Canonical Sources and Duality in Chiral Media

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Abstract—**Chiral media are characterized by the constitutive relations $\mathbf{D} = \epsilon\mathbf{E} + i\xi_c\mathbf{B}$ and $\mathbf{H} = \mathbf{B}/\mu + i\xi_c\mathbf{E}$ where ξ_c is the *chirality admittance* introduced to take into account macroscopic handedness or optical activity inherent in the media. In addition we define a *chirality impedance* and a dimensionless *chirality factor* to describe the wave properties of this medium. As known for some time, this medium supports the plane-wave propagation of circularly polarized waves of opposing handedness and differing wavenumbers. Here we examine the radiation of electromagnetic waves from a set of simple canonical arrays. This leads us to the notion of duality for chiral media which can be exhibited in a surprisingly simple form. We show that in the far field, both point and extended sources, whether electric or magnetic, radiate two electromagnetic eigenmodes which are of opposing handedness. We also demonstrate sources which access only one of the eigenmodes of the medium. Several applications of the results and array performance in chiral media are noted.**

I. Introduction

CHIRALITY[1] or handedness is common in a variety of naturally occurring and man-made objects. The former includes a diverse array of sugars, amino acids, DNA, and certain mollusks and winding vegetation while the latter encompasses such common objects as gloves, stringed instruments, and helices. This form of symmetry, or lack of bilateral symmetry, has been of interest to the scientific community since its discovery by Arago [1] in the early nineteenth century and subsequent experimentation by Biot [2] and Pasteur [3] in the mid-1800's. These researchers were concerned with the rotation of the plane of polarization of optical waves due to interaction with certain crystals and liquids. Since then, this phenomenon has been of interest to those in the electromagnetics' community starting with the simple but illuminating microwave experiments of Lindman [4], [5] and Pickering [6] performed in the early and middle part of the twentieth century, respectively.

Of more recent note are several papers by Bohren examining the reflection of electromagnetic waves from chiral spheres and cylinders [7], [8] and the book by Kong [9] and numerous references therein regarding general bianisotropic media. Shortly thereafter was the research by Jaggard *et al.* on relating the interaction of electromagnetic waves with chiral structures and the relation of microscopic and macroscopic chiral media [10], and the work on transition radiation at a chiral-achiral interface by Engheta and Mickelson [11]. In the most recent past is the work on the reflection of waves from chiral-achiral interfaces reported by Silverman [12] and Lakhtakia *et al.* [13] and the scattering of waves from nonspherical chiral objects by Lakhtakia *et al.* [14].

Here we shall examine the characteristics of antenna arrays embedded in unbounded chiral media using the Green's dyadic for electric sources and the Green's vector for magnetic sources. The former was recently found by Bassiri *et al.* [15] for electric sources. Our purpose is to bring to light the new characteristics of sources, both point and extended whether electric or magnetic, which interact with this medium and to examine general characteristics of sources located in a medium with handedness. These results may prove valuable in estimating the effect of bounded chiral media, such as lenses and radomes, on the performance of arrays. Of theoretical interest is the very simple duality relations that are characteristic of chiral media when the results are written in terms of the circular eigenmodes. Appropriate measures of chirality such as the chirality admittance and impedance and a dimensionless chirality factor are introduced as needed.

II. Problem Formulation

Chirality is introduced into electromagnetic theory through the constitutive relations [9]–[11], [15], [16] given by

$$\mathbf{D} = \epsilon\mathbf{E} + i\xi_c\mathbf{B} \tag{1}$$

$$\mathbf{H} = \mathbf{B}/\mu + i\xi_c\mathbf{E} \tag{2}$$

for the lossless case where boldface quantities denote vectors and lightface characters denote scalars. The ***chirality admittance*** ξ_c (a real number) is an indication of the degree of chirality of the medium and ϵ and μ are the usual permittivity and permeability, respectively. Intuitively, this chirality admittance is a result of electrical-like secondary sources being induced by magnetic fluxes and magnetic-like secondary sources being induced by electric fields as explained previously for the case of a medium composed of electrically small helices [10]. Alternative but equivalent constitutive relations have also been employed.[2]

Using the time-harmonic Maxwell equations for both electric sources $\mathbf{J}$ and ρ and magnetic sources $\mathbf{J_m}$ and ρ_m with

Manuscript received July 16, 1987; revised October 29, 1987.

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IEEE Log Number 8820229.

[1] By definition, an object is chiral if it cannot be brought into congruence with its mirror image by translation and rotation. The mirror image of a chiral object is denoted its enantiomorph. If a chiral object is right-handed (left-handed) then its enantiomorph is left-handed (right-handed). An object which is not chiral is said to be achiral.

[2] In [7], [8], [13], [14] the fields $\mathbf{D}$ and $\mathbf{B}$ are related to $\mathbf{E}$ and $\mathbf{H}$ and their derivatives through the relations $\mathbf{D} = \epsilon\mathbf{E} + \beta\epsilon\nabla\times\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H} + \beta\mu\nabla\times\mathbf{H}$ where β is measure of chirality. These relations can be shown to produce results equivalent to those obtained through (1) and (2).

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$e^{-i\omega t}$ excitation assumed

$$\nabla \times \mathbf{E} = i\omega \mathbf{B} - \mathbf{J}_m \tag{3}$$

$$\nabla \times \mathbf{H} = \mathbf{J} - i\omega \mathbf{D} \tag{4}$$

$$\nabla \cdot \mathbf{B} = \rho_m \tag{5}$$

$$\nabla \cdot \mathbf{D} = \rho \tag{6}$$

one can find the following inhomogeneous differential equations for the field quantities with the aid of (1) and (2):

$$\Box_c^2 \mathbf{E} = i\omega\mu[\mathbf{J} - i\xi_c \mathbf{J}_m] - \nabla \times \mathbf{J}_m \tag{7}$$

$$\Box_c^2 \mathbf{H} = i\omega\mu[i\xi_c \mathbf{J} + \mathbf{J}_m/\eta_c^2] + \nabla \times \mathbf{J} \tag{8}$$

$$\Box_c^2 \mathbf{B} = \mu\{\nabla \times [\mathbf{J} + i\xi_c \mathbf{J}_m] + i\omega\epsilon \mathbf{J}_m\} \tag{9}$$

$$\Box_c^2 \mathbf{D} = \mu\{i\omega\epsilon \mathbf{J} + \nabla \times [i\xi_c \mathbf{J} - \mathbf{J}_m/\eta_c^2]\} \tag{10}$$

where the chiral differential (Helmholtz-like) operator is given by the relation

$$\Box_c^2\{\ \} \equiv \nabla \times \nabla \times \{\ \} - 2\omega\mu\xi_c \nabla \times \{\ \} - k^2\{\ \} \tag{11}$$

and where

$$\eta_c \equiv \eta_0 \left[\frac{1}{\sqrt{1+\eta_0^2 \xi_c^2}}\right] \tag{12}$$

is a generalized *chiral impedance* with $\eta_0 (=\sqrt{\mu/\epsilon})$ as the background intrinsic wave impedance. The introduction of both the chiral impedance by (12) and the chiral admittance through (1) and (2) lead us naturally to the definition of a dimensionless *chirality factor* κ given by their product. Explicitly we write

$$\kappa \equiv \eta_c \xi_c \tag{13}$$

where the absolute value of κ is bounded by zero and unity. It is this parameter that is a quantitative measure of the degree of chirality of the medium, and we propose it here as the appropriate measure of the chirality of a medium. We note that ξ_c is bounded by relations involving the permittivity and permeability and cannot become large without limit [16].

Since the fields **E**, **B**, **D**, and **H** are linearly dependent on the current sources **J** and $\mathbf{J}_m$ one can write these fields in terms of integrals over the sources and appropriately defined Green's vectors and Green's dyads. Following an analogous procedure to that outlined by Papas [17] for achiral media, the time-harmonic fields can be written for unbounded chiral media as

$$\mathbf{E}(\mathbf{x}) = i\omega\mu \int \bar{\Gamma}(\mathbf{x}, \mathbf{x}') \cdot [\mathbf{J}(\mathbf{x}') - i\xi_c \mathbf{J}_m(\mathbf{x}')]\, d\mathbf{x}' - \int \Gamma_m(\mathbf{x}, \mathbf{x}') \times \mathbf{J}_m(\mathbf{x}')\, d\mathbf{x}' \tag{14}$$

$$\mathbf{H}(\mathbf{x}) = i\omega\mu \int \bar{\Gamma}(\mathbf{x}, \mathbf{x}') \cdot [\mathbf{J}_m(\mathbf{x}')/\eta_c^2 + i\xi_c \mathbf{J}(\mathbf{x}')]\, d\mathbf{x}' + \int \Gamma_m(\mathbf{x}, \mathbf{x}') \times \mathbf{J}(\mathbf{x}')\, d\mathbf{x}' \tag{15}$$

$$\mathbf{B}(\mathbf{x}) = \mu \left\{ i\omega\epsilon \int \bar{\Gamma}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{J}_m(\mathbf{x}')\, d\mathbf{x}' + \int \Gamma_m(\mathbf{x}, \mathbf{x}') \times [\mathbf{J}(\mathbf{x}') + i\xi_c \mathbf{J}_m(\mathbf{x}')]\, d\mathbf{x}' \right\} \tag{16}$$

$$\mathbf{D}(\mathbf{x}) = \mu \left\{ i\omega\epsilon \int \bar{\Gamma}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{J}(\mathbf{x}')\, d\mathbf{x}' - \int \Gamma_m(\mathbf{x}, \mathbf{x}') \times [\mathbf{J}_m(\mathbf{x}')/\eta_c^2 - i\xi_c \mathbf{J}(\mathbf{x}')]\, d\mathbf{x}' \right\} \tag{17}$$

where the functions Green's dyad $\bar{\Gamma}(\mathbf{x}, \mathbf{x}')$ and the Green's vector $\Gamma_m(\mathbf{x}, \mathbf{x}')$ are as yet undetermined. Here boldface quantities with overbars indicate dyads. It can be verified that (14)–(17) satisfy the constitutive relations (1) and (2).

The dyadic Green's function $\bar{\Gamma}(\mathbf{x}, \mathbf{x}')$ has been derived [15] and can be rewritten in the compact and instructive form

$$\bar{\Gamma}(\mathbf{x}, \mathbf{x}') = \bar{\Gamma}^+(\mathbf{x}, \mathbf{x}') + \bar{\Gamma}^-(\mathbf{x}, \mathbf{x}') = \beta\bar{\gamma}^+(k_+)G_+(\mathbf{x}, \mathbf{x}') + [1-\beta]\bar{\gamma}^-(k_-)G_-(\mathbf{x}, \mathbf{x}') \tag{18}$$

where the + and − superscripts refer to the first and second terms, respectively, on the right-hand side of (18) and the dyadic operators for the two eigenmodes are given in terms of the unit dyad $\bar{\mathbf{I}}$ by

$$\bar{\gamma}^\pm(k_\pm) = \{\bar{\mathbf{I}} \pm k_\pm^{-1} \bar{\mathbf{I}} \times \nabla + k_\pm^{-2} \nabla\nabla\} \tag{19}$$

and where

$$G_\pm(\mathbf{x}, \mathbf{x}') = \frac{\exp\,[ik_\pm|\mathbf{x}-\mathbf{x}'|]}{4\pi|\mathbf{x}-\mathbf{x}'|} \tag{20}$$

$$k_\pm = k_0[\sqrt{1+\eta_0^2\xi_c^2} \pm \eta_0\xi_c] = k_0\sqrt{1+\eta_0^2\xi_c^2}\,[1 \pm \kappa] \tag{21}$$

$$\beta = \frac{k_0^2 - k_+^2}{k_-^2 - k_+^2} = \frac{1}{2}[1+\kappa] \tag{22}$$

$$1-\beta = \frac{k_0^2 - k_-^2}{k_+^2 - k_-^2} = \frac{1}{2}[1-\kappa]. \tag{23}$$

The wavenumbers $k_\pm$ are the propagation constants for the two eigenmodes supported by the medium. We denote the factors β and $1 - \beta$ *handedness factors*. These quantities will play a role in the far-field radiation patterns of antennas and arrays and represent the relative amplitude of waves of each handedness. Here $k_0 (=\omega\sqrt{\mu\epsilon})$ is the background wavenumber of the achiral media with identical permittivity and permeability.

The Green's vector $\Gamma_m(\mathbf{x}, \mathbf{x}')$ can be found in a similar manner to be

$$\Gamma_m(\mathbf{x}, \mathbf{x}') = \beta\gamma_m^+(k_+)G_+(\mathbf{x}, \mathbf{x}') + [1-\beta]\gamma_m^-(k_-)G_-(\mathbf{x}, \mathbf{x}') \tag{24}$$

where the vector operators are given by

$$\gamma_m^\pm(k_\pm) = \{\nabla \pm k_\pm^{-1} \nabla \times \nabla\} \tag{25}$$

and the other quantities are defined as before.

We note that the background wavenumber satisfies the two-sided inequality $k_- \le k_0 \le k_+$ for positive ξ_c and $k_+ \le k_0 \le$

k_- for negative ξ_c. Likewise, the handedness factors β and $1-\beta$ satisfy the inequality $0 \leq \beta, 1-\beta \leq 1$. Clearly, the two modes, denoted by $^{+(-)}$ correspond to waves propagating with wavenumbers greater (less than) the background wavenumber for a positive chirality factor and it can be demonstrated that the former produce right-handed circularly polarized waves while the latter produce left circularly polarized waves in the far field. This form complements Bohren's results for source-free wave propagation [7], [8]. In the limit $\xi_c \to 0$, the medium becomes achiral since the wavenumbers $k_\pm \to k_0$ while β, $1-\beta \to 1/2$ and G_+ and G_- approach a common value, that of the background medium. Similarly, the Green's dyad and the Green's vector take on their achiral forms [17] in this limit.

III. Duality and Far-Field Radiation

Using (14)–(17) and the operators (18) and (24), the field expressions can be simplified so that appropriate field duality for chiral media becomes evident. Expressions (14)–(17) take on the simple form for each eigenmode,

$$\mathbf{E}(\mathbf{x})_\pm = i\omega\mu \int \bar{\Gamma}^\pm(\mathbf{x}, \mathbf{x}') \cdot [\mathbf{J}(\mathbf{x}') \pm i\mathbf{J}_\mathbf{m}(\mathbf{x}')/\eta_c]\, d\mathbf{x}' \tag{26}$$

$$\mathbf{H}(\mathbf{x})_\pm = \frac{-i\omega\mu}{\eta_c} \int \bar{\Gamma}^\pm(\mathbf{x}, \mathbf{x}') \cdot (\pm i)[\mathbf{J}(\mathbf{x}') \pm i\mathbf{J}_\mathbf{m}(\mathbf{x}')/\eta_c]\, d\mathbf{x}' \tag{27}$$

$$\mathbf{B}(\mathbf{x})_\pm = -i\mu k_\pm \int \bar{\Gamma}^\pm(\mathbf{x}, \mathbf{x}') \cdot (\pm i)[\mathbf{J}(\mathbf{x}') \pm i\mathbf{J}_\mathbf{m}(\mathbf{x}')/\eta_c]\, d\mathbf{x}' \tag{28}$$

$$\mathbf{D}(\mathbf{x})_\pm = \frac{i\mu k_\pm}{\eta_c} \int \bar{\Gamma}^\pm(\mathbf{x}, \mathbf{x}') \cdot [\mathbf{J}(\mathbf{x}') \pm i\mathbf{J}_\mathbf{m}(\mathbf{x}')/\eta_c]\, d\mathbf{x}' \tag{29}$$

where the subscript or superscript $\pm$ indicates the appropriate eigenmode and the total field is obtained by summing over the two modes.

We note that the particularly simple forms for the field quantities due to both electric and magnetic sources, as given by (26)–(29), are peculiar to the case of chiral media and its resulting circular polarization eigenmodes. These expressions are less cumbersome than their achiral counterparts which are more like relations (14)–(17). Thus we expect a certain simplicity in the field structure when both electric and magnetic sources are present.

From the symmetry of (26)–(29) it is apparent that **E** and **H** are dual field quantities as are **D** and **B**. Explicitly, one can show by substitution that if the original fields and their sources which satisfy (1)–(6) are denoted by unprimed quantities, their duals, denoted by primes, also satisfy Maxwell's equations and the constitutive relations providing

$$\mathbf{E}' = \pm\eta_c\mathbf{H} \qquad -\mathbf{H}' = \pm\mathbf{E}/\eta_c \qquad -\mathbf{B}' = \pm\eta_c\mathbf{D} \qquad \mathbf{D}' = \pm\mathbf{B}/\eta_c \tag{30}$$

and

$$-\mathbf{J}_\mathbf{m}' = \pm\eta_c\mathbf{J} \qquad \mathbf{J}' = \pm\mathbf{J}_\mathbf{m}/\eta_c \qquad -\rho_m' = \pm\eta_c\rho \qquad \rho' = \pm\rho_m/\eta_c. \tag{31}$$

These results show the extremely simple relationship that electric and magnetic sources have in chiral media due to the peculiar constitutive relations of (1) and (2) and evidenced in the field similarities demonstrated in (26)–(29). We use these results in the next section to examine the radiation from simple electric and magnetic sources. Also note the role of the chiral impedance η_c which is an intrinsic descriptor of the chiral medium.

From a far-field expansion of the Green's dyad (18) the electric field eigenmodes corresponding to (14) or (26) can be written in the form

$$\mathbf{E}(\mathbf{x})_\pm \underset{kr \gg 1}{\Rightarrow} i\omega\mu \begin{Bmatrix} \beta \\ 1-\beta \end{Bmatrix} \frac{e^{ik_\pm r}}{4\pi r} [-\hat{\mathbf{e}}_\mathbf{r}\times\hat{\mathbf{e}}_\mathbf{r}\times \pm i\hat{\mathbf{e}}_\mathbf{r}\times] \cdot \int e^{-ik_\pm \hat{\mathbf{e}}_\mathbf{r}\cdot\mathbf{x}'} [\mathbf{J}(\mathbf{x}') \pm i\mathbf{J}_\mathbf{m}(\mathbf{x}')/\eta_c]\, d\mathbf{x}' \tag{32}$$

for general current sources where $r = |\mathbf{x}|$, $\hat{\mathbf{e}}_\mathbf{r}$ is a unit vector along the position vector **x**. It is understood here and in the following equations that in the triple-cross product involving $\hat{\mathbf{e}}_\mathbf{r}$ the cross products are carried out right to left. Likewise, using (15) or (27) one can show that the magnetic field in this limit is related to (32) by the expression

$$\mathbf{H}(\mathbf{x})_\pm \underset{kr \gg 1}{\Rightarrow} \frac{i\omega\mu}{\eta_c} \begin{Bmatrix} \beta \\ 1-\beta \end{Bmatrix} \frac{e^{ik_\pm r}}{4\pi r} [\hat{\mathbf{e}}_\mathbf{r}\times \pm i\hat{\mathbf{e}}_\mathbf{r}\times\hat{\mathbf{e}}_\mathbf{r}\times] \cdot \int e^{-ik_\pm \hat{\mathbf{e}}_\mathbf{r}\cdot\mathbf{x}'} [\mathbf{J}(\mathbf{x}') \pm i\mathbf{J}_\mathbf{m}(\mathbf{x}')/\eta_c]\, d\mathbf{x}' \tag{33}$$

so that

$$\mathbf{H}(\mathbf{x})_\pm = \hat{\mathbf{e}}_\mathbf{r}\times\mathbf{E}(\mathbf{x})_\pm/\eta_c \tag{34}$$

where the chiral wave impedance η_c has been given by (12). Likewise, for the field pair **D** and **B** one finds

$$\mathbf{D}(\mathbf{x})_\pm \underset{kr \gg 1}{\Rightarrow} \frac{i\mu}{\eta_c} k_\pm \begin{Bmatrix} \beta \\ 1-\beta \end{Bmatrix} \frac{e^{ik_\pm r}}{4\pi r} [-\hat{\mathbf{e}}_\mathbf{r}\times\hat{\mathbf{e}}_\mathbf{r}\times \pm i\hat{\mathbf{e}}_\mathbf{r}\times] \cdot \int e^{-ik_\pm \hat{\mathbf{e}}_\mathbf{r}\cdot\mathbf{x}'} [\mathbf{J}(\mathbf{x}') \pm i\mathbf{J}_\mathbf{m}(\mathbf{x}')/\eta_c]\, d\mathbf{x}' \tag{35}$$

$$\mathbf{B}(\mathbf{x})_\pm \underset{kr \gg 1}{\Rightarrow} i\mu k_\pm \begin{Bmatrix} \beta \\ 1-\beta \end{Bmatrix} \frac{e^{ik_\pm r}}{4\pi r} [\hat{\mathbf{e}}_\mathbf{r}\times \pm i\hat{\mathbf{e}}_\mathbf{r}\times\hat{\mathbf{e}}_\mathbf{r}\times] \cdot \int e^{-ik_\pm \hat{\mathbf{e}}_\mathbf{r}\cdot\mathbf{x}'} [\mathbf{J}(\mathbf{x}') \pm i\mathbf{J}_\mathbf{m}(\mathbf{x}')/\eta_c]\, d\mathbf{x}' \tag{36}$$

and consequently

$$\mathbf{B}(\mathbf{x})_\pm = \eta_c\hat{\mathbf{e}}_\mathbf{r}\times\mathbf{D}(\mathbf{x})_\pm \tag{37}$$

in a manner reminiscent of (34).

We note that the results (34) and (37) are identical both to similar results in achiral media and the results given in [15] for chiral media with only electric sources present. That is, the introduction of magnetic sources in no way upsets the symmetry of the equations which characterize chiral medium. This is due in part to the fact that the additional terms in (14)–(17) due to magnetic sources can be absorbed through the use of chiral impedance to produce (26)–(29).

Of particular note from (26)–(29) is that either eigenmode can be excited while the other is suppressed through the appropriate choice of electric and magnetic sources. This concept is new with the introduction of magnetic sources and will be exploited in the following section concerning simple canonical arrays in chiral media. Also note from (32)–(33) and (35), (36) that each field has three parts, the first is an amplitude prefactor dependent on β, the second is a polarization term indicated by the bracketed cross products of unit vectors $\hat{\mathbf{e}}_r$, and the third is a propagation part due to the exponential kernel $e^{-ik_\pm \hat{\mathbf{e}}_r \cdot \mathbf{x}'}$ in the integrand. Each of these three parts of the fields is investigated in the following sections which apply these results to illuminating examples.

IV. Canonical Point Arrays

The expression for the electric field eigenmodes due to a point electric dipole $\mathbf{p}$ and point magnetic dipole $\mathbf{m}$ located at the origin is immediately found from (26) with the relations $\mathbf{J}(\mathbf{x}') = -i\omega\mathbf{p}\,\delta(\mathbf{x}')$ and $\mathbf{J}_m(\mathbf{x}') = -i\omega\mu\mathbf{m}\,\delta(\mathbf{x}')$ as

$$\mathbf{E}(\mathbf{x})_\pm = \omega^2\mu\bar{\Gamma}^\pm(\mathbf{x}, \mathbf{0}) \cdot [\mathbf{p} \pm i\mathbf{m}/v_c]. \quad (38)$$

In the far field this expression can be written as

$$\mathbf{E}(\mathbf{x})_\pm \underset{kr \gg 1}{\Rightarrow} \omega^2\mu \begin{Bmatrix} \beta \\ 1-\beta \end{Bmatrix} \{-\hat{\mathbf{e}}_r \times \hat{\mathbf{e}}_r \times [\mathbf{p} \pm i\mathbf{m}/v_c] \pm i\hat{\mathbf{e}}_r \times [\mathbf{p} \pm i\mathbf{m}/v_c]\} \frac{e^{ik_\pm r}}{4\pi r} \quad (39)$$

with

$$v_c \equiv \eta_c/\mu \equiv [\sqrt{\mu\epsilon}\sqrt{1+\eta_0^2\xi_c^2}]^{-1}$$

being the generalized *chiral velocity*. It is apparent that this expression suggests ways in which one or both of the eigenmodes of the medium can be excited or sensed. The two-element point arrays of interest examined here are those formed by coincident parallel electric and magnetic dipoles and the turnstyle antenna formed by two coincident orthogonal electric dipoles. These configurations are displayed in Fig. 1.

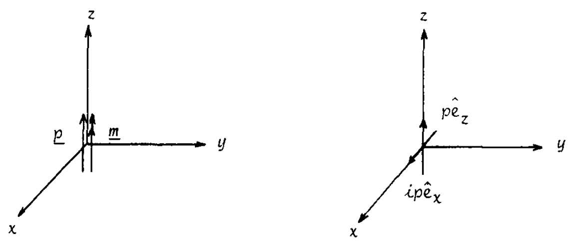

Fig. 1. Two-element point arrays. Coincident parallel electric and magnetic dipoles are shown on the left; a small turnstyle antenna is shown on the right.

Consider first the case of parallel electric and magnetic dipoles located at the origin. Two special cases are especially illuminating. Assume as the first special case the relation where the currents in the two dipoles are in phase and give rise to fields of equal magnitude. If $\mathbf{p} = i\mathbf{m}/v_c = p\hat{\mathbf{e}}_z$, only the positive eigenmode is excited and the total electric field is found to be

$$\mathbf{E}(\mathbf{x}) = \mathbf{E}(\mathbf{x})_+ \underset{kr \gg 1}{\Rightarrow} -2\sqrt{2}\omega^2\mu\beta p \sin\theta \frac{e^{ik_+ r}}{4\pi r} \hat{\mathbf{e}}_+ \quad (40)$$

while if $\mathbf{p} = -i\mathbf{m}/v_c = p\hat{\mathbf{e}}_z$, only the negative eigenmode is excited and the result is

$$\mathbf{E}(\mathbf{x}) = \mathbf{E}(\mathbf{x})_- \underset{kr \gg 1}{\Rightarrow} -2\sqrt{2}\omega^2\mu(1-\beta)p \sin\theta \frac{e^{ik_- r}}{4\pi r} \hat{\mathbf{e}}_- \quad (41)$$

for the total electric field where the circular polarization basis vectors are $\hat{\mathbf{e}}_\pm \equiv (\hat{\mathbf{e}}_\theta \pm i\hat{\mathbf{e}}_\phi)/\sqrt{2}$ and the angles θ and ϕ are the polar and azimuthal angles measured from the z and x axes, respectively. We note that this excitation of only a single mode of the chiral medium is particular to the case where both electric and magnetic sources are present since this cannot be accomplished in chiral medium with only electric sources. In particular, note that for each of these cases, the far field is perfectly circularly polarized, regardless of direction. As in the achiral case, however, the radiation pattern has the $\sin\theta$ dependence characteristic of all electrically small sources.

As the second special case, consider the case where the currents in the two dipoles are fed in phase quadrature so that the moments are in phase and are given by $\mathbf{p} = \mathbf{m}/v_c = p\,\hat{\mathbf{e}}_z$. The field calculation here yields both modes given as

$$\mathbf{E}(\mathbf{x})_\pm \underset{kr \gg 1}{\Rightarrow} -\sqrt{2}\omega^2\mu \begin{Bmatrix} \beta \\ 1-\beta \end{Bmatrix} \cdot p(1 \pm i) \sin\theta \frac{e^{ik_\pm r}}{4\pi r} \hat{\mathbf{e}}_\pm \quad (42)$$

in a manner similar to that of the electric dipole alone. Here the total electric field is not circularly polarized but instead is elliptically polarized. As $\xi_c \to 0$, the ellipse gets thin and the polarization becomes linear. The orientation angle of the ellipse is dependent here on the distance the receiver is from the source.

As the third case of this section, consider the turnstyle antenna where the electric current distribution is given by $\mathbf{J}(\mathbf{x}') = -i\omega p(\hat{\mathbf{e}}_z + i\hat{\mathbf{e}}_x)\,\delta(\mathbf{x}')$. Using (26) or (38) the total electric field exhibits the two circularly polarized eigenmodes as

$$\mathbf{E}(\mathbf{x})_\pm \underset{kr \gg 1}{\Rightarrow} -\sqrt{2}\omega^2\mu \begin{Bmatrix} \beta \\ 1-\beta \end{Bmatrix} \cdot p \frac{e^{ik_\pm r}}{4\pi r} [1 \pm \sin\theta \sin\phi] \hat{\mathbf{e}}'_\pm \quad (43)$$

where $\hat{\mathbf{e}}'_\pm$ denotes the right- and left-handed circular polarization vectors with axes about $\hat{\mathbf{e}}_r$. As for the case of scattering from small helices [10], the two eigenmodes possess considerably different angular dependences. These are displayed in Fig. 2 where it is clear that the two modes access two different half-spaces divided by the plane of the turnstyle antennas. Therefore, in this case, each half-space has essentially a circularly polarized wave of opposite handedness.

V. Canonical Distributed Array

As a final example, consider a distributed source which is a linear array of dipoles embedded in chiral media. Since there

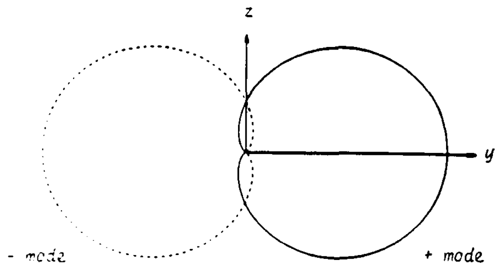

Fig. 2. The radiation pattern of the turnstyle in chiral media displaying the two eignmodes for positive chiral admittance ($\xi_c > 0$). The solid line represents the positive mode with right-hand circular polarization. The dashed line represents that of the negative mode with left-hand circular polarization.

is an inherent geometrical spacing which defines the array, it intuitively appears that the two eigenmodes of the medium will see an array of differing effective geometry. Here we examine the radiation characteristics of these two modes and examine the operation of this antenna when it is operated as a phased array.

Consider the case of a linear array of dipoles as displayed in Fig. 3 where N elements are spaced a distance d apart along the x axis. The phase shift per element is taken to be α. Calculations similar to those leading to (38) here produce the eigenmode expressions in the far field

$$\mathbf{E}(\mathbf{x})_{\pm}\Big|_{kr \gg 1} = \omega^2\mu\bar{\mathbf{\Gamma}}^{\pm}(\mathbf{x}, \mathbf{0}) \cdot [\mathbf{p} \pm i\mathbf{m}/v_c]AF_{\pm} \tag{44}$$

where the angular dependence of the array factor $AF_{\pm}$ is given by

$$AF_{\pm} = \frac{\sin\left[\dfrac{N}{2}(k_{\pm} d \cos\Omega + \alpha)\right]}{\sin\left[\dfrac{1}{2}(k_{\pm} d \cos\Omega + \alpha)\right]} \tag{45}$$

and where Ω is the angle between the array axis and the position vector of the observer. Here both modes play an important role except for the special case of $\pm\mathbf{p} = i\mathbf{m}/v_c$ when only one of the eigenmodes is excited as noted in the previous section. Now, consider the case of electric dipoles only which are given by $\mathbf{p} = p\,\hat{\mathbf{e}}_z$. The total electric field is found to be

$$\mathbf{E}(\mathbf{x})\Big|_{kr \gg 1} -\sqrt{2}\omega^2\mu p \sin\theta \left\{[\beta]\frac{e^{ik_+ r}}{4\pi r}AF_+\hat{\mathbf{e}}_+ + [1-\beta]\frac{e^{ik_- r}}{4\pi r}AF_-\hat{\mathbf{e}}_-\right\} \tag{46}$$

which displays an elliptically polarized wave (combined from the two circular eigenmodes) at broadside but for nonzero phase shifts, can also exhibit two beams of opposite handedness. For this case, $\cos\Omega = \sin\theta\cos\phi$.

As an example of the latter we plot in Fig. 4 the far-field

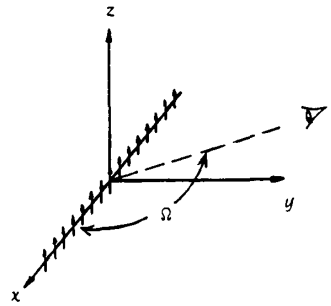

Fig. 3. A linear array of N-element dipoles spaced a distance d apart along the x axis.

radiation pattern of (46) for an array 15 elements ($N = 15$), spaced a half-wavelength apart ($k_0 d = \pi$). As the phase shift α is varied from nominally broadside to increasing values the beam splitting just mentioned is graphically noted. The criterion for beam splitting in which the main beam splits into two large beams is given by the expression

$$N|\alpha| = 2\pi/\kappa. \tag{47}$$

This indicates that for values of $N|\alpha|$ larger than those of (47), the array exhibits two distinct main beams, each circularly polarized with opposite handedness. Of course, grating lobes may also appear as in the case of achiral media.

The evolution of the beam splitting is clearly shown in Fig. 4 for six values of the phase shift α with positive chiral admittance ξ_c. First is shown the broadside case ($\alpha = 0$) in part (a) followed by increasing phase shift until condition (47) is met in part (b). As the phase shift is increased further, a grating lobe appears in the beam for the larger wavenumber k_+ as first noted in part (c). As the phase shift increases still further so that $\alpha > k_- d$, the visible range begins to exclude the main lobe and the negative eigenmode beam decreases in size as shown in part (e). In the limit as $\alpha \to \pi$, almost all of the beam energy in th negative eigenmode vanishes and is converted to the positive eigenmode beam as noted in part (f). This beam suppression is of interest for the case when the antenna is to be used as a source of circular polarization only.

VI. Discussion

Here we examine the radiation of canonical sources, whether point or distributed, in chiral media. This leads us to examine the effect of magnetic sources in this medium and to construct the appropriate duality relations. Each field quantity is the product of three terms as shown in (32), (33) and (35), (36). The first term is an amplitude term which is dependent upon the handedness factors, the second is the polarization and slowly varying pattern term given by the cross products of unit position vectors, while the third is dependent upon the characteristic pair of wavenumbers. The later term determines the propagation characteristics and is the dominant one for the problem of distributed sources since the beams of each eigenmode can access a different region of space. The effect of each of these three terms is examined in turn by the investigation of the electric and magnetic dipole combinations, the turnstyle antenna, and the linear array, respectively.

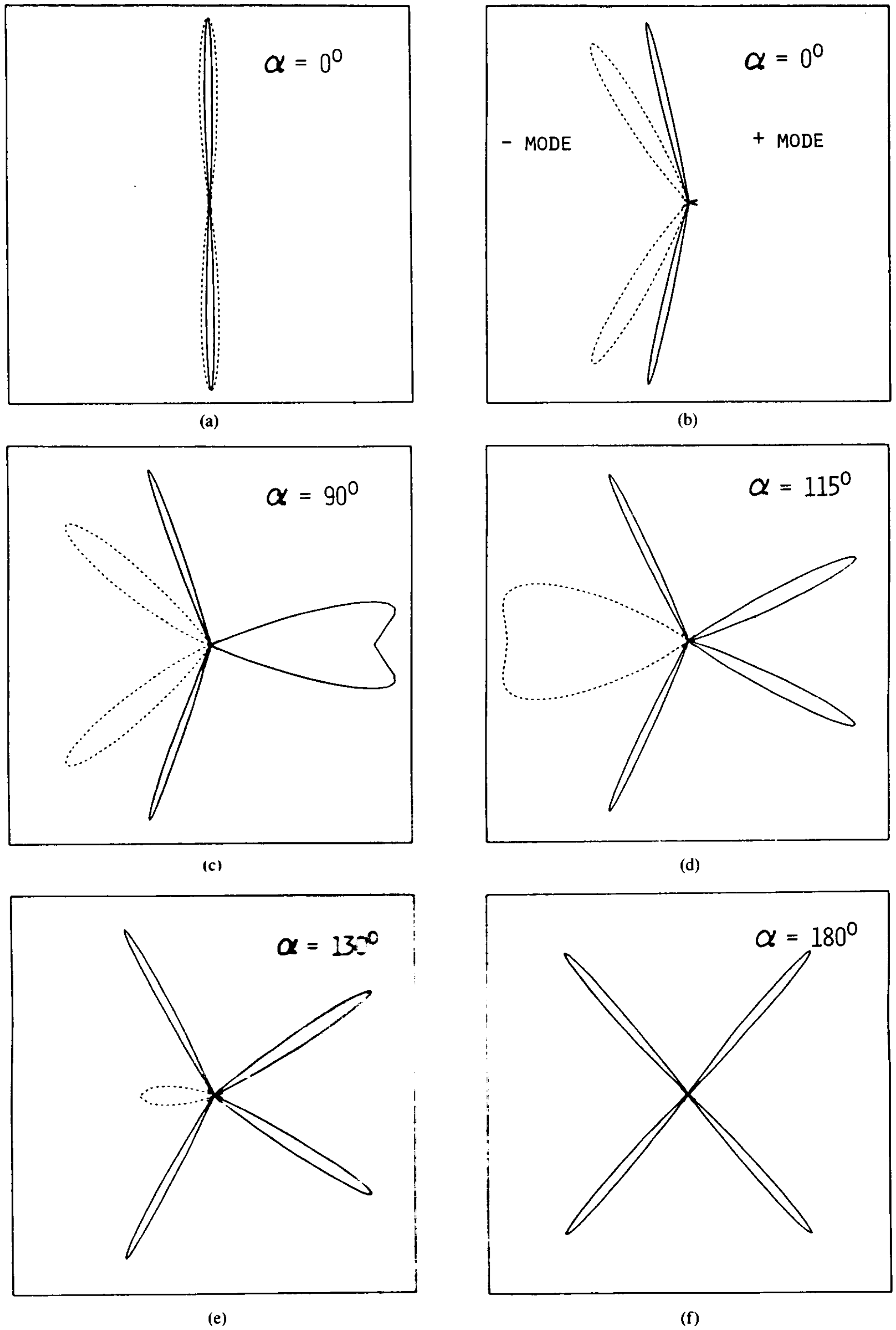

Fig. 4. Evolution of beam splitting and occurrence of mode supression for different phase shifts α in chiral media with $N = 15$ and $\kappa = 0.4$. Here the solid line represents the positive mode while the dashed line represents the negative mode. We use (a) $\alpha = 0°$. (b) $\alpha = 60°$ (beam splitting begains). (c) $\alpha = 90°$. (d) $\alpha = 115°$. (e) $\alpha = 130°$. (f) $\alpha = 180°$ (complete negative eigenmode suppression).

Field duality relations take on a particularly simple form for chiral media in the presence of electric and magnetic sources due, apparently, to the particular constitutive relations of this medium and the use of circular bases functions. These relations indicate that electric and magnetic sources in this medium produce identical results, with the exception of a quadrature phase factor, when viewed in circular bases vectors. In general it is an elliptically polarized wave. The surprising result is that the duality relations are of a form more simple than those of achiral media. The duality relations along with the complementary geometry in scattering problems can be used in diffraction problems in chiral media. The applications of duality relations and generalization of Babinet's principle in chiral media are under present consideration.

The canonical cases examined here are of practical interest in a variety of problems. First consider the design and analysis of antennas covered by chiral radomes or lenses for pattern and/or polarization control. The results given here for unbounded chiral media provide an upper bound or first-order calculation for the effect of finite nonresonant chiral slabs. (Naturally, some resonant structures could show increased chirality over that offered by unbounded media due to multiple reflections.) These applications and those to other polarization diverse arrays is under present consideration.

Second, these results are applicable to the remote sensing of chiral media and the identification of their parameters. Of particular interest is the problem of chirality measurement which involves either the specification of the chirality factor and sense of handedness or equivalently the specification of the chiral admittance. This has traditionally been accomplished in optics through a measurement of the plane of rotation over a differential path. However, at lower frequencies this may not be an easy measurement. Instead we propose a method involving the use of the parallel electric and magnetic dipoles to construct a point sensor in conjunction with the use of the turnstyle antenna as a source. By exciting the chiral medium by a turnstyle antenna, both the absolute degree of chirality and the handedness of the chiral medium can be measured by varying the output currents of the point sensor until a null is achieved. This condition indicates that the ratio p/m of the point sensor is $\pm i/v_c$. From this relation and knowledge of the background permittivity and permeability, the absolute value of the chiral admittance and the chirality factor of the medium can also be found. To find the sign of the chiral admittance or the sense of handedness of the medium, the turnstyle antenna can be used in two mirror-image orientations in which the plane of the antenna is perpendicular to the line connecting the transmitting turnstyle antenna to the point sensor. The larger signal will indicate the correct sense of handedness since the two eigenmodes of the turnstyle antenna access opposing half-spaces.

Finally, we note that describing the characteristics of sources in this medium requires an indication of polarization. The beamwidth, gain, directionality, and related quantities for these antennas depend significantly on which of the modes are excited and/or received. This becomes of particular importance for distributed sources such as phased arrays in which beam splitting is present and which, under certain phase delays, can cause beam suppression of one of the eigenmodes.

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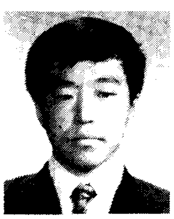

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