

Pricing Strategies for User-Provided Connectivity Services

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Abstract—User-provided connectivity (UPC) services offer a possible alternative, or complement, to existing infrastructure-based connectivity. A user allows other users to occasionally connect through its “home base” in exchange for reciprocation, or possibly compensation. This service model exhibits strong positive and negative *externalities*. A large user base makes the service more attractive, as it offers more connectivity options to roaming users, but it also implies a greater volume of (roaming) traffic passing through a user’s home base, which can increase congestion. These interactions make it difficult to predict the eventual success of such a service offering, and in particular how to effectively price it. This paper investigates a two-price policy where the first price is an introductory price that expires once service adoption reaches a certain level. The paper uses a simplified analytical model to investigate pricing strategies under this policy, and their sensitivity to changes in system parameters. The insight and practical guidelines this yields are validated numerically under more realistic conditions.

I. INTRODUCTION

User-provided connectivity (UPC) has emerged as a possible “organic” service model for ubiquitous connectivity. The approach was first made popular by FON (www.fon.com) that introduced a community-based Internet access model, where roaming FON users are allowed access to the Internet through the home connection of other FON users¹. Home access is through a dedicated device (the FONERA) that connects to the Internet over a standard broadband connection, and allows shared Wi-Fi access (for FON subscribers) to that connection. The main value of the service is in offering connectivity beyond a user’s home base, and this value grows with service coverage, *i.e.*, the first user derived no additional value beyond standard Internet connectivity, while a user joining a network with over 4 million FON spots worldwide (as of 2011) clearly sees a much higher value. On the flip-side, a large number of FON users also means greater odds of having to share one’s home connection. High levels of roaming traffic² may incite some of the more “sedentary” users who joined early (lack of coverage had little impact on them) to leave. If this happens, coverage may decrease enough to affect the service’s value to frequently roaming users, who may in turn start leaving.

In [1], we developed a simple model to elucidate adoption patterns for such a service. Our focus in this paper is on *pricing*

strategies and how prices should be set to maximize the profit of a UPC service provider. Pricing policies in general may vary and range from non-discriminatory (same price for all users) to various levels of price discrimination, *e.g.*, see [2] for a review. In this paper, we limit ourselves to a basic two-price policy, where one price is an introductory price that ends once service adoption reaches a certain level. This is an example of a discriminatory policy, where price discrimination is a function of when a user adopts.

The investigation of this paper is initially analytical in nature, and builds on the simple model of [1]. As argued in Section II, in spite of its simplicity the model captures key aspects of a UPC service adoption, and its analysis provides insight into how to price such a service. The applicability of this insight to more realistic settings is validated numerically for various relaxations and generalizations of the model’s assumptions. The paper’s contributions are not so much the derivation of “optimal” prices. Its simplified model may in fact not match any real system, so that exact prices are of limited practical relevance. Instead, the paper’s main contributions are in using the results of the analysis to develop qualitative pricing guidelines for successful UPC service deployments.

The rest of the paper is structured as follows. Section II introduces the model and its parameters. Solution methods and results are presented in Section III, while Section IV tests and demonstrates their robustness. Finally, Section V gives a brief overview of related works, and Section VI summarizes the paper’s results and points to possible extensions.

II. MODEL FORMULATION

This section first summarizes the simplified model introduced in [1] and its applicability to UPC services such as FON. The two-price strategy is then presented, together with the corresponding revenue maximization problem.

A. Service Utility Function

As commonly done [3], service adoption decisions are based on a *utility function*, with a user adopting when its utility is non-negative. The simple³ utility function we use is given in Eq. (1), whose terms we review next.

$$U(\theta) = \gamma + \theta(2x - 1) - cm - p_\theta. \quad (1)$$

³The utility function of Eq. (1) is simpler than that of [1], where users can be compensated for external traffic using their Internet access.

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¹See also KeyWifi (<http://keywifi.com>) for a similar service.

²The FONERA can control how much access bandwidth is available to other users, but that share must be high enough to deliver acceptable service to roaming users.

γ corresponds to the intrinsic utility that any user derives from home connectivity. In the case of FON this corresponds to the value of Internet connectivity and services through the FONERA itself. Because our focus is not on exploring how basic Internet connectivity entices users to sign-up for the service, we assume the same γ for all users. Differentiation between users is introduced through the parameter θ that measures a user's propensity to roam and, therefore, seek Internet access when away from home. θ is taken to be uniformly distributed in $[0, 1]$. A FON user with $\theta = 0$ never attempts to connect to the Internet through other users' FONERA, while a user with $\theta = 1$ is constantly on the road in need of nearby FONERAs to access the Internet. The former user sees little or no benefits from FON's shared connectivity, while the latter heavily depends on it. Conversely, a $\theta = 1$ user imposes its entire Internet traffic on other users', while a $\theta = 0$ user has no such impact. These two competing factors are reflected in the terms $\theta(2x - 1)$ and $-cm$, respectively.

The term $\theta(2x - 1)$ measures the service utility to roaming users. It depends on how often a user roams, θ , and the service coverage (adoption), x , that ranges from 0 to 100%. Note that when coverage is low ($x < 1/2$), roaming utility is negative⁴. In other words, until the odds of finding an available FONERA while roaming are high enough, the limited coverage of the service is a deterrent. As more users join, coverage improves and the service's value grows as it becomes easier for roaming users to locate Internet access. This valuation is weighed by a user's own θ value that specifies how often it takes advantage of it. The function $-cm$ is a negative, decreasing function of the volume m of roaming traffic that users generate, *i.e.*, traffic that is directed to other users' Internet access, which captures the negative impact this traffic can have on a user's home access. The parameter c is a normalization factor. Note that the volume of roaming traffic, m , grows not just as a function of the number x of adopters, but also as a function of *who* those adopters are, *i.e.*, their θ values. Finally, p_θ represents the cost of the service to users, which as the notation indicates could vary across users.

The two-price policy and the profit maximization problem are introduced next.

B. Two-Price Policy

The major differentiation of a UPC service over traditional home-base connectivity is the ability to connect while roaming. Because this added-value is realized only when adoption is high enough, the adoption timing of a UPC service displays a strong correlation with θ , a user's propensity to roam. Specifically, as a UPC service starts off with limited or no coverage, it is initially attractive only to sedentary users that make up the early adopters. As these users join and coverage correspondingly increases, frequently roaming users gradually find the service more appealing and start joining.

⁴As discussed in [1], an equivalent formulation uses a roaming utility that is a positive, increasing function of coverage, and instead scales down the valuation of home connectivity by $(1 - \theta)$.

The consequence of this adoption pattern is that a low introductory price is typically needed to facilitate the initial build-up of a user-base. However, keeping the price low as coverage increases does not allow the service provider to extract the value created by the higher coverage. This can be remedied by allowing the provider to switch to a higher price once coverage is high enough, *i.e.*, introduce time-based price discrimination. Note that implicit in this two-price policy is that a high final adoption level is realized, *i.e.*, late adopters can be charged more only if they derive greater value from the service because of high coverage. This implies an introductory price low enough to not only bootstrap adoption, but also to ensure that if price was kept unchanged adoption would eventually stabilize at a high level. It is only then, that a higher final price can be safely introduced to recoup some of the added value that the high coverage creates.

Specifically, under a two-price policy, the provider has three decision variables: p_i, p_f , and x_{th} , where p_i is the introductory price, and x_{th} is the service adoption level (coverage) at which pricing switches to the final price $p_f \geq p_i$. The provider's profit $\Pi(p_i, p_f, x_{th})$ is then of the form

$$\Pi(p_i, p_f, x_{th}) = (p_i - e)x_i + (p_f - e)x_f, \quad (2)$$

where x_i and x_f denote the number of adopters that at equilibrium pay p_i and p_f , respectively, for the service. e is the exogenous per-customer cost of offering the service. Note that $x_i \leq x_{th}$ as some of the initial adopters may ultimately drop out, *i.e.*, because of the increase in roaming traffic at higher adoption levels. The provider seeks to identify p_i^*, p_f^* and x_{th}^* such that

$$\Pi(p_i^*, p_f^*, x_{th}^*) = \max_{p_i, p_f, x_{th}} \{\Pi(p_i, p_f, x_{th})\}.$$

The provider's ability to extract the added value associated with higher coverage depends on both p_f and x_{th} . The higher x_{th} , the more valuable the service has become when the price is raised. This allows a higher p_f , but levied on few users. Conversely, lowering x_{th} ensures that more adopters are charged p_f , but it limits how much higher than p_i , p_f can be. In particular, when the price switch occurs, the θ value of the last user who joined prior to the switch is $\theta_{th} = x_{th}$ [1]. The next user considering joining the service has a θ value of x_{th}^+ , but will have to pay p_f . If p_f is too high, this user's utility may be negative, which can then stall adoption.

Ensuring that this does not arise can be formulated more formally as a constraint. It imposes that when switching to price p_f , the user with $\theta = x_{th}^+$ must have positive utility under the new price. This leads to the following two conditions (see again Appendix A of [4] for details) that depend on the position of x_{th} relative to a threshold function of c .

$$p_f \leq \begin{cases} \gamma - \frac{1}{8-2c} & \text{if } x_{th} < \frac{1}{4-c} \\ (2 - c/2)x_{th}^2 - x_{th} + \gamma & \text{if } x_{th} \geq \frac{1}{4-c} \end{cases} \quad (3)$$

The constraints of Eq. (3) are, however, insufficient to ensure that adoption ultimately converges to a high level after switching to p_f . As demonstrated in [1] for a single price, in

some instances adoption need not ever converge to a stable equilibrium. This can also arise under a two-price policy even when p_f satisfies the constraint of Eq. (3).

Ensuring convergence to a stable equilibrium calls for additional constraints beyond those of Eq. (3). Unfortunately, the complex dynamics of adoption under a two-price policy make the analysis under those constraints challenging. This is because, unlike what Fig. 1 suggests, the final adoption region under a two-price policy need not be “contiguous.” In particular, it is possible for the last adopter paying p_i , *i.e.*, the adopter with $\theta = x_{th}$, to have a positive utility, while its immediate neighbor, *i.e.*, the user with $\theta = x_{th}^+$, would *ultimately* (when adoption levels converge) have a negative utility because of the higher price it pays. The utility of users with higher θ values may, however, remain positive because of the greater roaming benefits they derive. This creates a “gap” in the range of θ values of adopters, which makes characterizing final adoption challenging.

As a result and in order to preserve some level of analytical tractability, we introduce the added constraint that the two-price policy achieves equilibrium while maintaining a contiguous (in θ) adoption region. Under this additional constraint, which is shown in Appendix A of [4] to have minimal impact on profit, it is possible to identify an analytically tractable sufficient condition that p_f and x_{th} must meet to ensure the feasibility of an equilibrium, and hence the policy. This condition is given in Eq. (4), where $x = x_i + x_f$ is the equilibrium adoption level (see details in Appendix A of [4]).

$$p_f \leq (2x - 1)x_{th} + \gamma - c/2(-x^2 + 2x), \quad (4)$$

III. SOLUTION AND GUIDELINES

Under the contiguity requirement, final adoption is as shown in Fig. 1. More importantly, final adoption is equal to that of a single-price policy with price p_i . Consequently and using results from [1], x is given by

$$x = \min \left[1, \frac{3 - c + \sqrt{(3 - c)^2 + 2(4 - c)(\gamma - p_i - 1)}}{4 - c} \right].$$

Furthermore, given a continuous adoption region, the set x_i and x_f of users that pay p_i and p_f , respectively, are as illustrated in Fig. 1 and easily seen to satisfy

$$x_i = x - (1 - x_{th}) \quad (5a)$$

$$x_f = 1 - x_{th}. \quad (5b)$$

This can then be used in Eq. (2) to compute profit as a function of x_{th} , p_i and p_f (and the exogenous parameters).

The next step involves maximizing this expression subject to the constraints of Eqs. (3) and (4). This is done in two phases. The first phase involves fixing p_i and identifying optimal values for p_f and x_{th} given p_i . In the second phase, p_i is varied to identify the overall optimal combination.

The outcome of this investigation is shown in Fig. 2, with details found in Appendix A of [4]. The figure reports as a function of the exogenous parameter c (x -axis) and for different γ values (different curves in each sub-figure)

the optimal values of the three parameters of the policy (p_i, p_f, x_{th}) and the resulting profit. Unsurprisingly, the profit decreases as γ decreases and c increases, since both negatively affect the overall service value. The more interesting aspect for our purpose is how changes in γ and c affect the parameters of the two-price policy. In spite of what may seem like a complex set of behaviors, relatively clear trends emerge that we can use to build guidelines for an effective two-price policy. In particular, one can identify three broad regions associated with increasing c values (low, medium, high), which give rise to distinct behaviors for selecting an “optimal” two-price policy.

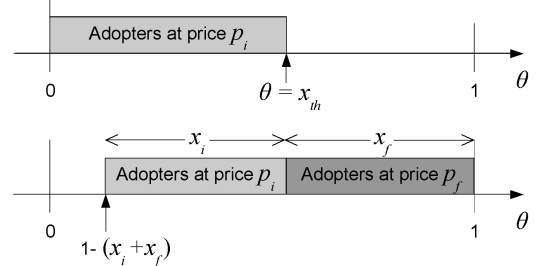


Fig. 1. Adoption under a two-price policy.

In the low c range, *i.e.*, $c \in [0, 0.5]$, which is where a UPC service is of most interest, p_i and p_f follow similar trends (Figs. 2a and 2b). Both prices are relatively insensitive to variations in c , and change linearly with γ (curves in each figure correspond to differences of 0.1 in γ , which all produce comparable price drops). Similarly and as long as c remains sufficiently low, the value of x_{th} (Fig. 2c) is mostly unaffected by changes in either c or γ . Hence, the structure of the policy that maximizes profit in the low c region is approximately constant: Both p_i and p_f prices are relatively insensitive to c and decrease roughly linearly with γ , while the adoption level at which p_f is introduced stays fixed at $x_{th} \approx 75\%$.

The insensitivity of x_{th} to both γ and c reflects the fact that in the low c region where the impact of roaming traffic is minimal, the service should operate at or near full adoption. This calls for building a reasonably large user-base before switching to the new higher price. The initial build-up of the user base depends on attracting sedentary users who are relatively insensitive to the low coverage. Realizing this is determined primarily by γ that represents the bulk of the service value for those users. Hence, the (linear) sensitivity of p_i to γ is intuitive. The limited sensitivity of p_i to c can also be explained by the fact that to build a large enough user base, p_i must be low enough to not only attract $\theta \approx 0$ users, but also users with $\theta > 0$. The service valuation of those users is negatively affected by the low coverage, and p_i must be set low enough to offset that disincentive. This then means that users with $\theta \approx 0$ actually have a positive utility surplus when they join at that price. It is this surplus that allows them to tolerate a (small) increase in c without calling for a corresponding decrease in p_i .

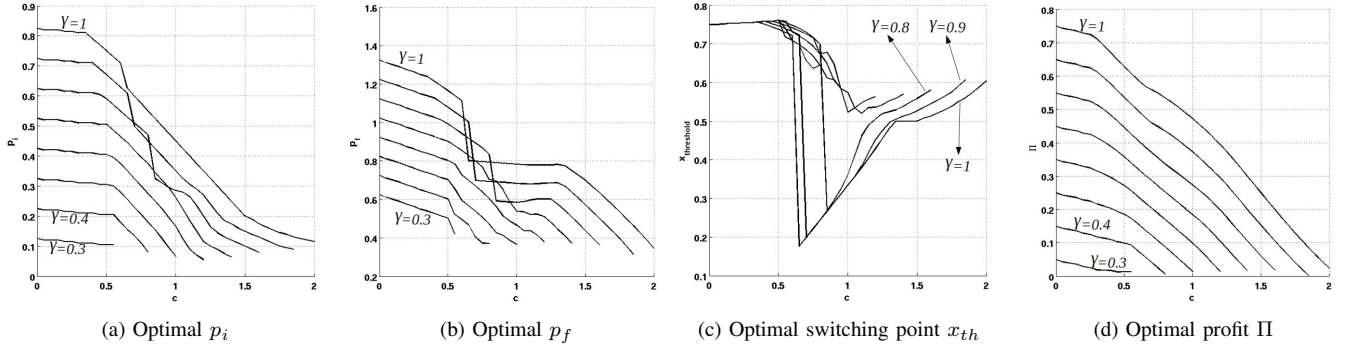


Fig. 2. Optimal p_i, p_f, x_{th} and profit as a function of γ at $e = 0.2$.

A similar situation arises for p_f , even if it affects different users, *i.e.*, the $\theta \geq x_{th}$ users. When adoption crosses x_{th} and the price switch triggers, Eq. (3) ensures that p_f is selected so that the user with $\theta = x_{th}^+$ has a positive utility. As adoption increases beyond x_{th} , that user also derives a greater utility from the improved coverage. This factor, even if it is somewhat offset by the negative impact of additional roaming traffic, provides some level of tolerance to increases in c . As shown in Fig. 2b, this is not always sufficient to completely avoid some small decrease in p_f , but it limits its magnitude.

The other two regions that emerge from Fig. 2, are in a range from medium to high c values. Intermediate c values (from $c = 0.5$ to about 1.25) see decreases in prices and switching threshold that can exhibit somewhat chaotic behaviors before they settle to some final low level once c is high enough. The c value at which this “settling down” occurs depends itself nearly linearly on γ , *i.e.*, a higher γ corresponds to a larger c . This is intuitive since the settling down is associated with configurations where the two-price policy stops being effective, *i.e.*, a high level of adoption is inefficient.

Furthermore, and as illustrated in Appendix A of [4], the chaotic behaviors superimposed on the otherwise relatively steady decline in parameters, can be essentially “ignored” without much impact on the actual profit. Specifically, rather than closely track the fluctuations of the switching threshold x_{th} , it is possible to simply select a threshold value of $x_{th} \approx 0.5$ that corresponds to the threshold used in the high c region, and similarly ignore fluctuations in either p_i and p_f and follow the mostly linear decreases that otherwise prevail. The reason why these fluctuations can be ignored has its root in the trade-off between x_{th} and p_f . Ignoring a sharp drop in price offsets the effect of ignoring a sharp drop in x_{th} .

The guidelines that result from the above discussion are summarized in Fig. 3 that displays three distinct regions. The region labeled ‘N’ corresponds to configurations where the two-price policy is unable to produce a positive profit. As mentioned earlier, this arises when “forcing” a high level of adoption calls for expensive subsidies (low p_i) to entice enough users to join and stay. These result in negative profits. This happens when γ is either very low (irrespective of c),

or too low relative to c . For the latter, the cross-over points lie on the boundary between the ‘N’ and ‘B’ regions, which shows a mostly linear relationship between c and γ . On that boundary, p_i, p_f , and x_{th} remain essentially constant, with $p_i \approx 0$ to attract and keep enough early adopters, $p_f \approx 0.3$, and $x_{th} \approx 0.5$ (under the simple model of Section II-A). In region ‘B’ prices increase nearly linearly when moving towards region ‘A’ (small c values). When entering ‘A’, x_{th} switches to a higher value ($x_{th} \approx 0.75$) and both p_i and p_f remain unaffected by further decreases in c and increase linearly with γ . Note that the fact that region ‘A’ does not extend all the way to $\gamma = 0$ is because unless $\gamma \gtrsim e$, a positive profit is not feasible.

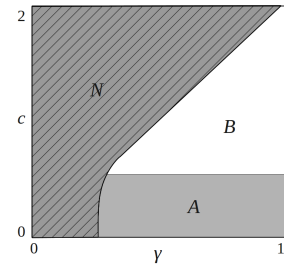


Fig. 3. Pricing guidelines for maximum profit

IV. ROBUSTNESS TESTS

This section offers numerical evidences of the relative robustness of the guidelines of Section III to errors in the estimation of exogenous parameters and to changes to the model’s assumptions, *e.g.*, non-linear externalities and roaming characteristics that follow a non-uniform distribution. Because of space limitations, we report only the results of a sample experiment, with additional results in Appendix B of [4].

Specifically, Fig. 4 displays the *cumulative* effect of modeling and estimation errors. It assumes that rather than a uniform distribution, θ follows a bimodal distribution with modes at 0.25 and 0.75, and that externalities are sub-linear rather than linear, *i.e.*, the users’ utility function is of the form

$$U(\theta) = \gamma + \theta(2x^{0.9} - 1) - cm^{0.9} - p_\theta.$$

Assuming exogenous parameters with values of $c = 0.3$, $\gamma = 0.6$ and $e = 0.2$, the guidelines of Section III give $p_i = 0.4$, $p_f = 0.9$ and $x_{th} = 0.75$. Fig. 4 plots profit (vertical axis), as c and γ vary around their assumed values. The thick black dot in the figure marks the profit in the absence of errors, *i.e.*, actual and estimated parameters coincide. The figure shows that while profit eventually degrades as the errors grow large, the policy is robust to a wide range of errors in both c and γ .

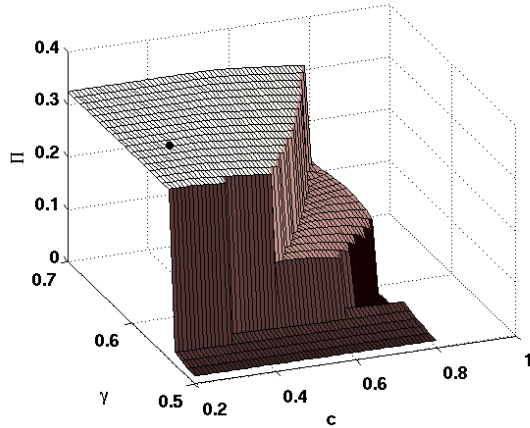


Fig. 4. Robustness of the two-price policy

V. RELATED WORKS

There is a vast literature on pricing strategies for products or services that exhibit externalities. It is not our intent to provide an exhaustive review. Instead we focus on a small subset of works most relevant to the problem targeted in this paper.

Early works on pricing in the presence of externalities include [5], [6] that investigated the evolution of prices for products with positive externalities in a monopoly environment. [5] investigated optimal pricing under linear externalities, while [6] was primarily interested in how introductory prices allowed the product to attract a critical mass of adopters.

Optimal pricing in the presence of general positive externalities was more recently investigated in [7] that considered full discriminatory pricing, and showed that an optimal strategy could be computed in polynomial time. Based on the results of its analysis, the paper also proposed what it termed an “influence-and-exploit” strategy that first gives the product away to some users to build critical mass before starting to charge the remaining users. [8] studied mostly the same problem, but under the assumption that prices could only be adjusted at discrete time instants.

The topic of optimal pricing for systems with *both* positive and negative externalities seems to be have been first addressed in [9] that sought to optimize a combination of provider’s profit and consumers’ surplus. Different pricing strategies were considered, including flat-rate pricing and pricing strategies that account for the product “amount” consumed by a user⁵.

⁵Our model does not have a notion of product amount consumed by a user, but this arises naturally when usage-based pricing is used.

More recently, [10] considered a similar problem with positive and negative externalities, which arises when introducing a new technology to replace an existing one with which it is compatible. The paper identifies that under the assumption of linear externalities, the optimal pricing strategy is essentially similar to the two-price strategy of Section III.

The main difference between this paper and earlier works is the fact that (negative) externalities depend not just on the *number* of users that have joined, but also on *which* users have joined, *i.e.*, the impact of a frequently roaming user differs from that of a sedentary user. The presence of heterogeneity in how users value the service and how they affect its value is a key aspect of a UPC service; one that influences its pricing.

VI. CONCLUSION

The paper relies on a simple model to develop guidelines for configuring a two-price policy so as to foster the successful deployment of a UPC service. The guidelines are then tested numerically for robustness to various relaxations of the model’s simplifying assumptions. The results help identify when a two-price policy that includes a low introductory price can contribute to the eventual success and profit of a UPC service, and conversely when a UPC service is unlikely to succeed irrespective of price.

An extension of interest is to explore service models that include compensation for users that offer access to roaming users. The latter is especially attractive in a context where a Wi-Fi based UPC service is used to offload traffic from a traditional 3G or 4G offering, *e.g.*, see [11].

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