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Hidden Symmetry in Topological Gravity

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1. Introduction

In this lecture we would like to review some recent developments in two-dimensional topological gravity discussed in the papers [1][2]; see also [3][4][5].[1] In these papers we were studying the construction of topological gravity from an ordinary local quantum field theory, *i.e.* following the lines of [6][7].[2] Briefly the point of ref. [1] is the following. Many familiar geometrical objects on a smooth manifold M are examples of a general construction called "G-structures" (where G is a Lie group), recalled below and systematically defined by E. Cartan [9]. For example if G is the group $O(n)$ then this structure is nothing but a Riemannian metric on M. The essence of Einstein's gravitation is that an $O(n)$ structure on M is taken to be a dynamical variable. Similarly we will argue here that the essence of topological gravity is that some *other* sort of G structure on a *super*manifold is taken to be a dynamical variable. This new G-structure we will call "semirigid" because it is a fragment of local $N = 2$ susy where half of the local supersymmetries G^+_n, G^-_n are lost, leaving only G^+_n, G^-_0.

Before giving any details, we should perhaps ask why this is an interesting result. First of all, we learn the precise role played by $N = 2$ susy in topological gravity. It has long been clear that there is some such role, since all known examples of topological matter theories in 2d arise by "twisting" some $N = 2$ superconformal model, and yet no hint of this underlying $N = 2$ structure is seen in previous formulations of the geometrical — *i.e.* ghost — sector [6][7]. Secondly, by finding the natural geometrical

[1] See these papers for many references not supplied here.

[2] The relation of this construction to the intersection-theory construction of [8] is a very interesting issue which we did not consider.

home for these theories, we gain a precise, explicit prescription for how to compute the observables. In particular we find out unambiguously how to deal with the zero modes of the bosonic ghosts, always a thorny problem in the fermionic string. In fact the situation here is far nicer than in the fermionic string; indeed we found a natural prescription to integrate out the odd coordinates and obtain a measure on ordinary moduli space [1]. We worked out this prescription explicitly in [2]. Third, we find that the mysterious "Liouville sector" appearing in [7] is unnecessary to get well-defined answers, and in particular to prove the dilaton [2] and puncture [5] equations. Finally, topological *super*gravity is an almost immediate generalization of our point of view [3].

2. G-structures

To get started let us recall the idea of a G-structure [10]. A smooth manifold M has at each point P a class of frames $\{E_a\}$ spanning $T_P M$. Any two frames are related by an invertible matrix. In other words any two frame *fields* are related by a *function* with values in $GL(n, \mathbf{R})$. Suppose now that I give you a smaller class of frames which I declare to be the "good" ones, and that any two fields of "good" frames are related by a function with values in some *subgroup* G of $GL(n, \mathbf{R})$. This smaller class is called a G-structure on M.[3] As we mentioned it is easy to see that for $G = SO(n)$ we can obtain from such a structure a *metric* on M just by taking any "good" frame field and declaring it to be orthonormal. Conversely, given a metric, the class of all orthonormal frames at any point is taken to itself by the action of $O(n)$.

Locally G-structures always exist for any G. We can specify one by taking any frame field near P and hitting it with every G-valued function to get the class of good frames. Clearly any other frame field in this class will then do as well. Thus even though most manifolds do not admit any global frame, we may still be able to define a global G-structure in this way by cutting M into patches and specifying on each patch a local frame field such that on patch overlaps our representatives always differ by G. Even this may prove impossible. As an extreme case if G is the trivial group containing only the identity, then we really do need to find a global frame, and this is generally impossible: most manifolds are not parallelizable. We will not discuss such global obstructions to choosing G-structures in the following.

Any two smooth manifolds of the same dimension are locally exactly alike: they are both diffeomorphic locally to $\mathbf{R}^n$. When we equip them with G-structures, however, something more interesting can happen. For example, some Riemannian metrics are *flat* while others are not; this is intuitively a purely local property.

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[3] Note that the embedding of G in $GL(n, \mathbf{R})$ is part of the structure.

To make the idea of flatness more precise, to every coordinate system $\{x^i\}$ we associate a standard G-structure specified by the frame family $\hat{E}_i^{\{x^i\}}$ given by some standard formula. For example, in Riemannian geometry we take $\hat{E}_i^{\{x^i\}} \equiv \frac{\partial}{\partial x^i}$. Clearly some other coordinate system $\{y^i\}$ will induce a different standard frame $\hat{E}_i^{\{y^i\}} \neq \hat{E}_i^{\{x^i\}}$. We can now ask when $\hat{E}_i^{\{x^i\}}$ and $\hat{E}_i^{\{y^i\}}$ specify the *same* G-structure, *i.e.*

$$\hat{E}_i^{\{y^i\}}|_P = g(P)_i^j \hat{E}_j^{\{x^j\}}|_P \tag{2.1}$$

for some function g in G. When they do we will say that $\{x^i\}$ and $\{y^i\}$ differ by a "good" diffeomorphism of $\mathbf{R}^n$. From the definitions we see that the "good" transformations always form a group. The smaller G is, the stronger (2.1) and the smaller this group. For example, for $O(n)$ the "good" transformations are the finite-dimensional group of Euclidean motions of $\mathbf{R}^n$. For larger G the situation can be much more interesting. For example in two dimensions we can take G to be not $O(2)$ but the group $\mathbf{C}^\times$ of invertible complex numbers; then as is well known a G structure is a conformal structure (plus an orientation) and the good diffeomorphisms are the orientation-preserving conformal maps of the plane — *i.e.* analytic maps of the *complex* plane.

Thus we arrive at another way to specify a G structure: cover M with coordinate charts all related by diffeomorphisms which are "good" for G. In each chart $\mathcal{U}_\alpha$ specify a G-structure using the standard frame for the chosen coordinate system. By hypothesis these all agree modulo G-transformations (*i.e.* they all specify the same G-structure) on the patch overlaps, since all the patching maps are "good" diffeomorphisms. In the case of $\mathbf{C}^\times$, for example, we get analytic patching maps, so that the G-structure is nothing but a complex structure making M into a Riemann surface.

The relation between our second way to specify a G-manifold (choose "good" patching maps and put the standard G-structure on each patch) and the first (choose a frame field on each patch all related on overlaps by functions with values in G) is not obvious. While it is clear that the second way always yields examples of the first, the converse may be false, because given an arbitrary G-structure near a point P there may be *no* local coordinate whose $\hat{E}_i^{\{x^i\}}$ generates the given structure. If there is such a coordinate (and hence several, obtained from any one by the "good" coordinate transformations) then we call the given G-structure *flat*. The terminology generalizes the case of $O(n)$, since if $\{\frac{\partial}{\partial x^i}\}$ are orthonormal then clearly the corresponding metric $\delta_{ij}\,\mathrm{d}x^i \otimes \mathrm{d}x^j$ is flat. The interesting question arises of finding necessary and sufficient conditions for flatness.

Various kinds of conditions to ensure flatness arise depending on G. For the two-dimensional case of $\mathbf{C}^\times$ above, we find that *every* G-structure is flat! This merely restates Gauss's theorem that every metric admits isothermal coordinates, when we recall that a $\mathbf{C}^\times$-structure is just a metric defined modulo Weyl rescalings. For the more general case of $G = GL(n/2, \mathbf{C})$ a G-structure is an almost-complex structure [11], and it is well known that a given almost-complex structure is flat (comes from a complex coordinate system) only if an integrability condition is met. Taking complex frames with the property that the last $\frac{n}{2}$ E's are the complex conjugates of the first $\frac{n}{2}$, this integrability condition requires that the Lie bracket of E_i with E_j, $i, j \leq \frac{n}{2}$ be itself in the span of the first $\frac{n}{2}$ E's, and so is a condition on the *first*-order derivatives of E_i. Finally for $G = O(n)$ it is well known that a metric is flat if and only if its curvature tensor vanishes — a condition on the *second*-order derivatives of the frame.

Our attitude towards these flatness conditions depends on the circumstances. In the case of $GL(\frac{n}{2}, \mathbf{C})$ the interesting category is not manifolds with almost-complex structures but rather those with *flat* AC structures; these are the usual complex manifolds [11]. In the case of $SO(n)$ the interesting category is instead the larger set of Riemannian manifolds — not the subset of flat ones. In the special case of $\mathbf{C}^\times$ we are spared this decision, since all structures are locally flat and the two categories agree: they are both the set of Riemann surfaces.

Moving on from classical examples to modern ones, consider supersymmetry. The basic objects in rigid susy are two vector fields $D_\alpha, D_{\dot\alpha}$. They have explicit definitions in terms of an a priori coordinate system on superspace. It turns out [12], however, that the appropriate invariant generalization consists of a G-structure, where G is the supergroup consisting of matrices of the form $\begin{pmatrix} \Lambda & \cdots \\ 0 & \rho(\Lambda) \end{pmatrix}$, where $\Lambda \in SO(n)$, ρ is the spin representation, and the ellipsis denotes arbitrary odd functions. Rigid susy then corresponds to the standard G-structure given by a coordinate system, while supergravity emerges when we take the G-structure to be dynamical.

In two (bosonic) dimensions we have an intermediate option. Recall that in $2|0$ dimensions above we saw that relaxing G from $SO(2)$ to $\mathbf{C}^\times$ gave us complex geometry. Similarly in $2|2$ real dimensions we can choose a complex structure to make our G-matrices 2×2 complex matrices, and enlarge our G to matrices of the form $\begin{pmatrix} a^2 & \cdots \\ 0 & c \end{pmatrix}$ where now $a, c \in \mathbf{C}^\times$, not $U(1)$, and again the dots are arbitrary. More generally we can consider complex supermanifolds of complex dimension $1|N$ and G-structures with

$$G \equiv \left\{ \begin{pmatrix} a^2 & \cdots \\ 0 & aM \end{pmatrix} \right\} \quad . \tag{2.2}$$

Here M is a complex, symmetric, invertible matrix and again $a \in \mathbf{C}^\times$. The group (2.2) defines a generalization of conformal structure which is called "N-superconformal" structure. (Ordinary conformal structure is recovered as the case $N = 0$.) The relevant fact we will need below is [13][4] that every N-superconformal structure is locally flat once a simple normalization condition is met, namely $\{E_A\} \equiv \{E_z, E_1, \ldots, E_N\}$ obeys[4]

$$[E_i, E_j] = 2\delta_{ij} E_0 \quad . \tag{2.3}$$

That is, there always exist complex coordinates $\mathbf{z} \equiv \{z, \vec{\theta}\}$ such that the given structure, normalized as above, is equivalent to

$$\{\hat{E}_A^{(\mathbf{z})}\} \equiv \left\{\frac{\partial}{\partial z}, D_i\right\}$$

where $D_i \equiv \frac{\partial}{\partial \theta_i} + \delta_{ij}\theta^j \frac{\partial}{\partial z}$. Thus the situation with superconformal structures is just like that of conformal structures: there are no flatness conditions at all.[5]

For example, in $N = 2$ we can choose a new basis $\{E_z, E_+, E_-\}$ in which the Kronecker delta is off diagonal: $\delta_{+-} = 1$. Then matrices in $SO(2, \mathbf{C})$ are diagonal, so matrices in G take the form

$$\begin{pmatrix} a^2 & \cdots & \\ 0 & ab & 0 \\ 0 & 0 & a/b \end{pmatrix} \quad . \tag{2.4}$$

3. Topological gravity

Let us now recall a few of the main features of pure 2d topological gravity. First, like any topological field theory it has a *scalar* supersymmetry charge Q_S, which therefore has to be nilpotent, $(Q_S)^2 = 0$ (unlike ordinary susy, where Q is a spinor so its square can be the momentum). All fields come in doublets related by Q_S, and in particular the fundamental fields are the usual b, c fields of the bosonic string and their partners of the *same* spin and opposite parity: β, γ.[6] The stress tensor is a total Q_S-variation: $T = \{Q_S, G\}$ for some local field G. And analogous to topological Yang-Mills there is another nilpotent operator $Q_T = Q_S + Q_B$, where Q_B is the ordinary BRST operator associated to conformal symmetry. The stress tensor is also a total Q_T-variation, $T = \{Q_T, b\}$.

To get nearly all the degrees of freedom (and *all* the propagating ones) to decouple, we need some very strong Kugo-Ojima type mechanism. This is provided by

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[4] We can always choose such a basis. This choice reduces the structure group a bit further, so that M in (2.2) is in $O(N, \mathbf{C})$.

[5] To be precise, we make this claim only after a complex structure is chosen. Regarding the complex and superconformal structures as being imposed at once we do get nontrivial flatness conditions which are just the famous "torsion constraints" of 2d supergravity [14].

[6] There is no room for an additional "Liouville" multiplet, since β, γ already contain the Liouville field. In fact β, γ are the bosonization of the latter together with the $c = -2$ CFT needed to get the topological point of the matrix model[15].

noting that the amplitudes descend to well-defined forms on the BRST cohomology of Q_T, and that this cohomology is very small. Indeed it is *empty.* Still the theory is not quite empty. It turns out that we want not the cohomology of Q_T but rather its *equivariant* cohomology relative to the subgroup of Virasoro generated by rigid rotations, *i.e.* by $L_0 - \overline{L}_0$, and this cohomology is small but not quite empty.

The construction sketched above has several mysteries. First, after a great deal of gauge-fixing and so on, one finds that the action of the b, c, β, γ system is *free*, though it did not start out that way. Couldn't we have seen this basic property more directly? Secondly this free action $S = \int [b\bar\partial c + \beta\bar\partial\gamma + \mathrm{c.c}]$ has symmetries

$$\begin{aligned}\delta b &= \pounds_v b + \pounds_\nu \beta \\ \delta\beta &= \pounds_v \beta + \epsilon b\end{aligned}$$

and similarly for c, γ. Here $\pounds$ is the Lie derivative appropriate to spin two, ϵ is an odd constant, v is a commuting vector field and ν an anticommuting one; their Laurent modes give rise to symmetry generators L_n, G_n. Together with the generator Q_S associated to ϵ, they generate the "twisted $N = 2$ algebra:"

$$\begin{aligned} [L_m, L_n] &= (m-n)L_{m+n} & [L_n, Q_S] &= 0 \\ [L_m, G_n] &= (m-n)G_{m+n} & \{G_n, Q_S\} &= -L_n \\ \{G_m, G_n\} &= 0 & \{Q_S, Q_S\} &= 0 \quad . \end{aligned} \tag{3.1}$$

But what is the geometric meaning of this algebra? It is a subalgebra of the full unbroken $N = 2$ superconformal algebra, but why did it arise? Is it just a coincidence that all known matter systems we can couple are actually $N = 2$ supersymmetric, and that (3.1) is always anomaly-free regardless of the original value of the central charge? Where did the rest of $N = 2$ go? More generally, how can we realize the ghosts and matter fields as superfields?

There are further mysteries when we examine the detailed form of the symmetry generators L_n, G_n. The L_n are just the familiar quadratic formulas for the bosonic string ghost sector plus terms of the opposite parity, but the G_n are crazy — in addition to the bilinear terms they have a linear bit! How can the Noether currents of a free field theory look like this? Similarly we have seen that the key BRST-like operator Q_T is also inhomogeneous, the sum of a quadratic bit Q_S and a cubic bit Q_B. Why?

Finally as we have mentioned the detailed algorithm to get answers is a mystery, in particular the disposition of the $\beta\gamma$ zero modes. How does one compute?

4. Some answers

Quite generally in any string theory we need more than just a world-sheet action to get answers. In addition we need some specification of the physical states, possibly as a subspace and/or quotient of the naive Fock space of states generated by the fields in the action. We also need a prescription for how to turn a collection of physical states into a volume form on some moduli space of curves, so that we may then integrate this volume and call the result the correlation function of the given states.

Remarkably, *one* geometric principle will deliver all three of these ingredients and along the way answer all the mysteries listed in the previous section!

To find it, let us focus on the lost symmetries. They will point us to a G structure where G is some supergroup smaller than the one used in $N=2$ supergravity.

Let us recall some more facts from $N=2$ superconformal geometry. One can readily work out the most general "good" coordinate transformation associated to the group (2.4). Infinitesimally such transformations are generated by vector fields of the form

$$V_v \equiv v\partial_z + \frac{1}{2}(D_+v)D_- + \frac{1}{2}(D_-v)D_+ \tag{4.1}$$

where $v \equiv v^z(\mathbf{z})$ is an even tensor field. We find $[V_{v_1}, V_{v_2}] = V_{[v_1,v_2]}$ where

$$[v_1, v_2] = v_1\partial v_2 - v_2\partial v_1 + \frac{1}{2}D_+v_1D_-v_2 + \frac{1}{2}D_-v_1D_+v_2 \quad . \tag{4.2}$$

The unique anomaly cocycle allowed by (4.2) is then the bilinear

$$C(v_1, v_2) = \oint [\mathrm{d}z|\mathrm{d}^2\theta]\, v_1[D_+, D_-]\partial_z v_2 \quad . \tag{4.3}$$

Let us consider the full, unbroken $N=2$ supergravity theory. It has a BRST symmetry, and as always there is a C superfield transforming like the infinitesimal generator of symmetry transformations, the function v above. There is also a B superfield transforming dually to v, and a stress tensor superfield T also dual to v. The stress tensor is determined by being a bilinear in B, C of the right weight, which generates the infinitesimal transformations (4.1) on B, C and (modulo a possible anomaly) itself. This fixes T to be

$$T = \partial(CB) - \tfrac{1}{2}D_iBD^iC \equiv J + \theta G^- - \xi G^+ + \theta\xi(T_B + \partial J) \quad .$$

We have decomposed T into components with names J, G^+, G^-, T_B; these peculiar linear combinations will be useful momentarily.

We now suggest imposing a further geometric structure on our manifold in addition to the given complex and superconformal structures. A surface with this stronger structure will be called a "semirigid super Riemann surface." Namely, consider the supergroup of matrices of the form

$$\begin{pmatrix} a^2 & \cdots & \\ 0 & 1 & 0 \\ 0 & 0 & a^2 \end{pmatrix} \quad . \tag{4.4}$$

Since this group is a subgroup of (2.4), any semirigid surface is in particular an $N = 2$ SRS of a special type. The collection of all inequivalent semirigid structures on a surface forms a moduli space in the usual way. Let us see what this construction buys us.

First of all, inspection of (4.4) shows that one of the odd frame vectors, E_+, is actually global, just as in rigid susy. The other one however, E_-, transforms like a -1-form as we change coordinate systems. Inspecting the residual "good" coordinate transformations, we correspondingly find that one of the odd coordinates θ^+ is global over the surface, so that any superfield will have an expansion in θ^+ which pairs its components into fields of opposite parity but like spin, as desired.

Examining the infinitesimal transformations we see that (4.1) makes a frame transformation of the form (4.4) only if $D_- v$ is an odd constant. One easily verifies from (4.2) that such v close under bracket. Moreover the central extension (4.3) is identically zero on pairs of vectors of this form. Indeed expanding the general solution for v one gets

$$v(\mathbf{z}) = v_0(z) + \theta\nu(z) + \xi\epsilon + \theta\xi\partial v_0(z) \quad . \tag{4.5}$$

Substituting into (4.2) indeed shows that we recover exactly the desired algebra (3.1), and shows why the latter is anomaly-free. The L_n are the Laurent modes of T_B, the G_n those of G^+, and $Q_S = G_0^-$.

What could break the full $N = 2$ symmetry down to the smaller (4.4)? Whatever it is, this mechanism must make sense even in the absence of any matter at all, so we haven't far to look: we can only impose symmetry-breaking constraints on B, C. Quite generally we break a geometrical symmetry by setting a tensor equal to a constant. A little thought shows that

$$D_- C = \text{const} \tag{4.6}$$

does the job. This constraint takes the $N = 2$ superfield C with four components down to just two independent components, the desired c, γ. The canonically conjugate constraint says that observables depend on B only through $D_- B$, again eliminating two components and leaving only the desired b, β as true degrees of freedom.

We claim that this constrained subset of the full $N = 2$ ghost system reproduces all the mysterious features of topological gravity. For one thing the full BC system is free, so the subset is too. Furthermore the full system is BRST-invariant. The operator $Q^{N=2}_{BRST}$ descends to the constrained theory, since it involves B only via D_B, and on the constrained theory it's nilpotent since the anomaly vanishes. Indeed it is precisely the mysterious operator Q_T; the inhomogeneity arises simply from terms in which components of C are replaced by the numerical value implied by (4.6). Furthermore the residual unbroken symmetries found above are all generated by operators which themselves descend to the constrained theory, and reproduce the generators of [7]. The mysterious inhomogeneous terms arise by the same prosaic mechanism as in Q_T.

The construction we have just given [1] generalizes at once to $N > 2$, where it yields topological supergravity [3]. For example with $N = 3$ we get the supergroup (2.2) consisting of matrices of the form[7]

$$\left(\begin{array}{c|ccc} a^2 & & \cdots & \\ \hline 0 & 1 & x & -x^2/2 \\ 0 & 0 & a & -xa \\ 0 & 0 & 0 & a^2 \end{array}\right)$$

As we mentioned earlier, once we have chosen a group G we have two ways to construct a category of G-manifolds. We can consider either the space of all G-structures, or the space of *integrable* ones. Recall also that manifolds in the latter category could be realized as collections of "good" patching maps. It turns out that for a string-like theory we want this latter category. For conformal and superconformal geometry there was no difference. Now however there are some interesting constraints to impose, as we work out in [4]. Just as in ordinary conformal geometry there still remains after restricting to the integrable structures an interesting but finite-dimensional moduli space $\mathcal{M}$ of semirigid surfaces. The free CFT described earlier then yields an integration density on this moduli space for each collection of physical states, much as in ordinary (super)string theory. The operator formalism can be used to show that the resulting measures are well defined [2]. Topological gravity then emerges as the integrals over the moduli space of these densities. At least some of the famous recursion relations of [7] are then very easy to deduce [2][5].

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[7] Note that we are again using the complex basis for two of the E's: E_+, E_3, E_-.

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