

ESSAYS ON CONTRACTS

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To my years at Penn.

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ABSTRACT

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This dissertation consists of two essays on contract theory. I investigate contracts under different economics contexts. In the first chapter, I consider a two-period model in which the success of the firm depends on the effort of a first-period manager (the incumbent) and the ability of a second-period manager. At the end of the first period, the board receives a noisy signal of the incumbent manager's ability and decides whether to retain or replace the incumbent manager. I show that the information technology the board has to assess the incumbent manager's ability is an important determinant of the optimal contract and replacement policy. The contract must balance providing incentives for the incumbent manager to exert effort and ensuring that the second-period manager is of high ability. I show that severance pay in the contract serves as a costly commitment device to induce effort. Unlike existing models, I identify conditions on the information structure under which both entrenchment and anti-entrenchment emerge in the optimal contract. In the second chapter, I use a dynamic model of life insurance with one-sided commitment and bequest-driven lapsation, as in Daily, Hendel and Lizzeri (2008) and Fang and Kung (2010), but with policyholders who may underestimate the probability of losing their bequest motive, to analyze how the life settlement market – the secondary market for life insurance – may affect consumer welfare in equilibrium. I show that life settlement

may increase consumer welfare in equilibrium when (i) policyholders are sufficiently overconfident; and (ii) the intertemporal elasticity of substitution of consumption (IES) of policyholders is greater than one.

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Chapter 1

Managerial Turnover and Entrenchment

This chapter is a joint work with Xi Weng.

1.1 Introduction

Designing compensation schemes in managerial contracts and deciding whether to replace a manager, such as a CEO, are important aspects of firm organization. These decisions are linked through the severance agreement, a key component of the contracts between a board and a manager. The severance agreement specifies payments to the manager upon his forced departure. Approximately 50% of the CEO compensation contracts implemented between 1994 and 1999 involved some form of severance agreement (Rusticus, 2006). The percentage of S&P firms that included a severance agreement in their CEO compensation contracts increased from 20% in 1993 to more

than 55% in 2007 (Huang, 2011). In general, a contract with a severance agreement adds an explicit cost to the board's retention decision and makes replacement more difficult relative to a compensation contract without such an agreement.

A widely held belief is that CEOs are replaced too infrequently, or entrenched.¹ Entrenchment may arise for many reasons. For example, it may be an instance of governance failure in the form of a captive board of directors (Inderst and Mueller, 2010; Shleifer and Vishny, 1989; Hermalin and Weisbach, 1998) or a way to mitigate a moral hazard problem (Almazan and Suarez, 2003; Casamatta and Guembel, 2010; Manso, 2011). Taylor (2010) makes the first attempt to measure the cost of entrenchment using a structural model of CEO turnover and finds suggestive evidence of the opposite. In particular, he finds that boards in large firms fire CEOs with higher frequency than is optimal. We refer to this phenomenon as anti-entrenchment. This finding cannot be rationalized by the existing models on CEO turnover and thus calls for a new model to better understand the determinants of managerial turnover.

This paper investigates how optimal design of the severance agreement influences managerial entrenchment. A manager is said to be entrenched if the board retains an incumbent manager who has an expected ability lower than that of a replacement manager. Anti-entrenchment occurs when the board fires some managers with higher than average expected ability. We propose a two-period principal-agent model of managerial turnover and identify conditions that predict the emergence of entrench-

¹Although evidence shows forced CEO turnover is increasing over time and indicates boards are using more aggressive replacement policies, it is widely believed that CEOs are rarely fired and thus are entrenched. For instance, Kaplan and Minton (2012) find that board-driven turnover increased steadily from 10.93% (1992 – 1999) to 12.47% (2000 – 2007) using data from publicly traded Fortune 500 companies.

ment and anti-entrenchment. Formally, we consider a setup in which the first-period manager is incentivized by a contract that contains performance-related pay and severance pay. The firm's success depends on the initial manager's effort and the second-period manager's ability. Thus, the board faces an ability selection problem and a moral hazard problem. After the initial manager exerts effort, the board observes a non-contractible signal regarding his ability. The board can fire the initial manager by paying the severance pay specified in the contract and hire a replacement manager. Since the board's information about the initial manager's ability is non-contractible, it lacks commitment power and cannot write a contract that specifies a retention decision contingent on the signal. Severance pay is used as a costly device to provide commitment to not firing the initial manager. By committing to a high severance pay, the board ensures a low expected profit for itself after replacement, which leads to a less aggressive replacement policy. The board's optimal replacement policy balances incentive provision, manager selection and commitment.

Our main result characterizes the optimal replacement policy and shows how it depends on the precision of the signal of the manager's ability. When this monitoring technology is noisy, entrenchment is optimal. In such a scenario, the board places higher priority on motivating the incumbent manager to exert effort rather than on maximizing the manager's ability. Setting an aggressive replacement policy will fire the incumbent of high ability too often and dis-incentivize the incumbent to exert effort, while saving little on severance pay. As a result, a contract that induces entrenchment is optimal for the board.

Anti-entrenchment is optimal when the board's monitoring technology is suffi-

ciently informative. The board is reluctant to provide commitment. On the one hand, a contract that favors the incumbent manager does not increase effort by much because of the low probability of replacement when the incumbent is of high ability. On the other hand, an aggressive replacement policy helps the board avoid paying the performance-related pay to the incumbent manager and can increase the firm's profit. Thus, anti-entrenchment is optimal for the board. To the best of our knowledge, we are the first to study the interaction between the board's monitoring technology and managerial turnover, and to show that a contract with anti-entrenchment is sometimes optimal.

Our model can be applied to a variety of real-world settings. For example, the model can be used to analyze the turnover of founder CEOs in venture-capital-backed companies where the venture capitalist is a large shareholder and engages in active monitoring. It could also be used to analyze the contracts between head coaches and professional sports teams.

Related Literature: This paper belongs to the literature on the principal-agent model with replacement.² One strand of research views entrenchment as a potential source of inefficiency that the board aims to mitigate. Consequently, anti-entrenchment cannot be observed. Inderst and Mueller (2010) solve the optimal contract for the incumbent manager who holds private information on the firm's future performance and can avoid replacement by concealing bad information. Consequently, the optimal contract is designed to induce the incumbent to voluntarily step down when evidence suggests low expected profit under his management. Sim-

²See Laux (2014) for a comprehensive survey of the theoretical models on this topic.

ilarly, entrenchment occurs if the incumbent can make manager-specific investments to create cost of replacement to the board (Shleifer and Vishny, 1989) or if there exist close ties between the board and manager (Hermalin and Weisbach, 1998).

Another strand of research views entrenchment as a feature of the optimal contract (board structure) that helps overcome the moral hazard problem. Manso (2011) shows that tolerance for early failure (entrenchment) can be part of the optimal incentive scheme when motivating a manager to pursue more innovative business strategies is important to the board. Casamatta and Guembel (2010) study the optimal contract for the incumbent manager with reputational concern. In their model, entrenchment is optimal because the incumbent manager would like to see his strategy succeed and is less costly to motivate than the replacement manager. Almazan and Suarez (2003) study the optimal board structure for incentivizing the incumbent manager. They show that it can be optimal for shareholders to relinquish some power and choose a weak board, where the incumbent can veto his departure, rather than a strong board, where the board can fire the incumbent at will. In the same spirit, Laux (2008) studies the optimal degree of board independence for shareholders. He shows that some lack of independence can increase shareholder value. In these papers, boards (shareholders) provide better job security to the incumbent by making dismissal more difficult to induce more effort. Our paper contributes to the existing literature by pointing out that despite all the incentive-providing merits of entrenchment, the cost of incentivizing can be high when the board's monitoring technology is sufficiently informative.

In terms of modeling, the paper is most similar to Taylor and Yildirim (2011).

They study the benefits and costs of different review policies and identify conditions under which the principal commits not to utilize the agent’s information and chooses blind review as optimal policy. We apply their model to analyze managerial turnover by adding a contract stage to endogenize the agent’s payoff and allowing the principal to replace the agent in the interim stage.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 defines entrenchment and anti-entrenchment and characterizes the optimal contract. Section 4 studies the impact of informativeness on optimal replacement policy. Section 5 discusses extensions of the model. Section 6 concludes. All proofs are in the Appendix.

1.2 Model

There are two periods $t = 1, 2$ and an initial contract stage.

Contract stage. The board (principal), hires a manager (agent) from a pool with unknown ability $\theta_i \in \{0, 1\}$ to work for the firm with common prior $\Pr(\theta_i = 1) = \frac{1}{2}$.³ The ability is unknown to both sides. The board offers a contract to the manager. We describe the contract details below.

Both the board and the managers are risk-neutral. Moreover, we assume that managers are protected by limited liability.⁴ Finally, we assume the value of the outside option to the manager is 0. This assumption guarantees that the individual

³The analysis is unchanged for a different prior of θ_i .

⁴This assumption is necessary because it excludes the possibility that the board sells the whole firm to the manager in order to provide the greatest possible incentive in the optimal contract.

rationality (IR) constraint never binds and simplifies the analysis.

Period 1. The manager exerts effort to create a project of quality q with cost $C(q) = \frac{1}{2}q^2$. Simultaneously, the board receives a signal $s \in \mathcal{S}$ of the manager's ability and decides whether to replace the incumbent manager. If the incumbent manager is fired, a replacement manager is hired and has ability θ_r randomly drawn from the same pool of managers.⁵

Period 2. The manager who stays in office implements the project with no additional effort and payoffs are realized. Implementation is assumed to be costless and depends only on manager's ability.⁶ To formalize this idea, we assume that the expected quality of the project is equal to $q\tilde{\theta}$, where q is the incumbent manager's choice of how much effort to exert and $\tilde{\theta}$ is the ability of the manager who stays in office at the beginning of period 2. With probability $q\tilde{\theta}$, the project is of high quality and yields outcome $y = 1$. With complementary probability $1 - q\tilde{\theta}$, the project is of low quality and yields outcome $y = 0$. After payoffs are realized, the incumbent manager receives payment according to the contract signed in period 0 and the game comes to an end.

In the optimal contract, the wage for low output is 0. A contract is defined by the tuple (w, k) , where w is the wage rate when $y = 1$ and k is the severance pay to the incumbent manager if he is fired. By the limited liability assumption, $w \geq 0$ and

⁵The project generation process can also be interpreted as a project selection process as in Casamatta and Guembel (2010). Assume some unknown state of the world $\eta \in [0, 1]$ is randomly drawn, and a manager is hired to select a project $a \in [0, 1]$ to match the underlying state. The quality of the project is 1 if $a = \eta$ and 0 otherwise. The manager incurs cost $C(q)$ to receive a signal ν of the true state. With probability q , the manager identifies η , that is, $\nu = \eta$, and with probability $1 - q$, ν is pure noise. Given q , the expected quality of the selected project is q . These two specifications lead to the same model.

⁶This assumption is relaxed in Section 5.2.

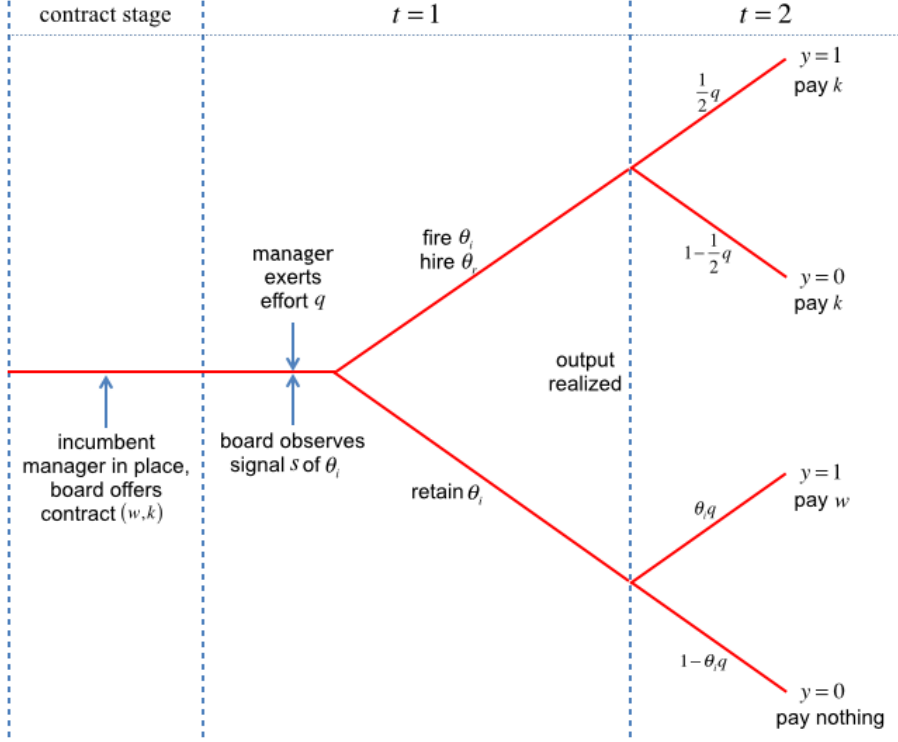


Figure 1.1: Timeline

$k \geq 0$.

Information structure. The board receives a noisy signal $s \in \mathcal{S}$ about incumbent manager's ability θ_i . s is drawn from distribution with cdf $F_{\theta_i}(\cdot)$ and pdf $f_{\theta_i}(\cdot)$ for $\theta_i \in \{0, 1\}$. Without loss of generality, we assume $\mathcal{S} = [0, 1]$ and normalize $s = \frac{1}{2}F_1(s) + \frac{1}{2}F_0(s)$ for $s \in [0, 1]$.⁷ The two conditional density functions $\{f_1(s), f_0(s)\}$ suffice to define an information structure under such normalization. Three assumptions are imposed on the information structure.

Assumption 1 *The monotone likelihood ratio property (MLRP): $\frac{f_1(s)}{f_0(s)}$ is strictly increasing in s for $s \in [0, 1]$.*

⁷This assumption is without loss of generality due to the fact any information structure can be normalized via integral probability transformation. See Appendix B for more details.

For binary states, the MLRP assumption is without loss of generality because signals can always be relabeled according to likelihood ratio to satisfy this assumption.

Assumption 2 *Perfectly informative at extreme signals:* $\lim_{s \rightarrow 0} \frac{f_1(s)}{f_0(s)} = 0$ and $\lim_{s \rightarrow 1} \frac{f_1(s)}{f_0(s)} = +\infty$.

Assumption 2 guarantees that support of the posterior belief is always $[0, 1]$. The last assumption imposed on the information structure is symmetry. This assumption allows us to define the first best replacement policy on the signal space.

Assumption 3 $f_1(s) = f_0(1 - s)$ for all $s \in [0, 1]$.

By Assumption 3, $f_1(\frac{1}{2}) = f_0(\frac{1}{2})$. Thus the likelihood ratio at $s = \frac{1}{2}$ is always 1 and the Bayesian update of the incumbent manager's ability at $\frac{1}{2}$ is equal to the prior.

Finally we introduce an index $\alpha \in (0, \infty)$ to parameterize the information structure. We assume that $f_{\theta_i}(s; \alpha)$ is continuous in s and α for $\theta_i \in \{0, 1\}$ and define the information structures for the two extreme values of α as follows.

Assumption 4 (Completely informative/uninformative information structure)

1. *The information structure becomes completely uninformative when $\alpha \rightarrow 0$, i.e.,*

$$\lim_{\alpha \rightarrow 0} [f_0(s; \alpha) - f_1(s; \alpha)] = 0 \text{ for } s \in (0, 1).$$

2. *The information structure becomes completely informative when $\alpha \rightarrow \infty$, i.e.,*

$$\lim_{\alpha \rightarrow \infty} f_1(s; \alpha) = 0 \text{ for } s \in [0, \frac{1}{2}) \text{ and } \lim_{\alpha \rightarrow \infty} f_0(s; \alpha) = 0 \text{ for } s \in (\frac{1}{2}, 1].^8$$

⁸Both completely informative and uninformative information structures are defined using point-wise convergence.

When the information structure becomes completely uninformative ($\alpha \rightarrow 0$), the two conditional density functions are the same. When the information structure becomes completely informative ($\alpha \rightarrow \infty$), the board will not observe a signal below $\frac{1}{2}$ when the incumbent manager is of high ability and a signal above $\frac{1}{2}$ when the incumbent manager's ability is low.

1.3 The Optimal Contract

1.3.1 The benchmark case: contractible effort

We first pin down the socially optimal replacement policy. By Assumption 1, the socially optimal replacement policy is a cutoff rule. Denote \hat{s} as the signal cutoff.

Lemma 1 (First best cutoff) *Suppose the board can contract on effort q of the incumbent manager. Then the optimal effort is $q^{\mathcal{FB}} = \frac{1}{2} + \frac{1}{4}[F_0(\hat{s}^{\mathcal{FB}}) - F_1(\hat{s}^{\mathcal{FB}})]$, where the optimal replacement cutoff $\hat{s}^{\mathcal{FB}} = \frac{1}{2}$.*

When effort is contractible, the board is able to optimize effort and selection separately. Thus, there is no tradeoff between the moral hazard problem and the selection problem. It is optimal to replace the incumbent manager when the posterior belief about the incumbent's ability falls below the expected value of the pool and retain the incumbent otherwise. By Assumption 3, the likelihood ratio $\frac{f_1(s)}{f_0(s)}$ at $s = \frac{1}{2}$ is always equal to 1. Consequently, the Bayesian update of the incumbent manager's ability is always equal to the prior independent of the informativeness α of the information structure. Consequently, the socially optimal cutoff $\hat{s}^{\mathcal{FB}} = \frac{1}{2}$ for all α .

Given the first best cutoff, we can now define entrenchment. Denote (w^*, k^*) as the optimal contract to the board. Let (\hat{s}^*, q^*) be the equilibrium replacement cutoff and effort of the continuation game induced by the optimal contract.

Definition 1 We define *entrenchment* as a cutoff $\hat{s}^* < \frac{1}{2}$ and *anti-entrenchment* as $\hat{s}^* > \frac{1}{2}$.

For the case where $\hat{s}^* = \frac{1}{2}$, we say that neither entrenchment nor anti-entrenchment is observed. The replacement policy coincides with the socially optimal policy. When $\hat{s}^* < \frac{1}{2}$, the replacement policy favors the incumbent manager: the board could have improved implementation by replacing the incumbent. Similarly, the replacement policy is considered aggressive and places the incumbent manager at a disadvantage when $\hat{s}^* > \frac{1}{2}$.

1.3.2 Characterizing the Optimal Contract

In this section, we solve the equilibrium outcome when effort is non-contractible. The board can only commit to the wage w and severance pay k in the contract. We are interested in the cutoff \hat{s}^* induced by the optimal contract.

Incentives under fixed contract (w, k)

A contract (w, k) induces a simultaneous move game. We first solve the sub-game in period 1, fixing contract (w, k) . The incumbent manager's effort q and the board's replacement policy \hat{s} will be determined in a Cournot-Nash equilibrium.

For contract (w, k) , the incumbent manager's best response to cutoff \hat{s} is effort q that maximizes:

$$\begin{aligned} \max_q \frac{1}{2} [1 - F_1(\hat{s})] qw + \frac{1}{2} [F_1(\hat{s}) + F_0(\hat{s})] k - C(q). \\ \Rightarrow q(\hat{s}; w, k) = \frac{1}{2} [1 - F_1(\hat{s})] w. \end{aligned} \quad (1.1)$$

The board can provide incentive on effort by increasing wage w or lowering equilibrium cutoff \hat{s} . For a fixed contract (w, k) the board's best response to the incumbent manager's effort level q is cutoff \hat{s} that maximizes:

$$\begin{aligned} \max_{\hat{s}} \frac{1}{2} [1 - F_1(\hat{s})] q(1 - w) + \frac{1}{2} [F_1(\hat{s}) + F_0(\hat{s})] \left(\frac{1}{2} q - k \right). \\ \Rightarrow \hat{s}(q; w, k) \text{ solves } \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} q(1 - w) = \frac{1}{2} q - k. \end{aligned} \quad (1.2)$$

Because a higher cutoff implies higher posterior belief about the incumbent manager's ability, the board chooses a cutoff such that the expected profit created by the marginal incumbent manager is equal to the expected profit under replacement in equilibrium.

Given contract (w, k) , the optimal cutoff and effort $(\hat{s}(w, k), q(w, k))$ are pinned down by equations (1.1) and (1.2). We can calculate the corresponding contract (w, k) that induces any tuple (\hat{s}, q) as follows,

$$w(\hat{s}, q) = \frac{q}{\frac{1}{2} [1 - F_1(\hat{s})]} \quad (1.3)$$

and

$$k(\hat{s}, q) = \frac{1}{2}q - \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})}q[1 - w(\hat{s}, q)].^9 \quad (1.4)$$

Derive the optimal contract for fixed replacement policy

The board chooses contract (w, k) to maximize expected profit:

$$\max_{\{w, k\}} \frac{1}{2}[1 - F_1(\hat{s})]q(1 - w) + \frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})]\left(\frac{1}{2}q - k\right)$$

s.t.

$$q = \frac{1}{2}[1 - F_1(\hat{s})]w$$

and

$$\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})}q(1 - w) = \frac{1}{2}q - k.$$

Equivalently, the board is maximizing expected profit over (\hat{s}, q) , with $w(\hat{s}, q)$ and $k(\hat{s}, q)$ as determined in equations (1.3) and (1.4). Substituting equations (1.3) and (1.4) into the board's profit function yields expected profit as a function of (\hat{s}, q) ,

$$q \left[1 - \frac{q}{\frac{1}{2}[1 - F_1(\hat{s})]} \right] \left\{ \frac{1}{2}[1 - F_1(\hat{s})] + \frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})] \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \right\}.$$

It can be verified that $q = \frac{1}{4}[1 - F_1(\hat{s})]$ under the optimal contract. Consequently, $w^* = \frac{1}{2}$. We summarize the results of the previous pages in a lemma.

⁹The non-negativity assumption on k is not always satisfied for all \hat{s} and q . We ignore this limited liability constraint for the moment and solve the unconstrained problem. This is not a big concern since it can be proved later that the optimal wage is $w^* = \frac{1}{2}$ and k is non-negative for all $\hat{s} \in [0, 1]$.

Lemma 2 *Fixing \hat{s} , the board maximizes expected profit by offering a contract,*

$$w = \frac{1}{2}$$

and

$$k(\hat{s}) = \frac{1}{4}[1 - F_1(\hat{s})] \left[\frac{1}{2} - \frac{1}{2} \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \right].$$

Moreover, in equilibrium, the incumbent manager chooses effort

$$q(\hat{s}) = \frac{1}{4}[1 - F_1(\hat{s})].$$

By Lemma 2, $k(\hat{s})$ is decreasing in the equilibrium cutoff \hat{s} . By committing to a higher severance pay, the board chooses a lower replacement cutoff in equilibrium and is able to induce more effort. The expected profit can be rewritten in terms of \hat{s} alone:

$$\pi(\hat{s}) := \frac{1}{8}[1 - F_1(\hat{s})] \left\{ \frac{1}{2}[1 - F_1(\hat{s})] + \frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})] \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \right\}.$$

The optimal cutoff depends on the informativeness of the information structure.

Rewrite the expected profit as follows:

$$\begin{aligned} \pi(\hat{s}) = \frac{1}{8} \underbrace{[1 - F_1(\hat{s})]}_{\text{incentive effect}} & \left\{ \underbrace{\left[\frac{1}{2}[1 - F_1(\hat{s})] + \frac{1}{4}[F_1(\hat{s}) + F_0(\hat{s})] \right]}_{\text{selection effect}} \right. \\ & \left. + \underbrace{\frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})] \left(\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} - \frac{1}{2} \right)}_{\text{commitment effect}} \right\}. \end{aligned}$$

Three effects play a role in determining the optimal cutoff. Because the outcome depends on the expected ability of the manager in period 2, the board faces a selection problem. This is captured by $\left[\frac{1}{2}[1 - F_1(\hat{s})] + \frac{1}{4}[F_1(\hat{s}) + F_0(\hat{s})]\right]$, which is called the *selection effect*. This is the expected ability of the manager in period 2. Increasing \hat{s} will increase the expected ability of the manager in office when $\hat{s} < \frac{1}{2}$ and decrease the expected ability when $\hat{s} \geq \frac{1}{2}$. To optimize selection independently, the board would choose $\hat{s} = \frac{1}{2}$.

Because the outcome also depends on the effort choice of the incumbent manager, the board faces a moral hazard problem and needs to incentivize the incumbent. This is captured by $[1 - F_1(\hat{s})]$, which is referred to as the *incentive effect*. As the equilibrium replacement cutoff \hat{s} increases, the incumbent manager expects a lower retaining probability in equilibrium and exerts less effort accordingly. The board provides more job security to better incentivize the incumbent manager in response. By this effect alone, the board sets $\hat{s} = 0$.

If the selection effect and the incentive effect were the only effects, a cutoff below $\frac{1}{2}$ is optimal to the board and entrenchment emerges under optimal contract. However, because the signal is non-contractible, board lacks commitment power on replacement policy. Severance pay serves as a costly commitment device that helps make replacement of the incumbent less likely. As the severance pay increases, it lowers the expected payoff of replacement, which creates a stronger incentive for the board to not replace the incumbent. In equilibrium the expected profit of replacement is equal to the expected profit created by the marginal incumbent manager. When board lowers the cutoff ($\hat{s} < \frac{1}{2}$) to provide more incentive on effort, it has to

increase severance pay to make the equilibrium replacement policy credible. This generates a net loss compared to the first best replacement policy. It is captured by $\frac{1}{2} \left[F_1(\hat{s}) + F_0(\hat{s}) \right] \left(\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} - \frac{1}{2} \right)$, which is referred to as the *commitment effect*. Compared to the first best cutoff $\hat{s} = \frac{1}{2}$, the board obtains a net commitment gain by providing less commitment and designing a contract that induces cutoff above $\frac{1}{2}$. Similarly, the board suffers a commitment loss by committing to a cutoff that is below $\frac{1}{2}$. The net commitment effect is shown by $\left(\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} - \frac{1}{2} \right)$. Multiplied by the probability of replacement, this yields the total net commitment gain/loss. By this effect alone, the board sets $\hat{s} = 1$.

If incentive effect dominates commitment effect, entrenchment is optimal to the board. Otherwise, anti-entrenchment is optimal.

1.4 The Optimal Replacement Policy

In this section, we study how the optimal replacement policy varies depending on the informativeness of the board's monitoring technology.

1.4.1 Replacement at limiting distribution

Proposition 1 *Suppose $\{f_1(\cdot; \alpha), f_0(\cdot; \alpha)\}$ satisfies Assumptions 1 - 4. Then there exist $\bar{\alpha}$ and $\underline{\alpha}$ such that,*

1. $\hat{s}^*(\alpha) > \frac{1}{2}$ for $\alpha > \bar{\alpha}$;

2. $\hat{s}^*(\alpha) < \frac{1}{2}$ for $\alpha < \underline{\alpha}$.

When the information structure is noisy, providing incentives is more profitable than obtaining more commitment. Choosing $\hat{s} > \frac{1}{2}$ reduces severance pay by a small amount because the Bayesian update around $\hat{s} = \frac{1}{2}$ changes very slowly and the expected ability of the incumbent manager at the cutoff is close to the expected ability at a cutoff of $\frac{1}{2}$. On the other hand, choosing $\hat{s} > \frac{1}{2}$ reduces the incumbent manager's incentive to exert effort. Consequently, it is optimal for the board to design a contract that leads to entrenchment.

When the board's monitoring technology is sufficiently informative, the benefit of commitment dominates and choosing $\hat{s} < \frac{1}{2}$ is not optimal for the board. Since the probability of firing a high ability manager is very small for all signals below $\frac{1}{2}$, lowering the equilibrium replacement cutoff does not have a large effect on the incumbent's effort. On the other hand, it is easy to obtain commitment gain. The expected ability of the manager in the right neighborhood of $\frac{1}{2}$ is very close to 1 when the information structure is sufficiently informative. That is, the board can largely reduce the severance pay by choosing a cutoff slightly above $\frac{1}{2}$. Thus, anti-entrenchment is optimal to the board.

1.4.2 Optimal replacement and informativeness

Proposition 1 does not characterize the equilibrium replacement policy for intermediate α . To do this, it is necessary to introduce an information order.

Distribution of posterior beliefs

Denote $p = \varphi(s)$ as the posterior belief of θ after observing signal s . Then $\varphi(s) = \frac{f_1(s)}{f_1(s)+f_0(s)}$. By Assumption 1, $\varphi(s)$ is strictly increasing in s . By Assumption 2, the support of p is $[0, 1]$. Denote $g(p)$ as the corresponding density function. Since $\mathbb{E}(\mathbb{E}(\theta|s)) = \frac{1}{2}$, the only constraint we impose on $g(\cdot)$ is that $\int_0^1 pg(p)dp = \frac{1}{2}$.

Given an information structure $\{f_1(\cdot), f_0(\cdot)\}$, the density function of posterior belief p can be calculated as follows:

$$g(p) = \frac{1}{2} \left[f_1(\varphi^{-1}(p)) + f_0(\varphi^{-1}(p)) \right] \frac{d\varphi^{-1}(p)}{dp}.$$

Lemma 3 *For any density function $g(\cdot)$ with support $[0, 1]$ that satisfies $\int_0^1 pg(p)dp = \frac{1}{2}$, there exists a unique information structure $\{f_1(\cdot), f_0(\cdot)\}$ that induces $g(\cdot)$.*

By Lemma 3, there exists a one-to-one mapping between $g(\cdot)$ and information structure $\{f_1(\cdot), f_0(\cdot)\}$. Thus working on the information structure $\{f_1(\cdot), f_0(\cdot)\}$ is equivalent to working on distribution of the posterior belief $g(\cdot)$. Consequently, we can define information order on $g(\cdot)$. By Assumption 3 on $\{f_1(\cdot), f_0(\cdot)\}$, $g(p) = g(1 - p)$ and $G(p) = 1 - G(1 - p)$ for $p \in [0, 1]$. Thus, it suffices to order different information structures based on $G(p)$ for $p \in [0, \frac{1}{2}]$.

The ρ -concave order

We use ρ -concavity to define the informativeness of the information structure.¹⁰ To the best of our knowledge, this is the first paper that defines information order using ρ -concavity.

Given $G(\cdot)$, define local ρ -concavity at p as,

$$\rho(p) := 1 - \frac{G(p)g'(p)}{g^2(p)}.$$

By definition, $\rho(p)$ is the power of $G(\cdot)$ such that the second order Taylor expansion at p drops out. Thus, $\rho(p)$ is a measure of the concavity of $G(\cdot)$ at point p . Log-concavity is equivalent to $\rho(p) \geq 0$ and concavity is equivalent to $\rho(p) \geq 1$. We focus on the distributions such that $\rho(p) \in (0, \infty)$. This assumption is a necessary condition to guarantee the initial condition $G(0) = 0$ is satisfied.¹¹

Definition 2 (ρ -concave order) $G_1(p)$ is said to be more informative than $G_2(p)$ in the ρ -concave order if $\rho(p|G_1) > \rho(p|G_2)$ for all $p \in [0, \frac{1}{2}]$.

By definition, $G_1(p)$ is more informative than $G_2(p)$ if $G_1(p)$ is everywhere more concave than $G_2(p)$ measured by local ρ -concavity. The ρ -concave order is a stronger condition than the rotation order and Blackwell's order: if a family of distributions is ordered according to the ρ -concave order, then it is rotation-ordered and ordered in the sense of Blackwell.¹²

¹⁰For more applications of ρ -concavity in economics, see Mares and Swinkels (2014) on auction theory; Anderson and Renault (2003), Weyl and Fabinger (2013) on industry organization.

¹¹Imposing this non-negativity assumption on $\rho(\cdot)$ is without loss of generality: a completely uninformative information structure can still be defined under this constraint.

¹²See Appendix C for more details.

Assume that $\max_{p \in [0, \frac{1}{2}]} \{\rho(p; \alpha)\}$ and $\max_{p \in [0, \frac{1}{2}]} \{\rho(p; \alpha)\}$ exist for all $\alpha \in (0, \infty)$. Denote $\bar{\rho}(\alpha) = \max_{p \in [0, \frac{1}{2}]} \{\rho(p; \alpha)\}$ and $\underline{\rho}(\alpha) = \min_{p \in [0, \frac{1}{2}]} \{\rho(p; \alpha)\}$ for notational convenience.

Lemma 4 *Suppose $0 < \underline{\rho} \leq \bar{\rho} < \infty$. Then $\frac{1}{2}(2p)^{\frac{1}{\underline{\rho}}} \leq G(p) \leq \frac{1}{2}(2p)^{\frac{1}{\bar{\rho}}}$ for $p \in [0, \frac{1}{2}]$.*

By Lemma 4, $G(p)$ can be bounded by two constant cumulative density functions with constant ρ -concavity. A completely informative information structure corresponds to the case where $\lim_{\alpha \rightarrow \infty} \underline{\rho}(\alpha) = \infty$ and a completely uninformative information structure is equivalent to $\lim_{\alpha \rightarrow \infty} \bar{\rho}(\alpha) = 0$.¹³ The following assumptions are imposed on the family of distribution $\{G(\cdot; \alpha)\}$ indexed by $\alpha \in (0, \infty)$.

Assumption 5 (a) *Log concavity: $\rho(p; \alpha) \in (0, \infty)$ for $(p, \alpha) \in [0, \frac{1}{2}] \times (0, \infty)$.*

(b) *ρ -concave order: If $\alpha_1 > \alpha_2$, $\rho(p; \alpha_1) > \rho(p; \alpha_2)$ for $p \in [0, \frac{1}{2}]$.*

(c) *Regularity 1: $\forall \alpha$, $\rho(p; \alpha)$ is weakly decreasing in p for $p \in [0, \frac{1}{2}]$.*¹⁴

(d) *Regularity 2: There exists α such that $\rho(p; \alpha) = 1$ for all $p \in [0, \frac{1}{2}]$.*

(e) *Normalization: $\lim_{\alpha \rightarrow \infty} \underline{\rho}(\alpha) = \infty$ and $\lim_{\alpha \rightarrow 0} \bar{\rho}(\alpha) = 0$.*

By Assumption 5(a), we focus on $G(p; \alpha)$ which is log-concave in $p \in [0, \frac{1}{2}]$. Together with Assumption 5(c), Assumption 5(d) guarantees that the concavity/convexity of $G(\cdot)$ will not change for given α . Assumption 5(e) restates Assumption 4 in the language of the ρ -concavity.

¹³See Appendix C for detailed proof.

¹⁴As will be clear later, this assumption generates a well-behaved profit function for $p \in [0, \frac{1}{2}]$.

The optimal replacement policy

Denote \hat{p} as the cutoff of the posterior belief and $\tilde{\pi}(\hat{p})$ as the board's profit as a function of \hat{p} . Then

$$\tilde{\pi}(\hat{p}) = \underbrace{\frac{1}{4} \int_{\hat{p}}^1 tg(t)dt}_{\text{incentive effect}} \left\{ \underbrace{\frac{1}{2}G(\hat{p}) + \int_{\hat{p}}^1 tg(t)dt}_{\text{selection effect}} + \underbrace{\left(\hat{p} - \frac{1}{2}\right) \int_0^{\hat{p}} g(t)dt}_{\text{commitment effect}} \right\}.$$

The profit function can be further simplified by combining the selection effect and the commitment effect,

$$\tilde{\pi}(\hat{p}) = \underbrace{\frac{1}{4} \int_{\hat{p}}^1 tg(t)dt}_{\text{incentive effect}} \left\{ \underbrace{\int_{\hat{p}}^1 tg(t)dt + \hat{p}G(\hat{p})}_{\text{selection+commitment effect}} \right\}.$$

The expression of the total selection and commitment effect is intuitive. In equilibrium, the board's expected profit of replacement is equal to the expected profit created by the marginal incumbent manager with expected ability \hat{p} . Hence the board is replacing the incumbent manager of ability $p \leq \hat{p}$ with \hat{p} taking commitment into consideration. It can be verified that the total of the selection effect and the commitment effect is increasing in \hat{p} and thus is maximized at $\hat{p} = 1$.

The first order derivative with respect to \hat{p} yields,

$$\tilde{\pi}'(\hat{p}) = \frac{1}{4} \left[-\hat{p}g(\hat{p}) \left(1 - \int_{\hat{p}}^1 G(t)dt \right) + G(\hat{p}) \int_{\hat{p}}^1 tg(t)dt \right].$$

$$\Rightarrow \tilde{\pi}'(\hat{p}) \begin{matrix} \leq \\ \geq \end{matrix} 0 \Leftrightarrow \frac{\hat{p}g(\hat{p})}{G(\hat{p})} \begin{matrix} \geq \\ \leq \end{matrix} \frac{\int_{\hat{p}}^1 tg(t)dt}{1 - \int_{\hat{p}}^1 G(t)dt}.$$

From the first order condition, $\hat{p}g(\hat{p})$ is the marginal incentive effect and $G(\hat{p})$ is the marginal selection plus commitment effect. Whether profit is increasing or decreasing in \hat{p} largely depends on the ratio of these two marginal effects, which is also the elasticity of $G(\cdot)$ at point \hat{p} . Since $\tilde{\pi}(1) = 0$, the incentive effect dominates the selection plus commitment effect when \hat{p} is close to 1. To relate ρ -concavity to the profit function, notice that $\frac{\hat{p}g(\hat{p})}{G(\hat{p})} = \left(\frac{\int_0^{\hat{p}} \rho(t)dt}{\hat{p}}\right)^{-1}$, which is the inverse of the average ρ -concavity of $G(\cdot)$ from 0 to \hat{p} . This ratio is weakly increasing if $\rho(p; \alpha)$ is weakly decreasing in p for $p \in [0, \frac{1}{2}]$ by Assumption 5(c). This assumption guarantees that the marginal incentive effect changes faster than the marginal selection plus commitment effect and yields a well-behaved profit function for $\hat{p} \in [0, \frac{1}{2}]$. Assumption 5(b) (ρ -concave order) guarantees that the marginal selection plus commitment effect changes faster than the marginal incentive effect for given $\hat{p} \in [0, \frac{1}{2}]$ as α increases. Consequently, the selection plus commitment effect takes over as board's monitoring technology improves and anti-entrenchment is more likely to emerge.

Proposition 2 *Suppose the family of distribution $\{G(\cdot; \alpha)\}$, indexed by $\alpha \in (0, \infty)$, satisfies Assumption 5. Then there exists α_1 and α_2 such that*

1. $\hat{s}^*(\alpha) = 0$ for $\alpha \in (0, \alpha_1]$;
2. $\hat{s}^*(\alpha) \in (0, \frac{1}{2})$ for $\alpha \in (\alpha_1, \alpha_2)$;
3. $\hat{s}^*(\alpha) \in (\frac{1}{2}, 1)$ for $\alpha \in (\alpha_2, \infty)$,

where α_1 satisfies $\rho(p; \alpha_1) = 1 \ \forall p \in [0, \frac{1}{2}]$ and $\alpha_2 > \alpha_1$.

Proposition 2 characterizes the optimal replacement policy for all α . When α is small, the board provides full job security and never fires the incumbent manager. When α is moderate, the board replaces the incumbent manager less frequently than the socially optimal level and entrenchment is optimal. When α is large, the board uses an aggressive replacement policy and anti-entrenchment emerges. There exists a clear cutoff between entrenchment and anti-entrenchment: once anti-entrenchment is optimal for informativeness level α' , the optimal replacement policy is never entrenchment under a more informative information structure $\alpha > \alpha'$.

A tractable example

Example 1 Suppose $G(p)$ has the following functional form,

$$G(p) = \begin{cases} \frac{1}{2}(2p)^{\frac{1}{\alpha}} & \text{for } \hat{p} \in [0, \frac{1}{2}] \\ 1 - \frac{1}{2}[2(1-p)]^{\frac{1}{\alpha}} & \text{for } \hat{p} \in (\frac{1}{2}, 1] \end{cases}.$$

Then the optimal cutoff is

1. for $\alpha \leq 1$, $\hat{p}^*(\alpha) = 0$;
2. for $1 < \alpha < \frac{\sqrt{5}+1}{2}$, $\hat{p}^* \in (0, \frac{1}{2})$;
3. for $\alpha > \frac{\sqrt{5}+1}{2}$, $\hat{p}^* \in (\frac{1}{2}, 1)$.

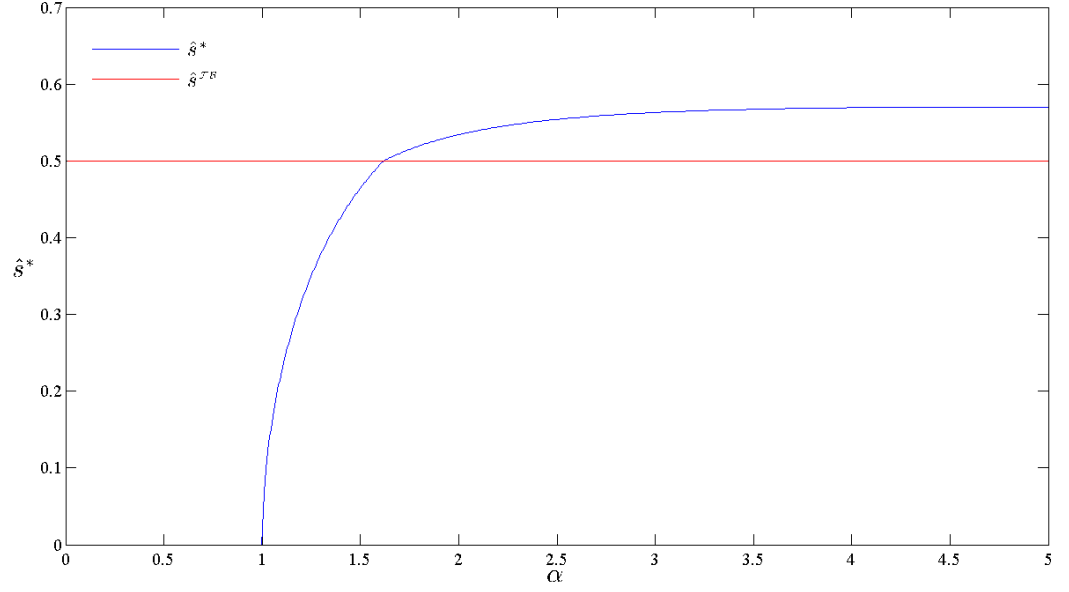


Figure 1.2: The optimal replacement policy

Given $G(\cdot)$, the two corresponding conditional density functions are

$$f_1(s) = \begin{cases} (2s)^\alpha & \text{for } s \in [0, \frac{1}{2}] \\ 2 - [2(1-s)]^\alpha & \text{for } s \in (\frac{1}{2}, 1] \end{cases}$$

and

$$f_0(s) = \begin{cases} 2 - (2s)^\alpha & \text{for } s \in [0, \frac{1}{2}] \\ [2(1-s)]^\alpha & \text{for } s \in (\frac{1}{2}, 1] \end{cases}.$$

Figure 1.2 shows the optimal cutoff for different informativeness levels of the monitoring technology. Turnover is increasing for $\alpha \in [1, \frac{\sqrt{5}+1}{2}]$ when the manager is entrenched. The relationship between turnover and the informativeness of the board's monitoring technology is an inverted-U shape for $\alpha > \frac{\sqrt{5}+1}{2}$. As α approaches infinity,

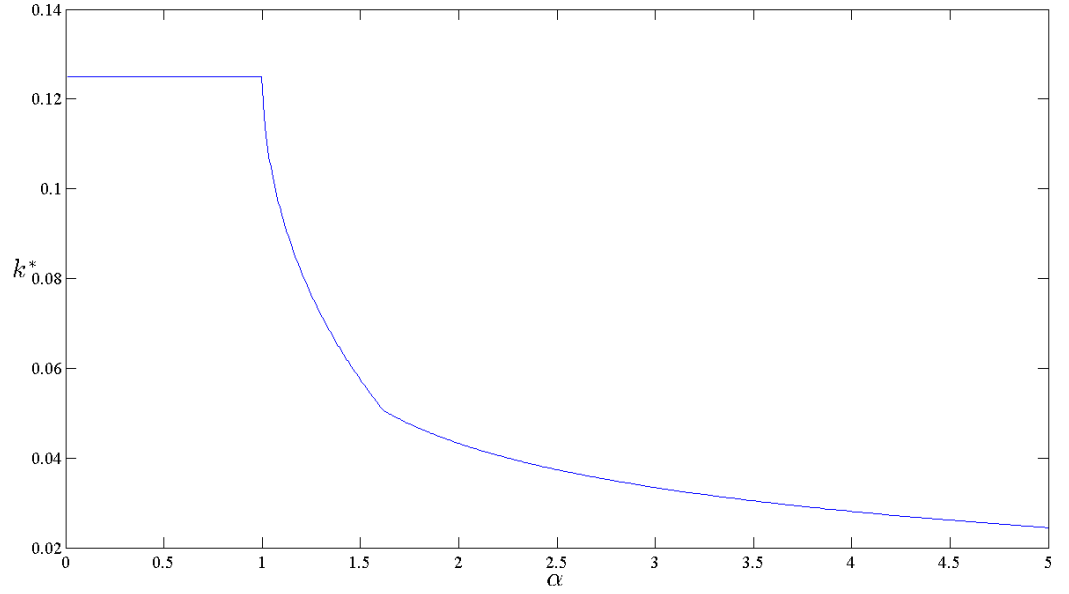


Figure 1.3: Severance pay in the optimal contract

the optimal cutoff converges to $\frac{1}{2}$.

Figure 1.3 shows that severance pay in the optimal contract is decreasing in the informativeness of the board's monitoring technology. When the information structure becomes more informative, it is easier for the board to obtain net commitment gain. Thus the board is less willing to commit to not replacing the incumbent manager and the size of severance pay offered in the optimal contract decreases as a result. This generates a testable implication of the model: the size of the severance package is decreasing in the informativeness of the board's monitoring technology.

1.4.3 Discussion

An optimal replacement policy that differs from the first best stems from two important assumptions: the signal is non-contractible and severance pay is constant with

respect to outcome, i.e., the board cannot provide performance-based severance pay. Without either of these assumptions, neither entrenchment nor anti-entrenchment emerges: the optimal replacement policy is always $\hat{s}^* = \frac{1}{2}$.

First consider what happens if the board's signal is contractible, while maintaining the assumption that severance pay is constant. A contract is fully characterized by $\{w(s), r(s), k(s)\}$, where $s \in [0, 1]$. $\{w(s), k(s)\}$ is the promised wage and severance pay after signal s . $r(s) \in [0, 1]$ specifies the retaining probability of the incumbent manager at signal s . In particular, $r(s) = 1$ indicates that the incumbent manager is retained while $r(s) = 0$ indicates that the incumbent is fired.¹⁵

Proposition 3 *Suppose that the signal is contractible and severance pay is constant with respect to outcome. Then $k^*(s) = 0$. Moreover, $r^*(s) = 1$ for $s \in [\frac{1}{2}, 1]$ and $r^*(s) = 0$ for $s \in [0, \frac{1}{2})$.*

Allowing the board to contract on signals gives the board commitment power on its retention decision at no cost. Severance pay is a costly commitment device, and is no longer used in the optimal contract.

The board can design a contract to induce any effort level without deviating from the socially optimal replacement cutoff. The manager is risk-neutral and only cares about the expected wage. Thus, the board can incentivize the incumbent manager by increasing the expected wage payment, which is determined by both the wage function $w(s)$ and the replacement policy $r(s)$. For given effort q and replacement

¹⁵Due to the board's risk neutrality, randomization is not optimal except for the case where the board is indifferent between retaining and firing the incumbent manager, in which we assume the incumbent manager is retained with probability 1.

policy $r(s)$, the board can adjust the wage function $w(s)$ to induce q without changing $r(s)$. That is, the board can optimize effort and selection separately if the signal is contractible, and the replacement cutoff is equal to $\frac{1}{2}$ in the optimal contract.

Next consider what happens when the board can condition the severance pay on the outcome, while maintaining the assumption that the signal is non-contractible. A contract is in the form of a triple (w_1, w_2, k) . w_1 is the wage rate when the incumbent manager stays as in the baseline model. The tuple (w_2, k) constitutes a severance package. w_2 is the payment to the incumbent manager if he is forced out and $y = 1$. k is the constant severance pay as in the baseline model.

Proposition 4 *Suppose that the signal is non-contractible and the board can provide performance-based severance pay. Then $k^* = 0$, $w_1^* = w_2^*$ and $\hat{s}^* = \frac{1}{2}$.*

Constant severance pay is less effective to the board than performance-based severance pay when the incumbent manager is no longer in office because a lump-sum payment rewards failure. Thus, only performance-based severance pay is employed in the optimal contract.

Again the board has no incentive to deviate from the first best cutoff. Due to manager's risk neutrality, the effort choice of the incumbent manager is only determined by the expected wage. For a given effort level q that the board wants to motivate, the expected wage is fixed, which is also the total cost to hire the incumbent manager. Since q is fixed, it remains to maximize the expected ability of the manager who stays in office in period 2. Hence the replacement cutoff stays at the first best in the optimal contract.¹⁶

¹⁶In practice, a severance package usually comes in the form of a combination of a lump-sum

1.5 Extensions

In this section, we show that the main result of the optimal replacement policy is robust to several different specifications.

1.5.1 More effort vs. better selection

It is interesting to study whether the main result on entrenchment (anti-entrenchment) remains optimal when effort becomes more important than ability. Anti-entrenchment is less likely to emerge when selection becomes less important. We model this by decreasing the variance of the manager's ability or increasing the importance of effort relative to ability in the success probability.

Assume the ability space is $\theta \in \{\frac{1}{2} - \delta, \frac{1}{2} + \delta\}$, where $\delta \in (0, \frac{1}{2}]$ and the success probability is equal to $q^{1+\tau}\theta$, where $\tau \in (-1, 1)$. δ is a measure of the variance of manager's ability ex ante while τ is a measure of the relative importance of effort compared to selection. The baseline model corresponds to $(\delta, \tau) = (\frac{1}{2}, 0)$. It is intuitive that entrenchment remains optimal as α approaches 0. Thus we focus on the case where α approaches infinity.

Proposition 5 (Comparative static)

1. if $\delta > \frac{1}{2} \frac{1-\tau}{2} - \frac{1}{2}$, there exists $\bar{\alpha}_A$ such that anti-entrenchment is optimal for

$$\alpha > \bar{\alpha}_A;$$

2. if $\delta < \frac{1}{2} \frac{1-\tau}{2} - \frac{1}{2}$, there exists $\bar{\alpha}_E$ such that entrenchment is optimal for $\alpha > \bar{\alpha}_E$.

payment and a stock option. This can be rationalized by the assumption that the manager is more risk-averse than the board. If that is the case, the optimal contract will involve some degree of lump-sum payment in response to risk sharing.

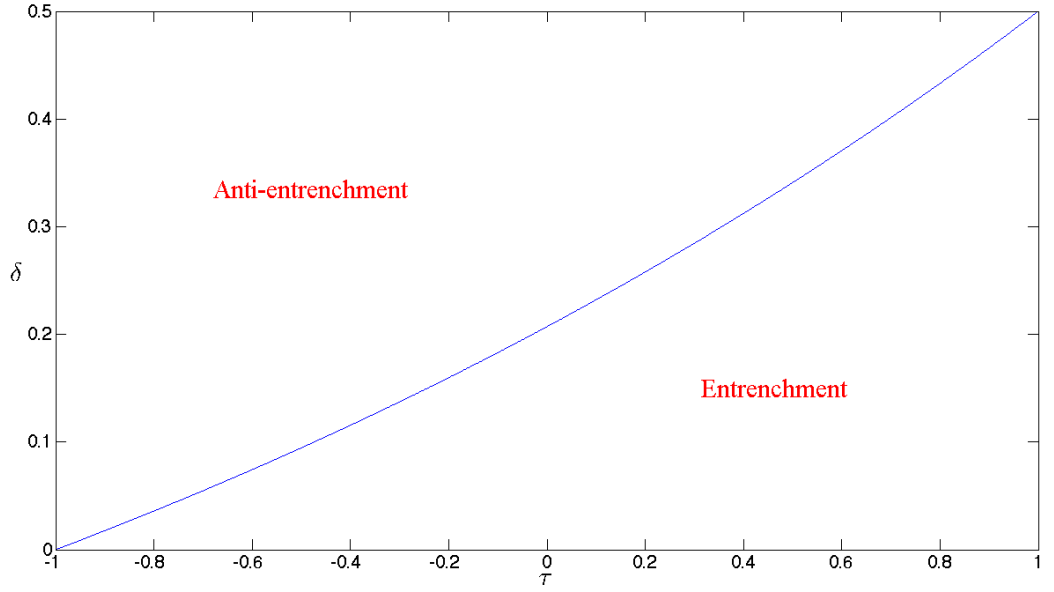


Figure 1.4: The optimal replacement policy at the limiting distribution

Fixing τ , selection becomes more important as the variance of the manager's ability increases. The intuition from the baseline model applies when δ is large, and anti-entrenchment emerges in the optimal contract. When δ is small, the marginal productivity of a low ability manager is close to that of a high ability manager. Since motivating the low ability manager is also important, the optimal contract leads to entrenchment as $\alpha \rightarrow \infty$.

1.5.2 Costly execution

Suppose the outcome depends on the incumbent manager's effort q in period 1, effort e of the manager in period 2 as well as the ability of the manager in office in period 2. Effort q can be interpreted as the project quality of the project selected by the incumbent manager and e can be interpreted as the effort required to execute the

project. Now the board needs to also offer a contract to the replacement manager. Moreover, the optimal contract with the incumbent manager must balance a two-dimensional moral hazard problem as well as the selection problem.

Assume the success probability is equal to $\tilde{\theta}[(1 - \lambda)q + \lambda\tilde{e}]$, where $\tilde{\theta} \in \{0, 1\}$ is the manager's ability in period 2, $q \in [0, 1]$ is the effort of the incumbent manager in period 1 and $\tilde{e} \in [0, 1]$ is the effort of the manager in period 2. Period 1 effort q and period 2 effort e are substitutes and $\lambda \in [0, 1]$ measures the relative importance of period 1 effort. When $\lambda = 0$ the model simplifies to the baseline model. After the board makes a retention decision, the manager in office at the beginning of period 2 exerts effort e to execute the project. The cost function to the incumbent manager is assumed to be separable and quadratic, i.e., $C_i(q, e) = \frac{1}{2}q^2 + \frac{1}{2}e^2$. The cost function to the replacement manager is assumed to be $C_r(q, e) = C_i(0, e)$.

Lemma 5 (First best outcome with costly execution) *The socially optimal cutoff is equal to $\frac{1}{2}$.*

The proof is similar to Lemma 1 and is omitted. With costly execution, the first best replacement cutoff remains at $\frac{1}{2}$. In fact, this result is very general. The optimal cutoff is always $\frac{1}{2}$ as long as the marginal impact of manager's ability is positive.

The board provides two contracts, contract (w, k) to the incumbent manager and a wage w_r to the replacement manager. Denote variables with subscript r as the variables related to the replacement manager after retention.

We first calculate the optimal contract with the replacement manager w_r after the incumbent's departure for a given belief of period 1 effort q . Given q and w_r , the

replacement manager chooses e_r to maximize:

$$\frac{1}{2}[(1-\lambda)q + \lambda e_r]w_r - \frac{1}{2}e_r^2 \Rightarrow e_r(w_r) = \frac{1}{2}\lambda w_r.$$

Note that the replacement manager's effort on execution is independent of q . This is because q and e are assumed to be substitutes. The board chooses w_r to maximize:

$$\frac{1}{2}[(1-\lambda)q + \lambda e_r](1-w_r) \Rightarrow w_r^* = \max \left\{ \frac{1}{2} - \frac{1-\lambda}{\lambda^2}q, 0 \right\}.$$

Since q and e are substitutes by assumption, the optimal wage to the replacement manager is weakly decreasing in the belief about q . When first period effort q is large or λ is small, the board provides a contract with $w_r = 0$ to the replacement manager.

Let $\underline{\pi}(q)$ be the board's expected profit under optimal contract after replacement. $\underline{\pi}(q)$ can be calculated as follows,

$$\underline{\pi}(q) = \begin{cases} \frac{1}{4} \left(\frac{1}{2}\lambda + \frac{1-\lambda}{\lambda}q \right)^2 & \text{for } q \leq \frac{1}{2} \frac{\lambda^2}{1-\lambda} \\ \frac{1}{2}(1-\lambda)q & \text{for } q > \frac{1}{2} \frac{\lambda^2}{1-\lambda} \end{cases}.$$

Next we calculate the equilibrium for a given contract (w, k) with the incumbent manager. For a fixed contract (w, k) and belief about cutoff \hat{s} , the incumbent manager chooses (q, e) to maximize:

$$\frac{1}{2}[1 - F_1(\hat{s})][(1-\lambda)q + \lambda e]w + \frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})]k - \frac{1}{2}q^2 - \frac{1}{4}[(1 - F_1(\hat{s})) + (1 - F_0(\hat{s}))]e^2.$$

$$\Rightarrow q = (1-\lambda) \frac{1 - F_1(\hat{s})}{2} w \quad \text{and} \quad e = \lambda \frac{1 - F_1(\hat{s})}{[1 - F_1(\hat{s})] + [1 - F_0(\hat{s})]} w.$$

Note that q is decreasing in \hat{s} while e is increasing in \hat{s} . A higher equilibrium replacement cutoff leads to a lower first period effort q and higher second period effort e by the incumbent manager. The first period effort q is decreasing in \hat{s} because a higher cutoff implies lower retaining probability in period 2 and dis-incentivizes the incumbent manager as in the baseline model. Conditional on the fact that the incumbent manager is retained, a higher cutoff yields a higher estimate of the incumbent's ability and thus the incumbent is willing to exert more effort in period 2 (θ and e are assumed to be compliments). As a result, e is increasing in equilibrium cutoff \hat{s} . Similarly to Casamatta and Guembel (2010), the incumbent is easier to motivate, but for different reasons. The incumbent manager is easier to motivate in Casamatta and Guembel (2010) due to his reputational concern while in our model it is due to the incumbent's learning of his ability. For a given wage rate w , the replacement manager chooses $e_r = \frac{1}{2}\lambda w$ while the incumbent manager chooses $e = \lambda \frac{1-F_1(\hat{s})}{[1-F_1(\hat{s})]+[1-F_0(\hat{s})]} w > \frac{1}{2}\lambda w$. The incumbent manager learns from his retention that his ability is above average. Since ability and effort are assumed to be compliments, a higher estimate of ability implies a higher marginal return on effort. Thus, the incumbent manager exerts more effort in period 2 than the potential replacement manager given the same wage.

For a fixed contract (w, k) and belief about effort (q, e) , the board chooses cutoff \hat{s} to maximize:

$$\frac{1}{2}[1 - F_1(\hat{s})][(1 - \lambda)q + \lambda e](1 - w) + \frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})][\underline{\pi}(q) - k].$$

$$\Rightarrow \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} [(1 - \lambda)q + \lambda e] (1 - w) = \underline{\pi}(q) - k.$$

Not every \hat{s} can be implemented. For instance, \hat{s} very close to 1 cannot be induced by a contract. This is due to the limited liability assumption of severance pay. Extremely high cutoff can only be induced if severance pay is allowed to be negative. This is different from the baseline model. Using an aggressive replacement policy results in a small q , making the board's outside option very unattractive. On the other hand, an aggressive replacement policy improves learning and makes e very high, increasing the value of keeping the current manager. Thus, unless the board is compensated by negative severance pay, a very aggressive replacement policy cannot be induced by a contract subject to the limited liability constraint.

Similarly to the baseline model, the expected profit can be written as a function of cutoff \hat{s} alone, assuming away the limited liability constraint of k ,¹⁷

$$\begin{aligned} \pi(\hat{s}) = & \frac{1}{8} \left\{ (1 - \lambda)^2 \underbrace{\frac{1 - F_1(\hat{s})}{2}}_{\text{incentive effect}} + \lambda^2 \underbrace{\frac{1 - F_1(\hat{s})}{[1 - F_1(\hat{s})] + [1 - F_0(\hat{s})]}}_{\text{learning effect}} \right\} \\ & \cdot \left\{ \underbrace{\left[[1 - F_1(\hat{s})] + \frac{1}{2} [F_1(\hat{s}) + F_0(\hat{s})] \right]}_{\text{selection effect}} + \underbrace{\left[F_1(\hat{s}) + F_0(\hat{s}) \right] \left(\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} - \frac{1}{2} \right)}_{\text{commitment effect}} \right\}. \end{aligned}$$

Proposition 6 (Optimal replacement policy with costly execution) 1. If $\lambda \in$

$[0, \sqrt{2} - 1)$, there exists $\bar{\alpha}$ such that $\hat{s}^*(\alpha) > \frac{1}{2}$ for $\alpha > \bar{\alpha}$.

2. For $\lambda \in [0, 1]$, there exists $\underline{\alpha}$ such that $\hat{s}^*(\alpha) < \frac{1}{2}$ for $\alpha < \underline{\alpha}$.

¹⁷The intuition can be clearly illustrated assuming that every \hat{s} can be induced. The limited liability constraint of severance pay is taken into consideration in the proof of Proposition 6.

A learning effect enters into the board's profit function along with the three aforementioned effects. When the information structure is sufficiently noisy ($\alpha \rightarrow 0$), the incumbent's learning is very slow for all $s \in (0, 1)$. The learning effect plays a minor role in determining the optimal replacement policy since $\frac{1-F_1(\hat{s})}{[1-F_1(\hat{s})]+[1-F_0(\hat{s})]}\lambda^2$ can be considered as a constant. Thus, entrenchment is expected to be optimal when α is sufficiently small independent of the size of λ .

When the information structure is sufficiently informative ($\alpha \rightarrow \infty$), the incumbent's learning becomes very fast. When execution becomes sufficiently important, entrenchment can be optimal to the board. When period 1 effort q is sufficiently important relative to period 2 effort e (i.e., $\lambda < \sqrt{2} - 1$), the incentive effect is more important than the learning effect in board's contractual problem. Thus, the main insight in the baseline model follows through and entrenchment is expected to emerge in the optimal contract.

1.5.3 Signal of outcome instead of ability

In the baseline model, it is assumed that the board observes a signal of the incumbent manager's ability rather than the outcome under the incumbent's management. Since the signal is not related to the incumbent manager's effort, the incumbent cannot increase his probability of retention by exerting more effort. When the board receives a signal related to effort, the incumbent manager is able to increase his probability of being retained by exerting more effort.

Suppose for outcome $y \in \{0, 1\}$, signal s is drawn from a distribution with density

$h_y(\cdot)$ and cdf $H_y(\cdot)$. Similarly, we assume $\{h_1(\cdot), h_0(\cdot)\}$ satisfies Assumptions 1 - 4. The signal provides information about the expected outcome and the incumbent manager's ability.

The social planner chooses (\hat{s}, q) to maximize:

$$\max_{\{\hat{s}, q\}} \frac{1}{2}q[1 - H_1(\hat{s})] + \frac{1}{2}q\left[\frac{1}{2}qH_1(\hat{s}) + \left(1 - \frac{1}{2}q\right)H_0(\hat{s})\right] - \frac{1}{2}q^2.$$

Lemma 6 (First best outcome) $\hat{s}^{\mathcal{FB}} = \frac{1}{2}$ and $q^{\mathcal{FB}} = \frac{1+H_0(\frac{1}{2})-H_1(\frac{1}{2})}{2+H_0(\frac{1}{2})-H_1(\frac{1}{2})}$ in the first best outcome.

The proof is similar to Lemma 1 and is omitted. Given effort level q , the Bayesian update of the incumbent manager's ability is

$$\varphi(\hat{s}, q) = \frac{\frac{1}{2}qh_1(\hat{s}) + \frac{1}{2}(1-q)h_0(\hat{s})}{\frac{1}{2}qh_1(\hat{s}) + \left(1 - \frac{1}{2}q\right)h_0(\hat{s})}.$$

It can be verified that $\varphi(\frac{1}{2}, q) = \frac{1}{2}$ independent of q and α . Thus it is socially optimal to replace the incumbent manager if and only if the posterior belief of his ability falls below the prior.

Given a contract (w, k) and belief about replacement cutoff \hat{s} , the manager chooses q to maximize:

$$\frac{1}{2}q[1 - H_1(\hat{s})]w + \left\{ \frac{1}{2}qH_1(\hat{s}) + \left(1 - \frac{1}{2}q\right)H_0(\hat{s}) \right\} k - \frac{1}{2}q^2.$$

The manager's best response is:

$$q = \max \left\{ \frac{1}{2}[1 - H_1(\hat{s})]w - \frac{1}{2}[H_0(\hat{s}) - H_1(\hat{s})]k, 0 \right\}.$$

Unlike the baseline model, here the incumbent manager is directly dis-incentivized by severance pay. An increase in severance pay increases the opportunity cost of exerting effort and leads directly to a decrease in effort. If the severance pay is high enough, the incumbent manager exerts no effort at all and is willing to be fired. Under this extension, severance pay is a double-edged sword. By the direct effect (better outside option if the incumbent manager is replaced), effort decreases. By the indirect effect (better job security with lower equilibrium replacement cutoff), effort increases. The design of the optimal contract should take this non-trivial incentive of k on q into consideration.

Fixed (w, k) and q , the board chooses \hat{s} to maximize:

$$\frac{1}{2}q[1 - H_1(\hat{s})](1 - w) + \left\{ \frac{1}{2}qH_1(\hat{s}) + \left(1 - \frac{1}{2}q\right) H_0(\hat{s}) \right\} \left(\frac{1}{2}q - k \right).$$

The board's indifference condition is:

$$\frac{\frac{1}{2}qh_1(\hat{s})}{\frac{1}{2}qh_1(\hat{s}) + \left(1 - \frac{1}{2}q\right)h_0(\hat{s})}(1 - w) = \frac{1}{2}q - k.$$

\hat{s} is the solution to $\zeta(\hat{s}, q) = \max \left\{ \min \left\{ \frac{\frac{1}{2}q - k}{1 - w}, 1 \right\}, 0 \right\}$, where $\zeta(\hat{s}, q)$ is the estimate of the outcome under the incumbent's management at \hat{s} given q ,

$$\zeta(\hat{s}, q) \equiv \frac{\frac{1}{2}qh_1(\hat{s})}{\frac{1}{2}qh_1(\hat{s}) + \left(1 - \frac{1}{2}q\right)h_0(\hat{s})}.$$

It is difficult to write the expected profit as a function of \hat{s} alone because q is now affected by (w, k) directly and by equilibrium cutoff \hat{s} indirectly. However, we can still discuss the optimal replacement policy under extreme information structure.

Multiple equilibria may exist for some contract (w, k) because incentive on effort q is not monotone in k as in the baseline model. For the same reason, equilibria may not be Pareto-ranked. We further assume that the equilibrium most favorable to the board is selected when multiple equilibria exist.

Proposition 7 (Optimal replacement policy) *There exist $\bar{\alpha}$ and $\underline{\alpha}$ such that,*

1. $\hat{s}^*(\alpha) > \frac{1}{2}$ for $\alpha > \bar{\alpha}$;
2. $\hat{s}^*(\alpha) < \frac{1}{2}$ for $\alpha < \underline{\alpha}$.

When the information structure is sufficiently noisy ($\alpha \rightarrow 0$), $H_0(s)$ is very close to $H_1(s)$ for $s \in [0, 1]$. The direct negative effect of severance pay on effort is small and the model is back to the baseline in the limit. Knowing that the board has noisy monitoring technology, the incumbent manager has little incentive to manipulate the realization of the signal. Entrenchment is expected to be optimal when α is sufficiently small.

When the information structure is sufficiently informative ($\alpha \rightarrow \infty$), the magnitude of the direct negative effect of k (i.e., $\frac{1}{2}[H_0(\hat{s}) - H_1(\hat{s})]$) is very large. Under this scenario, the board can simply avoid the disadvantage of k by setting $k = 0$. Moreover, this does not contradict the possibility of obtaining a net commitment gain. In fact, we can construct a contract with high wage and zero severance pay

that yields anti-entrenchment and dominates all possible contracts that yield entrenchment. Thus, anti-entrenchment emerges in the optimal contract as the board's information structure becomes sufficiently informative.

1.6 Conclusion

This paper explores how the problem of motivating the incumbent manager to exert effort and keeping the flexibility to choose a high ability manager interacts with the equilibrium replacement policy. We focus on the situation where the board observes a non-contractible signal after the incumbent manager exerts effort and solve for the optimal contract. We show that the information technology that the board uses to assess the incumbent manager's ability is an important determinant of the optimal contract and of managerial turnover. Unlike the existing literature on managerial turnover, which aims to rationalize entrenchment, we show that anti-entrenchment can also be optimal for shareholders in some situations. This result is robust to allowing costly execution and the possibility that the board observes a signal of the outcome rather than incumbent manager's ability. The model highlights the board's monitoring technology as an important determinant of managerial turnover.

There are several interesting questions that can be pursued using the stylized model introduced in this paper. For future research, it would be interesting to endogenize the informativeness of the board's monitoring technology. In practice, informativeness is often the choice of the board. Some boards actively monitor their CEOs while some tend to be passive monitors. Endogenizing the board's monitoring

technology could help us better understand the differences of monitoring intensity that occur across industries.

Another intriguing research avenue would be to incorporate voluntary departure into the model by allowing the possibility that manager possesses private information about the firm's profit. As Inderst and Mueller (2010) point out, managers sometimes have private information about a firm's performance. In such scenarios, the optimal contract needs to provide incentives for the incumbent manager to step down voluntarily. It would be interesting to build a unified model with both forced departure and voluntary departure and study the interaction between them.

Chapter 2

Life Settlement Market with Overconfident Policyholders

This chapter is a joint work with Hanming Fang.

2.1 Introduction

Life insurance is a prevalent long-term contract for people to keep their dependents from economic disaster when the policyholders die. The life insurance is a large and growing industry. According to Life Insurance Marketing and Research Association International (LIMRA international), 70% of U.S. households owned some type of life insurance in 2010. U.S. families purchased \$2.8 trillion of insurance coverage in 2013 and the total life insurance coverage in the US was \$19.7 trillion by the end of 2013. The average face amount of the individual life insurance policies purchased increased from \$81 thousand in 1993 to \$165 thousand in 2013 at an average annual growth

rate of 3.56%¹.

An important feature of the the life insurance market is that policyholders lapse their policies before the period of coverage and receive the cash surrender value (CSV) that is a substantially small fraction (typically 3-5%) of the policy's face value². Policyholders may lapse the contract if they lose the demand for life insurance (i.e., lost of bequest motive) or need for liquidity (i.e., negative income shock).³ Recently, the secondary market for life insurance – life settlement market – emerged and offers the policyholders the option of selling their unwanted policy for more than the CSV.

Although the life settlement market is in its infancy, it draws attentions from the life insurance firms who put intensive effort into lobbying to prohibit the securitization of life settlement contracts.⁴ They argue that the life insurance contract is designed taking into consideration of the fact that a fraction of policyholders lapse the contract without receiving the death benefit. The existence of the settlement market forces the insurance firms to pay death benefits on more policies than expected, which will lead to higher premiums for policyholders in the long run and hurt consumers eventually. The life settlement industry has been working hard to justify its existence, emphasizing its role of enhancing liquidity to policyholders.⁵ It is interesting to note that the life settlement industry has gained some success recently. In 2010, the General

¹See American Council of Life Insurers (2014).

²See <http://www.lisa.org/content/13/What-is-a-Life-Settlement.aspx/>

³Fang and Kung (2012) show that income shocks are relatively more important than bequest motive in explaining lapsation when policyholders are young. As policyholders age, bequest motive shocks become more important.

⁴See Martin (2010) for detailed dicussions of life insurance and life settlement market.

⁵As mentioned in Martin (2010): "In 2008, the executive of the life settlement industry's national trade organizaton testifies to the Florida Office of Insurance Regulation that the 'secondary market for life insurance has brought great benefits to consumers, unlocking the value of life insurance policies."

Assembly in Kentucky passed a bill requiring insurers to inform policyholders who are considering surrendering their policy that the settlement is a potential alternative.⁶

Should the life settlement industry be banned? To resolve this theoretically and empirically interesting question, it would be useful to understand the role of the life settlement market and its impact on policyholders' welfare. In this paper we extend the models of Daily, Hendel, and Lizzeri (2008) and Fang and Kung (2010) to study the welfare implication of the life settlement market by assuming that consumers are overconfident about the probability of losing their bequest motives at the time they purchase the contract.⁷ Fang and Kung (2010) show that when lapsation is due to lost of bequest motive, introducing life settlement market reduces consumer welfare. We prove that this result depends on the full rationality assumption on policyholders. When policyholders exhibit overconfidence, the presence of the settlement market provides them a channel to correct their mistakes and undercut the loss due to their misperception. This new role of the settlement market generates a potential welfare gain which is not present when consumers are fully rational and may lead to increase in consumer welfare in equilibrium. Our results may contribute to the debate over banning life settlement market.

This paper is related to the growing life insurance literature. In a seminal paper,

⁶Similar requirements exist in Maine, Oregon, Washington (See Martin, 2010) and U.K (See Januário and Naik, 2014).

⁷Many studies document that people are unrealistically optimistic about future life events. For instance, Weinstein (1980) finds strong evidence of over-optimism in a lab experiment setting with 258 college students. Subjects overwhelmingly predict themselves to be better than a median individual regarding positive events and below average regarding negative events. Robb et al. (2004) also detects underestimation of risk among patients who participated in cancer examination. They find that the self perceived risk is lower than the actual risk of colorectal cancer determined by flexible sigmoidoscopy screening.

Hendel and Lizzeri (2003) use a two-period model to analyze the role of commitment on long-term life insurance contract. In their model, risk-neutral life insurance firms compete to offer contracts to risk averse consumers who are subject to mortality risk. Consumers' health status may change over time and thus face *reclassification risk*. Insurance firms are able to commit to contractual terms while consumers can lapse the contract in the second period, lacking commitment power (i.e., one-sided commitment). They prove that the equilibrium contract is *front-loaded*: consumers are offered a contract with first period premium that is higher than actuarially fair in exchange for reclassification risk in the second period. Daily et al. (2008) and Fang and Kung (2010) investigate this problem further by introducing a settlement market and analyze its effect on the equilibrium contract and consumer welfare. In their models, policyholders may lose bequest motive in the second period, facilitating lapsation and the demand for the settlement market. Using a model similar to Hendel and Lizzeri (2003), Fang and Kung (2010) show that the shape of the equilibrium contract is fundamentally changed in the presence of the settlement market. Instead of flat premiums, a contract with premium discounts is offered in the second period. They conclude that consumer welfare is reduced in the presence of the settlement market. In recent independent research, Gottlieb and Smetters (2014) investigate the equilibrium life insurance contracts where lapsation is motivated by a negative income shock. Similar to our paper, consumers are overconfident in the sense that they place zero probability on the event of experiencing the liquidity shock. Unlike Hendel and Lizzeri (2003), the equilibrium contract is front-loaded because insurance firms have incentives to make the policy look better given policyholders' misperception of the

probability of lapsing the contract. They show that the presence of the settlement market would increase consumer welfare if lapsation is due to liquidity shock.

This paper also belongs to the strand of literature on behavioral contract theory.⁸ Most papers assume consumers exhibit some type of behavioral bias and investigate how firms design contracts accordingly. For instance, De la Rosa (2011) and Santos-Pinto (2008) study the incentive contract in a principal-agent model of moral hazard when agent is overconfident. Grubb (2009) proposes a model of contracting overconfident consumers in US cellular phone services market and confirms evidence of overconfidence. In the context of insurance market, Sandroni and Squintani (2007) modify the textbook Rothschild-Stiglitz model to study the equilibrium contract by assuming that part of the insurees are overconfident about their risk types. They find that when a significant fraction of individuals are overconfident, compulsory insurance serves as a transfer of income between different types of agents. Their results have much different implications than Rothschild and Stiglitz (1976) on government intervention in insurance market. Spinnewijn (2012) studies the optimal unemployment insurance contract under perfect competition where the insuree has misperception on the probability of finding a job. Our paper contributes to this strand of literature by pointing out the role of a secondary market in alleviating the negative consequences caused by consumers' behavioral bias.

The remainder of the paper is organized as follows. Section 2 presents the baseline model of dynamic life insurance without the life settlement market when policyholders exhibit overconfidence and characterizes the set of equilibrium contracts. Section

⁸See Kőszegi (2014) for a comprehensive survey on this topic.

3 incorporates the settlement market into the baseline model. Section 4 studies the welfare effect of the settlement market under different levels of policyholders' overconfidence. Section 5 concludes. All proofs are presented in the Appendix.

2.2 The Baseline Model without the Settlement Market

In this section, we propose a model of dynamic life insurance slightly modified from Fang and Kung (2010), Daily, Hendel, and Lizzeri (2008) and Hendel and Lizzeri (2003).

2.2.1 The Model

Consider a perfectly competitive life insurance market with insurance buyers (policyholders) and sellers (life insurance firms). The market operates for two periods.

Income, health and preference. The policyholder receives an income of $y - g$ in the first period and $y + g$ in the second period, where $g \in [0, y)$ is a measure of income growth. In the first period, the policyholder has a death probability of $p_1 \in (0, 1)$, which is common knowledge between policyholders and insurance firms. The death probability is interpreted as the health status of the policyholder. In the second period, the mortality risk $p_2 \in [0, 1]$ is randomly drawn from a distribution with continuous density $\phi(\cdot)$ and corresponding cdf $\Phi(\cdot)$. Health status p_2 is not known in the first period when the policyholder purchases the insurance and is symmetrically

learned by the insurance firms and the policyholders at the beginning of the second period.

A policyholder has two sources of utility: utility from his own consumption if he is alive and utility from his dependent's consumption if he dies. If the policyholder is alive and consumes $c \geq 0$, his utility is given by $u(c)$. If the policyholder dies, his utility is given by $v(c)$, where c is the consumption of his dependent. We assume that both $u(\cdot)$ and $v(\cdot)$ are strictly increasing, twice differentiable and strictly concave. Furthermore, we assume that $u(\cdot)$ and $v(\cdot)$ satisfy the Inada conditions: $\lim_{c \rightarrow 0} u'(c) = \infty$, $\lim_{c \rightarrow 0} v'(c) = \infty$ and $\lim_{c \rightarrow \infty} v'(c) = 0$.

Bequest motives and overconfidence. A policyholder does not lose his bequest motive in the first period. However, the policyholder may lose his bequest motive with probability $q \in (0, 1)$ at the beginning of period 2. If the policyholder loses his bequest motive, he no longer derives utility from his dependent's consumption. Under such scenario, his utility is $u(\cdot)$ if he is alive and some constant normalized to zero if he dies. The policyholder believes his probability of losing bequest motive is $\tilde{q} \leq q$. When $\tilde{q} = q$, the policyholder is rational and the model degenerates to Fang and Kung (2010). When $\tilde{q} < q$, the policyholder exhibits overconfidence and underestimates the probability of losing his bequest motive. Both \tilde{q} and q are assumed to be common knowledge. For ease of our exposition, denote $\frac{q-\tilde{q}}{q}$ by Δ . The variable $\Delta \in [0, 1]$ is a measure of policyholders' overconfidence. In particular, when $\Delta = 0$, policyholder is fully rational and forms correct belief about q . When $\Delta = 1$, a policyholder is extremely overconfident and never expects himself to lose bequest

motive in the second period.

Timing, commitment and contracts. At the beginning of the first period, the consumer may choose to purchase a long-term life insurance contract after he learns the period-1 health status p_1 . A long-term insurance contract is in the form of $\langle (Q_1, F_1), (Q_2(p_2), F_2(p_2)) : p_2 \in [0, 1] \rangle$, where $\langle Q_1, F_1 \rangle$ specifies a premium and face value for the first period, and $\langle Q_2(p_2), F_2(p_2) \rangle$ specifies the corresponding premium and face value for each health status $p_2 \in [0, 1]$ for the second period. Note that the second period premium and face value are state dependent.

At the end of the first period, a fraction p_1 of policyholders die and their dependents receive the face value F_1 . The remaining policyholders continue to period 2, where a perfectly competitive spot market exists. We assume one-sided commitment: insurance firms can commit to future premiums and face values while the policyholders are free to opt out of the contract. After the policyholder learns the period-2 health status p_2 , he can choose to continue with the long-term contract purchased in the first period, or terminate the contract and purchase a spot contract, or just lapse the contract and stay uninsured in the absence of the life settlement market.

2.2.2 Equilibrium contracts

We characterize the equilibrium contract without the settlement market. Competition among the life insurance firms drives their profits to zero in the long run. Therefore

the long-term equilibrium contract $\langle (Q_1, F_1), (Q_2(p_2), F_2(p_2)) : p_2 \in [0, 1] \rangle$ solves:

$$\max[u(y - g - Q_1) + p_1 v(F_1)] \quad (2.1)$$

$$+ (1 - p_1) \int_0^1 \{(1 - \tilde{q})[u(y + g - Q_2(p_2))] + p_2 v(F_2(p_2))\} + \tilde{q}u(y + g) d\Phi(p_2)$$

$$\text{s.t. } (Q_1 - p_1 F_1) + (1 - p_1)(1 - q) \int_0^1 [Q_2(p_2) - p_2 F_2(p_2)] d\Phi(p_2) = 0, \quad (2.2)$$

$$Q_2(p_2) - p_2 F_2(p_2) \leq 0 \text{ for all } p_2 \in [0, 1], \quad (2.3)$$

$$Q_2(p_2) \geq 0 \text{ for all } p_2 \in [0, 1]. \quad (2.4)$$

Note that the set of equilibrium contracts maximizes policyholders' expected perceived utility instead of the utility based on the actual probability of losing bequest motive.⁹ Constraint (2.2) is the zero-profit condition that captures the competition in the primary market. Constraint (2.3) is the no-lapsation condition for policyholders whose bequest motives remain in period 2.¹⁰ The intuition is as follows. For any contract $\langle Q_2(p_2), F_2(p_2) \rangle$ in the second period, $p_2 F_2(p_2) - Q_2(p_2)$ is the actuarial value of the contract for health state p_2 . Since the spot market is perfectly competitive, the actuarial value of the spot contract is zero. In order to avoid the policyholders to lapse the long-term contract and substitute for a spot contract, the primary insurance firms have to provide a contract with *actuarial value* no less than 0, i.e., $p_2 F_2(p_2) - Q_2(p_2) \geq 0$. Finally, constraint (2.4) simply states that the second period premium for any health status can not be negative.¹¹

⁹Policyholders' expected utility according to the correct probability of losing bequest motive is used when we evaluate the welfare effect of introducing the life settlement market.

¹⁰For a formal argument of constraint (2.3), see Hendel and Lizzeri (2003).

¹¹The non-negativity constraint of the period 2 face value $F_2(p_2)$ never binds by the Inada condition of $v(\cdot)$.

The first order conditions for problem (2.1) with respect to Q_1 , F_1 , $Q_2(p_2)$ $F_2(p_2)$ yield:

$$u'(y - g - Q_1) = \mu, \quad (2.5a)$$

$$v'(F_1) = \mu, \quad (2.5b)$$

$$(1 - \tilde{q})u'(y + g - Q_2(p_2)) = (1 - q)\mu + \frac{\lambda(p_2) + \gamma(p_2)}{(1 - p_1)\phi(p_2)}, \quad (2.5c)$$

$$(1 - \tilde{q})v'(F_2(p_2)) = (1 - q)\mu + \frac{\lambda(p_2)}{(1 - p_1)\phi(p_2)}, \quad (2.5d)$$

where μ , $\lambda(p_2)$ and $\gamma(p_2)$ are the Lagrange multipliers for constraint (2.2), (2.3) and (2.4), with $\mu > 0$, $\lambda(p_2) \leq 0$ and $\gamma(p_2) \geq 0$ satisfying complementary slackness conditions:

$$\lambda(p_2)[Q_2(p_2) - p_2 F_2(p_2)] = 0, \quad (2.6a)$$

$$\gamma(p_2)Q_2(p_2) = 0. \quad (2.6b)$$

The first order conditions (2.5) imply that:

$$u'(y - g - Q_1) = v'(F_1). \quad (2.7)$$

In equilibrium, the marginal utility of policyholder's consumption is equal to the marginal utility of his dependent's consumption in period 1. This is referred to as the *full-event insurance* in Fang and Kung (2010).

To characterize the equilibrium contracts, we follow Fang and Kung (2010) and divide the support of the second period health states p_2 into two subsets \mathcal{B} and

\mathcal{NB} : for $p_2 \in \mathcal{B}$, the no-lapsation constraint (2.3) binds; for $p_2 \in \mathcal{NB}$, the no-lapsation constraint (2.3) does not bind. It would also be useful to define the *fair premium and face amount* for the full-event second period insurance contract with health state $p_2 \in [0, 1]$, denoted by $Q_2^{FI}(p_2)$ and $F_2^{FI}(p_2)$ respectively, as the solution to the following pair of equations:

$$u'(y + g - Q_2^{FI}(p_2)) = v'(F_2^{FI}(p_2)), \quad (2.8a)$$

$$Q_2^{FI}(p_2) - p_2 F_2^{FI}(p_2) = 0. \quad (2.8b)$$

This is indeed the equilibrium spot contract with health state p_2 in period 2.¹²

Lemma 7 (Fang and Kung 2010) *If $p_2 \in \mathcal{B}$ and $p'_2 \in \mathcal{NB}$, then $p_2 < p'_2$, $Q_2(p_2) \leq Q_2(p'_2)$ and $F_2(p_2) \geq F_2(p'_2)$.*

The proof is similar to that of Lemma 1 in Fang and Kung (2010). Lemma 7 indicates that there exists a threshold p_2^* such that $p_2 \in \mathcal{B}$ if $p_2 < p_2^*$ and $p_2 \in \mathcal{NB}$ if $p_2 > p_2^*$.

Lemma 8 *If there exists one health state $p_2^i \neq 0$ such that $Q_2(p_2^i) = 0$, then $Q_2(p_2) = 0$ for all $p_2 \in [0, 1]$.*

Lemma 8 greatly narrows down the set of period 2 equilibrium premiums to two possibilities: either $Q_2(p_2) > 0$ or $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$. If $Q_2(p_2^i) > 0$ for

¹²The second period spot contract $\langle Q_2(p_2), F_2(p_2) \rangle$ solves $\max u(y + g - Q_2(p_2)) + p_2 v(F_2(p_2))$ subject to $Q_2(p_2) - p_2 F_2(p_2) = 0$, which leads to the same conditions as in (2.8).

some p_2^i , $\gamma(p_2) = 0$ for all $p_2 \in (0, 1]$ and policyholders obtain full-event insurance in period 2 for all health states p_2 :

$$u'(y + g - Q_2(p_2)) = v'(F_2(p_2)) \text{ for all } p_2 \in (0, 1]. \quad (2.9)$$

If $Q_2(p_2^i) = 0$ for some $p_2^i \neq 0$, $\gamma(p_2) = 0$ for all $p_2 \in (0, 1]$; together with the first order conditions (2.5b) and (2.5d), we must have:

$$(1 - \tilde{q})v'(F_2(p_2)) = (1 - q)v'(F_1) \text{ for all } p_2 \in (0, 1]. \quad (2.10)$$

If $p_2^* = 1$, the no-lapsation condition (2.3) binds for all period-2 health states, i.e., $Q_2(p_2) - p_2 F_2(p_2) = 0$ for all $p_2 \in (0, 1]$. Meanwhile, the Inada condition of $v(\cdot)$ implies that $F_2(p_2) > 0$ for all p_2 . Therefore $Q_2(p_2) = p_2 F_2(p_2) > 0$ for all $p_2 \in (0, 1]$, which in turn implies that conditions (2.9) hold. Thus, the equilibrium contract in period 2 is the set of spot contracts.

If $p_2^* = 0$, we must have:

$$Q_2(p_2) = 0 \text{ for all } p_2 \in (0, 1]. \quad (2.11)$$

To see this, suppose to the contrary that $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$. The first order conditions (2.5) imply that $u'(y + g - Q_2(p_2)) = v'(F_2(p_2)) = \frac{1-q}{1-\tilde{q}}u'(y - g - Q_1)$ for all $p_2 \in (0, 1]$. Thus, $F_2(p_2)$ and $Q_2(p_2)$ remain constant for all $p_2 \in (0, 1]$. When p_2 is sufficiently small, $Q_2(p_2) - p_2 F_2(p_2)$ is strictly positive, contradicting (2.3). Therefore the set of equilibrium contracts is fully characterized by (2.2), (2.7), (2.10) and (2.11).

If $0 < p_2^* < 1$, the no-lapsation constraint (2.3) binds and $\gamma(p_2) = 0$ ¹³ for $p_2 < p_2^*$. Thus, $u'(y + g - Q_2(p_2)) = v'(F_2(p_2))$ and $\langle Q_2(p_2), F_2(p_2) \rangle = \langle Q_2^{FI}(p_2), F_2^{FI}(p_2) \rangle$. For $p_2 > p_2^*$, $p_2 \in \mathcal{NB}$ and $\lambda(p_2) = 0$, which implies that $F_2(p_2)$ must remain constant by the first order condition (2.5d). Since $Q_2(p_2) > 0$ for $p < p_2^*$, $\gamma(p_2) = 0$ for all $p_2 \in (0, 1]$. This implies that $u'(y + g - Q_2(p_2)) = v'(F_2(p_2))$ for all $p_2 \in (0, 1]$. Thus, the second period equilibrium premium $Q_2(p_2)$ for $p_2 > p_2^*$ is *front-loaded* (i.e., $Q_2(p_2) < Q_2^{FI}(p_2)$) and remains constant. In equilibrium, the insurance firms charge the policyholders a low premium for health state $p_2 > p_2^*$ relative to the fair premium and insure the policyholders against *reclassification risk* via “level premiums”. The next lemma characterizes the equilibrium contract at $p_2 = p_2^*$.

Lemma 9 *Suppose $p_2^* \in (0, 1)$. The equilibrium contract at $p_2 = p_2^*$ solves:*

$$Q_2(p_2^*) = Q_2^{FI}(p_2^*), \quad (2.12)$$

$$(1 - \tilde{q})u'(y + g - Q_2^{FI}(p_2^*)) = (1 - q)u'(y - g - Q_1). \quad (2.13)$$

The proof of Lemma 9 replicates that of Lemma 2 in Fang and Kung (2010) and is omitted. When $p_2^* \in (0, 1)$, the equilibrium second period premiums $Q_2(p_2)$ must satisfy

$$Q_2(p_2) = \begin{cases} Q_2^{FI}(p_2) & \text{if } p_2 \leq p_2^* \\ Q_2^{FI}(p_2^*) & \text{if } p_2 > p_2^* \end{cases}, \quad (2.14)$$

and the set of equilibrium contracts is fully characterized by (2.2), (2.7), (2.9) and

¹³Because $F_2(p_2) > 0$ by the Inada condition of $v(\cdot)$, it follows immediately that $Q_2(p_2) = p_2 F_2(p_2) > 0$ for $p_2 \in (0, p_2^*)$.

(2.14).

Lemma 10 (Period-2 equilibrium premiums) *There exists a threshold $\bar{q} \in (0, 1)$ such that:*

1. *if $q < \bar{q}$, $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$ and $\Delta \in [0, 1]$.*

2. *if $q \geq \bar{q}$, there exists a threshold $\bar{\Delta}(q)$ such that*

(a) *if $\Delta < \bar{\Delta}(q)$, $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$;*

(b) *if $\Delta > \bar{\Delta}(q)$, $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$.*

When $\Delta = 0$, the non-negativity constraint of the second period premium $Q_2(p_2)$ never binds (i.e. $\gamma(p_2) = 0$).¹⁴ Hence policyholders always obtain full-event insurance in period 2 when they have correct belief about the probability of losing bequest motives. This result does not always hold when policyholders exhibit behavioral bias. In particular, for sufficiently high probability of losing bequest motive and overconfidence level, policyholders no longer obtain full-event insurance in period 2. Insurance firms instead offer contracts with zero premiums for all health states in period 2 and fully insure against reclassification risk. The intuition is as follows. Firstly, given that policyholders are rational (i.e. $\Delta = 0$), increasing q lowers equilibrium premiums in period 2 for all health states p_2 . The variable $1 - q$ can be interpreted as the cost for life insurance firms to provide long-term insurance contracts. When q increases, the insurance firms face a lower probability of paying the death benefits, which leads to a net profit if the set of equilibrium contract remains the same. From

¹⁴See Lemma B1 for the proof.

the perfect competition assumption, these profits are passed onto the policyholders in terms of [a higher degree of reclassification risk](#) in period 2: more health states are offered actuarially favorable premiums (i.e. a higher p_2^*). Secondly, fixing q , the second period premiums decrease for all health states when policyholders become more overconfident. This is because under such scenario the policyholders care more about their dependents' utility in period 2 as well as the expected utility generated by the period 2 contracts. As a result, the insurance firms respond by raising the first period premium and lowering the second period premiums. When these two effects are strong enough (i.e. for sufficiently high q and Δ), contracts with zero period-2 premiums can emerge in equilibrium.

The above discussions are summarized below:

Proposition 8 (Equilibrium contracts) *The set of equilibrium contracts satisfies the following properties:*

1. *All policyholders receive full-event insurance in period 1.*
2. *There is a period-2 threshold health state $p_2^* \in [0, 1]$ such that $p_2 \in \mathcal{B}$ if $p_2 < p_2^*$ and $p_2 \in \mathcal{NB}$ if $p_2 > p_2^*$.*
3. (a) *If $p_2^* = 0$, all policyholders lose full-event insurance in period 2. The set of equilibrium contracts solves (2.2), (2.7), (2.10) and (2.11).*
(b) *If $0 < p_2^* < 1$, all policyholders receive full-event insurance in period 2. The set of equilibrium contracts solves (2.2), (2.7), (2.9) and (2.14).*
(c) *If $p_2^* = 1$, the equilibrium contract is the set of spot contracts.*

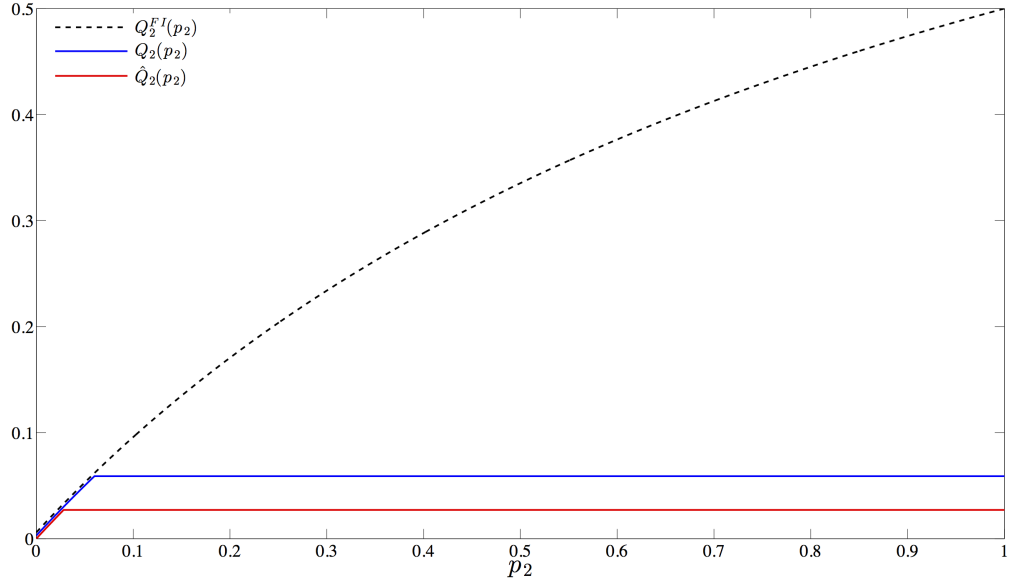


Figure 2.1: Equilibrium Period-2 Premium Profiles without the Settlement Market.

4. When q and Δ are sufficiently large, $p_2^* = 0$. Policyholders are fully insured against reclassification risk, receiving zero premiums in period 2 for all health states.

2.2.3 Effect of the policyholder's overconfidence

Proposition 9 (Comparative statics of equilibrium contracts) *Suppose $\hat{\Delta} < \Delta$. Let $\langle \hat{F}_1, \hat{Q}_1 \rangle$ and $\langle F_1, Q_1 \rangle$ be the equilibrium contract for the first period with $\hat{\Delta}$ and Δ respectively. If $0 < \hat{p}_2^* < 1$, then $\hat{F}_1 > F_1$, $\hat{Q}_1 < Q_1$ and $\hat{p}_2^* > p_2^*$.*

When the level of policyholders' overconfidence increases from $\hat{\Delta}$ to Δ , the first period premium will be higher and a higher degree of reclassification risk is offered in the second period. Figure 2.1 illustrates the equilibrium period 2 premiums under $\hat{\Delta}$ and Δ . The intuition is as follows. As policyholders become more overconfident,

they place more weight on the expected utility from the set of period 2 contracts and prefer a more actuarially favorable period 2 contract terms. To maximize the perceived expected utility of the policyholders, the life insurance firms respond by lowering the period 2 premiums and covering a higher degree of reclassification risk in the second period. Fixing q , this implies the insurance firms will suffer a greater loss in period 2. This loss is compensated by a more front-loading contract in the first period in equilibrium. This argument is reminiscent of Gottlieb and Smetters (2014): overconfidence leads to (more) front-loading life insurance contract in equilibrium.

Once the equilibrium contract $\langle (Q_1, F_1), (Q_2(p_2), F_2(p_2)) : p_2 \in [0, 1] \rangle$ is pinned down, consumer welfare can be calculated accordingly. Denote the consumer welfare with actual probability of losing bequest motive q and overconfidence level Δ by $W(q, \Delta)$. Then

$$W(q, \Delta) := [u(y - g - Q_1) + p_1 v(F_1)] \\ + (1 - p_1) \int_0^1 \{(1 - q)[u(y + g - Q_2(p_2))] + p_2 v(F_2(p_2))\} + qu(y + g) d\Phi(p_2).$$

Note that q instead of \tilde{q} enters into the calculation of consumer welfare.

Proposition 10 (Welfare implication of overconfidence) *Fixing $q \in (0, 1)$, $W(q, \Delta)$ is weakly decreasing in Δ .*

Consumer welfare is weakly reduced if consumers become more overconfident. When consumers have unbiased belief, the socially optimal contract and the set of equilibrium contracts proposed by the life insurance firms coincide. Therefore consumer

welfare is maximized. However, when consumers have biased belief about the probability of losing bequest motive, the set of equilibrium contracts departs from the socially optimal contract. With biased belief, policyholders obtain a higher expected utility in period 2 at the cost of obtaining a lower period 1 expected utility. A higher behavioral bias leads to more deviation from the socially optimal contract and the welfare loss in the first period exceeds the welfare gain in the second period. Thus, consumer welfare is decreasing in the level of policyholders' overconfidence.

2.3 Introducing the Life Settlement Market

In this section, we introduce the life settlement market at the beginning of period 2 to the baseline model. After the policyholders learn the period 2 health status p_2 and the realization of their bequest motives, they can sell the contracts to the settlement firms before the death uncertainty is realized. Instead of lapsing the contract, policyholders that lose their bequest motives in period 2 have a better option: they can now sell their contracts to the settlement market and receive a fraction $\beta \in (0, 1)$ of the *actuarial value* of the contract. The actuarial value of a life insurance contract is the expected cash benefit that is paid by the contract, as opposed to being paid out of pocket by the policyholders. The life settlement firms continue to pay the second period premiums for policyholders. In return, the life settlement firms become the beneficiaries of such policies and collect all the death benefits from life insurance firms once policyholders die by the end of period 2.

2.3.1 Equilibrium contracts with the settlement market

The presence of the settlement market shapes equilibrium life insurance contract in a different way. Without the settlement market, life insurance firms can avoid paying death benefits if policyholders lose their bequest motives. This is because lapsing the contract is the optimal choice for policyholders if they lose their bequest motives in the absence of settlement market. However, with the settlement market, they can cash out a fraction of the actuarial value of the contract by selling the contract to the settlement firms. Thus, the life insurance firms have to pay the death benefits no matter policyholders lose bequest motives or not. As will be clear later, this difference fundamentally varies the way insurance firms provide the long-term insurance contracts in equilibrium.

The life insurance firms solve the following maximization problem to provide the equilibrium contracts:

$$\max[u(y - g - Q_{1s}) + p_1 v(F_{1s})] \quad (2.15)$$

$$+ (1 - p_1) \int_0^1 \left\{ (1 - \tilde{q}) \left[\begin{array}{c} u(y + g - Q_{2s}(p_2)) \\ + p_2 v(F_{2s}(p_2)) \end{array} \right] + \tilde{q} u(y + g + \beta V_{2s}(p_2)) \right\} d\Phi(p_2)$$

$$\text{s.t. } (Q_{1s} - p_1 F_{1s}) + (1 - p_1) \int_0^1 [Q_{2s}(p_2) - p_2 F_{2s}(p_2)] d\Phi(p_2) = 0, \quad (2.16)$$

$$Q_{2s}(p_2) - p_2 F_{2s}(p_2) \leq 0 \quad \text{for all } p_2 \in [0, 1], \quad (2.17)$$

$$Q_{2s}(p_2) \geq 0 \quad \text{for all } p_2 \in [0, 1], \quad (2.18)$$

where $V_{2s}(p_2) \equiv p_2 F_{2s}(p_2) - Q_{2s}(p_2)$ is the *actuarial value* of the period 2 contract

with health status p_2 . From the no-lapsation condition (2.17), $V_{2s}(p_2)$ is always non-negative. The first order conditions with respect to Q_{1s} , F_{1s} , $Q_{2s}(p_2)$ $F_{2s}(p_2)$ are:

$$u'(y - g - Q_{1s}) = \mu, \quad (2.19a)$$

$$v'(F_{1s}) = \mu, \quad (2.19b)$$

$$(1 - \tilde{q})u'(y + g - Q_{2s}(p_2)) + \beta\tilde{q}u'(y + g + \beta V_{2s}(p_2)) = \mu + \frac{\lambda(p_2) + \gamma(p_2)}{(1 - p_1)\phi(p_2)}, \quad (2.19c)$$

$$(1 - \tilde{q})v'(F_{2s}(p_2)) + \beta\tilde{q}v'(y + g + \beta V_{2s}(p_2)) = \mu + \frac{\lambda(p_2)}{(1 - p_1)\phi(p_2)}. \quad (2.19d)$$

Note that q enters into neither the zero-profit condition (2.16) nor the first order conditions (2.19). Hence, fixing \tilde{q} , the set of equilibrium contracts is *independent* of q . This is because life insurance firms have to pay the face amount when policyholders die in period 2 no matter they lose bequest motives or not. Therefore the life insurance firms does not take into account the actual probability of losing bequest motive when they maximize policyholders' perceived utility.

Lemma 11 *Fixing $(q, \Delta) \in [0, 1) \times [0, 1]$, $Q_{2s}(p_2) > 0$ for all $p_2 \in (0, 1]$.*

Lemma 11 states that the non-negativity condition of $Q_{2s}(p_2)$ never binds. This result provides a stark contrast to Lemma 10. When the life settlement market is not present, contracts with zero premiums in the second period can be sustained in equilibrium when q and Δ are sufficiently large because insurance firms expect a large fraction of policyholders at the beginning of period 2 (i.e. $q\Delta$) to lapse the contract due to their misperception and the death benefits will not be paid. However, in the

presence of the settlement market, providing a set of contracts with zero premiums in period 2 is too costly for the insurance firms because the settlement firms collect all the death benefits from policyholders that lapse their contracts.

Lemma 11 implies immediately that $\gamma(p_2) = 0$ for all $p_2 \in (0, 1]$; together with the first order conditions (2.19), we have:

$$u'(y - g - Q_{1s}) = v'(F_{1s}) \quad (2.20)$$

$$u'(y + g - Q_{2s}(p_2)) = v'(F_{2s}(p_2)) \text{ for all } p_2 \in (0, 1]. \quad (2.21)$$

Thus, in the presence of the life settlement market, policyholders always obtain full-event insurance in both period 1 and all health states in period 2.

The characterization of the equilibrium contracts replicates that in Fang and Kung (2010) by replacing the variable q with \tilde{q} . Similar to the case without the settlement market, all the period 2 health states can be divided into two subsets \mathcal{B}_s and $\mathcal{N}\mathcal{B}_s$ depending on whether the no-lapsation constraint binds.

Lemma 12 (Fang and Kung 2010) *If $p_2 \in \mathcal{B}_s$ and $p'_2 \in \mathcal{N}\mathcal{B}_s$, then $p_2 < p'_2$ and $Q_2(p_2) < Q_2(p'_2)$.*

The proof is the same as in Fang and Kung (2010) and hence omitted. Lemma 12 implies that there is a threshold p_{2s}^* such that $p_2 \in \mathcal{B}_s$ if $p_2 < p_{2s}^*$ and $p_2 \in \mathcal{N}\mathcal{B}_s$ if $p_2 > p_{2s}^*$.

Lemma 13 *For all $(q, \Delta) \in [0, 1) \times [0, 1]$, $p_{2s}^* \geq p_1$.*

Lemma 13 rules out the possibility that $p_{2s}^* = 0$ ($p_{2s}^* \geq p_1 > 0$). If $p_{2s}^* = 1$, the set of equilibrium contracts in period 2 is the set of spot contracts. The following lemma characterizes the period-2 premiums $Q_{2s}(p_2)$ as a function of p_{2s}^* if $p_{2s}^* < 1$.

Lemma 14 *If $p_{2s}^* \in (0, 1)$, the equilibrium period 2 premiums $Q_{2s}(p_2)$ satisfy:*

1. *for $p_2 \leq p_{2s}^*$, $Q_{2s}(p_2) = Q_2^{FI}(p_2)$;*

2. *for $p_2 > p_{2s}^*$, $Q_{2s}(p_2)$ solves:*

$$\begin{aligned} & (1 - \tilde{q})u'(y + g - Q_{2s}(p_2)) + \beta\tilde{q}u'(y + g + \beta V_{2s}(p_2)) \\ &= (1 - \tilde{q})u'(y + g - Q_2^{FI}(p_{2s}^*)) + \beta\tilde{q}u'(y + g). \end{aligned} \quad (2.22)$$

By Lemma 14, the set of period-2 contracts is fully characterized by p_{2s}^* alone. Moreover, it can be proved from (2.22) that both $Q_{2s}(p_2)$ and $V_{2s}(p_2)$ are strictly increasing in p_2 .¹⁵ From the first order conditions (2.19a), (2.19c) and Lemma 14, the period 1 premium Q_{1s} is the solution to:

$$u'(y - g - Q_{1s}) = (1 - \tilde{q})u'(y + g - Q_2^{FI}(p_{2s}^*)) + \beta\tilde{q}u'(y + g). \quad (2.23)$$

To characterize the equilibrium insurance contract, it remains to pin down p_{2s}^* , which is determined by the zero-profit condition (2.16). The following proposition summarizes the above discussions.

Proposition 11 (Equilibrium contracts with the settlement market) *The set of equilibrium contracts satisfies the following properties:*

¹⁵See the proof of Proposition 3 in Fang and Kung (2010).

1. All policyholders receive full-event insurance in period 1 and 2 as defined by (2.20) and (2.21).
2. There exists a threshold $p_{2s}^* \in [p_1, 1]$ such that $p_2 \in \mathcal{B}_s$ if $p_2 < p_{2s}^*$ and $p_2 \in \mathcal{NB}_s$ if $p_2 > p_{2s}^*$.
3. (a) If $p_{2s} < 1$, the set of equilibrium contracts is determined by (2.16), (2.20), (2.21) and Lemma 14. Moreover, $Q_{2s}(p_2)$ and $V_{2s}(p_2)$ are strictly increasing in p_2 .
(b) If $p_{2s} = 1$, the equilibrium contract is the set of spot contracts.

In the presence of the settlement market, the life insurance firms no longer provide flat premiums in period 2. Instead, they provide partial insurance against reclassification risk in equilibrium. The set of period-2 equilibrium contract is in the form of *premium discounts* relative to the spot market contracts. Policyholders with higher mortality risk are charged higher premiums. The equilibrium contract is still in favor of higher p_2 in the sense that policyholders with higher p_2 are offered contracts with higher actuarial values.

2.3.2 Effect of overconfidence with settlement market

Proposition 12 Suppose $\hat{\Delta} < \Delta$. Let $(\hat{F}_{1s}, \hat{Q}_{1s})$ and (F_{1s}, Q_{1s}) be the equilibrium contract in period 1 with $\hat{\Delta}$ and Δ respectively. Then $\hat{F}_{1s} > F_{1s}$ and $\hat{Q}_{1s} < Q_{1s}$.

When consumers become more overconfident, the life insurance firms respond by offering a set of contracts with a higher degree of front-loading (i.e. a higher premium

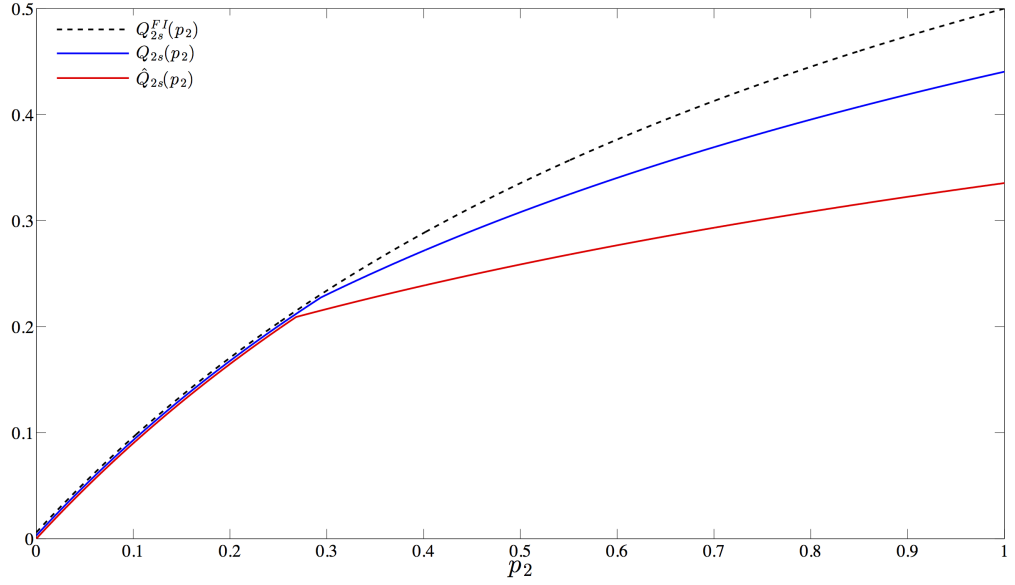


Figure 2.2: Equilibrium Period-2 Premium Profiles with the Settlement Market: Case I.

and lower face value) in the first period. The intuition is similar to that of Proposition 9. When policyholders become more overconfident, they demand actuarially more favorable contract terms in period 2 in expectation. To keep budget balanced, the first period premium increases as a result.

Different from Proposition 9, we can not obtain clean comparative statics on the threshold p_{2s}^* with respect to Δ , i.e., increasing Δ can not guarantee a higher degree of reclassification risk (i.e. lower p_{2s}^*). Figure 2.2 and 2.3 depict the change of equilibrium premiums in the second period as the level of policyholders' overconfidence increases from $\hat{\Delta}$ to Δ from simulations. Increasing Δ can lead to a lower/higher threshold p_{2s}^* as shown in Figure 2.2/Figure 2.3.

When there is no settlement market, the set of optimal period 2 contracts is offered in the form of flat premiums, which depend only on p_2^* . Therefore the only

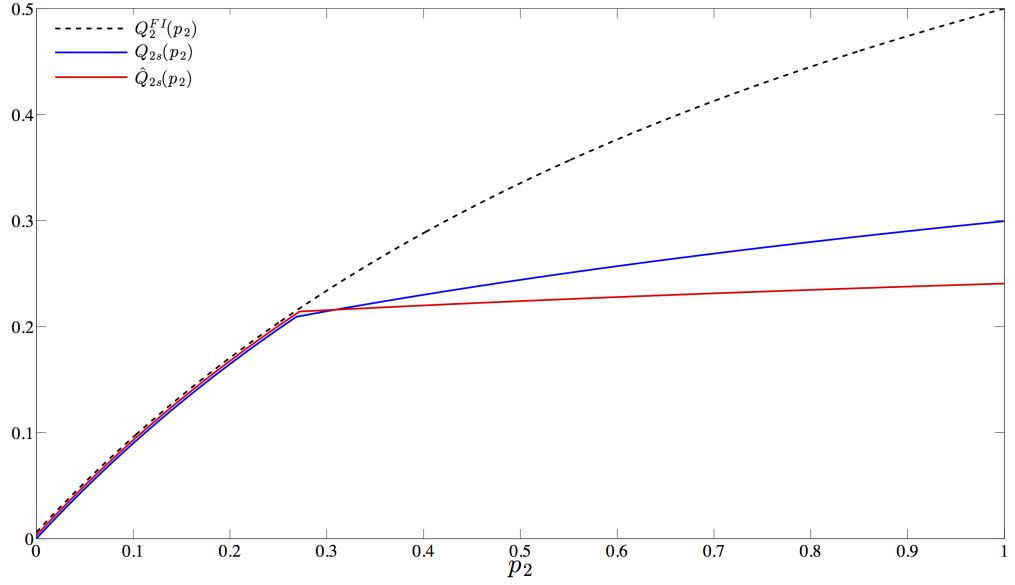


Figure 2.3: Equilibrium Period-2 Premium Profiles with the Settlement Market: Case II.

way to offer a better set of period 2 contracts is to decrease p_2^* . When the secondary life settlement market is introduced, insurance firms provide contracts with premium discounts rather than flat premiums, whose shape depends on not only p_{2s}^* but also $u(\cdot)$ and $v(\cdot)$ (see equation (2.22)). Unlike the situation where the settlement market is not present, without further assumptions on the utility functions, it is possible for the insurance firms to offer period 2 contracts without increasing the degree of reclassification risk as Δ increases. Specifically, as shown by Figure 2.3, insurance firms can provide a set of period 2 contracts of a higher expected actuarial value and a higher p_{2s}^* by lowering the premiums for some health states and increasing the premiums for the other.

Given the set of equilibrium contracts $\langle (Q_{1s}, F_{1s}), (Q_{2s}(p_2), F_{2s}(p_2)) : p_2 \in [0, 1] \rangle$,

the consumer welfare, denoted by $W_s(q, \Delta)$, can be derived as:

$$W_s(q, \Delta) := [u(y - g - Q_{1s}) + p_1 v(F_{1s})] \\ + (1 - p_1) \int_0^1 \left\{ (1 - q) \left[\begin{array}{c} u(y + g - Q_{2s}(p_2)) \\ + p_2 v(F_{2s}(p_2)) \end{array} \right] + qu(y + g + \beta V_{2s}(p_2)) \right\} d\Phi(p_2).$$

Again, q rather than \tilde{q} is used in the calculation of consumer welfare.

Proposition 13 (Welfare implication of overconfidence) *Fixing $q \in (0, 1)$, $W_s(q, \Delta)$ is weakly decreasing in Δ .*

Proposition 13 establishes a similar comparative statics of consumer welfare with respect to the level of policyholders' overconfidence. The intuition is similar to Proposition 10. In general, overconfidence reduces consumer welfare regardless of the presence of the settlement market.

2.4 Welfare Comparison

Lemma 15 (Fang and Kung 2010) $W(q, \Delta) \geq W_s(q, \Delta)$ if $\Delta = 0$.

When the policyholder has correct belief about the probability of losing his bequest motive, introducing the life settlement market weakly reduces consumer welfare in equilibrium. The proof replicates Proposition 7 in Fang and Kung (2010) and is omitted. Because policyholders lack commitment power of not lapsing the long-term contract in period 2, we deviate from complete markets and are in a second-best

world. As argued in Fang and Kung (2010), although the settlement market allows policyholders to access the actuarial value of their contracts and thus contribute to market completeness, it also contributes market incompleteness by weakening policyholders' ability to commit to not asking for a return when they lose their bequest motives. Therefore introducing the settlement market can lead to a decrease in consumer welfare. The next proposition provides sufficient conditions under which the welfare result in Lemma 15 can be overturned.

For ease of our exposition, let $\eta(c) = -\frac{v'(c)}{cv''(c)}$ denote the intertemporal elasticity of substitution (IES) of $v(\cdot)$ at c by $\eta(c)$.

Proposition 14 *Suppose $\eta(c) \geq \alpha > 1$ for all $c > 0$. There exists a threshold \bar{q} such that for $q \geq \bar{q}$, $W_s(q, \Delta) > W(q, \Delta)$ if Δ is sufficiently large.*

The intuition can be better explained by assuming utility $v(\cdot)$ with constant IES ρ , i.e., $v(c) = \frac{c^{\frac{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}}}$. From Lemma 10, when q and Δ are sufficiently large, $Q_2(p_2) = 0$ for all p_2 ; together with first order conditions (2.5b) and (2.5d), we must have $\frac{v'(F_2(p_2))}{v'(F_1)} = \frac{1-q}{1-\bar{q}}$ for all p_2 . This in turn implies that:

$$\frac{F_2(p_2)}{F_1} = \left(\frac{1-\bar{q}}{1-q} \right)^\rho = \left(1 + \frac{q}{1-q} \Delta \right)^\rho. \quad (2.24)$$

From (2.24) we know that the IES measures policyholders' propensity to smooth consumption for their dependents. Specifically, when $\rho > 1$, the equilibrium consumption growth is *sensitive* to changes in the level of policyholders' overconfidence Δ . This indicates that policyholders' consumption smoothing motive is weak relative to that under $\rho < 1$. In the absence of the settlement market, policyholders

obtain a contract with a very low face value and a high premium in the first period in exchange for the set of period 2 contracts of high actuarial values as they become sufficiently overconfident. This indeed harms policyholders especially when q and Δ are sufficiently large. In equilibrium, a large portion (i.e. $q\Delta$) of the expected utility promised by the set of the equilibrium contracts in the second period is not realized due to policyholders' misperception of the probability of losing bequest motives. To summarize, policyholders with a high value of IES are *more vulnerable* from their overconfidence and can potentially benefit more from the presence of the settlement market than those with a low value of IES.

In the presence of the settlement market, the set of equilibrium contracts do not deviate too much from the socially optimal contracts in terms of the degree of front-loading as policyholders become more overconfident. In fact, we can establish a lower bound of the expected utility for policyholders in the first period. To see this, note that $p_{2s}^* \geq p_1$ from Lemma 13. Therefore contracts with zero premiums in period 2 can not be sustained in equilibrium and the degree of reclassification risk insurance is limited by the threat of life settlement market. From the zero-profit condition (2.16), this in turn implies that there is an upper bound of the amount of front-loading. Thus the presence of the settlement market protects policyholders from obtaining contracts with too much front-loading in the first period as they become more overconfident. Such protection is more valuable to the vulnerable policyholders with high IES than those with a low value of IES and leads the consumer welfare with settlement market to be greater than that without.

Another way to understand the welfare result is the following. As argued by Fang

and Kung (2010), when policyholders are fully rational, introducing life settlement market further contributes to market incompleteness and reduces consumer welfare. When policyholders are overconfident, the settlement market has a new role: it helps policyholders correct their biased beliefs in the second period. In particular, a fraction $q\Delta$ of policyholders no long remain bequest motives in the second period as expected at the time of purchasing insurance policies. When there is no settlement market, they can only lapse the contract and suffer the utility loss caused by their misperception. However, with the settlement market, policyholders can cash out part of the actuarial value of their contracts and undercut the utility loss. This generates a potential welfare gain. If the magnitude of this welfare gain is large enough to offset the welfare loss due to the lower degree of market completeness, introducing a secondary settlement market can potentially benefit consumers in equilibrium.

2.5 Conclusion and Future Research

In this paper, we provide sufficient conditions under which consumer surplus can be higher in the presence of the life settlement market than in its absence. Specifically, we prove that introducing life settlement market can lead to an increase in consumer welfare when policyholders sufficiently underestimate the probability of losing their bequest motives. There are several directions for future research. First, it would be interesting to empirically test the existence of policyholders' overconfidence based on the predictions in this paper. It would also be interesting study the welfare effect of the settlement market when consumers are overconfident about their health status in

the second period. Another intriguing research avenue would be to analyze the welfare implications of the secondary market in a unified framework when lapsation is driven by negative income shocks in addition to policyholders' lost of bequest motives.

Appendix A

Appendix for Chapter 1

A.1 Appendix: Proofs of the propositions

Proof of Lemma 1. The first best outcome is the solution to the following maximization problem:

$$\max_{\{\hat{s}, q\}} \frac{1}{2} [1 - F_1(\hat{s})] q + \frac{1}{4} [F_1(\hat{s}) + F_0(\hat{s})] q - \frac{1}{2} q^2.$$

The first order condition with respect to \hat{s} yields:

$$f_1(\hat{s}) = f_0(\hat{s}) \Rightarrow \hat{s}^{\mathcal{FB}} = \frac{1}{2}.$$

The first order condition with respect to q yields:

$$q^{\mathcal{FB}} = \frac{1}{2} + \frac{1}{4} [F_0(\hat{s}^{\mathcal{FB}}) - F_1(\hat{s}^{\mathcal{FB}})].$$

■

Proof of Proposition 1. It is useful to first prove the two lemmas.

Lemma A1 (Uniform convergence of $F_1(\cdot)$ as $\alpha \rightarrow \infty$) *For any given $\epsilon > 0$, there exists N such that for $\alpha > N$, $F_1(s; \alpha) < \epsilon$ for $s \in [0, \frac{1}{2}]$ and $F_1(s; \alpha) < (2s - 1) + \epsilon$ for $s \in [\frac{1}{2}, 1]$.*

Proof. By the definition of the completely informative information structure, given $\epsilon' = \frac{1}{2}\epsilon$ and $\Delta \in (0, \frac{1}{2})$, there exists N such that $f_1(\Delta; \alpha) < \epsilon'$ for $\alpha > N$. Thus,

$$F_1\left(\frac{1}{2}; \alpha\right) = \int_0^{\frac{1}{2}} f_1(t; \alpha) dt = \int_0^{\Delta} f_1(t; \alpha) dt + \int_{\Delta}^{\frac{1}{2}} f_1(t; \alpha) dt \leq \Delta\epsilon' + \left(\frac{1}{2} - \Delta\right).$$

Let $\Delta = \frac{1}{2} - \epsilon'$. $F_1\left(\frac{1}{2}; \alpha\right)$ can be bounded above by

$$F_1(s; \alpha) \leq F_1\left(\frac{1}{2}; \alpha\right) \leq \epsilon' \left(\frac{1}{2} - \epsilon'\right) + \epsilon' < 2\epsilon' = \epsilon \text{ for } s \in [0, \frac{1}{2}].$$

Similarly, for all $s \in [\frac{1}{2}, 1]$,

$$F_1(s; \alpha) = 2s - F_0(s; \alpha) = (2s - 1) + F_1(1 - s; \alpha) < (2s - 1) + \epsilon \text{ for } \alpha > N.$$

■

Lemma A2 (Uniform convergence of $F_1(\cdot)$ as $\alpha \rightarrow 0$) *For any given $\epsilon > 0$, there exists N such that for $\alpha < N$, $F_1(s; \alpha) > s - \epsilon$ for all $s \in [0, 1]$.*

Proof. By the definition of the completely uninformative information structure, for

any given ϵ and $\Delta \in (0, \frac{1}{2})$, there exists N such that $f_1(\delta; \alpha) > 1 - \epsilon$. Thus,

$$\begin{aligned} s - F_1(s; \alpha) &= \int_0^s [1 - f_1(t; \alpha)] dt \leq \int_0^{\frac{1}{2}} [1 - f_1(t; \alpha)] dt \\ &= \int_0^{\Delta} [1 - f_1(t; \alpha)] dt + \int_{\Delta}^{\frac{1}{2}} [1 - f_1(t; \alpha)] dt \\ &\leq \Delta + \epsilon(\frac{1}{2} - \Delta) \text{ for } s \in [0, \frac{1}{2}]. \end{aligned}$$

Let $\Delta = \frac{1}{2}\epsilon$. $s - F_1(s; \alpha)$ can be bounded above by

$$s - F_1(s; \alpha) \leq \Delta + \epsilon(\frac{1}{2} - \Delta) < \frac{1}{2}\epsilon + \frac{1}{2}\epsilon = \epsilon \text{ for } s \in [0, \frac{1}{2}].$$

Similarly, for all $s \in [\frac{1}{2}, 1]$,

$$s - F_1(s; \alpha) = s - [2s - F_0(s; \alpha)] = (1 - s) - F_1(1 - s; \alpha) < \epsilon.$$

■

Recall that the expected profit function is

$$\pi(\hat{s}) = \frac{1}{8} [1 - F_1(\hat{s})] \left\{ \frac{1}{2} [1 - F_1(\hat{s})] + \frac{1}{2} [F_1(\hat{s}) + F_0(\hat{s})] \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \right\}.$$

By Assumption 3, $f_0(s) = f_1(1 - s)$ and $F_0(s) = 1 - F_1(1 - s)$. The expected profit function can be written as

$$\pi(\hat{s}) = \frac{1}{16} [1 - F_1(\hat{s})] \left[[1 - F_1(\hat{s})] + \hat{s} f_1(\hat{s}) \right].$$

1. Anti-entrenchment:

$$\pi(\hat{s}; \alpha) < \frac{1}{16}(1 + \hat{s}) < \frac{3}{32} \text{ for all } \hat{s} \in [0, \frac{1}{2}].$$

By Lemma A1, for any ϵ , there exists N such that for $\alpha > N$, $1 - F_1(\hat{s}; \alpha) > 2 - 2\hat{s} - \epsilon$ for $\hat{s} \in (\frac{1}{2}, 1)$. Moreover, given $\hat{s} \in (\frac{1}{2}, 1)$ and ϵ' , there exists N' such that $\frac{1}{2}f_1(\hat{s}) = \frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})} > 1 - \epsilon'$ for $\alpha > N'$.

Let $\bar{\alpha} = \max\{N, N'\}$. Then for $\alpha > \bar{\alpha}$,

$$\pi(\hat{s}; \alpha) > \frac{1}{16}(2 - 2\hat{s} - \epsilon) [(2 - 2\hat{s} - \epsilon) + 2\hat{s}(1 - \epsilon')].$$

Let $\hat{s} = \frac{1}{2}(1 + \epsilon)$ and $\epsilon' = \frac{\epsilon}{1+\epsilon}$. Then,

$$\begin{aligned} \pi\left(\frac{1}{2}(1 + \epsilon); \alpha\right) &> \frac{1}{16}(1 - 2\epsilon) [(1 - 2\epsilon) + (1 + \epsilon)(1 - \epsilon')] \\ &= \frac{1}{8}(1 - 2\epsilon)(1 - \epsilon). \end{aligned}$$

To prove the proposition, it suffices to find ϵ such that

$$\frac{1}{8}(1 - 2\epsilon)(1 - \epsilon) \geq \frac{3}{32}.$$

This inequality holds when $\epsilon \leq \frac{3-\sqrt{7}}{4}$.

2. Entrenchment:

It suffices to prove that there exists $\underline{\alpha}$ such that for $\alpha < \underline{\alpha}$ and, $\pi(\hat{s}) < \pi(0) = \frac{1}{16}$ for all $\hat{s} \in [\frac{1}{2}, 1]$. Since $f_1(s) < 2$ for $s \in [0, 1)$ by normalization, it directly

follows that $1 - F_1(s) < 2(1 - s)$. Thus, $1 - F_1(1 - \Delta) < 2\Delta$ for $\Delta \in (0, \frac{1}{2})$.

$$\Rightarrow \pi(\hat{s}) < \frac{1}{4}\Delta(\Delta + 1) \text{ for } \hat{s} \in [1 - \Delta, 1].$$

For Δ to be sufficiently small, $\frac{1}{4}\Delta(\Delta + 1) < \frac{1}{16}$. In particular, let $\Delta = \frac{\sqrt{2}-1}{2}$.

Then $\hat{s} \in [1 - \Delta, 1]$ cannot be optimal replacement policy.

It remains to prove that there exists $\underline{\alpha}$ such that for $\alpha < \underline{\alpha}$, $\pi(\hat{s}) < \pi(0)$ for all $\hat{s} \in [\frac{1}{2}, 1 - \Delta]$. By the definition of the completely uninformative information structure, for any ϵ' , there exists N' such that $\frac{1}{2}f_1(\hat{s}) = \frac{f_1(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})} < \frac{1}{2} + \epsilon'$ for $\hat{s} \in [\frac{1}{2}, 1 - \Delta]$ and $\alpha < N'$.

By Lemma A2, for any ϵ , there exists N such that for $\alpha < N$, $F_1(\hat{s}; \alpha) > \hat{s} - \epsilon$ for $\hat{s} \in [0, 1]$. Thus,

$$\pi(\hat{s}) < \frac{1}{16}(1 - \hat{s} + \epsilon)[(1 - \hat{s} + \epsilon) + (1 + 2\epsilon')].$$

Let $\epsilon' = \epsilon = \frac{\sqrt{3}}{3} - \frac{1}{2}$ and $\underline{\alpha} = \min\{N, N'\}$. Then for $\alpha < \underline{\alpha}$,

$$\pi(\hat{s}) < \frac{1}{16}(1 - \hat{s} + \epsilon)(2 - \hat{s} + 3\epsilon) \leq \frac{3}{16}\left(\frac{1}{2} + \epsilon\right)^2 = \frac{1}{16} \text{ for all } \hat{s} \in [\frac{1}{2}, 1].$$

■

Proof of Lemma 3. For existence, it suffices to construct an example. Suppose

$\{f_1(\cdot), f_0(\cdot)\}$ induces $g(\cdot)$. By the definition of the information structure,

$$\frac{1}{2}F_1(s) + \frac{1}{2}F_0(s) = s \text{ for all } s \in [0, 1] \Leftrightarrow \frac{1}{2}f_1(s) + \frac{1}{2}f_0(s) = 1 \text{ for all } s \in [0, 1].$$

Meanwhile, we have

$$g(p) = \left[\frac{1}{2}f_1(\varphi^{-1}(p)) + \frac{1}{2}f_0(\varphi^{-1}(p)) \right] \frac{d\varphi^{-1}(p)}{dp}.$$

Thus, $g(p)dp = d\varphi^{-1}(p) \Rightarrow \varphi(G(p)) = p \Rightarrow \tilde{f}_1(x) = 2G^{-1}(x)$ and $\tilde{f}_0(x) = 2[1 - G^{-1}(x)]$. This finishes the proof of existence.

For uniqueness, suppose two information structures $\{f_1(s), f_0(s)\}$ and $\{f_1^\dagger(s), f_0^\dagger(s)\}$ induce the same $g(p)$. By the definition of the information structure,

$$\frac{1}{2}f_1^\dagger(s) + \frac{1}{2}f_0^\dagger(s) = 1 = \frac{1}{2}f_1(s) + \frac{1}{2}f_0(s).$$

By the definition of p ,

$$\frac{1}{2}f_1(s) + \frac{1}{2}f_0(s) = \frac{f_1(s)}{2p} = \frac{f_0(s)}{2(1-p)}.$$

By the derivation of $g(p)$,

$$g(p) = \left[\frac{1}{2}f_1(\varphi^{-1}(p)) + \frac{1}{2}f_0(\varphi^{-1}(p)) \right] \frac{d\varphi^{-1}(p)}{dp}.$$

$$\Rightarrow pg(p) = \frac{1}{2}f_1(\varphi^{-1}(p)) \frac{d\varphi^{-1}(p)}{dp}.$$

$$\Rightarrow \int_0^p t g(t) dt = \frac{1}{2} F_1(\varphi^{-1}(p)) = \frac{1}{2} F_1^\dagger(\varphi^{\dagger-1}(p)).$$

Similarly,

$$(1-p)g(p) = \frac{1}{2} f_0(\varphi^{-1}(p)) \frac{d\varphi^{-1}(p)}{dp}.$$

$$\Rightarrow \int_0^p (1-t)g(t) dt = \frac{1}{2} F_0(\varphi^{-1}(p)) = \frac{1}{2} F_0^\dagger(\varphi^{\dagger-1}(p)).$$

Thus,

$$\frac{1}{2} F_1^\dagger(\varphi^{\dagger-1}(p)) + \frac{1}{2} F_0^\dagger(\varphi^{\dagger-1}(p)) = \frac{1}{2} F_1(\varphi^{-1}(p)) + \frac{1}{2} F_0(\varphi^{-1}(p)).$$

$$\Rightarrow \varphi^{\dagger-1}(p) = \varphi^{-1}(p) \Rightarrow f_1^\dagger(s) = f_1(s).$$

Since $\frac{1}{2} f_1^\dagger(s) + \frac{1}{2} f_0^\dagger(s) = \frac{1}{2} f_1(s) + \frac{1}{2} f_0(s)$, it follows directly that $f_0^\dagger(s) = f_0(s)$. This finishes the proof of uniqueness. ■

Proof of Lemma 4. By definition, $\underline{\rho} \leq 1 - \frac{G(p)g'(p)}{g^2(p)} \leq \bar{\rho}$. Integrating both sides from 0 to p yields

$$\underline{\rho} p \leq \frac{G(p)}{g(p)} - \frac{G(0)}{g(0)} \leq \bar{\rho} p.$$

$$\Rightarrow \frac{1}{\bar{\rho}} \frac{1}{p} \leq \frac{g(p)}{G(p)} \leq \frac{1}{\underline{\rho}} \frac{1}{p} \Leftrightarrow \frac{1}{\bar{\rho}} \leq \frac{pg(p)}{G(p)} \leq \frac{1}{\underline{\rho}}.$$

Integrating both sides from p to $\frac{1}{2}$ yields

$$\frac{1}{2} (2p)^{\frac{1}{\underline{\rho}}} \leq G(p) \leq \frac{1}{2} (2p)^{\frac{1}{\bar{\rho}}}.$$

■

Proof of Proposition 2.

Lemma A3 *If $G(p) \leq p$ for all $p \in [0, \frac{1}{2}]$, entrenchment is optimal to the board.*

Moreover, if $G(p)$ is convex in p for $p \in [0, \frac{1}{2}]$, $\hat{p}^* = 0$.

Proof. We finish the proof in two steps:

1. $\tilde{\pi}(1 - \hat{p}) < \tilde{\pi}(0)$ for $\hat{p} \in [0, \frac{1}{2}]$.

It is equivalent to prove that

$$\int_{1-\hat{p}}^1 tg(t)dt \left(1 - \int_{1-\hat{p}}^1 G(t)dt\right) < \int_0^1 tg(t)dt \left(1 - \int_0^1 G(t)dt\right).$$

Because $G(1 - \hat{p}) = 1 - G(\hat{p})$, $\int_0^1 G(t)dt = \frac{1}{2}$. Thus, the right-hand side can be further simplified as

$$\int_0^1 tg(t)dt \left(1 - \int_0^1 G(t)dt\right) = \left(1 - \int_0^1 G(t)dt\right)^2 = \frac{1}{4}.$$

For the left-hand side,

$$\begin{aligned} & \int_{1-\hat{p}}^1 tg(t)dt \left(1 - \int_{1-\hat{p}}^1 G(t)dt\right) \\ &= \left(1 - \int_{1-\hat{p}}^1 G(t)dt - (1 - \hat{p})G(1 - \hat{p})\right) \left(1 - \int_{1-\hat{p}}^1 G(t)dt\right) \\ &< \left(1 - \int_0^{\hat{p}} (1 - G(t))dt - \frac{1}{2}(1 - \hat{p})[1 - G(\hat{p})]\right)^2 \\ &= \left(\frac{1 - \hat{p}}{2}(1 + G(\hat{p})) + \int_0^{\hat{p}} G(t)dt\right)^2 \\ &\leq \left(\frac{1 - \hat{p}}{2}(1 + \hat{p}) + \int_0^{\hat{p}} tdt\right)^2 = \frac{1}{4}. \end{aligned}$$

2. $\tilde{\pi}(\hat{p})$ is strictly decreasing in \hat{p} for $\hat{p} \in [0, \frac{1}{2}]$ if $G(p)$ is convex in p for $p \in [0, \frac{1}{2}]$.

First, notice that

$$\int_{\hat{p}}^1 tg(t)dt = \int_{\hat{p}}^1 tdG(t) = 1 - \int_{\hat{p}}^1 G(t)dt - \hat{p}G(\hat{p}) < 1 - \int_{\hat{p}}^1 G(t)dt \text{ for } \hat{p} \in (0, \frac{1}{2}].$$

Second, when $g(\hat{p})$ is increasing in \hat{p} , we have

$$G(\hat{p}) = \int_0^{\hat{p}} g(t)dt \leq \int_0^{\hat{p}} g(t)dt = \hat{p}g(\hat{p}).$$

Thus, $\tilde{\pi}'(p) < 0$ for $p \in (0, \frac{1}{2}]$.

■

It directly follows that $\hat{p}^* = 0$ for $\alpha \leq \alpha_1$ by Lemma A3. For $\alpha > \alpha_1$, it is useful to first prove the following two lemmas.

Lemma A4 *If $\rho(p; \alpha)$ is weakly decreasing in p , $\frac{G(p)}{pg(p)}$ is weakly decreasing in p for $p \in [0, \frac{1}{2}]$.*

Proof. By the definition of ρ -concavity,

$$\rho(t) = 1 - \frac{G(t)g'(t)}{g^2(t)}.$$

Integrating both sides from 0 to p yields,

$$\begin{aligned} \int_0^p \rho(t)dt &= \frac{G(p)}{g(p)} \Rightarrow \frac{G(p)}{pg(p)} = \frac{\int_0^p \rho(t)dt}{p} \\ \Rightarrow \left(\frac{\int_0^p \rho(t)dt}{p} \right)' &= \frac{\rho(p)p - \int_0^p \rho(t)dt}{p^2} = \frac{\int_0^p [\rho(p) - \rho(t)]dt}{p^2} \leq 0. \end{aligned}$$

■

Lemma A5 For $\alpha_1 > \alpha_2$, $G(p; \alpha_1) > G(p; \alpha_2)$ for $p \in (0, \frac{1}{2})$.

Proof. By Lemma A4,

$$\int_0^p \rho(t) dt = \frac{G(p)}{g(p)} \Rightarrow \ln\left(\frac{1}{2}\right) - \ln G(p; \alpha) = \int_p^{\frac{1}{2}} \frac{1}{\int_0^\omega \rho(t; \alpha) dt} d\omega.$$

It can be verified that $\int_p^{\frac{1}{2}} \frac{1}{\int_0^\omega \rho(t; \alpha) dt} d\omega$ is decreasing in α by the definition of ρ -concave order. Thus, $G(p; \alpha)$ is increasing in α . ■

Rearranging the first order condition with respect to \hat{p} yields

$$\tilde{\pi}'(\hat{p}) \geq 0 \Leftrightarrow \frac{G(\hat{p})}{\hat{p}g(\hat{p})} \geq \frac{1 - \int_{\hat{p}}^1 G(t) dt}{\int_{\hat{p}}^1 tg(t) dt} = \frac{\frac{1}{2} + \int_0^{\hat{p}} G(t) dt}{\frac{1}{2} + \int_0^{\hat{p}} G(t) dt - pG(p)}.$$

By Lemma A4, the left-hand side is decreasing in \hat{p} . It is can be verified that the right-hand side is increasing in \hat{p} . Thus, the board's profit function for $\hat{p} \in [0, \frac{1}{2}]$ is well-behaved.

Notice that $\lim_{p \rightarrow 0} \frac{G(\hat{p})}{\hat{p}g(\hat{p})} = \rho(0) > 1$ for $\alpha > \alpha_1$, and $\lim_{p \rightarrow 0} \frac{1 - \int_{\hat{p}}^1 G(t) dt}{\int_{\hat{p}}^1 tg(t) dt} = 1$. It suffices to compare the end points of the two curves.

If $2 \int_0^{\frac{1}{2}} \rho(t; \alpha) dt > \frac{\frac{1}{2} + \int_0^{\frac{1}{2}} G(t) dt}{\frac{1}{4} + \int_0^{\frac{1}{2}} G(t) dt}$, $\tilde{\pi}(\hat{p})$ is increasing in $\hat{p} \in [0, \frac{1}{2}]$ and the optimal cutoff \hat{p}^* lies between $\frac{1}{2}$ and 1.

If $2 \int_0^{\frac{1}{2}} \rho(t; \alpha) dt < \frac{\frac{1}{2} + \int_0^{\frac{1}{2}} G(t) dt}{\frac{1}{4} + \int_0^{\frac{1}{2}} G(t) dt}$, $\tilde{\pi}(\hat{p})$ is first increasing and then decreasing in $\hat{p} \in [0, \frac{1}{2}]$. The maximal can be pinned down by the first order condition for $\hat{p} \in [0, \frac{1}{2}]$.

We further argue that this local maximal is indeed the global maximal for $\hat{p} \in [0, 1]$.

To see this, notice that second order derivative of the profit function with respect to \hat{p} is

$$\tilde{\pi}''(\hat{p}) = \frac{1}{4} \left[-\hat{p}g'(\hat{p}) \left(1 - \int_{\hat{p}}^1 G(t)dt \right) - 3\hat{p}g(\hat{p})G(\hat{p}) \right].$$

Because $G(p)$ is concave for $p \in [0, \frac{1}{2}]$ for $\alpha > \alpha_1$, $G(p)$ is convex for $p \in [\frac{1}{2}, 1]$. This directly implies $g'(p) > 0$ for $p \in [\frac{1}{2}, 1]$. Thus $\pi''(\hat{p}) < 0$ for $p \in [\frac{1}{2}, 1]$. Because $\tilde{\pi}'(\frac{1}{2}) < 0$, profit is decreasing in \hat{p} for $\hat{p} \in [\frac{1}{2}, 1]$.

By the definition of ρ -concavity, $2 \int_0^{\frac{1}{2}} \rho(t; \alpha) dt$ is increasing in α . By Lemma A5, $\int_0^{\frac{1}{2}} G(t; \alpha) dt$ is increasing in $\alpha \Rightarrow \frac{\frac{1}{2} + \int_0^{\frac{1}{2}} G(t) dt}{\frac{1}{4} + \int_0^{\frac{1}{2}} G(t) dt}$ is decreasing in α . By Assumptions 5(d) and 5(e),

$$\lim_{\alpha \rightarrow \alpha_1} 2 \int_0^{\frac{1}{2}} \rho(t; \alpha) dt = 1 \text{ and } \lim_{\alpha \rightarrow \infty} 2 \int_0^{\frac{1}{2}} \rho(t; \alpha) dt = \infty.$$

Thus, there exists $\alpha_2 > \alpha_1$ such that for $\alpha > \alpha_2$, anti-entrenchment is optimal; for $\alpha < \alpha_2$, entrenchment is optimal. ■

Proof of Example 1. Given the functional form of $g(\cdot)$, it can be verified that the board's profit function is

$$\tilde{\pi}(\hat{p}; \alpha) = \begin{cases} \frac{1}{4} \left[\frac{1}{2} + \frac{1}{4} \frac{\alpha}{\alpha+1} (2\hat{p})^{\frac{\alpha+1}{\alpha}} \right] \left[\frac{1}{2} - \frac{1}{4} \frac{1}{1+\alpha} (2\hat{p})^{\frac{\alpha+1}{\alpha}} \right] & \text{for } \hat{p} \in [0, \frac{1}{2}] \\ \frac{1}{16} [2(1-\hat{p})]^{\frac{\alpha+1}{\alpha}} \left[\frac{\alpha}{\alpha+1} + \frac{\hat{p}}{1-\hat{p}} \right] \left[\hat{p} + \frac{1}{4} \frac{\alpha}{\alpha+1} [2(1-\hat{p})]^{\frac{\alpha+1}{\alpha}} \right] & \text{for } \hat{p} \in (\frac{1}{2}, 1] \end{cases}.$$

It is obvious that $\alpha_1 = 1$. By the proof of Proposition 2, α_2 is the informativeness such that the first derivative of profit at $\hat{p} = \frac{1}{2}$ is equal to 0. $\Rightarrow \alpha_2 = \frac{\sqrt{5}+1}{2}$. ■

Proof of Proposition 3. Designing a contract based on the signal is equivalent to designing a contract based on the posterior belief about the incumbent's ability

$p \in [0, 1]$. By abuse of notation, denote $\{w(p), r(p), k(p)\}$ as the contract based on the board's posterior belief. It suffices to prove that $r^*(p) = 1$ for $p \in [\frac{1}{2}, 1]$ and $r^*(p) = 0$ for $p \in [0, \frac{1}{2})$ in the optimal contract.

Given contract $\{w(p), r(p), k(p)\}$, the incumbent manager chooses q to maximize:

$$\int_0^1 \{r(p)qpw(p) + [1 - r(p)]k(p)\}g(p)dp - C(q).$$

The first order condition with respect to q yields

$$C'(q) = \int_0^1 r(p)qw(p)g(p)dp.$$

Note that $k(p)$ cannot provide incentive on the effort level. Because the incumbent manager is protected by limited liability, $k^*(p) = 0$ in the optimal contract.

The board chooses $\{w(p), r(p)\}$ to maximize

$$\begin{aligned} & \int_0^1 \left\{ [r(p)qp(1 - w(p))] + \frac{1}{2}q(1 - r(p)) \right\} g(p)dp \\ &= q \left(\int_0^1 \left[r(p)p + \frac{1}{2}(1 - r(p)) \right] g(p)dp - \int_0^1 r(p)pw(p)g(p)dp \right). \end{aligned}$$

Given effort level q , $\int_0^1 r(p)pw(p)g(p)dp = C'(q)$ is a constant by the incumbent manager's first order condition. It is equivalent to maximize:

$$\int_0^1 \left[r(p)p + \frac{1}{2}(1 - r(p)) \right] g(p)dp.$$

The integral is maximized by setting $r(p) = 1$ for $p \in [\frac{1}{2}, 1]$ and $r(p) = 0$ for $p \in [0, \frac{1}{2})$.

■

Proof of Proposition 4. We first proved that $k^* = 0$ in the optimal contract.

Given (w_1, w_2, k) and belief of cutoff \hat{s} , the incumbent manager chooses q to maximize

$$\frac{1}{2}[1 - F_1(\hat{s})]qw_1 + \frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})]\left(\frac{1}{2}qw_2 + k\right) - C(q).$$

$$\Rightarrow q = \frac{1}{2}[1 - F_1(\hat{s})]w_1 + \frac{1}{4}[F_1(\hat{s}) + F_0(\hat{s})]w_2.$$

Given (w_1, w_2, k) and belief of project quality q , the board's indifference condition yields:

$$\frac{1}{2}q(1 - w_2) - k = \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})}q(1 - w_1).$$

The board chooses (w_1, w_2, k) to maximize expected profit:

$$\begin{aligned} & \frac{1}{2}[1 - F_1(\hat{s})]q(1 - w_1) + \frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})]\left[\frac{1}{2}q(1 - w_2) - k\right] \\ &= q(1 - w_1) \left\{ \frac{1}{2}[1 - F_1(\hat{s})] + \frac{1}{2}[F_1(\hat{s}) + F_0(\hat{s})] \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \right\}. \end{aligned}$$

Given (q, \hat{s}) the board would like to induce, it is obvious that profit is decreasing in w_1 . By the two equilibrium conditions, it is easy to verify that $w_1(k)$ is increasing in k and $w_2(k)$ is decreasing in k . Thus, $k^* = 0$.

The board's profit maximization problem can be written as

$$\max_{\{w_1, w_2, q, \hat{s}\}} \pi(w_1, w_2, q, \hat{s}) := \frac{1}{2}[1 - F_1(\hat{s})]q(1 - w_1) + \frac{1}{4}[F_1(\hat{s}) + F_0(\hat{s})]q(1 - w_2)$$

s.t.

$$q - \left(\frac{1}{2}[1 - F_1(\hat{s})]w_1 + \frac{1}{4}[F_1(\hat{s}) + F_0(\hat{s})]w_2 \right) = 0$$

and

$$\frac{1}{2}(1 - w_2) - \varphi(\hat{s})(1 - w_1) = 0.$$

Let \mathcal{L} be the Lagrangian and denote λ_1 and λ_2 as Lagrangian multipliers on the two constraints respectively.

$$\frac{\partial \mathcal{L}(w_1, w_2, q, \hat{s}, \lambda_1, \lambda_2)}{\partial w_1} = 0 \Rightarrow -\frac{1}{2}(q + \lambda_1)[1 - F_1(\hat{s})] + \varphi(\hat{s})\lambda_2 = 0.$$

$$\frac{\partial \mathcal{L}(w_1, w_2, q, \hat{s}, \lambda_1, \lambda_2)}{\partial w_2} = 0 \Rightarrow -\frac{1}{2}(q + \lambda_1)[F_1(\hat{s}) + F_0(\hat{s})] - \lambda_2 = 0.$$

It can be verified that $\hat{s} = 0$ is never optimal. Thus, $\varphi(\hat{x}) > 0$. Then $\lambda_2 = 0$ and $\lambda_1 = -q$ must be true. The first order condition of the Lagrangian with respect to \hat{s} yields

$$\frac{\partial \mathcal{L}(w_1, w_2, q, \hat{s}, \lambda_1, \lambda_2)}{\partial \hat{s}} = 0.$$

$$\begin{aligned} &\Rightarrow -q(1 - w_1)f_1(\hat{s}) + \frac{1}{2}[f_1(\hat{s}) + f_0(\hat{s})]q(1 - w_2) \\ &\quad + \lambda_1 \left(f_1(\hat{s})w_1 - \frac{1}{2}[f_1(\hat{s}) + f_0(\hat{s})]w_2 \right) = 0 \\ &\Rightarrow \varphi(\hat{s}) = \frac{1}{2} \Rightarrow \hat{s}^* = \frac{1}{2}. \end{aligned}$$

Because $\frac{1}{2}(1 - w_2) - \varphi(\hat{s})(1 - w_1) = 0$, it follows directly that $w_1^* = w_2^*$. ■

Proof of Proposition 5. Given contract (w, k) and \hat{s} , the incumbent manager's

best response is

$$q = \left\{ (1 + \tau) \left[\frac{1}{2} \left(\frac{1}{2} + \delta \right) [1 - F_1(\hat{s})] + \frac{1}{2} \left(\frac{1}{2} - \delta \right) [1 - F_0(\hat{s})] \right] w \right\}^{\frac{1}{1-\tau}}.$$

Similarly, the board's indifference condition is

$$\frac{1}{2} q^{1+\tau} - k = \left[\frac{1}{2} + \frac{f_1(\hat{s}) - f_0(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \delta \right] q^{1+\tau} (1 - w).$$

Plugging the two equilibrium conditions into the board's profit function yields

$$\begin{aligned} \pi(\hat{s}) = & M \left[\left(\frac{1}{2} + \delta \right) [1 - F_1(\hat{s})] + \left(\frac{1}{2} - \delta \right) [1 - F_0(\hat{s})] \right]^{\frac{1+\tau}{1-\tau}} \\ & \cdot \left\{ \left[\left(\frac{1}{2} + \delta \right) [1 - F_1(\hat{s})] + \left(\frac{1}{2} - \delta \right) [1 - F_0(\hat{s})] \right] + [F_1(\hat{s}) + F_0(\hat{s})] \left[\frac{1}{2} + \frac{f_1(\hat{s}) - f_0(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} \delta \right] \right\}, \end{aligned}$$

where $M = \frac{1}{4}^{\frac{2}{1-\tau}} (1 - \tau)(1 + \tau)^{\frac{2+2\tau}{1-\tau}}$.

Next, we calculate the board's expected profit for a given cutoff \hat{s} as $\alpha \rightarrow \infty$:

$$\lim_{\alpha \rightarrow \infty} \pi(\hat{s}; \alpha) = \begin{cases} M \left[1 - (1 - 2\delta)\hat{s} \right]^{\frac{1+\tau}{1-\tau}} & \text{for } \hat{s} \in [0, \frac{1}{2}) \\ M \left[\delta + \frac{1}{2} \right]^{\frac{1+\tau}{1-\tau}} (1 + \delta) & \text{for } \hat{s} = \frac{1}{2} \\ M \left[(1 + 2\delta)(1 - \hat{s}) \right]^{\frac{1+\tau}{1-\tau}} (1 + 2\delta) & \text{for } \hat{s} \in (\frac{1}{2}, 1] \end{cases}.$$

1. Entrenchment as $\alpha \rightarrow \infty$:

Notice that $\frac{f_1(\hat{s})-f_0(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})} \leq 1$. Then $\pi(\hat{s}; \alpha)$ can be bounded above by

$$\begin{aligned} \pi(\hat{s}; \alpha) \leq & M \left[\left(\frac{1}{2} + \delta \right) [1 - F_1(\hat{s})] + \left(\frac{1}{2} - \delta \right) [1 - F_0(\hat{s})] \right]^{\frac{1+\tau}{1-\tau}} \\ & \cdot \left\{ \left[\left(\frac{1}{2} + \delta \right) [1 - F_1(\hat{s})] + \left(\frac{1}{2} - \delta \right) [1 - F_0(\hat{s})] \right] + [F_1(\hat{s}) + F_0(\hat{s})] \left[\frac{1}{2} + \delta \right] \right\}. \end{aligned}$$

Denote the right-hand side as $\pi_E(\hat{s}; \alpha)$. By Lemma A1, $F_1(\hat{s}; \alpha)$ converges uniformly to $\max \{0, 2\hat{s} - 1\}$ as $\alpha \rightarrow \infty$. Thus, $\pi_E(\hat{s}; \alpha)$ converges uniformly to $M \left[(1 + 2\delta)(1 - \hat{s}) \right]^{\frac{1+\tau}{1-\tau}} (1 + 2\delta)$ for $\hat{s} \in [\frac{1}{2}, 1]$ as $\alpha \rightarrow \infty$. Because $\pi(0; \alpha) = M$, entrenchment is optimal for sufficiently large α if

$$M > \max_{\hat{s} \in [\frac{1}{2}, 1]} \left\{ M \left[(1 + 2\delta)(1 - \hat{s}) \right]^{\frac{1+\tau}{1-\tau}} (1 + 2\delta) \right\} \Rightarrow \delta < \frac{1}{2}^{\frac{1-\tau}{2}} - \frac{1}{2}.$$

2. Anti-entrenchment as $\alpha \rightarrow \infty$:

Notice that $\frac{f_1(\hat{s})-f_0(\hat{s})}{f_1(\hat{s})+f_0(\hat{s})} \leq 0$ for $\hat{s} \in [0, \frac{1}{2}]$. Then $\pi(\hat{s}; \alpha)$ can be bounded above by

$$\begin{aligned} \pi(\hat{s}; \alpha) \leq & M \left[\left(\frac{1}{2} + \delta \right) [1 - F_1(\hat{s})] + \left(\frac{1}{2} - \delta \right) [1 - F_0(\hat{s})] \right]^{\frac{1+\tau}{1-\tau}} \\ & \cdot \left\{ \left[\left(\frac{1}{2} + \delta \right) [1 - F_1(\hat{s})] + \left(\frac{1}{2} - \delta \right) [1 - F_0(\hat{s})] \right] + \frac{1}{2} [F_1(\hat{s}) + F_0(\hat{s})] \right\}. \end{aligned}$$

Denote the right-hand side as $\pi_A(\hat{s}; \alpha)$. By Lemma A1, $F_1(\hat{s}; \alpha)$ converges uniformly to $\max \{0, 2\hat{s} - 1\}$. Thus, $\pi_A(\hat{s}; \alpha)$ converges uniformly to $\xi(\hat{s}; \delta, \tau) = M \left[1 - (1 - 2\delta)\hat{s} \right]^{\frac{1+\tau}{1-\tau}} (1 + 2\delta\hat{s})$ for $\hat{s} \in [0, \frac{1}{2}]$ as $\alpha \rightarrow \infty$.

Given $(\delta, \tau) \in (0, \frac{1}{2}) \times (-1, 1)$, it can be verified that there exists $\nu(\delta, \tau) < \frac{1}{2}$ such that $\xi(\hat{s}; \delta, \tau) < M \left[\delta + \frac{1}{2} \right]^{\frac{1+\tau}{1-\tau}} (1 + 2\delta)$ for $\hat{s} \in [\nu(\delta, \tau), \frac{1}{2}]$. Thus, $\hat{s} \in [\nu(\delta, \tau), \frac{1}{2}]$

can never be optimal for sufficiently large α .

Because $f_1(\hat{s})$ is strictly increasing in \hat{s} and $\lim_{\alpha \rightarrow \infty} f_1(\hat{s}; \alpha) = 0$ for all $\hat{s} \in [0, \frac{1}{2})$, $f_1(\hat{s}; \alpha)$ converges uniformly to 0 for $\hat{s} \in [0, \nu(\delta, \tau)]$ as $\alpha \rightarrow \infty$. Then $\pi(\hat{s}; \alpha)$ converges uniformly to $M \left[1 - (1 - 2\delta)\hat{s} \right]^{\frac{1+\tau}{1-\tau}}$ for $\hat{s} \in [0, \nu(\delta, \tau)]$ as $\alpha \rightarrow \infty$. Thus, entrenchment is optimal for sufficiently large α if

$$\max_{\hat{s} \in [0, \nu(\delta, \tau)]} M \left[1 - (1 - 2\delta)\hat{s} \right]^{\frac{1+\tau}{1-\tau}} < M \left[\delta + \frac{1}{2} \right]^{\frac{1+\tau}{1-\tau}} (1 + 2\delta) \Rightarrow \delta > \frac{1}{2}^{\frac{1-\tau}{2}} - \frac{1}{2}.$$

■

Proof of Proposition 6. Given \hat{s} , a contract can always be constructed to induce \hat{s} . However, it is not necessarily $w = \frac{1}{2}$. $k \geq 0$ does not hold for all $\hat{s} \in [0, 1]$ when $w = \frac{1}{2}$. To see this, notice that the severance pay k is

$$k(\hat{s}, w) = \pi(q(\hat{s}, w)) - \frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} [(1 - \lambda)q(\hat{s}, w) + \lambda e(\hat{s}, w)](1 - w).$$

Given \hat{s} , letting w be sufficiently close to 1 generates a positive severance pay k . Define $\hat{\mathcal{S}} = \left\{ \hat{s} \mid k(\hat{s}, \frac{1}{2}) \geq 0 \text{ \& } \hat{s} \in [0, 1] \right\}$, which is the the set of cutoffs that can be induced by contracts that satisfy $w = \frac{1}{2}$ and $k \geq 0$. If $\hat{s} \in \hat{\mathcal{S}}$, the board's expected profit can be written as

$$\bar{\pi}(\hat{s}) = \frac{1}{16} \left[1 - F_1(\hat{s}) \right] \left[(1 - \lambda)^2 + \lambda^2 \frac{1}{1 - \hat{s}} \right] \left[[1 - F_1(\hat{s})] + \hat{s} f_1(\hat{s}) \right].$$

If $\hat{s} \notin \hat{\mathcal{S}}$, $w = \frac{1}{2}$ cannot be sustained. Define $\mathcal{W}(\hat{s}) = \left\{ w \mid k(\hat{s}, w) \geq 0 \text{ \& } w \in [0, 1] \right\}$,

which is the set of wages that can induce \hat{s} without violating the limited liability constraint of k .

1. Entrenchment:

It suffices to prove that $\pi(\hat{s}; \alpha) < \pi(0; \alpha)$ for all $\hat{s} \in [\frac{1}{2}, 1]$ for sufficiently small α . $\pi(0; \alpha)$ is independent of α and can be calculated as

$$\pi(0; \alpha) = \frac{1}{16} [(1 - \lambda)^2 + \lambda^2].$$

Lemma A6 *There exist $\Delta \in (0, \frac{1}{2})$ and N such that for $\alpha < N$, $\pi(\hat{s}; \alpha) < \pi(0; \alpha)$ for all $\hat{s} \in [1 - \Delta, 1]$.*

Proof. By Lemma A2, for any $\epsilon > 0$ there exists N such that for $\alpha < N$, $1 - F_1(1 - \Delta) < \Delta + \epsilon$ for all $\Delta \in [0, 1]$. Note that $\frac{1}{4} \left(\frac{1}{2} \lambda + \frac{1-\lambda}{\lambda} q \right)^2 \geq \frac{1}{2} (1 - \lambda) q$ for all q .

The expected profit of replacement can be bounded above by

$$\begin{aligned} \underline{\pi}(q) - k &\leq \frac{1}{4} \left(\frac{1}{2} \lambda + \frac{1-\lambda}{\lambda} q \right)^2 = \frac{1}{16} \left(\lambda + \frac{(1-\lambda)^2}{\lambda} [1 - F_1(\hat{s})] w \right)^2 \\ &\leq \frac{1}{16} \left(\lambda + \frac{(1-\lambda)^2}{\lambda} [1 - F_1(\hat{s})] \right)^2. \end{aligned}$$

Thus, the board's expected profit can be bounded for $\hat{s} \in [1 - \Delta, 1]$:

$$\begin{aligned}
\pi(\hat{s}) &\leq \frac{1}{2} [1 - F_1(\hat{s})] \left\{ \frac{1 - F_1(\hat{s})}{2} (1 - \lambda)^2 + \frac{1 - F_1(\hat{s})}{[1 - F_1(\hat{s})] + [1 - F_0(\hat{s})]} \lambda^2 \right\} w(1 - w) \\
&\quad + \frac{1}{32} [F_1(\hat{s}) + F_0(\hat{s})] \left(\lambda + \frac{(1 - \lambda)^2}{\lambda} [1 - F_1(\hat{s})] \right)^2 \\
&\leq \frac{1}{8} [1 - F_1(\hat{s})] \left\{ \frac{1 - F_1(\hat{s})}{2} (1 - \lambda)^2 + \frac{1 - F_1(\hat{s})}{[1 - F_1(\hat{s})] + [1 - F_0(\hat{s})]} \lambda^2 \right\} \\
&\quad + \frac{1}{16} \left(\lambda + \frac{(1 - \lambda)^2}{\lambda} [1 - F_1(\hat{s})] \right)^2 \\
&< \frac{1}{8} (\Delta + \epsilon) \left[\frac{\Delta + \epsilon}{2} (1 - \lambda)^2 + \lambda^2 \right] + \frac{1}{16} \left[\lambda + \frac{(1 - \lambda)^2}{\lambda} (\Delta + \epsilon) \right]^2.
\end{aligned}$$

Note that the last expression is increasing in $\Delta + \epsilon$. It suffices to prove that

$$\frac{1}{16} \lambda^2 < \frac{1}{16} [(1 - \lambda)^2 + \lambda^2].$$

Consequently, we can always find sufficiently small Δ and ϵ such that

$$\frac{1}{8} (\Delta + \epsilon) \left[\frac{\Delta + \epsilon}{2} (1 - \lambda)^2 + \lambda^2 \right] + \frac{1}{16} \left[\lambda + \frac{(1 - \lambda)^2}{\lambda} (\Delta + \epsilon) \right]^2 < \frac{1}{16} [(1 - \lambda)^2 + \lambda^2].$$

■

Next, notice that $\bar{\pi}(\hat{s})$ is the maximum expected profit without the limited liability constraint of k . Thus $\pi(\hat{s}) \leq \bar{\pi}(\hat{s})$ for all $\hat{s} \in [0, 1]$.

Given ϵ , the expected profit for $\alpha < N$ for all $\hat{s} \in [\frac{1}{2}, 1 - \Delta]$ is bounded above

by

$$\begin{aligned}
\pi(\hat{s}) &\leq \frac{1}{16} \left[1 - F_1(\hat{s}) \right] \left[(1 - \lambda)^2 + \lambda^2 \frac{1}{1 - \hat{s}} \right] \left[[1 - F_1(\hat{s})] + \hat{x} f_1(\hat{s}) \right] \\
&\leq \frac{1}{16} (1 - \hat{s} + \epsilon) \left[(1 - \lambda)^2 + \lambda^2 \frac{1}{1 - \hat{s}} \right] [(1 - \hat{s} + \epsilon) + \hat{s}(1 + \epsilon)] \\
&\leq \frac{1}{16} \left(1 + \frac{\epsilon}{\delta} \right) \left[\frac{1}{2} (1 - \lambda)^2 + \lambda^2 \right] (1 + 2\epsilon).
\end{aligned}$$

It remains to prove that $\frac{1}{16} \left[\frac{1}{2} (1 - \lambda)^2 + \lambda^2 \right] < \frac{1}{16} [(1 - \lambda)^2 + \lambda^2]$, which is obvious.

2. Anti-entrenchment:

For $\hat{s} \in [0, \frac{1}{2}]$, the expected profit can be bounded above by

$$\begin{aligned}
\pi(\hat{s}; \alpha) &\leq \frac{1}{16} \left[1 - F_1(\hat{s}; \alpha) \right] \left[(1 - \lambda)^2 + \lambda^2 \frac{1}{1 - \hat{s}} \right] \left[[1 - F_1(\hat{s}; \alpha)] + \hat{x} f_1(\hat{s}; \alpha) \right] \\
&< \frac{3}{32} [(1 - \lambda)^2 + 2\lambda^2].
\end{aligned}$$

It remains to find $\hat{s} > \frac{1}{2}$ that yields a profit no less than $\frac{3}{32} [(1 - \lambda)^2 + 2\lambda^2]$. For notational convenience, let $\psi = \left(\frac{\lambda}{1 - \lambda} \right)^2$. $\lambda < \sqrt{2} - 1$ directly implies that $\psi < \frac{1}{2}$.

By the limited liability constraint of k , we have

$$\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} [(1 - \lambda)q + \lambda e] (1 - w) \leq \pi(q).$$

Notice that $\frac{f_1(\hat{s})}{f_1(\hat{s}) + f_0(\hat{s})} < 1$, it suffices to satisfy

$$\frac{1 - F_1(\hat{s})}{4} (1 - \lambda)^2 w \geq \left\{ \frac{1 - F_1(\hat{s})}{2} (1 - \lambda)^2 + \frac{1 - F_1(\hat{s})}{[1 - F_1(\hat{s})] + [1 - F_0(\hat{s})]} \lambda^2 \right\} w(1 - w),$$

and

$$\frac{1 - F_1(\hat{s}; \alpha)}{2} w \geq \frac{1}{2} \psi.$$

The second inequality comes from the construction that the board will not induce effort from the replacement manager. Let $\hat{s} = \frac{1}{2} + \kappa(\lambda)$ and $w = \frac{1}{2} + \iota(\lambda)$. Then it suffices to find (κ, ι) that yields a higher expected profit given λ . Note that the first inequality is independent of α . By Lemma A1, $\frac{1 - F_1(\hat{s}; \alpha)}{2}$ can be arbitrarily close to $1 - \hat{s}$ when α is sufficiently large. Thus, these two conditions can be further simplified as

$$\frac{1}{2} \left(\frac{1}{2} - \kappa \right) \geq \left[\left(\frac{1}{2} - \kappa \right) + \psi \right] \left(\frac{1}{2} - \iota \right),$$

and

$$\begin{aligned} 2 \left(\frac{1}{2} + \iota \right) \left(\frac{1}{2} - \kappa \right) &\geq \psi. \\ \Rightarrow \iota &\geq \max \left\{ \frac{\frac{1}{2} \psi}{\frac{1}{2} - \kappa + \psi}, \frac{\psi}{1 - 2\kappa} - \frac{1}{2} \right\}. \end{aligned}$$

Let $\iota = \frac{\frac{1}{2} \psi}{\frac{1}{2} - \kappa + \psi}$. It can be verified that $\psi < \frac{1}{4}$ if $\kappa < \frac{1}{2} - \psi$. The board's expected profit from the contract with the incumbent manager (w, k) that induces $\hat{s} = \frac{1}{2} + \kappa$ with wage $w = \frac{1}{2} + \iota$ as $\alpha \rightarrow \infty$ is

$$\lim_{\alpha \rightarrow \infty} \pi(\hat{s}; \alpha) = \left[(1 - \lambda)^2 \left(\frac{1}{2} - \kappa \right) + \lambda^2 \right] \left(\frac{1}{4} - \frac{\frac{1}{2} \psi}{\frac{1}{2} - \kappa + \psi} \right)^2.$$

Note that

$$\begin{aligned}
\lim_{\kappa \rightarrow 0} \lim_{\alpha \rightarrow \infty} \pi(\hat{s}; \alpha) &= \lim_{\kappa \rightarrow 0} \left\{ \left[(1 - \lambda)^2 \left(\frac{1}{2} - \kappa \right) + \lambda^2 \right] \left[\frac{1}{4} - \left(\frac{\frac{1}{2}\psi}{\frac{1}{2} - \kappa + \psi} \right)^2 \right] \right\} \\
&= \frac{1}{2} [(1 - \lambda)^2 + 2\lambda^2] \left[\frac{1}{4} - \left(\frac{\frac{1}{2}\psi}{\frac{1}{2} + \psi} \right)^2 \right] \\
&> \frac{3}{32} [(1 - \lambda)^2 + 2\lambda^2].
\end{aligned}$$

Thus, we can find sufficiently small κ such that $\lim_{\alpha \rightarrow \infty} \pi(\hat{s}; \alpha) > \frac{3}{32} [(1 - \lambda)^2 + 2\lambda^2]$. That is, anti-entrenchment is optimal to the board when α is sufficiently large and $\lambda < \sqrt{2} - 1$.

■

Proof of Proposition 7.

1. Entrenchment:

It can be verified that $\pi(0; \alpha) = \frac{1}{16}$. Similarly, $\pi(1; \alpha) = 0$. Thus, $\hat{s} = 1$ is never optimal. It suffices to prove that there exists N such that for $\alpha < N$, $\pi(\hat{s}) < \pi(0)$ for all $\hat{s} \in [\frac{1}{2}, 1]$.

Lemma A7 *There exist $\Delta \in (0, \frac{1}{2})$ and N such that for $\alpha < N$, $\pi(\hat{s}; \alpha) < \pi(0; \alpha)$ for all $\hat{s} \in [1 - \Delta, 1]$.*

Proof. Since $q = \max \left\{ \frac{1}{2} [1 - H_1(\hat{s})]w - \frac{1}{2} [H_0(\hat{s}) - H_1(\hat{s})]k, 0 \right\}$, the effort level of the incumbent manager can be bounded above by

$$q \leq \frac{1}{2} [1 - H_1(\hat{s})]w.$$

Thus, the expected profit can be bounded above by

$$\begin{aligned}
\pi(\hat{s}, q) &\leq \frac{1}{2}q[1 - H_1(\hat{s})](1 - w) + \left\{ \frac{1}{2}qH_1(\hat{s}) + \left(1 - \frac{1}{2}q\right) H_0(\hat{s}) \right\} \left(\frac{1}{2}q - k \right) \\
&\leq \frac{1}{2}q[1 - H_1(\hat{s})](1 - w) + \frac{1}{2}q \left\{ \frac{1}{2}qH_1(\hat{s}) + \left(1 - \frac{1}{2}q\right) H_0(\hat{s}) \right\} \\
&\leq \frac{1}{2}q[1 - H_1(\hat{s}) + 1] \leq q < 1 - H_1(\hat{s}).
\end{aligned}$$

Let $\Delta = \frac{1}{32}$. By Lemma A2, for $\epsilon' = \frac{1}{32}$, there exists N such that for $\alpha < N$,

$H_1(\hat{s}) \geq \hat{s} - \epsilon'$ for all $\hat{s} \in [0, 1]$. Since $\hat{s} \geq 1 - \Delta$, we have

$$\pi(\hat{s}, q) < 1 - H_1(\hat{s}) \leq 1 - \hat{s} + \epsilon' \leq \Delta + \epsilon' = \frac{1}{16} = \pi(0; \alpha).$$

■

Lemma A8 *Given any $\Delta \in (0, \frac{1}{2})$ and $q \in [0, 1]$, for any $\epsilon > 0$, there exists N'*

such that for $\alpha < N'$, $\frac{\frac{1}{2}qh_1(\hat{s})}{\frac{1}{2}qh_1(\hat{s}) + (1 - \frac{1}{2}q)h_0(\hat{s})} \leq \frac{1}{2}q + \epsilon$ for $\hat{s} \in [\frac{1}{2}, 1 - \Delta]$.

Proof. For any $\epsilon > 0$, let $\epsilon' = \frac{\epsilon}{1 + \epsilon}$. By the definition of the completely

uninformative information structure, there exists N' such that for $\alpha < N'$,

$$h_1(1 - \delta; \alpha) < 1 + \epsilon'.$$

$$\begin{aligned}
\frac{\frac{1}{2}qh_1(\hat{s})}{\frac{1}{2}qh_1(\hat{s}) + (1 - \frac{1}{2}q)h_0(\hat{s})} - \frac{1}{2}q &= \frac{1}{2}q \left(1 - \frac{1}{2}q\right) \frac{h_1(\hat{s}) - h_0(\hat{s})}{\frac{1}{2}qh_1(\hat{s}) + (1 - \frac{1}{2}q)h_0(\hat{s})} \\
&\leq \frac{1}{2} \frac{h_1(\hat{s}) - h_0(\hat{s})}{h_0(\hat{s})} = \frac{h_1(\hat{s}) - 1}{2 - h_1(\hat{s})} \\
&\leq \frac{h_1(1 - \delta; \alpha) - 1}{2 - h_1(1 - \delta; \alpha)} \leq \frac{\epsilon'}{1 - \epsilon'} = \epsilon.
\end{aligned}$$

■

By Lemma A8, for all $\hat{s} \in [\frac{1}{2}, 1 - \Delta]$, $\pi(\hat{s}, q)$ can be bounded above by

$$\begin{aligned}
\pi(\hat{s}, q) &\leq \frac{1}{2}q[1 - H_1(\hat{s})](1 - w) + \left\{ \frac{1}{2}qH_1(\hat{s}) + \left(1 - \frac{1}{2}q\right) H_0(\hat{s}) \right\} \left(\frac{1}{2}q + \epsilon \right) (1 - w) \\
&\leq \frac{1}{2}q(1 - w) \left[[1 - H_1(\hat{s})] + \frac{1}{2}qH_1(\hat{s}) + \left(1 - \frac{1}{2}q\right) H_0(\hat{s}) \right] + \epsilon \\
&\leq \frac{1}{4}[1 - H_1(\hat{s})][2 - H_1(\hat{s})]w(1 - w) + \epsilon \\
&\leq \frac{1}{16}(1 - \hat{s} + \epsilon)(2 - \hat{s} + \epsilon) + \epsilon = \frac{1}{16}\left(\frac{1}{2} + \epsilon\right)\left(\frac{3}{2} + \epsilon\right) + \epsilon.
\end{aligned}$$

The last expression is strictly less than $\frac{1}{16}$ for sufficiently small ϵ .

2. Anti-entrenchment:

For $\hat{s} \in [0, \frac{1}{2}]$, $\zeta(\hat{s}, q) \leq \frac{1}{2}q$. Thus,

$$\begin{aligned}
\pi(\hat{s}, q) &= \frac{1}{2}q[1 - H_1(\hat{s})](1 - w) + \left\{ \frac{1}{2}qH_1(\hat{s}) + \left(1 - \frac{1}{2}q\right) H_0(\hat{s}) \right\} \left(\frac{1}{2}q - k \right) \\
&\leq \frac{1}{2}q[1 - H_1(\hat{s})](1 - w) + \zeta(\hat{s}, q)(1 - w)H_0(\hat{s}) \\
&\leq \frac{1}{2}q(1 - w)[1 + H_0(\hat{s}) - H_1(\hat{s})] \\
&\leq \frac{1}{4}w(1 - w)[1 - H_1(\hat{s})][1 + H_0(\hat{s}) - H_1(\hat{s})] \leq \frac{1}{8}.
\end{aligned}$$

Next, we consider a fixed contract $(w', k') = (\frac{4}{5}, 0)$. It can be verified that this contract will not yield an equilibrium with a replacement cutoff \hat{s} below $\frac{1}{2}$. To

see this, notice that the effort level under this contract is

$$q = \frac{2}{5}[1 - H_1(\hat{s})].$$

The expected profit of replacement is,

$$\frac{1}{2}q - k = \frac{1}{5}[1 - H_1(\hat{s})].$$

The expected profit created by the manager on the margin is

$$\frac{\frac{1}{2}qh_1(\hat{s})}{\frac{1}{2}qh_1(\hat{s}) + (1 - \frac{1}{2}q)h_0(\hat{s})}(1 - w) \leq \frac{1}{2}q(1 - w) = \frac{1}{25}[1 - H_1(\hat{s})], \text{ for } \hat{s} \in [0, \frac{1}{2}].$$

The indifference condition of the board never holds for $\hat{s} \in [0, \frac{1}{2}]$. Thus, the only possible equilibrium replacement policy under this contract is $\hat{s} > \frac{1}{2}$. It remains to prove that the profit of the contract is above $\frac{1}{8}$ for sufficiently large α .

Lemma A9 *For any $\Delta \in (0, \frac{1}{2})$, there exists N such that for $\alpha > N$, $\hat{s}(\alpha) < \frac{1}{2} + \Delta$ with contract $(w', k') = (\frac{4}{5}, 0)$.*

Proof. It suffices to prove that for any $\Delta \in (0, \frac{1}{2})$, there exists N such that for $\alpha > N$, the board's indifference condition never holds for all $\hat{s} \in [\frac{1}{2} + \Delta, 1]$ with contract $(w', k') = (\frac{4}{5}, 0)$.

The board's indifference condition can be simplified as

$$\frac{h_1(\hat{s}; \alpha)}{h_0(\hat{s}; \alpha)} = 1 + \frac{4}{H_1(\hat{s}; \alpha)}.$$

Since $H_1(\hat{s}) \geq 2\hat{s} - 1$,

$$1 + \frac{4}{H_1(\hat{s}; \alpha)} \leq 1 + \frac{4}{H_1(\frac{1}{2} + \Delta; \alpha)} \leq 1 + \frac{2}{\Delta}.$$

$\frac{h_1(\hat{s}; \alpha)}{h_0(\hat{s}; \alpha)}$ approaches infinity as $\alpha \rightarrow \infty$ while $1 + \frac{4}{H_1(\hat{s}; \alpha)}$ is bounded, which is a contradiction. ■

For notational convenience, define $\Lambda(\hat{s}; \alpha) = 1 - H_1(\hat{s})$. The board's expected profit can be written as

$$\begin{aligned} & \pi(\hat{s}(w', k'; \alpha), q(w', k'; \alpha)) \\ &= \frac{1}{5}\Lambda^2(\hat{s}; \alpha)\left(\frac{7}{5} - \frac{2}{5}\Lambda(\hat{s}; \alpha)\right) + \frac{1}{5}\Lambda(\hat{s}; \alpha)\left(1 - \frac{1}{5}\Lambda(\hat{s}; \alpha)\right)(2\hat{s} - 1) \\ &\geq \frac{1}{5}\Lambda^2(\hat{s}; \alpha)\left(\frac{7}{5} - \frac{2}{5}\Lambda(\hat{s}; \alpha)\right) \geq \frac{1}{5}\Lambda^2(\hat{s}; \alpha). \end{aligned}$$

By Lemma A1, given any $\epsilon > 0$, there exists N such that for $\alpha > N$, $\Lambda(\hat{s}; \alpha) > 2(1 - \hat{s}) - \epsilon$ for all $\hat{s} \in [\frac{1}{2}, 1]$. Thus,

$$\pi(\hat{s}(w', k'; \alpha), q(w', k'; \alpha)) \geq \frac{1}{5}[2(1 - \hat{s}) - \epsilon]^2 \geq \frac{1}{5}(1 - 2\Delta - \epsilon)^2.$$

Let $\Delta = \epsilon = \frac{1}{24}$. Then,

$$\pi(\hat{s}(w', k'; \alpha), q(w', k'; \alpha)) \geq \frac{1}{5}(1 - 2\Delta - \epsilon)^2 = \frac{49}{320} \geq \frac{1}{8}.$$

■

A.2 Appendix: Normalization of information structure

In this section we first show that normalizing the signal space \mathcal{S} to $[0, 1]$ and assuming $\frac{1}{2}F_1(s) + \frac{1}{2}F_0(s) = s$ are without loss of generality. Next we show that the three assumptions imposed on $\{f_1(\cdot), f_0(\cdot)\}$ can be derived from similar assumptions on information structures without such normalization.

Suppose instead the board receives a noisy signal $x \in \mathcal{X}$ about the incumbent manager's ability θ_i . x is drawn from distribution with cdf $\tilde{F}_{\theta_i}(\cdot)$ and pdf $\tilde{f}_{\theta_i}(\cdot)$ for $\theta_i \in \{0, 1\}$ with support $\mathcal{X} = [\underline{x}, \bar{x}]$, where $-\infty \leq \underline{x} < \bar{x} \leq \infty$. Together with the signal space \mathcal{X} , the two conditional distributions $\{\tilde{f}_1(\cdot), \tilde{f}_0(\cdot)\}$ define an information structure.

Given an information structure $\{\tilde{f}_1(\cdot), \tilde{f}_0(\cdot), \mathcal{X}\}$, define a new signal x by applying the probability integral transformation to $x = \frac{1}{2}\tilde{F}_1(x) + \frac{1}{2}\tilde{F}_0(x)$. Then the unconditional distribution of s is uniform on $[0, 1]$. Let $F_\theta(s)$ and $f_\theta(s)$ be the corresponding conditional cdf and pdf for $\theta_i \in \{0, 1\}$ respectively. It can be verified that $\frac{1}{2}F_1(s) + \frac{1}{2}F_0(s) = s$ for all $s \in [0, 1]$.

Assumption 6 *The monotone likelihood ratio property (MLRP): $\frac{\tilde{f}_1(x)}{\tilde{f}_0(x)}$ is strictly increasing in $x \in [\underline{x}, \bar{x}]$.*

Assumption 6 directly implies Assumption 1. For binary states, the MLRP assumption is without loss of generality since it can always be satisfied by relabeling signals according to the likelihood ratio.

Lemma A10 *Suppose two information structures $\{\tilde{f}_1(\cdot), \tilde{f}_0(\cdot), \mathcal{X}\}$ and $\{\tilde{f}_1^\dagger(\cdot), \tilde{f}_0^\dagger(\cdot), \mathcal{X}^\dagger\}$ generate the same distribution of posterior beliefs with prior $\Pr(\theta_i = 1) = \frac{1}{2}$. Then they yield the same distribution of posterior beliefs with all prior $\Pr(\theta_i = 1) \in (0, 1)$.*

The proof of Lemma A10 is similar to Lemma 3 and thus is omitted. Since entrenchment (anti-entrenchment) is defined by comparing the expected ability of the incumbent manager with that of the replacement manager, only the posterior belief about the incumbent manager matters. By Lemma A10, we can restrict attention to the information structures that satisfy $\frac{1}{2}F_1(s) + \frac{1}{2}F_0(s) = s$ for $s \in [0, 1]$ without loss of generality.

Assumption 7 *Perfectly informative at extreme signals: $\lim_{x \rightarrow \underline{x}} \frac{\tilde{f}_1(x)}{\tilde{f}_0(x)} = 0$ and $\lim_{x \rightarrow \bar{x}} \frac{\tilde{f}_1(x)}{\tilde{f}_0(x)} = +\infty$.*

Assumption 8 *There exists $\check{x} \in (\underline{x}, \bar{x})$ such that $\tilde{f}_0(x) = \tilde{f}_1(2\check{x} - x)$.*

Assumptions 7 and 8 directly imply Assumptions 2 and 3, respectively. We close this section by introducing two indexed families of information structures that satisfy Assumptions 6 – 8. The corresponding normalized signals after probability integral transformation also satisfy Assumption 4.

Example 2 (Normal Distribution) Suppose $x = \theta_i + \epsilon$ for $\theta_i \in \{0, 1\}$, where $\epsilon \sim \mathcal{N}(0, \alpha^{-1})$. Then $x|\theta \sim \mathcal{N}(\theta, \alpha^{-1})$.

Example 3 (Beta Distribution) Suppose $\tilde{f}_1(x; \alpha) = (1 + \alpha)x^\alpha$ and $\tilde{f}_0(x; \alpha) = (1 + \alpha)(1 - x)^\alpha$ for $x \in [0, 1]$. Then $\tilde{F}_1(x; \alpha) = x^{1+\alpha}$ and $\tilde{F}_0(s; \alpha) = 1 - (1 - x)^{1+\alpha}$. This example is borrowed from Taylor and Yildirim (2011).

For both examples, $\alpha \in (0, \infty)$ is interpreted as the informativeness of the information structure.

A.3 Appendix: Properties of the ρ -concave order

By Lemma A5, the ρ -concave order implies the rotation order first introduced by Johnson and Myatt (2006) with $\Pr(\theta_i = \frac{1}{2})$. It can be verified that for a different prior, the rotation order does not remain. Intuitively, if the information structure becomes more informative, more densities concentrate on $p = 0$ and $p = 1$, and the distribution becomes more disperse.

Lemma A11 (Bayesian update) Suppose $G_1(\cdot)$ is more informative than $G_2(\cdot)$ in the ρ -concave order. Then $\varphi(s|G_1) \geq \varphi(s|G_2)$ for $s \in (\frac{1}{2}, 1]$ and $\varphi(s|G_1) \leq \varphi(s|G_2)$ for $s \in (0, \frac{1}{2}]$.

Proof. Since $G_1(0) = G_2(0) = 0$ and $G_1(\frac{1}{2}) = G_2(\frac{1}{2}) = \frac{1}{2}$ and $G_1(p) \geq G_2(p)$ for $p \in [0, \frac{1}{2}]$ by Lemma A5, $G_1^{-1}(s) \leq G_2^{-1}(s)$ for $s \in [0, \frac{1}{2}]$. Thus $\varphi(s|G_1) = \frac{G_1^{-1}(s)}{G_1^{-1}(s) + [1 - G_1^{-1}(s)]} \leq \frac{G_2^{-1}(s)}{G_2^{-1}(s) + [1 - G_2^{-1}(s)]} = \varphi(s, |G_2)$. The proof for $s \in (\frac{1}{2}, 1]$ is similar. ■

Lemma A11 shows the implication of the ρ -concave order on the Bayesian update of the incumbent manager's ability. The posterior belief $\varphi(x; \alpha)$ rotates counter-clockwise via $(\frac{1}{2}, \frac{1}{2})$ the as information structure becomes more informative. In other words, a fixed signal x has more information value to the board as the information structure becomes more informative.

Lemma A12 (Comparison with Blackwell's sufficiency) *If $G_1(\cdot)$ is more informative than $G_2(\cdot)$ in the ρ -concave order, $G_1(\cdot)$ is more informative than $G_2(\cdot)$ in the sense of Blackwell.*

Proof.

Lemma A13 $F_1(s|G_1) \leq F_1(s|G_2)$ and $F_0(s|G_1) \geq F_0(s|G_2)$ for $s \in [0, 1]$.

Proof. From the proof of Lemma A11, $G_1^{-1}(s) \leq G_2^{-1}(s)$ for $s \in [0, \frac{1}{2}]$.

1. For $s \in [0, \frac{1}{2}]$,

$$F_1(s|G_1) = \int_0^s f_1(t|G_1)dt = \int_0^s 2G_1^{-1}(t)dt \leq \int_0^s 2G_2^{-1}(t)dt = F_1(s|G_2).$$

2. For $s \in (\frac{1}{2}, 1]$,

$$\begin{aligned} F_1(s|G_1) &= \int_0^s f_1(t|G_1)dt = \int_0^{1-s} f_1(t|G_1)dt + \int_{1-s}^s f_1(t|G_1)dt \\ &= \int_0^{1-s} f_1(t|G_1)dt + \frac{1}{2}(2s - 1) \\ &\leq \int_0^{1-s} f_1(t|G_2)dt + \int_{1-s}^s f_1(t|G_2)dt = F_1(s|G_2). \end{aligned}$$

Thus, $F_1(s|G_1) \leq F_1(s|G_2)$ for $s \in [0, 1]$. Similarly, $F_0(s|G_1) \geq F_0(s|G_2)$. ■

Note that for binary states, Blackwell's order is equivalent to Lehmann's order.

Thus, it suffices to prove that for $\omega \in (0, 1)$,

$$F_1(F_0^{-1}(\omega|G_1)|G_1) \leq F_1(F_0^{-1}(\omega|G_2)|G_2).$$

Suppose we have the contrary, then there exists ω' such that,

$$F_1(F_0^{-1}(\omega'|G_1)|G_1) > F_1(F_0^{-1}(\omega'|G_2)|G_2).$$

By Lemma A13, it follows directly that $F_0^{-1}(\omega'|G_1) > F_0^{-1}(\omega'|G_2)$. However, $F_0^{-1}(\omega'|G_1) >$

$F_0^{-1}(\omega'|G_2)$ cannot be true. To see this, let $s_1 = F_0^{-1}(\omega'|G_1)$ and $s_2 = F_0^{-1}(\omega'|G_2)$.

Then $s_1 > s_2$ and $F_0(s_1|G_1) = F_0(s_2|G_1) = \omega'$. Again by Lemma A13, we have

$F_0(s_1|G_1) > F_0(s_2|G_1) \geq F_0(s_2|G_2)$, which is a contradiction. ■

Appendix B

Appendix for Chapter 2

B.1 Appendix: Proofs of the propositions

Proof of Lemma 7. Suppose $p_2 \in \mathcal{B}$ and $p'_2 \in \mathcal{NB}$, conditions (2.6) implies that $\lambda(p_2) \leq 0$ and $\lambda(p'_2) = 0$. Thus, $Q_2(p_2) = p_2 F_2(p_2) > 0 \Rightarrow \gamma(p_2) = 0$. The first order conditions (2.5c) imply that:

$$\begin{aligned} (1 - \tilde{q})u'(y + g - Q_2(p_2)) &= (1 - q)\mu + \frac{\lambda(p_2) + \gamma(p_2)}{(1 - p_1)\phi(p_2)} \\ &\leq (1 - q)\mu + \frac{\lambda(p'_2) + \gamma(p'_2)}{(1 - p_1)\phi(p'_2)} = (1 - \tilde{q})u'(y + g - Q_2(p'_2)). \end{aligned}$$

Since $u(\cdot)$ is strictly concave, it must be that $Q_2(p_2) \leq Q_2(p'_2)$. Similarly, it can be proved that $F_2(p_2) \geq F_2(p'_2)$.

To prove $p_2 < p'_2$, suppose $p_2 \geq p'_2$ instead. Then we have

$$Q_2(p_2) \leq Q_2(p'_2) < p'_2 F_2(p'_2) \leq p_2 F_2(p_2),$$

which is a contradiction to $p_2 \in \mathcal{B}$. ■

Proof of Lemma 8. Suppose there exist two health states $p_2^i \neq 0$ and $p_2^j \neq 0$ such that $Q_2(p_2^i) > 0$ and $Q_2(p_2^j) = 0$, then $\gamma(p_2^i) = 0$ and $\gamma(p_2^j) \geq 0$. Moreover, $\lambda(p_2^i) \leq 0$ and $\lambda(p_2^j) = 0$. From the first order conditions (2.5c) we have:

$$\begin{aligned} (1 - \tilde{q})u'(y + g - Q_2(p_2^i)) &= (1 - q)\mu + \frac{\lambda(p_2^i) + \gamma(p_2^i)}{(1 - p_1)\phi(p_2^i)} \\ &\leq (1 - q)\mu + \frac{\lambda(p_2^j) + \gamma(p_2^j)}{(1 - p_1)\phi(p_2^j)} = (1 - \tilde{q})u'(y + g - Q_2(p_2^j)). \end{aligned}$$

Thus, $u'(y + g - Q_2(p_2^i)) \leq u'(y + g - Q_2(p_2^j))$. By the strict concavity of $u(\cdot)$, $Q_2(p_2^i) \leq Q_2(p_2^j)$, which is a contradiction. ■

Proof of Lemma 10.

Lemma B1 *If $\Delta = 0$, $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$.*

Proof. Suppose there exists a health state $\hat{p}_2 \in (0, 1]$ such that $Q_2(\hat{p}_2) = 0$, then $\gamma(\hat{p}_2) > 0$. Since $F_2(p_2) > 0$ for all $p_2 > 0$, $\hat{p}_2 F_2(\hat{p}_2) - Q_2(\hat{p}_2) > 0 \Rightarrow \lambda(\hat{p}_2) = 0$. Since $\Delta = 0$, combining first order conditions (2.5a) and (2.5c) yields $u'(y + g) = u'(y + g - Q_2(\hat{p}_2)) \geq \mu = u'(y - g - Q_1)$, which is a contradiction. ■

Lemma B2 *Fixing q , if there exist Δ and Δ' such that $Q_2(p_2) > 0$ and $Q'_2(p_2) = 0$ for all $p_2 \in (0, 1]$, then $\Delta < \Delta'$.*

Proof. Suppose $\Delta \geq \Delta'$ instead, then $\tilde{q} \leq \tilde{q}'$. Since $Q'_2(p_2) = 0$ for all $p_2 \in (0, 1]$, $\gamma'(p_2) \geq 0$ and $\lambda'(p_2) = 0$ for all $p_2 \in (0, 1]$. Similarly, $\gamma(p_2) = 0$ and $\lambda(p_2) \leq 0$ for all

$p_2 \in (0, 1]$. From (2.5c) and (2.5d), we have:

$$\begin{aligned} v'(F'_2(p_2)) &= u'(y + g - Q'_2(p_2)) - \frac{\gamma'(p_2)}{(1 - \tilde{q}')(1 - p_1)\phi(p_2)} \\ &< u'(y + g - Q_2(p_2)) = v'(F_2(p_2)) \text{ for all } p_2 \in (0, 1]. \end{aligned}$$

The last strict inequality follows from that $\gamma(p_2) = 0$ and $Q_2(p_2) > Q'_2(p_2) = 0$. Thus, $F'_2(p_2) > F_2(p_2)$ for all $p_2 \in (0, 1]$ by the strict concavity of $v(\cdot)$.

Combining conditions (2.5b) and (2.5c) yields:

$$\begin{aligned} (1 - q)v'(F_1) &= (1 - \tilde{q})u'(y + g - Q_2(p_2)) - \frac{\lambda(p_2) + \gamma(p_2)}{(1 - p_1)\phi(p_2)} \\ &\geq (1 - \tilde{q})u'(y + g - Q_2(p_2)) \\ &> (1 - \tilde{q}')u'(y + g - Q'_2(p_2)) \\ &\geq (1 - \tilde{q}')u'(y + g - Q'_2(p_2)) - \frac{\lambda'(p_2) + \gamma'(p_2)}{(1 - p_1)\phi(p_2)} = (1 - q)v'(F'_1). \end{aligned}$$

Therefore $(1 - q)v'(F'_1) < (1 - q)v'(F_1) \Rightarrow F'_1 > F_1$ and $Q'_1 < Q_1$.

Hence,

$$\begin{aligned} 0 &= (Q'_1 - p_1 F'_1) + (1 - p_1)(1 - q) \int_0^1 [Q'_2(p_2) - p_2 F'_2(p_2)] d\Phi(p_2) \\ &< (Q_1 - p_1 F_1) + (1 - p_1)(1 - q) \int_0^1 [Q_2(p_2) - p_2 F_2(p_2)] d\Phi(p_2) = 0, \end{aligned}$$

which is a contradiction. ■

Lemma B3 *Fixing \tilde{q} , there exists at least one $q \in [\tilde{q}, 1)$ such that $Q_2(p_2^i) = 0$ for*

some $p_2^i \neq 0$.

Proof. Suppose to the contrary that there exists a \tilde{q} such that $Q_2(p_2) > 0$ for all $q \in [\tilde{q}, 1)$. This implies that $\lambda(p_2) \leq 0$ and $\gamma(p_2) = 0$ for all $p_2 \in (0, 1]$. We must have $v'(F_2(p_2)) = u'(y + g - Q_2(p_2)) > u'(y + g)$ for all $p_2 \in (0, 1]$. Hence $F_2(p_2)$ is bounded from above by $v'^{-1}(u'(y + g))$. The first period profit is bounded from above by,

$$0 \leq Q_1 - p_1 F_1 < (1 - p_1)(1 - q)\bar{p}_2 v'^{-1}(u'(y + g)), \quad (\text{B.1})$$

where, $\bar{p}_2 \equiv \int_0^1 p_2 d\Phi(p_2)$ is the expected mortality rate in period 2. Taking left limit of (B.1) yields:

$$0 \leq \lim_{q \rightarrow 1^-} (Q_1 - p_1 F_1) \leq \lim_{q \rightarrow 1^-} (1 - p)(1 - q)\bar{p}_2 v'^{-1}(u'(y + g)) = 0.$$

Thus, $\lim_{q \rightarrow 1^-} (Q_1 - p_1 F_1) = 0 \Rightarrow \lim_{q \rightarrow 1^-} F_1 = F_1^{FI}$ and $\lim_{q \rightarrow 1^-} Q_1 = Q_1^{FI}$, where $\langle Q_1^{FI}, F_1^{FI} \rangle$ is the solution to the following pair of equations:

$$u'(y - g - Q_1^{FI}) = v'(F_1^{FI}),$$

$$p_1 F_1^{FI} - Q_1^{FI} = 0.$$

Plugging (2.5b) into (2.5d) yields:

$$(1 - \tilde{q})v'(F_2(p_2)) = (1 - q)v'(F_1) + \frac{\lambda(p_2)}{(1 - p_1)\phi(p_2)}.$$

Note that $\lim_{q \rightarrow 1^-} (1 - \tilde{q})v'(F_2(p_2)) \geq \lim_{q \rightarrow 1^-} (1 - \tilde{q})u'(y + g) > 0$ while $\lim_{q \rightarrow 1^-} (1 -$

$q)v'(F_1) + \frac{\lambda(p_2)}{(1-p_1)\phi(p_2)} \leq 0$, which is a contradiction. ■

Lemma B4 *Fixing \tilde{q} , there exists a threshold $q_0(\tilde{q}) < 1$ such that $Q_2(p_2)=0$ for all $p_2 \in (0, 1]$ if $q > q_0(\tilde{q})$ and $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$ if $q < q_0(\tilde{q})$. Moreover, $q_0(\tilde{q})$ is weakly increasing in \tilde{q} .*

Proof. Lemma B1 states that $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$ if $q = \tilde{q}$. Similarly, Lemma B3 together with Lemma B2 tells that there exists at least one q with $q > \tilde{q}$ such that $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$. To prove the existence of threshold $q_0(\tilde{q})$, suppose to the contrary that there exist q' and q with $q' > q$ such that $Q_2(p_2) = 0$ and $Q'_2(p_2) > 0$ for all $p_2 \in (0, 1]$. Then $\lambda(p_2) = 0$, $\lambda'(p_2) \leq 0$, $\gamma(p_2) \geq 0$ and $\gamma'(p_2) = 0$.

From (2.5c) and (2.5d), we have:

$$\begin{aligned} v'(F_2(p_2)) &= u'(y + g - Q_2(p_2)) - \frac{\gamma(p_2)}{(1 - \tilde{q})(1 - p_1)\phi(p_2)} \\ &< u'(y + g - Q'_2(p_2)) = v'(F'_2(p_2)) \text{ for all } p_2 \in (0, 1]. \end{aligned}$$

The last strict inequality follows from that $\gamma(p_2) \geq 0$ and $Q'_2(p_2) > Q_2(p_2) = 0$.

Therefore $F_2(p_2) > F'_2(p_2)$ for all $p_2 \in (0, 1]$ by the strict concavity of $v(\cdot)$. Combining

conditions (2.5b) and (2.5c) yields:

$$\begin{aligned}
v'(F_1) &= \frac{1-\tilde{q}}{1-q}u'(y+g-Q_2(p_2)) - \frac{\lambda(p_2)+\gamma(p_2)}{(1-p_1)(1-q)\phi(p_2)} \\
&\leq \frac{1-\tilde{q}}{1-q}u'(y+g-Q_2(p_2)) \\
&< \frac{1-\tilde{q}}{1-q'}u'(y+g-Q'_2(p_2)) \\
&\leq \frac{1-\tilde{q}}{1-q'}u'(y+g-Q'_2(p_2)) - \frac{\lambda'(p_2)+\gamma'(p_2)}{(1-p_1)(1-q')\phi(p_2)} = v'(F'_1).
\end{aligned}$$

Thus, $F_1 > F'_1$ and $Q_1 < Q'_1$. Hence,

$$\begin{aligned}
0 &= (Q'_1 - p_1 F'_1) + (1-p_1)(1-q') \int_0^1 [Q'_2(p_2) - p_2 F'_2(p_2)] d\Phi(p_2) \\
&> (Q_1 - p_1 F_1) + (1-p_1)(1-q) \int_0^1 [Q_2(p_2) - p_2 F_2(p_2)] d\Phi(p_2) = 0,
\end{aligned}$$

which is a contradiction.

To prove that $q_0(\tilde{q})$ is weakly increasing in \tilde{q} , suppose there exists $\tilde{q}_1 > \tilde{q}_2$ such that $q_0(\tilde{q}_1) < q_0(\tilde{q}_2)$. It follows directly that $\tilde{q}_2 < \tilde{q}_1 < q_0(\tilde{q}_1) < q_0(\tilde{q}_2)$. Fix $q = \frac{q_0(\tilde{q}_1) + q_0(\tilde{q}_2)}{2}$. Because $q < q_0(\tilde{q}_2)$, all period 2 premiums except health state $p_2 = 0$ are positive if $\tilde{q} = \tilde{q}_2$. Similarly, because $q > q_0(\tilde{q}_1)$, all period 2 premiums are zero if $\tilde{q} = \tilde{q}_1$. However, Lemma B2 implies that $\tilde{q}_2 > \tilde{q}_1$, which is a contradiction. ■

Let $\bar{q} = q_0(0)$. Suppose $q < \bar{q}$. We want to show that $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$ and $\Delta \in [0, 1]$. Suppose to the contrary that there exists a \tilde{q} such that $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$. By Lemma B4, $q \geq q_0(\tilde{q}) \geq q_0(0) = \bar{q}$, which is a contradiction.

Suppose $q > \bar{q}$. If $\tilde{q} = 0$ (i.e. $\Delta = 1$), Lemma B4 implies that $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$. If $\tilde{q} = q$ (i.e. $\Delta = 0$), Lemma B1 implies that $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$. Thus, from Lemma B2 there exists a threshold $\bar{\Delta}(q) < 1$ for $q > \bar{q}$ such that $Q_2(p_2) = 0$ for all $p_2 \in (0, 1]$ if $\Delta > \bar{\Delta}(q)$ and $Q_2(p_2) > 0$ for all $p_2 \in (0, 1]$ if $\Delta < \bar{\Delta}$. ■

Proof of Proposition 9. Suppose to the contrary that $\hat{\Delta} < \Delta$ ($\hat{q} > \tilde{q}$) and $\hat{Q}_1 \geq Q_1$. Equation (2.7) implies that $\hat{F}_1 \leq F_1$. If health state p_2 binds under $\hat{\Delta}$, then $\hat{Q}_2(p_2) - p_2\hat{F}_2(p_2) = 0 \geq Q_2(p_2) - p_2F_2(p_2)$.

If health state p_2 does not bind under $\hat{\Delta}$, we have:

$$\begin{aligned} (1 - \tilde{q})v'(F_2(p_2)) &= (1 - q)v'(F_1) + \frac{\lambda(p_2)}{(1 - p_1)\phi(p_2)} \\ &\leq (1 - q)v'(\hat{F}_1) = (1 - \hat{q})v'(\hat{F}_2(p_2)). \end{aligned}$$

Because $\hat{q} > \tilde{q}$, we must have $(1 - \hat{q})v'(\hat{F}_2(p_2)) \geq (1 - \tilde{q})v'(F_2(p_2))$. Thus, $v'(\hat{F}_2(p_2)) > v'(F_2(p_2))$. By the strict concavity of $v(\cdot)$, $\hat{F}_2(p_2) < F_2(p_2)$ if $p_2 \in \mathcal{NB}$ under $\hat{\Delta}$.

If $\gamma(p_2) = 0$, we have $(1 - \hat{q})u'(y + g - \hat{Q}_2(p_2)) \geq (1 - \tilde{q})u'(y + g - Q_2(p_2)) \Rightarrow \hat{Q}_2(p_2) \geq Q_2(p_2)$. If $\gamma(p_2) > 0$, then we have $\hat{Q}_2(p_2) \geq Q_2(p_2) = 0$. Either way, $\hat{Q}_2(p_2) \geq Q_2(p_2)$. The profit under $\hat{\Delta}$ is:

$$\begin{aligned} &(\hat{Q}_1 - p_1\hat{F}_1) + (1 - p_1)(1 - q) \int_0^1 [\hat{Q}_2(p_2) - p_2\hat{F}_2(p_2)]d\Phi(p_2) \\ &> (Q_1 - p_1F_1) + (1 - p_1)(1 - q) \int_0^1 [Q_2(p_2) - p_2F_2(p_2)]d\Phi(p_2) = 0, \end{aligned}$$

where the last inequality follows from postulated $p_2^* < 1$. This is a contradiction to the zero-profit condition (2.2).

To prove that $\hat{p}_2^* > p_2^*$, suppose instead $\hat{p}_2^* \leq p_2^*$. It follows immediately that $\hat{Q}_2^* \leq Q_2^*$ and $\hat{F}_2^* \geq F_2^*$. Moreover, we have $\hat{Q}_1 < Q_1$ and $\hat{F}_1 > F_1$. The profit under $\hat{\Delta}$ is bounded above by

$$\begin{aligned}
& (\hat{Q}_1 - p_1 \hat{F}_1) + (1 - p_1)(1 - q) \left\{ \int_0^{\hat{p}_2^*} [\hat{Q}_2 - p_2 \hat{F}_2] d\Phi(p_2) + \int_{\hat{p}_2^*}^1 [\hat{Q}_2 - p_2 \hat{F}_2] d\Phi(p_2) \right\} \\
&= (\hat{Q}_1 - p_1 \hat{F}_1) + (1 - p_1)(1 - q) \int_{\hat{p}_2^*}^1 [\hat{Q}_2^* - p_2 \hat{F}_2^*] d\Phi(p_2) \\
&= (\hat{Q}_1 - p_1 \hat{F}_1) + (1 - p_1)(1 - q) \left\{ \int_{\hat{p}_2^*}^{p_2^*} [\hat{Q}_2^* - p_2 \hat{F}_2^*] d\Phi(p_2) + \int_{p_2^*}^1 [\hat{Q}_2^* - p_2 \hat{F}_2^*] d\Phi(p_2) \right\} \\
&< (Q_1 - p_1 F_1) + (1 - p_1)(1 - q) \int_{p_2^*}^1 [Q_2^* - p_2 F_2^*] d\Phi(p_2) = 0,
\end{aligned}$$

which again is a contradiction to the zero-profit condition (2.2). ■

Proof of Proposition 10. Similar to the proof of Proposition 9, we can show that if $p_2^* = 1$ for some $\hat{\Delta}$, $p_2^* = 1$ for all $\Delta < \hat{\Delta}$. Therefore it suffices to discuss the following three cases.

Case I: If the period 2 equilibrium contracts for all p_2 are spot contracts under some (q, Δ) . It is obvious that decreasing Δ does not change the equilibrium contracts and consumer welfare stays constant.

Case II: Suppose $p_2^* = 0$. By Lemma 10, $p_2^{*'} = 0$ for $\Delta' > \Delta$. In this case, $Q_2(p_2) = 0$ and $F_2(p_2)$ remain constant. Define F_2 by $F_2 \equiv F_2(p_2)$. The optimal contracts

can be pinned down by the following system of equations:

$$(1 - \tilde{q})v'(F_2) = (1 - q)v'(F_1), \quad (\text{B.2a})$$

$$v'(F_1) = u'(y - g - Q_1), \quad (\text{B.2b})$$

$$(Q_1 - p_1 F_1) - (1 - p_1)(1 - q)\bar{p}_2 F_2 = 0. \quad (\text{B.2c})$$

Taking derivative with respect to Δ for (B.2c) yields:

$$\frac{dQ_1}{d\Delta} - p_1 \frac{dF_1}{d\Delta} = (1 - p_1)(1 - q)\bar{p}_2 \frac{dF_2}{d\Delta}. \quad (\text{B.3})$$

The derivative with respect to Δ for $W(q, \Delta)$ can be simplified as:

$$\begin{aligned} & \frac{\partial W(q, \Delta)}{\partial \Delta} \\ &= -u'(y - g - Q_1) \frac{dQ_1}{d\Delta} + p_1 v'(F_1) \frac{dF_1}{d\Delta} + (1 - p_1)(1 - q)\bar{p}_2 v'(F_2) \frac{dF_2}{d\Delta} \\ &= v'(F_1) \left(-\frac{dQ_1}{d\Delta} + p_1 \frac{dF_1}{d\Delta} \right) + (1 - p_1)(1 - q)\bar{p}_2 v'(F_2) \frac{dF_2}{d\Delta} \\ &= -(v'(F_1) - v'(F_2)) \left(\frac{dQ_1}{d\Delta} - p_1 \frac{dF_1}{d\Delta} \right). \end{aligned}$$

Noting that $v'(F_1) - v'(F_2) = \frac{q - \tilde{q}}{1 - q} v'(F_2) \geq 0$ by (B.2a) and $\frac{dQ_1}{d\Delta} > 0$ and $\frac{dF_1}{d\Delta} < 0$

by Proposition 9, we must have $\frac{\partial W(q, \Delta)}{\partial \Delta} \leq 0$.

Case III: Suppose $0 < p_2^* < 1$, p_2^* is strictly decreasing Δ by Proposition 9. Thus,

there exists a one-to-one mapping between Δ and p_2^* . Once p_2^* is determined,

the optimal contract is pinned down. Hence, to show $W(q, \Delta)$ is decreasing in

Δ is equivalent to show that consumer welfare is increasing in p_2^* .

Denote $W^p(p_2^*)$ by:

$$\begin{aligned} W^p(p_2^*) = & [u(y - g - Q_1(p_2^*)) + p_1 v(F_1(p_2^*))] \\ & + (1 - p_1)(1 - q) \int_0^{p_2^*} [u(y + g - Q_2^{FI}(p_2)) + p_2 v(F_2^{FI}(p_2))] d\Phi(p_2) \\ & + (1 - p_1)(1 - q) \int_{p_2^*}^1 [u(y + g - Q_2^{FI}(p_2^*)) + p_2 v(F_2^{FI}(p_2^*))] d\Phi(p_2), \end{aligned}$$

where $\langle Q_1(p_2^*), F_1(p_2^*) \rangle$ is the solution to the following pair of equations:

$$u'(y - g - Q_1(p_2^*)) = v'(F_1(p_2^*)), \quad (\text{B.4a})$$

$$Q_1(p_2^*) - p_1 F_1(p_2^*) = (1 - p_1)(1 - q) \int_{p_2^*}^1 [p_2 F_2^{FI}(p_2^*) - Q_2^{FI}(p_2^*)] d\Phi(p_2). \quad (\text{B.4b})$$

Taking derivative with respect to p_2^* for (B.4b) yields:

$$(1 - p_1)(1 - q) \int_{p_2^*}^1 \left(p_2 \frac{dF_2^{FI}}{dp_2^*} - \frac{dQ_2^{FI}}{dp_2^*} \right) d\Phi(p_2) = - \left(p_1 \frac{dF_1}{dp_2^*} - \frac{dQ_1}{dp_2^*} \right).$$

Taking derivative with respect to p_2^* for $W^p(p_2^*)$ yields:

$$\begin{aligned}
& \frac{dW^p(p_2^*)}{dp_2^*} \\
&= v'(F_1) \left(p_1 \frac{dF_1}{dp_2^*} - \frac{dQ_1}{dp_2^*} \right) + (1-p_1)(1-q)[u(y+g-Q_2^{FI}(p_2^*)) + p_2 v(F_2^{FI}(p_2^*))] \\
&\quad + (1-p_1)(1-q) \int_{p_2^*}^1 v'(F_2) \left(p_2 \frac{dF_2^{FI}}{dp_2^*} - \frac{dQ_2^{FI}}{dp_2^*} \right) d\Phi(p_2) \\
&\quad - (1-p_1)(1-q)[u(y+g-Q_2^{FI}(p_2^*)) + p_2 v(F_2^{FI}(p_2^*))] \\
&= v'(F_1) \left(p_1 \frac{dF_1}{dp_2^*} - \frac{dQ_1}{dp_2^*} \right) + (1-p_1)(1-q) \int_{p_2^*}^1 v'(F_2) \left(p_2 \frac{dF_2^{FI}}{dp_2^*} - \frac{dQ_2^{FI}}{dp_2^*} \right) d\Phi(p_2) \\
&= [v'(F_1) - v'(F_2)] \left(p_1 \frac{dF_1}{dp_2^*} - \frac{dQ_1}{dp_2^*} \right).
\end{aligned}$$

By Proposition 9, p_2^* and F_1 are decreasing in Δ , and Q_1 is increasing in Δ .

Thus, $\frac{dF_1}{dp_2^*} \geq 0$ and $\frac{dQ_1}{dp_2^*} \leq 0$. Thus, $\frac{dW^p(p_2^*)}{dp_2^*} \geq 0 \Rightarrow \frac{\partial W(q, \Delta)}{\partial \Delta} \geq 0$.

■

Proof of Lemma 11. Suppose to the contrary that there exists a tuple (q, Δ) such that $Q_{2s}(p_2^i) = 0$ for some health state $p_2^i \in (0, 1]$. This implies that $\gamma(p_2^i) \geq 0$ and $\lambda(p_2^i) = 0$. From (2.19a) and (2.19c), we have:

$$(1-\tilde{q})u'(y+g) + \beta\tilde{q}u'(y+g+\beta V_{2s}(p_2^i)) = u'(y-g-Q_{1s}) + \frac{\lambda(p_2^i) + \gamma(p_2^i)}{(1-p_1)\phi(p_2^i)} \geq u'(y-g-Q_{1s}),$$

which is a contradiction since $(1-\tilde{q})u'(y+g) + \beta\tilde{q}u'(y+g+\beta V_{2s}(p_2^i)) \leq [1 - (1-\beta)\tilde{q}]u'(y+g) < u'(y-g-Q_{1s})$. ■

Proof of Lemma 13. Suppose there exists a tuple (q, Δ) such that $p_{2s}^* < p_1$, then

the no-lapsation condition (2.17) of the period 2 health state $p_2 = p_1$ does not bind (i.e. $\lambda(p_1) = 0$) and $Q_{2s}(p_1) - p_1 F_{2s}(p_1) < 0$.

Noting that $u'(y+g-Q_{2s}(p_1)) > (1-\tilde{q})u'(y+g-Q_{2s}(p_1)) + \beta\tilde{q}u'(y+g+\beta V_{2s}(p_1)) = u'(y-g-Q_{1s})$, we must have $Q_{2s}(p_1) > Q_{1s} + 2g$ and $F_{2s}(p_1) < F_{1s}$.

Hence, $0 > Q_{2s}(p_1) - p_1 F_{2s}(p_1) > Q_{1s} + 2g - p_1 F_{1s} \Rightarrow Q_{1s} - p_1 F_{1s} < -2g \leq 0$, which is a contradiction to (2.16). ■

Proof of Proposition 12. Suppose $\hat{\Delta} < \Delta$ ($\hat{q} > \tilde{q}$) and $\hat{F}_{1s} \leq F_{1s}$. This implies directly that $\hat{Q}_{1s} \geq Q_{1s}$.

If health state p_2 binds under $\hat{\Delta}$, then $\hat{Q}_{2s}(p_2) - p_2 \hat{F}_{2s}(p_2) = 0 \geq Q_{2s}(p_2) - p_2 F_{2s}(p_2)$.

If health state p_2 does not bind under $\hat{\Delta}$, from (2.19b) and (2.19d) we have:

$$\begin{aligned}
& (1 - \tilde{q})v'(F_{2s}(p_2)) + \beta\tilde{q}u'(y + g + \beta V_{2s}(p_2)) \\
&= v'(F_{1s}) + \frac{\lambda(p_2)}{(1 - p_1)\phi(p_2)} \\
&\leq v'(\hat{F}_{1s}) \\
&= (1 - \hat{q})v'(\hat{F}_{2s}(p_2)) + \beta\hat{q}u'(y + g + \beta \hat{V}_{2s}(p_2)). \tag{B.5}
\end{aligned}$$

Next, we prove that $\hat{F}_{2s}(p_2) < F_{2s}(p_2)$. Suppose not, then we must have $\hat{F}_{2s}(p_2) \geq$

$F_{2s}(p_2)$ and $\hat{Q}_{2s}(p_2) \leq Q_{2s}(p_2)$. This implies that $\hat{V}_{2s}(p_2) \geq V_{2s}(p_2)$. Thus,

$$\begin{aligned} & (1 - \tilde{q})v'(F_{2s}(p_2)) + \beta\tilde{q}u'(y + g + \beta V_{2s}(p_2)) \\ & > (1 - \hat{\tilde{q}})v'(F_{2s}(p_2)) + \beta\hat{\tilde{q}}u'(y + g + \beta V_{2s}(p_2)) \\ & \geq (1 - \hat{\tilde{q}})v'(\hat{F}_{2s}(p_2)) + \beta\hat{\tilde{q}}u'(y + g + \beta \hat{V}_{2s}(p_2)), \end{aligned}$$

which is a contradiction to (B.5). Thus, when health state p_2 does not bind under $\hat{\Delta}$, we must have $\hat{F}_{2s}(p_2) < F_{2s}(p_2)$ and $\hat{Q}_{2s}(p_2) > Q_{2s}(p_2) \Rightarrow \hat{Q}_{2s}(p_2) - p_2\hat{F}_{2s}(p_2) > Q_{2s}(p_2) - p_2F_{2s}(p_2)$. Hence,

$$\begin{aligned} & (\hat{Q}_{1s} - p_1\hat{F}_{1s}) + (1 - p_1) \int_0^1 [\hat{Q}_{2s}(p_2) - p_2\hat{F}_{2s}(p_2)]d\Phi(p_2) \\ & > (Q_{1s} - p_1F_{1s}) + (1 - p_1) \int_0^1 [Q_{2s}(p_2) - p_2F_{2s}(p_2)]d\Phi(p_2) = 0, \end{aligned}$$

which is a contradiction to (2.16). ■

Proof of Proposition 13.

Lemma B5 *Fixing $q \in [0, 1)$, if $p_{2s}^* < 1$ under Δ and $\hat{p}_{2s}^* = 1$ under $\hat{\Delta}$, then $\Delta > \hat{\Delta}$.*

Proof. Suppose instead $\Delta \leq \hat{\Delta}$ (i.e. $\tilde{q} \geq \hat{\tilde{q}}$). The threshold $\hat{p}_{2s}^* = 1$ implies that the period 2 contracts with $\hat{\Delta}$ are spot contracts for all $p_2 \in [0, 1]$. Thus, $\hat{Q}_{2s}(p_2) = Q_2^{FI}(p_2)$. From (2.19a) and (2.19c), we have:

$$(1 - \hat{\tilde{q}})u'(y + g - Q_2^{FI}(p_{2s}^*)) + \beta\hat{\tilde{q}}u'(y + g) \leq u'(y - g - \hat{Q}_1).$$

Similarly, we have:

$$(1 - \tilde{q})u'(y + g - Q_2(p_{2s}^*)) + \beta\tilde{q}u'(y + g + \beta V_2(p_{2s}^*)) = u'(y - g - Q_1).$$

Since $p_{2s}^* < 1$ and $\hat{p}_{2s}^* = 1$, we must have $Q_1 > \hat{Q}_1$ from (2.16), which implies that $u'(y - g - Q_1) > u'(y - g - \hat{Q}_1)$. Thus,

$$(1 - \hat{q})u'(y + g - Q_2^{FI}(p_{2s}^*)) + \beta\hat{q}u'(y + g) < (1 - \tilde{q})u'(y + g - Q_2(p_{2s}^*)) + \beta\tilde{q}u'(y + g + \beta V_2(p_{2s}^*)),$$

which is a contradiction since

$$\begin{aligned} & (1 - \hat{q})u'(y + g - Q_2^{FI}(p_{2s}^*)) + \beta\hat{q}u'(y + g) \\ & \geq (1 - \tilde{q})u'(y + g - Q_2^{FI}(p_{2s}^*)) + \beta\tilde{q}u'(y + g) \\ & = (1 - \tilde{q})u'(y + g - Q_2(p_{2s}^*)) + \beta\tilde{q}u'(y + g + \beta V_2(p_{2s}^*)), \end{aligned}$$

where the inequality follows from postulated $\tilde{q} \geq \hat{q}$ and the equality follows from Lemma 14. ■

By Lemma B5, it suffices to discuss two cases:

Case I: For all Δ , $p_{2s}^*(\tilde{q}) = 1$. Because the period 2 equilibrium contracts are spot contracts for all $p_2 \in [0, 1]$, $\langle Q_{2s}(p_2), F_{2s}(p_2) : p_2 \in [0, 1] \rangle$ are independent of Δ and $W_s(q, \Delta)$ is constant with respect to Δ .

Case II: There exists a threshold $\bar{\Delta}$ such that $p_{2s}^* < 1$ if $\Delta > \bar{\Delta}$ and $p_{2s}^* = 1$ if $\Delta < \bar{\Delta}$. If $\Delta < \bar{\Delta}$, the argument in Case I applies. If $\Delta > \bar{\Delta}$, by implicit

function theorem, $p_{2s}^*(\Delta)$ is continuous and differentiable. Notice that $\lambda(p_2) \leq 0$ if $p_2 \in \mathcal{B}_s$ and $\lambda(p_2) = 0$ if $p_2 \in \mathcal{NB}_s$. The zero-profit condition (2.16) can be written as:

$$(Q_{1s} - p_1 F_{1s}) + (1 - p_1) \int_{p_{2s}^*}^1 [Q_{2s}(p_2) - p_2 F_{2s}(p_2)] d\Phi(p_2) = 0.$$

Taking derivative with respect to Δ for the above equation yields:

$$\left(\frac{\partial Q_{1s}}{\partial \Delta} - p_1 \frac{\partial F_{1s}}{\partial \Delta} \right) + (1 - p_1) \int_{p_{2s}^*}^1 \left(\frac{\partial Q_{2s}(p_2)}{\partial \Delta} - p_2 \frac{\partial F_{2s}(p_2)}{\partial \Delta} \right) d\Phi(p_2) = 0. \quad (\text{B.6})$$

Taking derivative with respect to Δ for $W_s(q, \Delta)$ yields:

$$\begin{aligned} & \frac{\partial W_s(q, \Delta)}{\partial \Delta} \\ &= v'(F_{1s}) \left(p_1 \frac{\partial F_{1s}}{\partial \Delta} - \frac{\partial Q_{1s}}{\partial \Delta} \right) \\ & \quad + (1 - p_1) \int_{p_{2s}^*}^1 \left[\begin{array}{c} (1 - q)v'(F_{2s}) \\ + \beta q u'(y + g + \beta V_{2s}) \end{array} \right] \left(p_2 \frac{\partial F_{2s}}{\partial \Delta} - \frac{\partial Q_{2s}}{\partial \Delta} \right) d\Phi(p_2) \\ &= (1 - p_1) \int_{p_{2s}^*}^1 \left(\frac{\partial Q_{2s}}{\partial \Delta} - p_2 \frac{\partial F_{2s}}{\partial \Delta} \right) \left[\begin{array}{c} v'(F_{1s}) - (1 - q)v'(F_{2s}) \\ - \beta q u'(y + g + \beta V_{2s}) \end{array} \right] d\Phi(p_2) \\ &= (1 - p_1)(q - \tilde{q}) \int_{p_{2s}^*}^1 \left(\frac{\partial Q_{2s}}{\partial \Delta} - p_2 \frac{\partial F_{2s}}{\partial \Delta} \right) \left[\begin{array}{c} u'(y + g - Q_{2s}) \\ - \beta u'(y + g + \beta V_{2s}) \end{array} \right] d\Phi(p_2), \end{aligned}$$

where the second equality follows from (B.6) and the third equation follows from the fact that $(1 - \tilde{q})v'(F_{2s}(p_2)) + \beta \tilde{q}u'(y + g + \beta V_{2s}(p_2)) = \mu = v'(F_{1s})$

if $p_2 \in \mathcal{NB}$. Denote $x(p_2) \equiv \frac{\partial Q_{2s}(p_2; \Delta)}{\partial \Delta} - p_2 \frac{\partial F_{2s}(p_2; \Delta)}{\partial \Delta}$ and $y(p_2) \equiv u'(y + g - Q_{2s}(p_2; \Delta)) - \beta u'(y + g + \beta V_{2s}(p_2; \Delta))$. Because $Q_{2s}(p_2)$ and $V_{2s}(p_2)$ are both non-negative, $y(p_2) \geq (1 - \beta)u'(y + g) > 0$. Divide \mathcal{NB}_s into two subsets $\mathcal{NB}_s^+ \equiv \{p_2 | p_2 \in \mathcal{NB}_s, x(p_2) \geq 0\}$ and $\mathcal{NB}_s^- \equiv \{p_2 | p_2 \in \mathcal{NB}_s, x(p_2) < 0\}$ depending on the sign of $x(p_2)$. For $p_2 \in \mathcal{NB}_s$, we must have $\lambda(p_2) = 0$. Notice that $\tilde{q} = q(1 - \Delta)$. Combing (2.19b) and (2.19c) yields:

$$(1 - \tilde{q})u'(y + g - Q_{2s}) + \beta \tilde{q}u'(y + g + \beta V_{2s}) = v'(F_{1s}). \quad (\text{B.7})$$

Taking derivative with respect to Δ for (B.7) and rearranging yields:

$$\begin{aligned} & q \left[u'(y + g - Q_{2s}) - \beta u'(y + g + \beta V_{2s}) \right] \\ &= v''(F_{1s}) \frac{\partial F_{1s}}{\partial \Delta} + \beta^2 \tilde{q} u''(y + g + \beta V_{2s}) \left(\frac{\partial Q_{2s}}{\partial \Delta} - p_2 \frac{\partial F_{2s}}{\partial \Delta} \right) \\ &+ (1 - \tilde{q}) u''(y + g - Q_{2s}) \frac{\partial Q_{2s}}{\partial \Delta}. \end{aligned} \quad (\text{B.8})$$

Suppose $p_i \in \mathcal{NB}_s^+$ and $p_j \in \mathcal{NB}_s^-$, $x(p_i) \geq 0 > x(p_j)$ by definition. From (B.8)

we have $y(p_i) < y(p_j)$. Denote $\bar{y} \equiv \sup_{p_2 \in \mathcal{NB}_s^+} y(p_2)$ and $\underline{y} \equiv \inf_{p_2 \in \mathcal{NB}_s^-} y(p_2)$.

It follows directly that $\underline{y} \geq \bar{y}$. The derivative with respect to Δ for $W_s(q, \Delta)$

can be further simplified as:

$$\begin{aligned}
& \frac{\partial W_s(q, \Delta)}{\partial \Delta} \\
&= (1 - p_1)(q - \tilde{q}) \int_{p_{2s}^*}^1 x(p_2)y(p_2)d\Phi(p_2) \\
&= (1 - p_1)(q - \tilde{q}) \left(\int_{p_2 \in \mathcal{NB}_s^+} x(p_2)y(p_2)d\Phi(p_2) + \int_{p_2 \in \mathcal{NB}_s^-} x(p_2)y(p_2)d\Phi(p_2) \right) \\
&\leq (1 - p_1)(q - \tilde{q}) \left(\int_{p_2 \in \mathcal{NB}_s^+} x(p_2)\bar{y}d\Phi(p_2) + \int_{p_2 \in \mathcal{NB}_s^-} x(p_2)\underline{y}d\Phi(p_2) \right) \\
&\leq (1 - p_1)(q - \tilde{q})\bar{y} \int_{p_{2s}^*}^1 x(p_2)d\Phi(p_2) \\
&= (q - \tilde{q})\bar{y} \left(p_1 \frac{\partial F_{1s}}{\partial \Delta} - \frac{\partial Q_{1s}}{\partial \Delta} \right),
\end{aligned}$$

where the last equality follows from (B.6). Proposition 12 implies that $p_1 \frac{\partial F_{1s}}{\partial \Delta} -$

$\frac{\partial Q_{1s}}{\partial \Delta} \leq 0$. Together with the fact that $\bar{y} > 0$, we must have $\frac{\partial W_s(q, \Delta)}{\partial \Delta} \leq 0$.

■

Proof of Proposition 14.

Lemma B6 *Fixing \tilde{q} , $\lim_{q \rightarrow 1} (1 - p_1)(1 - q) \int_0^1 [u(y + g - Q_2(p_2)) + p_2 v(F_2(p_2))] d\Phi(p_2) = 0$.*

Proof. The result is obvious if $v(\cdot)$ is bounded. Suppose $\lim_{c \rightarrow \infty} v(c) = \infty$. By Lemma B4, $Q_2(p_2) = 0$ for all $p_2 \in [0, 1]$ if $q > q_0(\tilde{q})$. Thus, $\lim_{q \rightarrow 1} Q_2(p_2) = 0$ and $\lim_{q \rightarrow 1} u(y + g - Q_2(p_2)) = u(y + g)$. The zero-profit condition (2.2) can be rewritten as:

$$(1 - p_1)(1 - q) \int_0^1 F_2(p_2) d\Phi(p_2) = Q_1 - p_1 F_1,$$

where $0 \leq Q_1(q) - p_1 F_1(q) \leq y - g$. Thus,

$$\begin{aligned} 0 &\leq \int_0^1 p_2 v(F_2(p_2)) d\Phi(p_2) \\ &\leq \int_0^1 v(p_2 F_2(p_2)) d\Phi(p_2) \\ &\leq v \left(\int_0^1 p_2 F_2(p_2) d\Phi(p_2) \right) \leq v \left(\frac{y - g}{(1 - p_1)(1 - q)} \right), \end{aligned}$$

where the second and third inequalities come from the concavity of $v(\cdot)$. Hence,

$$\begin{aligned} 0 &\leq \lim_{q \rightarrow 1} (1 - p_1)(1 - q) \int_0^1 p_2 v(F_2(p_2)) d\Phi(p_2) \\ &\leq \lim_{q \rightarrow 1} (1 - p_1)(1 - q) v \left(\frac{y - g}{(1 - p_1)(1 - q)} \right) = 0. \end{aligned}$$

The last equality holds due to L'Hospital rule and the assumption that $\lim_{c \rightarrow \infty} v'(c) = 0$. To see this,

$$\lim_{q \rightarrow 1} (1 - p_1)(1 - q) v \left(\frac{y - g}{(1 - p_1)(1 - q)} \right) = \lim_{x \rightarrow 0} \frac{v \left(\frac{y - g}{x} \right)}{\frac{1}{x}} = (y - g) \lim_{x \rightarrow 0} v' \left(\frac{y - g}{x} \right) = 0.$$

Therefore $\lim_{q \rightarrow 1} (1 - p_1)(1 - q) \int_0^1 p_2 v(F_2(p_2)) d\Phi(p_2) = 0 \Rightarrow \lim_{q \rightarrow 1} (1 - p_1)(1 - q) \int_0^1 [u(y + g - Q_2(p_2)) + p_2 v(F_2(p_2))] d\Phi(p_2) = 0$. ■

Lemma B7 *Let $W^\dagger(q, \tilde{q}) \equiv W(q, \frac{q - \tilde{q}}{q})$. If $\eta(c) \geq \alpha > 1$ for all c , $\lim_{q \rightarrow 1} W^\dagger(q, \tilde{q}) = [u(0) + p_1 v(0)] + (1 - p_1)u(y + g)$ for all $\tilde{q} \in [0, 1]$.*

Proof. Fix \tilde{q} . When $q > q_0(\tilde{q})$, $Q_2(p_2) = 0$ and $\lambda(p_2) = 0$ for all $p_2 \in (0, 1]$ by Lemma

B4. Combing (2.5b) and (2.5d) yields:

$$(1 - \tilde{q})v'(F_2(p_2)) = (1 - q)v'(F_1). \quad (\text{B.9})$$

Notice that $F_2(p_2) \geq F_1$ because $\tilde{q} \leq q$. Because $\eta(c) = -\frac{v'(c)}{cv''(c)} \geq \alpha$, $\frac{1}{\alpha}v'(c) + cv''(c) \geq 0 \Rightarrow c^{\frac{1}{\alpha}}v'(c)$ is weakly increasing in c . Thus,

$$F_2^{\frac{1}{\alpha}}(p_2)v'(F_2(p_2)) \geq F_1^{\frac{1}{\alpha}}v'(F_1). \quad (\text{B.10})$$

Equation (B.9), together with (B.10), implies that:

$$\frac{1 - q}{1 - \tilde{q}} = \frac{v'(F_2(p_2))}{v'(F_1(q))} \geq \left(\frac{F_1}{F_2(p_2)} \right)^{\frac{1}{\alpha}}.$$

Rearranging the above inequality yields:

$$F_2(p_2) \geq F_1 \left(\frac{1 - \tilde{q}}{1 - q} \right)^{\alpha}.$$

From the zero-profit condition (2.2) and the inequality above, we have:

$$\begin{aligned} & p_1 F_1 + (1 - p_1)(1 - q)\bar{p}_2 F_1 \left(\frac{1 - \tilde{q}}{1 - q} \right)^{\alpha} \\ & \leq p_1 F_1 + (1 - p_1)(1 - q) \int_0^1 p_2 F_2(p_2) d\Phi(p_2) = Q_1 \leq y - g, \end{aligned}$$

where \bar{p}_2 is defined as $\bar{p}_2 \equiv \int_0^1 p_2 d\Phi(p_2)$. Thus,

$$0 \leq F_1 \leq \frac{y - g}{p_1 + (1 - p_1)(1 - q)\bar{p}_2 \left(\frac{1 - \tilde{q}}{1 - q}\right)^\alpha}.$$

Taking limit of the above inequality yields:

$$0 \leq \lim_{q \rightarrow 1} F_1(q) \leq \lim_{q \rightarrow 1} \frac{y - g}{p_1 + (1 - p_1)(1 - q)\bar{p}_2 \left(\frac{1 - \tilde{q}}{1 - q}\right)^\alpha} = 0.$$

$$\Rightarrow \lim_{q \rightarrow 1} F_1 = 0 \text{ and } \lim_{q \rightarrow 1} Q_1 = y - g.$$

Thus, $\lim_{q \rightarrow 1} W^\dagger(q, \tilde{q}) = (u(0) + p_1 v(0)) + (1 - p_1)u(y + g)$. ■

Lemma B8 Let $W_s^\dagger(q, \tilde{q}) \equiv W_s(q, \frac{q - \tilde{q}}{q})$. Suppose $\eta(c) \geq \alpha > 1$ for all c . Fixing $\tilde{q} \in [0, 1)$, there exists a threshold \underline{q} such that for $q \geq \underline{q}$, $W_s^\dagger(q, \tilde{q}) > W^\dagger(q, \tilde{q})$.

Proof. Fixing \tilde{q} , note that the equilibrium contract with the presence of the settlement market does not depend on q . Hence, $\lim_{q \rightarrow 1} W_s^\dagger(q, \tilde{q}) = [u(y - g - Q_{1s}) + p_1 v(F_{1s})] + (1 - p_1) \int_0^1 u(y + g + \beta V_{2s}(p_2)) d\Phi(p_2)$.

Because $(F_{1s}, Q_{1s}) = (0, y - g)$ is possibly the most front-loading contract in period 1, we must have $u(y - g - Q_{1s}) + p_1 v(F_{1s}) > u(0) + p_1 v(0)$. Together with the fact

that $u(y + g) \leq u(y + g + \beta V_{2s}(p_2))$, we must have

$$\begin{aligned}
& \lim_{q \rightarrow 1} W^\dagger(q, \tilde{q}) \\
&= [u(0) + p_1 v(0)] + (1 - p_1)u(y + g) \\
&< [u(y - g - Q_{1s}) + p_1 v(F_{1s})] + (1 - p_1) \int_0^1 u(y + g + \beta V_{2s}(p_2)) d\Phi(p_2) \\
&= \lim_{q \rightarrow 1} W_s^\dagger(q, \tilde{q}).
\end{aligned}$$

Notice that $W^\dagger(\tilde{q}, \tilde{q}) \geq W_s^\dagger(\tilde{q}, \tilde{q})$ by Lemma 15 and $\lim_{q \rightarrow 1} W^\dagger(q, \tilde{q}) < \lim_{q \rightarrow 1} W_s^\dagger(q, \tilde{q})$.

Fixing \tilde{q} , by the continuity of $W^\dagger(\cdot, \cdot)$ and $W_s^\dagger(\cdot, \cdot)$, there exist a threshold $\underline{q}(\tilde{q}) \in (\tilde{q}, 1)$

such that $W_s^\dagger(q, \tilde{q}) > W^\dagger(q, \tilde{q})$ for $q \geq \underline{q}$. ■

Let $\bar{q} \equiv \underline{q}(0)$. On the one hand, Lemma B8 implies that $W_s(q, 1) = W_s^\dagger(q, 0) > W^\dagger(q, 0) = W(q, 1)$ for $q \geq \underline{q}(0)$. On the other hand, Lemma 15 implies that $W(q, 0) \geq W_s(q, 0)$. Fix $q \geq \bar{q}$. Because $W(q, \Delta)$ and $W_s(q, \Delta)$ are both continuous in Δ , there exists a threshold $\bar{\Delta}$ such that $W_s(q, \Delta) > W(q, \Delta)$ if $\Delta > \bar{\Delta}$.

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