleifer, J. (1971). "The Private and Social Value of Information and the Reward to Inventive Activity." American Economic Review 61, 561-574.
- Jackson, M. (1988). "Equilibrium, Price Formation and the Value of Information in Economies with Privately Informed Agents." Working Paper. Northwestern University.
- Kihlstrom, R. E. and L. J. Mirman, (1975). "Information and Market Equilibrium." Bell Journal of Economics 6, 357-376.
- Kihlstrom, R. E. and A. Postlewaite (1983). "Equilibrium in a Securities Market with a Dominant Trader Possessing Inside Information." Working Paper. University of Pennsylvania.
- Kreps, D. M. and R. Wilson (1982). "Sequential Equilibrium." Econometrica 50, 863-894.
- Kyle, A. S. (1984). "Market Structure, Information, Futures Markets, and Price Formation," in International Agricultural Trade: Advanced Readings in Price Formation, Market Structure, and Price Stability, edited by Storey, G. G., A. Schmitz and A. H. Sarris. Westview Press, Boulder, CO. 45-84.

- Kyle, A. S. (1985). "Continuous Auctions and Insider Trading." Econometrica 53, 1315-1335.
- Kyle, A. S. (1986). "Informed Speculation with Imperfect Competition." Working Paper. Princeton University.
- Laffont, J.-J. and E. Maskin (1988). "The Efficient Market Hypothesis and Insider Trading on the Stock Market." Working Paper. Harvard University.
- Matthews, S. (1984). "Information Acquisition in Discriminatory Auctions." in Bayesian Models in Economic Theory, edited by Boyer M. and R. Kihlstrom. North-Holland Press, Amsterdam, 181-207
- Milgrom, P. R. (1979). "A Convergence Theorem for Competitive Bidding with Differential Information." Econometrica 47, 679-688.
- Milgrom, P. R. (1981). "Rational Expectations, Information Acquisition and Competitive Bidding." Econometrica 49, 921-943.
- Miller, M. and K. Rock (1985). "Dividend Policy Under Asymmetric Information." Journal of Finance 40, 1031-1052.
- O'Hara M. and G. S. Oldfield (1986). "The Microeconomics of Market Making." Journal of Financial and Quantitative Analysis 21, 21, 361-376.
- Ross, S. A. (1977). "The Determination of Financial Structure: The Incentive Signalling Approach." Bell Journal of Economics 8, 23-40.
- Ross, S. A. (1979). "Disclosure Regulation in Financial Markets: Implications of Modern Finance Theory and Signaling Theory," in Issues in Financial Regulation, edited by Edwards, F. McGraw-Hill, New York, N.Y. 177-202.

- Verrecchia, R. E. (1982a). "Information Acquisition in a Noisy Rational Expectations Economy." Econometrica 50, 1415-1430.
- Verrecchia, R. E. (1982b). "The Use of Mathematical Models in Financial Accounting." Journal of Accounting Research 20 Supplement, 1-42.
- Wilson, R. (1979). "Auctions of Shares." Quarterly Journal of Economics 93, 675-689.

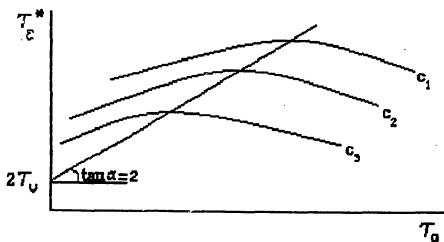


Figure 1 - Production of Private Information

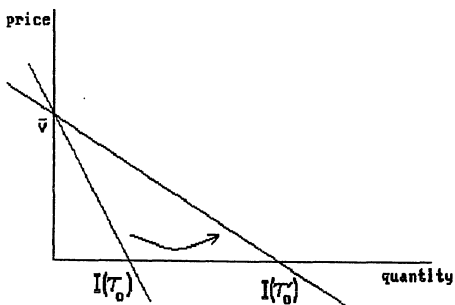


Figure 2 - Expected Demand Function

Part 2

INSIDER TRADING AND ASSET PRICING IN AN
IMPERFECTLY COMPETITIVE MULTI-SECURITY MARKET

(with Murugappa Krishnan)

1. INTRODUCTION

The central purpose of this paper is to develop a theory of insider trading (i.e. trading based on private information), in the context of an imperfectly competitive multi-security market. Imperfect competition allows us to consider strategic behavior, and a multi-security market lets us study the effect of a correlated environment on equilibrium. A salient feature of our model — in contrast to traditional multi-asset models — is the manner in which we create a link between demands for different securities by the informed. Rather than focusing on the incentive to reduce portfolio variance, or the effect of short-selling restrictions or budget constraints, we employ an informational assumption — that the market maker can observe all order flows, and so, given correlated fundamentals, can potentially learn about every security from each order flow. This causes even a perfectly informed risk neutral trader who does not face short-selling restrictions to refrain from determining the demand for each security independently. While this assumption does not exclude the traditional ways of generating a link between demands for different securities, a basic premise of this paper is that a priori this hypothesis is at least as important and interesting, and deserves to be studied independently.

Our principal results include an explicit characterization of the unique equilibrium given a linear pricing rule, as a function of three general variance-covariance matrices (associated with returns,

"noise trading" and errors in private signals). Under imperfect competition, correlation has two effects. One, *ceteris paribus*, it allows a market maker (or, more generally, the uninformed) to learn from additional variables — order flows in our model. Note, however, that each order flow could potentially have information about all returns. On the other hand, it creates an incentive for, and enables, manipulation by an informed trader, who would like to minimize what others can learn from public information. It is important to realize that a priori we cannot tell who has more "power", or what kind of equilibrium will result, given that both the informed trader and the market maker behave strategically. In the context of our model, we shall study how these two effects balance each other in equilibrium.

The most striking feature of the model is the extent to which strategic behavior can "neutralize" the effect of a correlated environment, and distill "joint" effects into "pure" effects. This is in sharp contrast to analyses under perfect competition which ignore, by definition, the possibility of manipulating inferences and focus only on the possibility of learning.

Admati (1985) considers a correlated multi-asset environment, as we do in this paper, but under the assumption of perfect competition. The main lesson is that it is possible to have a variety of "perverse" results (e.g. asset demands may increase in their own prices), by virtue of a well-known result in linear statistical inference: the use of correlated regressors leads to response coefficients that are ambiguous in sign (since they generate an "indirect effect" that could swamp the "direct effect"). This can sometimes generate "abnormal" predictions

about terminal values, and also "abnormal" effects on demands and prices, since, in a perfect competition rational expectations equilibrium (with exponential utility and Gaussian fundamentals), these are just linear functions of such predictions.

While in general such "perverse" possibilities also exist under imperfect competition — since conditional expectations about terminal values continue to be a key ingredient — what is interesting are the additional effects that can arise, solely as a consequence of strategic behavior. To emphasize that these results are peculiar to imperfect competition — and cannot arise in an Admati (1985) world — we study the case of perfect private information, which eliminates the possibility of any correlated-regressors effect. This allows us to focus on the impact of "cross-effects" that arise when the informed trader tries to "manipulate" the market maker's strategy, even as the market maker tries to "learn" from order flows (which are potentially correlated despite perfect private information, because of correlated fundamentals). In a Gaussian setting, under completely general correlation structures describing returns and liquidity noise, we show that strategic behavior is enough to restore various theoretical regularities previously associated only with asset pricing models such as the CAPM, which did not recognize the existence of private information, and allowed only for exogenous, homogeneous beliefs. Market makers set prices "as if" each price is affected only by its own return, and all public information about a security's return is contained in its own price, regardless of the underlying correlation structure. The informed trader's trade off between two incentives — to

trade (and derive advantage from private information) and to refrain from trading (and so reveal less) — can however create new reasons for "perverse" possibilities, such as asset demands being negative even with good news.

We begin by studying the equilibrium in the more general case, i.e., with imperfect private information, in which we have both the correlated-regressors effect and the strategic effect. (To the extent that we can allow for perfect private information, on some or all assets, our specification is slightly richer than that in Admati (1985), for under perfect competition, a rational expectations equilibrium cannot even exist with perfect private information. See, e.g., Hellwig (1980)). Even in this case, we find that all public information about a security's return is contained in its own price. Also, the relationship between prices and returns is always independent of the variance-covariance matrix of liquidity noise: the effect of noise trading is always exactly "balanced" by informed trading. We show that restoring predictive content where Admati (1985) found ambiguities is valuable for a class of empirical applications in accounting and finance, referred to as "event studies". It helps justify procedures originally used in conjunction with the CAPM even in settings characterized by the existence of private information.

The plan of the rest of the paper is as follows. In Section 2, we introduce our model, and discuss the role of key assumptions. We derive the equilibrium in the most general case we consider — with imperfect private information — in Section 3, and identify some general properties in Section 4. To highlight the role of the strategic effect,

distinct from the correlated-regressors effect referred to above, we analyze the case of perfect private information in Section 5, and consider a two-asset example in detail in Section 6. Section 7 provides concluding remarks.

2. MODEL

The model of a multi-security market that we develop is inspired by, and is a generalization of, the single-security model in Kyle (1985). It can be regarded as a model of a multi-good auction: the price is determined in the last stage of the game¹, after traders have made their quantity choices. This means that the informed trader selects a quantity based on not an actual but an expected price, which captures the essence of a setting with "market orders". Of course, in equilibrium the trader correctly anticipates the pricing rule (though not the actual price) followed by market makers.

Our parametric assumptions are guided in part by Admati (1985) who studies a multi-asset market under perfect competition, with a rich correlation structure. Keeping the same correlation structure makes it easier to see the impact of strategic behavior.

¹ These models are not, strictly speaking, game-theoretic, for the market maker is merely assumed to use a "pricing rule" rather than maximize a well-specified objective function. However, as Kyle (1985) pointed out, they can easily be made consistent with rigorously defined games by assuming Bertrand-type competition among risk-neutral market makers.

Assumptions

A1 There are n securities in the market, which will be indexed by j , $j = 1, 2, \dots, n$, yielding a multivariate return vector, $\tilde{v} = (\tilde{v}_1, \tilde{v}_2, \dots, \tilde{v}_n)$, which is distributed normally with mean vector \bar{v} and a nonsingular variance-covariance matrix Σ_v .

The assumption of nonsingularity is primarily for convenience. Extending the analysis to the case of singular distributions does not promise any gain in intuition: it involves only technical considerations adequately dealt with in Admati (1985).

As in Kyle (1985), Hellwig (1980), Grossman and Stiglitz (1980), etc., we assume that this return distribution is exogenous — in other words, we abstract from moral hazard considerations, which are beyond the scope of this paper.

A2 There is a single risk-neutral trader (who faces no short-selling restrictions) with access to private information about each security, i.e., this trader observes a vector of signals, $\tilde{s} = s$, where $\tilde{s} = \tilde{v} + \tilde{e}$, with $\tilde{e} \sim N(0, \Sigma_e)$. The error distribution is independent of the return distribution, but the variance-covariance matrix of errors is completely general, and possibly singular, to allow for perfect information on some or all assets. The trader's demands will be denoted by a vector \tilde{x} , which is a function of the signal vector \tilde{s} .

This assumption is also found in Kyle (1985), and allows a more tractable analysis. With several informed traders, we would also have

to reckon with competition among informed traders, which does not seem germane to the theme of this paper, and would only increase analytical complexity². Assuming a single informed trader also captures our belief that the strategic interaction between the informed and the uninformed is more critical than competition among the informed. We consider the almost exclusive focus of public policy on the former effect to be indirect evidence consistent with this belief³. Thus, assuming a single informed trader, though literally untrue, nevertheless helps model a feature that is plausible on empirical grounds.

A3 There are noise traders who generate a vector of random "liquidity demands",

$$\tilde{z} \sim N(\bar{z}, \Sigma_z)$$

with Σ_z nonsingular. This is important in providing camouflage for informed trading. An alternative way to introduce noise is with unobservable preferences, as in Allen (1987) and Ausubel (1988): to do so we would have to give up risk-neutrality, which is useful in maintaining tractability, in a model already overburdened by computation.

² The reader should consult Caballe (1988) for a detailed treatment of a Kyle market with many informed traders. Besides considering the effect of competition among traders under both common and diverse information regimes, it also provides results pertaining to entry, with or without the possibility of voluntary syndication or pooling.

³ To state this in a slightly different way, public concern centers around whether people like Ivan Boesky can — singly or collectively — gain an unfair advantage in their interaction with the uninformed, and not around whether they may ruin themselves with competition. Nor is public policy complacently assuming that such competition will alleviate the disadvantage of the uninformed.

A4 The price vector \tilde{p} is determined by the following rule:

$$p \equiv P(\tilde{w}) = E(\tilde{V}|\tilde{w}) \quad (1)$$

where $\tilde{w} = \tilde{x} + \tilde{z}$

Thus, the pricing rule is such that conditional on any set of public signals (order flows) market makers can expect to make zero profits, in each market. This assumes that we have Bertrand-type competition among risk-neutral market makers even before order flows are observed (i.e. when pricing strategies, rather than prices, are determined), and captures the notion that markets for market making are "perfectly contestable" (see, e.g. Baumol, Panzar and Willig (1982)), so that cross-subsidization is not possible in equilibrium⁴.

This condition is also important for a technical reason: if we imposed the weaker requirement that only overall expected profits should be zero, we would have to consider pricing rules that are possibly

⁴ It is helpful to have in mind a description of the game among market makers, to convince oneself that an argument analogous to the traditional argument underlying a Bertrand equilibrium is indeed valid. Before order flows are determined, market makers decide on "pricing rules". A rule promising positive expected profits cannot be an equilibrium since it can be "undercut" by an alternative rule that yields lower expected profits (which will offer traders more attractive terms).

To see that there will be no incentive to deviate from a zero-expected-profits pricing rule, one should note that given rational expectations each trader would know every market maker's pricing rule, and can direct orders to any market maker, without revealing her identity (by, say, using brokers), and so a market maker who uses a rule that offers a higher price than the zero-expected-profits pricing rule will expect to attract the trader only when she wishes to sell; with a lower price, only when she wishes to buy — so a deviation could only lead to negative expected profits.

non-linear in order flows⁵. This would also result in an objective function for the informed trader that is more complex — in which case even the existence of an optimum cannot be taken for granted, and we will have to impose additional restrictions.

The assumption that each price is set by an agent who can observe all order flows is critical for our analysis. It is not only plausible on empirical grounds⁶, but is also crucial in generating a link between demands for different securities. To see this, consider what would happen under our other assumptions if the price of a security was related only to the order flow of that security. If we have a risk-neutral trader with private information about each return, and without short-selling restrictions, even if the underlying returns are correlated, the informed trader will only be concerned about the sign of the net expected return of each security. While this will not lead to an infinite position in any security — for the trader will still reckon with the possibility of pushing up the price of that security — there are no "cross-effects" on prices due to order flows, and what we will have are simply several "Kyle markets" functioning independently. (Correlation among signals will make the trader use all the information

⁵ Expected overall profits are given by:

$$\sum_j [p_j - E(\tilde{v}_j | w)] w_j$$

If each such term in the sum is required to be zero, then $p_j = E(\tilde{v}_j | w)$, $\forall j$. Given normality, each price must be linear in order flows. When only the sum must be zero, prices need not equal expected returns, and so need not be linear in order flows any longer.

⁶ Most modern exchanges have a "big screen" or TV monitors that provide information on all, or at least many, order flows.

in forming conditional expectations about each return, but once these expectations are formed, demands will be determined independently).

To be sure, this is not the only way of generating cross-effects. Traditional models do so in at least two ways. Imperfect information and risk-aversion make traders concerned about the portfolio variance. Alternatively, with short-selling restrictions (or a budget constraint), traders equate marginal net benefits per dollar invested across securities⁷. In either case, demands are jointly determined.

Our preference for an informational assumption about the market maker, to create a link between demands, is due only in part to its novelty. We consider the practice of popular business commentators who refer to groups of securities (e.g., "chemical stocks", "food companies", etc.) as indirect evidence consistent with the view that a market maker can learn significantly more about one security by using even the order flows of other securities.

A5 Both the informed trader and the market maker have rational expectations, i.e. the informed trader correctly anticipates the market maker's pricing rules, and the market maker has a correct conjecture about the informed trader's strategy.

⁷ In passing, we would like to note that shortselling restrictions or discretionary cross-sectional liquidity trading would generate a link between demands even when underlying returns are uncorrelated.

Time Structure

The time structure of the model is summarized in Table 1 below.

Table 1 — Time Structure of Basic Model

Date	Event
$t = 0$	The informed trader receives private information
$t = 1$	The informed trader submits demands, taking into account the market makers' pricing strategy, while noise traders submit their random demands. The market maker observes only aggregate order flows.
$t = 2$	Market makers set prices conditional on order flows, and absorb any excess supply or meet any excess demand.
$t = 3$	Terminal values are realized, and terminal payoffs are consumed.

3. EQUILIBRIUM

In this section, because of the normality of all random

variables (and following Kyle (1985) and Admati and Pfleiderer (1988)), we consider an equilibrium with linear pricing rules, given by:

$$P(w) = A_0 + A_1 w \quad (2)$$

We show that such an equilibrium is unique, and provide an explicit characterization. We also demonstrate formally that it is a generalization of Kyle (1985).

We use the notation for the strategies of the informed trader and the market maker given in the previous section. Profits of the informed trader are given by:

$$\tilde{\pi} = (\tilde{v} - \tilde{p})^T \tilde{x} \quad (3)$$

where the superscript 'T' denotes the transpose, and the vector of prices, \tilde{p} , depends on market maker's pricing rules, i.e.

$$\tilde{p} = P(\tilde{w}) = P(\tilde{x} + \tilde{z}) \quad (4)$$

The informed traders strategies X are functions from the realization of the random variable $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_n)$ to quantities traded:

$$\tilde{x} = X(\tilde{s}) \quad (5)$$

So we may write

$$\tilde{\pi} = \pi(X(\tilde{s}), P(\tilde{w})) \quad (6)$$

Definition of Equilibrium

An equilibrium is a pair X, P , such that the following conditions hold.

(a) Profit maximization:

For any signal vector s , and for any alternate trading strategy X'

$$E(\pi(X(\tilde{s}), P(\tilde{w}) | \tilde{s} = s)) \geq E(\pi(X'(\tilde{s}), P(\tilde{w}) | \tilde{s} = s)) \quad (7)$$

(b) Market efficiency:

The pricing rule P satisfies:

$$P(w) = E(\tilde{v} | \tilde{w} = w) \quad (8)$$

We derive the equilibrium via a series of lemmas.

LEMMA 3.1: If pricing rules are linear (affine), i.e. $P(w) = A_0 + A_1 w$, then, if an equilibrium exists,

(a) the optimal strategies of the informed trader (if they exist) are linear in the signal vector s :

$$x = B_0 + B_1 s \quad (9)$$

(b) $(A_1 + A_1^T)$ and B_1 are both nonsingular.

Proof: (a) For any $\tilde{s} = s$, the trader's optimization problem is given by:

$$\text{Max}_x E \{ (\tilde{v} - P(x + \tilde{z}))^T x | \tilde{s} = s \} = \quad (10)$$

$$= \text{Max}_x E \{ (\tilde{v} - A_0 - A_1 x - A_1 \tilde{z})^T x | \tilde{s} = s \} \quad (11)$$

First-order conditions imply:

$$(A_1 + A_1^T)x = [E(\tilde{v} | \tilde{s} = s) - A_0 - A_1 \tilde{z}] = \quad (12)$$

$$= [\bar{v} + \Sigma_v (\Sigma_v + \Sigma_e)^{-1} (s - \bar{v}) - A_0 - A_1 \tilde{z}] \quad (13)$$

Since the RHS is linear in s , x must be linear in s .

(b) Using (9) and equating coefficients, we get

$$(A_1 + A_1^T)B_1 = \Sigma_v (\Sigma_v + \Sigma_e)^{-1} \quad (14)$$

Since the RHS of (14) is nonsingular, an equilibrium solution exists only if $(A_1 + A_1^T)$ and B_1 are both nonsingular. (This is important because we can restrict our search for an equilibrium accordingly, and this is enough to guarantee invertibility of all subsequent expressions assumed invertible. Of course, we must check in the end that our candidate A_1^* and B_1^* do satisfy the above requirement). Then the informed trader's optimal strategies are given by (9), provided the following second-order condition hold: $(A_1 + A_1^T)$ should be positive semi-definite. Since $(A_1 + A_1^T)$ is nonsingular, we can assert that $(A_1 + A_1^T)$ must be positive definite (i.e. A_1 must be positive quasi-definite). These second-order conditions will be critical in

helping us establish both existence and uniqueness of equilibrium. ■

LEMMA 3.2: In an equilibrium, we must have:

$$B_1 = (A_1 + A_1^T)^{-1} \Sigma_v (\Sigma_v + \Sigma_e)^{-1} \quad (15)$$

$$B_0 = (A_1 + A_1^T)^{-1} [\bar{v} - \Sigma_v (\Sigma_v + \Sigma_e)^{-1} \bar{v} - A_0 - A_1 \bar{z}] \quad (16)$$

$$A_1 = [B_1 (\Sigma_v + \Sigma_e) \Sigma_v^{-1} + \Sigma_z (B_1^T)^{-1} \Sigma_v^{-1}]^{-1} \quad (17)$$

$$A_0 = \bar{v} - A_1 [B_0 + B_1 \bar{v} + \bar{z}] \quad (18)$$

and A_1 must be positive quasi-definite.

Proof: Solving (13) for x , we get:

$$x = (A_1 + A_1^T)^{-1} [\bar{v} + \Sigma_v (\Sigma_v + \Sigma_e)^{-1} (s - \bar{v}) - A_0 - A_1 \bar{z}] \quad (19)$$

Using (9) and equating coefficients (15) and (16) follow. (17) and (18) follow from the market makers' inference problem: details are consigned to the Appendix. As we said, positive quasi-definiteness comes from the second order condition. ■

Remark: (15) $\Rightarrow (A_1 + A_1^T)^{-1} = B_1 (\Sigma_v + \Sigma_e) \Sigma_v^{-1}$ (20)

Note that this means that $B_1 (\Sigma_v + \Sigma_e) \Sigma_v^{-1}$ is symmetric positive definite.

Remark: The second-order condition in Lemma 3.1, and the definition of B_1 in (15), tell us that under perfect private information (i.e. if $\Sigma_e = 0$) B_1 is a symmetric positive definite matrix, regardless of the

structure of A_1 . In the more general case of imperfect private information, however, the lemma tells us little about the structure of A_1 or B_1 , and it will turn out that though A_1 is always symmetric positive definite, B_1 has little special structure.

LEMMA 3.3: If an equilibrium exists A_1 must be symmetric.

Proof: See the Appendix. ■

LEMMA 3.4: The equilibrium A_1 is given by the positive definite solution for A_1 in:

$$A_1 = \frac{1}{4} \Sigma_v (\Sigma_v + \Sigma_c)^{-1} \Sigma_v A_1^{-1} \Sigma_z^{-1} \quad (21)$$

Proof: See the Appendix. ■

Remark: While the general strategy of our proof of existence and uniqueness of equilibrium exploits the linear-Gaussian structure of the model, as does Kyle (1985), there is a small but important technical point that deserves to be noted.

If we did not exploit matrix structure, (21), as a fixed-point problem in terms of the elements of A_1 , is a formidable non-linear problem. Even with $n = 2$, it involves in general a system of four third-degree multivariate polynomial equations.

This would lead us to regard the discovery of a unique equilibrium as, in the words of Kyle (1985), "fortuitous". However, the matrix structure makes it clear that the problem of finding an

equilibrium is equivalent to finding the positive definite square root of a positive definite matrix; hence existence and uniqueness follow. The effort in the proof is mainly to define the positive definite matrix for which we need to find the square root.

Before we state the main theorem of this paper, let us state one more lemma.

LEMMA 3.5: The unique solution for A_1 in (21) is given by:

$$A_1 = (M^T)^{-1} D^{1/2} M^{-1} \quad (22)$$

where M is an eigenmatrix which simultaneously diagonalizes

$S = \frac{1}{4} \Sigma_v (\Sigma_v + \Sigma_e)^{-1} \Sigma_v$ and Σ_z^{-1} , and D is the positive definite diagonal matrix with diagonal elements being the roots of $\det(\Sigma_z^{-1} - \lambda S)$.

Proof: Details are given in the Appendix. What is interesting is that A_1 is unique even if the eigenvalues are not distinct, i.e. even when the diagonalizing M is not unique. ■

We now provide an explicit characterization of the unique equilibrium.

THEOREM 1: Assume that returns \tilde{v} , liquidity noise \tilde{z} and errors in private signals \tilde{e} , are all multinormal random vectors, and are defined by:

$$(i) \tilde{v} \sim N(\bar{v}, \Sigma_v) \quad (ii) \tilde{z} \sim N(\bar{z}, \Sigma_z) \quad (iii) \tilde{e} \sim N(0, \Sigma_e)$$

with \tilde{v} , \tilde{z} and $\tilde{\epsilon}$ mutually independent, Σ_v and Σ_z positive definite and Σ_c positive semi-definite.

Given a linear pricing rule, $P(\tilde{w} = w)$ there exists a unique equilibrium defined as follows:

$$(a) \quad S = \frac{1}{4} \Sigma_v (\Sigma_v + \Sigma_c)^{-1} \Sigma_v$$

(b) M is a nonsingular matrix such that:

$$(i) \quad M^T S M = I \quad (ii) \quad M^T \Sigma_z^{-1} M = D$$

where D is the positive definite diagonal matrix whose diagonal elements are the eigenvalues of Σ_z^{-1} in the metric of S , i.e. the roots of $\det(\Sigma_z^{-1} - \lambda S)$.

The price vector is $p = A_0 + A_1 w$, where

$$A_1 = (M^T)^{-1} D^{1/2} M^{-1} \quad (23)$$

$$A_0 = \tilde{v} - A_1 \tilde{z} \quad (24)$$

and demand strategies $x = B_0 + B_1 s$, where

$$B_1 = 2 \Sigma_z A_1 \Sigma_v^{-1} = \frac{1}{2} A_1^{-1} \Sigma_v (\Sigma_v + \Sigma_c)^{-1} \quad (25)$$

$$B_0 = -B_1 \tilde{v} \quad (26)$$

Proof: Lemma 3.5 gives us (23). (A22) in the Appendix gives us (25). (24) and (26) come from (16) and (18), after some algebra. It is then straightforward to verify that $(A_1 + A_1^T)$ and B_1 are both nonsingular, which completes the proof. ■

Remark: For the single asset case, (21) can be solved easily for:

$$A_1 = \frac{1}{2} \cdot \frac{\sigma_v^2}{\left[(\sigma_v^2 + \sigma_\varepsilon^2) \sigma_z^2 \right]^{1/2}} \quad (27)$$

and

$$B_1 = \left[\frac{\sigma_z^2}{(\sigma_v^2 + \sigma_\varepsilon^2)^{1/2}} \right]^{1/2} \quad (28)$$

which tally with Admati and Pfleiderer (1988, p. 10), equations (5) and (4).

Putting $\sigma_\varepsilon^2 = 0$ gives us the original Kyle (1985, p. 1319) result⁸.

4. GENERAL PROPERTIES OF EQUILIBRIUM

We shall now identify some general properties of this model. It is important to note that in the general case — i.e. when we allow imperfect private information with a general covariance matrix of errors, in addition to a general covariance structure for returns and noise — we have both the correlated-regressors effect and a strategic

⁸ The actual statement of Theorem 1 in Kyle (1985, p. 1319) contains a typographical error. The coefficient in λ (equivalent to our A_1) should be $1/2$, and not 2. Subsequent computations in that paper are however based on the correct value.

effect. Thus, as in Admati (1985) we still have some ambiguities which do not exist in the case of asset pricing models like the CAPM with exogenous, homogeneous beliefs. However, strategic behavior ensures that the equilibrium in our model has simpler structure, relative to Admati (1985), in a sense that we shall make more precise in this section. In particular, under the same general restriction that prices be linear in information, we find that our model exhibits some theoretical regularities, even with imperfect private information, which are of value for an important class of empirical applications in accounting and finance, a class of "event studies".

One property of the equilibrium in this model — as we have already noted in Lemma 3.3 — is that A_1^* is symmetric. This tells us that regardless of the extent of asymmetry across assets, the i^{th} price responds to the j^{th} order flow exactly as the j^{th} price responds to the i^{th} order flow, for any i and j . While this ultimately reflects the balance between various complex interactive effects, it will help build intuition to consider a heuristic explanation of how strategic behavior helps achieve this balance.

Assume that asset "i" is characterized by a very high level of liquidity noise; this makes the trader more aggressive in trading asset "i", relative to some other asset "j" which has less noise, since there is more camouflage. This makes the informativeness of order flows the same for both assets: order flow "i" is as useful in predicting return "j" as order flow "j" in predicting return "i". So a market maker's priors are modified "in the same way" for every asset. The market makers' matrix of response coefficients are like a ratio of

prior-to-posterior precisions (remember that in equilibrium, by virtue of the market efficiency requirement, prices equal expected returns, i.e. prices are like regression functions). Since the prior variance-covariance matrix is symmetric, given the same degree of improvement in precision from observing order flows, this symmetry is preserved in the market maker's pricing rule.

This is an argument to show that A_1^* is symmetric even when liquidity noise varies across assets. A similar argument can be constructed to account for differences in return variances and error variances. This symmetry property, which holds quite generally in our model, is a testable proposition, given the recent availability of transaction data, which permits us to construct measures of order flows.

PROPOSITION 4.1: The equilibrium price vector, as a function of the signal vector \tilde{s} and the noise vector \tilde{z} is given by:

$$\tilde{p} = \bar{v} + \frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_c)^{-1} (\tilde{s} - \bar{v}) + A_1^* \tilde{z} \quad (29)$$

Proof: Notice that

$$\tilde{p} = A_0^* + A_1^* \tilde{w} = A_0^* + A_1^* B_0^* + A_1^* B_1^* \tilde{s} + A_1^* \tilde{z}$$

and the proposition follows from the equilibrium values of A_0^* , A_1^* , B_0^* , and B_1^* , after some algebra. In particular, the coefficient matrix $A_1^* B_1^*$ \blacksquare

$$= \frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_c)^{-1}.$$

PROPOSITION 4.2: The conditional distribution of the i^{th} price, \tilde{p}_1 , given the i^{th} signal, \tilde{s}_1 , is stochastically increasing in the realization of \tilde{s}_1 : if $s_1^1 > s_1^2$, then $(\tilde{p}_1 | s_1^1)$ dominates $(\tilde{p}_1 | s_1^2)$ in the sense of strict first-order stochastic dominance. The same is true for the conditional distribution of \tilde{p}_1 given the i^{th} return, \tilde{v}_1 , and for $(\tilde{s}_1 | \tilde{p}_1)$ and $(\tilde{v}_1 | \tilde{p}_1)$.

Proof: Since $(\tilde{s}_1, \tilde{p}_1)$ and $(\tilde{v}_1, \tilde{p}_1)$ are both bivariate normal random variables, to establish the claim, we only need to consider the covariance matrix of (\tilde{s}, \tilde{p}) and, respectively, (\tilde{v}, \tilde{p}) , and check if the diagonal elements are positive. The first covariance matrix is $\frac{1}{2} \Sigma_v$; the second, $\frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_e)^{-1} \Sigma_v$. Since both matrices are positive definite, the diagonal elements must be positive. ■

Proposition 4.2 tells us about the relationship between the i^{th} price and the i^{th} signal, allowing other variables to vary freely. However, it is the diagonal elements of the coefficient matrix in the price function of Proposition 4.1, $\frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_e)^{-1}$ that are the "partial derivatives" of each price with respect to its own signal: they measure the change in price "holding other things fixed".

Remark: As in Admati (1985), these diagonal elements are not necessarily positive: the coefficient matrix $\frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_e)^{-1}$ is the product of two symmetric positive definite matrices, and therefore does not have any special structure itself. The following examples show that the diagonal elements can be negative: this means that the i^{th} price could, ceteris paribus, be decreasing in the i^{th} signal.

Consider the following numerical example.

Example #1

$$\Sigma_z = I \quad \Sigma_v = \begin{bmatrix} 1 & -9 \\ -9 & 100 \end{bmatrix} \quad \Sigma_e = \begin{bmatrix} 1 & -100 \\ -100 & 10001 \end{bmatrix}$$

This yields the coefficient matrix:

$$\frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_e)^{-1} = \frac{1}{2} \begin{bmatrix} 1.0962 & 0.0109 \\ -9.6165 & -0.0939 \end{bmatrix}$$

and the following equilibrium matrices A_1^* and B_1^* .

$$A_1^* \equiv \{a_{1j}\} = \begin{bmatrix} 0.0681 & -0.4951 \\ -0.4951 & 4.3670 \end{bmatrix}$$

$$B_1^* \equiv \{b_{1j}\} = \begin{bmatrix} 0.2474 & 0.0124 \\ -1.0744 & -0.0093 \end{bmatrix}$$

To understand this example it is useful to begin by considering the effect of correlated error terms in the two private signals. If the value of, say, signal 2, increases, then the trader knows that on average the return on asset 2 would increase: this is the "direct effect" of signal 2 on the trader's belief about asset 2. But if errors are, as in our example, highly correlated, then signal 2 is also informative about the error in signal 1. Since the correlation among errors is negative, a higher value of signal 2 could also imply a lower value of the error in signal 1. This implies that the return on asset 1

could be higher, and since it is strongly negatively correlated with the return on asset 2, this means a lower return on asset 2. This is the "indirect effect" of signal 2 on the trader's belief about asset 2, which, in our example, is opposite in sign from the "direct effect". In our example, the "direct effect" is very weak, because signal 2 is very imprecise (the variance of $\tilde{\epsilon}_2$ is 10001) and the "indirect effect" dominates (after taking into account the "strategic effect", of revelation via order flows) to the point of making b_{22} negative. The market maker realizes that there is a possibility of a less aggressive response from the trader to his own information, and compensates for this by placing substantially more weight (4.3670) on the second order flow. Since in equilibrium $A_1^* B_1^* = \frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_\epsilon)^{-1}$ (the coefficient matrix), when this weight is sufficiently large, the response of price 2 to the signal on asset 2 can become negative.

It is interesting to note that even if this "indirect effect" is not so strong as to make b_{22} negative, the response of price 2 to the return on asset 2 may still be negative. This is demonstrated in the following example.

Example #2

$$\Sigma_z = I \quad \Sigma_v = \begin{bmatrix} 1 & -9 \\ -9 & 100 \end{bmatrix} \quad \Sigma_\epsilon = \begin{bmatrix} 1 & -20 \\ -20 & 10001 \end{bmatrix}$$

This yields the coefficient matrix:

$$\frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_e)^{-1} = \frac{1}{2} \begin{bmatrix} 0.5082 & 0.0005 \\ -4.5457 & -0.0032 \end{bmatrix}$$

$$A_1^* \equiv \{a_{1j}\} = \begin{bmatrix} 0.0497 & -0.3511 \\ -0.3511 & 3.1659 \end{bmatrix}$$

$$B_1^* \equiv \{b_{1j}\} = \begin{bmatrix} 0.1907 & 0.0101 \\ -0.6966 & 0.0006 \end{bmatrix}$$

The parameters for this example are the same as for the previous example, except for the weaker negative correlation among errors. In this case, b_{22} is positive (and the weight the market maker places on order flow 2 is also smaller). Yet the coefficient on signal 2 in price 2 is negative.

This highlights the other source of a negative effect. Given the negative correlation among returns and errors, the market maker places negative weights on cross-order flows, while the trader still places a positive weight on signal 2 in forming a demand for asset 1, because of the "indirect effect". The negative contribution from this term $(-0.3511)(0.0101)$ dominates the other (positive) contribution.

Thus we see from these two examples, that though A_1^* is always symmetric positive definite, little can be said in general about the coefficient matrix $A_1^* B_1^* = \frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_e)^{-1}$. Also, since B_1^* can be non-symmetric, we cannot identify any special properties of B_1^* beyond its existence, uniqueness, and an explicit form in terms of primitive

parameter matrices.

While the technical reason for this ambiguity result is the same as in Admati (1985) — we have a product of symmetric matrices — some reflection suggests that the coefficient matrix in our model is simpler. For one thing, it is independent of Σ_z , the noise covariance matrix. This generalizes the Kyle (1985) result, that more noise leads to more aggressive trading, so that the informativeness of order flows is the same, and independent of the level of noise. For another, as we shall demonstrate in Proposition 4.3 below, we can identify simple sufficient conditions on Σ_v and Σ_c to ensure positive partial derivatives. In the model of Admati (1985), all three parameter matrices are involved, and it is hard to obtain such simple sufficient conditions.

PROPOSITION 4.3: (Sufficient Conditions For Positive Partial Derivatives Of Prices With Respect To Their Own Signals)

The diagonal elements of the coefficient matrix of Proposition 4.1, $\frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_c)^{-1}$, will be positive if any of the following conditions hold:

- (a) $\Sigma_c = \lambda \Sigma_v$, $\lambda \geq 0$
- (b) Σ_v is diagonal.
- (c) Σ_c is diagonal.

Proof: (a) The proof is trivial.

(b) The formula for the i^{th} diagonal element is (denoting $\sigma_{v(i,j)}$ as the (i,j) element in Σ_v , and so on):

$$\sum_j \sigma_{v(i,j)} \cdot (\sigma_v + \sigma_e)^{-1}_{(j,i)} = \sigma_{v(i,1)} \cdot (\sigma_v + \sigma_e)^{-1}_{(1,i)} > 0$$

since $\sigma_{v(i,j)} = 0$ for $i \neq j$, and since $\Sigma_v + \Sigma_e$ is symmetric positive definite.

(c) For this part, it is useful to first state a lemma.

LEMMA 4.1: Let A and B be symmetric positive semi-definite matrices. Then $\det(A + B) \geq \det(A)$.

Proof: See Bellman (1970, p. 117, Theorem 3). ■

The formula for the i^{th} diagonal element of $\Sigma_v(\Sigma_v + \Sigma_e)^{-1}$ is:

$$\sum_j \sigma_{v(i,j)} \cdot (\sigma_v + \sigma_e)^{-1}_{(j,i)} \quad (30)$$

Define a matrix $C_1 \equiv \{c_{tj}\}$ $\begin{cases} \sigma_{v(t,j)} & t = j = i \\ (\sigma_v + \sigma_e)_{(t,j)} & \text{otherwise} \end{cases}$

In other words, C_1 is identical to $\Sigma_v + \Sigma_e$ except possibly for the i^{th} diagonal element. Therefore, elements in the i^{th} column of $(\Sigma_v + \Sigma_e)^{-1}$ are just the corresponding elements of the i^{th} column of the adjoint of C_1 , divided by a scale factor, $\det(\Sigma_v + \Sigma_e)$. Thus (30) can be interpreted as the dot product of the i^{th} row of C_1 , and the i^{th} column of the adjoint of C_1 (modified by a scale factor). Then, using the well-known result that for any matrix J, $J \cdot \text{adjoint}(J) = \det(J) \cdot I$, (30) is equivalent to $\det(C_1) / \det(\Sigma_v + \Sigma_e)$, which is positive since both the numerator and the denominator are positive. The denominator is positive

since $\Sigma_v + \Sigma_e$ is positive definite; the numerator, since Lemma 4.1 tells us that $\det(C_1) \geq \det(\Sigma_v) > 0$. ■

While (a) seems to be a strong condition, it is important to note the class of admissible matrices for which it holds extends beyond the perfect information case ($\lambda = 0$). The condition stipulates that large variances or covariances among returns are associated with large variances or covariances among errors. Parts (b) and (c) show that diagonal elements in the price coefficient matrix can be positive, even without requiring it to be symmetric positive definite. It is also important to note that since the product of two continuous functions is a continuous function, even if the conditions in Proposition 4.3 are not met exactly, for matrices "sufficiently close" to the matrices in these conditions, the diagonal elements in the price coefficient matrix will still be positive. Proposition 4.3 should be useful for applications as in experimental economics, where they provide an easy way of obtaining a price coefficient matrix with desirable properties.

The general possibility of ambiguous response coefficients was noted in the context of perfect competition by Admati (1985). Her results were both important and disturbing. They were important because they were the result of extending the model of Hellwig (1980) to a multi-asset setting with a general correlation structure, which is presumably the more realistic assumption to make. While Admati (1985) predicted that a general model would pave the way for applications, our considered view is that this has not happened, for a simple reason. The generalization involved a serious loss of predictive content.

To understand this, it will help to consider a well-defined class of applications in accounting and finance called "event studies". A good example would be studies examining the informational efficiency of prices with respect to earnings announcements (see e.g. Conedes, Dopuch, Penman (1976)). In the context of an asset pricing model like the CAPM — with exogenous and homogeneous beliefs — it is easy to assert that "good news", i.e. predictions about above-average terminal values captured in above-average earnings should cause prices to go up, since each price will always be increasing in its own return.

Given the vast increase in the number of models with private information, a natural question to ask is: if we make the more realistic assumption that significant private information exists, can we continue to use traditional event study procedures, in particular pooling of (unexpected return, unexpected earnings) observations to assess "information content"? Admati's work shows that the expected price reaction to a prediction of above-average terminal values can be ambiguous. So we cannot justify pooling of observations any more, even if our concern is only with the sign of the association.

One way to resolve this problem would be to estimate primitive parameters, and then compute the estimated price coefficient matrix, which would give us the sign of the expected price reaction. Given the explosive growth in the number of parameters in these models, the general problem has never even been attempted.

We know that even in our case, unless we impose appropriate restrictions, the problem remains. The next proposition tells us, however, that our model which adds to the realistic assumption of

private information another dose of realism, strategic behavior, can be useful for a slightly different class of event studies (sometimes called "reverse regression models"). Technically, it is a simple consequence of having market makers condition on all order flows in setting each price, and of the equilibrium A_1^* being invertible. This means that for an econometrician to observe ex post the vector of prices is equivalent to observing the vector of order flows. Since market makers extract information optimally from order flows in setting each price, to learn about the i^{th} return it is sufficient to condition on the i^{th} price.

PROPOSITION 4.4: (a) All public information about the i^{th} return is always contained in the i^{th} price.

(b) The informativeness of prices, measured by the reduction in the prior variance-covariance matrix of the return vector, when we condition on the vector of prices, $\text{Var}(\tilde{v}) - \text{Var}(\tilde{v}|\tilde{p})$, is given by $\frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_e)^{-1} \Sigma_v$.

Proof: What part (a) asserts is that $\text{Var}(\tilde{v}_i | p_i) = \text{Var}(\tilde{v}_i | p)$. To see why this is true we must compute the variance-covariance matrix of the partitioned vector $[\tilde{v}, \tilde{p}]$. This is given by the appropriately partitioned matrix below.

$$\Sigma_{v,p} = \begin{bmatrix} \Sigma_v & \frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_e)^{-1} \Sigma_v \\ \frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_e)^{-1} \Sigma_v & \Sigma_p \end{bmatrix}$$

But using (29), we get:

$$\begin{aligned} \Sigma_p &= \frac{1}{4} \Sigma_v (\Sigma_v + \Sigma_\varepsilon)^{-1} (\Sigma_v + \Sigma_\varepsilon) (\Sigma_v + \Sigma_\varepsilon)^{-1} \Sigma_v \\ &\quad + \frac{1}{4} \Sigma_v (\Sigma_v + \Sigma_\varepsilon)^{-1} B_1^{-1} \Sigma_z (B_1^T)^{-1} (\Sigma_v + \Sigma_\varepsilon)^{-1} \Sigma_v \end{aligned} \quad (31)$$

Note that from (A20) and (A21) in the Appendix, $B_1^T = (\Sigma_v + \Sigma_\varepsilon)^{-1} B_1^{-1} \Sigma_z$, so (31) implies

$$\Sigma_p = \frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_\varepsilon)^{-1} \Sigma_v \quad (32)$$

Denote by $\hat{\sigma}_{11}$ the (i, i) -element of $\Sigma_v (\Sigma_v + \Sigma_\varepsilon)^{-1} \Sigma_v$ and by $\hat{\sigma}_1$, the i^{th} column vector of the same matrix. Then

$$\text{Var}(\tilde{v}_1 | p_1) = \sigma_{v(1,1)} - \frac{1}{2} \hat{\sigma}_{11} \left(\frac{1}{2} \hat{\sigma}_{11} \right)^{-1} \cdot \frac{1}{2} \hat{\sigma}_{11} = \sigma_{v(1,1)} - \frac{1}{2} \hat{\sigma}_{11}$$

and

$$\text{Var}(\tilde{v}_1 | p) = \sigma_{v(1,1)} - \frac{1}{2} (\hat{\sigma}_1)^T \left(\frac{1}{2} \Sigma_v (\Sigma_v + \Sigma_\varepsilon)^{-1} \Sigma_v \right) \cdot \frac{1}{2} \hat{\sigma}_1 = \sigma_{v(1,1)} - \frac{1}{2} \hat{\sigma}_{11}$$

Part (b) follows from joint normality, and our definition of

$\Sigma_{v,p}$. ■

Note that Σ_p is identical to the two off-diagonal blocks of $\Sigma_{v,p}$. Thus expected returns \tilde{v} given prices \tilde{p} is simply:

$$E(\tilde{v} | \tilde{p} = p) = \bar{v} + I(p - \bar{v}) = p \quad (33)$$

and we see that only the i^{th} price is useful in predicting the i^{th} return.

Though a regression of prices on terminal values involved a non-diagonal coefficient matrix, the corresponding reverse regression

involves only a diagonal matrix. This is one subtle difference between the multivariate case and univariate case, in which a non-zero coefficient in a regression implies a non-zero coefficient in the corresponding reverse regression. The practical value of this proposition lies in the justification it provides for reverse-regression based event studies (e.g. Beaver, Lambert, Ryan (1986)), even in settings assumed to allow for private information.

Part (b), which follows from the computation of the posterior variance-covariance matrix, tells us that with perfect information ($\Sigma_{\epsilon} = 0$), as in Kyle (1985), only half the information is incorporated in prices. From a purely theoretical perspective this is interesting in view of the great scarcity of robust results in models with private information. Given a linear pricing rule, regardless of the underlying correlation structure, as market makers try and learn from all order flows, the trader tries to minimize such learning (to the extent consistent with optimization) so that, in equilibrium, market makers learn about each return only as much as they would in a single-security world.

Finally, we record a general proposition about profits.

PROPOSITION 4.5: The informed trader's ex ante expected profits are given by:

$$\text{trace}(B_1^* \Sigma_v) - \frac{1}{2} \text{trace} \left\{ (\Sigma_v + \Sigma_{\epsilon})^{-1} \Sigma_v B_1^* (\Sigma_v + \Sigma_{\epsilon}) \right\} \quad (34)$$

Proof:

$$E(\tilde{\pi}) = E\{[\tilde{v} - A_0 - A_1 X(\tilde{s}) - A_1 \tilde{z}]^T X(\tilde{s})\}$$

Substituting equilibrium values, and simplifying we get an expectation of a sum of two quadratic forms, in $(\tilde{v} - \bar{v})$ and $(\tilde{s} - \bar{s})$. The expectation of these quadratic forms, with respect to \tilde{v} and \tilde{s} respectively, is easily accomplished using Graybill (1983, p. 341). ■

5. PERFECT PRIVATE INFORMATION

In this section we continue to allow the correlation structure associated with returns and with noise trading to be completely general, but we now assume that private information is perfect. This special case is important because it is a setting in which we can rule out the "correlated-regressors effect" which can create "perverse" predictions of asset returns. This highlights the power of the "strategic effect", which, we shall see, can neutralize all of the correlation due to returns and noise, and distill "joint" effects into "pure" effects.

PROPOSITION 5.1: Under perfect private information, the equilibrium solution is given by:

- (i) $\frac{1}{4} M^T \Sigma_V M = I$
- (ii) $M^T \Sigma_Z^{-1} M = D$
- (iii) $A_1^* = (M^T)^{-1} D^{1/2} M^{-1}$

$$(iv) B_1^* = \frac{1}{2} A_1^{-1}$$

$$(v) A_0^* = \bar{v} - A_1^* \bar{z}$$

$$(vi) B_0^* = -B_1^* \bar{v}$$

(Proofs are omitted in this section since they all follow from setting $\Sigma_e = 0$.)

Proposition 5.1 tells us that under perfect private information, we are in a position to assert not only the symmetry and positive definiteness of A_1^* , but also the symmetry and positive definiteness of B_1^* , despite possible asymmetries across assets pertaining to returns and liquidity noise.

PROPOSITION 5.2: The equilibrium variance-covariance matrix of the partitioned vector $[\tilde{v}, \tilde{p}]$ is given by the following (conformably partitioned) matrix:

$$\begin{bmatrix} \Sigma_v & \frac{1}{2} \Sigma_v \\ \frac{1}{2} \Sigma_v & \frac{1}{2} \Sigma_v \end{bmatrix}$$

This demonstrates the remarkable robustness of the original Kyle (1985) result, that with one informed trader and perfect private information, the posterior precision is exactly twice the prior precision.

PROPOSITION 5.3: The equilibrium price vector \tilde{p} , as a function of the return vector \tilde{v} and the noise vector \tilde{z} , is given by:

$$\tilde{p} = \bar{v} + \frac{1}{2} (\tilde{v} - \bar{v}) + A_1^* \tilde{z} \quad \blacksquare$$

Regardless of the correlation structure, prices are always strictly increasing in their own returns. Also, market makers, who do not have any access to private information, and who must rely exclusively upon (imperfect and correlated) order flow information, set prices "as if" each price is determined only by its own return.

The relative robustness of these results stands in stark contrast to the general flavor of results under imperfect information, where a slight perturbation in the information structure can often sharply change the nature of equilibrium. Essentially, the only real restriction in our analysis is that of a linear equilibrium (which is also true of most models in the literature on perfect-competition rational expectations equilibrium).

6. A TWO-ASSET EXAMPLE

The matrix treatment we have adopted so far in this paper has been convenient for handling the general n -asset case. However, this obscures the role of individual elements of different variance-covariance matrices in the analysis, and in particular, it tells us very little about the effect of correlation parameters (which

are after all what makes the multi-asset case qualitatively different from the single-asset case). To rectify this and to build more intuition, we now study in detail a two-asset example, with perfect private information and with all variances equal to unity. This enables us to examine limits and comparative statics involving the return correlation parameter, ρ_v , and the noise correlation parameter, ρ_z . We also illustrate all of the propositions in this paper.

Given the parameter matrices below:

$$\Sigma_v = \begin{bmatrix} 1 & \rho_v \\ \rho_v & 1 \end{bmatrix} \quad \Sigma_z = \begin{bmatrix} 1 & \rho_z \\ \rho_z & 1 \end{bmatrix} \quad \Sigma_\epsilon = 0$$

with both ρ_v and $\rho_z \in (-1, 1)$, we obtain the following diagonal matrix D , where the diagonal elements are the eigenvalues of Σ_z^{-1} in the metric of $\frac{1}{4} \Sigma_v$.

$$D = \begin{bmatrix} \frac{1}{(1 + \rho_v)(1 + \rho_z)} & 0 \\ 0 & \frac{1}{(1 - \rho_v)(1 - \rho_z)} \end{bmatrix}$$

These eigenvalues yield the following eigenmatrix M :

$$M = \begin{bmatrix} \frac{1}{\sqrt{2(1 + \rho_z)}} & \frac{1}{\sqrt{2(1 - \rho_z)}} \\ \frac{-1}{\sqrt{2(1 + \rho_z)}} & \frac{-1}{\sqrt{2(1 - \rho_z)}} \end{bmatrix}$$

We now state various propositions pertaining to this example, the proofs of which are mainly computational.

PROPOSITION 6.1: In equilibrium,

(i) the "own-coefficients" in the pricing rule are:

$$a_{11} = a_{22} = \frac{1}{4} \left[\sqrt{\frac{1 - \rho_v}{1 - \rho_z}} + \sqrt{\frac{1 + \rho_v}{1 + \rho_z}} \right]$$

(ii) the "cross-coefficients" in the pricing rule are:

$$a_{12} = a_{21} = -\frac{1}{4} \left[\sqrt{\frac{1 - \rho_v}{1 - \rho_z}} - \sqrt{\frac{1 + \rho_v}{1 + \rho_z}} \right]$$

(iii) the "own-coefficients" in the demand strategies are:

$$b_{11} = b_{22} = \frac{1}{2} \left[\sqrt{\frac{1 + \rho_z}{1 + \rho_v}} + \sqrt{\frac{1 - \rho_z}{1 - \rho_v}} \right]$$

(iv) the "cross-coefficients" in the demand strategies are:

$$b_{12} = b_{21} = \frac{1}{2} \left[\sqrt{\frac{1 + \rho_z}{1 + \rho_v}} - \sqrt{\frac{1 - \rho_z}{1 - \rho_v}} \right] \quad \blacksquare$$

Table 2 below collects information about limits and Table 3 about comparative statics. Figures 1-4 illustrate the behavior of the four sets of coefficients for different values of ρ_z and ρ_v . As the trader disguises her strategy by placing less weight on the i^{th} security, the market maker compensates for this by putting more weight

on the i^{th} order flow in determining the i^{th} price. As both $\rho_v, \rho_z \rightarrow 0$, we have 'n' independent "Kyle markets", as can be verified by comparing our coefficients with the corresponding coefficients in Kyle (1985, p. 1319).

Table 2 — Limits

	$\rho_z \rightarrow 1$	$\rho_z \rightarrow -1$	$\rho_z \rightarrow 0$
b_{11}	$\frac{1}{\sqrt{2(1+\rho_v)}}$	$\frac{1}{\sqrt{2(1-\rho_v)}}$	$\frac{1}{2} \left[\frac{1}{\sqrt{1+\rho_v}} + \frac{1}{\sqrt{1-\rho_v}} \right]$
b_{1j} $i \neq j$	$\frac{1}{\sqrt{2(1+\rho_v)}}$	$\frac{-1}{\sqrt{2(1-\rho_v)}}$	$\frac{1}{2} \left[\frac{1}{\sqrt{1+\rho_v}} - \frac{1}{\sqrt{1-\rho_v}} \right]$
a_{11}	∞	∞	$\frac{\sqrt{1+\rho_v} + \sqrt{1-\rho_v}}{4}$
a_{1j} $i \neq j$	$-\infty$	∞	$\frac{\sqrt{1+\rho_v} - \sqrt{1-\rho_v}}{4}$

Table 2 — Limits (continued)

	$\rho_v \rightarrow 1$	$\rho_v \rightarrow -1$	$\rho_v \rightarrow 0$	$\rho_v, \rho_z \rightarrow 0$
b_{11}	∞	∞	$\frac{\sqrt{1+\rho_z} + \sqrt{1-\rho_z}}{2}$	1
b_{1j} $i \neq j$	$-\infty$	∞	$\frac{\sqrt{1+\rho_z} - \sqrt{1-\rho_z}}{2}$	0
a_{11}	$\frac{1}{2\sqrt{2(1+\rho_z)}}$	$\frac{1}{2\sqrt{2(1-\rho_z)}}$	$\frac{1}{4} \left[\frac{1}{\sqrt{1+\rho_z}} + \frac{1}{\sqrt{1-\rho_z}} \right]$	$\frac{1}{2}$
a_{1j} $i \neq j$	$\frac{1}{2\sqrt{2(1+\rho_z)}}$	$\frac{-1}{2\sqrt{2(1-\rho_z)}}$	$\frac{1}{4} \left[\frac{1}{\sqrt{1+\rho_z}} - \frac{1}{\sqrt{1-\rho_z}} \right]$	0

Table 3 — Comparative Statics

Own-coefficients

$\frac{\partial b_{11}}{\partial \rho_v} > 0$ if $\rho_v > 0, \rho_v > -\rho_z$ $\frac{\partial b_{11}}{\partial \rho_v} < 0$ if $\rho_v < 0, \rho_v < -\rho_z$	$\frac{\partial a_{11}}{\partial \rho_v} < 0$ if $\rho_v > -\rho_z$ $\frac{\partial a_{11}}{\partial \rho_v} > 0$ if $\rho_v < -\rho_z$
$\frac{\partial b_{11}}{\partial \rho_z} < 0$ if $\rho_z > -\rho_v$ $\frac{\partial b_{11}}{\partial \rho_z} > 0$ if $\rho_z < -\rho_v$	$\frac{\partial a_{11}}{\partial \rho_z} > 0$ if $\rho_z > 0, \rho_z > -\rho_v$ $\frac{\partial a_{11}}{\partial \rho_z} < 0$ if $\rho_z < 0, \rho_z < -\rho_v$

Table 3 — Comparative Statics (continued)

Cross-coefficients

For $i \neq j$

$\frac{\partial b_{1j}}{\partial \rho_z} > 0$	$\frac{\partial a_{1j}}{\partial \rho_z} < 0$
$\frac{\partial b_{1j}}{\partial \rho_v} < 0$	$\frac{\partial a_{1j}}{\partial \rho_v} > 0$

(INSERT FIGURES 1-4 HERE)

Cross-effects on prices provide a new explanation for why the nature of news (good or bad) may not always be related to the demand for a security in an intuitive way. Even if there is very good news about a particular security, if it is strongly positively correlated with another security, a trader would choose to "dampen" demand in a bid to keep both prices down. This is why the cross-coefficients b_{1j} , $i \neq j$, become negative. It is important to realize that there is a cost — foregone opportunities — in trying to reduce what the market maker may learn, and therefore this "cross-effect" may not always dominate the "own-effect". In Proposition 6.2 below, we record the regions in (v_1, v_j) -space, in which such "counterintuitive" effects on demand will occur.

PROPOSITION 6.2: For $\rho_v > \rho_z$, $\forall i, i \neq j$

(i) If $(v_j - \bar{v}_j) > (-b_{11}/b_{1j})(v_1 - \bar{v}_1)$ then $x_1 < 0$ even if $v_1 > \bar{v}_1$

(ii) If $(v_j - \bar{v}_j) < (-b_{11}/b_{1j})(v_1 - \bar{v}_1)$ then $x_1 > 0$ even if $v_1 < \bar{v}_1$.

For $\rho_v < \rho_z$, $\forall i, i \neq j$

(iii) If $(v_j - \bar{v}_j) > (-b_{11}/b_{1j})(v_1 - \bar{v}_1)$ then $x_1 > 0$ even if $v_1 < \bar{v}_1$

(iv) If $(v_j - \bar{v}_j) < (-b_{11}/b_{1j})(v_1 - \bar{v}_1)$ then $x_1 < 0$ even if $v_1 > \bar{v}_1$

$$\text{where } \frac{-b_{11}}{b_{1j}} = \frac{\sqrt{(1 - \rho_v)(1 + \rho_z)} + \sqrt{(1 + \rho_v)(1 - \rho_z)}}{\sqrt{(1 + \rho)(1 - \rho)} - \sqrt{(1 - \rho)(1 + \rho)}}$$

In (i) above, the cross-coefficient $b_{1j} < 0$, hence $(-b_{11}/b_{1j}) > 0$. Thus, for very large $v_j - \bar{v}_j$, even if $v_1 - \bar{v}_1$ is positive, the informed trader would minimize revelation about v_j by holding a negative position in asset i , and try to reap large returns by holding asset j . Conversely, if $v_j - \bar{v}_j$ is very negative, as in (ii), even if $v_1 - \bar{v}_1$ is negative, the trader would seek a positive position in asset i , which would boost both prices, and make profits by selling asset j .

It is worth stressing that these "cross-effects" arise in our model solely as a consequence of strategic behavior, i.e. as a result of the interactions between the market maker's incentive to learn, and the informed trader's incentive to reduce such learning. Analyses of

correlated environments under perfect competition (e.g. Admati (1985)) ignore — by the very definition of perfect competition — the latter effect, of trying to reduce what others can learn, and consider only the possibility of learning. Thus we see that while with perfect private information we cannot have the various ambiguities cited in Admati (1985) — all of which stem from the ambiguities associated with predictions of terminal values — strategic behavior can help generate other kinds of "non-intuitive" results. Whether (in a more general model) the purely statistical effect identified by Admati (1985) and the strategic effect we have focused on will reinforce or neutralize each other is, ultimately, an empirical question: it will depend on the actual parametrization.

The equilibrium variance-covariance matrix of $(\tilde{v}_1, \tilde{v}_2, \tilde{p}_1, \tilde{p}_2)$ is given by:

$$\begin{bmatrix} 1 & \rho_v & \frac{1}{2} & \frac{\rho_v}{2} \\ \rho_v & 1 & \frac{\rho_v}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\rho_v}{2} & \frac{1}{2} & \frac{\rho_v}{2} \\ \frac{\rho_v}{2} & \frac{1}{2} & \frac{\rho_v}{2} & \frac{1}{2} \end{bmatrix}$$

This illustrates Proposition 5.2.

Some features of the relationship between returns and prices are very simple. For example,

$$\text{Cov}(\tilde{v}_1, \tilde{p}_1) = \frac{1}{2}$$

$$\text{sign}(\text{Cov}(\tilde{v}_1, \tilde{p}_j)) = \text{sign}(\rho_v) \quad \forall i, i \neq j$$

$$\text{Var}(\tilde{v}_1 | \tilde{p}_1) = \text{Var}(\tilde{v}_1 | \tilde{p}_1, \tilde{p}_j) = \frac{1}{2} \quad \forall i, i \neq j$$

all of which are intuitive. Allowing other variables to vary freely, good news about one security's return would increase its own price, and the other price as well, if the returns are positively correlated. Also, all information about the i^{th} return is contained in the i^{th} price.

We now use this to compute expected profits.

PROPOSITION 6.3: The ex ante expected profits of the informed trader are given by:

$$E(\tilde{\pi}) = \frac{1}{2} \text{trace}(B_i^* \Sigma_v) = \frac{1}{2} \left[\sqrt{(1 - \rho_z)(1 - \rho_v)} + \sqrt{(1 + \rho_z)(1 + \rho_v)} \right] \quad (35)$$

(INSERT FIGURE 5 HERE)

If $\rho_z = 0$, we see in Figure 5 that profits are highest when $\rho_v = 0$. As $|\rho_v|$ increases, order flows become more revealing due to "cross-effects", and so profits decline. If $\rho_v = 0$, profits are highest when $\rho_z = 0$. As $|\rho_z|$ increases, liquidity traders provide less "camouflage", and hence profits decline. In general, the trader benefits the most if $\rho_v = \rho_z$, i.e. revelation is minimized when returns and liquidity noise correlations change similarly, and so preserve the level of camouflage.

One interesting issue arises when we recognize that even with two risky projects, a firm may sometimes choose to issue only one security. How does a firm make this securitization decision (or, for an existing firm, the capital restructuring decision)? A general answer is beyond the scope of this paper — it will involve, for instance, also an analysis of the allocation of voting rights, as in Grossman and Hart (1988). However, for the limited case of established firms (controlled by purely speculative managers) deciding whether to merge or spinoff, Proposition 6.4 identifies one incentive. We assume that managers are able to participate in the market, and anticipate access to private information after the securitization decision has been made.

Assume the same fundamentals as the rest of this section. But assume that there is only one security, and that all liquidity traders now deal in this one security, i.e. $\tilde{z} = \tilde{z}_1 + \tilde{z}_2 \sim N(\bar{z}_1 + \bar{z}_2, \sigma_z^2)$ where $\sigma_z^2 = \sigma_{z1}^2 + \sigma_{z2}^2 + 2\rho_z \sigma_{z1} \sigma_{z2} = 2(1 + \rho_z)$, and the return $\tilde{v} = \tilde{v}_1 + \tilde{v}_2 \sim N(\bar{v}_1 + \bar{v}_2, \sigma_v^2)$ where $\sigma_v^2 = \sigma_{v1}^2 + \sigma_{v2}^2 + 2\rho_v \sigma_{v1} \sigma_{v2} = 2(1 + \rho_v)$.

PROPOSITION 6.4: The trader's ex ante expected profits with one security would be:

$$E(\tilde{\pi}) = \sqrt{\frac{1 + \rho_v}{1 + \rho_z}}$$

This exceeds the expected profits with two securities (given in (35)), if and only if $\rho_v > \rho_z$. ■

(INSERT FIGURE 6 HERE)

Figure 6 illustrates the comparison for $\rho_z = 0$. When $\rho_v > \rho_z$, i.e. when the returns are sufficiently positively correlated (or insufficiently negatively correlated), the manager is better off with a single security because less is then revealed of her private information about each project. When $\rho_v < \rho_z$, a different effect dominates: by issuing two securities, she is able to better discriminate or exploit divergent opportunities (i.e. she can go long in one while going short in the other), which are more likely to arise when $\rho_v < \rho_z$ i.e. when the returns are sufficiently negatively correlated (or insufficiently positively correlated), and this benefit exceeds the negative impact of greater revelation.

7. CONCLUSION

This paper has developed a model of multi-asset pricing. In contrast to the traditional CAPM our model recognizes the existence of private information, and strategic interaction between the informed and uninformed. Unlike the perfect competition rational expectations model which has little predictive content because of the dominance of the correlated-regressors effect, in our model strategic behavior restores some regularities, even in the general case with imperfect private information. All public information about a security's return is contained in its own price. In fact, all of the correlatedness due to

returns and noise can be neutralized. This is because an informed trader realizes that market makers can learn from correlated variables, and so has an incentive to manipulate order flows, and "lower" the correlation.

APPENDIX

In this appendix we provide details pertaining to various results in the paper.

Note: Whenever we refer to a positive definite (or semi-definite) matrix in this paper, it should be understood to mean a symmetric positive definite (or semi-definite) matrix.

Before we consider our model itself, it will be useful to state a well-known result in linear algebra which we use in the sequel.

LEMMA A1: Let A be an $n \times n$ positive definite matrix; and B , an $n \times n$ positive semi-definite matrix.

There exists a nonsingular real matrix F such that:

$$(i) F^T A F = I \quad \text{and} \quad (ii) F^T B F = D$$

where I refers to the identity matrix and D to a positive definite diagonal matrix whose diagonal values are given by the roots (eigenvalues) of the characteristic equation of B in the metric of A :

$$\det(B - \lambda A)$$

The matrix F is unique if the eigenvalues are distinct. If the eigenvalues are not distinct, the eigenvectors corresponding to repeated roots may be replaced by any independent linear combination.

Proof: See Franklin (1968, p. 106). ■

The reader should note that the variance-covariance matrices associated with returns and liquidity trading noise, Σ_v and Σ_z , are assumed to be positive definite, while the variance-covariance matrix of errors in private signals, Σ_e , is positive semi-definite (i.e., we allow for perfect private information on some or all assets).

Proof of Lemma 3.2: Since the signal vector is given by $\tilde{s} = \tilde{v} + \tilde{e}$, and the order flow vector by $\tilde{w} = B_0 + B_1 \tilde{s} + \tilde{z}$, the variance-covariance matrix of (\tilde{v}, \tilde{w}) is

$$\begin{bmatrix} \Sigma_v & \Sigma_v B_1^T \\ B_1^T \Sigma_v & B_1 (\Sigma_v + \Sigma_e) B_1^T + \Sigma_z \end{bmatrix}$$

Then,

$$\begin{aligned} P(\tilde{w}) &= E(\tilde{v} | \tilde{w} = w) = \\ &= \tilde{v} + \Sigma_v B_1^T \left[B_1 (\Sigma_v + \Sigma_e) B_1^T + \Sigma_z \right]^{-1} (w - B_0 - B_1 \tilde{v} - \tilde{z}) \end{aligned} \quad (A1)$$

So we have, using (2) and equating coefficients

$$\begin{aligned} A_1 &= \Sigma_v B_1^T \left[B_1 (\Sigma_v + \Sigma_e) B_1^T + \Sigma_z \right]^{-1} = \left\{ \left[B_1 (\Sigma_v + \Sigma_e) B_1^T + \Sigma_z \right] (B_1^T)^{-1} \Sigma_v^{-1} \right\}^{-1} = \\ &= \left[B_1 (\Sigma_v + \Sigma_e) \Sigma_v^{-1} + \Sigma_z (B_1^T)^{-1} \Sigma_v^{-1} \right]^{-1} \end{aligned} \quad (A2)$$

(Note, this means that A_1 is invertible.)

and

$$A_0 = \tilde{v} - A_1 [B_0 + B_1 \tilde{v} + \tilde{z}] \quad (A3)$$

Proof of Lemma 3.3: Let us solve for B_1 and A_1 . From (A2) and (20), and since A_1 and $(A_1 + A_1^T)$ are both invertible, we get:

$$(A_1 + A_1^T)^{-1} = A_1^{-1}(A_1^{-1} + (A_1^T)^{-1})(A_1^T)^{-1} \quad (A4)$$

where

$$A_1^T = \left[B_1(\Sigma_v + \Sigma_e)\Sigma_v^{-1} + \Sigma_v^{-1}B_1^{-1}\Sigma_z \right]^{-1} \quad (A5)$$

$$\begin{aligned} (A_1 + A_1^T)^{-1} &= \\ &= \left\{ \left[B_1(\Sigma_v + \Sigma_e)\Sigma_v^{-1} + \Sigma_z(B_1^T)^{-1}\Sigma_v^{-1} \right]^{-1} + \left[B_1(\Sigma_v + \Sigma_e)\Sigma_v^{-1} + \Sigma_v^{-1}B_1^{-1}\Sigma_z \right]^{-1} \right\}^{-1} = \\ &= \left[B_1(\Sigma_v + \Sigma_e)\Sigma_v^{-1} + \Sigma_z(B_1^T)^{-1}\Sigma_v^{-1} \right] \cdot \\ &\quad \cdot \left[2B_1(\Sigma_v + \Sigma_e)\Sigma_v^{-1} + \Sigma_z(B_1^T)^{-1}\Sigma_v^{-1} + \Sigma_v^{-1}B_1^{-1}\Sigma_z \right]^{-1} \cdot \\ &\quad \cdot \left[B_1(\Sigma_v + \Sigma_e)\Sigma_v^{-1} + \Sigma_v^{-1}B_1^{-1}\Sigma_z \right] = \end{aligned} \quad (A6)$$

(we have used above the fact that $B_1(\Sigma_v + \Sigma_e)\Sigma_v^{-1}$ is symmetric.)

$$= B_1(\Sigma_v + \Sigma_e)\Sigma_v^{-1} \quad (A7)$$

The last equality follows from (20). Postmultiply both sides of the last equality by $\Sigma_v(\Sigma_v + \Sigma_e)^{-1}B_1^{-1}$. Then premultiply each side by $[B_1(\Sigma_v + \Sigma_e)^{-1}\Sigma_v^{-1} + \Sigma_z(B_1^T)^{-1}\Sigma_v^{-1}]^{-1}$ and postmultiply each side by $[I + \Sigma_v^{-1}B_1^{-1}\Sigma_z\Sigma_v(\Sigma_v + \Sigma_e)^{-1}B_1^{-1}]^{-1}$. Then take inverses on both sides, and

simplify. We will get:

$$B_1(\Sigma_v + \Sigma_c)\Sigma_v^{-1} = \left[\Sigma_v^{-1}B_1^{-1}\Sigma_z \right] \cdot \left[\Sigma_v(\Sigma_v + \Sigma_c)^{-1}B_1^{-1} \right] \cdot \left[\Sigma_z(B_1^T)^{-1}\Sigma_v^{-1} \right] \quad (A8)$$

$$\text{Define } P = B_1(\Sigma_v + \Sigma_c)\Sigma_v^{-1} \quad (A9)$$

(P is symmetric positive definite, by (20).)

$$\Rightarrow P^{-1} = \Sigma_v(\Sigma_v + \Sigma_c)^{-1}B_1^{-1} \quad (A10)$$

$$\Rightarrow B_1^{-1} = (\Sigma_v + \Sigma_c)\Sigma_v^{-1}P^{-1} \quad (A11)$$

and

$$\Rightarrow (B_1^T)^{-1} = P^{-1}\Sigma_v^{-1}(\Sigma_v + \Sigma_c) \quad (A12)$$

Using (A9)-(A12), (A8) can be rewritten as:

$$P = \Sigma_v^{-1}(\Sigma_v + \Sigma_c)\Sigma_v^{-1}P^{-1}\Sigma_zP^{-1}\Sigma_zP^{-1}\Sigma_v^{-1}(\Sigma_v + \Sigma_c)\Sigma_v^{-1} \quad (A13)$$

Define $H = \Sigma_v^{-1}(\Sigma_v + \Sigma_c)\Sigma_v^{-1}$ (H is positive definite). Then

(A13) becomes:

$$H^{-1}PH^{-1} = P^{-1}\Sigma_zP^{-1}\Sigma_zP^{-1} \quad (A14)$$

Since P is positive definite, there exists an orthogonal matrix F such that:

$P = F^T\Lambda F$, where Λ is a diagonal matrix with the (positive) eigenvalues of P along the diagonal. Substituting into (A14), we get:

$$H^{-1}F^T\Lambda FH^{-1} = F^T\Lambda^{-1}F\Sigma_zF^T\Lambda^{-1}F\Sigma_zF^T\Lambda^{-1}F \quad (A15)$$

Factoring Λ and Λ^{-1} into their unique positive definite square

roots, premultiplying by $\Lambda^{1/2}F$ and postmultiplying by $F^T\Lambda^{1/2}$, we get:

$$(\Lambda^{1/2}F^T H^{-1} F \Lambda^{1/2})(\Lambda^{1/2}F H^{-1} F^T \Lambda^{1/2}) = (\Lambda^{-1/2}F \Sigma_z F^T \Lambda^{-1/2})(\Lambda^{-1/2}F \Sigma_z F^T \Lambda^{-1/2}) \quad (A16)$$

$$\text{Define } L = \Lambda^{1/2}F^T H^{-1} F \Lambda^{1/2} \text{ and } R = \Lambda^{-1/2}F \Sigma_z F^T \Lambda^{-1/2},$$

so that (A16) becomes

$$LL^T = RR^T \quad (A17)$$

Not only are LL^T and RR^T symmetric positive definite but so are L and R . Since the symmetric positive definite square root of a symmetric positive definite matrix is unique (see, e.g. Bellman (1970), pp. 93-94), it is legal to write:

$$L = R \Rightarrow \Lambda^{1/2}F^T H^{-1} F \Lambda^{1/2} = \Lambda^{-1/2}F \Sigma_z F^T \Lambda^{-1/2} \quad (A18)$$

Premultiplying by $F\Lambda^{1/2}$, postmultiplying by $\Lambda^{1/2}F^T$, and using the orthogonality of F , we get:

$$F A F^T H^{-1} F A F^T = \Sigma_z \quad (A19)$$

$$\Rightarrow PH^{-1}P = \Sigma_z$$

$$\Rightarrow \left[B_1 (\Sigma_v + \Sigma_e) \Sigma_v^{-1} \right] \cdot \left[\Sigma_v (\Sigma_v + \Sigma_e)^{-1} \Sigma_v \right] \cdot \left[B_1 (\Sigma_v + \Sigma_e) \Sigma_v^{-1} \right] = \Sigma_z$$

$$\Rightarrow B_1 \Sigma_v B_1 (\Sigma_v + \Sigma_e) \Sigma_v^{-1} = \Sigma_z$$

$$\Rightarrow B_1 (\Sigma_v + \Sigma_e) \Sigma_v^{-1} = \Sigma_v^{-1} B_1^{-1} \Sigma_z = \quad (A20)$$

and, by symmetry of the LHS (implied by (20))

$$= \Sigma_v^{-1}(\Sigma_v + \Sigma_c)B_1^T = \Sigma_z(B_1^T)^{-1}\Sigma_v^{-1} \quad (A21)$$

(A21), together with (A2), establishes that

$$A_1 = \frac{1}{2} \Sigma_v(\Sigma_v + \Sigma_c)^{-1}B_1^{-1} = \frac{1}{2} \Sigma_z^{-1}B_1\Sigma_v = \frac{1}{2} (A_1 + A_1^T) \quad (A22)$$

i.e., A_1 is symmetric (and positive definite). ■

Proof of Lemma 3.4: To get an expression for A_1 in terms of the primitive parameter matrices Σ_v , Σ_z , and Σ_c , note that from (A22)

$$B_1 = 2\Sigma_z A_1 \Sigma_v^{-1} \quad (A23)$$

and

$$A_1 = \frac{1}{2} \Sigma_v(\Sigma_v + \Sigma_c)^{-1}B_1^{-1} \quad (A24)$$

Substituting (A23) in (A24), we get:

$$A_1 = \frac{1}{4} \Sigma_v(\Sigma_v + \Sigma_c)^{-1}\Sigma_v A_1^{-1}\Sigma_z^{-1} \quad (A25)$$

Proof of Lemma 3.5:

(a) Existence

Define $S = \frac{1}{4} \Sigma_v(\Sigma_v + \Sigma_c)^{-1}\Sigma_v$ (S is positive definite).

By Lemma A1, there exists a nonsingular matrix M such that:

$$(i) \quad M^T S M = I \Rightarrow S = (M^T)^{-1} M^{-1} \\ \Rightarrow S^{-1} = M M^T \quad (A26)$$

$$(ii) \quad M^T \Sigma_z^{-1} M = D \Rightarrow \Sigma_z^{-1} = (M^T)^{-1} D M^{-1} \quad (A27)$$

(where D is a positive definite diagonal matrix with diagonal elements being eigenvalues of Σ_z^{-1} in the metric of S, i.e., the roots of $\det(\Sigma_z^{-1} - \lambda S)$).

Substituting (A26) into (A25), we get:

$$A_1 = (M^T)^{-1} M^{-1} A_1^{-1} \Sigma_z^{-1} \Rightarrow A_1 M M^T A_1 = \Sigma_z^{-1}$$

Premultiplying by M^T , postmultiplying by M, and using (A27), we get:

$$\Rightarrow M^T A_1 M M^T A_1 M = M^T \Sigma_z^{-1} M = D \quad (A28)$$

Factoring D into its unique positive definite square root, we get:

$$M^T A_1 M = D^{1/2} \quad (A29)$$

$$\Rightarrow A_1 = (M^T)^{-1} D^{1/2} M^{-1} \quad (A30)$$

(b) Uniqueness

If the eigenvalues in D above are distinct, M is unique, and this is sufficient to establish uniqueness of A_1 , (and, consequently, B_1 , A_0 and B_0).

But in general the eigenvalues may not be unique, and consequently, besides a matrix M, there may be another matrix \bar{M} that accomplishes the simultaneous diagonalization of S and Σ_z^{-1} . We show below that though M may not be unique, A_1 (and consequently B_1 , A_0 and B_0) will be unique.

For any two matrices M and \bar{M} which accomplish the simultaneous

diagonalization, define

$$K = M^{-1}\bar{M} \quad (A31)$$

From the invertibility of M and \bar{M} , K is invertible.

If corresponding to M , we get A_1^* , and corresponding to \bar{M} , we get \bar{A}_1^* , to prove uniqueness we must show that $\bar{A}_1^* = A_1^*$. We do so in three steps.

Step 1

K is orthogonal.

Proof: Since both \bar{M} and M must diagonalize S into the identity, we have:

$$\begin{aligned} S &= (M^T)^{-1}M^{-1} = (\bar{M}^T)^{-1}\bar{M}^{-1} = (K^T M^T)^{-1}(MK)^{-1} \\ &\Rightarrow (M^T)^{-1}M^{-1} = (M^T)^{-1}(K^T)^{-1}K^{-1}M^{-1} \\ &\Rightarrow (K^T)^{-1}K^{-1} = I \\ &\Rightarrow KK^T = I \Rightarrow K^T = K^{-1} \quad \blacksquare \end{aligned}$$

Step 2

$$D^{1/2} = KD^{1/2}K^T$$

Proof: Since both M and \bar{M} must diagonalize Σ_z^{-1} into D , we get:

$$\begin{aligned} \Sigma_z^{-1} &= (M^T)^{-1}DM^{-1} = (\bar{M}^T)^{-1}\bar{D}\bar{M}^{-1} = (M^T)^{-1}(K^T)^{-1}DK^{-1}M^{-1} \\ &\Rightarrow D = (K^T)^{-1}DK^{-1} \end{aligned}$$

Using Step 1, we get:

$$D = KDK^T$$

$$\Rightarrow D^{1/2} \cdot D^{1/2} = KD^{1/2} K^T K D^{1/2} K^T$$

(where each side has been factored into its unique positive definite square root)

$$\Rightarrow D^{1/2} = KD^{1/2} K^T \quad \blacksquare$$

Step 3

From (A30), given \bar{M} ,

$$\begin{aligned} \bar{A}_1^* &= (\bar{M}^T)^{-1} D^{1/2} \bar{M}^{-1} = (M^T)^{-1} (K^T)^{-1} D^{1/2} K^{-1} M^{-1} \quad (\text{using A31}) \\ &= (M^T)^{-1} K D^{1/2} K^T M^{-1} \quad (\text{using Step 1}) \\ &= (M^T)^{-1} D^{1/2} M^{-1} \quad (\text{using Step 2}) \\ &= \bar{A}_1^* \quad \blacksquare \end{aligned}$$

REFERENCES

- Admati, A. R. (1985). "A Noisy Rational Expectations Equilibrium for Multi-Asset Securities Markets." Econometrica 53, 629-657.
- Admati, A. R. and P. Pfleiderer (1988). "A Theory of Intraday Trading Patterns: Volume and Price Variability." Review of Financial Studies 1, 3-40.
- Allen, F. A. (1987). "Contracts to Sell Information." Working Paper. University of Pennsylvania.
- Ausubel, L. M. (1988). "Insider Trading in a Rational Expectations Economy." Working Paper. Northwestern University.
- Baumol, W. J., J. C. Panzar and R.D. Willig (1982). Contestable Markets and the Theory of Industry Structure. Harcourt Brace Jovanovich, Inc., New York.
- Beaver, W.H., R. A. Lambert and S. G. Ryan (1987). "The Information Content of Security Prices: A Second Look." Journal of Accounting and Economics 9, 139-157.
- Bellman, R. (1970). Introduction to Matrix Analysis (Second Edition). McGraw-Hill Book Company.
- Caballe, J. (1988). "Strategic Behavior and Asymmetric Information in Financial Markets." Department of Economics. University of Pennsylvania.
- Franklin, J. N. (1968). Matrix Theory. Prentice-Hall, Inc., Englewood Cliffs, NJ.

- Gonedes, N., N. Dopuch and S. Penman (1976). "Disclosure Rules, Information Production, and Capital Market Equilibrium. The Case of Forecast Disclosure Rules." Journal of Accounting Research 14, 89-137.
- Graybill, F. A. (1983). Matrices with Applications in Statistics. Wadworth International Group, Belmont, California.
- Grossman, S.J. and O. Hart (1988). "One Share/One Vote and the Market for Corporate Control." Working Paper. Department of Economics, Princeton University.
- Grossman, S. J. and J. E. Stiglitz (1980). "On The Impossibility of Informationally Efficient Markets." American Economic Review, 70, 393-408.
- Hellwig, M. F. (1980). "On the Aggregation of Information in Competitive Markets." Journal of Economic Theory 22, 477-498.
- Kyle, A. S. (1985). "Continuous Auctions and Insider Trading." Econometrica 53, 1315-1335.

