

COMMUNICATION AND THE GENESIS OF STRUCTURE

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Introduction

In the following an attempt is made to state and give evidence for a fairly definite, basic, and entirely general law involving communication processes in animals, machines, man, or society. In the context of society, the law could read: any communication process, once initiated and maintained, leads to the genesis of social structure—whether or not such structure is anticipated or deemed desirable. Without elaboration such a statement might be misinterpreted, particularly since agreement on the basic terms—communication process and structure—can hardly be presupposed. Also some of the implications of the law might otherwise be overlooked.

Let me therefore proceed more carefully. First, I will illustrate the content of the proposed law by intuitively obvious examples. Second, I will discuss the methodological basis of the approach taken in this paper. Third and fourth I will treat the concepts of "structure" and "communication" as formally as necessary for the argument, leaving the technical definitions for an appendix. Fifth, I will report the findings. Sixth, I will consider and discount several apparent counter-examples and, finally, I will suggest some consequences for theory construction and philosophical perspective.

Natural History of Systems of Communicating Components

Let us see what happens to individuals or social organizations when they communicate with each other over time.

Suppose one has a chance of observing the interaction between newlyweds. There is much experimentation, much to talk about, much quarreling. Neither party has experience with what a marriage demands. Consequently, many risks are taken and many foolish mistakes are made. Their results communicate how far each can go and what each can do best. As time passes, say, ten, twenty, thirty years, one observes that the daily activity has become routinized, a division of responsibility has emerged and has become rigid, and their life has become increasingly interdependent. In addition, conversation is now quite stereotyped and in fact there is little to discuss. What has developed over time is a pattern of mutually acceptable interdependencies: a structure. How strong such interdependencies can become is felt when one partner dies.

Suppose one forms an ad hoc group of individuals that may be characterized by their possession of certain attitudes, technical expertise,

and communicative skills, and let them interact freely, say, a month or longer. The result is similar to the above example. Experientially we know that, under conditions of communication, attitudes may not only be modified but they may also become uniform or polarized, whatever the case may be. In addition, functional and status differentiations appear. For example, a leader may emerge together with a hierarchical form of organization. Rules of procedure are adopted, patterns of punishment and rewards develop and may become legitimized. These in turn coordinate the behavior of the group members towards each other and towards common aims. This is not to imply that such a group will work without conflict or as a rational unit at the termination point of the experiment, but that cognition and behavior become interdependent, correlated across individuals and patterned.

Suppose one observes a newspaper starting with new ideas for publishing and seeking to attract a large audience. Initially, everything seems possible. The editors explore new topics and the few readers that do respond are happy to find something radically different. But as time passes, both readers and non-readers begin to develop expectations, and editors experience more and more what sells and what does not. Disappointments experienced by readers have consequences for further subscription and purchases at newsstands, and apparent editorial mistakes as measured through a drop in sales impose constraints on the newspaper's subsequent make-up and content. In order to survive, the paper's editorial formula may thus become increasingly stereotyped, its content predictable, its readership stable, etc., which are all marks of the emergence of a structure.

Suppose one were to consider the history of political conventions in the United States from the time before television was admitted to date. The alleged purpose of such a convention is to elect the next presidential candidate and to attend to various party business. Because of its claim to provide first-hand and undistorted coverage of public events, television seems to be the ideal observer. However, as several studies have shown, people who are aware of being on television behave quite differently than they would otherwise. In front of a camera, individuals tend to smile, and to act important; crowds show greater excitement and become more dramatic in gesture; speakers behave as though they were addressing the mass audiences they envision. Conversely, the television camera—or rather the crew that operates it—tends to seek

out television-friendly individuals, groups of people who are engaged in dramatic acts, and political personalities that have something to say beyond the group of the small number of delegates present. In addition, as the television camera is known to focus more likely on action and on sensational events than on ordinary procedure, groups participating in a convention employ rather strange devices and exhibit highly unusual behaviors to attract the attention of the camera and to make themselves more visible than their political opponents. As a consequence, television, whose existence and active search for news is communicated to the delegates and the latter's reactions to seeing themselves in the news, in part creates the events which are finally transmitted to the public at large. Not only are the events at a convention thus partly artificial; the very organization of such conventions has over the years been incrementally transformed so as to accommodate both the demands of the delegates to be exposed and of the networks to catch events that are capable of attracting large audiences. Thus, a political decision-making process has restructured itself to become at least to a large extent a public ritual, a show. The structure that has emerged in the cause of communication is evident in the resulting complex interdependence of convention participants, including television people and the press, regarding the scheduling of events, regarding their reliance on information, regarding their gaining satisfactory experiences and, finally, regarding their political success.

Another class of situations relating the genesis of structure to communication processes is considered by several theories of relatively short interpersonal encounters, notably theories advanced by Ruesch and Bateson (1951), Watzlawick, et al. (1967), and Goffman (1959). They all recognize one distinction as crucial which Watzlawick, et al. call the content aspect of communication and the relationship aspect of communication. Thus the two messages: "It is important to release the clutch gradually and smoothly," and "Just let the clutch go, it'll ruin the transmission in no time" convey approximately the same content but define different relationships between speaker and listener. This is also the basis for distinctions among levels of communication. Relative to the content aspect of communication, the relationship aspect of communication is regarded as meta-communication. In this sense, the message "This is play" specifies how subsequent messages are to be interpreted; it refers to a relationship among individuals and must therefore be regarded as meta-communication, while the message "The mail has arrived" conveys information about an event outside the communicators and thus primarily conveys content.

It seems agreed not only that the relationship aspect of communication qua meta-communication is logically prior to the content aspect in the sense

of classifying, constraining, or controlling the interpretation of the latter, but also that mutual agreement on the meta-communication level is a precondition for conveying any information on the content level of communication. For example, Goffman analyzes many interpersonal encounters which all show that the parties of a spontaneous encounter must first engage in defining a situation which includes the establishment of binding relationships among themselves and can then—only after such a framework for discussion has been agreed upon—exchange substantial information. Initiating debates about procedural matters, e.g., about the shape of a table preceding substantive peace negotiations, provide familiar examples. Forms of greetings, the first 30 seconds of telephone conversations, or certain visible gestures and signs presumably have similar meta-communicative functions. Unless the situation including the relationships between the parties concerned is sufficiently defined and mutually understood, the assertion "play," for example, can have a thousand different meanings, ranging from a request to use the piano, a suggestion to go to the theater, to merely pronouncing what was just completed on the scrabble board.

Since structure is rare and nature is often seen as tending towards states of ever increasing entropy, emergent structures are likely to be thought of as serving some basic human needs, as man-made, as rationally comprehensible, and as the result of will. Thus, the cognitive structuring of an interpersonal communication situation down the logical hierarchy seems easily explainable as a device for reducing ambiguity (for increasing communicative efficiency) in the search for subjective meaningfulness. Also, the desire to define or establish beneficial social relationships with the other party of an encounter—on which Goffman's explanations primarily rest—implies purpose, intentionality, and successful control.

While the last example concerned subjective and cognitive structures and dealt with communications as conscious correlates thereof, the other four concern inter-individual structures which are more central to the argument. Here, too, intentionality seems to be the primary source of explanation. This form of reasoning is manifest when one or a few individuals are held responsible for the emergence of an undesirable group structure, e.g., a tyrannical domination by one. It is evident in the press's defense of mediocre journalism in terms of "this is what the public wants." And it is the sort of explanation that underlies the reasoning which brings one rather than both parties of a marriage to the psychiatrist. Reference to certain individual needs and their satisfaction is also implied when the subject concerns such abstract functional prerequisites of social groups as the predictability of group processes as an explanation for the social structure that develops.

Even though it seems subjectively satisfying to make the intentionality of the human components of complex communication situations responsible for generative aspects of social life—a view that has a long tradition in Western philosophy—it is my contention that such explanations are neither necessary nor sufficient to account for the fact of structure growth. Accordingly, the significance of purposiveness is overshadowed by other processes, though its influence on the direction of the growth is not to be denied.

Methodological Basis of the Proposed Law

The question thus posed is which of the many characteristics of such communication situations as described above are necessary and sufficient to account for the observed tendency of structure to develop. In terms of traditional behavioral methodology, the most important prerequisite for deciding among possible answers is not only the investigator's ability to operationally distinguish the candidates for such explanations, but also to make them vary independently of each other. This is the approach which psychological experiments and survey research have taken either when observing the behavior of subjects in various forms of physical isolation or when eliminating the effects of variables on each other statistically. However, by taking individuals, small groups, or social organizations as objects of observation in the very nature of which such notions as purpose, preference for order or for meaningfulness, and communications are so confusedly correlated with each other, an absence of either characteristics hardly occurs in realistic settings. Thus, traditional behavioral methods may well be incapable of an unequivocal answer.

However, there is another investigative strategy available. This is simulation. It involves, first, a mere conceptual isolation of those characteristics of a process which are of interest to the investigator from that complex of interacting social variables that are observable in principle. Second, the dynamic properties of the characteristics in question must be formalized, and unambiguously and completely stated in an algorithmic language which can serve as the instruction to a computer. This constitutes the simulation model of the process. Third, the consequences of the dynamic properties are explored by repeated computer runs, possibly on different sets of initial data configurations or under different conditions. And, finally, real world events must be matched with the obtained results so as to assess the validity of the model and its interpretations.

This method allows one to study the logical and dynamic consequences of any algorithmically specifiable characteristic—social or otherwise—without either unduly disturbing the object of analysis (as is the case with interviewees) or artificially confining the behavior in question (as is the

case of subjects in psychological experiments). In fact, the object of analysis to which the simulation model refers can be left entirely untouched by the analysis. The significance of the results is testable just as in any psychological experiment, except that the validity of a test depends on the degree to which the simulation model is a homomorphic representation of the process in question.

It is thereby possible to study in isolation those variables that are in reality highly correlated and difficult if not impossible to control separately. It is also conceivable that the resultant findings are more definite and conclusive than those obtained by traditional behavioral methods, because confounding effects of extraneous variables which occur in nearly every social situation are excluded from a simulation model. Although one can thus not presume that the variables studied by this means do occur in as pure a form in real situations as they appear in a simulation model, the model leads at least to definite conclusions about their dynamic consequences. This is often precluded in verbal reasoning.

Another advantage of this investigative strategy lies in the requirement to make explicit what components and which dynamic properties of a situation are to be considered. This imposes on the analyst the discipline of clarifying the concepts he uses. The simulation model can in principle be used by other interested scholars and may thus serve to establish unambiguous inter-scientific communication about a chosen subject matter. Although simulation models can be complex and difficult to track down mathematically, their proper use makes the findings transparent and avoids many of the uncertainties due to subjectively disparate interpretations.

The reader will recognize the above steps in the following presentation.

Structure

How can "structure" be defined so that its presence is measurable?

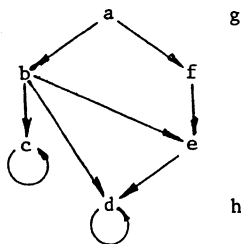
Nominal definitions of structure often refer to sets of relationships. Hence, with such a definition in mind, the structure of a family, for example, should be described in terms of the relationships that hold at any one time between its members. Typical names for such relations are "is married to," "is descendant of," "is mother of," "dominates," "controls," "has more prestige than," "is less likely to get what he wants than," "conspires with," etc. An elaborate network of such relationships would no doubt be illustrative of how a family is organized, but is analytically not very fruitful. Let me therefore approach the concept of structure via a rigorous definition of relation.

One of Wiener's (1914) early contributions to the communication sciences is the equation of relation with a particular kind of set. More specifically, according to what mathematicians now accept

everywhere, a relation is a proper subset of the Cartesian product of two or more sets. Since the connection between this concept and its more intuitive notion is not obvious, let me make it clear. Take the well known relation "is greater than" as an example. The Cartesian product of the set A of real numbers with itself is the set of all ordered pairs of real numbers, say, (a_1, a_2) . Whether $a_1 > a_2$ is true or not can be established for any pair. The subset for which this is true defines the relation "is greater than."

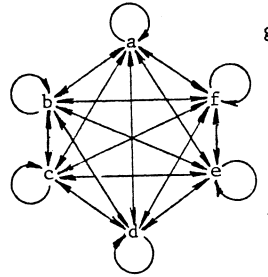
Note that relations do not require magnitudes such as implied by numbers. The relations "sent a letter to," "is responsible for," "is partner of," define subsets of pairs of individuals. Note further that relations need not be binary, i.e., they need not consist of pairs only. In the context of human genetics, the relation "is descendant of" requires at least three individuals, the two parents and the offspring, and is at least tertiary. When it is said of a person that "he votes democrat provided that his neighbor voted republican," a quaternary relation (involving two voters and two votes) is asserted, etc. Finally, note that relations do not require names to be defined. A political institution which acts so as to prevent the occurrence of certain combinations of individual behaviors in effect defines a relation (between individual behaviors). And an interviewee establishes a relation between sets of questions and sets of answers by his very response, regardless of whether an investigator has or will find a name for this behavior.

Relations can be distinguished by their properties, for example, symmetry, reflexivity, transitivity, equivalence. Of particular interest is the rectangularity of a relation (Ashby 1964) which is synonymous with the notion of orthogonality and of decomposability. Roughly, a relation is rectangular if it can be regarded as the Cartesian product of its components and can thus be decomposed into parts without loss. Formally, a relation R is decomposable into two components if for all pairs: (a,b) and (c,d) of R implies that (a,d) and (c,b) is in R. This property is again easily illustrated. Let us pair all individuals a, b, c, d, e, f, g, and h, and ascertain whether the relation "is superordinate to" holds true. For purposes of illustration, let the hypothetical data be represented both as a directed graph and as a subset:



	a	b	c	d	e	f	g	h
a								
b		■						
c			■					
d				■				
e					■			
f						■		
g								
h								

Clearly, a is directly superordinate to b and f and indirectly superordinate to all others except g and h which is nowhere included in the relation. Since "b→c" and "f→e" does not imply "b→e" and "f→c", the relation is not decomposable. Now suppose the findings were to indicate that within the set (a, b, c, d, e, f) everyone is superordinate to everyone else including himself, then the graph would appear maximally connected and the subset would appear rectangular:

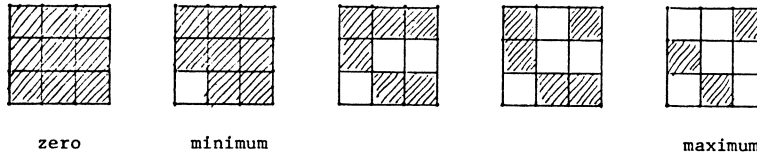


	a	b	c	d	e	f	g	h
a	■	■	■	■	■	■		
b	■	■	■	■	■	■		
c	■	■	■	■	■	■		
d	■	■	■	■	■	■		
e	■	■	■	■	■	■		
f	■	■	■	■	■	■		
g								
h								

Hence, the relation is decomposable into one set of which some individuals do and some others do not have the property of, say, being members of a social organization. The relation "is superordinate to" would then appear quite artificial if not without meaning. It is apparent that the non-decomposability of a relation is a very crucial property. I take it to delineate the notion of structure. Furthermore, I would like to restrict this notion to situations in which no dynamic is implied and the component sets all refer to the same point in time. Thus, in the context of this paper, a structure is a non-decomposable relation at any one moment in time or with possible time differences between the relata ignored.

Illustrations of structure are numerous. A truth table in logic defines a structure: if a & b and c & d are true then a & b and c & d cannot be true. The balance theories of affective cognition make assumptions about cognitive structure. In its most primitive form, they suggest that "(+) associated with (+)" and "(-) associated with (-)" will occur, while "(+) associated with (-)" and "(-) associated with (+)" will not. In the context of society, social roles are often thought to complement each other. Thus, situations of policeman-traffic violator, salesman-customer, mother-child encounters possess structure to the extent that the parties' actions are mutually constrained and not freely reversible. Similarly, when a group of individuals cooperate in completing a task which is such that the deviation from a prescribed course of action by only one of its members will prevent the group from completing it, the group exhibits a certain structure precisely because it is not decomposable into individuals parts. One can also assess the strength of a structure or talk about the

amount of structure. When a relation is fully decomposable, e.g., the group's task is such that every member's behavior is free and mutually unrestrained, the amount of structure is zero. When neither (a,d) nor (c,b) of the above definition is true (occurs or is permissible), then, in the context of a social group, its member's behavior is fully determined without freedom of choice and the amount of structure is at its maximum. Between the two extremes many shades exist. Intuitively different amounts of structure may be depicted as follows:



But since this paper does not require a calculus of this quantity, I will not settle on any of the possible measuring functions. I suppose psychologists would prefer to express such quantities by familiar measures of the strength of an association, information theorists may define it in terms of the quantity of transmission between variables, and cyberneticians may talk about the strength of a constraint. Here I merely suggest that references to the amount of structure are not merely nominal; they can be quantified at any moment.

Those technical details of the definition of structure which are necessary to this paper may be found in Appendix A.

Communication

Communication is another concept that requires a more formal definition. The nominal definitions of some scholars restrict it to human participants and thereby exclude the involvement of animals and machines in the process. The definitions of others stress the reliance on language or on technology as a medium, and still others insist on the presence of intentionality, awareness of social contexts and the like. I prefer to regard communication as a process of transmission of information between two or more parts of a system that are identifiable in time and in space regardless of the materiality of the parts. And I would demand that any such definition of communication lend itself to objective tests for the existence of the process once adequate data are gathered. Most nominal definitions are quite inadequate in this respect, and I have argued elsewhere that the data which are intended to preserve the evidence about communication processes must have a particular form and require analytical procedures that are different from those employed by most traditional behavioral scientists (Krippendorff 1970).

Three notions are basic to any definition of communication. The first is that it involves information, pattern structure, a configuration, a mes-

sage, however this is measured or described. The material manifestation of information is less important than the variability. The second is that it refers to a process and involves time, i.e., information is maintained throughout a transformation whether this is one of coding, transmission, memory or of computation. Communication is asymmetrical with respect to time and has a direction. The third is that it mediates between different points in space, i.e., information is translocated in the process. This notion is probably dispensable when one talks about memory as a process of

transmission of information from the past to the present. But, generally, communication requires reference to the behavior of two or more individuals, it states a dynamic dependency between an organization's input and its output, or it relates processes realized in machines with cognitive processes in the mind of their operators, each occupying different geographical locations.

Intuitively, when all members of a social group behave and talk in such a way that none of the previous gestures or utterances has an influence on their current behavior, then, in spite of the verbal noise that is generated, the situation does not require communication as an explanatory construct. The individuals could have been observed in isolation from each other with the same result. Mediation is not evident in the situation. Similarly, if the individuals comprising a market have access to television but their political attitudes, their kind of work, or their consumer behavior is not the least influenced by the presence or absence of television or by particular shows, then this medium can hardly be said to be one of communication. When communication is present, one should expect that the individuals comprising a social group are at least sometimes influenced in their behavior by what other group members did in the past, so that the joint trajectory of behavior, for example, the course of a conversation, is influenced if not determined by more than one communicator at different points in time. If television is a medium of communication—and I have no doubt that it is—then one should expect that the information presented on the screen should at least sometimes be absorbed and transformed by its audience so that its current behavior cannot be explained in full without considering to what it was exposed in the past. The study of communication cannot stop at describing what is said or how something is represented but also must consider the consequences of transmitting information, how it is utilized, what becomes of it, which changes are affected or

prevented. If the time dimension is omitted from analysis, then the process character of communication is lost. If references to sender and receiver are omitted in the data, then the mediating character of communication cannot be analyzed. If data are such that they are capable of containing all defining characteristics of communication and neither the maintenance over time of information nor the mediation of influence across individuals or social organizations is evident, then communication is dispensable in accounting for what goes on in a situation.

It should be emphasized that the basic tests for the existence of communication have been worked out by Ashby (1962). However, in the context of this paper, I can neither go into their formalisms or state them rigorously, but I will outline the essential idea. Suppose one observes the trajectory of behavior of k joint components of a system, say, of a social organization involving k offices. One could record this trajectory, for example, as a set of transitions from one k -tuple to the next, each k -tuple describing a complex state involving all k components. Given such a record, one can now project this set onto each distinguishable component, by which one would obtain the state to state transition of each office just as if it were observed isolated from the remainder, and determine whether and to what extent the joint behavior can be explained as the mere sum of its parts. If this cannot be so explained, then communication must be said to be present, and the extent to which this is not so explainable serves as a measure of the amount of information transmitted between those components, i.e., between the offices of the organization. Thus, "communication is absent" implies that the set of transitions that is obtained by aggregating the transitions of separated components is equal to the set of transitions that have been observed on all components jointly.

$$\text{Communication is absent} \iff \left\{ \begin{array}{l} \text{Trajectories of behavior obtained by} \\ \text{aggregating the trajectories observed} \\ \text{on each component separately} \end{array} \right\} = \left\{ \begin{array}{l} \text{Trajectories of behavior observed} \\ \text{on all components jointly} \end{array} \right\}$$

This test is implicit in several notions of information theory, satisfies the basic notions set forth above, and can be adapted to a variety of different situations and measures. The testing for communication is thus perfectly general and straightforward. (Technical details may be found in Appendix A.)

Procedure and Results

To ascertain whether and under what condi-

tions structure develops among communicators, a computer program was written. This incorporated as its most essential ingredient two artificial communicators which could be given a large number of behaviors and be communicatively linked in numerous ways. This effort corresponds to the third step as described above.

The behavior of each of the two artificial communicators was specified by a three-dimensional transition matrix with entries indicating the probability that a communicator will emit a certain message given that he occupied a certain internal state and received a certain message. The communication between the two artificial communicators was specified in terms of the probabilities that a certain message sent results in a certain message received. This accounted for the possibility of transmission errors, uncertainty of interpretation, etc.

The program was designed to perform many experiments, each with randomly chosen behavior and communication links. In each case, the amount of communication present in the situation and the emergent quantity of structure was assessed by means of multivariate information transmission measures (McGill 1954). The aim was to construct a simple two-by-two table of frequencies which would indicate the strength of the relation between communication and the genesis of structure. In case the results were to turn out discouraging, it was planned to make the communicators' behavior goal-oriented, to add to their memory, and the like.

Writing the program turned out to be more difficult than anticipated and required an undue amount of computer time. In addition, during the debugging stage, test runs produced only situations in which both communication was present and structure developed. This was for a long time discouraging. An error could not be found.

It was at this point that another stab at the problem was taken, and an attempt was made to shortcut the simulation somewhat by considering random mappings. To make this a manageable task, it was assumed that the two communicators behave deterministically and that noise is absent during the transmission process. Thus, if $a \in A^t$ is the message received by the first communicator, $b \in B^t$ is his internal state at the same time t and $b' \in B^{t+1}$ is both the message he will subsequently emit and his next internal state, the first communicator's behavior comes to be described by the mapping:

$$A \times B^t \longrightarrow B^{t+1}$$

and the second communicator's behavior can be defined analogously by:

$$Y \times Z^t \longrightarrow Z^{t+1}$$

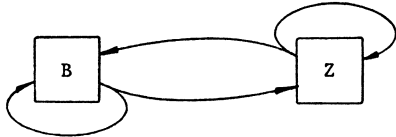
Since no noise is to enter the transmission process

(though equivocation need not be absent), the following two mappings specify what is transmitted from the first to the second communicator and back, respectively:

$$B^t \rightarrow Y^t$$

$$Z^t \rightarrow A^t.$$

These four mappings define a system with the capability of communication between its two components:



the arrows from B and Z to itself respectively denoting the possibility of internal memory and define one or more joint trajectories of behavior:

$$\langle b,z \rangle^0 \rightarrow \langle b,z \rangle^1 \rightarrow \dots \rightarrow \langle b,z \rangle^u \rightarrow \langle b,z \rangle^{u+1} \rightarrow \dots \rightarrow \langle b,z \rangle^{u+c}$$

which depend on the initial state $\langle b,z \rangle^0$ and typically reach a cycle after, say, u transient states at which point the messages that are exchanged become repetitious.

One can now proceed by sampling either from the population of possible mappings or from the population of possible trajectories, one by one, and establish in each case whether or not information was transmitted between B and Z, and whether or not the set of states in the cycle toward which the joint behavior converges does or does not exhibit a structure as defined above. The result can be presented in a two-by-two contingency table, with the occurrence or non-occurrence of communication, c or \bar{c} , and the subsequent emergence or non-emergence of structure, s or \bar{s} as coordinates and probabilities as entries.

	c	\bar{c}
s	P_{cs}	$P_{\bar{c}s}$
\bar{s}	$P_{c\bar{s}}$	$P_{\bar{c}\bar{s}}$

$$P_{cs} + P_{\bar{c}s} + P_{c\bar{s}} + P_{\bar{c}\bar{s}} = 1$$

Let me consider testing the population of possible mappings first. Here we would take one mapping at a time, apply this on all possible initial states simultaneously, and thus generate all joint trajectories in parallel. With each communicator having n states or n different messages to send, the number of such mappings is nn^{nn} . This can be a sizable number. For example, if $n = 10$,

which is unrealistically small compared to most real situations, then $nn^{nn} = 100100 = 10^{200}$ behaviors, which is a number starting with a one following by two hundred zeros. This population is too large to be generated and tested one by one, and to draw conclusions on only a few may introduce a sampling error of indeterminable magnitude. Faced with this insurmountable problem, I tried to bypass parts of the simulation by computing the probability for all possible behaviors in any one of the four categories. The formulas for this task are developed in Appendix A. Their evaluation was not possible without the aid of an electronic computer. Results are reproduced in Appendix B. For $k = 2$ communicators and $n = 2, 3, 10$ states or messages available to each, the probabilities are

	c	\bar{c}		c	\bar{c}		c	\bar{c}
s	.4063	0	s	.7527	0	s	$\approx 1 \cdot 10^{-2}$	0
\bar{s}	.5313	.0625	\bar{s}	.2473	$\approx 10^{-6}$	\bar{s}	$\approx 10^{-2}$	$\approx 10^{-180}$
	$N = 256$			$N = 3.9 \cdot 10^8$			$N = 10^{200}$	

When the situation is extremely simple and each of the two communicators has just $n = 2$ messages to send, the communication-structure cell contains about 41% of all possible behaviors, which is barely above the expected value. But as soon as the situation becomes only moderately complex, this cell contains nearly all behaviors. When $k = 2$ and $n = 10$, $p_{cs} = .99597$.

The complexity can also be increased regarding the number of communicators involved. With $n = 2$ and the number of communicators, $k = 2, 3, 10$, the probabilities are

	c	\bar{c}		c	\bar{c}		c	\bar{c}
s	.4063	0	s	.7613	0	s	$\approx 1 \cdot 10^{-3}$	0
\bar{s}	.5313	.0625	\bar{s}	.2387	$\approx 10^{-6}$	\bar{s}	$\approx 10^{-3}$	$\approx 10^{-3076}$
	$N = 256$			$N = 1.7 \cdot 10^7$			$N = 10^{3082}$	

Now let me apply the same procedure on the population of possible trajectories of behavior. Here we would start with an initial state and generate all possible trajectories that could be described by a mapping, separately and one at a time. The number of possible trajectories that the communicators can follow jointly or separately, leading toward a structure or not, is not as easily expressible as the number of mappings. But this number is only slightly less than the latter. Formulas for the number of possible trajectories are again presented and justified in Appendix A, and from the results listed in Appendix B the following

is obtained. For $k = 2$ communicators and $n = 2, 3, 10$ states or messages available to each, the probabilities are

	c	\bar{c}		c	\bar{c}		c	\bar{c}
s	.2872	0	s	.5605	0	s	$\approx 1-10^{-1}$	0
\bar{s}	.5798	.1330	\bar{s}	.4395	$\approx 10^{-6}$	\bar{s}	$\approx 10^{-1}$	$\approx 10^{-180}$
	N = 188			N = $1.6 \cdot 10^8$			N = 10^{199}	

For $n = 2$ states or messages available to $k = 2, 3, 10$ communicators each, the probabilities are

	c	\bar{c}		c	\bar{c}		c	\bar{c}
s	.2872	0	s	.5708	0	s	$\approx 1-10^{-2}$	0
\bar{s}	.5798	.1330	\bar{s}	.4292	$\approx 10^{-5}$	\bar{s}	$\approx 10^{-2}$	$\approx 10^{-3072}$
	N = 188			N = $7.4 \cdot 10^6$			N = $1.8 \cdot 10^{3079}$	

Thus, while the tendency of communication to generate structure is not quite as strong when one observes each line of behavior separately as when one observes all possible lines of behavior simultaneously, in both cases only a slight increase of the complexity of the situation, either in terms of the number of messages each communicator can exchange or in terms of the number of communicators involved or both, makes the emergence of structure virtually certain. In real life situations where n is certainly very large and k too can take values larger than here considered, the two by two table of probabilities is likely to look like this:

	Communication present	Communication not present
Structure evolves	practically one	zero
Structure does not evolve	practically zero	not noticeable different from zero

To summarize these findings: whenever a situation is only moderately complex, either in terms of the number of communicators involved or in terms of the numbers of messages exchangeable among them or both, it is the very existence of communication processes which is the necessary and causal prerequisite for the genesis of structure and accounts for such genesis with extremely high probability.

The explanation which this communication law

offers is causal because the emergence of structure is always preceded by at least one incident of communication between communicators. When no such incident is encountered, structure cannot evolve. Because there are cases in which structure does not develop subsequent to existing communication, the presence of communication is not sufficient to explain the genesis of structure. However, as I have shown, for moderately complex situations this probability is practically zero, and the probability of communication causing structure to develop is practically one. To link the emergence of structure to purposes, intentions, preferences, or to the needs, emotions, or rational faculties of the individuals involved in the process of communication is thus neither necessary nor sufficient. The law is as general as here stated and is as such extremely powerful.

Apparent Counter-Examples

Let me now consider three primary classes of apparent counter-examples, i.e., situations that seem incongruent with the law, and show in each case the reasons for the apparent disparity. Such a discussion might prevent possible misinterpretation and may serve to qualify the law in the context of practical situations.

The first class of apparent counter-examples can be dealt with briefly. Suppose the artificial communicators produce messages entirely at random, i.e., with no detectable relation to what they have access to. This would correspond to a gathering of individuals all of whom talk without any evidence of listening to anyone other than themselves. Clearly, no structure among such individuals can develop under these conditions, even though messages may be generated in large numbers. However, this does not contradict the proposed law, because the definition of communication requires that some dynamic dependency across different points in space and time be demonstrated. If one individual's messages are unrelated to the messages he receives, then the messages he produces are also unrelated to any other individual's previously produced communications and a communication process does not exist. Hence structure cannot evolve. The same would apply when each individual responds to what he receives in a determinable way, but the channels between individuals produce only noise.

The point is that the law holds only when communication is chained—without break—involving possibly many steps both in space and in time. Such is predominantly the case when communication is circular, i.e., whenever information emitted at one point is returned to that point in space in whatever form, regardless of the number of intermediary communicators involved and regardless of the length of time required. Of course, the fewer the transmission errors, the more deterministic the behavior of the communicators and the shorter

the loops, the faster will a structure emerge. The law holds wherever the transmission channels and the communicator's behavior exhibit at least some statistical regularity between input and output, some preferences that are conditional on the situation as perceived, some rationality, however primitive, as long as it accounts for systematic selections among messages in response to other messages. In other words, any non-random behavior with chain-like effects, particularly on itself, will lead to the genesis of structure.

Common to my second class of apparent counter-examples is the existence of communication from the outside of a circular flow of information.

Peterson's (1964) account of the history of magazines in the United States—how they rise and why they die—may be instructive here. Briefly, his analysis focuses attention on the "editorial formula" of a magazine, which is a summary term for the policies governing choices of stories for print, the make-up, style of writing and of photography, etc., and it is usually reflected in the magazine's management, in the division of labor among special editors, for example. According to Peterson, an editorial formula which has been successful in the past not only becomes an increasingly pervasive guideline for the publication and a reflection of presumed readership characteristics, but it also becomes increasingly resistant to change. If changes in the market situation, which might be regarded as induced by forces outside the circular editor-reader communication, cannot be absorbed by the established editorial formula, the magazine's existence is doomed. In going over the many accounts of successful adjustments to changing market situations that Peterson collected, it seems that the publisher's primary response to an experience of decay is to search for a solution which lies outside of the established communication flow: a new editor is hired, a different management is employed, either of which have not been part of the previous communication process. Sometimes, just before a magazine is about to fade away, recovery is thus achieved through a new editorial formula which is often so radically different from the old that all that remains of the old magazine is its name.

A similar breaking of a slowly evolved structure may be observed when one part of the married couple decides to consult a psychotherapist who can provide the information necessary to unfreeze the established inter-dependency, e.g., by asking pertinent questions, by making marital problems transparent, by reorganizing patterns of communication, by introducing a new perspective. Here again it is outside interference which can destroy the structure that has evolved. More generally, if one or both partners are in continuous communication with agents or events outside their circular flow of information, not just with a psychotherapist, but also with stimulating colleagues, neighbors, or through participation (for example, in the women's

liberation movement), an emerging structure may be prevented from becoming as quickly rigid as it would otherwise become. But as long as communication is circular and is maintained over some time, a structure will emerge even though the relative openness to communications from the outside may now make its own demands as to how the relation among the communicators becomes defined.

Thus, while it is apparently possible to either consciously destroy or unintentionally alter a structure among communicators that has evolved in time, or to prevent such a structure from becoming too rigid, such changes never seem to be affected from the inside of the system. A new editor for the magazine or a psychotherapist is coupled with the established pattern of communication precisely because they might provide new ideas or help to overcome apparent barriers. The law therefore holds only for systems that are relatively invariant with respect to their degree of openness, i.e., for systems that maintain some circular flow of information over time in spite of continuous exposure to communication from the outside. It does not make assertions regarding how initially outside agents are drawn into communication or how new sources of information are incorporated in the circular flow. A new baby in a family, changing membership of social groups, the biological replacements of members of society all have the effects of changing the communication situation and introducing new agents to a circular flow of information. However, once such initially outside agents are brought into communication with the old patterns, then according to the law they are likely to be entangled in new interdependencies. Thus, the new editor will soon experience and consider the confinements of his situation, become increasingly dependent on his readers for rewards and become increasingly enmeshed. The psychotherapist may become a paid member of the family. The examples are thus totally consistent with what the law claims to explain.

This situation is often characterized by the saying "you can't change the system from within." As long as the components of a system behave and communicate among each other in somewhat regular ways, each can merely influence the speed and the direction of the process of which it is a part, but it cannot escape the communication law which suggests that communicative involvement converges with near perfect certainty towards coordinated activity and mutually imposed constraints regardless of prevailing intentions.

It is not altogether unreasonable to consider whether it is possible to escape the law in yet another direction through an awareness of the systemic properties of the communication process. This introduces a third class of apparent counter-examples which I will consider below. One important ability of man is to reflect, at least from time to time, not only on his own behavior relative

to that of others, but also to step out of such narrow confinements and to consider himself in the context of a large picture. Bateson (1969) has alluded to this level of understanding, and the notion of meta-communication referred to above implies this possibility.

Let me illustrate this with the familiar example of an arms race. There are two countries. Each determines its rate of armament production according to its own level of armament and that achieved by the other. As the levels of armament expenditures are credibly communicated across the two countries, a process is set in motion which induces a spiral of mutual stimulation, often ending in such disasters as war, which makes these countries mutually dependent in ways that perhaps neither country wants or can afford. The situation is not different from many interpersonal encounters in which one person might express his dissatisfactions to a second who in turn responds by complaining to the former. The exchange of complaints and counter-complaints may thus develop into an exchange of insults. Soon, before they realize it, the individuals have had a fight and become bitter enemies without ever intending to go this far.

If both countries or individuals were to respond not merely to each message as received but would further consider what sort of dependencies their exchange of messages will generate in the end, then it appears possible to escape the determinism inherent in the way messages are exchanged, and, if this were so, the claimed generality of the law would have to be abandoned. However, upon closer analysis this would be an erroneous conclusion.

Recall that the artificial communicator's behavior was specified by a transformation of both the received message and his internal state into a future internal state and the message that he will send. A country's behavior in the arms race can be mapped onto this framework without loss of generality. Messages sent and received refer either to its own or to its opponent's level of armament expenditures and the transformation determines the rate of change. It is important to note that each country's behavior is described by just one transformation, i.e., one of the many possible policies relating the opponent's armament expenditures to its own.

Now, it is suggested that an awareness of the systemic properties of this communication process has changed the situation insofar as one or both communicators consider the very policies that determine the interaction among countries on the armament expenditure level as variable and subject to choice according to some higher order objective. One of the many objectives may be to keep the expenditure below a critical level. Another may be to effect disarmament or to keep the two armament expenditures independent of each other. The internal states of such a country now consist of its

policies together with its levels of armament expenditure. What is received or to what such a country now responds is information about the opponent's presumed choice of policy and level of armament expenditure. The rationality of its own decision-makers together with their collective capacity to predict the mutual outcomes of choices among policies define the transformation of such a country.

What has changed is the level of communication. Relative to communications conveying information about levels of armament expenditures, messages conveying information about policies of responding to armament expenditures are meta-communications. But the abstract structure of the behavior, the fact that some states or policies adopted by one communicator affect the changes of policies observed on the other, still conforms to the framework as discussed above. Thus, even if the objective was set to make armament expenditures independent of each other, when there is communication on the policy level an interlocking of policies finally surviving the process is again very likely to emerge. This is all the more likely as there are theoretically many more policies among which choices may be made than levels of armament expenditures. However, structures on the policy level may not be generated as quickly as in the armament level because changes of policies tend to require much more time, and communication is presumably slower on this level.

Thus, the awareness of the systemic properties of communication processes merely leads the communicators out of one trap into another which is located on a higher level. On this level a structure will then develop with even greater certainty, though perhaps requiring a longer period of time.

The consideration of apparent counter-examples seems to have merely reinforced the confidence in the validity of the communication law. Let me summarize it:

Whenever the behavior of the individual components of a system exhibit some regularity, e.g., can be characterized by conditional preferences, by statistical biases in response to stimuli, by rational choices, etc.;

whenever communication is circular though not necessarily closed against communication from the outside, irrespective of the number of components or the temporal length of the circle;

whenever communication is somewhat permanent and maintained without significant alteration of the communicators involved,

then the very process of communication causes a structure among the communicating parts of a system to emerge,

whereby the certainty of structural genesis increases with the complexity of the situation,

both in terms of the number of communicators involved and the number of states or messages distinguishable by each;

whereby the speed of structural genesis decreases with the level of awareness and the number of messages controlled by each communicator,

irrespective of whether the result is anticipated or desirable.

The law seems inescapable.

Implications

Let me draw a few separate conclusions and offer some programmatic remarks regarding the task of communication research.

1. I think simulation as an investigative strategy can contribute in important ways to the understanding of some of the dynamic consequences of communication processes. It is fair to state that sufficiently powerful methods for studying complex and freely evolving social communication situations do not exist, and the confinement which experiments impose on a situation primarily restricts the communication among individuals. This leaves the study of communication to intuitive speculations to an extent that cannot be afforded. Although the problem I have dealt with is admittedly narrow, the results seem to be at least conclusive and, hopefully, convincing both of the potentialities of simulation and of the existence of fairly definite lawfulness associated with communication processes. Regarding the latter, I would like to emphasize that, because the simulation model omitted references to such extraneous factors as the materiality of the communicators involved, the findings hold for communication among animals, man, machines, social organizations and nations alike. The only condition is that they behave roughly as modeled, which here means that there must be some information transmitted through and across each communication and back. This is probably the most general notion of communication conceivable. The simulation revealed that the genesis of structure is one of the consequences of this notion.

2. I think many theories of human communication have stopped short at either providing conceptual frameworks for merely talking about communication or at listing variables that correlate with individual or organizational coding behavior. The dynamic consequences of communication are essentially alien to such theories. What is needed are communication theories that can be stated algorithmically, that make explicit the evolutionary nature of communication processes, and that recognize the fact that many social sequences of behavior are to a significant extent non-ergodic. In other words, unless social institutions are old and unchanging and have found stable relations between their components, adequate theories of communication must account for a converging process in which properties, relations, and structures emerge rather

than are fixed. The uncritical reliance on theories that depict or implicitly assume the world as essentially unchanging not only prohibits the scientific use of communication processes and technology as agents of social change, but also bars the development of an understanding of the dynamic changes that the very process sets in motion.

3. There exists a common (and as I would argue wrong) generalization of the Second Law of Thermodynamics, which suggests that the tendency toward more entropic states means a decrease of order. Clearly, the truth of such a generalization hinges on the definition of "order," which can be understood both as an unequal distribution of energy quanta and as a constraint on the information transmittable between communicating agents. However, these two notions are not readily interchangeable. While some of the energy which an ongoing communication process requires is presumably always lost to entropy, the findings indicate that this is most likely to coincide with a genesis of structure which is nothing but a manifestation of order in the second of the above senses. This does not contradict the Second Law of Thermodynamics, but it warns against its uncritical generalization. The proposed communication law suggests that on the level of information the world tends to become increasingly complex and its elements increasingly interdependent as long as the supply of energy is sufficient to support the ongoing processes of communication and as long as the communicators maintain their own identities.

4. I showed that the tendency of communication processes to develop structure does not require notions of intentionality or other human characteristics. Though structural genesis is autonomous to the possible purposiveness of the communicators, this does not deny that the latter may contribute to the direction which the process may take. In processes of circular communication the effects of any systematic behavior, including relatively stable forms of rationality, may be extended beyond their respective local and temporal origins and may subsequently become imprinted on the structure on which the communicators ultimately settle. In a sense, communication dynamically implies a mutually selective amplification of patterns beyond the scope of the individual communicators involved. It is a process which does not need to be and probably rarely ever is comprehensible and fully controllable by the participating individuals. I think that one implication of the proposed law is that one will have to abandon the comforting idea that social structures are man-made and hence rationally comprehensible just because thinking individuals are involved. As I could demonstrate, the primary mechanism that accounts for the genesis of structure is communication, which is to a significant degree super-individual in character. Communication processes can thus produce structures vastly more complex than

the communicators are able to consider and may relate human and non-human participants in the process in ways neither can understand in isolation.

Finally, if the interpretation of the findings proves to be correct and as general as claimed, then the communication processes we initiate and facilitate for whatever reasons are largely responsible for having complicated the world which we now have to come to grips with. Some of the complex structures that have emerged are not just the end products of interaction but have developed their own life: dynamic properties and purposes. A world that incorporates such structures is difficult to comprehend not only for those involved in the process but also for the scientific observer who stands outside. As Jay Forrester noted:

"Evolutionary processes have not given us the mental skills needed to properly interpret the dynamic behavior of the systems of which we have now become a part." In the jargon of the mathematician, or systems analyst, these systems belong to a class known as multiloop nonlinear feedback sys-

tems in which a great many factors interact to determine the outcome.... "The problem... is not shortage of data but rather our failure to understand the laws and behavior patterns determining the implications of the data." (Sullivan 1970)

Given that nearly every act of communication contributes to the development of some interdependencies, and that "improvements" of communication technology link more and more people more speedily and richly with each other and with their bio-technical world, we, in our social role as communication scientists, may have to examine our latent motivations and sources of rewards. At this stage of socio-technological development, the almost ideological trust in the desirability of communication (and conversely in the undesirability of the lack of communication) may well be a dangerously activist attitude. As communication scientists, we may have to help in curbing the growth of structures that become increasingly unmanageable and are already quite out of control by finding ways not to communicate or to communicate less and more selectively; otherwise . . .

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APPENDICES

APPENDIX A

The following technical note uses the notations and definitions in Ashby's (1964) Set Theory of Mechanism and Homeostasis. In its terms, let the over-all behavior of a system with k components be described by a mapping of the set X of all of their joint states, $X = X_1 \times X_2 \times \dots \times X_k$, into itself:

$$\Gamma(x^t) \subseteq x^{t+1}$$

where the superscript denotes time. The multiple

operation of Γ on X defines one or more successions, $x^0 \rightarrow x^1 \rightarrow x^2 \rightarrow x^3 \rightarrow \dots$, called trajectories of behavior of states $x \in X$, where each state is a k-tuple of states of each component. When this succession becomes repetitious, i.e., the behavior is trapped in a cycle, an equilibrium is reached.

We consider two ways of generating behaviors that are governed by random mapping functions of the kind described above and define tests for the existence of communication and the emergence of structure for each.

(a) By operating on all initial states X_0

jointly, all trajectories are generated and can be followed simultaneously. The set E of all equilibrium states of the system is then defined by:

$$E \text{ is the set of equilibrium states } \langle \text{====} \rangle \{ x \mid x \in X, \Gamma^w(x) = x \}$$

where w is a positive whole number indicating the number of times the mapping is applied.

Structure is defined as a non-decomposable relation without reference to time. The absence of structure is then indicated when the set E of all joint equilibrium states can be regarded as the Cartesian product of the equilibria of each of the system's components. The latter may be obtained by projection:

Structure does not evolve

$$\langle \text{====} \rangle \text{pr}_1(E) \times \text{pr}_2(E) \times \dots \times \text{pr}_k(E) \subseteq E.$$

Communication within a system is manifest when the behavior of the whole cannot fully be accounted for by the behavior of its components in isolation. Thus, if the mapping is regarded as a subset, $\Gamma \subset X \times X$, then the projection $\text{pr}_i(\Gamma)$ describes the behavior of the i-th component, and the absence of communication is indicated by:

Communication not present

$$\langle \text{====} \rangle \text{pr}_1(\Gamma) \times \text{pr}_2(\Gamma) \times \dots \times \text{pr}_k(\Gamma) \subseteq \Gamma.$$

Now, let the k components have n states each. The number of joint states in X is n^k . The population of possible mappings of X into itself then becomes:

$$M = \binom{n^k}{n^k} = n^{kn^k}.$$

Of this number, the number of mappings that can be defined without communication present is the product of the number of mappings that can be defined on each component separately. This number is:

$$M_c = (n^n)^k = n^{kn}.$$

The number of mappings that do not produce a structure is not as easily expressible. Without simplification, this number is:

$$M_s = \sum_{c_1=1}^n \binom{n}{c_1} \sum_{c_2=1}^n \binom{n}{c_2} \dots \alpha_1 \alpha_2 \dots \sum_{c_k=1}^n \binom{n}{c_k} \underbrace{(\prod_{i=1}^k \Pi_{c_i})! \prod_{i=1}^k (n^k)^{n^k - \prod_{i=1}^k c_i - 1}}_{\alpha_k \beta \gamma}$$

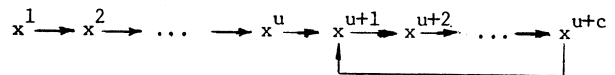
where α_i is the number of combinations of n taken c_i at a time, i.e., the number of ways c_i states can be selected from the n states of the i-th component, so that the product $\alpha_1 \cdot \alpha_2 \cdot \dots \cdot \alpha_k$ is the number of rectangular (decomposable) equilibrium sets, β , i.e., the factorial of the number of states in the equilibrium set is the number of one-to-one mappings definable on this set. The rightmost component, γ , expresses the number of ways the $n^k - \prod_{i=1}^k c_i$ possible transient states, outside the equilibrium set, can reach the rectangular set of size $\prod_{i=1}^k c_i$. This number needs further justification which will be given at the end of this Appendix.

Considering that an absence of communication always implies rectangular equilibrium sets, the number $M_{\bar{c}s}$ of mappings that generate structure when communication is absent is zero. These four values, $M, M_c, M_s,$ and $M_{\bar{c}s}$, fully specify the two-by-two table below:

	c	\bar{c}	
s	$M_{cs} = M - M_s$	$M_{\bar{c}s} = 0$	$M_s = M - M_{\bar{c}s}$
\bar{s}	$M_{c\bar{s}} = M_s - M_c$	$M_{\bar{c}\bar{s}} = M_c$	$M_{\bar{s}} = M - M_{c\bar{s}}$
	$M_c = M - M_{\bar{c}s}$	$M_{\bar{c}} = M_c$	M

This table represents a breakdown of the total population of mappings according to whether each incorporates communication and/or structure emerges in the end.

(b) By operating on each initial state $x^0 \in X^0$ separately, one trajectory is generated and can be followed at a time. Such a trajectory reaches an equilibrium after u transitions where it enters a cycle of length c:



That portion of the mapping Γ which accounts just for the trajectory starting with x^0 is denoted by Γ_{x^0} . It should be noted that $\Gamma = \bigcup_{x^0 \in X^0} \Gamma_{x^0}$. The set of E_{x^0} of equilibrium states of the trajectory starting with x^0 is defined by:

E_{x^0} is the set of equilibrium states of a trajectory starting with x^0

$$\langle \text{====} \rangle \{ x \mid x \in X, \Gamma^w(x^0) = x, \Gamma^c(x) = x \}$$

It should be noted that similar to the above $E = \bigcup_{x^0 \in X^0} E_{x^0}$. Referring for the details of the

argument to the foregoing, the absence of structure is defined by:

Structure
does not evolve

$$\langle====\rangle \text{pr}_1(E_x^o) \times \text{pr}_2(E_x^o) \times \dots \times \text{pr}_k(E_x^o) \subseteq E_x^o.$$

And the absence of communication is indicated when the condition

Communication
not present

$$\langle====\rangle \text{pr}_1(\Gamma_x^o) \times \text{pr}_2(\Gamma_x^o) \times \dots \times \text{pr}_k(\Gamma_x^o) \subseteq \Gamma_x^o$$

holds true.

Since one mapping may produce one or more distinct trajectories and one trajectory can appear in different mappings, the population of possible trajectories is not equivalent to the population of possible mappings. The total number of different trajectories is expressed by:

$$T_c = \left[\sum_{c=1}^n \binom{n}{c} (c-1)! \sum_{u=1}^{n-c} c(u+c)^{u-1} \right]^k$$

which can of course be simplified considerably. α is again the number of ways an equilibrium set of size c can be selected out of the nk possible joint states. β is the number of mappings involving c states in one cycle, and the expression γ is the number of ways u transient states can be arranged outside the equilibrium set of size c . Analogously, the number of trajectories that exhibit no communication between communicators is the product of the number of trajectories that could be observed on each component separately. This number is:

$$T = \sum_{c=1}^n \binom{nk}{c} (c-1)! \sum_{u=0}^{nk-c} c(u+c)^{u-1}$$

$\alpha \quad \beta \quad \gamma$

and is so similar to T that it does not require further interpretation. Finally, the number of trajectories that end up in a rectangular set of equilibrium states is expressed as follows:

$$T_{\bar{s}} = \sum_{c_1=1}^n \binom{n}{c_1} \sum_{c_2=1}^{n-c_1} \binom{n-c_1}{c_2} \dots$$

$\alpha_1 \quad \alpha_2$

$$\sum_{c_k=1}^n \binom{n}{c_k} (\prod_{i=1}^k c_i - 1)! \sum_{u=0}^{n-\prod_{i=1}^k c_i} \prod_{i=1}^k c_i (u + \prod_{i=1}^k c_i)^{u-1}$$

$\alpha_k \quad \beta \quad \gamma$

where α_i expresses again the number of ways c_i states can be selected from the i -th component's n possible states, β is the number of ways $\prod_{i=1}^k c_i$

states can be arranged in a cycle, and γ expresses the number of ways u possible transient states can be arranged outside the cycle. Again, the number T_{cs} of trajectories leading to a structure when communication is absent is zero. Thus a two-by-two table of frequencies is specified as above:

	c	\bar{c}	
s	$T_{cs} = T - T_{\bar{s}}$	$T_{\bar{c}s} = 0$	$T_s = T - T_{\bar{s}}$
\bar{s}	$T_{c\bar{s}} = T_{\bar{s}} - T_{\bar{c}}$	$T_{\bar{c}\bar{s}} = T_{\bar{c}}$	$T_{\bar{s}}$
	$T_c = T - T_{\bar{c}}$	$T_{\bar{c}}$	T

To justify the expression γ which appears repeatedly in the above formul, let us consider it as it occurs in the expression for T . Here the length of a cycle is denoted by $c=1, 2, \dots, n$, and the number of transient states is $u=0, 1, \dots, n-c$. The transient states may be thought of as arranged in orbits around the cycle with the number m_1 of states being one transition away from the cycle, the number m_2 of states being two transitions away from the cycle, etc., until m_p . The number of ways m_1 can be selected out of u is $\binom{u}{m_1}$, and the number of mappings of m_1 states into c states is c^{m_1} . Similarly, the number of ways m_2 can be selected out of the remaining $u-m_1$ states is $\binom{u-m_1}{m_2}$, and the number of mappings of m_2 states into m_1 states is $m_1^{m_2}$, etc. With S_p denoting the set of all nonempty p -partitions m_1, m_2, \dots, m_p , the number of arrangements of u transient states in p orbits is:

$$\sum_p \sum_{S_p} \binom{u}{m_1} c^{m_1} \binom{u-m_1}{m_2} m_1^{m_2} \dots \binom{m_p}{m_p} m_{p-1}^{m_p}$$

This expression may be rewritten in a simplified form and by hypothesis the following equality should hold:

$$c \, u! \left\{ \sum_p \sum_{S_p} \frac{c^{m_1} m_1^{m_2} \dots m_{p-1}^{m_p}}{c^{m_1} m_1! m_2! \dots m_p!} \right\} = c (u+c)^{u-1}$$

That the expression in parenthesis is $\frac{(u+c)^{u-1}}{u!}$ has been shown by Katz (1955) who presented the following proof: First, the binomial of the above expression is expanded:

$$\frac{(u+c)^{u-1}}{u!} = \sum_{m_1=1}^u \binom{u-1}{m_1-1} \frac{1}{u!} c^{m_1-1} u^{u-m_1}$$

Second, the number of transient states with those up to the i-th orbit removed is expressed recursively by:

$$u_i = u_{i-1} - m_i.$$

When the above expression is simplified and u_i is inserted for the number of transient states with the first orbit removed, this becomes:

$$\frac{(u+c)^{u-1}}{u!} = \sum_{m_1=1}^u \frac{c^{m_1-1}}{(m_1-1)!} \frac{(u_1+m_1)^{u_1-1}}{u_1!}$$

Clearly, the second part of the summand is similar to the left expression and may be iteratively expanded accordingly. Thus with p arbitrary:

$$\frac{(u+c)^{u-1}}{u!} = \sum_{m_1=1}^u \frac{c^{m_1-1}}{(m_1-1)!} \sum_{m_2=1}^{u_1} \frac{m_1^{m_2-1}}{(m_2-1)!} \dots \sum_{m_p=1}^{u_{p-1}} \frac{m_{p-1}^{m_p-1}}{(m_p-1)!} m_p^{-1}.$$

Since the summations on the right are equivalent to the expression in parenthesis with sums over all p -partitions of u , the above equality is proven.

APPENDIX B

Following is the list of numerical results from which the two-by-two tables of probabilities have been computed.

k	n	log ₁₀ (M)	log ₁₀ (M _c)	log ₁₀ (M _s)	log ₁₀ (T)	log ₁₀ (T _c)	log ₁₀ (T _s)
2	2	2.4082	1.2041	2.1818	2.2741	1.3979	2.1271
2	3	8.5882	2.8627	7.9814	8.2036	2.8299	7.8465
2	10	200.0000	20.0000	198.147	199.051	19.1798	198.043
3	2	7.2247	1.8062	6.6026	6.8697	2.0969	6.5024
3	3	38.6467	4.2941	37.3867	38.0107	4.2449	37.3028
3	10	3000.0000	30.0000	2996.95	2996.74	28.7697	2996.33
10	2	3082.5472	6.0206	3079.39	3079.26	6.9897	3077.77

