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Direct simulation of the motion of solid particles in Couette and Poiseuille flows of viscoelastic fluids

By P. Y. HUANG[1], J. FENG[2], H. H. HU[3] and D. D. JOSEPH[1]

[1] Department of Aerospace Engineering and Mechanics and the Minnesota Supercomputer Institute, University of Minnesota, Minneapolis, MN 55455, USA

[2] Department of Chemical Engineering, University of California, Santa Barbara, CA 93106, USA

[3] Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, PA 19104, USA

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This paper reports the results of direct numerical simulation of the motion of a two-dimensional circular cylinder in Couette flow and in Poiseuille flow of an Oldroyd-B fluid. Both neutrally buoyant and non-neutrally buoyant cylinders are considered. The cylinder's motion and the mechanisms which cause the cylinders to migrate are studied. The stable equilibrium position of neutrally buoyant particles varies with inertia, elasticity, shear thinning and the blockage ratio of the channel in both shear flows. Shear thinning promotes the migration of the cylinder to the wall while inertia causes the cylinder to migrate away from the wall. The cylinder moves closer to the wall in a narrower channel. In a Poiseuille flow, the effect of elastic normal stresses is manifested by an attraction toward the nearby wall if the blockage is strong. If the blockage is weak, the normal stresses act through the curvature of the inflow velocity profile and generate a lateral force that points to the centreline. In both cases, the migration of particles is controlled by elastic normal stresses which in the limit of slow flow in two dimensions are compressive and proportional to the square of the shear rate on the body. A slightly buoyant cylinder in Couette flow migrates to an equilibrium position nearer the centreline of the channel in a viscoelastic fluid than in a Newtonian fluid. On the other hand, the same slightly buoyant cylinder in Poiseuille flow moves to a stable position farther away from the centreline of the channel in a viscoelastic fluid than in a Newtonian fluid. Marked effects of shear thinning are documented and discussed.

1. Introduction

We have developed a numerical scheme to simulate the motion of fluids and particles in two dimensions. The fully nonlinear nature of the problem is preserved and treated directly. This method was first applied to particle motion in Newtonian fluids (Hu, Joseph & Crochet 1992; Feng, Hu & Joseph 1994*a*, *b*; Huang, Feng & Joseph 1994). Sedimentation in an Oldroyd-B fluid has been studied using the same scheme (Feng, Huang & Joseph 1996). The present paper deals with the motion of particles in Couette and Poiseuille flows of an Oldroyd-B fluid.

In a celebrated experiment, Segré & Silberberg (1961) discovered that neutrally buoyant spherical particles in a pipe flow of a Newtonian fluid migrate to a radial position roughly midway between the axis and the wall. The perturbation analysis of Ho & Leal (1974) demonstrated that inertia causes a sphere to assume an off-centre position in a planar Poiseuille flow. Karnis & Mason (1966) studied the migration of a sphere in a viscoelastic liquid in a pipe flow when inertia is negligible. The sphere approaches the centre of the pipe regardless of its initial position. Ho & Leal (1976) showed that normal stresses in a second-order fluid cause a sphere to migrate to the centre of a planar Poiseuille flow. A few experiments on the motion of particles in Couette flows have been reported (Gauthier, Goldsmith & Mason 1971; Bartram, Goldsmith & Mason 1975). The purpose of these experiments was to study the migration of spheres in the slightly non-uniform shear flow in the gap between two concentric cylinders. No observation of particle motions in uniform shear flows of viscoelastic fluids has been reported.

Direct numerical simulations are not restricted to small nonlinearity and the motion of particles may be followed in real time. Feng *et al.* (1996) simulated the sedimentation of particles in a vertical channel filled with an Oldroyd-B fluid and they showed that particles are forced towards or away from walls depending on the particle–wall separation and the width of the channel. The purpose of the present paper is to extend the same line of study to shear flows. We will investigate the effects of normal stresses, inertia and solid walls on the motion of a circular particle in a Couette or Poiseuille flow bounded by two parallel walls.

The motion of the incompressible viscoelastic fluid is governed by the equations of motion:

$$\left.\begin{aligned} &\nabla \cdot \boldsymbol{u} = 0, \\ &\rho_f\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = -\nabla p + \nabla\cdot\boldsymbol{T}, \end{aligned}\right\} \tag{1}$$

where ρ_f is the fluid density. The constitutive equation for an Oldroyd-B fluid is

$$\boldsymbol{T} + \lambda_1 \overset{\triangledown}{\boldsymbol{T}} = 2\eta(\boldsymbol{D} + \lambda_2 \overset{\triangledown}{\boldsymbol{D}}), \tag{2}$$

where $\boldsymbol{D} = (\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T)/2$ is the strain-rate tensor; λ_1 and λ_2 are constant relaxation and retardation times; η is the viscosity. The triangle denotes the upper-convected time-derivative

$$\overset{\triangledown}{\boldsymbol{T}} = \frac{\partial \boldsymbol{T}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{T} - (\nabla\boldsymbol{u})^T\cdot\boldsymbol{T} - \boldsymbol{T}\cdot(\nabla\boldsymbol{u}), \tag{3}$$

where $(\nabla\boldsymbol{u})_{ij} = \partial u_j/\partial x_i$. Shear thinning can be easily added to the Oldroyd-B model by using the Carreau–Bird viscosity law:

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = [1 + (\lambda_3\dot{\gamma})^2]^{(n-1)/2}, \tag{4}$$

where $\dot{\gamma}$ is the strain rate defined in terms of the second invariant in the usual way and $0 < n \leqslant 1$. In our computation, we fixed the fluid relaxation time ratio $\lambda_2/\lambda_1 = 1/8$ and viscosity ratio $\eta_\infty/\eta_0 = 0.1$.

Joseph & Feng (1996) did an analysis of the forces that move particles in slow plane flows of a second-order fluid:

$$\boldsymbol{T} = -p\boldsymbol{I} + \eta\boldsymbol{A}_1 + \alpha_1\boldsymbol{A}_2 + \alpha_2\boldsymbol{A}_1^2, \tag{5}$$

where $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are the first and second Rivlin–Ericksen tensors. Using the Giesekus–Tanner theorem they showed that

$$p = p_N + \frac{\alpha_1}{\eta}\frac{\mathrm{D}p_N}{\mathrm{D}t} + (\alpha_2 + \tfrac{3}{2}\alpha_1)\dot{\gamma}^2, \tag{6}$$

where p_N is the pressure of the Stokes flow for the prescribed values of velocity at the boundary of the flow domain. At the boundary of a solid body with outward normal $\boldsymbol{n}$, the normal component T_{nn} of $\boldsymbol{T}$ is given by

$$T_{nn}+p = (2\alpha_1+\alpha_2)\,\dot{\gamma}^2 \tag{7}$$

and

$$T_{nn} = -p_N - \frac{\alpha_1}{\eta}\frac{\mathrm{D}p_N}{\mathrm{D}t} + \tfrac{1}{2}\alpha_1\,\dot{\gamma}^2. \tag{8}$$

This shows that the normal component of the nonlinear elastic part of the normal stress

$$\tfrac{1}{2}\alpha_1\,\dot{\gamma}^2 = \tfrac{1}{4}\Psi_1(0)\,\dot{\gamma}^2, \tag{9}$$

where $\Psi_1(0)$ is the coefficient of the first normal stress, is always compressive. These compressive normal stresses are such as to turn long bodies into the stream and cause circular particles to aggregate and chain.

In the case of the Oldroyd-B fluid studied here, we find that $\alpha_1 = -\eta(\lambda_1-\lambda_2)$ and $\alpha_2 = -2\alpha_1$ in the limit of slow flows, hence the coefficient of the second normal stress vanishes: $2\alpha_1+\alpha_2 = 0$. Equation (7) shows that the entire contribution to T_{nn} comes from the pressure and none from the extra stress with

$$T_{nn} = -p = -p_N + (\lambda_1-\lambda_2)\frac{\mathrm{D}p_N}{\mathrm{D}t} - \tfrac{1}{2}\eta(\lambda_1-\lambda_2)\,\dot{\gamma}^2. \tag{10}$$

Of course, shear thinning does not appear at second-order in the asymptotic analysis leading to the second-order form of the Oldroyd-B model. We can stimulate some thoughts about the important role of shear thinning in the motion of particles by a heuristic argument due to Joseph suggested by writing the last term of (10) as

$$\kappa(\lambda_1-\lambda_2)\,\dot{\gamma}^{n+1}. \tag{11}$$

The term (11) shows that the normal stresses in the shear-thinning form of the Oldroyd-B fluid still increase more rapidly than $\dot{\gamma}$ so that if the effect of shear thinning is to decrease the viscosity and increase the shear rate at places of high $\dot{\gamma}$ on the body, the overall effect would be to reinforce migration effects due to compressive normal stresses over and above what they would be without shear thinning. Another way to say this is that in a pipe flow with a prescribed pressure gradient, the pressure force balances the shear force at the wall so the shear stress $\tau_w = \eta(\dot{\gamma})\,\dot{\gamma}$ is the same for all viscosity functions. If the fluid thins in shear, the viscosity η goes down and the shear rate $\dot{\gamma}$ up, keeping the product constant. Then $\eta(\dot{\gamma})\,\dot{\gamma}^2 = \tau_w\,\dot{\gamma}$ is larger than it would be if the fluid did not shear thin because $\dot{\gamma}$ is larger.

Normal stresses are not the only forces which can cause a particle to migrate in a shear-thinning fluid: a lateral imbalance of the shear stresses can also cause lateral migration.

In general the shear rates are large near the places where the velocities are large and the pressures due to inertia are small. On the other hand, where the stagnation pressures are high the compressive effects of the elastic normal stresses are small, and the places to which particles migrate are worked out in this competition. This argument seems to be consistent with the results of the numerical experiments described below.

2. Numerical method

The equations of motion and the constitutive equation are solved at each time step using an EVSS (elastic-viscous-split-stress) formulation. The main solver was adapted from that of the popular code POLYFLOW. The solid–fluid coupling is treated by an arbitrary Lagrangian–Eulerian method with mesh velocities. The numerical method

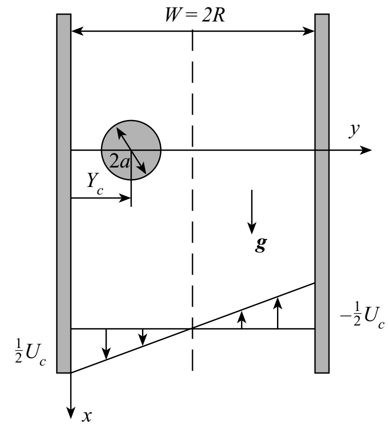

FIGURE 1. Lateral migration of a solid cylindrical particle in Couette flow between two walls moving in opposite directions. W is the width of the channel. Y_c is the distance of the cylinder centre from the wall. We call the final equilibrium value of Y_c/W the standoff distance.

has been explained before (Huang & Feng 1995; Feng *et al.* 1996) and the details will not be discussed here. Because of the lower-order scheme used, our computations are limited to low Deborah numbers; above a critical De value, convergence is lost. This critical De depends on the Reynolds number Re, the degree of shear thinning in the fluid and the blockage ratio of the channel β: higher De can be reached for smaller Re, smaller β and milder shear thinning. As an example, we have obtained convergent results up to $De = 3$ at $Re = 5$, $\beta = 0.25$ and $n = 1$ in a Poiseuille flow.

In our simulations we use an unstructured mesh with triangular elements. A typical mesh used in most of our computations for $\beta = 0.10$ has 1520 nodes and 720 elements. We refined the mesh to study convergence. For instance, at $Re = 5$ and $De = 0.2$ in a Poiseuille flow (see figure 13) refining the mesh to 2560 nodes and 1230 elements results in a 1.2% difference in the final standoff distance. Thus we think the mesh used is adequate. For wider channels ($\beta = 0.025$), we use up to 2780 nodes and 1360 elements.

In the following sections, numerical results for the motion of a two-dimensional circular cylinder in a Couette flow and in a Poiseuille flow will be presented. Both neutrally buoyant and non-neutrally buoyant particles are considered.

3. Particle migration in a Couette flow

The lateral migration of a circular particle in a two-dimensional Couette shear flow is studied. In our set-up, the walls are moving in opposite directions with speed $\pm\frac{1}{2}U_c$ (figure 1). The motion of a solid particle in the channel is determined by the following dimensionless parameters: Reynolds number $Re = \rho_f U_c a/\eta$, Deborah number $De = U_c \lambda_1/a$, blockage ratio of the channel $\beta = a/R$ and ratio of solid particle to fluid density ρ_s/ρ_f. Here a is the radius of the particle, R is the half-width of the channel.

3.1. *Neutrally buoyant particles*

A neutrally buoyant circular cylinder is released at different initial positions in a channel with $\beta = 0.25$, $Re = 5$ and $De = 1.0$, as shown in figure 2. The particle reaches an equilibrium position at the centreline of the channel, regardless of its initial position and velocity. This is the same as in a Newtonian fluid (Feng *et al.* 1994*b*).

The numerical solution of Feng *et al.* (1994*b*) and the perturbation solutions of Ho & Leal (1974) and Vasseur & Cox (1976) showed that the centreline of a channel is the

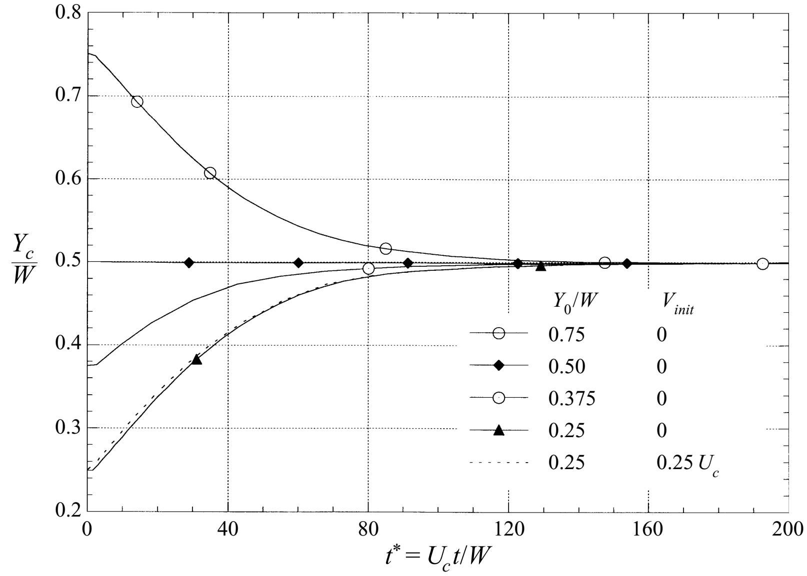

FIGURE 2. Lateral migration of a neutrally buoyant particle released from different initial positions in Couette flow of an Oldroyd-B fluid ($\beta = 0.25, Re = 5, De = 1.0$). Y_0 indicates the initial position of the particle. V_{init} is the initial velocity of the particle.

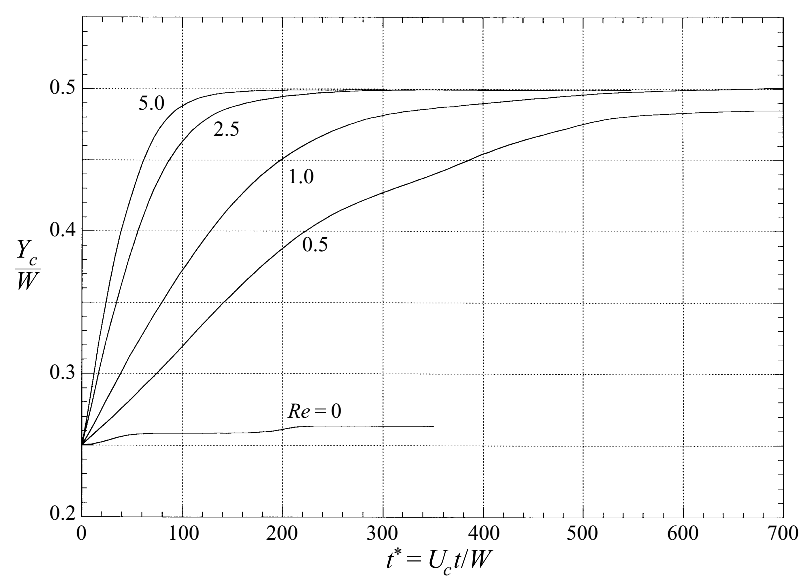

FIGURE 3. The effect of the Reynolds number on the migration of a neutrally buoyant particle in Couette flow of an Oldroyd-B fluid ($\beta = 0.25, De = 1.0$). The centreline of the channel is no longer a stable equilibrium when the Reynolds number is small.

stable equilibrium position for a neutrally buoyant particle in a Couette flow of a Newtonian fluid regardless of the Reynolds number. For a particle sedimenting in an Oldroyd-B fluid, Feng *et al.* (1996) found that the particle is pushed further away from the sidewall as the Reynolds number is increased. To study the effect of Reynolds number on the motion of a solid particle in a Couette flow of an Oldroyd-B fluid, we fixed the Deborah number $De = 1.0$ and changed the Reynolds number Re by

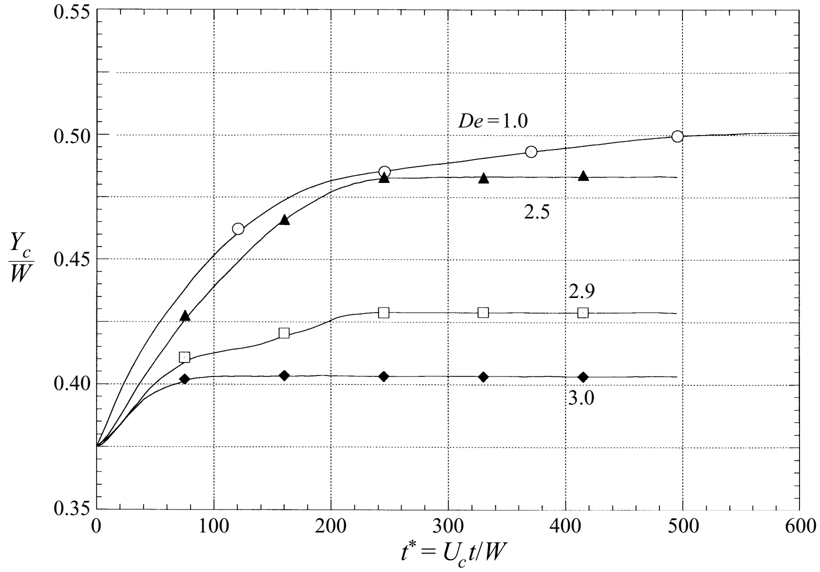

FIGURE 4. The effect of Deborah number on the migration of a neutrally buoyant particle in Couette flow of an Oldroyd-B fluid ($\beta = 0.25$, $Re = 1.0$).

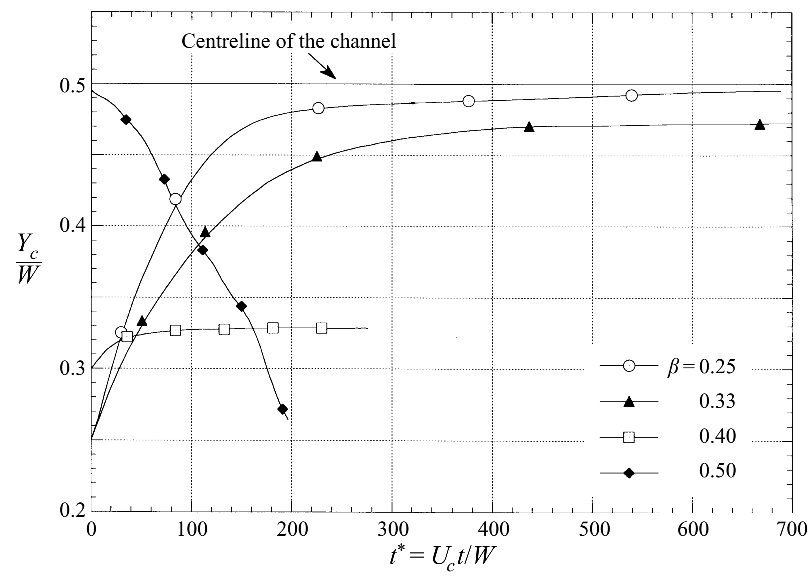

FIGURE 5. The effect of the blockage ratio $\beta = a/R$ on the migration of a neutrally buoyant particle in Couette flow of an Oldroyd-B fluid ($De = 1.0$, $Re = 0$). The cylinder will touch the wall when the centre of the cylinder is at $Y_c/W = 0.5\beta$ in the channel.

changing both the Couette inflow velocity U_c and relaxation time λ_1. Figure 3 shows that the particle moves toward the centreline very fast when inertia is strong ($Re = 5$). As Re is decreased the centreline of the channel is apparently no longer a global attractor of trajectories of the neutrally buoyant particle; normal stress effects become dominant as Re is decreased. If we set Re to zero by removing the inertia term $\boldsymbol{u} \cdot \boldsymbol{\nabla u}$ in equation (10), the particle no longer moves to the centreline but stays very close to the wall (see figure 3). Particle migrations in viscoelastic Couette flows and

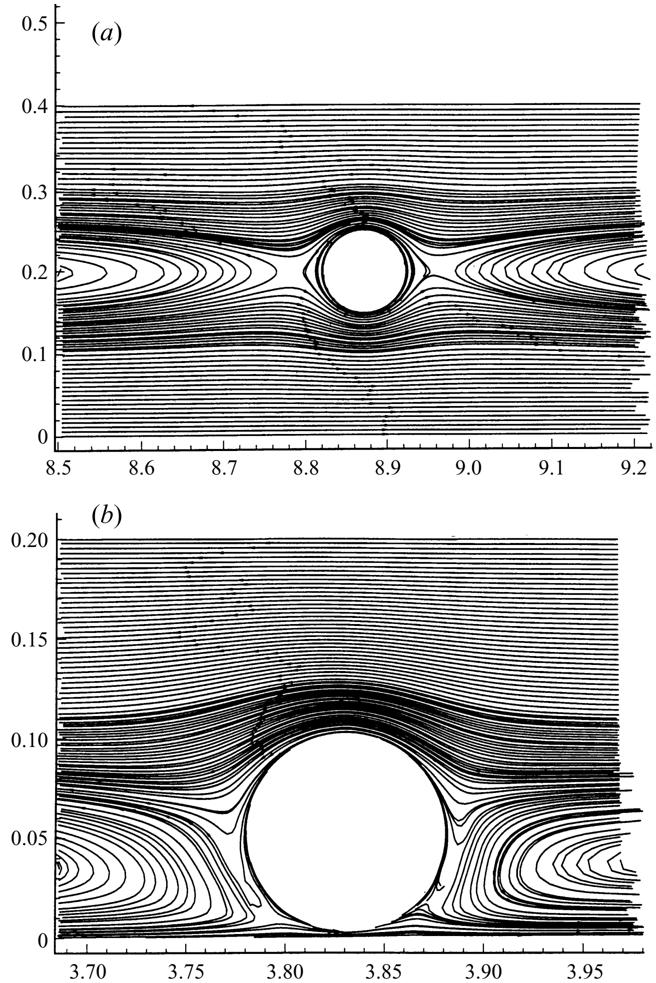

FIGURE 6. Streamlines for the Couette flow around a cylinder under conditions specified in figure 5. (*a*) $\beta = 0.25$ and $t^* = 689$: at this t^* the particle has drifted to its equilibrium position. (*b*) $\beta = 0.50$ and $t^* = 196.5$: the particle is still migrating to the wall, the computation fails for larger t^*.

sedimentation are such as to have normal stresses generated on the side of the particle away from the wall, forcing it to the wall, and the migration is opposed by lubrication forces associated with slow flow in the smaller gap near the wall (see Feng *et al.* 1996).

In figure 4, we study the effects of *De* with a fixed Reynolds number $Re = 1.0$. When $De = 1.0$, the particle moves to the centreline of the channel. As the Deborah number is increased, the particle migrates toward the sidewall. Perhaps there is a critical *De* for any Reynolds number such that the centre attracts for *De* smaller than critical and a position nearer the wall attracts when *De* is greater than critical.

The effect of normal stresses is strengthened by the effect of the blockage. To emphasize this, we suppressed inertia by setting $Re = 0$. For a wide channel with $\beta = 0.25$ or smaller, the particle goes to the centreline eventually regardless of its initial position. When the gap is smaller than critical, the centreline of the channel is no longer a stable equilibrium position as can be seen in figure 5: the particle is pushed toward the sidewall. For $\beta = 0.5$, the particle migrates until it almost touches the wall. At this point our numerical computation fails. Figure 6 gives the contours of streamlines as

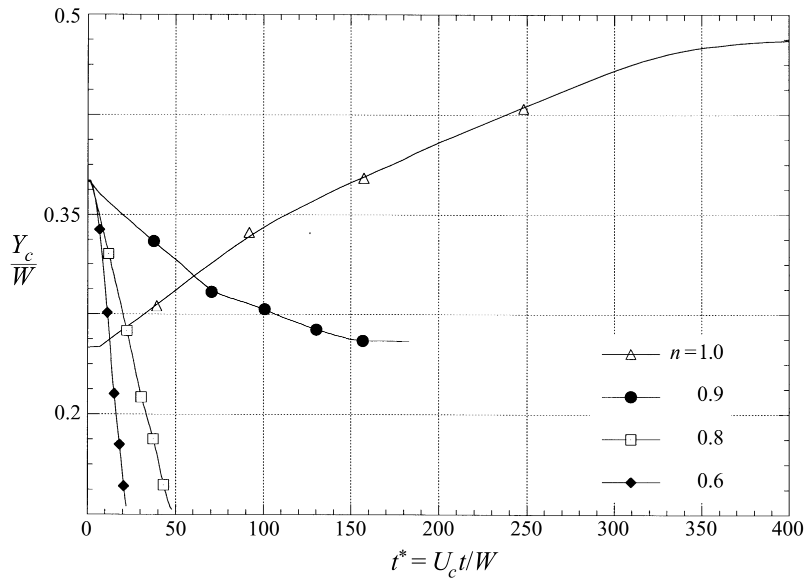

FIGURE 7. The effect of shear thinning on a neutrally buoyant particle in Couette flow of an Oldroyd-B fluid ($\beta = 0.25$, $Re = 0$, $De = 2.8$). The cylinder will touch the wall when the centre of the cylinder is at $Y_c/W = 0.125$ in the $\beta = 0.25$ channel.

seen in a coordinate system fixed on the particle. As shown in figure 6(*a*) with $\beta = 0.25$, the particle at the time t^* has drifted to its equilibrium position which is determined by a balance of compressive normal stresses on the side near the centreline where the streamlines are crowded and lubrication forces on the other side. This feature is even clearer in figure 6(*b*) with $\beta = 0.5$.

In the sedimentation problem studied by Feng *et al.* (1996), the final equilibrium position of the particle is also closer to the sidewall when the channel is narrower but the particle never approaches the wall so closely that the computation fails even when $Re = 0$.

The position of equilibrium is determined by a balance of the normal compressive stresses where the streamlines are crowded and the lubrication forces which arise in the gap between the cylinder and the wall. Lubrication forces are associated with the change from high to low pressures, linear in velocity, when the fluid is forced slowly through the gap between the cylinder and the wall. For faster flow through the gap, stagnation pressures proportional to the square of the velocity are induced by inertia, hence lubrication forces are at work.

The effect of shear thinning is also to drive the particle closer to the wall. This effect has already been noted by Huang & Feng (1995). In figure 7, we used a Carreau–Bird viscosity law with $\lambda_3/\lambda_1 = 10$; the power index $n = 1.0$ corresponds to no shear thinning, and smaller n indicates more shear thinning. The inertia effect was removed in these computations. Shear thinning enhances the migration of the cylinder to the wall. The results are consistent with the argument we gave in the first section; shear thinning promotes the migration towards the wall because of the action of the compressive normal stresses on the side of the particle away from the wall, as shown in (11). For $n = 0.8$ and $n = 0.6$, the particle approaches the sidewall rapidly and so closely that our computation fails.

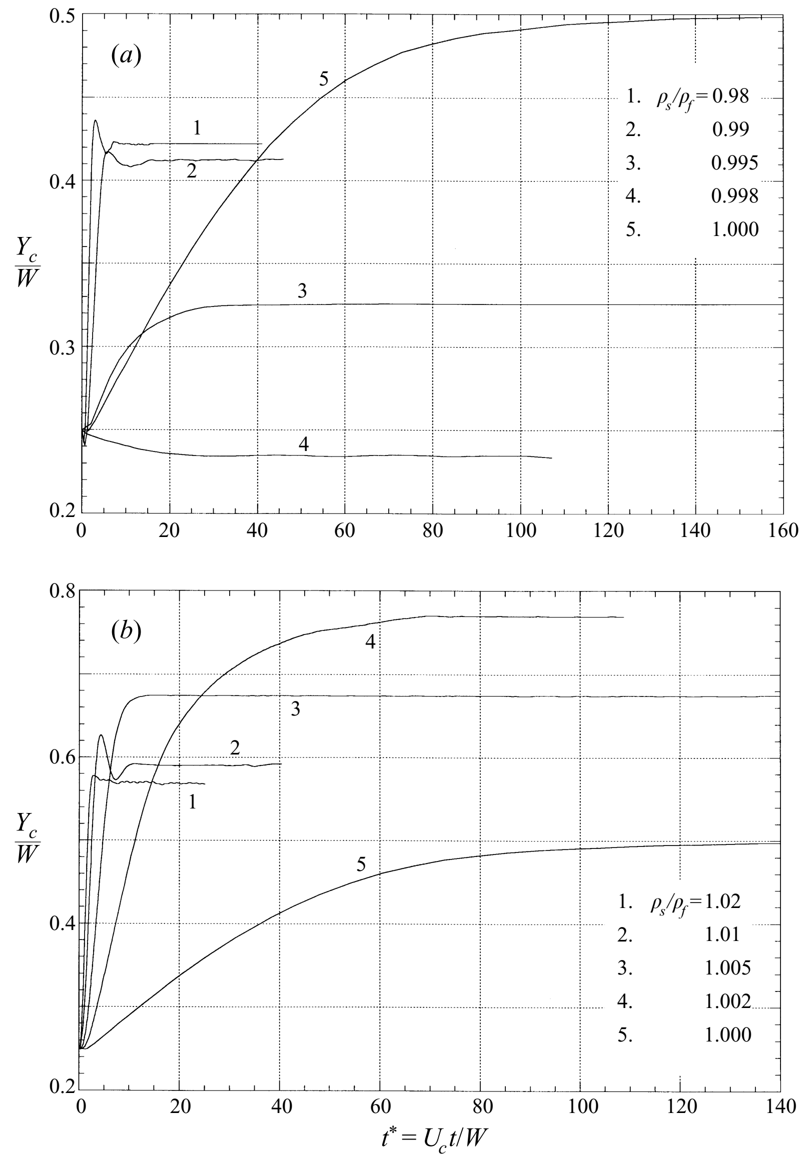

FIGURE 8. Trajectories of non-neutrally buoyant particles in Couette flow of an Oldroyd-B fluid ($\beta = 0.25, Re = 5, De = 1.0$). (*a*) Particles with density ρ_s smaller than the fluid ρ_f; (*b*) particles with density ρ_s larger than the fluid ρ_f.

3.2. *Non-neutrally buoyant particles*

Now we focus our attention on the motion of non-neutrally buoyant particles. In the following simulations, we keep these parameters unchanged: $\beta = 0.25$, $Re = 5$ and $De = 1.0$, and release particles with different densities. The direction of gravity $\boldsymbol{g}$ is along the axis $\boldsymbol{x}$, as shown in figure 1.

Figure 8 shows that the particle reaches a stable position close to either of the walls when it is only slightly buoyant ($\rho_s/\rho_f = 0.998$ and $\rho_s/\rho_f = 1.002$). A light particle is always at the side of the channel in which the flow is down and a heavy particle is at the other side. As the density difference increases, the slip velocity of the particle

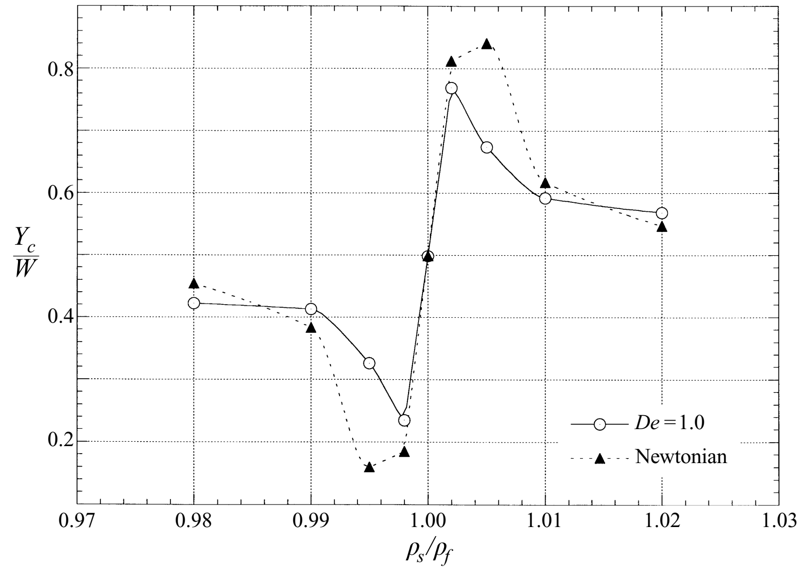

FIGURE 9. Equilibrium position of a non-neutrally buoyant particle in Couette flow of an Oldroyd-B fluid with $De = 1.0$ and a Newtonian fluid ($\beta = 0.25$, $Re = 5$).

increases, enhancing the effects of inertia which push particles away from the wall. As in a Newtonian fluid (Feng *et al.* 1994*b*), this effect is not quite symmetric with respect to the sign of the density difference because the solid inertia varies (see figure 9).

A slightly buoyant ($0.99 < \rho_s/\rho_f < 1.01$) particle in an Oldroyd-B fluid moves closer to the centreline of the channel than in a Newtonian fluid whereas a more buoyant particle moves closer to the wall. This is similar to the shift of the equilibrium position in sedimentation studied by Feng *et al.* (1996).

4. Particle migration in a Poiseuille flow

Now let us consider the lateral migration of a circular particle in a two-dimensional Poiseuille flow of an Oldroyd-B fluid (figure 10). The geometry of the channel is the same as that in a Couette flow. All the dimensionless parameters can be defined as in Couette flow, except that the velocity U_c is replaced by the maximum velocity of the Poiseuille inflow U_0.

4.1. *Neutrally buoyant particles*

The motion of a neutrally buoyant particle in a viscoelastic fluid varies with the Reynolds number *Re*, the Deborah number *De*, the blockage ratio β and the parameters of shear thinning; to study these effects, we change one of the parameters while keeping the others fixed.

First a neutrally buoyant circular cylinder is released from different initial positions in a channel with $\beta = 0.25$, $Re = 5$ and $De = 0.2$. The behaviour is similar to that in a Newtonian fluid; all the particles reach the same equilibrium position roughly halfway between the wall and the centre, as shown in figure 11. The final position of a particle is independent of the initial conditions.

Figure 12 shows that the effects of inertia, at higher *Re*, are such as to push the particle away from the wall after an initial overshoot as in the Couette flow of an Oldroyd-B fluid (cf. figure 3). In Poiseuille flow of a Newtonian fluid, the effect of inertia, at higher *Re*, is such as to push the particle toward the wall (Feng *et al.* 1994*b*).

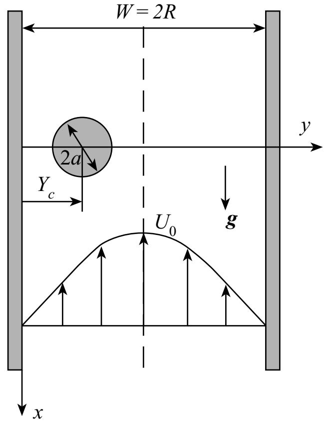

FIGURE 10. Lateral migration of a solid particle in a plane Poiseuille flow. U_0 is the maximum velocity at the centreline. (X_c, Y_c) is the centre of the cylinder.

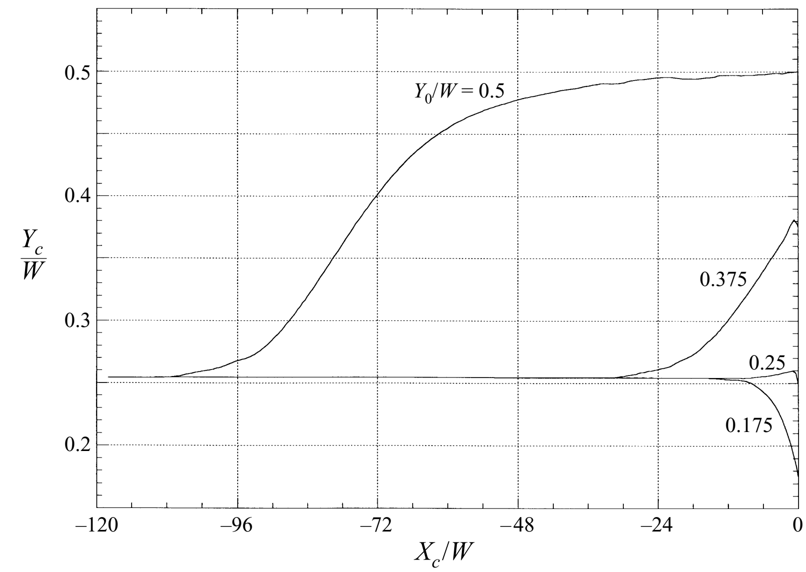

FIGURE 11. Trajectories of a neutrally buoyant particle released at different initial positions in Poiseuille flow of an Oldroyd-B fluid ($\beta = 0.25, Re = 5, De = 0.2$). Y_0 is the initial position of the particle.

The width of the channel plays a very important role in the migration of the solid particles. Karnis & Mason (1966) observed the migration of a sphere in a viscoelastic fluid in a pipeline flow when inertia is negligible. They found that the sphere moves to the centre of the pipe regardless of its initial position. The perturbation results of Ho & Leal (1976) showed that normal stresses of a second-order fluid cause the neutrally buoyant particle to migrate to the centre of the channel. On the other hand, Dhahir & Walters (1989) measured the lateral force acting on a circular cylinder fixed in a slow channel flow of a viscoelastic fluid. The force pulls the cylinder towards the nearby wall. Their results were later verified by the numerical results of Carew & Townsend (1991) and Feng *et al.* (1996).

The principal difference between the two groups of results is the blockage ratio. Dhahir & Walters (1989) and Carew & Townsend (1991) used a very large cylinder in

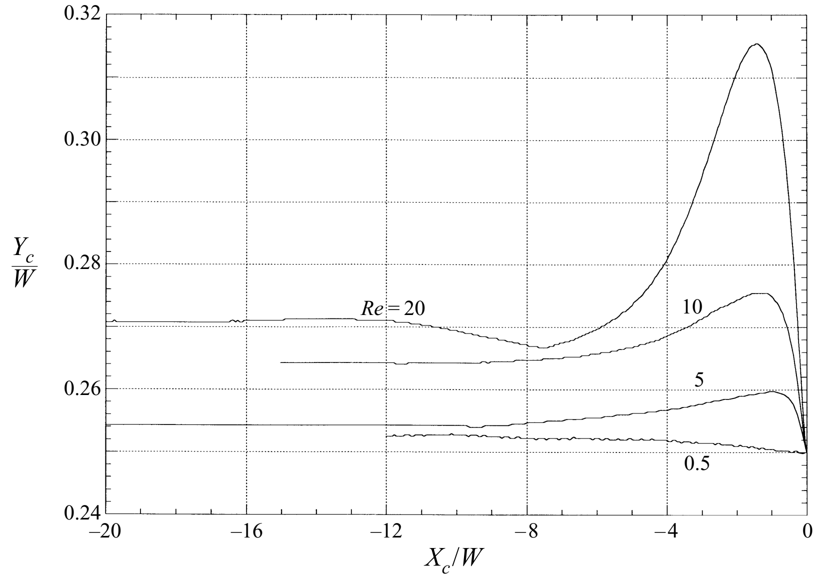

FIGURE 12. The effect of the Reynolds number on a neutrally buoyant particle released in Poiseuille flow of an Oldroyd-B fluid ($\beta = 0.25, De = 0.2$).

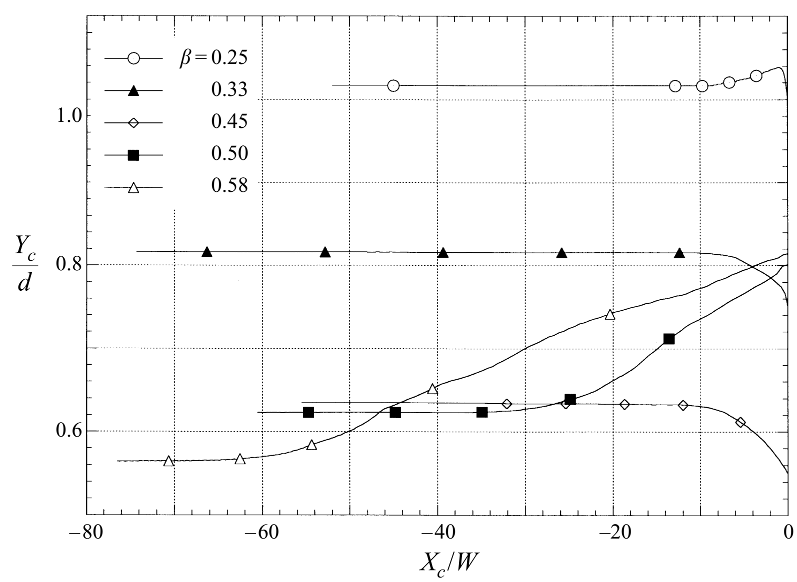

FIGURE 13. The effect of the blockage ratio $\beta = a/R$ on the motion of a neutrally buoyant particle in Poiseuille flow of an Oldroyd-B fluid ($Re = 5, De = 0.2$). The cylinder will touch the wall when the centre of the cylinder is at $Y_c/d = 0.5$ where d is the diameter of the particle.

their planar Poiseuille flow: $\beta = 7/12$. In their perturbation analysis Ho & Leal (1976) restricted the solid particle to be very small as $\beta = a/R \ll 1$. The effect of the wall is merely to generate the undistributed flow field and makes no direct contribution to the lateral migration of the particle. The experiment of Karnis & Mason (1966) also used very small particles, though their results might have been affected by the density mismatch as discussed in the next subsection. It appears that the effects of viscoelastic

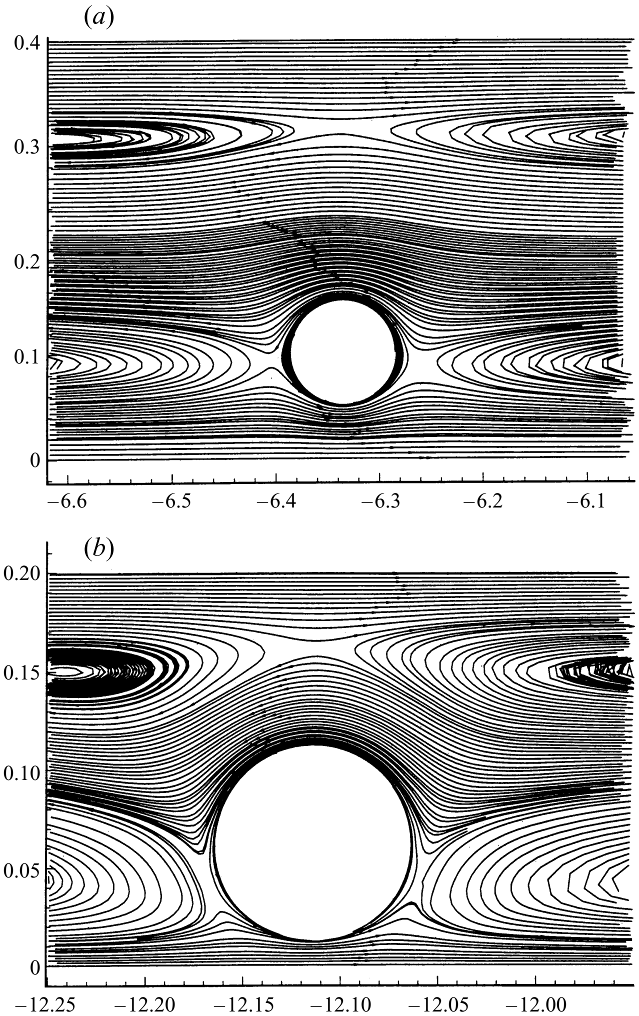

FIGURE 14. Streamlines for the Poiseuille flow around a cylinder when $Re = 5$ and $De = 0.2$. (*a*) $\beta = 0.25$: the particle has reached its equilibrium position which is determined by a balance of compressive normal stresses on the open side near the centreline where the streamlines are crowded and lubrication forces on the other side. (*b*) $\beta = 0.50$: the particle has drifed to an equilibrium position very close to the wall. Streamlines on the open side near the centreline are crowded.

stresses are modulated by the wall blockage; a particle experiences an attraction toward the nearby wall at large β, but feels a lateral force toward the centre of the Poiseuille flow at small β.

The effects of the blockage ratio β are illustrated in figure 13. As the width of the channel is decreased (larger β), the particle stabilizes closer to the wall because the streamlines in a narrower channel crowd in such a way as to intensify the shear rates in the more open portions of the channel away from the wall (see figures 14*a* and 14*b*), intensifying compressive normal stresses there.

To explore the limit of small β, we have done a series of computations with $\beta = 0.025$ (figure 15*a*). The inertia term is put to zero so we can focus our attention on the effects of the normal stresses induced by the curvature of the inflow velocity. Now the particle approaches the centre of the channel where the shear rate of the undisturbed flow is

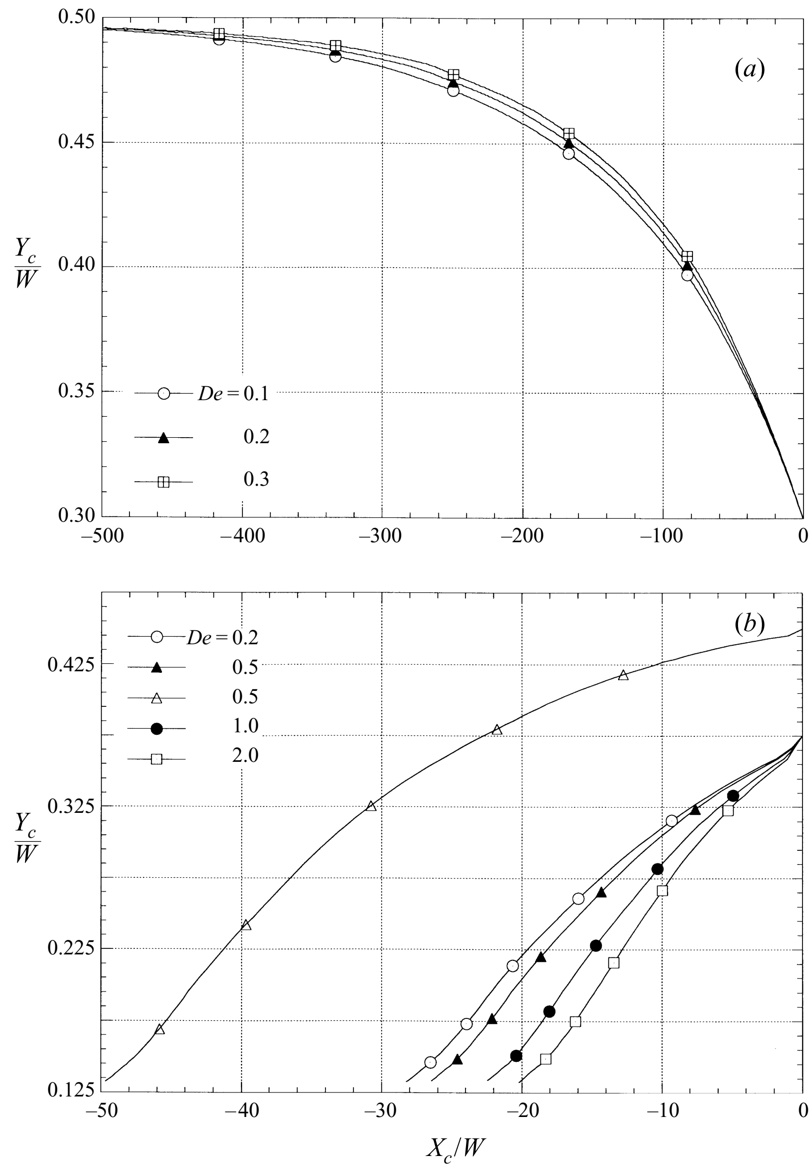

FIGURE 15. The effect of normal stresses on a neutrally buoyant particle released in Poiseuille flow of an Oldroyd-B fluid without inertia ($Re = 0$). (*a*) $\beta = 0.025$. (*b*) $\beta = 0.25$.

zero. Without wall effects, the migration of the particle is controlled by the normal stresses through the curvature of the velocity profile. The larger the Deborah number *De* is, the faster the particle goes toward the centre of the channel. The results agree with the perturbation solution of Ho & Leal (1976). Our simulation is in two dimensions where the wall effect can be even stronger than in three dimensions.

The effects of Deborah number *De* at relatively large β are shown in figure 15(*b*). The inertia is again put to zero ($Re = 0$). The particle moves all the way to the wall and this migration is faster when *De* is larger. Without inertia effects, the strong effects of the wall blockage overwhelm the effects of the curvature of the velocity profile and increase

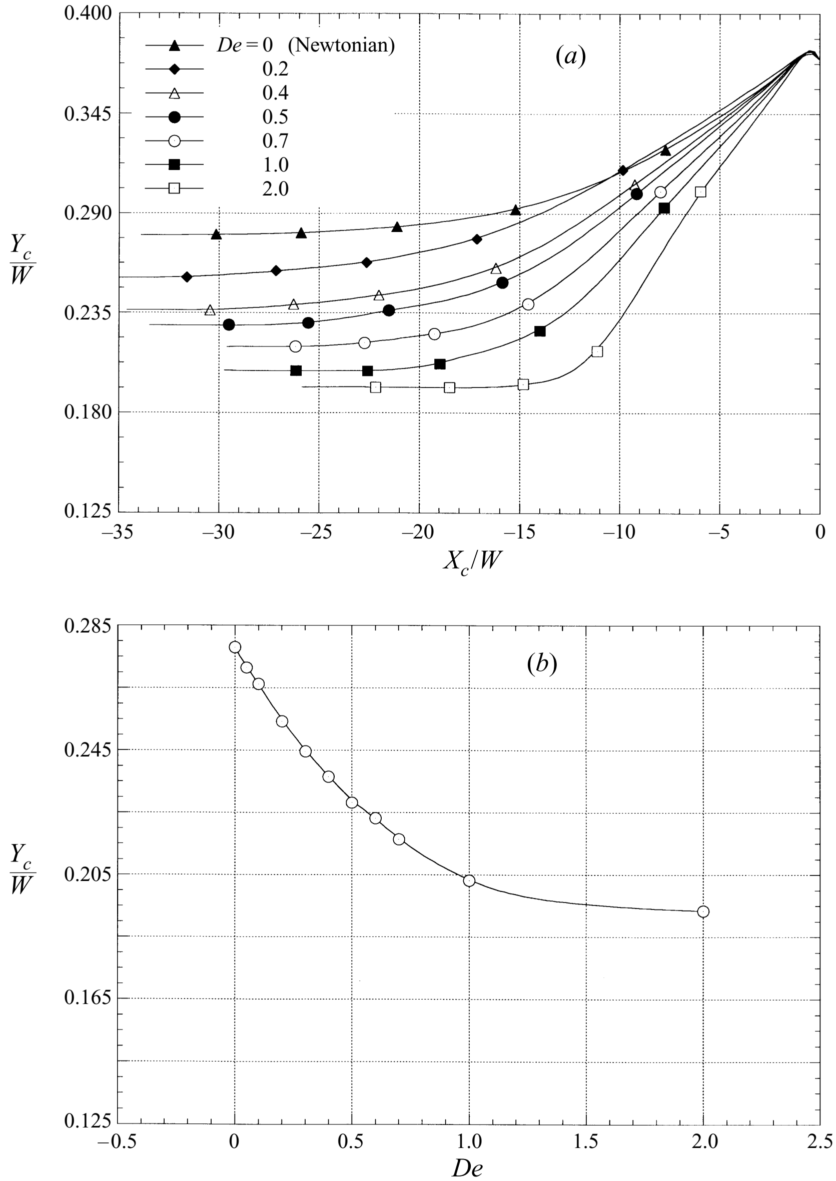

FIGURE 16. The effect of normal stresses on a neutrally buoyant particle released in Poiseuille flow of an Oldroyd-B fluid ($\beta = 0.25$, $Re = 5$). The cylinder will touch the wall when the centre of the cylinder is at $Y_c/W = 0.125$ in this $\beta = 0.25$ channel. (*a*) The particle trajectories; (*b*) the standoff distances.

the effects of the compressive normal stresses acting on the particle surface so as to drive the particle toward the wall. This is consistent with the experimental results of Dhahir & Walters (1989) and the numerical results of Carew & Townsend (1991).

The effect of inertia on the interplay between β and De is such that the particle never touches the wall. Figure 16 shows a series of competition. At $\beta = 0.25$, the elastic stresses push the particle toward the wall. But unlike in figure 15(*b*), the particle will stop at a standoff distance from the wall. This standoff distance Y_c/W decreases as De increases, and is expected to increase with Re.

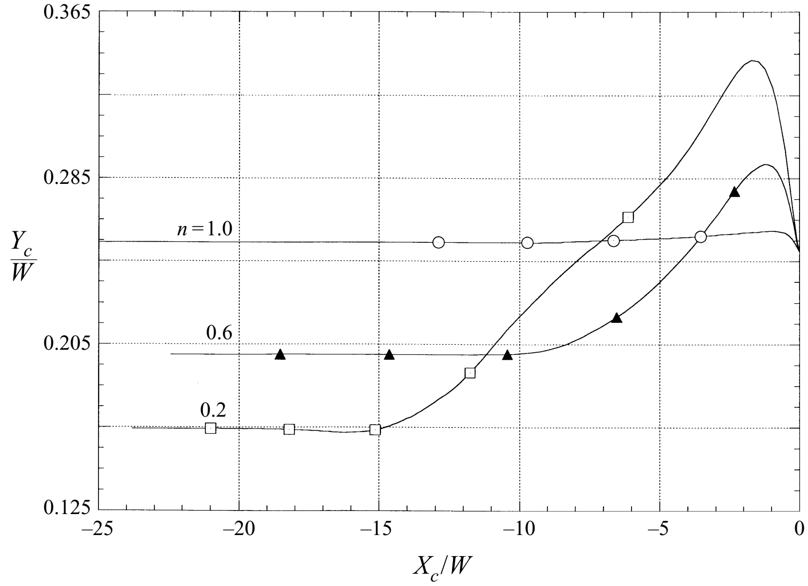

FIGURE 17. The effect of shear thinning on the migration of a neutrally buoyant particle in Poiseuille flow of an Oldroyd-B fluid ($\beta = 0.25$, $Re = 5$, $De = 0.2$, $\lambda_3/\lambda_1 = 100$). The cylinder will touch the wall when the centre of the cylinder is at $Y_c/W = 0.125$ in the $\beta = 0.25$ channel.

From figures 15 and 16 we can draw the conclusion that inertia always pushes the particle away from the wall. The normal stresses push the particle toward the wall at strong blockage. At weak blockage, the wall attraction becomes minimal and the normal stresses generate a lift force, through the curvature of the velocity profile, that leads the particle to the centre of the channel where the shear rate is zero. The final equilibrium position of the particle is determined by a competition between inertia and normal stresses. In the sedimentation problem studied by Feng *et al.* (1996), the particles assume a finite standoff distance close to the wall even for $Re = 0$.

The effect of shear thinning always promotes the particle's motion toward the wall. A heuristic argument, associated with (11), was given in §1 whose main thrust is that compressive normal stresses are intensified by shear thinning because of the increase of the shear rate at the walls where the resistance to flow is weakened. The shear stress τ_w may not change much at a particle surface, but $\tau_w \dot{\gamma}$ will definitely increase there. This idea is consistent with the numerical results displayed in figure 17 where it is shown that the final standoff distance of the particle comes closer to the wall as the power law index n is decreased. More shear thinning also gives rise to a large transient overshoot.

It is necessary to explain why particles in Couette flow are pushed all the way to the wall in situations where particles in Poiseuille flow do not touch. It is true, as seen in figure 15(*b*), that particles in Poiseuille flow will drift all the way to the wall without the intervention of inertia ($Re = 0$). For $Re > 0$, the compressive normal stresses pushing the particle from the open channel side are balanced by inertia forces in the gap. Obviously inertia forces and normal stresses act all around the particle and not only on one or the other side. We can understand the difference between Couette and Poiseuille flow by noting that the shear rates at the particle are due to the main flow and a perturbation, and the main flow shear is uniform across the particle in Couette flow and greater in the gap and weaker in the open part of the channel in Poiseuille flow

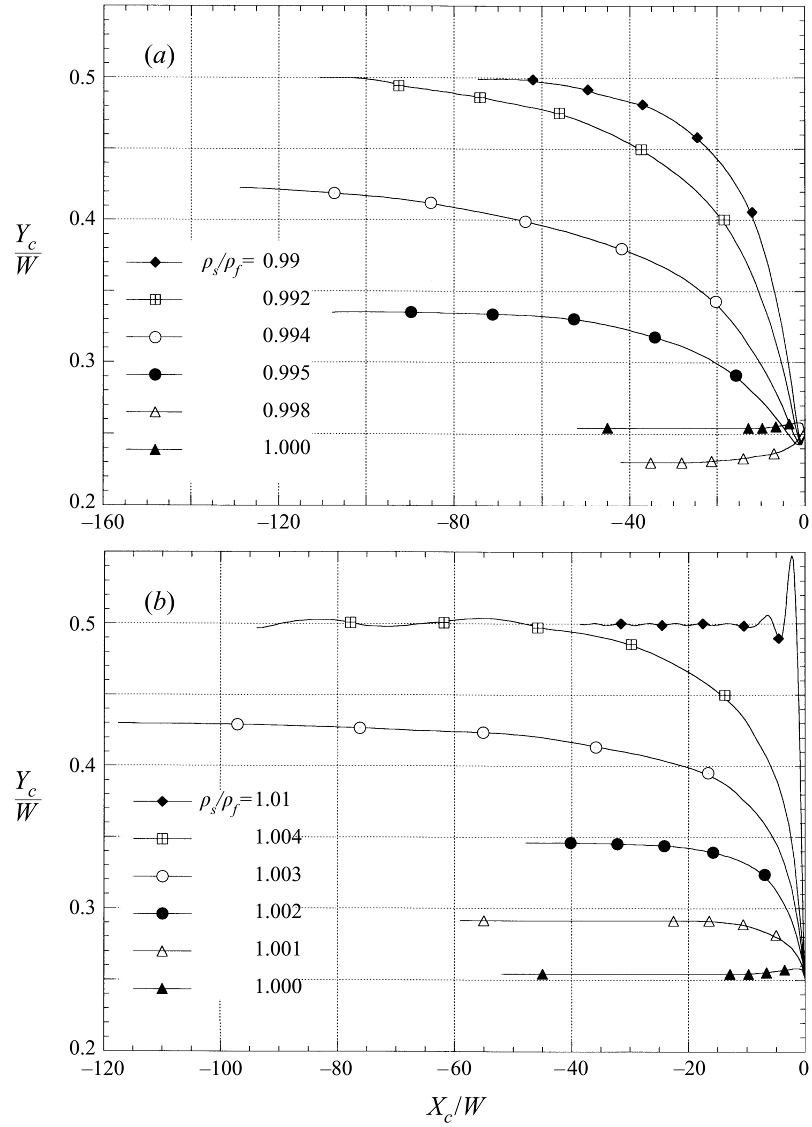

FIGURE 18. Trajectories of non-neutrally buoyant particles in Poiseuille flow of an Oldroyd-B fluid ($\beta = 0.25, Re = 5, De = 0.2$) when (*a*) the density of the particle ρ_s is smaller than the density of the fluid ρ_f and (*b*) the density of the particle ρ_s is larger than the density of the fluid ρ_f.

(compare the streamlines in figure 14*b* to figure 6*b*). So there is a greater action of compressive normal stresses on the gap side of the particle in Poiseuille flow and this helps inertia to keep the particle off the wall.

4.2. *Non-neutrally buoyant particles*

Particles with densities larger or smaller than the fluid were released midway between the wall and the centreline of the channel with $\beta = 0.25$. The trajectories are shown in figure 18 for $Re = 5, De = 0.2$. Here the direction of gravity $\boldsymbol{g}$ is opposite to the inflow, along the axis $\boldsymbol{x}$, as shown in figure 10.

If the density difference is large, the particle moves to the centre of the channel no

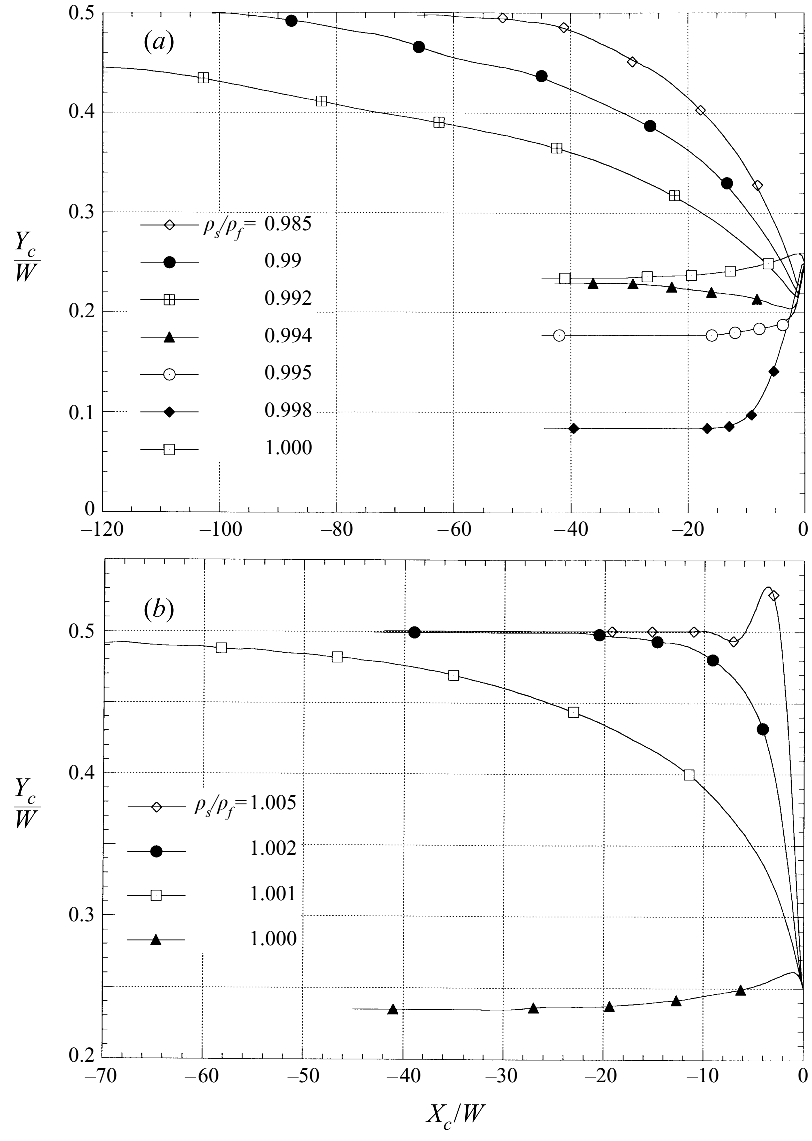

FIGURE 19. As figure 18 but for $\beta = 0.10$.

matter whether the particle is lighter or heavier than the fluid, but lighter particles with ρ_s/ρ_f close to 1 ($\rho_s/\rho_f = 0.998$ in figure 18(*a*)) move to the wall. In many cases there is a transient in which the particle first drifts away from its equilibrium position in narrow channels (figure 18) and wider ones (figure 19). The effects of density differences are greater in wider channels. Particles only slightly light than the fluid ($\rho_s/\rho_f = 0.998$) move closer to the wall than neutrally buoyant particles. The equilibrium positions of heavier particles are closer to the centreline.

Figure 20 shows how the equilibrium positions of particles vary with density. Heavy particles drift closer to the centre in a wider channel while light particles drift further from the centre in a wider channel. It is interesting to note the difference between the migration of the particle in an Oldroyd-B fluid and that in a Newtonian fluid. A particle with slight density difference (between $\rho_s/\rho_f = 0.998$ and $\rho_s/\rho_f = 1.001$) moves

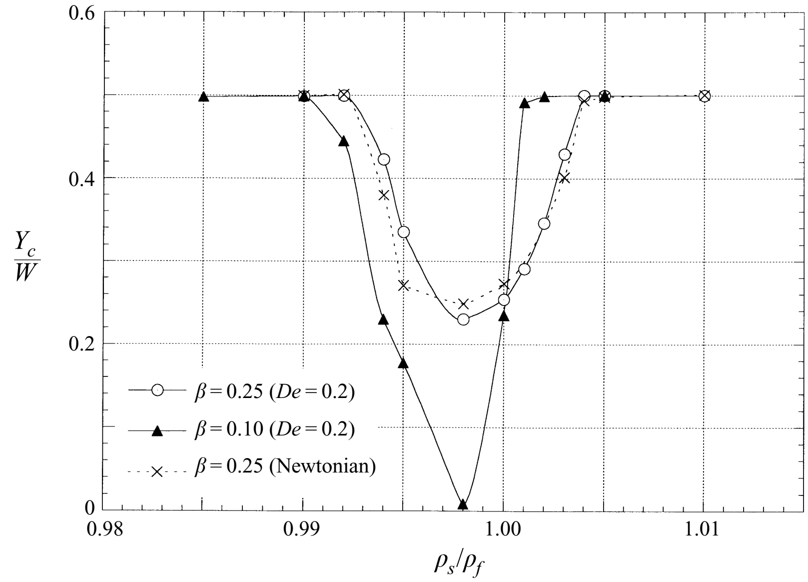

FIGURE 20. The stable position of a non-neutrally buoyant particle in Poiseuille flow of an Oldroyd-B fluid. $Re = 5$. A Newtonian curve for $\beta = 0.25$ is also shown for comparison.

a bit closer to the wall in a viscoelastic fluid but a particle with larger density difference stabilizes closer to the centre of the channel. This trend is opposite to that in a Couette flow (figure 9).

Karnis & Mason (1966) matched the density of the fluid and particles in their experiments on Couette flow but they did not report procedures for density matching in their experiments on Poiseuille flow even though the experiments were for different concentrations of polymer using the same particles. They observed that particles move toward the centre of the channel in Poiseuille flow but did not say that these particles were neutrally buoyant. It seems to us that the particles used in their experiments on Poiseuille flow could not all be neutrally buoyant, though the mismatch of density might be small. Our results in figure 20 shows that the particle will approach the centre of the channel if the density of the particle ρ_s is 0.5% larger or 1% smaller than the density of the carrying fluid ρ_f.

5. The effects of shear thinning in sedimentation

In this section we present some results on the effects of shear thinning in the sedimentation of the particles in an Oldroyd-B fluid which was not given in the previous study of Feng *et al.* (1996). All the dimensionless parameters are defined in the same way as in Couette and Poiseuille flows except that the characteristic velocity is replaced by the particle velocity U_p.

Shear thinning leads to a larger sedimentation velocity of the particle. This means larger inertial effects on one hand, which drive the particle away from the wall, and larger effects of the compressive normal stresses on the other hand, which push it toward the wall. These features are clearly seen in figure 21 where the particle is released in a $\beta = 0.25$ channel. The dashed curves show that the effects of inertia are greater than the effects of normal stresses due to the increasing velocity induced by the shear thinning so the particle moves to an equilibrium position closer to the centre. To

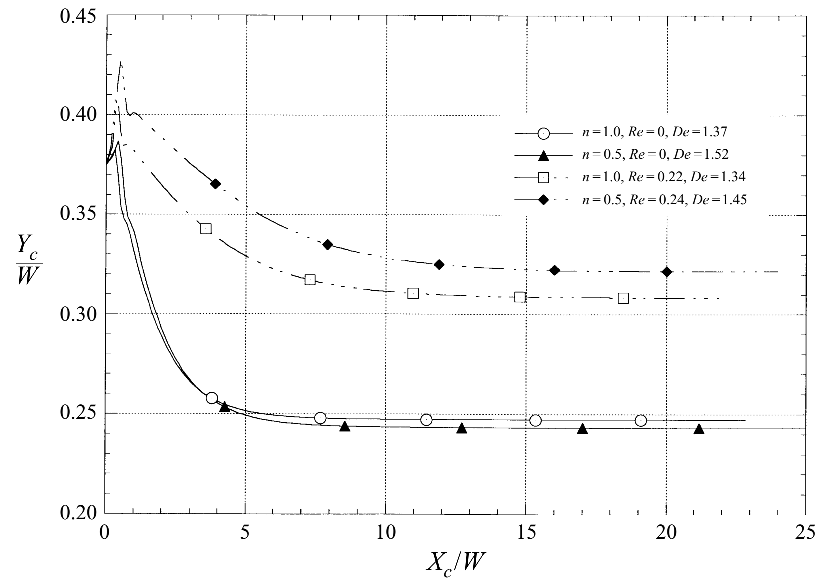

FIGURE 21. The effects of shear thinning on the sedimentation of particle in an Oldroyd-B fluid. $\beta = 0.25$.

emphasize the effects of the normal stresses, we remove the inertial term so that $Re = 0$. The solid curves show that the compressive normal stresses push the particle closer to the wall as a result of shear thinning.

6. Conclusions

In this paper we have studied the unsteady motion of both neutrally buoyant and non-neutrally buoyant particles in a Couette flow and in a Poiseuille flow. Using direct numerical simulation, we can follow the particle's motion and verify our concepts of the mechanisms which move the solid particles in the viscoelastic fluids.

For a neutrally buoyant particle in a viscoelastic fluid, the equilibrium position varies as a result of inertia, elasticity, the blockage ratio and shear thinning.

(*a*) *Effects of inertia*. Lower Reynolds number leads to stronger attraction to the wall on a particle in a viscoelastic fluid so the particle migrates closer to the wall in both a Couette flow and a Poiseuille flow. This is similar to the attraction toward the wall on a particle sedimenting in a viscoelastic fluid but is different from the migration in a Newtonian fluid. In a Newtonian fluid, a particle will move to the centreline of the channel in a Couette flow regardless of Reynolds number and move closer to the centreline in a Poiseuille flow as Reynolds number is decreased.

(*b*) *Effects of the blockage ratio*. A particle moves closer to the wall as the width of the channel becomes smaller, as in the sedimentation of a particle. The streamlines in a narrower channel crowd in such a way as to intensify the shear rates in the more open portions of the channel away from the wall so as to intensify the compressive normal stresses.

(*c*) *Effects of shear thinning*. Shear thinning strengthens the attraction to the wall on the particle in both Couette and Poiseuille flows. In a Couette flow, a particle migrates and finally touches the nearby wall in the absence of inertia. This is because the compressive normal stresses at places of high shear rate on the body increase as a result of shear thinning. In sedimentation, the settling velocity of the particle increases because of shear thinning, increasing both the effect of the inertia which drives the particle away from the wall and the effect of the compressive normal stresses which pushes it toward the wall.

(*d*) *Effects of elasticity*. Elasticity gives rise to the compressive normal stresses on the particle and attracts it to the wall in Couette flow and in sedimentation. If the blockage ratio is large enough ($\beta = 0.25$), the effect of the curvature of the inflow velocity profile is overwhelmed by the effect of the blockage ratio, and the particle in Poiseuille flow moves toward the sidewall until it touches the wall if the fluid inertia is put to zero. The compressive normal stresses on the surface of the particle where the streamlines are crowded cause the particle to move closer to a sidewall. This is different from sedimentation where a particle goes closer to the wall with larger elasticity but never touches the wall since the elasticity generates a repulsion from the wall in the region next to the wall. If the blockage ratio is very small ($\beta = 0.025$), the particle in Poiseuille flow moves to the centre of the channel where the shear rate of the undisturbed flow is zero. In this case, the migration of the particle is controlled by the normal stresses induced by the curvature of the inflow velocity profile. Larger Deborah number *De* will drive the particle toward the centre of the channel faster.

In general, a particle's motion is controlled by the competition between the effects of inertia and the effects of the compressive normal stresses. The effects of the normal stresses come from the local shear rate on the particle surface induced by the motion of the particle when the effect of the blockage ratio is strong and by the curvature of the inflow velocity profile when the effect of the blockage ratio is weak.

For the Deborah numbers tested, all non-neutrally buoyant particles migrate toward the centreline of the channel when they are 1 % lighter or 0.5 % heavier than the fluid, as in a Newtonian fluid.

(*e*) A slightly buoyant particle in a Couette flow migrates closer to the centreline of the channel in a viscoelastic fluid than it does in a Newtonian fluid. A more buoyant particle in a Couette flow moves closer to the sidewalls than it does in a Newtonian fluid.

(f) In a Poiseuille flow, a slightly buoyant particle moves closer to the sidewall and a very buoyant particle moves closer to the centreline of the channel in a viscoelastic fluid than it does in a Newtonian fluid.

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