Investor Sentiment and the Mean-Variance Relation*

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Abstract

This study documents the influence of investor sentiment on the market's mean-variance tradeoff. We find that the stock market's expected excess return is positively related to the market's conditional variance in low-sentiment periods but unrelated to variance in high-sentiment periods. These findings are consistent with sentiment traders who, during the high-sentiment periods, undermine an otherwise positive mean-variance tradeoff. We also find that the negative correlation between returns and contemporaneous volatility innovations is much stronger in the low-sentiment periods. The latter result is consistent with the stronger positive ex ante relation during such periods.

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1 Introduction

Theories of rational asset pricing typically imply a positive relation over time between the market's expected return and variance (Merton (1980)). Yet numerous studies over the past three decades find rather mixed empirical evidence of such a relation. The results appear sensitive to methodology, especially the volatility models.[1] Theories departing from rational asset pricing often posit the influence of investor sentiment (e.g., De Long et al. (1990)), and recent empirical studies find evidence that sentiment impacts expected stock returns (e.g., Baker and Wurgler (2006)).[2] This paper analyzes whether investor sentiment influences the mean-variance relation and explores whether sentiment attenuates the link between the conditional mean and variance of returns.

We discover a critical role for investor sentiment in the mean-variance relation. In particular, there is a strong positive tradeoff when sentiment is low but little if any relation when sentiment is high. These results are consistent with greater participation of sentiment-driven traders in the market when sentiment is high, thereby perturbing prices away from levels that would otherwise reflect a positive mean-variance tradeoff.

Despite some debate with respect to the overall importance of sentiment traders, one can reasonably make the following two cases. First, sentiment traders exert greater influence during high-sentiment periods than during low-sentiment periods, due to their reluctance to take short positions in low-sentiment periods.[3] Empirical

[1]Section 3 reviews this literature, which dates from the classic study by French, Schwert, and Stambaugh (1987).

[2]Baker and Wurgler (2006) construct an investor sentiment index and find that the cross-section of expected stock returns displays opposite patterns in low- and high-sentiment periods. Other studies find that investor sentiment predicts market returns in both the short run (Simon and Wiggins (1999), Brown and Cliff (2004), Kaniel, Saar, and Titman (2006)) and the long run (Brown and Cliff (2005), Yuan (2005)). Taken together, these studies support the general hypothesis that sentiment moves stock prices and, in turn, influences expected returns.

[3]For example, in the study of the individual investors from a large discount brokerage firm, Barber and Odean (2006) document that only 0.29% of positions are short positions.

studies find consistent evidence that sentiment-driven investors participate and trade more aggressively in high-sentiment periods (e.g. Karlsson, Loewenstein, and Seppi (2005), Yuan (2008)). Second, because sentiment traders tend to be inexperienced and naive investors, they are likely to have a poor understanding of how to measure risk and hence are likely to misestimate the variance of returns, weakening the mean-variance relation.[4] Together, these two arguments suggest that the increased presence and trading of sentiment investors during high-sentiment periods should undermine an otherwise positive mean-variance tradeoff in the stock market.

Using the investor sentiment index proposed by Baker and Wurgler (2006), we identify high- and low-sentiment periods and then analyze the mean-variance relation within both regimes. In low-sentiment periods, we find a positive tradeoff that is not only statistically significant but also economically important: a one-standard-deviation increase in conditional variance is associated with a 1% roughly increase in expected monthly excess return. In contrast, during high-sentiment periods, we find the mean-variance tradeoff to be significantly lower and nearly flat.

Further evidence that sentiment plays a key role in the mean-variance tradeoff appears in the reactions of prices to volatility innovations. During low-sentiment periods, there is a strong negative correlation between returns and contemporaneous volatility innovations. This result is consistent with the positive mean-variance tradeoff we document during low-sentiment periods, since rational investors who require compensation for bearing volatility should push prices down when unfavorable volatility innovations arrive. The negative correlation between returns and volatility innovations is significantly weaker in high-sentiment periods, consistent with investors on the whole being less averse to volatility during such periods, in that prices respond less to unfavorable volatility shocks.

[4] As we discuss in the next section and the appendix, in alternative settings in which naive sentiment traders are subject to cognitive biases, we can also obtain the same conclusion.

One striking feature of our empirical results is their robustness across four widely used volatility models. In particular, we conduct our empirical tests using the rolling window model, the mixed data sampling approach, GARCH, and asymmetric GARCH. In previous studies these models often yield different conclusions about the mean-variance relation, but our results are remarkably consistent across all four models.

Finally, we investigate whether similar two-regime mean-variance results obtain when regimes are formed using alternative variables, specifically, the interest rate, the term premium, the default premium, the dividend-price ratio, and the consumption surplus ratio defined in Campbell and Cochrane (1999). We show that only investor sentiment is able to distinguish a regime that exhibits a strong mean-variance tradeoff from a regime that does not.

The bottom line is that the mean-variance relation – perhaps the fundamental risk-return tradeoff in finance – exhibits a strong two-regime pattern, and that investor sentiment has a unique capacity to distinguish these two regimes. It is hard to explain these results within the traditional asset pricing theories. Early models like Merton's ICAPM generally predict a constant mean-variance relation, which contradicts the time-varying relation in our empirical findings. More recently, motivated by empirical evidence of significant time variation of expected returns over the business cycle (see, for example, Keim and Stambaugh (1986) and Fama and French (1989)), researchers have proposed theoretical models with cyclical variation in risk aversion (e.g., Campbell and Cochrane (1999)). Such time-varying risk aversion could produce a counter-cyclical risk-return tradeoff. However, our empirical results show that macroeconomic variables containing business cycle information have far less ability than investor sentiment to distinguish the high and low mean-variance tradeoff regimes. Overall, it seems very difficult for our empirical results to fit in the existing hypotheses with either constant or time-varying risk aversion.

In our opinion, a simple and realistic explanation for these results is that greater market participation of sentiment-driven traders when sentiment is high perturbs prices away from levels that would otherwise reflect a positive mean-variance tradeoff.

The rest of the paper is organized as follows. Section 2 develops our hypothesis. Section 3 introduces the four volatility models. Sections 4 reports the main empirical results and Section 5 gives the results of robustness tests. Section 6 concludes.

2 Hypothesis Development

In this paper we argue that investor sentiment attenuates the mean-variance relation during high-sentiment periods. This argument is based on the following three assumptions.

First, sentiment investors, who are optimistic or pessimistic about the market's prospects, are present in the market. Lee, Shleifer, and Thaler (1991) document that prices of close-end funds are different from their NAVs, which is likely caused by sentiment-driven individual investors. Similarly, Ritter (1991) finds evidence of long-run reversal of IPO stocks, which is likely to be a consequence of overoptimistic sentiment towards IPO firms. Baker and Wurgler (2006) go one step further to document the impact of sentiment on many types of cross-sectional returns and conclude that sentimental traders impact the prices of most stocks.

Second, sentiment traders are reluctant to take short positions. Empirical evidence shows that individual traders, the primary candidates for sentiment traders, seldom short. For example, Barber and Odean (2006) document that only 0.29% of positions of individual investors are short positions. Moreover, empirical studies also find consistent evidence that these traders are more active in the market during high-sentiment periods. Karlsson, Loewenstein, and Seppi (2005) and Yuan (2008) document that significantly more individual investors check their portfolios and trade their positions during market run-ups.

Third, sentiment traders misestimate variance. Sentiment traders, who tend to be inexperienced and naive investors, are likely to have a poor understanding of how to measure risk. As a result, sentiment traders are expected to misestimate the variance of returns.

Putting these three assumptions together leads to two intermediate implications: The first implication is that because sentiment traders misestimate variance, the mean-variance relation is weaker when sentiment traders purchase more stocks and have stronger influence on stock prices. The second implication is that due to sentiment traders' reluctance to short, they hold more stocks and have a stronger impact on the equity market when aggregate sentiment is high. These two intermediate implications lead to our paper's main argument: The heavy presence of sentiment investors during high-sentiment periods should undermine an otherwise positive mean-variance tradeoff in the stock market.

Note that the same set of implications will obtain if sentiment traders correctly estimate variance but are subject to cognitive biases.[5] The intuition for biased sentiment traders to undermine the mean-variance relation is as follows. In contrast to rational investors, who invest based on risk compensation, biased sentiment traders may sacrifice risk compensation (risk-adjusted returns) to pursue benefits or avoid costs derived from their cognitive biases. For example, using a general equilibrium model to analyze investors with the cognitive biases of loss aversion, mental accounting, and probability weighting, Barberis and Huang (2007) find that the utility of these investors improves if they hold securities with positively skewed returns. As a result, these investors demand lower risk compensation for positively skewed stocks,

[5] Many empirical studies find that individual investors, who are more likely to be sentiment traders, are subject to different cognitive biases. See, for example, Lewellen, Lease, and Schlarbaum (1977), Shefrin and Statman (1985), Odean (1998, 1999), Barber and Odean (2000, 2006), Odean (2001), and Yuan (2008).

thus weakening the risk-return tradeoff.

In the Appendix, we provide a theoretical model that formalizes the intuition in this section.

3 Volatility Models

In this section we present the four volatility models used in the study: the rolling window model (RW), the mixed data sampling approach (MIDAS), GARCH(1,1), and asymmetric GARCH(1,1).

Previous studies have found that empirical conclusions on the mean-variance tradeoff rely heavily on the conditional variance models selected, which leads the overall evidence to be inconclusive. French, Schwert, and Stambaugh (1987), Baillie and DeGennaro (1990), Campbell and Hentschel (1992), Ghysel, Santa-Clara, and Valkanov (2005), Lundblad (2005), Guo and Whitelaw (2006), Pastor, Sinha, and Swaminathan (2006), Brandt and Wang (2007) find a positive mean-variance relation. Campbell (1987), Nelson (1991), Whitelaw (1994), Lettau and Ludvigson (2003), and Brandt and Kang (2004) find a negative relation. Turner, Starts, and Nelson (1989), Glosten, Jagannathan, and Runkle (1993), Harvey (2001), and MacKinlay and Park (2004) find both a positive and a negative relation.

As we show in the rest of the paper, after taking sentiment influence into account, the results are impressively robust across all the conditional variance models.

3.1 Rolling Window Model

A natural method to estimate the conditional variance is to use the rolling window model (RW) (e.g., French, Schwert, and Stambaugh (1987)). This model uses the realized variance of the current month as the conditional variance for the next month's return:

$$Var_t(R_{t+1}) = \sigma_t^2 = \frac{22}{N_t} \sum_{d=1}^{N_t} r_{t-d}^2 \ ,$$

where r_{t-d} is the demeaned daily return[6] in month t, the corresponding subscript $t-d$ is the date t minus d days, N_t is the number of trading days in month t, and 22 is the approximate number of trading days in one month.

In addition to estimating the conditional variance to analyze the mean-variance relation, we also need to calculate the variance innovation to explore the correlation between returns and contemporaneous volatility innovations. There are two ways to measure volatility innovation, as the unexpected change in current return volatility and as the unexpected change in future return volatility. Evidently these two measures are highly correlated, since the volatility process is persistent. The unexpected change in future variance is theoretically more plausible because it is future volatility that affects investors' utility. However, to estimate the conditional variance after the next period (that is, $Var_t(R_{t+2})$, $Var_t(R_{t+3})$, etc.), some econometric models (e.g., RW and MIDAS) need additional assumptions, which increases the risk of misspecification. This paper employs the following strategy in selecting the measure for volatility innovation: If the future variance can be estimated without additional econometric assumptions, the unexpected change in future variance is selected as the proxy; otherwise we use the unexpected change in current variance.

Following French, Schwert, and Stambaugh (1987), we use the unexpected change in current volatility, that is, the change in the realized variance, as the proxy for volatility innovation[7]:

$$Var(R_{t+1})^u = \sigma_{t+1}^2 - Var_t(R_{t+1}) = \sigma_{t+1}^2 - \sigma_t^2 \ .$$

[6]The daily demeaned return is computed by subtracting the within-month mean return from the daily raw return.

[7]With the additional assumption that volatility follows a random walk process, this measure is also the unexpected change in future volatility.

3.2 Mixed Data Sampling Approach

Ghysel, Santa-Clara, and Valkanov (2005) propose the mixed data sampling approach (MIDAS). Compared to RW, which calculates conditional variance using the prior month's daily returns with equal weights, MIDAS has a long horizon and a different weighting system. The MIDAS estimator of conditional variance is as follows:

$$Var_t(R_{t+1}) = 22 \sum_{d=0}^{250} w_d r_{t-d}^2 \ ,$$

where

$$w_d(\kappa_1, \kappa_2) = \frac{exp\{\kappa_1 d + \kappa_2 d^2\}}{\sum_{i=0}^{250} exp\{\kappa_1 i + \kappa_2 i^2\}} \ ,$$

r_{t-d} is the demeaned daily return[8] and the corresponding subscript $t-d$ is for the date t minus d days. The daily data of the previous 250 days, approximately 1 year, is used to estimate the conditional variance, w_d is the weight on r_{t-d}^2, and κ_1 and κ_2 are the parameters in the weight function. Ghysel, Santa-Clara, and Valkanov (2005) argue that the weight function provides a flexible weight structure with the two parameters, κ_1 and κ_2, estimated using the maximum likelihood method.

Under this setting, we calculate the volatility innovation as the unexpected change in current variance, that is, the difference between the realized variance and the conditional variance:

$$Var(R_{t+1})^u = \sigma_{t+1}^2 - Var_t(R_{t+1}) \ .$$

[8]The daily demeaned return is computed by subtracting the within-month mean return from the daily raw return. This specification is consistent with the realized variance estimator and RW. The empirical results are robust if we use the daily raw returns.

3.3 GARCH and Asymmetric GARCH

The GARCH-type models have been extensively used in modeling the volatility of stock market returns. Bollerslev (1986) proposes the GARCH model based on the ARCH model developed by Engle (1982). Glosten, Jagannathan, and Runkle (1993) build an asymmetric GARCH model to allow different impacts from positive and negative residuals.

GARCH(1,1) and asymmetric GARCH(1,1) are the third and fourth volatility models in this paper. GARCH(1,1) models the conditional variance as

$$Var_t(R_{t+1}) = \omega + \alpha\epsilon_t^2 + \beta Var_{t-1}(R_t) \ ,$$

where $Var_t(R_{t+1})$ is the conditional variance and ϵ_t is the residual, the difference between the realized return and its conditional mean. In asymmetric GARCH(1,1), the conditional variance is modeled as

$$Var_t(R_{t+1}) = \omega + \alpha_1\epsilon_t^2 + \alpha_2 I_t\epsilon_t^2 + \beta Var_{t-1}(R_t) \ ,$$

where I_t is the dummy variable for positive shocks, that is, I_t is 1 when ϵ_t is positive.

Future variance innovations are used in GARCH(1,1) and asymmetric GARCH(1,1), since the variance of future returns can be calculated without any additional assumptions. Moreover, daily data are used to improve the volatility estimation. The details are as follows. We first fit GARCH(1,1) with daily return data:

$$r_{t+1}^{raw} = \mu + \epsilon_{daily,t+1} \ ,$$

$$h_{t+1} = \omega + \alpha\epsilon_{daily,t}^2 + \beta h_t \ ,$$

where r_{t+1}^{raw} is the daily raw return and h_{t+1} is the conditional variance of the daily returns. With the estimates from the daily GARCH(1,1), the daily variance process, h_t, is calculated. The monthly variance process and monthly volatility innovations are then calculated as follows:

$$Var_t(R_{t+1}) = E_t(\sum_{d=1}^{22} h_{t+d}) \ ,$$

$$Var(R_{t+1})^u = Var_{t+1}(R_{t+2}) - Var_t(R_{t+2}) = E_{t+1}(\sum_{d=1}^{22} h_{t+1+d}) - E_t(\sum_{d=23}^{44} h_{t+d}) \ ,$$

where R_t is the monthly excess return, h_t is the conditional variance of the daily returns, and the corresponding subscript $t + d$ represents the date t plus d days. For asymmetric GARCH(1,1), the procedures are the same except that the daily conditional variance is modeled as asymmetric GARCH(1,1).

4 Main Empirical Results

In this section we test whether investor sentiment affects the relation between expected returns and variance, using the models described in Section 3. Before doing so, we first describe the sentiment index that we use to identify the low- and high-sentiment regimes in Section 4.1 and we provide summary information on the data in Section 4.2.

4.1 Investor Sentiment Index

Baker and Wurgler (2006) form a composite sentiment index that is the first principal component of six measures of investor sentiment. The principal component analysis filters out idiosyncratic noise in the six measures and captures their common component – investor sentiment. The six measures are the closed-end fund discount, the NYSE share turnover, the number of IPOs, the average first-day return of IPOs, the equity share in new issues, and the dividend premium.[9] To remove business cycle information, Baker and Wurgler (2006) first regress each of the raw sentiment measures on a set of macroeconomics variables[10] and use the residuals to build the sentiment index.

The composite sentiment index is plotted in Figure 1. This index captures most anecdotal accounts of fluctuations in sentiment. Immediately after the 1961 crash of growth stocks, investor sentiment was low but rose to a subsequent peak in the 1968 and 1969 electronic bubble. Sentiment fell again by the mid-1970's, but picked up and reached a peak in the biotech bubble of the late 1970's. In the late 1980's, sentiment dropped but rose again in the early 1990's, reaching its most recent peak in the Internet bubble.

Using the index, we identify the late 1960's, early and mid 1980's, and mid and late 1990's as high-sentiment periods. These have been widely perceived as high-sentiment periods by both anecdotal analysis and academic research.[11] Since our main empirical results are based on two regimes as identified by the sentiment index, not on the detailed levels of the index, our conclusions should be robust beyond Baker and Wurgler's index. Furthermore, our empirical patterns continue to hold if we identify the two regimes with alternative investor sentiment indicators, like the closed-end fund discount rate, IPO activity, trading volume, and the Michigan Consumer Sentiment Index.

[9]Many studies argue that these six variables should be related to investor sentiment. See, for example, Lee, Shleifer, and Thaler (1991) for the closed-end fund discount, Baker and Stein (2004) for turnover, Stigler (1964) and Ritter (1991) for the number of and first-day returns of IPOs, Baker and Wurgler (2000) for the equity share in new issues, and Baker and Wurgler (2004a, b) for the dividend premium.

[10]The set includes the industrial production index growth, durable consumption growth, non-durable consumption growth, service consumption growth, employment growth, and a dummy variable for NBER recessions.

[11]For example, Malkiel (1990), Brown (1991), Siegel (1998), Shiller (2000), Cochrane (2003), and Ljungqvist and Wilhelm (2003).

4.2 Data and Summary Statistics

In this paper, we use the NYSE-AMEX equal-weighted and value-weighted returns as proxies for stock market returns, and the one-month T-bill returns as the interest rate. These data are obtained from CRSP for the period January 1963 to December 2004.

The sentiment literature has long focused on the equal-weighted index. Equal-weighted returns provide an excellent and accommodating stage to explore the impact of investor sentiment because they are more influenced by small-cap stocks. As pointed out by Baker and Wurgler (2006), small stocks are likely to be young, unprofitable, and extreme-growth potential, which makes them more vulnerable to broad shifts in the propensity to speculate. Moreover, the arbitrage force is relatively weak in small stocks because of their high idiosyncratic risk and their high costs to sell short.[12]

Besides exploring the equal-weighted index, our study also analyzes the value-weighted index and finds strong empirical results with it. Accordingly, sentiment influence on the mean-variance tradeoff is pervasive through the entire stock universe.

The summary statistics of market excess returns and realized variance are reported in Table 1. The moments of returns and realized variance are different between the low- and high-sentiment regimes. The mean of the equal-weighted returns in the low-sentiment regime is 1.396%, which is much higher than its counterpart in the high-sentiment regime (0.150%). This pattern is consistent with economic intuition – high sentiment drives up the price and depresses the return – and has been documented by the existing literature.

We find interesting patterns in the skewness of stock returns. It has been well

[12]High idiosyncratic risk makes relative-value arbitrage especially risky (Wurgler and Zhuravskaya (2002)). High costs to sell short reduce the profits of arbitrage strategies and, in some cases, cause them to become completely unprofitable (Geczy, Musto, and Reed (2002), Jones and Lamont (2002), Duffie, Garleanu, and Pedersen (2002)).

documented that stock returns show negative skewness. Table 1 shows that the overall negative skewness results from the negatively skewed returns in the high-sentiment periods (-0.654 for the equal-weighted index and -0.471 for the value-weighted index), while in the low-sentiment periods the return skewness could be positive (0.95 for the equal-weighted index) or close to zero (-0.048 for the value-weighted index).

Such patterns of divergent skewness in the two regimes are consistent with the sentiment hypothesis. It is widely perceived that investor sentiment should be mean-reverting, based on both empirical and theoretical evidence.[13] Given the mean-reverting property of sentiment, the distribution of sentiment conditional on the high-sentiment regime should have a longer right tail. Since higher sentiment pushes up current prices and depresses expected returns, the return distribution is left skewed in such a regime. Hence, we expect significantly negative skewness from the high-sentiment regime.

The right half of Table 1 reports the moments of realized variance, and provides support for one of our key arguments: In high-sentiment periods, sentiment traders have more impact on stock prices. All the moments of realized variance in the high-sentiment regime are dramatically higher than their counterparts in the low-sentiment regime. Such results indicate that prices are more volatile in high-sentiment periods, which is consistent with the large influence of sentiment traders during such periods.

4.3 Mean-Variance Relation

The mean-variance relation has been intensively analyzed in the equation

$$R_{t+1} = a + bVar_t(R_{t+1}) + \epsilon_{t+1} \ ,$$

[13] Baker and Wurgler's index evidently follows a mean-reverting process. The mean-reverting property of investor sentiment also has a theoretical foundation. For example, Scheinkman and Xiong (2003) argue that overconfidence would lead to a mean-reverting difference of opinions among different investors.

where R_{t+1} is the monthly excess return and $Var_t(R_{t+1})$ is the conditional variance. To test our hypothesis that the tradeoff is undermined in the high-sentiment regime, we analyze the following two-regime equation:

$$R_{t+1} = a_1 + b_1 Var_t(R_{t+1}) + a_2 D_t + b_2 D_t Var_t(R_{t+1}) + \epsilon_{t+1} \ ,$$

where D_t is a dummy variable for the high-sentiment regime, that is, D_t equals 1 if month t is in a high-sentiment period. The details to define it are as follows. Since the BW index is an annual index, we classify a year as a high-sentiment year if the prior year's sentiment – also the beginning-of-period value of the current year – is positive. In our 1963 to 2004 sample period, 21 years, or half of the sample period, falls into the high-sentiment regime.

We expect b_2 to be negative since high sentiment should weaken the mean-variance relation, and we expect b_1 to be positive since there should exist positive compensation for bearing volatility during low-sentiment periods without too much turbulence caused by sentiment traders.

Table 2 reports the estimates and t-statistics with the rolling window model as the conditional variance model. In the one-regime equation that has been analyzed in the existing literature, the mean-variance tradeoff is weak and ambiguous. The mean-variance relation, b, is -0.299 with a t-statistic of -0.33. The R^2 of the regression is low, less than 0.1%.

The empirical results from the two-regime equation strongly support the view that the mean-variance tradeoff varies with investor sentiment. In the low-sentiment periods, we find a significantly positive tradeoff (b_1 is 13.075 with a t-statistic of 2.45), whereas in the high-sentiment periods, such a tradeoff is dramatically weakened (b_2 is -13.714 with a t-statistic of -2.64). As a result, the mean-variance slope in the high-sentiment periods is nearly flat ($b_1 + b_2$ is -0.639 with a t-statistic -1.06). The above estimates are not only statistically significant but also economically impressive. The magnitude of b_1 implies that a one-standard-deviation increase in variance is associated with a 1.017% increase in monthly excess returns during the low-sentiment periods. Moreover, the two-regime equation accommodates the data much better than the one-regime equation, with R^2 increasing from less than 0.1% in the old equation to 3.1%.

We also find significant results with value-weighted returns in Panel B. The mean-variance relation in the low-sentiment periods is 8.650 with a t-statistic of 2.22 and the difference between the two regimes is -9.361 with a t-statistic of -2.38. Such results indicate that the sentiment effect on the mean-variance tradeoff is not limited to small-cap stocks but also spreads to the large-cap stocks. However, the influence on large stocks is weaker than that on small stocks. This is consistent with the well-established pattern: Investor sentiment has a stronger impact on small stocks than large ones.

Table 3 reports the coefficients and t-statistics with MIDAS as the conditional variance model. Ghysel, Santa-Clara, and Valkanov (2005) argue that MIDAS models conditional variance better than RW because MIDAS use a longer history of past returns and a more flexible weighting system. Moreover, they find that the mean-variance coefficient with MIDAS as the variance model is different from that with RW. In our sample period, the mean-variance coefficient in the one-regime equation with MIDAS, b, is 3.246, which is different from that in RW (-0.299).

However, including the sentiment influence, MIDAS yields the same set of conclusions as RW. The coefficient in the low-sentiment regime, b_1, is 19.814 and the difference between the two regimes, b_2, is -18.102. The t-statistics are 4.54 and -3.52, respectively. The expected returns are positively correlated with conditional variance in the low-sentiment periods. The relation is significantly lower and close to zero in the high-sentiment periods. Similarly, the two-regime equation explains the expected return better than the one-regime equation, with R^2 increasing from 0.4% to 2.9%. The results with value-weighted returns are reported in Panel B, which also show the same two-regime pattern.

Tables 4 and 5 report the results from GARCH (1,1) and asymmetric GARCH (1,1), respectively. GARCH-type models have been extensively used to explore the mean-variance relation.[14] Glosten, Jagannathan, and Runkle (1993) find that the standard GARCH and the asymmetric GARCH can produce conflicting conclusions. Under the two-regime setting, however, these two models reach the same set of conclusions that has been shown above.

Our empirical results are strikingly robust across different conditional variance models. While they lead to different results in the one-regime setting, the four volatility models yield the same set of empirical conclusions under the two-regime setting: There is a positive mean-variance tradeoff in the low-sentiment periods, but this tradeoff is significantly undermined in the high-sentiment periods. These results strongly support our hypothesis that the large impact of sentiment traders in the high-sentiment periods undermines an otherwise positive mean-variance tradeoff, and also provide evidence for the long-standing intuition that risk is compensated with a positive price when rational investors dominate the stock market.

Moreover, the four volatility models yield similar economic implications: A one-standard-deviation increase in conditional variance during the low-sentiment periods is associated with an approximately 1% increase in monthly equal-weighted returns (1.017%, 0.951%, 1.173%, and 1.210%, respectively) and a 0.7% increase in monthly value-weighted returns (0.637%, 0.617%, 0.656%, and 0.810%, respectively). Evidently, the mean-variance slopes in the low-sentiment regime are not only statistically

[14]For example, French, Schwert, and Stambaugh (1987), Nelson (1991), Campbell and Hentschel (1992), Ghysel, Santa-Clara, and Valkanov (2005), and Lundblad (2005).

significant but also economically significant.

To further show the economic significance, we also calculate the annual Sharpe ratios implied from the above mean-variance coefficients, which are also impressive. The annual Sharpe ratio in the low-sentiment periods is 1.078, 1.791, 1.355, and 2.000 for the equal-weighted index, and 0.830, 1.050, 1.321, and 2.116 for the value-weighted index.[15]

None of the mean-variance coefficients in the high-sentiment regime, that is, b_1+b_2, are significantly different from zero.[16] The nearly flat mean-variance relation might seem to suggest that the market is dominated by irrationality during those periods, but this need not be the case. In a unreported simulation analysis that can be provided upon request, we find that sentiment-driven traders can move the price rather modestly but still render the mean-variance tradeoff undetectable in the sample sizes at hand.

Another noteworthy empirical pattern is that the predictive ability of the sentiment dummy (a_2) is not significant while the predictive ability of the interaction between the sentiment dummy and conditional variance (b_2) is significant. The predictive ability of the sentiment dummy corresponds to the widely understood intuition that when sentiment is high (low), the stock market is overvalued (undervalued). Since sentiment eventually returns to its long-run mean and the price comoves with sentiment, we expect a lower (higher) future return. Our empirical results, however, suggest that such predictive ability is weak at the one-month horizon. This result is broadly consistent with Brown and Cliff (2005) and Yuan (2005), who document that due to sentiment's high persistence, its long-run ability to predict market returns is

[15] To obtain the implied annual Sharpe ratio, we first calculate the implied monthly Sharpe ratio by multiplying the mean-variance coefficients by the mean of the conditional standard deviations, and then multiply the monthly Sharpe ratio by $\sqrt{12}$.

[16] With the four volatility models, the estimates are -0.639, 1.712, 4.038, and 3.424. The t-statistics are -1.06, 0.632, 1.18, and 1.18, respectively.

stronger than its short-run predictive ability. Turning to the predictive ability of the interaction between the sentiment dummy and conditional variance, we argue in this paper that sentiment also predicts returns by influencing the compensation for bearing variance risk. Our results indicate that such predictive ability emerges in a more quickly manner than that of sentiment dummy alone, which requires that sentiment revert to its long-run mean.

4.4 Return-Innovation Relation

In this subsection we examine the relation between realized returns and contemporaneous volatility innovations. Such a relation is called an "indirect" test of the mean-variance tradeoff by French, Schwert, and Stambaugh (1987) – if a high conditional volatility depresses the current price and boosts the expected future return, an unexpected volatility shock should also push the realized return down.

Our hypothesis predicts a two-regime pattern in the return-innovation relation too. In low-sentiment periods without sentiment trader bungling, there should be strong negative market reactions to variance shocks. We expect weaker reactions during the high-sentiment periods with more sentiment traders who forgo risk compensation.

To explore the above hypothesis, we examine the following equation:

$$\begin{aligned} R_{t+1} &= c_1 + d_1 Var_t(R_{t+1}) + e_1 Var(R_{t+1})^u \\ &\quad + c_2 D_t + d_2 D_t Var_t(R_{t+1}) + e_2 D_t Var(R_{t+1})^u + \epsilon_{t+1} \ , \end{aligned}$$

where $Var_t(R_{t+1})$ is the conditional variance, $Var(R_{t+1})^u$ is the contemporaneous volatility innovation, and D_t is the dummy variable for the high-sentiment regime. Our hypothesis predicts a negative e_1 and a positive e_2.

Table 6 reports the estimates and t-statistics for the equal-weighted returns in the four volatility models. In RW, e_1, the coefficient on the volatility innovation in the low-sentiment periods, is -22.625 with a t-statistic of -3.10. The volatility shocks depress the contemporaneous price levels significantly in the low-sentiment regime. The difference between the two regimes, e_2, is 15.543 with a t-statistic of 2.09. The significantly positive e_2 shows that reactions to volatility innovations are weaker during high-sentiment periods. Such two-regime patterns in the return-innovation relation provide further support for the empirical conclusion in the last subsection: Investor sentiment plays a crucial role in the mean-variance tradeoff.

The above two-regime patterns in the return-innovation relation are robust across the other three volatility models. The return-innovation relation is significantly negative in the low-sentiment regime: The coefficient in the low-sentiment regime, e_1, is -24.674 in MIDAS, -21.656 in GARCH, and -26.198 in asymmetric GARCH. The reactions are significantly weaker during the high-sentiment periods: the difference, e_2, is 17.456 in MIDAS, 12.760 in GARCH, and 15.395 in asymmetric GARCH. The results with the value-weighted returns, reported in Table 7, exhibit the same set of patterns.

Moreover, the magnitude of the estimated return-innovation coefficient during the low-sentiment periods (e_1) is also close to the value implied by our estimated mean-variance coefficients. We conduct a calibration based on the following intuition. Because of the persistence of the variance process, a volatility shock in the current period should have an impact on future variances. Since the future conditional mean and the conditional variance are correlated, the current volatility shock changes the future expected returns - the discount rates - by magnitudes depending on the mean-variance slope and the persistence of the volatility process. With the assumption of unchanged future dividends, we calibrate the change in the current price caused by a volatility shock and then obtain the implied ratio of the return-innovation coefficient over the mean-variance slope, $\frac{e_1}{b_1}$. The implied $\frac{e_1}{b_1}$ from the calibration is -1.89, which is fairly close to the ratios obtained from the four econometric models: -1.73 in RW, -1.25 in MIDAS, -2.86 in GARCH, and -2.29 in asymmetric GARCH. The details of the calibration can be provided upon request.

Overall, investor sentiment impacts not only the tradeoff between expected returns and conditional variance, but also the reactions of contemporaneous returns to variance shocks. Furthermore, the two sets of results are consistent both in signs and in magnitudes. Such two-regime patterns provide solid evidence for sentiment influence on the link between return and volatility.

In the high-sentiment regime, despite weaker reactions to variance, we still get significantly negative return-innovation relations. The return-innovation slope in the high-sentiment regime, $e_1 + e_2$, is -7.082 in RW, -7.218 in MIDAS, -8.897 in GARCH, and -10.802 in asymmetric GARCH. These results indicate that the overall market is still averse to volatility in the high-sentiment regime. Such results appear to imply a weaker but positive mean-variance tradeoff, which may be difficult to detect directly with the sample sizes at hand.[17]

We also explore the relation among innovations in sentiment, returns, and volatility. The correlation between innovations in annual sentiment and innovations in annual returns is positive. This result is consistent with the long-standing intuition that stock prices comove with investor sentiment. The correlation between absolute values of sentiment innovations and volatility innovations is positive. A large change in beliefs of sentiment traders seems to raise the volatility of the equity market. This result suggests that, in addition to the first moment of returns, investor sentiment also influences the second moment, variance, which has not been documented in the

[17] According to Lundblad (2005), if the mean-variance slope is as low as 2, the significantly positive relation can hardly be detected, even with a 50-year sample, owing to high volatility of the stock returns.

academic literature. However, due to the scope of this paper, we leave further analysis to future research.

5 Robustness Checks

In this section, we present the results of robustness checks. We first investigate whether the two-regime pattern above exists for macroeconomic variables containing business cycle information in Section 5.1. We then analyze the setting where the mean-variance coefficient is assumed to be a linear function of sentiment value in Section 5.2.

5.1 Comparing Sentiment With Macro Variables

In the last section we show that investor sentiment has a strong ability to distinguish two regimes with different degrees of aversion to volatility. In this subsection we explore the following empirical question: Does any macroeconomic variable show similar ability?

Early models like ICAPM generally suggest a constant mean-variance relation, which contradicts our empirical findings – the time-varying risk-return tradeoff. More recent empirical studies such as Keim and Stambaugh (1986) and Fama and French (1989), who document significant time variation of the expected returns over business cycle, posit the presence of counter-cyclical risk aversion, which could suggest a time-varying mean-variance relation. A leading theoretical model along this line is the external habit model of Campbell and Cochrane (1999), in which the representative agent's utility depends on the difference between current consumption and habit which is the average of past consumption. The risk aversion of the representative agent moves in the opposite direction of the time-varying surplus ratio, which is the above difference normalized by current consumption. Since the surplus ratio varies with the business cycle, such time-varying risk aversion could produce a countercyclical risk-return tradeoff. Hence, the alternatives along this line have the empirical implications that the business cycle variables and the consumption surplus ratio drive the time-varying mean-variance relation.

To explore these alternatives, in this subsection we analyze macroeconomic variables containing business cycle information – interest rate, term premium, default premium, dividend-price ratio, and the consumption surplus ratio defined in Campbell and Cochrane (1999).[18] For every macroeconomic variable, we run identical empirical tests as for investor sentiment, to explore whether any variable has the ability to distinguish two regimes with different degrees of aversion to volatility.

As we show next, no macroeconomic variable has such an ability. Hence, investor sentiment has a unique capacity to distinguish periods that are strongly averse to volatility from those that are not.[19] Overall, it seems very difficult for our empirical results to fit in the existing theoretical models with either constant or time-varying risk aversion.

The details of the tests are as follows. First, we divide the whole sample period into two regimes, according to whether the macro variable's level is above or below its median. Next we analyze the mean-variance tradeoff in these two regimes and define

[18]The 1-year T-bill return is used as a proxy for the interest rate. The term premium is defined as the return difference between the 30-year and 1-month T bills. The default premium is defined as the return difference between AAA and BAA corporate bonds. The dividend-price ratio is defined as the ratio of the total dividend to the market value of all stocks on the NYSE-AMEX index. The surplus ratio is approximated by a smoothed average of the past 40-quarter consumption growth as in Wachter (2006). The AAA and BAA corporate bonds data and the consumption data are downloaded from the website of the Federal Reserve at St. Louis. The remaining data are obtained from CRSP.

[19]Note that the business cycle information has been removed from the sentiment index in Baker and Wurgler (2006) by regressing each of the raw sentiment measures on industrial production index growth, durable consumption growth, nondurable consumption growth, service consumption growth, employment growth, and a dummy variable for NBER recessions.

the dummy variable – $D_t = 1$ – for the low mean-variance regime:

$$R_{t+1} = a_1 + b_1 Var_t(R_{t+1}) + a_2 D_t + b_2 D_t Var_t R_{t+1} + \epsilon_{t+1}.$$

By construction, b_2 should be negative, although the significance level could vary across different macroeconomic variables, and such a regime should correspond to the periods less averse to volatility. Finally, we also examine the relation between returns and contemporaneous variance innovations:

$$R_{t+1} = c_1 + d_1 Var_t(R_{t+1}) + e_1 Var(R_{t+1})^u + c_2 D_t + d_2 D_t Var_t R_t + e_2 D_t Var(R_{t+1})^u + \epsilon_{t+1}.$$

We expect e_2 to be positive, which implies that reactions to variance innovations are weaker in the low mean-variance regime.

For a variable capable of distinguishing two regimes with different degrees of aversion to volatility, we expect significant results across the four volatility models and consistent evidence in both the mean-variance and return-innovation relations.

We summarize the above tests with the equal- and value-weighted returns shown in Table 8, which reports the counts of significant estimates for the four key parameters (b_1, b_2, e_1, and e_2) with the four variance models. We also divide the estimates into those with correct signs and those with wrong signs according their consistency. For example, a negative estimate for e_2 is classified as a wrong sign, since it indicates stronger reactions to variance innovations in the second regime, which is constructed as the weak mean-variance regime. In the last line we report the summary score, which is calculated by subtracting the sum of wrong counts from the sum of correct counts. A high score indicates a robust and consistent two-regime pattern for this variable.

Table 8 shows that sentiment has the highest score across all the variables. Since we estimate the four parameters with the four variance models for the equal- and value-weighted returns, the full score is 32. For sentiment, 29 out of 32 parameters are significant and none of the significant estimates shows inconsistency. No macro variable could produce a set of similarly strong and consistent results. For interest rate, default premium, and dividend price ratio, the estimates from the mean-variance and return-innovation relations yield contradicting implications. Seven out of eight estimates for e_2 are significantly negative. Such results indicate that in the low mean-variance regime, prices react more strongly to variance innovations – a clearly inconsistent set of results. The term premium and consumption surplus ratio show little ability to distinguish the high and low mean-variance regimes.

In summary, the empirical patterns of sentiment are strong and consistent, which is unique compared with macro variables. No other variable shows a similar ability to distinguish the two regimes having different degrees of aversion to volatility. It is unlikely that our empirical results are driven by business cycle information.

5.2 A Linear Function of Sentiment

In this subsection, we explore an alternative specification where the mean-variance slope is assumed to be a linear function of investor sentiment. Specifically, we analyze the following empirical model:

$$R_{t+1} = f_0 + f_1\delta_t + (g_0 + g_1\delta_t)Var_t(R_{t+1}) + \epsilon_{t+1} \ ,$$

where δ_t is the sentiment value at the beginning of the month and $Var_t(R_{t+1})$ is the conditional variance of the monthly excess returns. Baker and Wurgler (2007) build a monthly sentiment index with the similar methodology as their annual index. The monthly index, which starts in January 1966, has a shorter sample period. But it contains more time-varying information. In the above test, we use this monthly index. The empirical results are similar if we use the annual index instead. The conditional variance is estimated by the four volatility models. In GARCH(1,1) and asymmetric GARCH(1,1), both monthly and daily data are used to estimate the conditional variance.

Table 9 reports the empirical results. All the estimates of g_1 are negative and most are significant, which indicates that investor sentiment undermines the mean-variance tradeoff. These results confirm the central intuition of our hypothesis: High sentiment causes a large impact of sentiment traders, which undermines the risk-return tradeoff. Moreover, the impact of sentiment is economically significant. A one-standard-deviation increase in sentiment leads to a 2.065 to 5.927 decrease in the mean-variance slope.[20]

We also find that on average, the mean-variance relation is positive. Since the sentiment index has a zero mean, g_0 is the mean-variance coefficient on average through the entire sample period. All the estimates of g_0 are positive. Accordingly, the fundamental intuition in finance – the risk-return tradeoff – holds on average, although it could be much weaker in high-sentiment periods.

6 Conclusions

This study documents that the mean-variance tradeoff is strongly impacted by investor sentiment: High sentiment undermines an otherwise positive tradeoff. This result is further supported by the weaker negative correlation between contemporaneous volatility innovations and returns in high-sentiment periods. The empirical evidence is consistent with a larger influence of sentiment traders during high-sentiment

[20]Since the sentiment index has unit variance, g_1 is the magnitude of the change in the mean-variance coefficient associated with a one-standard-deviation change in sentiment.

periods and with the intuition that sentiment traders can undermine the risk-return tradeoff.

Our paper suggests a novel mechanism for sentiment to move stock prices. Existing studies focus on its direct impact, where stock price levels comove with sentiment. However, in this paper, we documents a new way where sentiment influences the compensation for volatility first and then, in turn, price levels.

Our intuition should extend beyond the mean-variance relation and apply to general risk-return tradeoffs. The hypothesis predicts that high market sentiment reduces risk premiums by activating biased sentiment traders who demand lower price of risk. Such a result should have a huge impact on asset allocation decisions. For example, asset management firms should consider reducing their holdings on high-risk stocks during high-sentiment periods, since the risk tolerated during these periods is poorly compensated.

Finally, this study contributes to the mean-variance literature. Despite intensive analysis, empirical researchers have not reached a consensus on the tradeoff. Taking the role of investor sentiment into account, we find that there exists a robust positive mean-variance tradeoff in low-sentiment periods, periods during which sentiment traders have small impact. Such results provide evidence for the long-standing intuition that rational investors require positive compensation for bearing volatility. Moreover, we find that sentiment traders undermine this positive tradeoff. These results suggest that models of stock prices and risk-return tradeoff should integrate investor sentiment and assign it a significant role.

Appendix

In this appendix, we develop a two-period model to formalize the intuition in Section 2. In our model, there are two types of investors: rational traders (i.e. arbitrageurs) and sentiment traders. Sentiment traders are different with the arbitrageurs in the following three ways. First, while arbitrageurs have correct beliefs with respect to the fundamental, that is, dividend growth, sentiment traders over- or underestimate it, depending on their sentiment. Second, while arbitrageurs can take both long and short positions, sentiment traders are assumed to be unable to short stock. This is in line with the evidence that individual investors – the primary sentiment trader candidates – rarely hold short positions.[21] Finally, in contrast to rational investors who precisely estimate variance, the estimates of sentiment traders contain noise.[22] The intuition here is that sentiment traders are more likely to be naive investors and hence they may have a poor understanding of how to measure risk. We formalize these differences below.

The investment opportunities are represented by two securities, namely, a risky stock and a riskless bond with gross risk-free rate R_f and zero net supply. The stock pays dividend D_0 in period 0 and $D_1 \sim N(\mu_D, \sigma_D^2)$ in period 1. Without loss of generality, we assume $D_0 = 1$, and hence μ_D is the expected growth rate of the dividend. There are a total of $N = N_1 + N_2$ investors in the economy. The first N_1 investors are arbitrageurs, who have correct beliefs about the dividend in period 1, D_1. The remaining N_2 investors are sentiment traders, who have divergent beliefs about the expectation of D_1. In the model, we assume that sentiment trader i's belief is such that the expected dividend is $\mu_D + \eta_i$. Accordingly, if η_i is positive (negative), investor i is optimistic (pessimistic). Market sentiment is defined as the average of

[21] See, for example, Odean (1998), Barber and Odean (2006).

[22] We highly appreciate the referee for this suggestion.

sentiment traders' beliefs, that is, $\delta_t = \frac{1}{N_2}\sum_{i=1}^{N_2}\eta_i$. Note that our model requires no specific assumption over the distribution of η_i across sentiment traders.

As we discuss above, sentiment traders misestimate variance. We formalize this assumption by assuming that sentiment trader i's belief with respect to the variance of dividend D_1 is equal to $\sigma_D^2(1+\epsilon_i)$. Here, the noise term ϵ_i is independently distributed across all sentiment traders with zero mean and $\epsilon_i > -1$. Note that sentiment traders have unbiased estimates of the variance on average.

All investors have the same CARA utility functions with the same risk aversion coefficient. Similar results can be reached if other utility functions (e.g., like CRRA) are used instead. At time 0, trader i owns ω^i shares of the stock market, with $\sum_{i=1}^{N}\omega^i = 1$. Investor i maximizes his two-period utility over consumption by choosing his current consumption allocation C_0^i and the portion of wealth to be invested in the stock market π^i at time 0:

$$
\begin{aligned}
&\max_{\pi^i, C_0^i} E^i\left(u\left(C_0^i\right)+u\left(C_1^i\right)\right)\\
s.t.\quad C_1^i &= \left(W_0^i - C_0^i\right)\left(\left[1-\pi^i\right]R_f + \pi^i\frac{D_1}{S_0}\right)\\
W_0^i &= \omega^i\left(D_0+S_0\right),
\end{aligned}
$$

where S_0 is the ex-dividend stock price at time 0, $E^i(\cdot)$ is the expectation operator under the belief of agent i, and $W_0^i = \omega^i(D_0+S_0)$ is the initial wealth of agent i at time $t=0$ before period 0 consumption. We also have market clearing conditions for the stock $(\sum_{i=1}^{N}\pi^i(W_0^i - C_0^i) = S_0)$ and bond $(\sum_{i=1}^{N}(W_0^i - C_0^i)[1-\pi^i] = 0)$.

When the short-sale constraint is not binding, the first-order condition with respect to agent i's investment in stock π^i gives investor i's demand for the stock as

$$
\left(\omega^i\left(D_0+S_0\right)-C_0^i\right)\pi^i = \frac{\frac{\mu_D+\eta_i}{S_0}-R_f}{\alpha\frac{\sigma_D^2(1+\epsilon_i)}{S_0^2}},
$$

where α is the risk aversion coefficient and $\epsilon_i = 0$ for arbitrageurs ($i = 1, 2, \cdots N_1$).

More importantly, the above equation provides us the following two results. First, the demand of a trader who misestimates the variance as $\sigma^2(1+\epsilon_i)$ is the same as the demand of an investor who correctly estimates the variance and has a risk aversion coefficient of $\alpha(1+\epsilon_i)$. In other words, misestimating variance is equivalent to having a noisy risk aversion coefficient. Second, the short-sale constraint of sentiment trader i is binding if and only if $\frac{\mu_D+\eta_i}{S_0} < R_f$. In the model, sentiment traders stay out of the stock market if they are pessimistic and believe that the stock return is lower than the risk-free rate.

Combining the above first-order condition with the market clearing condition for the stock, we can derive the following relationship between the expected excess return and variance.

Proposition 1: With CARA utility, if the last K sentiment investors' short-sale constraints are binding, then we have the following mean-variance relation:

$$\begin{aligned} E\left(R_1^{ex}\right) &= \sigma^2\left(R_1^{ex}\right) \cdot \frac{\left\{\frac{S_0}{N_1+N_2-K}\right\}}{\left[\frac{1}{\alpha}\frac{N_1}{N_1+N_2-K} + \frac{N_2-K}{N_1+N_2-K} \cdot \frac{1}{N_2-K}\sum_{i=N_1+1}^{N_1+N_2-K}\frac{1}{(1+\epsilon_i)\alpha})\right]} \\ &\quad - \frac{1}{S_0}\frac{\frac{1}{N_2-K}\sum_{i=N_1+1}^{N-K}\frac{\eta_i}{1+\epsilon_i}}{\frac{N_1}{N_2-K} + \frac{1}{N_2-K}\sum_{i=N_1+1}^{N-K}\frac{1}{1+\epsilon_i}} \end{aligned}$$

where η_i is investor i's sentiment, α is the CARA risk-aversion coefficient, and ϵ_i is the noise in investor i's variance estimation.

The above proposition shows that the risk-return tradeoff is mostly determined by two factors: the average of the inverse risk-aversion attitudes of stock market participants (the item in $[\cdot]$), and the average of stock market participants' stock holdings (the item in $\{\cdot\}$). High sentiment weakens the mean-variance tradeoff by influencing both of these factors, as we discussed next.

High market sentiment undermines the mean-variance relation by increasing the average of the inverse risk-aversion attitudes of stock market participants (the item in $[\cdot]$). The intuition for this result is as follows[23]. Following Jensen's inequality, the last term inside $[\cdot]$, $\frac{1}{N_2-K}\sum_{i=N_1+1}^{N-K}\frac{1}{(1+\epsilon_i)\alpha} \approx \frac{1}{\alpha}E\left(\frac{1}{1+\epsilon_i}\right) > \frac{1}{\alpha}$. Thus misestimation of variance makes sentiment traders collectively less averse to variance. When more sentiment traders participate in the stock market, the weight on sentiment traders, $\frac{N_2-K}{N_1+N_2-K}$, is higher, which results in a larger value for the item in $[\cdot]$. As a consequence, the mean-variance relation is weaker during periods when more sentiment traders participate in the stock market. In addition, because of short sale constraints, fewer sentiment traders participate in the stock market when market sentiment is low. Taken together, we have that the mean-variance relation is weaker when aggregate sentiment is high.

In addition, high market sentiment should further undermine the mean-variance relation by reducing the average stock holdings of market participants (the item in $\{\cdot\}$). The intuition for this result – the risk premium is negatively related to market participation – is consistent with limited participation studies, e.g., Basak and Cuoco (1998). During high-sentiment periods, with more participants in the market sharing the same amount of aggregate risk, participants demand a lower risk premium.

Note that we will obtain the same set of conclusions if sentiment traders correctly estimate variance but are subject to some cognitive biases. In the above setting, misestimation of variance induces the representative sentiment trader to demand less compensation for risk. More generally, many cognitive biases can cause sentiment investors to agree to a lower price for risk by compensating them with benefits derived from the biases. In other words, cognitive biases can lower the effective risk aversion of

[23] This result also derives from the following economic intuition. Since the demand function for stock is convex in the noise of traders' variance estimation, dispersion in this noises increases sentiment traders' aggregate demand, which pushes up the equilibrium stock price and in turn decreases the expected return required to compensate for the same amount of variance. Yan (2009) uses the same intuition to show that noise trading cannot be canceled out by aggregation.

sentiment traders even though they share similar aversion to risk as rational investors. Theoretical results for biased sentiment traders can be provided upon request.

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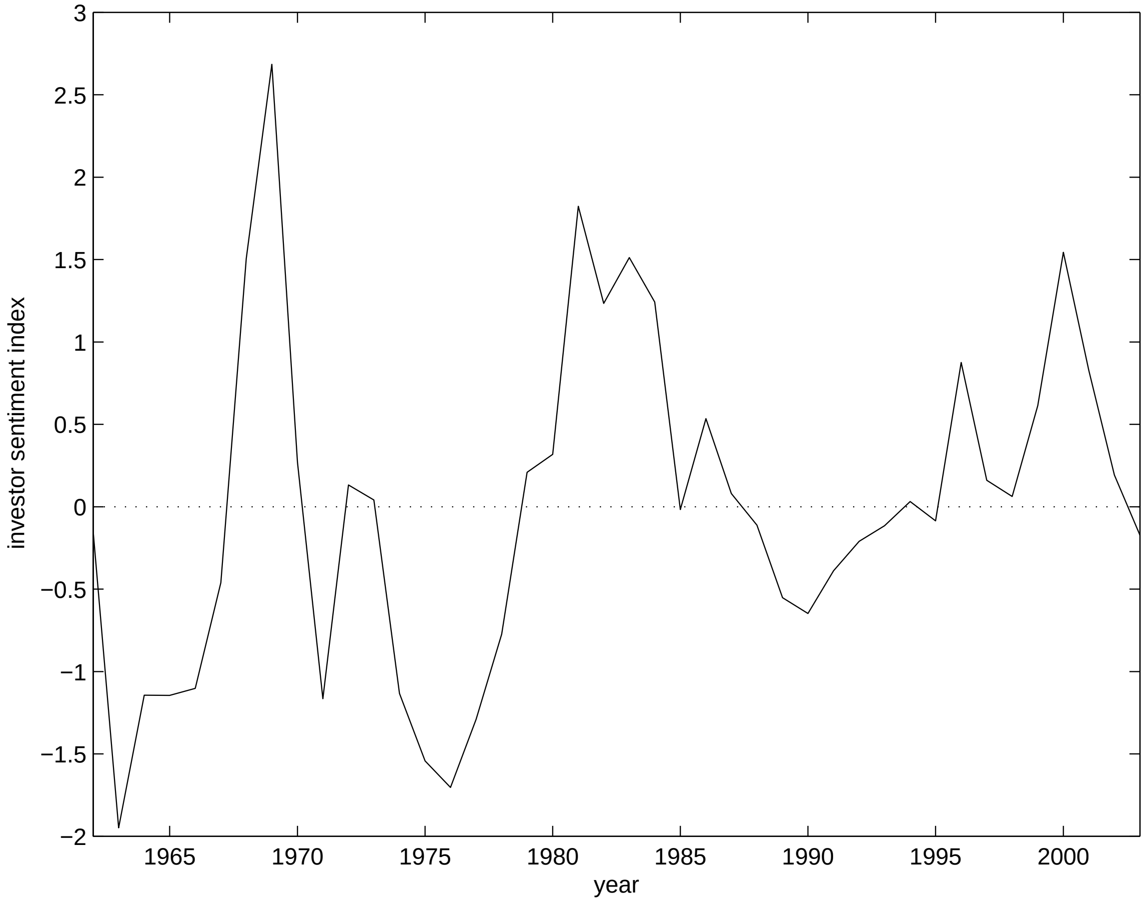
Figure 1: The Investor Sentiment Index From 1962 to 2003



The sentiment index is the first principal component of the six measures. The six measures are the closed-end fund discount, the NYSE share turnover, the number of and the average first-day returns on IPOs, the equity share in new issues, and the dividend premium. To control for macro conditions, we regress the six raw sentiment measures on the growth of industry production, the growth of durable consumption, the growth of nondurable consumption, the growth of service consumption, the growth of employment, and a dummy variable for NBER recessions.

Table 1: Summary Statistics of Monthly Excess Returns and Realized Variance

Panel A Equal-Weighted Index								
	Excess Returns				Realized Variance			
	Mean $\times 10^2$	Variance $\times 10^2$	Skewness	Kurtosis	Mean $\times 10^3$	Variance $\times 10^6$	Skewness	Kurtosis
Whole sample	0.773	0.294	-0.067	7.479	1.069	5.832	15.013	282.091
Low sentiment	1.396	0.250	0.958	10.547	0.689	0.565	2.793	12.551
High sentiment	0.150	0.333	-0.654	5.149	1.449	10.833	11.535	158.755

Panel B Value-Weighted Index								
	Excess Returns				Realized Variance			
	Mean $\times 10^2$	Variance $\times 10^2$	Skewness	Kurtosis	Mean $\times 10^3$	Variance $\times 10^6$	Skewness	Kurtosis
Whole sample	0.489	0.182	-0.432	5.195	1.545	8.122	13.930	257.010
Low sentiment	0.789	0.124	-0.048	4.154	0.881	0.540	2.132	8.953
High sentiment	0.189	0.238	-0.471	4.799	2.210	14.850	10.789	146.744

The excess returns are computed from the monthly returns on the NYSE-AMEX index and the returns on the 1-month T-bill. The realized variance is computed from the within-month daily returns. The sample period is January 1963 to December 2004.

Table 2: Monthly Excess Returns Against Conditional Variance in Rolling Window Model

Panel A Equal-Weighted Returns					
	a (a_1)	b (b_1)	a_2	b_2	R^2
One-regime	0.008	-0.299			0.000
(1), (3)	(3.10)	(-0.33)			
Two-regime	0.005	**13.075**	-0.002	**-13.714**	0.031
(2), (3)	(1.38)	(2.45)	(-0.37)	(-2.64)	

Panel B Value-Weighted Returns					
	a (a_1)	b (b_1)	a_2	b_2	R^2
One-regime	0.006	-0.581			0.002
(1), (3)	(2.83)	(-0.87)			
Two-regime	-0.000	**8.650**	0.003	**-9.361**	0.019
(2), (3)	(-0.00)	(2.22)	(0.72)	(-2.38)	

$$R_{t+1} = a + bVar_t(R_{t+1}) + \epsilon_{t+1} \quad (1)$$

$$R_{t+1} = a_1 + b_1Var_t(R_{t+1}) + a_2D_t + b_2D_tVar_tR_{t+1} + \epsilon_{t+1} \quad (2)$$

$$Var_t(R_{t+1}) = 22\sum_{d=1}^{N_t}\frac{1}{N_t}r_{t-d}^2 \quad (3)$$

R_{t+1} is the monthly excess return on the NYSE-AMEX index. $Var_t(R_{t+1})$ is the conditional variance. D_t is the dummy variable for the high-sentiment periods. r_{t-d} is the daily demeaned NYSE-AMEX equal-weighted index return (the daily return minus the within-month mean). N_t is the number of trading days in month t. The sample period is January 1963 to December 2004. The numbers in parentheses are t-statistics from the Newey-West standard error estimator.

Table 3: Monthly Excess Returns Against Conditional Variance in MIDAS

Panel A Equal-Weighted Returns

	a (a_1)	b (b_1)	a_2	b_2	κ_1	κ_2 $\times 10^3$	R^2
One-regime	0.004	3.246			-0.018	0.049	0.004
(4), (6)	(2.20)	(1.48)			(-2.37)	(1.65)	
Two-regime	-0.001	**19.814**	-0.000	**-18.102**	-0.020	0.053	0.029
(5), (6)	(-0.22)	(4.54)	(-0.08)	(-3.52)	(-2.75)	(1.88)	

Panel B Value-Weighted Returns

	a (a_1)	b (b_1)	a_2	b_2	κ_1	κ_2 $\times 10^3$	R^2
One-regime	0.002	1.571			-0.048	0.148	0.005
(4), (6)	(1.06)	(0.90)			(-3.57)	(2.57)	
Two-regime	-0.002	**10.340**	0.002	**-9.524**	-0.043	0.127	0.016
(5), (6)	(-0.66)	(2.60)	(0.46)	(-2.13)	(-3.33)	(2.23)	

$$R_{t+1} = a + bVar_t(R_{t+1}) + \epsilon_{t+1} \tag{4}$$

$$R_{t+1} = a_1 + b_1 Var_t(R_{t+1}) + a_2 D_t + b_2 D_t Var_t R_{t+1} + \epsilon_{t+1} \tag{5}$$

$$Var_t(R_{t+1}) = 22 \sum_{d=0}^{250} w_d r_{t-d}^2 \quad w_d(\kappa_1, \kappa_2) = \frac{exp\{\kappa_1 d + \kappa_2 d^2\}}{\sum_{i=0}^{250} exp\{\kappa_1 i + \kappa_2 i^2\}} \tag{6}$$

R_{t+1} is the monthly excess return on the NYSE-AMEX index. $Var_t(R_{t+1})$ is the conditional variance. D_t is the dummy variable for the high-sentiment periods. r_{t-d} is the daily demeaned return (the daily return minus the within-month mean). The sample period is January 1963 to December 2004. The numbers in parentheses are t-statistics.

Table 4: Monthly Excess Returns Against Conditional Variance in GARCH(1,1)

			Panel A	Equal-Weighted Returns				
	a (a_1)	b (b_1)	a_2	b_2	ω $\times 10^3$	α	β	R^2
One-regime	-0.003	4.461			0.150	0.086	0.868	0.017
(7), (9)	(-0.51)	(1.95)			(2.13)	(3.06)	(22.93)	
Two-regime	-0.006	**7.566**	-0.001	**-3.531**	0.167	0.087	0.859	0.036
(8), (9)	(-0.74)	(2.28)	(-0.06)	(-0.86)	(2.19)	(3.14)	(22.13)	

			Panel B	Value-Weighted Returns				
	a (a_1)	b (b_1)	a_2	b_2	ω $\times 10^3$	α	β	R^2
One-regime	-0.001	4.120			0.092	0.084	0.869	0.006
(7), (9)	(-0.23)	(1.25)			(1.78)	(2.97)	(25.13)	
Two-regime	-0.007	**9.692**	-0.005	**-2.084**	0.106	0.079	0.865	0.021
(8), (9)	(-0.68)	(1.50)	(-0.43)	(-0.29)	(1.81)	(2.86)	(24.55)	

$$R_{t+1} = a + bVar_t(R_{t+1}) + \epsilon_{t+1} \tag{7}$$
$$R_{t+1} = a_1 + b_1Var_t(R_{t+1}) + a_2D_t + b_2D_tVar_t(R_{t+1}) + \epsilon_{t+1} \tag{8}$$
$$Var_t(R_{t+1}) = \omega + \alpha\epsilon_t^2 + \beta Var_{t-1}(R_t) \tag{9}$$

R_{t+1} is the monthly excess return on the NYSE-AMEX index. $Var_t(R_{t+1})$ is the conditional variance. D_t is the dummy variable for the high-sentiment periods. The sample period is January 1963 to December 2004. The numbers in parentheses are t-statistics.

Table 5: Monthly Excess Returns Against Conditional Variance in Asymmetric GARCH(1,1)

		Panel A	Equal-Weighted Returns						
	a (a_1)	b (b_1)	a_2	b_2	ω $\times 10^3$	α_1	α_2	β	R^2
One-regime	-0.003	4.080			0.241	0.126	-0.097	0.838	0.017
(10), (12)	(-0.45)	(1.84)			(2.45)	(2.86)	(-1.86)	(18.39)	
Two-regime	-0.017	**11.453**	0.011	**-8.029**	0.266	0.124	-0.103	0.831	0.035
(11), (12)	(-1.46)	(2.59)	(0.88)	(-1.76)	(3.29)	(3.16)	(-2.63)	(21.84)	

		Panel B	Value-Weighted Returns						
	a (a_1)	b (b_1)	a_2	b_2	ω $\times 10^3$	α_1	α_2	β	R^2
One-regime	0.001	2.641			0.316	0.183	-0.228	0.721	0.005
(10), (12)	(0.10)	(0.73)			(1.60)	(2.10)	(-1.92)	(5.06)	
Two-regime	-0.019	**15.703**	0.028	**-17.109**	0.278	0.174	-0.220	0.772	0.019
(11), (12)	(-1.54)	(2.02)	(2.34)	(-2.41)	(3.16)	(2.37)	(-2.35)	(13.78)	

$$R_{t+1} = a + bVar_t(R_{t+1}) + \epsilon_{t+1} \tag{10}$$
$$R_{t+1} = a_1 + b_1Var_t(R_{t+1}) + a_2D_t + b_2D_tVar_t(R_{t+1}) + \epsilon_{t+1} \tag{11}$$
$$Var_t(R_{t+1}) = \omega + \alpha_1\epsilon_t^2 + \alpha_2I_t\epsilon_t^2 + \beta Var_{t-1}(R_t) \tag{12}$$

R_{t+1} is the monthly excess return on the NYSE-AMEX index. $Var_t(R_{t+1})$ is the conditional variance. D_t is the dummy variable for the high-sentiment periods. I_t is the dummy variable for positive shocks. The sample period is January 1963 to December 2004. The numbers in parentheses are t-statistics.

Table 6: Monthly Excess Returns Against Conditional Variance and Volatility Innovations

	Panel A						
	c_1	d_1	e_1	c_2	d_2	e_2	R^2
Rolling window	0.014	-1.395	**-22.625**	0.002	-5.664	**15.543**	0.165
(13), (14)	(2.11)	(-0.12)	(-3.10)	(0.35)	(-0.49)	(2.09)	
MIDAS	0.001	16.885	**-24.674**	0.005	-19.281	**17.456**	0.186
(13), (15)	(0.06)	(1.54)	(-3.54)	(0.53)	(-1.66)	(2.46)	
GARCH	0.001	6.277	**-21.656**	0.010	-11.633	**12.760**	0.232
(13), (16), (17)	(0.07)	(0.57)	(-4.14)	(0.99)	(-1.05)	(2.31)	
Asymmetric GARCH	0.009	-0.008	**-26.198**	0.002	-5.089	**15.395**	0.264
(13), (16), (18)	(1.36)	(-0.00)	(-8.47)	(0.25)	(-0.66)	(3.95)	

	Panel B Daily Estimates				
	μ $\times 10^3$	ω $\times 10^6$	$\alpha(\alpha_1)$	α_2	β
GARCH	1.090	1.802	0.181		0.794
(16), (17)	(19.56)	(9.49)	(17.25)		(68.93)
Asymmetric GARCH	1.009	1.869	0.231	-0.123	0.797
(16), (18)	(18.25)	(9.73)	(16.17)	(-10.68)	(67.61)

$$R_{t+1} = c_1 + d_1 Var_t(R_{t+1}) + e_1 Var(R_{t+1})^u + c_2 D_t + d_2 D_t Var_t R_{t+1} + e_2 D_t Var(R_{t+1})^u + \epsilon_{t+1} \tag{13}$$

$$Var_t(R_{t+1}) = 22 \sum_{d=1}^{N_t} \frac{1}{N_t} r_{t-d}^2 \tag{14}$$

$$Var_t(R_{t+1}) = 22 \sum_{d=0}^{250} w_d r_{t-d}^2 \quad w_d(\kappa_1, \kappa_2) = \frac{exp\{\kappa_1 d + \kappa_2 d^2\}}{\sum_{i=0}^{250} exp\{\kappa_1 i + \kappa_2 i^2\}} \tag{15}$$

$$r_{t+1}^{raw} = \mu + \epsilon_{daily,t+1} \tag{16}$$

$$h_{t+1} = \omega + \alpha \epsilon_{daily,t}^2 + \beta h_t \tag{17}$$

$$h_{t+1} = \omega + \alpha_1 \epsilon_{daily,t}^2 + \alpha_2 I_t \epsilon_{daily,t}^2 + \beta h_t \tag{18}$$

R_{t+1} is the monthly excess return on the equal-weighted NYSE-AMEX index. $Var_t(R_{t+1})$ is the conditional variance. $Var(R_{t+1})^u$ is the unpredictable component of the variance (the realized variance minus the conditional variance) for the rolling window model and MIDAS and the innovation of future volatility implied by daily GARCH(1,1) or asymmetric GARCH(1,1). D_t is the dummy variable for the high-sentiment periods. r_{t-d} is the daily demeaned NYSE-AMEX equal-weighted index return (the daily return minus the within-month mean). N_t is the number of trading days in month t. κ_1 and κ_2 are estimated from the MIDAS model in Table 3 ($\kappa_1 = -0.018$ and $\kappa_2 = 0.049 \times 10^{-3}$). r_{t+1}^{raw} is the daily raw return and h_{t+1} is the conditional variance of daily returns. I_t is the dummy variable for a positive shock. The sample period is January 1963 to December 2004. The numbers in parentheses are t-statistics from the Newey-West standard error estimator in Panel A and t-statistics from the MLE standard error estimator in Panel B.

Table 7: The Return-Innovation Relation of the Value-Weighted Index

	c_1	d_1	e_1	c_2	d_2	e_2	R^2
Rolling window	0.009	-1.411	**-15.679**	0.002	-2.699	**11.578**	0.117
	(1.95)	(-0.24)	(-4.41)	(0.38)	(-0.47)	(3.28)	
MIDAS	0.003	4.150	**-21.052**	0.005	-6.911	**16.765**	0.137
	(0.72)	(0.91)	(-6.08)	(0.86)	(-1.47)	(4.85)	
GARCH	0.005	-2.051	**-19.944**	0.006	-1.931	**14.748**	0.150
	(0.89)	(-0.32)	(-3.71)	(0.86)	(-0.31)	(2.73)	
Asy GARCH	0.010	-7.496	**-30.901**	0.000	3.901	**25.637**	0.251
	(2.10)	(-1.55)	(-8.59)	(0.06)	(0.81)	(6.70)	

$$\begin{aligned} R_{t+1} &= c_1 + d_1 Var_t(R_{t+1}) + e_1 Var(R_{t+1})^u \\ &\quad + c_2 D_t + d_2 D_t Var_t R_{t+1} + e_2 D_t Var(R_{t+1})^u + \epsilon_{t+1} \end{aligned}$$

R_{t+1} is the monthly excess return on the NYSE-AMEX value-weighted index. $Var_t(R_{t+1})$ is the conditional variance. $Var(R_{t+1})^u$ is the volatility innovation. D_t is the dummy variable for the high-sentiment periods. The sample period is January 1963 to December 2004. The numbers in parentheses are t-statistics from the Newey-West standard error estimator.

Table 8: The Numbers of Significant Estimates

	Parameters	Sent	Int	Term	Def	$\frac{D}{P}$	S
Correct sign	Positive b_1	7	3	0	3	3	1
	Negative b_2	6	1	0	1	3	1
	Negative e_1	8	8	8	8	8	8
	Positive e_2	8	0	2	0	0	2
Wrong sign	Negative b_1	0	1	0	0	1	0
	Positive b_2	0	0	0	0	0	0
	Positive e_1	0	0	0	0	0	0
	Negative e_2	0	7	0	7	7	0
Correct-wrong	**Score**	**29**	**4**	**10**	**5**	**6**	**12**

$$
\begin{aligned}
R_{t+1} &= a_1 + b_1 Var_t(R_{t+1}) + a_2 D_t + b_2 D_t Var_t R_{t+1} + \epsilon_{t+1} \\
R_{t+1} &= c_1 + d_1 Var_t(R_{t+1}) + e_1 Var(R_{t+1})^u \\
&\quad + c_2 D_t + d_2 D_t Var_t R_t + e_2 D_t Var(R_{t+1})^u + \epsilon_{t+1}
\end{aligned}
$$

We report the numbers of estimates that are significant at the 10% level in the four volatility models for the equal- and value-weighted returns. R_{t+1} is the monthly excess return on the NYSE-AMEX index. $Var_t(R_{t+1})$ is the conditional variance. $Var(R_{t+1})^u$ is the volatility innovation. D_t is the dummy variable for the second regime. Sentiment is the composite sentiment index. Int is the return on the 1-year T bill. Term is the return difference between 30-year and 1-month T bills. Def is the return difference between the AAA and BAA corporate bonds. $\frac{D}{P}$ is the dividend-price ratio of the stocks on NYSE-AMEX. S is the surplus consumption ratio from Campbell and Cochrane (1999). The sample period is January 1963 to December 2004.

Table 9: Linear Function of Investor Sentiment

	f_0	f_1	g_0	g_1	R^2
Rolling window	0.006	-0.002	1.357	**-3.438**	0.019
	(3.74)	(-1.02)	(2.73)	(-3.08)	
MIDAS	0.003	0.000	3.908	**-5.927**	0.027
	(1.39)	(0.17)	(1.67)	(-2.11)	
Daily GARCH(1,1)	0.005	0.000	1.782	**-4.263**	0.026
	(2.72)	(0.02)	(2.85)	(-3.15)	
Daily Asy GARCH(1,1)	0.007	-0.003	0.322	**-2.578**	0.018
	(5.67)	(-0.97)	(0.73)	(-1.97)	
Monthly GARCH(1,1)	0.001	0.001	2.042	**-2.065**	0.020
	(0.18)	(0.17)	(0.67)	(-0.77)	
Monthly Asy GARCH(1,1)	-0.002	0.003	3.332	**-2.933**	0.031
	(-0.29)	(0.40)	(1.18)	(-1.02)	

$$R_{t+1} = f_0 + f_1\delta_t + (g_0 + g_1\delta_t)Var_t(R_{t+1}) + \epsilon_{t+1}$$

R_{t+1} is the monthly excess return on the NYSE-AMEX equal-weighted index. δ_t is investor sentiment at the beginning of the month, from the monthly sentiment index in Baker and Wurgler (2007). $Var_t(R_{t+1})$ is the conditional variance. The sample period is January 1966 to December 2004. The numbers in parentheses are t-statistics from the Newey-West standard error estimator or from the MLE standard error estimator.