# ESSAYS ON ECONOMIC UNCERTAINTY AND MACRO-FINANCE

Yang Liu

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

in Partial Fulfillment of the Requirements

for the Degree of Doctor of Philosophy

2017

Supervisor of Dissertation

Amir Yaron, Robert Morris Professor of Banking and Finance

Graduate Group Chairperson

Jesús Fernández-Villaverde, Professor of Economics

Dissertation Committee

Enrique G. Mendoza, Presidential Professor of Economics

Frank Schorfheide, Professor of Economics

Ivan Shaliastovich, Associate Professor of Finance

ESSAYS ON ECONOMIC UNCERTAINTY AND MACRO-FINANCE

ACKNOWLEDGEMENT

I would like to extend my sincere gratitude to my advisor and mentor Amir Yaron for his invaluable guidance and support. I am indebted to my dissertation committee members, Enrique Mendoza, Frank Schorfheide, and Ivan Shaliastovich for their advice and encouragement. I also thank Rabah Arezki, Ravi Bansal, Ric Colacito, Hal Cole, Max Croce, Frank Diebold, Hanming Fang, Jesús Fernández-Villaverde, João Gomes, Urban Jermann, Dirk Krueger, Nick Roussanov, Tom Sargent, Lukas Schmid for helpful discussion and direction.

I am fortunate to study and work with a great cohort. I am thankful for my classmates and friends. Especially, I thank my long-time friend Xiang Fang who has been inspiring me since freshmen year in college and coauthored the third chapter.

I thank the Federal Reserve Bank of Philadelphia and the International Monetary Fund for their hospitality while writing part of this dissertation.

Finally, I am extremely thankful for my parents.

# ABSTRACT

ESSAYS ON ECONOMIC UNCERTAINTY AND MACRO-FINANCE

Yang Liu

Amir Yaron

This dissertation studies topics in macro-finance with a focus on economic uncertainty.

The first chapter (Government Debt and Risk Premia) studies the implications of government debt for asset prices. I document a set of new facts that government debt is related to risk premia in various asset markets. First, the debt-to-GDP ratio positively predicts excess stock returns. The forecast power is compelling, and it outperforms many popular predictors. Second, higher debt-to-GDP ratio is correlated with higher credit risk premia in both corporate bond excess returns and yield spreads. Third, higher debt-to-GDP ratio is associated with lower real risk-free rate. Fourth, higher debt-to-GDP ratio predicts lower average returns on government debt. Expected return variation contributes to a sizable amount of the volatility of the debt-to-GDP ratio. Fifth, debt-to-GDP ratio positively comoves with fiscal policy uncertainty. Fiscal uncertainty also has direct effects on the asset prices consistent with the effect of debt-to-GDP ratio. I rationalize these empirical findings in a general equilibrium model featuring recursive preferences, endogenous growth, and time-varying fiscal uncertainty. In the model, the tax risk premium is sizable and its time variation is driven by fiscal uncertainty. Furthermore, the model generates an endogenous positive relationship between the debt-to-GDP ratio and fiscal uncertainty: fiscal uncertainty increases debt valuation through discount rate channel whereas higher debt conversely raises uncertainty in future fiscal consolidations.

In the second chapter (Volatility Risk Pass-Through), we estimate and explain the international transmission of output volatility shocks to both currencies and international quantity dynamics. We produce novel empirical evidence on the relevance of output volatility (vol)

shocks for both currency and international quantity dynamics. Focusing on G-17 countries, we document several facts: (1) consumption and output vols are imperfectly correlated within countries; (2) across countries, consumption vol is more correlated than output vol; (3) the pass-through of relative output vol shocks onto relative consumption vol is moderate, especially if the uncertainty shocks originate from small countries; and (4) consumption differentials vol and exchange rate vol are disconnected, in contrast to the perfect correlation implied by a model of perfect risk-sharing with time-additive preferences. We rationalize these findings in a frictionless model with multiple goods and recursive preferences featuring a novel-and-rich risk-sharing of vol shocks.

The third chapter (Volatility, Intermediaries, and Exchange Rates) studies how financial market volatility drives exchange rates through the risk management practice of financial intermediaries. We build a model in which the major participants in the international financial market are levered intermediaries subject to Value-at-Risk constraints. Higher portfolio volatility translates into tighter funding conditions and increased marginal value of wealth. Thus, foreign currency is expected to appreciate. Our model can resolve the Backus-Smith puzzle, the forward premium puzzle, and the exchange rate volatility puzzle quantitatively. Our empirical test verifies two implications of the model that both financial market volatility and funding condition measurement have predictive power on exchange rates.

TABLE OF CONTENTS

LIST OF TABLES

# LIST OF ILLUSTRATIONS

# CHAPTER 1 : Government Debt and Risk Premia

## 1.1. Introduction

The government debt is of great importance to the economy, policymaking, and financial markets. This paper documents a set of new facts about the effects of government debt on asset prices in the United States. High debt-to-GDP ratios are related to high equity risk premia, high credit risk premia, low risk-free rates, low expected returns on government debt, and high fiscal policy uncertainty. I rationalize these facts in a general equilibrium model featuring a fiscal uncertainty channel that links government debt and asset prices.

The importance of government debt is manifested in equity, credit and treasury markets. First, high debt-to-GDP ratio corresponds to high equity premium. The debt-to-GDP ratio positively predicts excess stock returns at horizons from one quarter to five years. The ratio contains useful information beyond a large number of existing predictors, thus improving the predictive power. In a univariate predictive regression using debt-to-GDP ratio, the out-of-sample $R^2$ is 10% at an annual horizon and reaches 30% at a five-year horizon. In comparison, the out-of-sample $R^2$ of many popular predictors are marginally positive. A strategy that times the market using debt-to-GDP ratio can generate an excess return of 14.71% per annum with a Sharpe ratio of 0.66, while a buy and hold strategy of the market portfolio yields a Sharpe ratio of 0.3.

In credit markets, I observe a similar pattern that high debt-to-GDP ratios are related to high credit risk premia. One measure of credit risk premia is the expected excess return on corporate bonds. The debt-to-GDP ratio positively predicts excess returns on investment-grade and high-yield corporate bonds. The magnitude is close to the stock return predictability. Another measure of credit risk premium is a yield spread. I document that government debt raises the credit premium component of yield spreads.

The first two findings show that high debt-to-GDP ratio implies high cost of capital for firms. Regarding the cost of capital for government, however, high debt-to-GDP ratios are associated with low real risk-free rates and low expected returns on government debt. Both 1-month and 3-month real risk-free rates are negatively related to the debt-to-GDP ratio, controlling for expected growth and inflation. Furthermore, I examine the discount rate of the government. Since the government does not only issue short-term debt, the government discount rate or effective borrowing cost is the average return across terms to maturity on all the Treasury securities. In the default-free case, the government budget constraint implies that a high debt-to-GDP ratio can stem from three channels: (i) high expected future primary surplus to pay off the debt, (ii) high expected future growth to stabilize the ratio, and (iii) low expected future returns on government debt. The previous studies mainly focus on the first two channels. Here I document that the third discount rate channel is empirically important:[1] the debt-to-GDP ratio negatively predicts returns on government debt. I use a present value decomposition in a vector autoregression to quantify the relative contribution of these three components. The variation of expected returns accounts for 25% of the variation of the debt-to-GDP ratio.

Why does government debt have such significant effects on asset prices? Major existing channels of government debt such as liquidity, safety, and crowding out are silent or inconsistent with these facts. I propose a new channel—fiscal uncertainty—that can rationalize the empirical findings jointly. I propose a broad-based measure of fiscal policy uncertainty by utilizing 169 macro variables and estimating a dynamic factor model with stochastic volatility. In the data-rich environment, fiscal policy consists of 37 variables regarding various types of tax, spending and transfer. Fiscal uncertainty is measured as the common component of the conditional forecast error volatility of these fiscal policy instruments. Empirically, fiscal uncertainty fluctuates over time and positively comoves with the debt-to-GDP ratio with a correlation of 0.5. Therefore, government debt encodes the risks in fiscal policy that drive the variation of risk premia. I present direct evidence that fiscal un-

[1] This discount rate channel is addressed differently in several papers. Hall and Sargent (2011) show variations in realized returns affect the evolution of the debt-GDP ratio. Berndt et al. (2012) find that part of a fiscal spending shock is financed with decreases in the discount rate.

certainty affects asset prices in equity, credit, and treasury markets in the same directions and has similar magnitudes as the debt-to-GDP ratio.

Within a general equilibrium model, I quantify the effects of government debt and fiscal uncertainty on asset prices. The key ingredients of the model include recursive preferences, endogenous growth through innovation, and fluctuations in the volatility of distortionary corporate income tax. Tax hikes depress innovation and economic growth so that persistent tax changes are a source of endogenous long-run risks. Stock prices drop with tax hikes because of the tax payment and the lower cash flow growth. For fear of the joint decrease of growth prospects and stock prices, agents demand a large equity premium for tax risks. This risk compensation is even larger when the "quantity" of risk increases in times of high fiscal uncertainty. Hence, time variation in equity premium is driven by fiscal uncertainty. In contrast, non-defaultable government bonds rally in times of high tax, because lower expected growth induces the agents to purchase safe bonds. Thus, government bonds hedge against tax risks for investors and have negative risk premia. In time of high fiscal uncertainty, the hedging motive drives down the government bond premium. Moreover, uncertainty increases the precautionary saving motive and lowers the risk-free rate.

The model generates a positive comovement between the debt-to-GDP ratio and fiscal uncertainty through two mechanisms. Uncertainty lowers both risk-free rate and bond risk premium and thus the expected return on government debt. The declining expected return leads to the rise of the bond price. Therefore, the debt-to-GDP ratio increases with uncertainty through the discount rate channel. Conversely, debt generates uncertainty in future fiscal policy. The government implements fiscal consolidations from time to time to reduce deficits and debt accumulation. The consolidation policy is uncertain and anticipated to be more active when debt is high. As a result, high debt-to-GDP ratio brings more uncertainty in fiscal consolidations. The two mechanisms reinforce each other. In equilibrium, the debt-to-GDP ratio reveals fiscal uncertainty and has implications for asset prices that are consistent with the empirical findings. Calibrated to fiscal policy data, the model quantitatively explains many features of macroeconomics dynamics and asset markets such as equity premium and risk-free rate, as well as the novel facts regarding the government debt.

**Relation to Literature.** There is a long-lasting debate on the effects of government debt on interest rate (Elmendorf and Mankiw, 1999; Engen and Hubbard, 2005; Laubach, 2009). Few papers consider the importance of risk premia across different interest-bearing instruments. Distinguishing between real risk-free rate, return on equity, corporate bonds, and government debt, I show that high government debt is associated with high cost of capital for firms and low cost of capital for government. Krishnamurthy and Vissing-Jorgensen (2012) find that high government debt is related to lower spreads between assets with different liquidity and safety attributes.[2] My evidence of the effect of government debt on credit risk premia is complementary to their evidence of liquidity premia. I document differential effects that government debt enhances credit risk premia in corporate bond market and diminishes liquidity premia in money market. Croce et al. (2016) show that debt-to-GDP ratio predicts the spreads between innovation-sorted stock portfolios in the time series and cross section, while I focus on the aggregate asset markets. Greenwood and Vayanos (2014) document that the maturity structure of government debt affects nominal bond risk premia and term spreads.

I contribute to the voluminous literature of stock return predictability by analyzing debt-to-GDP ratio as a predictor. The results have little bias from the high persistence of the debt-to-GDP ratio (Campbell and Yogo, 2006; Stambaugh, 1999). The out-of-sample predictive power is compelling (Welch and Goyal, 2008). Debt-to-GDP ratio outperforms the popular predictors regarding out-of-sample mean squared error.[3]

A long theoretical literature links government debt to macroeconomic dynamics. Ricardian

[2] Graham et al. (2014) document a similar negative relationship between debt-to-GDP ratio and Baa-Aaa spread. They also find that government debt has large impacts on corporate financing and investment policies.

[3] Some of the major predictors are the dividend-price ratio (Campbell and Shiller, 1988), book-to-market ratio (Kothari and Shanken, 1997), term spread (Fama and French, 1989), short rate (Hodrick, 1992; Ang and Bekaert, 2007), investment rate (Cochrane, 1991), the consumption-wealth ratio (Lettau and Ludvigson, 2001), output gap (Cooper and Priestley, 2009), and government investment rate (Belo and Yu, 2013).

Equivalence states that government debt has no effect in a frictionless standard representative-agent model (Barro, 1974). However, in the presence of liquidity and safety needs, government debt plays a special role and has significant effects on macroeconomic quantities and asset prices (Bansal and Coleman, 1996; Krishnamurthy and Vissing-Jorgensen, 2012; Gorton and Ordonez, 2013; Drechsler et al., 2014; Greenwood et al., 2015). The impact of government debt is also large in heterogeneous agent incomplete market models (Gomes et al., 2013). These theories are either silent on the new empirical findings or have counterfactual implications that high government debt is related to low equity premium and high risk-free rate. I contribute to the understanding of government debt by proposing a new fiscal policy uncertainty channel which operates through the government discount rate and also affects other risk premia. Because debt-to-GDP ratio encodes the variation of fiscal uncertainty, it explains risk premium variation, which complements the existing explanations of time-varying risk aversion (Campbell and Cochrane, 1999), time-varying consumption volatility (Bansal and Yaron, 2004), and time-varying risk of disasters (Wachter, 2013).

My analysis of fiscal uncertainty also relates to the recent literature examining the role of economic uncertainty both in the data and models (Bloom, 2009a; Bansal et al., 2014a; Jurado et al., 2015b, among others). Pástor and Veronesi (2013) and Baker et al. (2015) study the asset pricing and macroeconomic impacts of general economic policy uncertainty. Fernández-Villaverde et al. (2015) and Born and Pfeifer (2014) show the importance of fiscal uncertainty on economic activities. I propose a new broad-based measure of fiscal policy uncertainty and illustrate its importance for asset prices.

More broadly, this article belongs to the growing literature studying asset prices in a production economy (Jermann, 1998; Croce, 2014). Similar to Kung and Schmid (2015) and Kung (2015), I endogenize the long run risks (Bansal and Yaron, 2004) in a expanding variety endogenous growth model (Romer, 1990).[4] The long run risks are purely driven by productivity shocks in Kung and Schmid (2015), whereas part of the long run risks

[4] Comin and Gertler (2006) study business cycle and long-run dynamics in a unified endogenous growth model.

rise from tax policy in my model. Croce et al. (2012) demonstrate a sizable tax risk premium in a model with capital structure choice where tax rate drives the technology growth exogenously.

The remainder of the paper is organized as follows. Section 1.2 documents the empirical findings. Section 1.3 lays out the model. Section 1.4 presents the economic mechanism and the quantitative implications of the model. Section 1.5 concludes.

## 1.2. Empirical Evidence

In this section, I document several new facts relating government debt-to-GDP ratio. First, I show that debt-to-GDP ratio positively predicts market equity risk premia both in sample and out of sample. This results is robust to a large set of controls, sub-sample analysis, different definitions of government debt, international data, and persistent predictor issues. To the best of my knowledge, this paper is the first to discover this relationship. I also show the connection between government debt and credit risk premia. Next, I document that high government debt is associated with low risk-free rate and low expected return on government debt. I propose a present-value decomposition to illustrate the importance of variation of government discount rate in driving debt-to-GDP ratio. Finally, as the economic force that I proposed underlines the rise in risk premia due to increased government debt is fiscal policy uncertainty, I use two empirical measures of fiscal uncertainty and show that they are positively correlated with government debt-to-GDP ratio.

### *1.2.1. Data Description*

Government debt is defined as the market value of the federal government debt held by the public. The market value of government debt is constructed by summing up the market value of all the credit market instruments across maturities (Treasury bonds, Treasury notes, Treasury bills, TIPS, etc). Government debt data are from Dallas Fed, Flow of Funds and, George Hall (Hall and Sargent, 2011). Figure 1.1 demonstrates the time series plot of the debt-to-GDP ratio. The ratio doubled from 20% to 40% during the Great

Depression and jumped to 100% around the second world war. It declined gradually in the peacetime expansion until the early 1970s. Congress increased its control on the government budget process after the Congressional Budget and Impoundment Control Act of 1974, leading to deficits and rising debt. In the 1980s, President Reagan's tax cuts and military buildup further increased the debt. The fiscal balance returned to a surplus in the term of President Clinton, due to tax increases, military spending decreases, and an economic boom. Finally, the Great Recession, combined with Bush tax cuts, caused expanding government expenditure and declining revenue. In 2014, the ratio reached its post-war peak. One crucial feature is that the debt-to-GDP ratio is driven mainly by military and political issues and fiscal policy reforms. While the debt-to-GDP ratio rises in NBER recessions, it does not tend to decline in normal times. In fact, the business cycle only accounts for a small proportion of its variance.

The asset price data are obtained from CRSP, Barclay and Fred. The average return on government debt is from George Hall. The stock return predictors are from Amit Goyal's website. The data on macroeconomic and fiscal variables are from NIPA and FRED-QD database (McCracken and Ng, 2016). The detailed explanations of the data are in the appendix.

### *1.2.2. Equity Premium*

After studying the time series property of the debt-to-GDP ratio, I show that it strongly predicts future excess stock returns in sample and out of sample.

**In-sample Tests** Table 1.1 reports the results from OLS regressions of future excess stock returns on log debt-to-GDP ratio. Excess stock return is the log market return subtracting the log risk-free rate. Long-horizon excess returns are the cumulative summation of the one-period excess returns. In the sample from 1926 to 2014, higher debt-to-GDP ratio forecasts higher stock returns in the future. The forecasting power becomes stronger at longer horizons as $R^2$ rises from 11% at the annual horizon to 38% at the five-year horizon.

As in the previous findings, returns are more predictable at longer horizons, since the high-frequency noises are canceled out, and the slow-moving expected return is reflected more clearly in the data. The coefficients across all the horizons are statistically significant at 99%. In Figure 1.2, I plot the 5-year ahead ex-post and expected excess return. The expected excess return is the fitted value of the predictive regressions. It is evident that higher debt-to-GDP ratio implies higher subsequent returns. The expected excess return rises from the 1930s to the 1950s, declines from the 1950s to the 1970s, and rises again from the 1970s to the 1990s and during the Great Recession.

Beyond the statistical significance, the economic impact of debt-to-GDP ratio on the expected excess return is substantial. A one percentage point increase in debt-to-GDP ratio indicates a 38 basis-point increase in expected excess return per annum.[5] Taking the Great Recession as an example, we observe a rapid increase in the debt-to-GDP ratio from 30% to 60%. This swing implies that the expected return is 11.4% higher than its pre-crisis level. It is acknowledged that excess return predictability is equivalent to time-varying equity premium in a standard rational pricing model.[6] Thus, the rise of debt-to-GDP ratio indicates that investors require a high premium to compensate equity risks. The classic equity premium puzzle emphasizes the difficulty in rationalizing the 6% average equity premium given the lower risk in the consumption profile. It is now more puzzling in that the equity premium is largely time-varying, from 2% in 2007 to 13% in 2014.

From an asset management point of view, this large time variation of expected return is valuable for investors. Consider a mean-variance investor who solves a static portfolio choice problem between aggregate stock and risk-free rate. As is shown in Campbell and Thompson (2008), observing the predictor increases the expected excess return by a factor of $(S^2 + R^2)/((1 - R^2)S^2)$, where $S$ is the Sharpe ratio of the market return. In the sample

[5] The debt-to-GDP ratio enters the regressions in log units. Given debt-to-GDP ratio has a mean of 0.40, a 1% increase is equivalent to a 0.40 percentage point increase of debt-to-GDP ratio.

[6] Equity premium is defined as the expected excess return of the stock market. If it can be predicted by some variable $x$, then in a simple regression case $E_t[r_{m,t+1} - r_{f,t}] = \beta_0 + x_t'\beta$. As a result, equity premium comoves with the predictor $x_t$.

1926-2014, the equity premium is 6.03% and the Shape ratio is 0.3. A strategy that times the market using debt-to-GDP ratio can generate an excess return of 14.71% per annum and a Sharpe ratio of 0.66.[7]

It is believed that debt-to-GDP ratio could be driven by several apparent factors: recessions, wars, and political parties. I show that debt-to-GDP is a new predictor instead of a proxy for business cycle, wars, and parties. First, in economic downturns, low GDP and counter-cyclical surplus raise the debt-to-GDP ratio and meanwhile the counter-cyclical expected return is high. To alleviate this concern, I show that the results are similar if a recession dummy is included in the regression. The debt-to-GDP ratio is acyclical in Figure 1.1. The average ratio is 40% in normal time and 37% in recessions. In the recent decade, even though the Great Recession ends in 2010, the ratio keeps rising afterwards. This reassures that government debt cycle and business cycle are distinct phenomena. Excluding the dramatic increase of the ratio after the recession from 2007 to 2014 doesn't alter the results. Second, the evident link between debt and wars leads to the conjecture that the forecasting power of the debt-to-GDP ratio is related to wars. I include a war-time dummy in the regression. The insignificance of the coefficients across horizons and time periods shows that the forecasting power remains in peacetime and wartime. Third, different political stance might simply determine the tightness of government debt policy. The high stock return under Democratic presidents are documented by Santa-Clara and Valkanov (2003) and Pastor and Veronesi (2017). However, debt-to-GDP still contributes to the explanatory power as before after I include a dummy of president's party.

Beyond the debt-to-GDP ratio, I have identified many other return predictors. Price-dividend ratio is arguably the most popular predictor that is both theoretically grounded and empirically successful. Controlling for price-dividend ratio, both the coefficients and significance of debt-to-GDP ratio are unchanged. As is seen in Figure 1.2, debt-to-GDP has distinct movements from the price-dividend ratio. Moreover, I consider a large set of

[7] The higher expected return is partially from taking on greater risk. The portfolio volatility increases by $1/(1-R^2)$ on average. Therefore, the portfolio Sharpe ratio increases by a factor of $(S^2+R^2)/S^2$.

alternative predictors: price-earning ratio (pe), dividend-earning ratio (de), stock return volatility (svar), book-to-market ratio (bm), net equity expansion (ntis), Treasury bill rate (tbl), long-term yield (lty), long-term return (ltr), term spread (tms), default yield spread (dfy), inflation (infl), investment-capital ratio (ik), consumption-wealth ratio (cay), GDP gap (gap), and government investment-capital ratio (gik). From the set of predictors, I extract the first three principal components that capture 97% of the variation. This parsimonious model is less subject to the concern of in-sample overfitting.[8] Conditioning on a large information set, the debt-to-GDP ratio still contributes to the prediction at a 99% significant level. The point estimates remain similar. The principal components do not drive out the explanatory power of debt-to-GDP ratio, suggesting that the ratio contains extra information.

To assess the stability of the results further, I run the same regressions on quarterly frequency post-war data. The results are reported in Table 1.2. Even at a short horizon of one quarter, the debt-to-GDP ratio significantly predicts excess return.[9] Moreover, the regression coefficients are highly significant and very close to the pre-war coefficients, and the $R^2$ are similar at the annual horizon and the five-year horizon. The significance is robust with several control variables that were mentioned before.

**Out-of-sample Tests** The literature documents considerable in-sample predictability, but out-of-sample performance is usually unsatisfactory (Welch and Goyal, 2008). The poor out-of-sample predictive power raises the concern of data snooping. Debt-to-GDP ratio has strong out-of-sample predictive power. I use out-of-sample $R^2$ to evaluate the predictive accuracy.

$$R^2_{os} = 1 - \frac{MSE_1}{MSE_0}$$

$MSE_0$ and $MSE_1$ are the mean square error using historical mean and the predictive model. In Table 1.3, the $R^2_{os}$ of univariate regression using debt-to-GDP ratio is 0.10 at the annual

[8]I explore each of the predictors in the out-of-sample tests.

[9]The predictability results hold at the one-month horizon as well.

horizon and 0.29 at the five-year horizon, indicating that debt-to-GDP generates smaller MSE than the historical mean. The test for equal predictive accuracy (Clark and West 2007) shows that $MSE_1$ is statistically significantly smaller than $MSE_0$.

Debt-to-GDP ratio outperforms many predictors regarding out-of-sample predictive power. I consider the set of predictors used in the in-sample tests. Debt-to-GDP ratio has the largest $R^2_{os}$ among all the predictors. In fact, most predictors have negative $R^2_{os}$, showing that they are not better than the historical mean. Furthermore, including debt-to-GDP ratio in a bi-variate regression with existing predictors yields positive $R^2_{os}$. The p-values of the equal-predictability test show that debt-to-GDP ratio significantly improves the performance of available predictors. This test can also be interpreted as an encompassing test. Table 1.4 reports the results in the post-war sample. Several variables have better performance than the historical mean in this period, including price-dividend ratio, price-earning ratio, investment-capital ratio, consumption-wealth ratio, GDP gap, and government investment/capital ratio. The last two predictors are documented after the critic of Welch and Goyal (2008). Particularly, they are related to debt-to-GDP ratio. The result shows that debt-to-GDP still has the largest $R^2_{os}$. The improvement is significant at all horizons.

**Overlapping Observations** When we do long-term prediction, the common practice is to predict cumulative return using overlapping observations and to adjust standard errors accordingly. Given many critics on the construction of standard errors, I try to use a different specification by predicting single-period excess return in period $t+h$.

$$r_{m,t+h} - r_{f,t+h-1} = \beta_0 + by_t\beta_1 + x_t\beta + u_{t+h}$$

This approach ensures that the error term $u_{t+h}$ is not serially correlated by construction. Figure 1.3 shows $\beta_1(h)$ and $R^2(h)$ across 40 quarters using quarterly sample. As a result, debt-to-GDP ratio can predict one-period excess return 15 quarters ahead at a 95% significance. This analysis not only shows the robustness of the benchmark results, but also illustrate the term structure of predictability. Both downward slopping coefficients and R-squared imply that it is easier to predict returns in the near future than returns in the far future. We get zero R-squared when trying to predict excess returns in 10 years.

**Persistence** One key concern is the high persistence of the predictor. The debt-to-GDP ratio has a persistence of 0.957 at annual frequency. The stationarity of debt-to-GDP ratio is examined thoroughly in the literature, and there is no convincing evidence of a unit root (Bohn, 2005). If the debt-to-GDP ratio is nonstationary, with probability one it will implausibly diverge to infinity.

The high persistence of the predictor leads to potential invalidity of the inference in two ways. First, if the innovation to the predictor and innovation to the return are correlated, there is a small-sample bias (Stambaugh, 1999). Second, the high degree of persistence results in a nonstandard asymptotic distribution. I use the efficient test by Campbell and Yogo (2006) to address both problems. They propose a Bonferroni test that corrects for the endogeneity and provides an accurate approximation to the finite-sample distribution of test statistics under flexible degrees of persistence (stationary, local-to-unity, and unit root). In Table 1.5, the test results confirm that the conventional t-test in Table 1.1 is valid. The 95% confidence interval does not include zero at all horizons and sample periods. The main reason is that the correlation between innovations of debt-to-GDP ratio and return $\rho_{ue}$ is close to zero, while the correlations are very high in a variety of valuation ratios.

**Components of Government Debt** In the benchmark case, the government debt is defined as the market value of net debt held by the public. There are other definitions and components of debt that have different economic interpretations. The gross level includes debt held by the government accounts that does not represent what the government owes.[10] A portion of the debt is non-marketable and cannot be traded in secondary markets. Given quantitative easing and the rapid growth of foreign investors, the debt held by the Federal

[10] These accounts include the Social Security Trust Fund, federal employee retirement funds, the Unemployment Trust Fund, etc.

Reserve System and foreigners are of interest on their own. Moreover, the commonly reported value is the par value not marked to market. In comparison, the benchmark definition is a more accurate measure in that net debt is more relevant than gross debt to measure the indebtedness of the government and market value reflects up-to-date information in yields changes. Nevertheless, I entertain all the definitions in the following analysis. The correlations of different debt-to-GDP ratios are around 0.9. Table 1.6 reports the results. The difference between various definitions is small in that all the coefficients are around 0.15, and $R^2$ around 10% at the annual horizon. Results are similar at other horizons. Therefore, the different definitions and decomposition of the government debt share the same forecasting power. As we see, the choice of benchmark is innocuous. In the last row, I define debt-to-GDP ratio as the ratio of net debt and potential GDP that capture the ideal level of trend GDP without business cycles, measured by Congress Budget Office. The predictability results remain in this setting. In fact, the denominator plays the role of normalization.[11]

**International Evidence** I verify the forecasting power using data from other countries. I use the debt-to-GDP ratio of each country to predict the excess return on the country's MSCI Index over 3-month treasury bill rate. The finding in the US market also shows up in Canada and UK in Table 1.7. The coefficients are statistically significant, and the magnitude is around 0.2, close to 0.15 in the US. Nonetheless, this empirical relationship is not expected to hold in a wide range of countries, because several country-specific mechanisms: default, inflation, denomination, etc. Specifically, one mechanism is that default probability increases with debt-to-GDP ratio, thus affecting the economic dynamics and asset prices on all the markets. This sovereign default premium should be of first order in most countries. Even treasury bonds of France and Italy have significant yield spreads over Germany treasury bonds. Furthermore, the interactions of monetary and fiscal policy often inflate away the effective government liability in some countries. In the US, we do not observe

[11] Debt-to-Consumption ratio and Debt-to-Industrial-Production ratio yield similar results. Therefore, the forecasting information is in debt and price instead of the various denominators. Similarly, the price-dividend ratio and price-earning ratio have similar forecasting power.

high inflation after high debt. In fact, debt-to-GDP ratio even negatively predicts future inflation. Also, in many countries, a large portion of government debt is denominated in foreign currencies and held by foreign investors. All these channels complicate the cross-country analysis of government debt and risk premia. In this paper, I focus on the countries that are similar to the US conditions that has little default and inflation risks and issues in domestic currency. I left the interesting mechanisms of default, inflation and foreign exposure for future research. Besides the supportive evidence of UK and Canada, I also find insignificance in Germany and Japan. The two countries are different econometrically and economically. In Germany and Japan, the debt-to-GDP ratios have a clear upward trend in the sample. It is clear that a trend does not serve the role of a predictor. This trending debt ratio could be due to small sample problem, monetary union for Germany, and zero lower bound for Japan.

**Dividend Predictability** In Campbell and Shiller (1988) decomposition, price-dividend ratio reflects the expected variation in future excess return and cash flow. This identity justifies why price-dividend ratio is the most widely acknowledged predictor.

$$pd_t \approx E_t[\underbrace{\Sigma_{j=0}\kappa_1^j \Delta d_{t+1+j}}_{cash\ flow} - \underbrace{\Sigma_{j=1}\kappa_1^j r_{t+1+j}}_{discount\ rate}]$$

If one variable has predictive power beyond price dividend ratio, then expected return changes but the price-dividend ratio does not reflects it. From the identity, there are two possibilities that price-dividend ratio fails to capture the expected return news. First, the change of dividend growth in the first term offsets the change in the second term. Second, the expected returns in different time move in the opposite directions. The second term remains unchanged because the variation in $r_{t+j}$ offset the variation in $r_{t+i}$. This dynamics is not common in economics and financial data, and it is not the case for debt-to-GDP ratio as can be seen from Figure 1.3. The predictability of debt-to-GDP ratio falls into the first case. In Table 1.8, the ratio predicts dividend growth significantly with $R^2$ as large as 30%

in a five year horizon. The results are similar after I controlling for price-dividend ratio.

### *1.2.3. Credit Premium and Liquidity Premium*

As shown in Section 1.2.2, debt-to-GDP ratio contains important information about risks in the equity market and *positively* predicts excess stock returns. Corporate bonds are another important asset class that reflect the risk premium for firms. Given the commonality of risk premia fluctuations, we expect to see similar results also in the credit market: the debt-to-GDP ratio (i) *positively* predicts excess returns on corporate bonds; (ii) *positively* relates to corporate bond yield spreads.

Excess returns and yield spreads between corporate and treasury bonds can capture the difference in several factors such as credit risk premium, liquidity premium, collateral premium, inflation premium, etc. A large literature argues that government debt plays a key role in liquidity and safety provision (Krishnamurthy and Vissing-Jorgensen, 2012). In this line of thought, investors value liquidity because of market frictions. Assets that provide liquidity attributes at different levels should have different premia. Time-varying liquidity premium depends on the outstanding amount of highly liquid assets such as government debt. Therefore, high government debt lubricates the economy and decreases the liquidity premium. These theories imply that the debt-to-GDP ratio (i) *negatively* predicts excess returns on equity;[12] (ii) *negatively* predicts excess returns on corporate bonds; (iii) *negatively* relates to corporate bond yield spreads. These implications of the liquidity channel are in sharp contrast to those of the risk channel.[13] Next, I test the two channels in the data.

#### 1.2.3.1. Stock Excess Return

In stock return predictability, I address the liquidity and safety channel of government debt by controlling yield spreads that account for the time-varying liquidity premium. These

[12] Bansal and Coleman (1996) and Krishnamurthy and Vissing-Jorgensen (2012) argue that part of the equity premium is liquidity premium. This liquidity premium channel can partially solve the equity premium puzzle.

[13] He and Xiong (2012) model that the interactions between liquidity and credit risk. Debt market illiquidity increases in not only liquidity premium but also credit risk. This mechanism amplifies the liquidity channel. The three implications have the same signs but larger magnitudes.

variables include spread between Moody's AAA bond and 30-year Treasury bond yield (ats) (Krishnamurthy and Vissing-Jorgensen, 2012) and spread between general collateral repo rate[14] and 3-month treasury bill (liqs) (Nagel, 2014). The results are in Table 1.2. The liquidity premium does not conceal the strong forecasting power of debt-to-GDP ratio. The sign is negative for ats, in contrast to the hypothesis that liquidity premium drives the excess return. Therefore, the time variation in equity premium cannot be explained only through the liquidity channel.

#### 1.2.3.2. Bond Excess Return

In bond return predictability, Table 1.9 shows that debt-to-GDP ratio positively predicts excess returns on corporate bonds, similar to the predictability of stock returns. In a one-year horizon, the coefficients are 0.09 and 0.12 for excess returns on investment-grade and high-yield bond portfolios, similar to the magnitude of coefficient of stock returns (0.15). Controlling for price-dividend ratio and market realized volatility does not weaken the effect of government debt. This predictability implies that debt-to-GDP ratio contains information about credit risk premium.

#### 1.2.3.3. Yield Spreads

Next, I consider a broad range of yield spreads that measure credit risk premia. We expect to see a positive relationship between debt-to-GDP ratio and yield spread if government debt increases the credit risk premia. Gilchrist and Zakrajšek (2012) construct a spread index (GZ spread) from individual corporate bonds traded in the secondary market. They carefully match the duration and maturity between each corporate bond and treasury bond. Their bond also covers the entire maturity spectrum from 1 year to 30 years. In contrast, the standard Moody's seasoned bond yield focuses on bonds that have remaining maturities from 20 to 30 years and unknown duration. In Table 1.10, debt-to-GDP ratio is positively related to GZ spread. The result is significant at 99% confidence level, controlling for

[14] The general collateral repo rate is available from 1991. As in Nagel (2014), I use the banker's acceptance rate before 1991.

the realized volatility and term spread. Realized volatility partially measures the default probability. The term spread controls for the effect of any potential maturity mismatch in the yield spreads on the left-hand side. This relationship is not significant for the spreads between Moody's Aaa, Aa, A, Baa bond yield and 30-year treasury bond yield. Since both debt-to-GDP ratio and spreads are persistent, I specify the regression model in first difference to further explore the dynamic interactions. Both credit risk and liquidity risk channels have the same implications for regressions in levels and first differences. GZ spread and spreads from Moody's all show a positive and significant relationship, supporting the credit risk channel.

In a longer sample from 1919 to 2008, Krishnamurthy and Vissing-Jorgensen (2012) find that Aaa-Treasury spread is negatively related to the debt-to-GDP ratio in a level regression. As seen in Table 1.10, the result is not significant in the sample of 1973-2014 when GZ spread is available. One reason could be sub-sample stability.[15] Another possible reason is that credit risk premium and liquidity premium offset each other. In fact, different yield spreads capture the two sources of premium with different weights. On one hand, Longstaff et al. (2005) document that the majority of long-term bond spreads are due to credit risks. On the other hand, some spreads in money market capture mostly liquidity premium and a priori have few default risks. These include spreads between general collateral repo rate, certificate of deposits rate, AA commercial paper rate, federal funds rate and T-bill rate (Drechsler et al., 2014; Nagel, 2014). Therefore, we could roughly categorize yield spreads into two groups: credit spreads (GZ, Aaa-Treasury, Aa-Treasury, A-Treasury, Baa-Treasury) are mainly in corporate bond market and liquidity spreads (Repo-Bill, CD-Bill, Paper-Bill, FFR-Bill) are in money market. These two categories are not only economically motivated but also empirically grounded. After a factor analysis, I find that each group of spreads has a single factor structure. The first principal component of the spreads within each group explains more than 80% of the variations. However, the two common factors

[15] Yield spreads in the early sample have larger measurement error. Given the data availability, it is not possible to match maturity and duration of government and corporate bonds as in Gilchrist and Zakrajšek (2012).

have a low correlation of 0.15. The factor analysis shows that the time-varying liquidity premium and credit premium are different phenomena.

I verify the liquidity channel in the group of liquidity spreads in Table 1.10. Higher debt-to-GDP ratio is associated with lower spreads that have more weight on liquidity premium. The results are significant both in the level and first difference specifications.

1.2.3.4. A VAR Analysis of Yield Spreads

I further study the dynamics relationship between debt-to-GDP ratio and yield spreads by analyzing the impulse response functions and variance decomposition in a vector autoregression framework. I estimate a five-variable VAR

$$Z_t = \Phi Z_{t-1} + u_t$$

$$Z_t = [\Delta gdp_t,\ svar_t,\ by_t^{book},\ by_t,\ spread_t]$$

The VAR includes GDP growth, stock market volatility, book-value debt-to-GDP ratio, market-value debt-to-GDP ratio and a yield spread. I use an identification strategy that recursively orders the variables as above. The third "book shock" increases the book value of debt but is orthogonal to output and market volatility contemporaneously. I interpret this shock as an exogenous net issuance of government bond that is non-discretionary and not based on the economic and financial conditions. The fourth "market shock" is a shock to the market value of government debt, holding book value constant. In Section 1.4.3, I interpret these two shock through the lens of the model. Though the two shocks capture different mechanisms, both of them link government debt to uncertainty and raise risk premia. In comparison, if liquidity channel dominates, these two shocks decrease risk premia.

I estimate the impulse response of yield spreads to the two debt-to-GDP shocks. I switch different spread into the VAR to keep the parsimony of the system. Figure 1.4 shows the impulse response of the spreads in corporate bond market where credit risks are important.

Both book and market shock increase Gilchrist and Zakrajšek (2012) spread and Moody's spreads. The effects are statistically significant, especially in the market shock. Figure 1.5 shows the impulse response of the spreads in money market where credit risks are less important (Repo-Bill, CD-Bill, Paper-Bill, FFR-Bill). The effect of both shocks are negative and significant on all the spreads. These results confirm the analysis in Section 1.2.3.3 that government debt has differential effects on different markets.

Next, I measure the importance of the effects by variance decomposition. Table 1.11 presents how much of the forecast error variance of the yield spreads can be attributed to the book shock and the market shock. The debt-to-GDP ratio shocks explain 9% of the one-year forecast error variance of GZ spread, 7% of the Aaa-Treasury spread, 15% of the Baa-Aaa spread and around 20% of yield spreads in money market. Therefore, the effect of government debt is quantitatively important to the dynamics of the yield spreads in both corporate bond and money markets.

Therefore, empirical evidence suggests that both channels of credit and liquidity risk are present. High debt-to-GDP ratio is associated with high credit risk premia and low liquidity risk premia. Debt-to-GDP ratio increases yield spreads that mainly capture credit risk and decreases yield spreads that mainly capture liquidity risk.

### *1.2.4. Real risk-free rate*

Government debt could have impacts on the interest rate. This is a long-standing empirical question with little consensus in the literature. In contrast to major previous work, I focus on the short-term real rate. This choice avoids two issues that: (i) the long-term inflation expectation is hard to measure, and (ii) long-term inflation premium is quantitatively important (Ang et al., 2008). To measure the short-term inflation expectation, I use the four-quarter moving average of past inflation and Livingston survey. These two measures are acknowledged to have superior out-of-sample forecasting power. The real risk-free rate is the nominal risk-free rate subtracting the inflation expectation. To control for expected growth, expected inflation, and time-varying risk aversion, I include in the regression the current and lagged consumption growth and inflation and price-dividend ratio. Table 1.12 shows that debt-to-GDP ratio is significantly negatively related to real risk-free rate. The results hold for both pre-war and post-war samples.

### *1.2.5. Return on Government Debt*

After studying the short-term real risk-free rate, I explore the effect of government debt on its aggregate return. The return is defined as the average return across terms to maturity on all the Treasury securities.[16] This return measures the effective borrowing cost or discount rate of the government. Unless the government only issues one-period debt, the return differs from the risk-free rate. In the government budget constraint, the evolution of government debt $B_t$ depends on the government receipts $T_{t+1}$, total outlay net of interest $G_{t+1}$, and the holding period return on government debt $R_{b,t+1}$.

$$B_{t+1} + T_{t+1} - G_{t+1} = B_t R_{b,t+1} \tag{1.1}$$

Similar to Campbell and Shiller (1988), dividing Equation (1.1) by GDP, log-linearizing, iterating forward, and taking expectation, we obtain the following present value decomposition.

$$by_t \approx E_t[\underbrace{\Sigma_{j=0}\kappa_1^j(\kappa_2\tau y_{t+j} - \kappa_3 g y_{t+j})}_{surplus} + \underbrace{\Sigma_{j=0}\kappa_1^j \Delta y_{t+j}}_{real\ growth} - \underbrace{\Sigma_{j=0}\kappa_1^j r_{b,t+j}}_{discount\ rate}] + \kappa_0 \tag{1.2}$$

where $\kappa$ are some constants.[17] The terminal term converges to zero under the assumption of no default. This condition has a intuitive interpretation. A high debt-to-GDP ratio

[16] Define $Q_t^{(n)}$ the price and $b_t^{(n)}$ the amount of n-period discount bond. A coupon bond can be effectively decomposed into discount bonds. The holding period return $R_{b,t}^{(n)} = Q_t^{(n-1)}/Q_{t-1}^{(n)}$. The total market value of debt $B_t = \Sigma_n Q_t^{(n)} b_t^{(n)}$ is the summation of all the outstanding debt. The return on government bond is the average return weighted by the bond value $R_{b,t} = \Sigma_n \frac{Q_{t-1}^{(n)} b_{t-1}^{(n)}}{B_{t-1}} R_{b,t}^{(n-1)}$.

[17] Define $by_t = log(B_t/Y_t)$, $\tau y_t = log(T_t/Y_t)$, $gy_t = log(G_t/Y_t)$. Dividing Equation (1.1) by GDP and log-linearizing,

$$\kappa_0 + \kappa_1 by_{t+1} + \kappa_2 \tau y_{t+1} - \kappa_3 g y_{t+1} = by_t + r_{b,t+1} - \Delta y_{t+1}$$

where $\kappa_1 = \frac{B}{B+T-G}$, $\kappa_2 = \frac{T}{B+T-G}$, $\kappa_3 = \frac{G}{B+T-G}$.

Interating forward,

$$by_t = \Sigma_{j=0}\kappa_1^j(\kappa_2\tau y_{t+j} - \kappa_3 g y_{t+j}) + \Sigma_{j=0}\kappa_1^j \Delta y_{t+j} - \Sigma_{j=0}\kappa_1^j r_{b,t+j} + \kappa_0 + lim_{j\to\infty}\kappa_1^j by_{t+j}$$

The term $lim_{j\to\infty}\kappa_1^j by_{t+j} = 0$ because of the no-Ponzi condition and the assumption of no default.

is rationalized by three channels: (i) high expected future primary surplus to pay off the debt, (ii) high expected future growth to stabilize the ratio, and (iii) low discount rates. The early studies mainly focus on the surplus channel. I find that the often-neglected discount rate channel is empirically important. In predictive regressions in Panel A of Table 1.13, higher debt-to-GDP ratio predicts lower return on government debt from 1 year to 20 years. Moreover, a variance decomposition illustrates the importance of discount rate channel. Take the covariance between Equation (1.2) and $by_t$ on both sides, the variance of debt-to-GDP ratio can be attributed to the three sources.

$$\begin{aligned}var(by_t) =&cov(E_t[\Sigma_{j=0}\kappa_1^j(\kappa_2\tau y_{t+j} - \kappa_3 gy_{t+j})],\ by_t) + cov(E_t[\Sigma_{j=0}\kappa_1^j\Delta y_{t+j}],\ by_t)\\ &-cov(E_t[\Sigma_{j=0}\kappa_1^j r_{b,t+j}],\ by_t)\end{aligned}$$

I estimate a vector autoregression model with five variables $[by_t,\ gy_t,\ \tau y_t,\ \Delta y_t,\ r_{b,t}]'$ to decompose the variance. Panel B of Table 1.13 shows that higher debt-to-GDP ratio precedes higher surplus, higher growth, and lower return. The variance of debt-to-GDP ratio corresponds to variations of all three sources. The discount rate channel accounts for 0.25 of the total variance. The importance is close to the growth channel and half of the surplus channel.[18]

### *1.2.6. Fiscal Uncertainty*

Why does government debt have such significant effects on asset prices? I propose a new channel–fiscal uncertainty–that can rationalize the facts jointly. In this section, I establish the evidence that government debt and fiscal uncertainty positively comove with each other. Furthermore, fiscal uncertainty drives asset prices in equity, credit, and treasury markets.

Throughout history, there exist many periods when people had little consensus about future fiscal policy. In Congress and the White House, policymakers had heated debates on issues such as military expenditures, tax reforms, entitlement, debt limit, and consolidations. In

[18] Cochrane (2011) documents that the importance of the discount rate channel is pervasive in a variety of asset markets.

other periods, fiscal policy was relatively stable, and households and firms reacted accordingly with more confidence. Fiscal policy uncertainty measures how precisely the agents can predict future fiscal policy.

High debt level results in debt-ceiling crises and large fiscal uncertainty. After 1939, Congress use an aggregate debt limit to restrict federal borrowing. If the debt limit binds, the government and Congress have to negotiate reforms on expenditure and tax in short period to avoid the cost of government shut down. These negotiations lead to large fiscal policy uncertainty. It is generally believed that the debt limit does not impose constraints on deficits or surpluses after 1939 (Hall and Sargent, 2015). However, there are a few exceptions. The first debt-ceiling crises is in 1953 (Garbade, 2016). The request of the Eisenhower administration to increase the limit was initially declined. After three temporary increases in 1954, 1955, and 1956, the limit reverted to its 1953 level. Another famous case is the government shutdown in 1995–1996. Recently, we have witnessed the fiscal cliff and multiple debt-ceiling crises. In every crisis, Congress was reluctant to increase the limit unless some balanced-budget amendments were added. The fiscal turmoil raised deep concerns about fiscal policy. Evidently, these crises took place when the government was highly indebted. When the debt-to-GDP ratio is low, the government has more room for its budget with few concerns of a binding debt limit. Therefore, debt-to-GDP ratio determines the probability of a debt-ceiling crisis and encodes fiscal policy uncertainty.

More formally, it is ideal to have some empirical measures of unobserved fiscal uncertainty to examine its effects. I propose a new measure of fiscal uncertainty that utilizes the dynamic factor model in a data-rich environment. This method follows Jurado et al. (2015b), who measure macroeconomic and financial uncertainty. Formally, the h-period ahead uncertainty $U_i(h)$ of a variable $y_{i,t}$ is defined as its conditional volatility.

$$U_i(h) = \sqrt{E_t[(y_{j,t+h} - E_t[y_{j,t+h}])^2]}$$

One main challenge is to correctly compute the conditional mean by including all the variables in the information set. Especially, since the government announces many of policy changes before implementation, accounting for such expected news as forecasting error will lead to a biased uncertainty measure. I collect 169 macroeconomics variables and fit them into a factor model (Equation 1.3-1.4) to capture the conditional mean dynamics of each variable. These variables are related to national income, industrial production, employment, inventories, orders and sales, prices, earning and productivity, and money and credit. Details of the data set are in the appendix. To filter out the conditional volatility, I specify the model with stochastic volatility (Equation 1.5) in both factor shocks ($\sigma^F_{j,t}$) and idiosyncratic shocks ($\sigma^y_{j,t}$) . I estimated the model by the Markov Chain Monte Carlo method.

$$y_{j,t+1} = \phi^y_j(L)y_{j,t} + \gamma^F_j(L)F_t + \sigma^y_{j,t}\varepsilon^y_{j,t+1} \tag{1.3}$$

$$F_{j,t+1} = \Phi^F F_{j,t} + \sigma^F_{j,t}v^F_{j,t+1} \tag{1.4}$$

$$log(\sigma^i_{j,t+1}) = \alpha^i_j + \beta^i_j log(\sigma^i_{j,t}) + \sigma^i_j\eta^i_{t+1},\ \eta^i_{j,t+1} \sim N(0,1),\ i = \{y, F\} \tag{1.5}$$

Another challenge is to determine what variables reveal the uncertainty on fiscal policy. The policy-making process is not separate in fiscal instruments. Therefore, I consider fiscal policy uncertainty as the first principal component of the uncertainty of 37 variables related to fiscal policies, ranging from different taxes to government consumption and investment.

The second measure of fiscal uncertainty is the Economic Policy Uncertainty Index (EPU) in Baker et al. (2015). These indices combine the newspaper coverage of policy-related economic uncertainty, the number of federal tax code provisions set to expire in future years, and disagreement among economic forecasters. The indices begin in year 1985 and span 11 specific policies such as monetary policy, fiscal policy, and health care policy.

Taking the two measures of fiscal uncertainty, I study whether they are related to the debt-to-GDP ratio. Figure 1.6 reports that when the debt-to-GDP ratio is high, the fiscal uncertainty is high. In Table 1.14, the correlation is 0.5 between 3-year fiscal uncertainty and the debt-to-GDP ratio. Furthermore, fiscal uncertainty is distinct from a broad measure of macroeconomic uncertainty. The macro uncertainty is the common component of the 132 variables excluding fiscal-related variables, similar to the measure in Jurado et al. (2015b). The correlation between macro uncertainty and the debt-to-GDP ratio is less than 0.1. In a more recent sample from 1985 to 2014, the debt-to-GDP ratio is still positively related to the fiscal uncertainty measures but not to macro uncertainty. The results are the same as the uncertainty measures from the very different narrative approach. The correlation between debt-to-GDP ratio and fiscal uncertainty in EPU indices is 0.36. This positive relationship is observed in a variety of fiscal-related policies, such as taxes, government spending, health care, and entitlement. On the contrary, debt-to-GDP ratio not is related to the non-fiscal policies, such as monetary, national security, and trade policy. Therefore, the results robustly show that debt-to-GDP largely captures the fiscal uncertainty.

If this risk channel exists, the fiscal uncertainty should have a direct impact on asset prices. In Table 1.15, I demonstrate that fiscal uncertainty affects the asset price in the same way as the debt-to-GDP ratio. Fiscal uncertainty positively predicts excess returns on stocks and corporate bonds. The $R^2$ is around 25% at the five-year horizon. This amount of predictability is large given the difficulty of measuring uncertainty. Fiscal uncertainty is positively related to GZ spread and negatively related to real risk-free rate and return on government debt. The results hold in both the broad-based measure and the EPU measure.

In sum, the evidence shows that high debt-to-GDP ratio is related to high equity risk premium, high credit risk premium, low risk-free rate, and low expected return on government debt. Furthermore, debt-to-GDP ratio positively reflects fiscal policy uncertainty. Fiscal uncertainty also has direct effects on the asset prices consistent with the effects of debt-to-GDP ratio.

## 1.3. Model

In this section, I propose a general equilibrium model to understand why fiscal uncertainty affects risk premia and why the debt-to-GDP ratio is positively correlated with fiscal uncertainty.Building on a standard expanding variety endogenous growth model (Romer, 1990), I study the implications of recursive preferences as in Kung and Schmid (2015). Besides, I augment the model with fiscal policy. The model quantifies the importance of the fiscal uncertainty channel and matches the novel facts.

*1.3.1. Preference*

The discrete-time economy is populated by measure one of representative agent with Epstein-Zin recursive preferences. These preferences break the link between relative risk aversion ($\gamma$) and intertemporal elasticity of substitution (IES) ($\psi$) in CRRA preferences. $\delta$ is the time discount factor. $\theta \equiv \frac{1-\gamma}{1-/\psi}$. Agents maximize the utility function

$$U_t = [(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}$$

subject to the budget constraint,

$$C_t + V_t^D s_t + B_t = (V_t^D + D_t)s_{t-1} + B_{t-1}R_{b,t} + w_t L_t(1-\tau_{l,t})$$

where $V_t^D$ and $D_t$ are the stock price and dividend. $R_{b,t}$ is the return on government debt. The households supply labor inelastically and receive wage bill subject to income tax. As shown in Epstein and Zin (1989), the stochastic discount factor is given by,

$$M_{t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\frac{1}{\psi}}(\frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}])})^{\frac{\psi-\gamma}{1-\gamma}}$$

The agents' marginal utility depends not only on the current consumption but also the continuation utility.

*1.3.2. Final good producer*

Final goods are produced using capital $K_t$, labor $L_t$, and intermediate inputs $X_{i,t}$, according to the following production function.

$$Y_t = (K_t{}^{\alpha}(A_tL_t)^{1-\alpha})^{1-\xi}\left[(\int_0^{N_t} X_{i,t}^{\nu}di)^{\frac{1}{\nu}}\right]^{\xi}$$

where $N_t$ is the number of varieties that the final good producer purchases from the intermediate good producers. $\nu$ affects the substitution between different inputs. $A_t$ is the exogenous technology process and follows AR(1) process.

$$log(A_{t+1}) = (1-\rho)log(A) + \rho log(A_t) + \sigma_a \epsilon_{t+1}$$

The firm owns capital and makes investment decisions subject to investment adjustment cost.

$$K_{t+1} = (1-\delta)K_t + \Phi(\frac{I_t}{K_t})K_t$$

The corporate income tax is levied at rate $\tau_{c,t}$ on the revenue $Y_t$ net cost of labor $w_tL_t$ and inputs $\int_0^{N_t} P_{i,t}X_{i,t}di$ . For tractability, I doesn't include taxes on distributions. The tax here can be viewed as both corporate income tax and individual income tax in the US tax code. The managers also take into account the tax burden on dividends of the shareholders. The free cash flow equals the net profit subtracting investment.

$$D_t = (1-\tau_{c,t})\left[Y_t - w_tL_t - \int_0^{N_t} P_{i,t}X_{i,t}di\right] - I_t$$

The firm maximizes equity value.

$$V_t(K_t) = max_{I_t,K_{t+1},L_t,X_{i,t}} D_t + E_t[M_{t+1}V_{t+1}(K_{t+1})]$$

#### *1.3.3. Intermediate good producer*

Intermediate good producers use a specific patent to build one unit of intermediate good using one unit of the final good. Thanks to the patent, they have monopoly power and set the price of the intermediate good to maximize profits. They face a downward-sloping demand curve implied by the cost minimization of the final goods producer. The optimality conditions are standard that $P_{i,t} = \frac{1}{\nu}$. In equilibrium, the profits depend on the demand elasticity

$$\Pi_{i,t} = (\frac{1}{\nu} - 1)X_{i,t}$$

These firms also have to pay the corporate income tax. Each patent has finite expected lifespan determined by the depreciation rate $\phi$. The value of the intermediate firm equals its discounted profit.

$$V^I_{i,t} = (1 - \tau_{c,t})\Pi_{i,t} + (1 - \phi)E_t[M_{t+1}V^I_{i,t+1}]$$

#### *1.3.4. Innovation*

Agents use final goods to conduct R&D. $S_{i,t}$ is the R&D expenditure. The stock of intangible capital or patents is accumulated through R&D and depreciates every period as follows:

$$N_{t+1} = S_{i,t} + (1 - \phi)N_t$$

Free entry to innovation pins down the optimality condition of R&D. One unit of R&D expenditure yields one unit of intermediate firm that has value $V_{i,t}$.

$$E_t[M_{t+1}V^I_{i,t+1}] = 1$$

*1.3.5. The government*

The government levies tax, arranges spending, and borrows from the households. As in most positive studies, I do not model the policymaking behavior of the fiscal authority. Instead, tax rate and government spending are assumed exogenously to match the observed data. Both spending and tax follow AR(1) processes.

$$log(\frac{G_{t+1}}{Y_{t+1}}) \equiv gy_{t+1} = \mu_{gy}(1-\rho_g) + \rho_g gy_t + \sigma_{g,0}\sigma_{\tau,t}u_{g,t+1}$$

$$\tau_{c,t+1} = \mu_{\tau c}(1-\rho_\tau) + \rho_\tau \tau_{c,t} + \sigma_{\tau,0}\sigma_{\tau,t}u_{\tau,t+1} + u_{c,t+1}$$

I introduce the time-varying volatility of the tax and spending shock $\sigma_{\tau,t}$, modeled as an AR(1) process (Fernández-Villaverde et al., 2015).

$$log(\sigma_{\tau,t+1}) = \nu_\tau log(\sigma_{\tau,t}) + \sigma_{\tau,w}w_{\tau,t+1} \tag{1.6}$$

A positive volatility shock $w_{\tau,t+1}$ leads to a higher conditional volatility of tax rate and fiscal uncertainty. For example, the tax cut expiration and presidential debate are shocks that raise the uncertainty.

The second source of fiscal uncertainty comes from the fiscal consolidations.

$$u_{c,t+1} = \frac{B_t}{Y_t}\phi_{\tau,t+1},\ \phi_{\tau,t+1} = \bar{\phi}_\tau(1-\rho_{\phi\tau}) + \rho\phi_{\tau,t} + \sigma^2_{\phi\tau}u_{\phi\tau,t}$$

In response to high debt-to-GDP ratio, the government tends to increase tax ($\bar{\phi}_\tau > 0$). D'Erasmo et al. (2016) systematically document that primary balance responds to the outstanding debt. However, when and how the government will consolidate is uncertain. We do not expect a fixed parameter $\phi_\tau$ which means the tax rate moves one-for-one with the debt ratio. In fact, we sometimes witnessed large fiscal consolidations at low debt level and no consolidations at high debt levels. The time variation of $\phi_{\tau,t}$ captures this uncertainty in consolidation. This specification is similar to Bi et al. (2013) that assumes that probability of a fiscal consolidation is rising in the debt-to-GDP ratio. In their specification, $u_{c,t+1}$ is zero if the debt is lower than a random threshold and is positive for four quarters if debt exceeds the threshold.

Two economic channels support the importance of the uncertainty in consolidation. On one hand, this uncertainty comes from the politics of policymaking. The government favors expansionary fiscal policies if supporters benefit more from high spending and transfer, low tax and high debt. Depending on the political stance, political cost of consolidation, and the patience (time discount factor) of the policymakers, the government sometimes is strict with the debt-to-GDP ratio and always discusses reducing the debt while sometimes is lenient with the debt-to-GDP ratio and allows it to grow. Song et al. (2012) build a political economy model to endogenize the debt policy in respond to the fundamentals and demographics. On the other hand, the uncertainty is associated with tax smoothing. When debt-to-GDP ratio is close to its debt limit defined as maximum possible discounted surplus, the tax rate is close to the peak of the Laffer Curve with a flatter slope. Thus, government cannot easily issue debt to smooth the spending and output shocks. The tax rate is more responsive to shocks and has larger volatility. Even though the government behavior is exogenous in the model, I formalize this idea of dynamic tax smoothing in a Pareto problem in the appendix.

Since there is no distortion on labor, the labor tax rate is set to be fixed. Given the tax rates, the total tax receipts equal the tax revenue from three sources of income.

$$T_t = \tau_{c,t}\left[Y_t - w_t L_t - \int_0^{N_t} P_{i,t} X_{i,t} di\right] + \tau_{c,t}\int_0^{N_t} (\frac{1}{\nu} - 1) X_{i,t} di + \tau_{l,t} w_t L_t$$

The government can issue a full menu of default-free zero-coupon debt across maturities. Define $Q_t^{(n)}$ the price and $b_t^{(n)}$ the amount of n-period discount bond. The total market value of debt $B_t = \Sigma_n Q_t^{(n)} b_t^{(n)}$ is the summation of all the outstanding debt. For tractability, the government actively manages the maturity structure to achieve a fixed geometrically-decaying maturity. $b_t^{(n)} = \phi_b^{n-1} b_t$. $\phi_b < 1$ determines the maturity structure. The quantity of debt depends on a single factor $b_t$. The government financing policy is specified as exogenous. Each period, it issues $b_t^{(n)}$ amount of bonds given the market price.

$$b_{t+1} = \rho b_t + u_{b,t+1} \tag{1.7}$$

The law of motion of debt is,

$$B_t = B_{t-1} R_{b,t} + G_t - T_t + Tr_t$$

where $R_{b,t} = \frac{\Sigma_n Q_{t-1}^{(n)} b_{t-1}^{(n)} R_t^{(n-1)}}{B_{t-1}}$ is the total return on government debt including matured principal and capital gains. $Tr_t$ is the lump-sum transfer that guarantees the holding of the government budget constraint at each period, since spending, tax, and financing policy are exogenous.[19]

*1.3.6. Definition of Equilibrium*

I focus on the symmetric equilibrium that all the intermediate good producers make identical decisions. $P_{i,t} = P_t$, $\Pi_{i,t} = \Pi_t$, $X_{i,t} = X_t$, $V_{i,t} = V_t$. A competitive equilibrium is allocations $\{C_t,\ Y_t,\ I_t,\ S_t,\ X_t,\ D_t,\ L_t,\ U_t,\ V_t,\ M_t,\ G_t,\ T_t,\ B_t\}_{t=0}^{\infty}$, prices $\{P_t,\ W_t,\ R_{f,t},\ R_{b,t},\ R_{m,t}\}_{t=0}^{\infty}$, exogenous technology $\{A_t\}_{t=0}^{\infty}$, fiscal policy $\{gy_t,\ \tau y_t,\ b_t,\ \sigma_{\tau,t},\ \phi_{\tau,t}\}_{t=0}^{\infty}$, law of motion $\{K_{t+1},\ N_{t+1}\}_{t=0}^{\infty}$ and initial value $\{K_0,\ N_0\}$ such that

1. the state variables $\{K_{t+1},\ N_{t+1}\}_{t=0}^{\infty}$ satisty their law of motions.

2. households, final goods producers, and intermediate goods producer solve their problems.

3. goods, labor, stock, and treasury markets clear.

[19] If the government debt is state contingent, the return will adjust to guarantee the holding of the budget constraint. This is not the case in the current structure of long-term debt.

## 1.4. Model Implications

### *1.4.1. Equilibrium Growth*

In equilibrium, the output has the familiar Cobb-Douglas form.

$$Y_t = K_t^{\alpha}(Z_t L_t)^{1-\alpha}$$

TFP $Z_t$ is driven not only by the exogenous force $A_t$ but also the intangible capital stock $N_t$,

$$Z_t = (\xi\nu)^{\frac{\xi}{(1-\xi)(1-\alpha)}} A_t N_t$$

The insight of the endogenous growth beyond standard exogenous growth model is that the economic growth is determined in part by the growth of the intangible capital, which in turn is determined by the discounted profit of intermediate good producers. Expecting larger profits, innovators exert more effort in R&D, which results in more innovation and faster economic growth.

$$\frac{N_{t+1}}{N_t} = (1-\phi) + E_t\left[\Sigma_{i=1}^{\infty}(1-\phi)^{i-1} M_{t,t+i}(1-\tau_{c,t+i})\Pi_{t+i}\right]^{\frac{\eta}{1-\eta}}$$

It is apparent that fiscal policy plays a role in the innovation process. Part of the profits are taken by the government in the form of corporate income tax. Figure 1.7 plots the impulse response functions to a one-standard-deviation positive tax shock $u_{\tau,t}$. A tax hike reduces future monopoly profits and innovation incentive, leading to lower intangible capital value $V_t$ and innovation growth $\Delta n_t$. This slowdown of innovation transforms into lower consumption growth $\Delta c_t$. The increase of consumption on impact is due to the reduction in investment.[20] Through this tax mechanism, the model features an endogenous persistent and predictable component in the growth rate as in the long-run risks model (Bansal and Yaron, 2004). The negative growth effect of distortionary taxation is well-documented in

[20] The aggregate output doesn't change given the fixed labor supply.

the endogenous growth literature. Gemmell et al. (2011) find strong empirical support for this mechanism. Djankov et al. (2010) further document the adverse effect of the corporate tax on aggregate investment and entrepreneurial activity.

### *1.4.2. Asset Prices and Fiscal Uncertainty*

Stocks and bonds are priced by the agents in the model. The aggregate equity market value is the sum of value of final and intermediate goods producers. The aggregate dividend also comes from the two sectors.

$$V_t^A = V_t + \int_0^{N_t} V_{i,t} di$$

$$D_t^A = D_t + \int_0^{N_t} \Pi_{i,t} di - S_t$$

The risk premium on an asset is related to the covariance between its return $R_{i,t+1}$ and stochastic discount factor $M_{t+1}$. The risk premium is the sum of risk premia of all the shocks. In the beta representation, the premium of each shock depends on the price of risk $\lambda$, risk exposure $\beta$, and the quantity of risk. Focusing on the tax risk premium,

$$E_t[R_{i,t+1} - R_{f,t}] = \frac{Cov_t[M_{t+1}, R_{i,t+1}]}{E_t[M_{t+1}]} \approx \underbrace{\lambda_\tau \beta_{\tau,i} Var_t(\tau_{c,t+1})}_{tax\ risk\ premium} + other\ premia \tag{1.8}$$

High marginal utility after tax hikes is a standard property in macroeconomic models. In Figure 1.7, the stochastic discount factor increases after the positive tax shock. The negative price of risk $\lambda_\tau$ does not rely on endogenous growth or the Epstein-Zin preferences. Related to the risk premium puzzle, the key issue is to have a large price of risk in the model to match asset price facts. In our economy, the agents have Epstein-Zin preferences so that they are sensitive to the persistent shifts in growth rate. Furthermore, tax rates are also highly persistent. As a result, tax variation is a large source of risk for investors and is manifested in asset prices. The price of risk is negative and sizable.

Upon tax hikes, stock prices fall as in Figure 1.7, because of two reasons: (i) higher tax payment, and (ii) lower cash flow growth in the future.[21] Thus, stocks have negative tax risk exposure $\beta_{\tau,m} < 0$. Because stocks perform poorly in bad times of high tax, investors require positive excess returns on average. Thus, tax risk premium is positive and large. When fiscal uncertainty increases and "quantity" of risk is larger, investors require higher compensation for this risk and equity premium increases. Hence, time variation in equity premium is driven by the fiscal uncertainty.

The implication is different for government bonds. Facing high tax and low expected growth, agents have high marginal rate of substitution. Meanwhile, bond yield decreases with growth rate and the government bonds rally, so that government debt is a hedge against tax risks for investors and has a negative risk premium. $\beta_{\tau,b} > 0$. In time of high fiscal uncertainty, the high hedging motive drives down the bond risk premium. Furthermore, risk-free rate is affect by uncertainty. When uncertainty is high, agents have a precautionary saving motive that lowers the risk-free rate.

### *1.4.3. Debt-to-GDP ratio and Fiscal Uncertainty*

After analyzing the determination of risk premium and how it is affected by fiscal uncertainty, I relate fiscal uncertainty to debt-to-GDP ratio. From Equation (1.2), the debt-to-GDP ratio varies from the variation of expected future primary surplus, growth, and discount. The importance of the discount rate channel has been documented in the empirical section.

In the model, the positive comovement between the debt-to-GDP ratio and the fiscal uncertainty is generated endogenously through two channels. As is stated in the last subsection, fiscal uncertainty decreases both risk-free rate and risk premium on debt. In total, fiscal uncertainty reduces the expected return on debt and raises debt valuation through the discount rate channel. An exogenous fiscal uncertainty shock will raise the fiscal uncertainty and debt-to-GDP ratio.

[21] As documented in Sialm (2009), stock valuation declines with tax burden.

The second channel is due to the uncertain fiscal consolidations. The several reasons for uncertain fiscal consolidation have been discussed in Section 1.3.5. Having this feature, the conditional volatility of the tax rate comes from regular tax shocks and consolidation shocks.

$$var_t(\tau_{c,t+1}) = \sigma^2_{\tau,0}\sigma^2_{\tau,t} + (\frac{B_t}{Y_t})^2\sigma^2_{\phi\tau}$$

A volatility shock increases the fiscal uncertainty, lowers expected return on government debt and raises the market value of debt. Conversely, an increase in the government debt raises the second term, the consolidation uncertainty. The two channels reinforce each other. Consequently, debt-to-GDP ratio positively reflected the fiscal uncertainty. In Section 1.2.3.4, I present evidence of both channels. In the VAR system, the book shock is $u_b$, the shock to the amount of bonds, in Equation (1.7). This shock raises debt-to-GDP ratio and thus uncertainty of fiscal consolidations, leading to higher risk premia. The market shock is $w_\tau$, the shock to the exogenous stochastic volatility on the tax rate, in Equation (1.6). This shock lowers the return on debt, raises debt-to-GDP ratio through the discount rate channel, and increases quantity of risk and thus risk premia.

### *1.4.4. Calibration*

The uncertainty channel qualitatively explains the facts. Next, the model is calibrated to evaluate the quantitative importance. I report the benchmark calibration in Table 1.16. The model is calibrated in quarterly frequency. Panel A refers to the preference and technology parameters. In line with the estimated value in Schorfheide et al. (2014), the risk aversion is set to 10 and the intertemporal elasticity of substitution is 2 so that agents have preferences for early resolution of uncertainty. The time discount factor is chosen to match the real risk-free rate. The calibration values of the technology parameters are in line with the class of endogenous growth models (Kung and Schmid, 2015). Capital share is set to 0.33 and the intermediate inputs share is 0.5. Through the balanced growth path, these parameters imply a markup of 1.6, consistent with the evidence in micro data. The depreciation of physical capital is 0.025. The depreciation of R&D capital is 0.075, matching the recent estimate in

Li and Hall (2016). The capital adjustment cost function $\Phi(\frac{I}{K}) = [\frac{a_{1,k}}{1-1/\xi_k}(\frac{I}{K})^{1-1/\xi_k} + a_{2,k}]$. $\xi_k$ is the same as Kung and Schmid (2015), and $a_{1,k}$, $a_{2,k}$ is set such that $\Phi = I/K$ and $\Phi' = 0$ at the steady state. The mean of the productivity is chosen to match the mean of the growth. The persistent and volatility of productivity shocks are set to match the consumption volatility. This is less persistent and volatile than the productivity in Kung and Schmid (2015), since a crucial source of endogenous long-run risk is taxation that is not in their model.

The lower panel includes parameters of fiscal processes. Most of the values are from direct data estimates. Federal corporate income tax is 36% on average. Tax rate has a persistence of 0.99, in the confidence interval [0.91, 0.99] of the effective tax rate. The statutory tax rate in the data tends to be more persistent. The calibration is conservative in that the volatility of tax rate is set to be 0.03, less than half of the volatility of the data counterpart. The persistence of the fiscal volatility follows the persistence of the broad-based fiscal uncertainty measure. The volatility of the volatility shock is in the estimated range of Fernández-Villaverde et al. (2015). Labor tax is set to be 10% to match the total tax receipt over GDP. Spending-to-GDP ratio has a mean of 0.17. Spending includes federal and state and local government and excludes transfers. For simplicity, state and local government has no debt and levy lump sum transfer to cover their spending needs. The average maturity is set to be 7 years, consistent with Greenwood and Vayanos (2014). The volatility and persistence of bond quantity inherit the persistence of debt-to-GDP ratio.

In the benchmark case, I focus on the effect of the tax volatility shock. Therefore, all the parameters about fiscal consolidation and government spending shock are set to be zero in panel C. In the extended model, I set the mean and volatility of the fiscal consolidation to be 0.001 and 0.0025, in line with the estimated value in Fernández-Villaverde et al. (2015). The persistence is set to be 0.92, so the consolidation intensity has a half-life of 2 years, which is half of a president term. The consolidation shock is negatively correlated with the productivity shock. The correlation is 0.5 so that half of the consolidations are attributed to tax base concerns. To allow for the effect of government spending shock, I choose the volatility and persistence of the spending process as in the data.

### *1.4.5. Quantitative Results*

I solve the model by third-order perturbation to account for the effects of time-varying volatility. A pruning method is applied to ensure the stablity of sample paths (Andreasen et al., 2013). Table 1.17 shows the unconditional moments of the key financial variables. The reported model moments are the mean, 5% and 95% quantile of the short-sample simulation. The moments implied by the model are largely consistent with the data. The mean (standard deviation) of consumption growth is 1.80% (2.70%) in the data and 1.80% (2.62%) in the model. The output growth is less volatile than the data mainly because the model abstracts away from the labor supply margin that allows immediate adjustment of output. In the model, the stock return is measured as the total return on tangible and intangible capital and levered by a factor of 2. The model generates a large equity premium (5.19%). The model undershoots the volatility of excess return as the production economy does not generate volatile enough endogenous cash flow. In the model, the log price-dividend ratio has a very similar mean (3.63) and volatility (0.43) as the data. Furthermore, the model matches the small and stable risk-free rates. In the data, the return on government return is larger and more volatile than the risk-free rate. It is commonly acknowledged that long-term bonds compensate for expected inflation and also inflation premium. Ang et al. (2008) show that the inflation premium for a five-year bond is 1.14% on average. Since the real model is silent on the inflation premium, I add this premium on the model-implied return. Model-implied government debt return has similar mean and volatility as the data. Finally, the debt-to-GDP ratio has a mean of 0.52 and volatility of 0.08, similar to the data. Even though I only consider the corporate income tax, the overall tax-to-GDP ratio is close to the data. This guarantees that the model does not imply a counterfactual high and volatility tax burden on the economy as a whole.

I evaluate the effect of debt-to-GDP ratio in the model in Table 1.18. In a univariate predictive regression, the benchmark model matches the stock return predictability in the data. The positive coefficients, ranging from 0.03 in 1 quarter to 0.65 in five years, close to the coefficients in the data. The 90% interval of $R^2$ of the model covers the data estimates. Because of the short sample, the distribution of the $R^2$ is variable. Moreover, the model generates observed evidence that debt-to-GDP ratio is negatively related to real risk-free rate and bond return. Both the coefficient and the $R^2$ are similar to the data counterparts. Especially, the long-run bond return regression implies that higher debt-to-GDP ratio predicts lower discount rate on debt. In other words, the expected return variation contributes to the debt-to-GDP variation to a large extent. This verifies the importance of the discount rate channel. Next, I investigate the impact on stock return itself instead of excess return. The model implies comparable magnitude to the data. Thus, the model successfully matches not only the extent of excess return predictability but also the amount of predictability of stock return and risk-free rate separately.

Finally, I directly test the implications of fiscal uncertainty in the data and the model in Panel B. The model implies a positive correlation of 0.43 between debt-to-GDP ratio and fiscal uncertainty. The fiscal uncertainty is measured as the conditional volatility of the tax rate $var_t(\tau_{c,t+1})$. This shows that the discount rate channel itself will endogenize a positive comovement of debt-to-GDP ratio and fiscal uncertainty. Consistent with Equation (1.8), fiscal uncertainty increases the equity premium, and decreases the risk-free rate and bond returns. The magnitude of the channel is close to both the broad-based measure and the measure in Economic Policy Uncertainty Index.

The benchmark model only has the exogenous fiscal volatility channel. In Table 1.19, I entertain the other potential channel: fiscal consolidations. The parameters of the fiscal consolidations are set as in Table 1.16. First, introducing the uncertain fiscal consolidations will magnify the importance of the fiscal uncertainty. Debt-to-GDP ratio has stronger impacts on stock return, risk-free rate, and government bond return in terms of both coefficients and $R^2$. The five-year $R^2$ goes up from 14% to 23%. Second, I shut down the stochastic volatility ($\sigma_{\tau,w} = 0$). With only fiscal consolidations, the model does a good job in matching the effect of debt-to-GDP ratio. The $R^2$ on stock returns are two-third of the ones with only stochastic volatility. The magnitude of risk-free rates and government bond returns are close to the data. However, this channel implies a perfect correlation between debt-to-GDP ratio and fiscal uncertainty. By construction, the only reason fiscal uncertainty fluctuates is that the strength of fiscal consolidations is related to debt-to-GDP ratio. Third, the model abstracts away from both stochastic volatility and fiscal consolidations. In this case, the risk premium is fixed and the predictability in the model is tiny. The positive $R^2$ are from small sample bias since $R^2$ is restricted to be non-negative. There is no movement in fiscal uncertainty and no relationship between uncertainty and debt. Finally, I introduce government spending shock. This shock does not change the asset pricing implications and has a small quantitative impact in the third decimal place.

One implication of the model is the large and persistent effect of tax on the growth rate. The size of this effect is both model dependent and empirically controversial. Gemmell et al. (2011) is the recent contribution to this question and they argue for the existence of significant effects. They document that 1% increase of Tax-to-GDP ratio reduces GDP by 5.8% in 10 years in the US and 3.2% in OECD countries. I also find a negative significant impact of tax rate on 10-year output growth. In Table 1.20, the impact is 3.7%, consistent with their estimates. The model matches the impact of the tax. The predictive $R^2$ in the data (model) are 0.13 (0.17) at the annual horizon and 0.21 (0.22) at the 10-year horizon. The point estimates of coefficients in the data are within the 90% set of the model. This result holds in consumption and TFP growth, too. Hence, the calibration does not exaggerate this endogenous long-run risk channel.

As a result, the model quantitatively matches macroeconomics and asset prices moments. More importantly, the model replicates the relationship between debt-to-GDP ratio, various asset prices and fiscal uncertainty.

## 1.5. Conclusion

This paper documents a set of novel facts that government debt is related to risk premia in various asset markets. First, the debt-to-GDP ratio positively predicts excess stock returns. The forecasting power is compelling, and it outperforms many popular predictors. Second, higher debt-to-GDP ratio is correlated with higher credit risk premia in both corporate bond excess returns and yield spreads. Third, higher debt-to-GDP ratio is associated with lower real risk-free rate. Fourth, higher debt-to-GDP ratio predicts lower average returns on government debt. Expected return variation contributes to a sizable amount of the volatility of the debt-to-GDP ratio. Fifth, debt-to-GDP ratio positively comoves with fiscal policy uncertainty. Fiscal uncertainty also has direct effects on the asset prices consistent with the effect of debt-to-GDP ratio.

I rationalize these empirical findings in a general equilibrium model featuring recursive preferences, endogenous growth, and time-varying fiscal uncertainty. In the model, the tax risk premium is sizable and its time variation is driven by fiscal uncertainty. Furthermore, the model endogenize a positive relationship between the debt-to-GDP ratio and fiscal uncertainty: fiscal uncertainty increases debt valuation through discount rate channel whereas higher debt conversely raises uncertainty because of future fiscal consolidations. Through this channel, the government debt has asset pricing implications consistent with the facts. However, major existing channels of government debt such as liquidity, safety, and crowding out are silent or inconsistent with these facts. The empirical findings and theory shed new light on how government debt is related to the cost of capital for firms and the government.

Figure 1.1: Debt-to-GDP Ratio

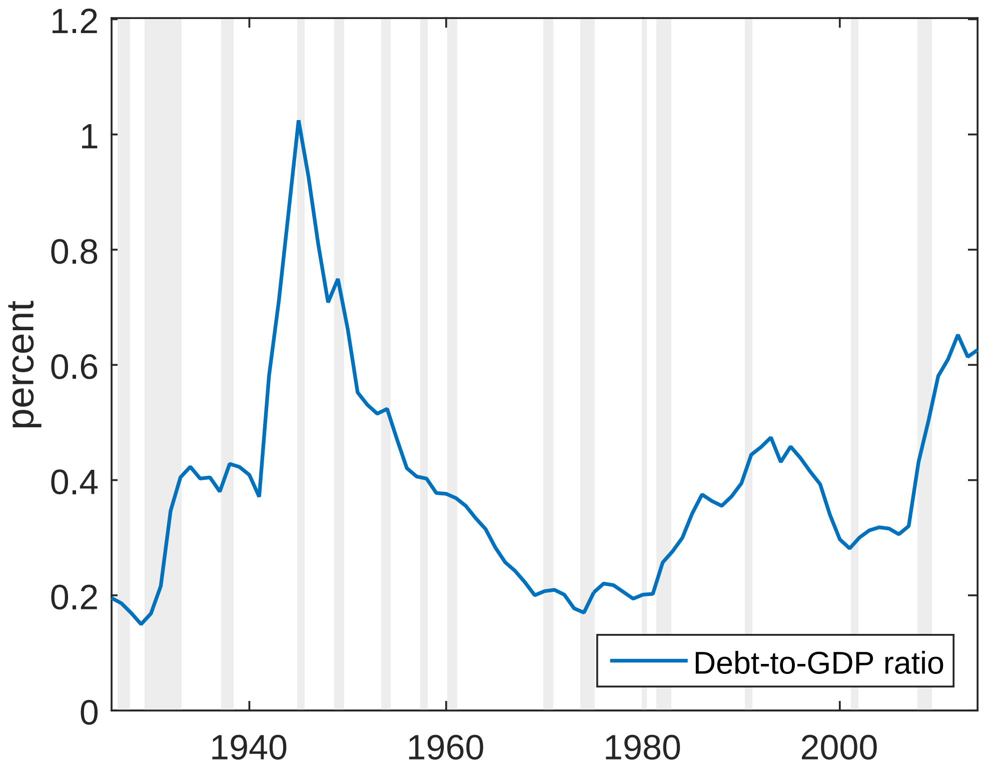


Figure 1.2: Debt-to-GDP Ratio and Expect Return

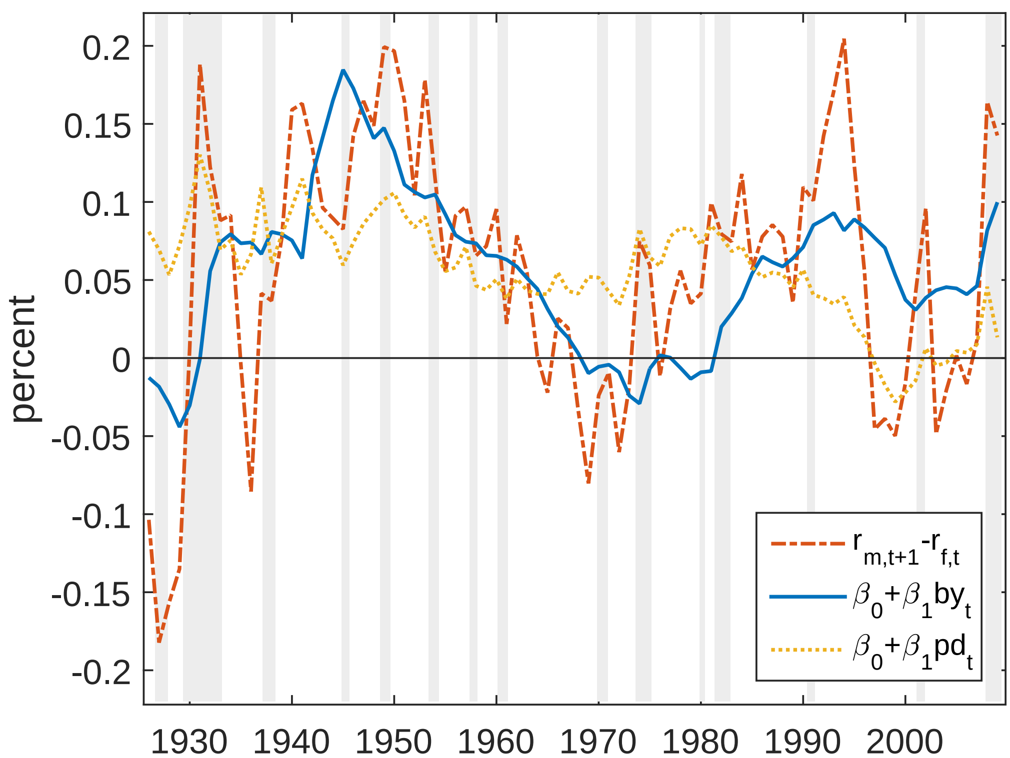


Figure 1.3: Debt-to-GDP Ratio and Expect Return Term Structure

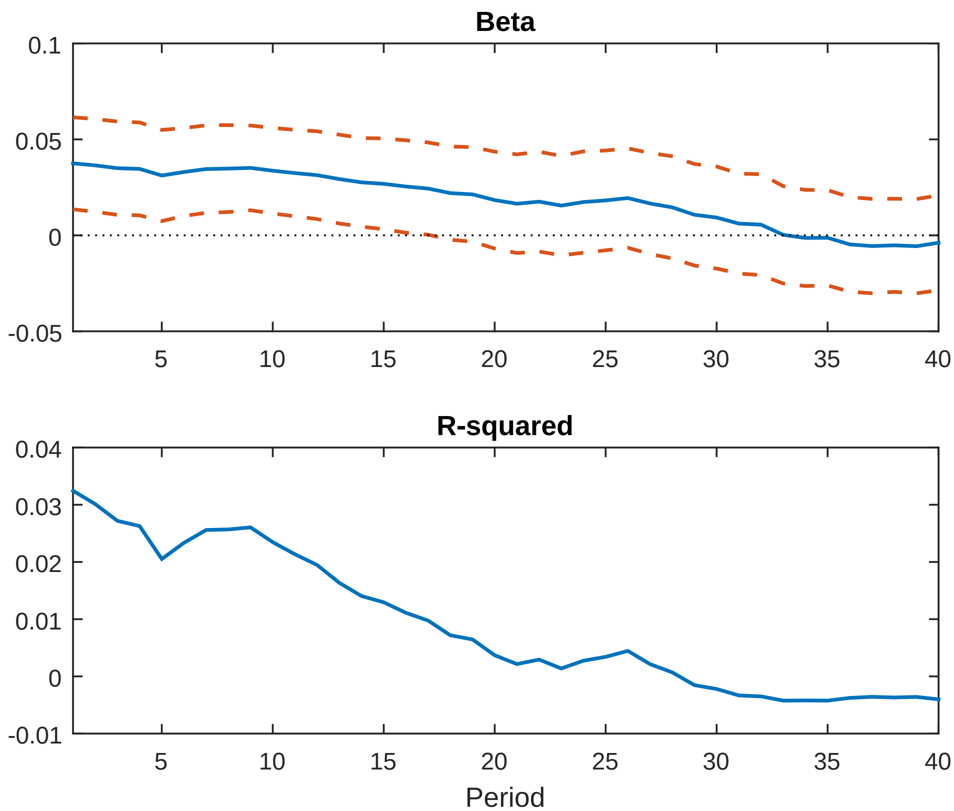


Figure 1.4: Impulse Response Functions to a Debt Shock

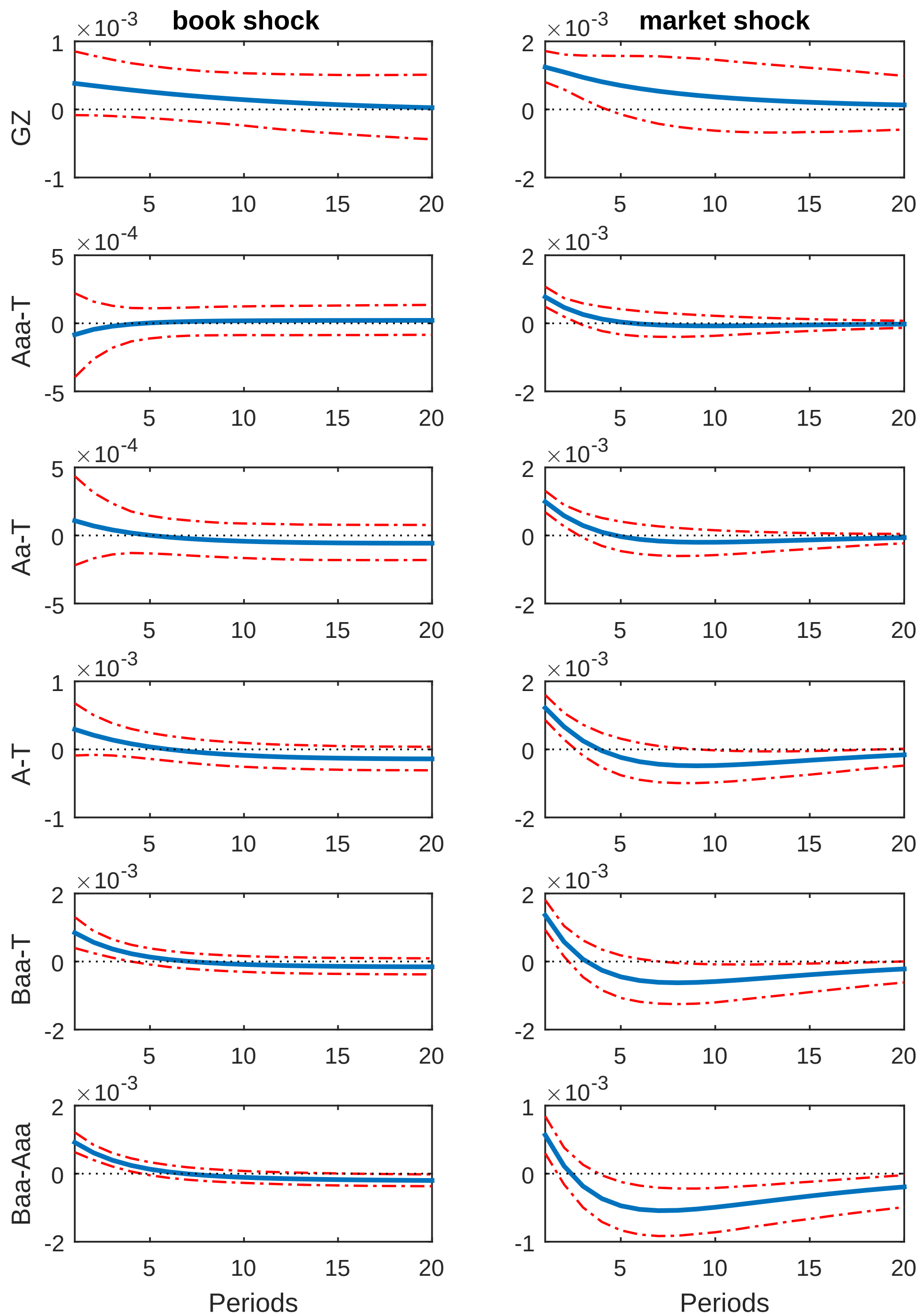


Figure 1.5: Impulse Response Functions to a Debt Shock

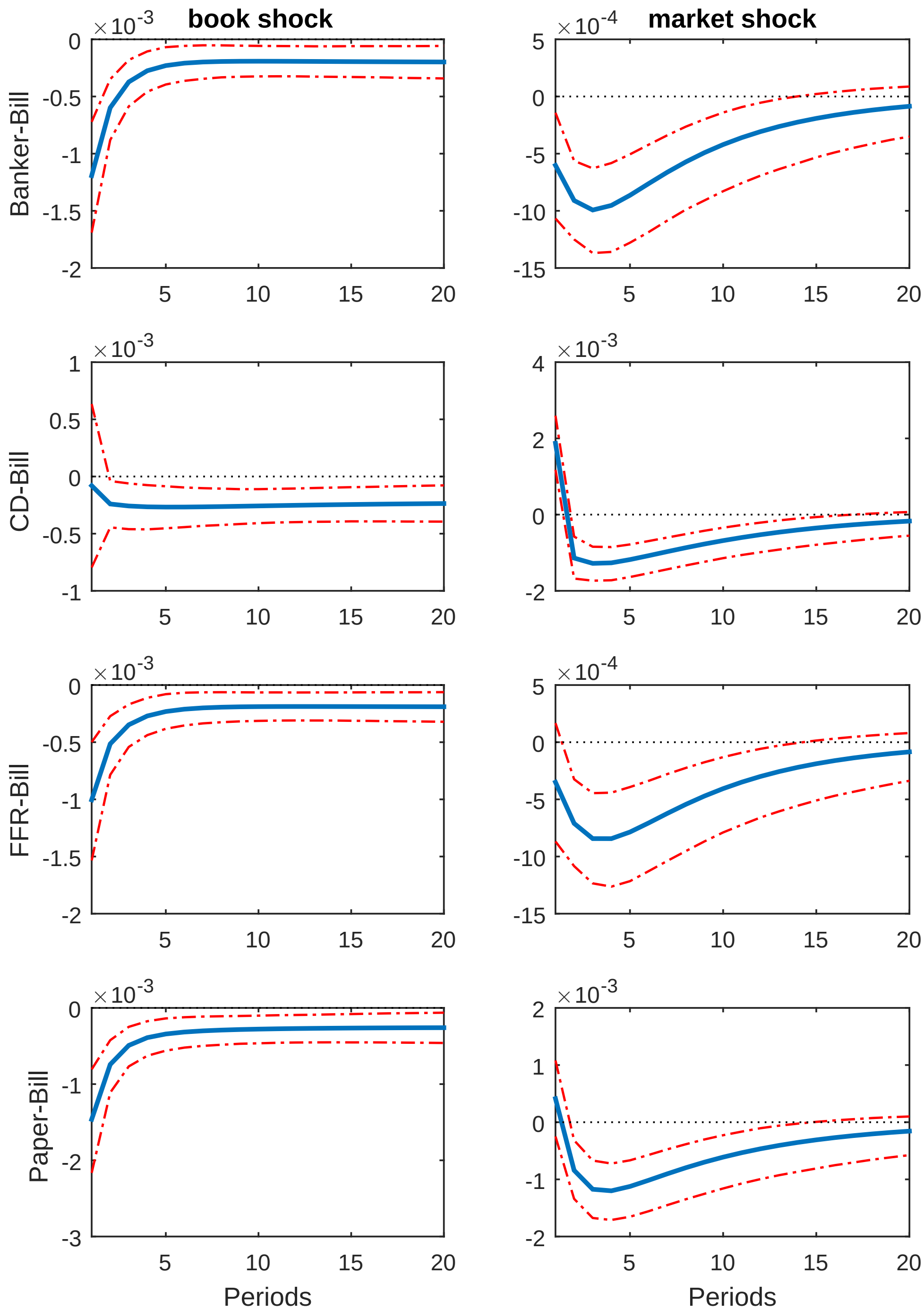


Figure 1.6: Debt-to-GDP Ratio and Fiscal Uncertainty

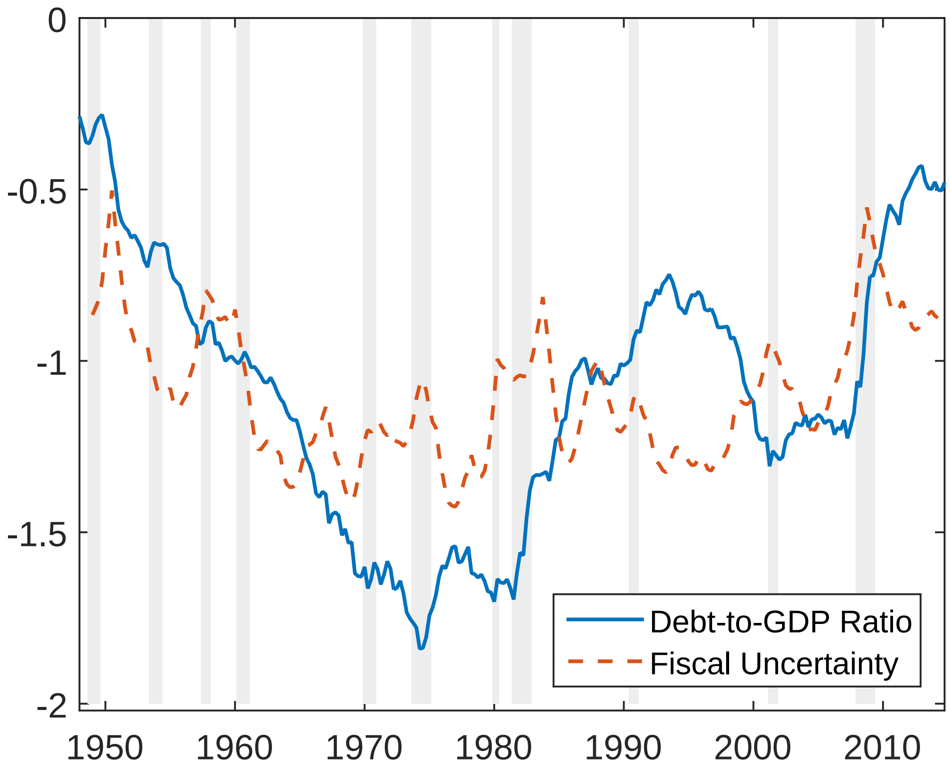


Figure 1.7: Impulse Response Functions to a Tax Shock

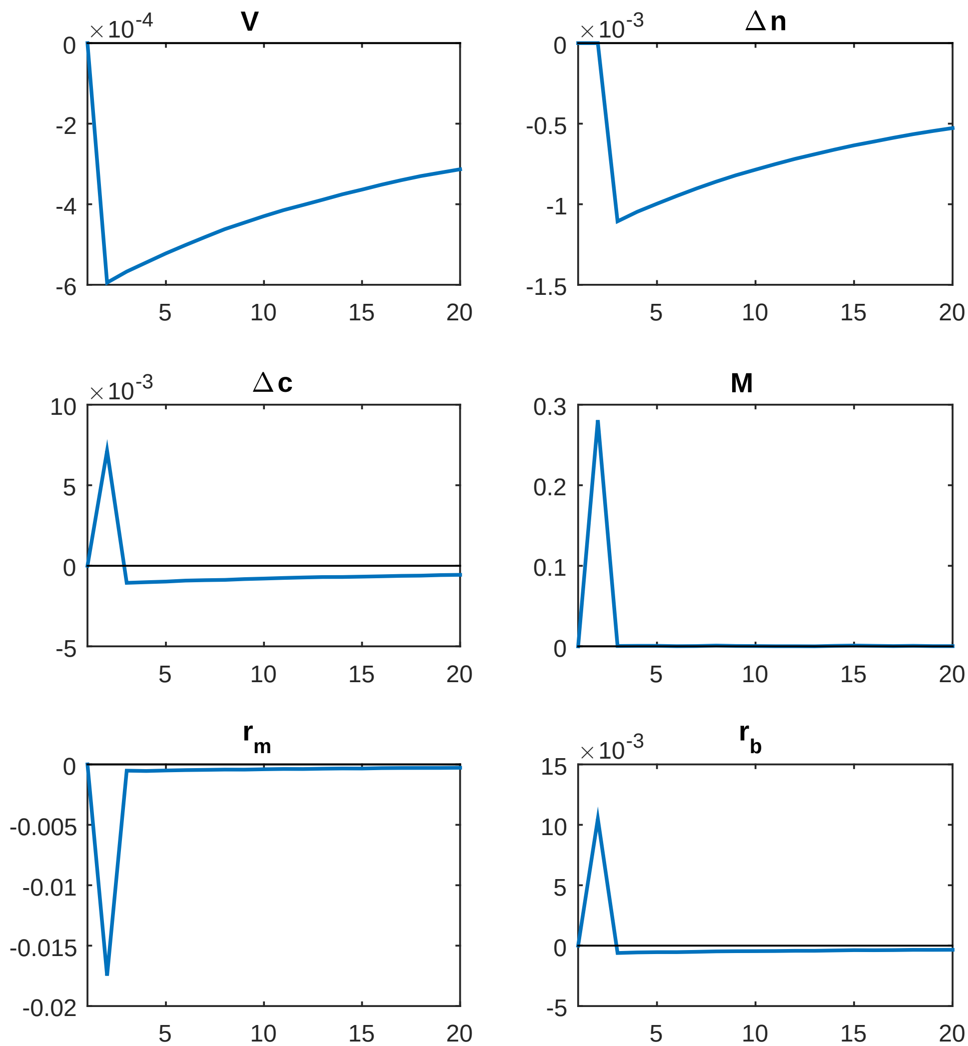


Table 1.1: Predictability of Excess Stock Returns

The table reports estimates from OLS regressions of future excess stock returns on log debt-to-GDP ratio and other control variables.

$$\Sigma_{i=1}^{h}(r_{m,t+i} - r_{f,t+i-1}) = \beta_0 + by_t\beta_1 + x_t\beta + u_{t+h}$$

Excess stock return is the log market return $r_m$ subtracting the log risk free rate $r_f$. Long-horizon excess returns are the cumulative summation of the one-period excess returns. *by* is the log debt-to-GDP ratio. "recession" is a dummy that equals 1 when US is in recession reported by NBER. "war" is a dummy that equals 1 when US is in war time. "party" is a dummy that equals 1 under Democratic presidency. *pd* is the price-dividend ratio. *PC* are the first three principal components of a set of predictors including price-dividend ratio, price-earning ratio, dividend-earning ratio, stock return volatility, book-to-market ratio, net equity expansion, treasury bill rate, long-term yield, long-term return, term spread, default yield spread and inflation. *h* is the predictive horizon. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors. The sample is from 1926 to 2014.

| *h* | *by* | (*t*-stat) | *recession* | (*t*-stat) | *war* | (*t*-stat) | *party* | (*t*-stat) | *pd* | (*t*-stat) | excl. 07-14 | *PC* | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.15 | (2.92) | | | | | | | | | | | 0.11 |
| | 0.15 | (2.98) | 0.02 | (0.34) | | | | | | | | | 0.11 |
| | 0.15 | (2.89) | | | 0.00 | (-0.08) | | | | | | | 0.11 |
| 1Y | 0.17 | (3.07) | | | | | -0.03 | (-0.73) | | | | | 0.11 |
| | 0.14 | (2.61) | | | | | | | | | Y | | 0.10 |
| | 0.15 | (2.87) | | | | | | | -0.07 | (-1.54) | | | 0.13 |
| | 0.15 | (2.98) | | | | | | | | | | Y | 0.15 |
| | 0.41 | (3.41) | | | | | | | | | | | 0.27 |
| | 0.42 | (3.29) | 0.05 | (0.62) | | | | | | | | | 0.28 |
| | 0.41 | (3.37) | | | 0.04 | (0.55) | | | | | | | 0.28 |
| 3Y | 0.43 | (3.52) | | | | | -0.02 | (-0.35) | | | | | 0.27 |
| | 0.40 | (3.11) | | | | | | | | | Y | | 0.26 |
| | 0.40 | (3.09) | | | | | | | -0.20 | (-2.60) | | | 0.34 |
| | 0.41 | (3.31) | | | | | | | | | | Y | 0.35 |
| | 0.60 | (4.46) | | | | | | | | | | | 0.38 |
| | 0.61 | (4.43) | 0.14 | (1.38) | | | | | | | | | 0.40 |
| | 0.60 | (4.42) | | | 0.03 | (0.29) | | | | | | | 0.38 |
| 5Y | 0.63 | (4.58) | | | | | -0.07 | (-0.83) | | | | | 0.38 |
| | 0.57 | (4.27) | | | | | | | | | Y | | 0.38 |
| | 0.57 | (3.90) | | | | | | | -0.32 | (-4.28) | | | 0.50 |
| | 0.58 | (4.08) | | | | | | | | | | Y | 0.49 |

Table 1.2: Predictability of Excess Stock Returns

The table reports estimates from OLS regressions of future excess stock returns on log debt-to-GDP ratio and other control variables.

$$\Sigma_{i=1}^{h}(r_{m,t+i} - r_{f,t+i-1}) = \beta_0 + by_t\beta_1 + x_t\beta + u_{t+h}$$

Excess stock return is the log market return $r_m$ subtracting the log risk-free rate $r_f$. Long-horizon excess returns are the cumulative summation of the one-period excess returns. *by* is the log debt-to-GDP ratio. *war* is a dummy that equals 1 when US is in recession reported by NBER. *pd* is the price-dividend ratio. *PC* are the first three principal components of a set of predictors including price-dividend ratio, price-earning ratio, dividend-earning ratio, stock return volatility, book-to-market ratio, net equity expansion, treasury bill rate, long-term yield, long-term return, term spread, default yield spread, inflation, investment-capital ratio, consumption-wealth ratio, GDP gap and government investment-capital ratio. *liqs* is the spread between general collateral repo and 3-month T-bill rate. *ats* is the spread between Moody's AAA bond and 30-year Treasury bond yield. *h* is the predictive horizon. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors. The sample is from 1947:I to 2014:IV.

| $h$ | $by$ | ($t$-stat) | $war$ | ($t$-stat) | $pd$ | ($t$-stat) | $PC$ | $liqs$ | ($t$-stat) | $ats$ | ($t$-stat) | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.04 | (3.26) | | | | | | | | | | 0.04 |
| | 0.04 | (4.11) | 0.00 | (0.03) | | | | | | | | 0.04 |
| 1Q | 0.04 | (3.30) | | | -0.02 | (-2.30) | | | | | | 0.06 |
| | 0.06 | (3.94) | | | | | Y | | | | | 0.08 |
| | 0.04 | (2.99) | | | | | | 0.70 | (0.35) | 0.38 | (0.30) | 0.03 |
| | 0.15 | (3.78) | | | | | | | | | | 0.12 |
| | 0.17 | (4.38) | 0.06 | (0.70) | | | | | | | | 0.17 |
| 1Y | 0.15 | (3.75) | | | -0.10 | (-2.79) | | | | | | 0.20 |
| | 0.21 | (4.45) | | | | | Y | | | | | 0.22 |
| | 0.17 | (4.03) | | | | | | 4.60 | (1.25) | -0.44 | (-0.13) | 0.12 |
| | 0.41 | (6.59) | | | | | | | | | | 0.33 |
| | 0.46 | (6.74) | 0.02 | (0.16) | | | | | | | | 0.40 |
| 3Y | 0.40 | (5.94) | | | -0.26 | (-3.22) | | | | | | 0.49 |
| | 0.50 | (6.14) | | | | | Y | | | | | 0.48 |
| | 0.35 | (4.98) | | | | | | 6.74 | (1.49) | -13.87 | (-2.30) | 0.37 |
| | 0.61 | (5.11) | | | | | | | | | | 0.42 |
| | 0.67 | (5.76) | 0.03 | (0.24) | | | | | | | | 0.49 |
| 5Y | 0.56 | (5.92) | | | -0.40 | (-6.20) | | | | | | 0.66 |
| | 0.61 | (4.71) | | | | | Y | | | | | 0.59 |
| | 0.57 | (4.80) | | | | | | 12.14 | (2.06) | -12.10 | (-1.80) | 0.44 |

Table 1.3: Out-of-sample Test

The table reports in-sample and out-of-sample $R^2$ from OLS regressions of future excess stock returns on log debt-to-GDP ratio and other control variables.

$$\Sigma_{i=1}^{h}(r_{m,t+i} - r_{f,t+i-1}) = \beta_0 + by_t\beta_1 + x_t'\beta + u_{t+h}$$

Excess stock return is the log market return $r_m$ subtracting the log risk-free rate $r_f$. Long-horizon excess returns are the cumulative summation of the one-period excess returns. $by$ is the log debt-to-GDP ratio. The other regressors are as follows: price-dividend ratio (pd), price-earning ratio (pe), dividend-earning ratio (de), stock return volatility (svar), book-to-market ratio (bm), net equity expoansion (ntis), treasury bill rate (tbl), long-term yield (lty), long-term return (ltr), term spread (tms), default yield spread (dfy) and inflation (infl). The first column shows the regressor. The out-of-sample period starts in 20 periods from the beginning of the sample. $R^2_{OS}$ is from univariate regressions and $R^2_{OS,by}$ is from bi-variate regression with debt-to-GDP ratio and a regressor. Column "p" shows p value of testing hypothesis $H_0: MSE_1 > MSE_0$ against $H_1: MSE_1 < MSE_0$. In the row of "by", $MSE_0$ is the mean squared error from historical mean. $MSE_1$ is the mean squared error from predictive model with by. In other rows, $MSE_0$ is the mean squared error from a predictor. $MSE_1$ is the mean squared error from predictive model with the predictor and by. The sample is from 1926 to 2014.

| | $R^2_{OS}$ | $R^2_{OS,by}$ | p | $R^2_{OS}$ | $R^2_{OS,by}$ | p | $R^2_{OS}$ | $R^2_{OS,by}$ | p |
|---|---|---|---|---|---|---|---|---|---|
| | | 1Y | | | 3Y | | | 5Y | |
| $by$ | 0.10 | | 0.00 | 0.29 | | 0.00 | 0.29 | | 0.00 |
| $pd$ | 0.00 | 0.10 | 0.00 | -0.27 | -0.08 | 0.00 | -0.32 | -0.14 | 0.00 |
| $dy$ | -0.16 | -0.02 | 0.00 | -0.73 | -0.38 | 0.00 | -0.25 | 0.20 | 0.00 |
| $pe$ | 0.00 | 0.10 | 0.00 | -0.05 | 0.30 | 0.00 | -0.16 | 0.36 | 0.00 |
| $de$ | -0.04 | 0.07 | 0.01 | -0.20 | 0.10 | 0.00 | -0.12 | 0.03 | 0.01 |
| $svar$ | -0.05 | 0.07 | 0.00 | -0.09 | 0.22 | 0.00 | -0.20 | 0.06 | 0.00 |
| $bm$ | -0.07 | 0.11 | 0.00 | -0.55 | 0.06 | 0.00 | -0.56 | 0.21 | 0.00 |
| $ntis$ | -0.10 | 0.02 | 0.00 | -0.27 | 0.11 | 0.00 | -0.25 | 0.23 | 0.00 |
| $tbl$ | -0.04 | 0.04 | 0.00 | -0.52 | 0.12 | 0.00 | -1.00 | 0.06 | 0.00 |
| $lty$ | -0.06 | 0.01 | 0.00 | -0.28 | 0.27 | 0.00 | -1.24 | 0.35 | 0.00 |
| $ltr$ | -0.07 | 0.06 | 0.00 | -0.05 | 0.27 | 0.00 | -0.16 | 0.21 | 0.00 |
| $tms$ | -0.01 | 0.07 | 0.00 | -0.35 | 0.02 | 0.00 | -0.31 | 0.12 | 0.00 |
| $dfy$ | -0.01 | 0.10 | 0.00 | -0.16 | 0.10 | 0.00 | -0.19 | 0.10 | 0.00 |
| $infl$ | -0.07 | 0.06 | 0.01 | -0.07 | 0.25 | 0.00 | -0.11 | 0.20 | 0.00 |

Table 1.4: Out-of-sample Test

The table reports in-sample and out-of-sample $R^2$ from OLS regressions of future excess stock returns on log debt-to-GDP ratio and other control variables.

$$\Sigma_{i=1}^{h}(r_{m,t+i} - r_{f,t+i-1}) = \beta_0 + by_t\beta_1 + x_t'\beta + u_{t+h}$$

Excess stock return is the log market return $r_m$ subtracting the log risk-free rate $r_f$. Long-horizon excess returns are the cumulative summation of the one-period excess returns. *by* is the log debt-to-GDP ratio. The other regressors are as follows: price-dividend ratio (pd), price-earning ratio (pe), dividend-earning ratio (de), stock return volatility (svar), book-to-market ratio (bm), net equity expoansion (ntis), treasury bill rate (tbl), long-term yield (lty), long-term return (ltr), term spread (tms), default yield spread (dfy), inflation (infl), investment-capital ratio (ik), consumption-wealth ratio (cay), GDP gap (gap), government investment-capital ratio (gik). The first column shows the regressor. The out-of-sample period starts in 20 periods from the beginning of the sample. $R^2_{OS}$ is from univariate regressions and $R^2_{OS,by}$ is from bi-variate regression with debt-to-GDP ratio and a regressor. Column "p" shows p value of testing hypothesis $H_0: MSE_1 > MSE_0$ against $H_1: MSE_1 < MSE_0$. In the row of "by", $MSE_0$ is the mean squared error from historical mean. $MSE_1$ is the mean squared error from predictive model with by. In other rows, $MSE_0$ is the mean squared error from a predictor. $MSE_1$ is the mean squared error from predictive model with the predictor and by. The sample is from 1947:I to 2014:IV.

| | $R^2_{OS}$ | $R^2_{OS,by}$ | p | $R^2_{OS}$ | $R^2_{OS,by}$ | p | $R^2_{OS}$ | $R^2_{OS,by}$ | p | $R^2_{OS}$ | $R^2_{OS,by}$ | p |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1Q | | | 1Y | | | 3Y | | | 5Y | |
| *by* | 0.02 | | 0.02 | 0.06 | | 0.02 | 0.27 | | 0.00 | 0.34 | | 0.01 |
| *pd* | -0.01 | 0.01 | 0.02 | -0.05 | -0.01 | 0.01 | 0.01 | 0.32 | 0.00 | 0.10 | 0.55 | 0.00 |
| *dy* | 0.00 | 0.02 | 0.01 | -0.02 | 0.03 | 0.01 | 0.01 | 0.30 | 0.00 | 0.11 | 0.55 | 0.00 |
| *pe* | -0.01 | 0.00 | 0.04 | -0.07 | 0.00 | 0.00 | -0.08 | 0.31 | 0.00 | -0.12 | 0.43 | 0.01 |
| *de* | -0.02 | 0.00 | 0.01 | -0.06 | -0.01 | 0.02 | -0.13 | 0.11 | 0.00 | -0.12 | 0.22 | 0.01 |
| *svar* | -0.01 | 0.01 | 0.02 | -0.05 | 0.02 | 0.00 | -0.16 | 0.23 | 0.00 | -0.23 | 0.27 | 0.01 |
| *bm* | -0.02 | 0.01 | 0.02 | -0.07 | 0.05 | 0.00 | -0.27 | 0.24 | 0.00 | -0.49 | 0.48 | 0.02 |
| *ntis* | -0.03 | 0.00 | 0.02 | -0.07 | -0.01 | 0.01 | -0.18 | 0.18 | 0.00 | -0.31 | 0.28 | 0.01 |
| *tbl* | -0.01 | 0.00 | 0.02 | -0.05 | -0.07 | 0.08 | -0.31 | 0.05 | 0.03 | -0.79 | 0.29 | 0.04 |
| *lty* | -0.02 | -0.02 | 0.05 | -0.14 | -0.11 | 0.00 | -0.45 | 0.18 | 0.01 | -1.24 | 0.22 | 0.04 |
| *ltr* | -0.02 | 0.01 | 0.02 | -0.01 | 0.07 | 0.00 | -0.09 | 0.26 | 0.00 | -0.14 | 0.35 | 0.01 |
| *tms* | -0.02 | 0.00 | 0.02 | -0.07 | -0.10 | 0.16 | -0.10 | 0.24 | 0.00 | -0.31 | 0.34 | 0.01 |
| *dfy* | -0.02 | 0.00 | 0.01 | -0.06 | 0.00 | 0.00 | -0.28 | 0.03 | 0.00 | -0.46 | 0.41 | 0.00 |
| *infl* | 0.00 | 0.01 | 0.03 | 0.02 | 0.05 | 0.02 | -0.05 | 0.26 | 0.00 | -0.17 | 0.36 | 0.01 |
| *ik* | 0.02 | 0.02 | 0.22 | 0.05 | 0.05 | 0.09 | 0.15 | 0.26 | 0.01 | 0.07 | 0.40 | 0.02 |
| *cay* | 0.00 | -0.01 | 0.05 | 0.03 | -0.02 | 0.06 | 0.09 | 0.20 | 0.01 | 0.15 | 0.28 | 0.04 |
| *gap* | 0.02 | 0.04 | 0.01 | 0.02 | 0.10 | 0.00 | 0.04 | 0.36 | 0.00 | 0.00 | 0.46 | 0.00 |
| *gik* | -0.01 | 0.01 | 0.01 | -0.02 | -0.01 | 0.06 | 0.03 | 0.26 | 0.00 | -0.05 | 0.30 | 0.00 |

Table 1.5: Bonferroni Test of Predictability

The table reports the Bonferroni test of predictability in Campebll and Yogo (2006).

$$r_{m,t+1} - r_{f,t} = \alpha + \beta by_t + u_{t+1}$$
$$by_{t+1} = \mu + \rho by_t + e_{t+1}$$

Excess stock return is the log market return $r_m$ subtracting the log risk-free rate $r_f$. $by$ is the log debt to GDP ratio. $h$ is the predictive horizon. The log debt-to-GDP ratio is modeled as an autoregressive process. The table shows the point estimate $\beta$ and 95% confidence interval based on the Bonferroni-Q test. $\rho_{ue} = corr(u_{t+1}, e_{t+1})$.

| Period | $h$ | $\beta$ | 95% CI | $\rho_{ue}$ |
|---|---|---|---|---|
| | 1Y | 0.15 | [0.06, 0.24] | 0.03 |
| 1929-2014 | 2Y | 0.30 | [0.09, 0.49] | -0.12 |
| | 3Y | 0.37 | [0.12, 0.63] | 0.08 |
| 1947-2014 | 1Q | 0.04 | [0.02, 0.06] | -0.01 |
| | 1Y | 0.16 | [0.06, 0.26] | 0.06 |

Table 1.6: Predictability of Excess Stock Returns

The table reports estimates from OLS regressions of future excess stock returns on log debt-to-GDP ratio and other control variables.

$$\Sigma_{i=1}^{h}(r_{m,t+i} - r_{f,t+i-1}) = \beta_0 + by_t\beta_1 + pd_t\beta_2 + u_{t+h}$$

Excess stock return is the log market return $r_m$ subtracting the log risk-free rate $r_f$. Long-horizon excess returns are the cumulative summation of the one-period excess returns. *by* is the log debt-to-GDP ratio. The predictive horizon is indicated in the name of the panel. The first column show the component of debt used to construct the debt-to-GDP ratio. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors. The sample is from 1947:I to 2014:IV.

| Component of Debt | *by* | ($t$-stat) | $R^2$ | *by* | ($t$-stat) | $R^2$ |
|---|---|---|---|---|---|---|
| | | 1Q | | | 1Y | |
| Net Debt | 0.04 | (2.89) | 0.03 | 0.14 | (3.38) | 0.10 |
| Gross Debt | 0.03 | (2.29) | 0.02 | 0.12 | (2.74) | 0.06 |
| Marketable Debt | 0.04 | (2.77) | 0.03 | 0.15 | (3.39) | 0.09 |
| Net Debt Book Value | 0.04 | (2.85) | 0.03 | 0.15 | (3.40) | 0.10 |
| Gross Debt Book Value | 0.03 | (2.26) | 0.02 | 0.13 | (2.83) | 0.07 |
| Marketable Debt Book Value | 0.04 | (2.75) | 0.03 | 0.16 | (3.45) | 0.10 |
| Net Debt exclud. Fed Holding | 0.03 | (2.53) | 0.03 | 0.13 | (2.97) | 0.09 |
| Net Debt exclud. Foreign Holding | 0.04 | (2.89) | 0.03 | 0.14 | (2.87) | 0.09 |
| Net Debt/Potential GDP | 0.04 | (2.94) | 0.03 | 0.15 | (3.53) | 0.11 |
| | | 3Y | | | 5Y | |
| Net Debt | 0.37 | (5.70) | 0.30 | 0.55 | (4.57) | 0.39 |
| Gross Debt | 0.33 | (3.81) | 0.19 | 0.50 | (2.87) | 0.25 |
| Marketable Debt | 0.40 | (5.28) | 0.25 | 0.60 | (3.67) | 0.31 |
| Net Debt Book Value | 0.40 | (5.78) | 0.31 | 0.59 | (4.96) | 0.40 |
| Gross Debt Book Value | 0.37 | (4.01) | 0.20 | 0.56 | (3.14) | 0.27 |
| Marketable Debt Book Value | 0.45 | (5.44) | 0.27 | 0.67 | (4.01) | 0.34 |
| Net Debt exclud. Fed Holding | 0.33 | (4.35) | 0.24 | 0.43 | (3.34) | 0.28 |
| Net Debt exclud. Foreign Holding | 0.35 | (4.52) | 0.25 | 0.42 | (4.20) | 0.26 |
| Net Debt/Potential GDP | 0.38 | (4.81) | 0.26 | 0.55 | (3.68) | 0.34 |

Table 1.7: Predictability of Excess Stock Returns

The table reports estimates from OLS regressions of future excess stock returns on log debt-to-GDP ratio.

$$\Sigma_{i=1}^{h}(r_{m,t+i} - r_{f,t+i-1}) = \beta_0 + by_t\beta_1 + u_{t+h}$$

Excess stock return is the log market return $r_m$ subtracting the log risk-free rate $r_f$. Long-horizon excess returns are the cumulative summation of the one-period excess returns. $by_t$ is the log debt-to-GDP ratio. $h$ is the predictive horizon. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors. The sample from 1980 to 2013.

| | | Gross Debt | | | Net Debt | | |
|---|---|---|---|---|---|---|---|
| | $h$ | $by$ | ($t$-stat) | $R^2$ | $by$ | ($t$-stat) | $R^2$ |
| Canada | 1Y | 0.26 | (1.97) | 0.10 | 0.10 | (1.46) | 0.07 |
| | 3Y | 0.43 | (2.92) | 0.14 | 0.17 | (2.37) | 0.10 |
| UK | 1Y | 0.15 | (1.83) | 0.06 | 0.17 | (1.97) | 0.09 |
| | 3Y | 0.35 | (1.94) | 0.06 | 0.36 | (2.15) | 0.08 |

Table 1.8: Predictability of Dividend Growth

The table reports estimates from OLS regressions of future dividend growth on log debt-to-GDP ratio and other control variables.

$$\Sigma_{i=1}^{h}\Delta d_{t+i} = \beta_0 + by_t\beta_1 + x_t\beta + u_{t+h}$$

$\Delta d$ is the dividend growth rate. Long-horizon dividend growth are the cumulative summation of the one-period growth rate. $by$ is the log debt-to-GDP ratio. $pd$ is the price-dividend ratio. $h$ is the predictive horizon. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors. The sample is from 1926 to 2014.

| $h$ | $by$ | ($t$-stat) | $pd$ | ($t$-stat) | $R^2$ |
|---|---|---|---|---|---|
| 1Y | 0.05 | (1.97) | | | 0.04 |
| 3Y | 0.18 | (2.71) | | | 0.16 |
| 5Y | 0.27 | (3.98) | | | 0.30 |
| 10Y | 0.30 | (4.94) | | | 0.34 |
| 1Y | 0.05 | (2.03) | 0.07 | (2.21) | 0.12 |
| 3Y | 0.18 | (2.90) | 0.10 | (1.52) | 0.21 |
| 5Y | 0.28 | (4.16) | 0.10 | (1.50) | 0.35 |
| 10Y | 0.31 | (4.30) | 0.09 | (1.29) | 0.37 |

Table 1.9: Predictability of Corporate Bond Excess Return

The table reports estimates from OLS regressions of future excess corporate bond returns on log debt-to-GDP ratio and other control variables.

$$\Sigma_{i=1}^{h}(r_{corp,t+i} - r_{f,t+i-1}) = \beta_0 + \beta_1 by_t + \beta_2 pd_t + \beta_3 svar_t + u_{t+h}$$

Excess corporate bond return is the log Barclay's corporate bond portfolio return $r_{corp}$ subtracting the log risk free rate $r_f$. Longer horizon excess returns are the cumulative summation of the one-period excess returns. $by_t$ is the log debt-to-GDP ratio. $by$ is the log debt to GDP ratio. $pd$ is the log price-dividend ratio. $svar$ is the stock return realized volatility. $h$ is the predictive horizon. The corporate bond portfolio is indicated in the name of the panel. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors. The sample is from 1973:I to 2014:IV for investment-grade bond and from 1983:I to 2014:IV for high-yield bond.

| $h$ | $by$ | ($t$-stat) | $pd$ | ($t$-stat) | $svar$ | ($t$-stat) | $R^2$ |
|---|---|---|---|---|---|---|---|
| | | | Investment Grade | | | | |
| 1q | 0.03 | (3.08) | -0.01 | (-1.72) | 0.01 | (2.06) | 0.06 |
| 1y | 0.09 | (2.73) | -0.04 | (-1.42) | 0.02 | (2.62) | 0.13 |
| 2y | 0.12 | (2.29) | -0.04 | (-0.91) | 0.06 | (3.85) | 0.20 |
| | | | High Yield | | | | |
| 1q | 0.04 | (3.16) | -0.03 | (-2.36) | 0.01 | (1.40) | 0.10 |
| 1y | 0.12 | (2.59) | -0.11 | (-2.75) | 0.04 | (1.80) | 0.24 |
| 2y | 0.13 | (1.55) | -0.18 | (-2.07) | 0.07 | (2.03) | 0.28 |

Table 1.10: Yield Spreads and Debt-to-GDP ratio

The table reports estimates from OLS regressions of yield spreads on log debt-to-GDP ratio and other control variables.

$$spread_t = \beta_0 + \beta_1 by_t + \beta_3 svar_t + \beta_3 tms_t + u_t$$
$$\Delta spread_t = \beta_0 + \beta_1 \Delta by_t + \beta_3 \Delta svar_t + \beta_3 \Delta tms_t + u_t$$

The spreads include: Gilchrist and Zakrajŝek (2012) spread index (GZ spread), the spreads between Moody's Aaa, Aa, A, Baa bond yield and 30-year treasury bond yield, the spread between general collateral repo rate (Repo), Certificate of Deposits rate (CD) , AA commercial paper (Paper) rate, federal funds rate (FFR) and treasury bill rate. *by* is the log debt-to-GDP ratio. *svar* is the stock return realized volatility. *tms* is the spread between 10-year treasury bond and 3-month T-bill. Panel A shows the regression in level and Panel B shows the regression in first difference. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors.

| $h$ | $by$ | ($t$-stat) | $R^2$ | $by$ | ($t$-stat) | $svar$ | ($t$-stat) | $tms$ | ($t$-stat) | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | A. Level | | | | | |
| GZ spread | 0.83 | (1.80) | 0.10 | 0.72 | (3.92) | 0.85 | (3.59) | 7.97 | (1.14) | 0.59 |
| Aaa-Treasury | 0.16 | (1.25) | 0.03 | 0.24 | (1.75) | 0.28 | (5.72) | -3.73 | (-1.26) | 0.41 |
| Aa-Treasury | -0.03 | (-0.15) | 0.00 | -0.02 | (-0.09) | 0.34 | (5.44) | -0.35 | (-0.10) | 0.39 |
| A-Treasury | -0.21 | (-0.70) | 0.02 | -0.25 | (-1.02) | 0.40 | (4.99) | 3.41 | (0.69) | 0.35 |
| Baa-Treasury | -0.19 | (-0.52) | 0.01 | -0.28 | (-1.02) | 0.52 | (3.90) | 6.04 | (1.04) | 0.38 |
| Baa-Aaa | -0.36 | (-1.22) | 0.08 | -0.52 | (-2.17) | 0.23 | (2.14) | 9.79 | (2.17) | 0.29 |
| Repo-Bill | -0.83 | (-3.84) | 0.31 | -0.59 | (-3.04) | 0.10 | (1.44) | -13.80 | (-3.29) | 0.42 |
| CD-Bill | -0.87 | (-3.69) | 0.18 | -0.81 | (-3.11) | 0.29 | (2.98) | -2.52 | (-0.35) | 0.28 |
| Paper-Bill | -1.10 | (-3.78) | 0.29 | -0.71 | (-2.73) | 0.09 | (1.47) | -22.45 | (-3.59) | 0.43 |
| FFR-Bill | -0.79 | (-3.86) | 0.26 | -0.59 | (-3.11) | 0.11 | (1.62) | -11.24 | (-2.55) | 0.35 |
| | | | | | B. First Difference | | | | | |
| GZ spread | 3.00 | (1.73) | 0.06 | 2.18 | (1.51) | 0.32 | (2.81) | -5.77 | (-2.22) | 0.33 |
| Aaa-Treasury | 1.55 | (2.56) | 0.04 | 1.15 | (2.18) | 0.18 | (5.37) | -10.83 | (-3.31) | 0.35 |
| Aa-Treasury | 2.06 | (2.82) | 0.06 | 1.60 | (2.26) | 0.20 | (4.75) | -9.06 | (-2.25) | 0.33 |
| A-Treasury | 2.56 | (2.56) | 0.07 | 2.00 | (2.09) | 0.23 | (4.27) | -7.83 | (-2.16) | 0.30 |
| Baa-Treasury | 3.65 | (2.58) | 0.09 | 2.98 | (2.18) | 0.27 | (3.65) | -7.90 | (-1.45) | 0.28 |
| Baa-Aaa | 2.10 | (2.20) | 0.08 | 1.83 | (1.90) | 0.10 | (1.75) | 2.91 | (1.11) | 0.14 |
| Repo-Bill | -3.43 | (-2.85) | 0.07 | -3.71 | (-2.92) | 0.13 | (2.21) | -9.31 | (-1.02) | 0.15 |
| CD-Bill | 1.21 | (0.52) | 0.00 | 0.10 | (0.05) | 0.31 | (4.01) | 35.23 | (2.19) | 0.20 |
| Paper-Bill | -1.58 | (-1.13) | 0.01 | -1.50 | (-0.80) | 0.02 | (0.17) | -17.72 | (-1.18) | 0.08 |
| FFR-Bill | -3.43 | (-2.55) | 0.06 | -3.75 | (-2.69) | 0.14 | (1.77) | -6.86 | (-0.74) | 0.12 |

Table 1.11: Variance Decomposition

The table reports variance decomposition of a VAR that includes GDP growth, stock market volatility, book-value debt-to-GDP ratio, market-value debt-to-GDP ratio and a yield spread. The spreads include: Gilchrist and Zakrajŝek (2012) spread index (GZ spread), the spreads between Moody's Aaa, Aa, A, Baa bond yield and 30-year treasury bond yield, the spread between general collateral repo rate (Repo), Certificate of Deposits rate (CD) , AA commercial paper (Paper) rate, federal funds rate (FFR) and treasury bill rate. $h$ shows the horizon of the forecast error decomposition. The reported values are in percentage point.

| | book shock | | | | market shock | | | |
|---|---|---|---|---|---|---|---|---|
| $h$ | 1 | 4 | 12 | 20 | 1 | 4 | 12 | 20 |
| GZ spread | 0.81 | 0.84 | 0.78 | 0.74 | 8.83 | 8.20 | 6.85 | 6.50 |
| Aaa-Treasury | 0.10 | 0.07 | 0.08 | 0.10 | 8.34 | 6.63 | 6.38 | 6.49 |
| Aa-Treasury | 0.13 | 0.10 | 0.14 | 0.28 | 11.65 | 8.15 | 8.52 | 8.99 |
| A-Treasury | 0.75 | 0.62 | 0.66 | 1.12 | 13.01 | 8.01 | 11.79 | 13.33 |
| Baa-Treasury | 4.13 | 3.27 | 2.85 | 3.20 | 10.81 | 6.14 | 11.00 | 12.53 |
| Baa-Aaa | 14.41 | 10.88 | 8.83 | 9.98 | 5.53 | 3.91 | 14.63 | 16.96 |
| Repo-Bill | 9.60 | 9.20 | 9.15 | 10.08 | 2.55 | 14.67 | 23.16 | 23.09 |
| CD-Bill | 0.02 | 0.47 | 1.50 | 2.37 | 10.28 | 18.94 | 27.78 | 28.07 |
| Paper-Bill | 5.97 | 6.08 | 6.52 | 7.36 | 0.74 | 8.61 | 16.00 | 16.12 |
| FFR-Bill | 7.36 | 7.68 | 7.95 | 8.82 | 0.57 | 9.52 | 18.55 | 18.82 |

Table 1.12: Risk-free Rate and Debt-to-GDP Ratio

The table reports estimates from OLS regressions of real risk-free rate on log debt-to-GDP ratio and other control variables.

$$r_{f,t} = \beta_0 + \beta_1 by_t + \beta_2 \Delta c_t + \beta_3 \Delta c_{t-1} + \beta_4 \pi_t + \beta_5 \pi_{t-1} + \beta_6 pd_t + u_t$$

The real risk-free rate is the nominal risk-free rate minus the four-quarter moving average of past inflation. The first column shows 3-month and 1-month real risk-free rate. "$r_f$, survey" calculate the real risk-free rate using Livingston survey on inflation. $by$ is log debt-to-GDP ratio. $\Delta c$ is consumption growth. $\pi$ is inflation. $pd$ is price-dividend ratio. "$r_f$, survey" is observed bi-annually. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors.

| Period | | $by_t$ | $\Delta c_t$ | $\Delta c_{t-1}$ | $\pi_t$ | $\pi_{t-1}$ | $pd_t$ | $R^2$ |
|---|---|---|---|---|---|---|---|---|
| 1947:I–2014:IV | $r_f$, 3M | -0.03 | 0.39 | 0.32 | -0.76 | -0.86 | 0.00 | 0.33 |
| | | (-2.94) | (2.29) | (1.37) | (-2.56) | (-2.86) | (-0.32) | |
| | $r_f$, 1M | -0.03 | 0.42 | 0.32 | -0.78 | -0.88 | 0.00 | 0.35 |
| | | (-3.09) | (2.60) | (1.45) | (-2.74) | (-3.14) | (-0.21) | |
| | $r_f$, survey | -0.02 | 0.02 | 0.00 | -0.22 | -0.78 | -0.01 | 0.24 |
| | | (-2.45) | (0.07) | (-0.01) | (-0.99) | (-2.12) | (-1.17) | |
| 1926–2014 | $r_f$, 3M | -0.05 | 0.01 | -0.09 | -0.79 | 0.19 | 0.00 | 0.71 |
| | | (-4.97) | (0.12) | (-0.71) | (-8.09) | (1.69) | (0.21) | |

Table 1.13: Government Debt Return and Debt-to-GDP Ratio

Panel A reports estimates from OLS regressions of government debt return on log debt-to-GDP ratio.

$$\Sigma_{i=1}^{h} r_{b,t+i} = \beta_0 + \beta_1 by_t + u_{+h}$$

The government debt return $r_b$ is the log average return across terms to maturity on all the government debt outstanding subtracting the realized inflation. Longer horizon returns are the cumulative summation of the one-period return. $by$ is log debt-to-GDP ratio. The t-statistics are based on heteroscedasticity and autocorrelation consistent standard errors. The sample is from 1929 to 2014. Panel B reports the variance decomposition. Surplus is $E_t[\Sigma_{j=0}\kappa_1^j(\kappa_2 \tau y_{t+j} - \kappa_3 g y_{t+j})]$. Growth is $E_t[\Sigma_{j=0}\kappa_1^j \Delta y_{t+j}]$. Return is $E_t[\Sigma_{j=0}\kappa_1^j r_{b,t+j}]$. The conditional expectation is computed from a first order vector autoregression model with five variables $[by_t,\ gy_t,\ \tau y_t,\ \Delta y_t,\ r_{b,t}]'$.

| A. Predictive Regression | | | |
|---|---|---|---|
| $h$ | $by_t$ | (t-stat) | $R^2$ |
| 1Y | -0.04 | (-2.29) | 0.09 |
| 5Y | -0.14 | (-1.74) | 0.15 |
| 10Y | -0.24 | (-2.00) | 0.17 |
| 20Y | -0.44 | (-3.59) | 0.28 |
| **B. Variance Decomposition** | | | |
| | surplus | growth | discount rate |
| $cov(by_t, E_t[.])/var(by_t)$ | 0.49 | 0.30 | -0.25 |

Table 1.14: Fiscal Uncertainty and Debt-to-GDP Ratio

This table reports the correlation between log debt-to-GDP ratio and measures of uncertainty. Panel A shows the broad-based measure of fiscal and macro uncertainty. Panel B shows the Economic Policy Uncertainty Index from Baker et al. (2015).

| | $corr(by,\ uncertainty)$ | |
|---|---|---|
| | 1947:I–2014:IV | 1985:I:2014:IV |
| **A. Broad-based Uncertainty Measure** | | |
| 1Y Fiscal Uncertainty | 0.28 | 0.21 |
| 3Y Fiscal Uncertainty | 0.50 | 0.29 |
| 5Y Fiscal Uncertainty | 0.54 | 0.29 |
| 1Y Macro Uncertainty | 0.06 | 0.15 |
| 3Y Macro Uncertainty | 0.06 | 0.06 |
| 5Y Macro Uncertainty | 0.06 | -0.26 |
| **B. Economic Policy Uncertainty** | | |
| Economic Policy Uncertainty | | 0.26 |
| Monetary policy | | -0.18 |
| Fiscal Policy | | 0.36 |
| Taxes | | 0.33 |
| Government spending | | 0.37 |
| Health care | | 0.58 |
| National security | | -0.14 |
| Entitlement programs | | 0.45 |
| Regulation | | 0.51 |
| Financial Regulation | | 0.27 |
| Trade policy | | -0.01 |
| Sovereign debt, currency crises | | 0.31 |

Table 1.15: Fiscal Uncertainty and Asset Prices

Panel A reports estimates from OLS regressions of asset prices on log debt-to-GDP ratio.

$$r_t = \beta_0 + \beta_1 Uncertainty_t + u_t$$

Excess stock return is the log market return $r_m$ subtracting the log risk-free rate $r_f$. $by$ is the log debt-to-GDP ratio. Excess corporate bond return is the log Barclay's corporate investment-grade bond portfolio return $r_{corp}$ subtracting the log risk free rate $r_f$. GZ spread is the credit spread in Gilchrist and Zakrajŝek (2012). The real risk free rate $r_f$ is the nominal risk free rate minus the four-month moving average of past inflation. The government debt return $r_b$ is the log average return across terms to maturity on all the government debt outstanding subtracting the realized inflation. Uncertainty is measured as the broad-based 3-year fiscal uncertainty or the fiscal policy uncertainty in Economic Policy Uncertainty Index from Baker et al. (2015).

| | Broad-based | ($t$-stat) | $R^2$ | EPU | ($t$-stat) | $R^2$ |
|---|---|---|---|---|---|---|
| $r_{m,t+1} - r_{f,t}$, 1Y | 0.021 | (1.26) | 0.02 | 0.048 | (2.39) | 0.08 |
| $r_{m,t+1} - r_{f,t}$, 3Y | 0.073 | (2.14) | 0.07 | 0.153 | (2.99) | 0.25 |
| $r_{m,t+1} - r_{f,t}$, 5Y | 0.168 | (2.93) | 0.21 | 0.188 | (4.79) | 0.24 |
| $r_{corp,t+1} - r_{f,t}$, 1Y | 0.025 | (2.43) | 0.07 | 0.018 | (2.42) | 0.09 |
| GZ spread | 0.007 | (2.51) | 0.43 | 0.003 | (1.10) | 0.06 |
| $r_{f,t}$ | -0.002 | (-1.54) | 0.09 | -0.002 | (-2.19) | 0.10 |
| $r_{b,t+1}$ | -0.002 | (-0.41) | 0.00 | -0.012 | (-1.99) | 0.09 |

Table 1.16: Calibration

The table reports the calibration of the model. Panel A contains the preferences and technology paraemters. Panel B contains fiscal policy parameters. Panel C contains parameters in fiscal consolidation and government spending policies.

| Description | Parameter | Value |
|---|---|---|
| | A. Preferences and Technology | |
| Subject discount factor | $\beta$ | 0.994 |
| Intertemporal elasticity of substitution | $\psi$ | 2 |
| Relative risk aversion | $\gamma$ | 10 |
| Capital share | $\alpha$ | 0.33 |
| Intermediate inputs share | $\xi$ | 0.50 |
| Depreciation rate of R&D capital | $\phi$ | 0.075 |
| Depreciation rate of capital | $\delta$ | 0.025 |
| Investment adjustment cost | $\xi_k$ | 0.80 |
| Exogenous productivity shock volatility | $\sigma_a$ | 0.008 |
| Exogenous productivity persistence | $\rho$ | 0.97 |
| | B. Fiscal Policy | |
| Corporatetax rate mean | $\mu_{\tau c}$ | 0.36 |
| Corporate tax rate shock volatility | $\sigma_{\tau.0}/\sqrt{1-\rho_\tau}$ | 0.03 |
| Corporate tax rate persistence | $\rho_\tau$ | 0.99 |
| Fiscal uncertainty shock volatility | $\sigma_{\tau,w}$ | 0.7 |
| Fiscal uncertainty persistence | $\nu_\tau$ | 0.995 |
| G-Y ratio mean | $\mu_{gy}$ | 0.17 |
| Labor tax rate mean | $\mu_{\tau l}$ | 0.10 |
| Long-term debt maturity | $\phi_b$ | 0.99 |
| Bond quantity persistence | $\rho_b$ | 0.99 |
| Bond quantity volatility | $\sigma_b$ | 0.01 |
| | C. Fiscal Consolidation and Government Spending Policy | |
| Fiscal consolidation | $\bar{\phi}_\tau$ | 0.001 |
| Fiscal consolidation volatility | $\sigma^2_{\phi\tau}$ | 0.006 |
| Fiscal consolidation cyclicality | $corr(\phi_{\tau,t+1}, \epsilon_{t+1})$ | -0.5 |
| G-Y ratio shock volatility | $\sigma_{g,0}/\sqrt{1-\rho_g}$ | 0.01 |
| G-Y ratio persistence | $\rho_g$ | 0.99 |

Table 1.17: Macroeconomic Dynamics and Asset Prices

The table reports the macroeconomic and asset price moments in the data and the model. The reported model moments are the mean, 5% and 95% quantile of short-sample simulations. The simulated sample period is 85 years.

| | Data | Model | 5% | 95% |
|---|---|---|---|---|
| $E[\Delta y]$ | 1.80 | 1.80 | -0.40 | 3.59 |
| $\sigma(\Delta y)$ | 5.00 | 2.80 | 2.55 | 3.15 |
| $\sigma(\Delta c)$ | 2.70 | 2.62 | 1.82 | 4.16 |
| $E[r_m - r_f]$ | 5.59 | 5.19 | 2.34 | 8.54 |
| $\sigma(r_m)$ | 20.04 | 7.32 | 5.81 | 10.12 |
| $E[r_f]$ | 0.45 | 1.48 | -0.46 | 2.98 |
| $\sigma(r_f)$ | 3.75 | 1.42 | 0.90 | 2.28 |
| $E[r_b]$ | 1.48 | 0.75 | -1.86 | 2.97 |
| $\sigma(r_b)$ | 5.31 | 4.54 | 3.69 | 5.96 |
| $E[pd]$ | 3.39 | 3.63 | 3.01 | 4.16 |
| $\sigma(pd)$ | 0.45 | 0.43 | 0.24 | 0.74 |
| $E[B/Y]$ | 0.40 | 0.52 | 0.41 | 0.63 |
| $\sigma(B/Y)$ | 0.18 | 0.08 | 0.05 | 0.12 |
| $E[T/Y]$ | 0.20 | 0.20 | 0.19 | 0.21 |
| $\sigma(T/Y)$ | 0.02 | 0.01 | 0.00 | 0.02 |

Table 1.18: Asset Prices, Debt-to-GDP and Fiscal Uncertainty

Panel A reports estimates from OLS regressions of asset prices on log debt-to-GDP ratio.

$$r_t = \beta_0 + \beta_1 by_t + u_t$$

Excess stock return is the log market return $r_m$ subtracting the log risk free rate $r_f$. $by$ is the log debt-to-GDP ratio. The real risk-free rate $r_f$ is the nominal risk-free rate minus the four-quarter moving average of past inflation. The government debt return $r_b$ is the log average return across terms to maturity on all the government debt outstanding subtracting the realized inflation. The standard errors are heteroscedasticity and autocorrelation consistent. Panel B reports estimates from OLS regressions of asset prices on the broad-based 3-year fiscal uncertainty. Model is from the benchmark calibration. The reported model moments are the mean, 5% and 95% quantile of $\beta$ and $R^2$ in short-sample simulations. The simulated sample period is 85 years.

| | Data | | | Model | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\beta$ | *s.e.* | $R^2$ | $\beta$ | 5% | 95% | $R^2$ | 5% | 95% |
| | | | | A. Debt-to-GDP ratio | | | | | |
| $r_{m,t+1} - r_{f,t}$, 1Q | 0.04 | (0.01) | 0.04 | 0.02 | 0.00 | 0.05 | 0.01 | 0.00 | 0.03 |
| $r_{m,t+1} - r_{f,t}$, 1Y | 0.15 | (0.05) | 0.11 | 0.08 | -0.01 | 0.18 | 0.04 | 0.00 | 0.10 |
| $r_{m,t+1} - r_{f,t}$, 3Y | 0.41 | (0.12) | 0.27 | 0.22 | -0.04 | 0.51 | 0.10 | 0.00 | 0.24 |
| $r_{m,t+1} - r_{f,t}$, 5Y | 0.60 | (0.13) | 0.38 | 0.33 | -0.08 | 0.80 | 0.14 | 0.00 | 0.35 |
| $r_{f,t}$ | -0.04 | (0.02) | 0.21 | -0.05 | -0.09 | -0.01 | 0.26 | 0.01 | 0.56 |
| $r_{b,t+1}$ | -0.04 | (0.02) | 0.09 | -0.09 | -0.16 | -0.03 | 0.10 | 0.01 | 0.22 |
| $r_{b,t+1}$, 10Y | -0.24 | (0.12) | 0.17 | -0.61 | -1.15 | -0.09 | 0.33 | 0.03 | 0.68 |
| $r_{m,t+1}$, 1Y | 0.11 | (0.05) | 0.05 | 0.03 | -0.07 | 0.14 | 0.02 | 0.00 | 0.06 |
| $r_{m,t+1}$, 3Y | 0.29 | (0.12) | 0.15 | 0.03 | -0.07 | 0.14 | 0.02 | 0.00 | 0.06 |
| $r_{m,t+1}$, 5Y | 0.43 | (0.15) | 0.22 | 0.15 | -0.33 | 0.64 | 0.08 | 0.00 | 0.23 |
| | | | | B. Fiscal Uncertainty | | | | | |
| $corr(by, uncertainty)$ | 0.50 | | | 0.43 | -0.15 | 0.86 | | | |
| $r_{m,t+1} - r_{f,t}$, 1Y | 0.02 | (0.02) | 0.02 | 0.02 | 0.01 | 0.03 | 0.08 | 0.00 | 0.18 |
| $r_{m,t+1} - r_{f,t}$, 5Y | 0.17 | (0.05) | 0.21 | 0.09 | 0.03 | 0.17 | 0.30 | 0.03 | 0.59 |
| $r_{f,t}$ | -0.002 | (0.001) | 0.09 | -0.01 | -0.01 | 0.00 | 0.19 | 0.00 | 0.53 |
| $r_{b,t+1}$ | -0.002 | (0.01) | 0.00 | -0.01 | -0.02 | -0.01 | 0.11 | 0.03 | 0.23 |

Table 1.19: Mechanisms of Fiscal Uncertainty

Panel A reports estimates from OLS regressions of asset prices on log debt-to-GDP ratio.

$$r_t = \beta_0 + \beta_1 by_t + u_t$$

Excess stock return is the log market return $r_m$ subtracting the log risk free rate $r_f$. $by$ is the log debt-to-GDP ratio. The real risk-free rate $r_f$ is the nominal risk-free rate minus the four-quarter moving average of past inflation. The government debt return $r_b$ is the log average return across terms to maturity on all the government debt outstanding subtracting the realized inflation. The standard errors are heteroscedasticity and autocorrelation consistent. Panel B reports estimates from OLS regressions of asset prices on the broad-based 3-year fiscal uncertainty.
"Tax Vol" is the benchmark calibration that has stochastic volatility. "Vol and Cons" includes both stochastic volatility and uncertain fiscal consolidations. "Cons." has uncertain fiscal consolidations but no stochastic volatility. "Spending" introduce spending shocks in the benchmark. "No Vol" has no stochastic volatility and uncertain fiscal consolidations. The reported model moments are the mean of $\beta$ and $R^2$ in short-sample simulations. The simulated sample period is 85 years.

| | Data | | Tax Vol | | Vol & Cons. | | Cons. | | Spending | | No Vol | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\beta$ | $R^2$ | $\beta$ | $R^2$ | $\beta$ | $R^2$ | $\beta$ | $R^2$ | $\beta$ | $R^2$ | $\beta$ | $R^2$ |
| **A. Debt-to-GDP ratio** | | | | | | | | | | | | |
| $r_{m,t+1} - r_{f,t}$, 1Q | 0.04 | 0.04 | 0.02 | 0.01 | 0.03 | 0.02 | 0.02 | 0.01 | 0.02 | 0.01 | 0.00 | 0.00 |
| $r_{m,t+1} - r_{f,t}$, 1Y | 0.15 | 0.11 | 0.08 | 0.04 | 0.12 | 0.07 | 0.09 | 0.04 | 0.09 | 0.04 | 0.01 | 0.01 |
| $r_{m,t+1} - r_{f,t}$, 3Y | 0.41 | 0.27 | 0.22 | 0.10 | 0.32 | 0.16 | 0.23 | 0.11 | 0.23 | 0.11 | 0.04 | 0.03 |
| $r_{m,t+1} - r_{f,t}$, 5Y | 0.60 | 0.38 | 0.33 | 0.14 | 0.48 | 0.23 | 0.35 | 0.16 | 0.35 | 0.16 | 0.05 | 0.05 |
| $r_{f,t}$ | -0.04 | 0.21 | -0.05 | 0.26 | -0.07 | 0.44 | -0.06 | 0.40 | -0.06 | 0.40 | -0.03 | 0.16 |
| $r_{b,t+1}$ | -0.04 | 0.09 | -0.09 | 0.10 | -0.14 | 0.16 | -0.12 | 0.14 | -0.12 | 0.14 | -0.04 | 0.04 |
| $r_{b,t+1}$, 10Y | -0.24 | 0.17 | -0.61 | 0.33 | -1.00 | 0.54 | -0.81 | 0.53 | -0.81 | 0.53 | -0.19 | 0.17 |
| $r_{m,t+1}$, 1Y | 0.11 | 0.05 | 0.03 | 0.02 | 0.05 | 0.03 | 0.02 | 0.02 | 0.02 | 0.02 | -0.02 | 0.02 |
| $r_{m,t+1}$, 3Y | 0.29 | 0.15 | 0.03 | 0.02 | 0.03 | 0.02 | 0.01 | 0.02 | 0.01 | 0.02 | -0.01 | 0.02 |
| $r_{m,t+1}$, 5Y | 0.43 | 0.22 | 0.15 | 0.08 | 0.17 | 0.09 | 0.07 | 0.07 | 0.07 | 0.07 | -0.05 | 0.06 |
| **B. Fiscal Uncertainty** | | | | | | | | | | | | |
| $corr(by_t, var_t(\tau_{c,t+1}))$ | 0.50 | | 0.43 | | 0.38 | | 1.00 | | 0.43 | | 0.00 | |
| $r_{m,t+1} - r_{f,t}$, 1Y | 0.02 | 0.02 | 0.02 | 0.08 | 0.02 | 0.06 | 0.01 | 0.02 | 0.02 | 0.08 | | |
| $r_{m,t+1} - r_{f,t}$, 5Y | 0.17 | 0.21 | 0.09 | 0.30 | 0.10 | 0.26 | 0.04 | 0.10 | 0.09 | 0.30 | | |
| $r_{f,t}$ | -0.002 | 0.09 | -0.01 | 0.19 | -0.01 | 0.18 | -0.01 | 0.32 | -0.01 | 0.19 | | |
| $r_{b,t+1}$ | -0.002 | 0.00 | -0.01 | 0.11 | -0.02 | 0.08 | -0.01 | 0.10 | -0.01 | 0.11 | | |

Table 1.20: Tax Impact on Growth

The table shows the OLS regressions of economic growth on log tax-to-GDP ratio.

$$\Sigma_{i=1}^{h}\Delta y_{t+i} = \beta_0 + \tau y_t \beta_1 + u_{t+h}$$

The growth $\Delta y$ is measured as real output, real private consumption or total factor productivity. $\tau y$ is the log tax-to-GDP ratio. Tax is the federal tax receipts. $h$ is the predictive horizon. The standard errors are heteroscedasticity and autocorrelation consistent. The sample is from 1947:I to 2014:IV. The reported model moments are the mean, 5% and 95% quantile of $\beta$ and $R^2$ in short-sample simulations. The simulated sample period is 85 years.

| | | Data | | | Model | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $h$ | | $\beta$ | *s.e.* | $R^2$ | $\beta$ | 5% | 95% | $R^2$ | 5% | 95% |
| 1Y | output, | -0.15 | (0.04) | 0.13 | -0.29 | -0.62 | 0.07 | 0.16 | 0.00 | 0.46 |
| | consumption | -0.09 | (0.03) | 0.06 | -0.41 | -0.80 | -0.04 | 0.24 | 0.01 | 0.55 |
| | TFP | -0.12 | (0.03) | 0.16 | -0.32 | -0.74 | 0.13 | 0.11 | 0.00 | 0.33 |
| 10Y | output | -0.66 | (0.19) | 0.21 | -1.43 | -4.81 | 1.85 | 0.19 | 0.00 | 0.55 |
| | consumption | -0.35 | (0.15) | 0.09 | -2.04 | -5.19 | 1.19 | 0.26 | 0.00 | 0.67 |
| | TFP | -0.32 | (0.15) | 0.13 | -1.31 | -5.42 | 2.96 | 0.15 | 0.00 | 0.47 |

# CHAPTER 2 : Volatility Risk Pass-Through

(with Riccardo Colacito, Mariano M. Croce, and Ivan Shaliastovich)

## 2.1. Introduction

The end of the Great Moderation period has highlighted once more the relevance of uncertainty shocks as key determinants of economic activity. In this paper, we estimate and explain the international transmission of output volatility shocks to both currencies and international quantity dynamics. More precisely, focusing on a large cross section of major industrialized countries, we identify news to the conditional volatility of output, consumption, and real exchange rates.

From this investigation we document several novel empirical findings. First, consumption and output volatilities are imperfectly correlated within countries. This implies that the growth rate of consumption in each country can experience changes in its conditional volatility that go beyond the arrival of endowment volatility shocks. Second, consumption volatility is more cross-country correlated than output volatility, suggesting that the output volatility shocks of one country propagate to the consumption of other countries.

To formalize the international propagation of output volatility shocks, we construct an index of volatility pass-through between two countries. Our index is equal to zero if a local output volatility shock results exclusively in an increase of local consumption volatility, without spilling over to the other country. Conversely, our index takes the value of one if a local output volatility shock results in an equal adjustment of consumption volatility in both countries.

We find that the pass-through of output volatility is sizeable, especially when the uncertainty shocks originate from the smallest countries in our cross section. Specifically, when we focus on G7 countries, the pass-through is on the order of 50%, regardless of the country in which the output volatility shock materializes. When we also include the next 10 countries according to their share of world GDP (henceforth G17), we find that the pass-through from bigger countries to smaller countries declines, whereas the pass-through of a volatility shock originating from small countries to large ones becomes as great as 70%. That is, smaller countries can better share volatility shocks compared to larger countries, by redistributing a bigger fraction of their uncertainty shocks to their trading partners.

Our last empirical finding refers to the disconnect between the volatility of consumption differentials and the volatility of exchange rates. We document that the correlation of these volatilities is about 20% for the set of countries that we consider in our empirical investigation. This extent of comovement constitutes an anomaly from the standpoint of a frictionless model with time-additive preferences, since this setting prescribes an almost perfect correlation. This is a novel observation that goes beyond the low correlation of the levels of consumption differentials and exchange rates (the Kollmann (1991) and Backus and Smith (1993a) puzzle).

In the second part of this manuscript, we show that our main findings are an anomaly in the context of an equilibrium risk-sharing model with time-additive preferences. In contrast, when agents have recursive preferences, news about both future growth rates and future uncertainty are priced, and thus they can jointly affect trade and volatility dynamics in a manner consistent with the data.

Specifically, we consider an economy with two countries, each populated by one agent with Epstein and Zin (1991) preferences (henceforth EZ preferences). Each agent is endowed with the stochastic supply of one country-specific good, whose dynamics are characterized by the presence of time-varying volatility shocks. Preferences feature a bias for the consumption of the domestic good. Trade occurs in frictionless goods markets and in financial markets featuring a complete set of state- and date-contingent securities.

Preferences are calibrated so that our agents dislike volatility of their continuation utilities. Since continuation utilities are a reflection of the entire future streams of consumption, we say that agents dislike long-run consumption variance. When news shocks hit the economy, agents have an incentive to trade in order to reduce the uncertainty of their future utility. Specifically, a country affected by a positive news shock will receive a smaller share of resources and have lower volatility of continuation utility going forward, but it will also have higher short-run consumption volatility.

When news pertains to future expected growth rates, the international reallocation of resources results in an international exchange of both short-run and long-run consumption volatility across countries. That is, variances are characterized by negative comovements. We call this force the reallocation effect. News to output volatility, in contrast, produces a positive comovement in consumption volatilities across all countries: changes in output volatility spread in the cross section of countries, with the reallocation channel only partially mitigating the effects of local shocks on local consumption volatility.

The recursive risk-sharing arrangement that we described above is the key driver of our main results. Since agents dislike time variation in the volatility of their consumption, they actively trade with each other in order to dampen the associated change in the volatility of consumption following an output volatility shock. This reallocation results in a marked degree of volatility pass-through, which brings our model closer to the data.

Because of the concavity of the utility function with respect to country size, the reallocation channel is more pronounced for small countries than for large countries. As a result, our model predicts that shocks to output volatility should come with a larger pass-through when they affect small countries, consistent with the data. In a model with CRRA preferences, however, this result is missing, as volatility shocks are not directly priced and the associated risk-sharing motive is absent.

Furthermore, the model can account for the small extent of positive comovement between the volatility of consumption differentials and the volatility of exchange rate fluctuations thanks to two opposite forces. Volatility shocks tend to create a positive correlation between the two volatilities, as they increase the uncertainty of all the variables in the economy. Long-run shocks, in contrast, generate a large negative comovement.

To better understand the role of long-run shocks, we note that they are responsible for most of the fluctuations of the wealth distribution, that is, our reallocation channel. As the wealth distribution becomes more unequal, our countries depend more on each other in order to share risks. In equilibrium, they engage in more active trading, and their stochastic discount factors become more correlated. By no arbitrage, the real exchange rate becomes less volatile. Simultaneously, the reallocation effect makes the cross-country difference of the consumption growth rates more volatile, as the pass-through of consumption volatility is not symmetric across countries with different wealth shares.

In a model without shocks to output volatility (e.g., Colacito and Croce (2013a)), the volatility of the exchange rate and that of the international differential of consumption growth rates would be strongly negative because of the dominance of the reallocation channel. In contrast, exogenous output volatility shocks increase the conditional volatility of all macroeconomic aggregates and hence endogenously produce positive comovements. Under our benchmark calibration, these opposite forces end up producing a positive but moderate correlation between consumption differentials and exchange rate volatility, as in the data.

The international long-run risk literature has already documented the ability of long-lasting consumption news shocks to account for several empirical regularities of international asset prices (see, among others, Colacito (2008); Nakamura et al. (2012); Colacito and Croce (2013a); and Bansal and Shaliastovich (2013)). We differ from this literature in at least two dimensions. First, we provide novel evidence on the diffusion of fundamental output volatility shocks to consumption and currencies. Second, we provide an equilibrium explanation of our findings through the lens of a frictionless risk-sharing scheme in which volatility shocks are priced.

**Related literature.** Our manuscript contributes to a recently growing literature that studies uncertainty shocks in an international setting. In an early contribution, Ramey and Ramey (1995) show that countries with higher volatility of GDP have lower growth in the future. Consistent with their cross-sectional evidence, we find that higher domestic output volatility is associated with a decline in relative consumption in the future. We develop a general equilibrium model to study the implications of volatility risk sharing for quantities and prices.

Fogli and Perri (2015) link macroeconomic volatility to trends in external imbalances in a neoclassical international production economy. Novy and Taylor (2014) nest uncertainty shocks in a model with endogenous production, international trade of intermediate inputs, and inventory concerns. They find that uncertainty shocks explain a relevant share of the cyclical behavior of trade and abstract away from asset pricing considerations. In contrast to these approaches, we take output as given and link the diffusion of consumption uncertainty to currency behavior.

Fernandez-Villaverde et al. (2011) study interest rate uncertainty shocks in the context of a rich, small open economy model with time-additive preferences. We study the propagation of uncertainty shocks in a general equilibrium exchange economy in which agents have recursive preferences and volatility shocks are priced. By doing so, we set the stage for a future class of macrofinance international business cycle models in which volatility shocks drive both international quantities and asset prices.

More broadly, our analysis relates to the recent literature examining the role of uncertainty both in the data and in economic models (see, among others, Jones et al. (2005); Justiniano and Primiceri (2008); Bloom et al. (2007); Bloom (2009b); Bloom et al. (2016); Basu and Bundick (2012); Jurado et al. (2015a); Kollmann (2016); and Gilchrist et al. (2014)).

Although our attention is focused on a frictionless risk-sharing setting with symmetric countries, we regard the introduction of frictions and heterogeneity into our model as an important direction for future research in this area (see, e.g., Gabaix and Maggiori (2015a); Ready et al. (2012); Backus et al. (2010); Maggiori (2011); and Lustig et al. (2011a)). These frictions may be important in addressing the empirical link between uncertainty and international capital flows documented by Gourio et al. (2014).

Our study is also related to the growing body of literature that has investigated the macroeconomic foundations of international financial markets' fluctuations (see, inter alia, Pavlova and Rigobon (2007), Pavlova and Rigobon (2010), Farhi and Gabaix (2008), Hassan (2013), Stathopoulos (2012), Heyerdahl-Larsen (2015), Verdelhan (2010a), Mueller et al. (2015), and Pavlova and Rigobon (2013) for an extensive review of the literature). We differ from these papers by explicitly introducing time-varying uncertainty in macroeconomic fundamentals and studying its effects on the optimal international risk-sharing arrangement.

Additionally, several papers have documented the relevance of higher-order moments in sharpening our understanding of currency dynamics. Gavazzoni et al. (2013) argue that non-Gaussian dynamics of the stochastic discount factors are needed to reconcile the riskiness of currencies with the level of the interest rates. Berg and Mark (2016) show that the cross-country high-minus-low conditional skewness of the unemployment gap is a measure of global macroeconomic uncertainty and it constitutes a factor that is robustly priced in currency excess returns. Zviadadze (2015) analyzes the relationship between shocks to the stochastic variance of US consumption and the cross section of currency risk premia. Relative to this literature, we document how volatility shocks spread in the cross section of G-17 countries and propose a model that accounts for the way that volatility risk is internationally shared. Farhi et al. (2015) and Chernov et al. (2014) study the role of crash risk for currency risk premia. We regard the introduction of rare events as an important generalization of this framework.

The manuscript is organized as follows. In section 2.2 we describe our empirical strategy and our novel findings concerning the cross section of volatilities of major industrialized countries. Section 2.3 describes our model, whose results are presented in section 2.4.

Section 2.5 concludes the paper. The appendix contains additional robustness checks and the model's extensions.

## 2.2. Empirical Evidence

In this section, we describe the econometric framework that we adopt to measure comovements in macroeconomic volatility within and across major industrialized countries. Focusing on the volatility of shocks to the growth rates of macroeconomic variables, we provide novel empirical evidence on the extent to which shocks to the relative volatility of GDP are transmitted to the relative volatility of consumption. We refer to this concept as the *volatility pass-through*. Further, we provide evidence linking volatility comovements to trade dynamics.

### *2.2.1. Data Description*

**Sources and sample.** Our empirical analysis is based on the cross section of the following 17 major industrialized countries, ranked by GDP size: the United States, Canada, France, Germany, Italy, Japan, the United Kingdom, Australia, Belgium, Denmark, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, and Switzerland. In this study, we refer to the group of the first seven countries as G7 and to the expanded set of countries as G17. We collect the national accounts, population, and CPI data for these countries from the Organization for Economic Cooperation and Development (henceforth OECD) database. The exchange rates, quoted as the US dollar price of the foreign currency, are from the Federal Reserve Economic Database (henceforth FRED) database. The macroeconomic data are seasonally adjusted, real, and per capita.

To be consistent with the endowment economy that we analyze in sections 2.3 and 2.4, we abstract away from both investment and public expenditure and compute aggregate output as the sum of consumption and net exports. Since our model is based on a frictionless risk-sharing scheme, we follow the common practice of letting our quarterly dataset range from 1971:q1 to 2013:q4, a period of substantial financial integration across all major industrialized countries (see, among others, Quinn (1997), Obstfeld (1998), Taylor (2002), and Quinn and Voth (2008)).[1]

**Cross-sectional similarities and differences.** In table 2.1, we shows key moments of our international data. For ease of exposition, we report cross-sectionally aggregated moments, as opposed to country-level values. Specifically, we look at moments for both G7 and G17 countries. For G7 countries, we report the simple average of our aggregates. For G17 countries, we present both simple and GDP-weighted cross-sectional averages of our moments. To assess the extent of cross-country heterogeneity, for each moment we also report its $1^{st}$ and $4^{th}$ quintiles in the G17 group.

We highlight three relevant facts. First, the moments for the G7 group very much resemble those that are typically encountered for the US. As an example, consumption growth has a mean of about 2% per year and a volatility of about 1.75%. In the G17 aggregate, the average growth rate declines, whereas the unconditional volatility of both output and consumption increases. In both cases, however, changes are relatively modest. Both quarterly consumption and output growth are almost serially uncorrelated.

Second, the average change in the net-export-to-output ratio is distributed nearly symmetrically around zero. In the group of G17 countries, this moment ranges from −30% to +34%. Since smaller countries have more volatile output than bigger countries, they also tend to have more volatile net-export-to-output ratios. In our model, we abstract away from this source of heterogeneity and focus on the volatility of net-export-to-output ratios relative to output volatility. In the data this ratio is about 0.80 for both G7 and G17 countries.

Third, in both the G7 and G17 groups, consumption growth rates feature low international correlations.[2] Further, output and consumption growth rates are imperfectly correlated

[1] Due to data availability and quality issues, the data for Belgium, Norway, and Spain start in 1981; for New Zealand in 1986; and for Portugal in 1991. Our Bayesian methods can easily be applied to an unbalanced panel.

[2] The quantity anomaly in Backus et al. (1994) does not apply to our dataset, as our measured output excludes both investment and government expenditure.

Table 2.1: Data Summary Statistics

Notes: This table shows summary statistics for consumption growth, output growth, change in net-export-to-output ratio, and consumption and output volatility. 'G7 Avg.' ('G17 Avg.') refers to simple (both simple and GDP-weighted) averages of key moments for G7 (G17) countries. The rightmost two columns show the first and fourth quintiles of the moments of interest in the G17 cross section. Macroeconomic variables are seasonally adjusted, real, and per capita. Means and volatilities are annualized, in percentages. Quarterly observations are from the 1971:Q1–2013:Q4 sample.

| | G7 Avg. | G17 Avg. | | G17 Quintile | |
|---|---|---|---|---|---|
| | Simple | Simple | Weighted | $1^{st}$ | $4^{th}$ |
| Mean | 1.91 | 1.63 | 1.89 | 1.26 | 2.02 |
| Std. Dev. | 1.75 | 1.99 | 1.67 | 1.34 | 2.47 |
| AR(1) | 0.11 | 0.07 | 0.17 | -0.16 | 0.31 |
| *Output growth:* | | | | | |
| Mean | 1.94 | 1.71 | 1.93 | 1.43 | 2.00 |
| Std. Dev. | 2.21 | 2.97 | 2.02 | 2.01 | 4.43 |
| AR(1) | 0.00 | -0.09 | 0.07 | -0.26 | 0.09 |
| $\Delta$*Net Exports over Output:* | | | | | |
| Mean | 0.03 | 0.08 | 0.04 | -0.30 | 0.34 |
| Std. Dev. | 1.60 | 2.48 | 1.45 | 1.79 | 3.24 |
| AR(1) | 0.00 | -0.09 | 0.07 | -0.26 | 0.09 |
| *Within-Country Correlations:* | | | | | |
| Consump. and output growth | 0.67 | 0.51 | 0.71 | 0.35 | 0.72 |
| Consump. and output vol | 0.54 | 0.47 | 0.65 | 0.26 | 0.80 |
| *Across-Country Correlations:* | | | | | |
| Consump. growth | 0.27 | 0.24 | 0.25 | 0.13 | 0.33 |
| Output growth | 0.15 | 0.14 | 0.14 | 0.06 | 0.20 |
| Consump. vol | 0.51 | 0.47 | 0.45 | 0.35 | 0.66 |
| Output vol | 0.32 | 0.30 | 0.30 | 0.18 | 0.45 |

within countries. Both of these empirical facts are consistent with the predictions of our recursive risk-sharing model.

In the next sections, we describe in detail our identification of the time-varying volatility components and address their comovements within and across countries.

*2.2.2. Volatility Measurement and Comovements*

We extract the volatility of the series of interest, $z_t$, by estimating the following specification:

$$\begin{aligned} z_t &= \mu(1-\rho) + \rho z_{t-1} + e^{\sigma_t(z)/2}\eta_t, \\ \sigma_t(z) &= \mu_\sigma(1-\nu) + \nu\sigma_{t-1}(z) + \sigma_w w_t, \end{aligned} \tag{2.1}$$

where $\sigma_t(z)$ is a latent process equal to the logarithm of the variance of macroeconomic shock to $z_t$. The innovations $\eta_t$ and $w_t$ are independent Gaussian shocks to the level and the volatility of $z_t$, respectively. The parameters $\rho$ and $\nu$ govern the persistence of $z_t$ and $\sigma_t(z_t)$, respectively, whereas $\mu$ and $\mu_\sigma$ represent the average level and volatility of $z_t$ and $\sigma_t(z_t)$, respectively. The parameter $\sigma_w$ captures the volatility of volatility.

Similar volatility specifications are employed in Cogley and Sargent (2005) and Primiceri (2005) in the context of macroeconomic volatility, and in Cortet et al. (2009) for financial volatility modeling. According to our specification, the variance of $z_t$ is guaranteed to take on positive values. In untabulated tests we directly estimated volatility in levels, with very similar results. For this reason, in the remainder of this manuscript we refer to $\sigma_t$ as either log-volatility or volatility interchangeably.

We estimate the system of equations (2.1) following the Bayesian methods in Kim et al. (1998). For each country, we fit our volatility specification to aggregate consumption and output growth separately. To check the robustness of our results, we also employ a specification in which the volatility parameters are restricted to be common across countries and are jointly estimated in our cross section of countries. For parsimony, a complete summary of the estimation details is provided in the appendix.

**Volatilities: aggregate time pattern.** In figure 2.2.2, we show our fitted volatilities aggregated across both G7 and G17 countries. For the G17 group, we also plot the first and the fourth cross-sectional volatility quintiles. Consistent with the findings reported in table 2.1, consumption volatility is systematically lower than output volatility. Further, our estimation procedure captures the well-documented Great Moderation phenomenon, as both our estimated consumption and output volatilities slowly decline from the 1980s to the mid-2000s. These findings are consistent with those documented by Lettau et al. (2008); Stock and Watson (2002); and McConnell and Quiros (2000) for the United States and support the plausibility of the results obtained so far.

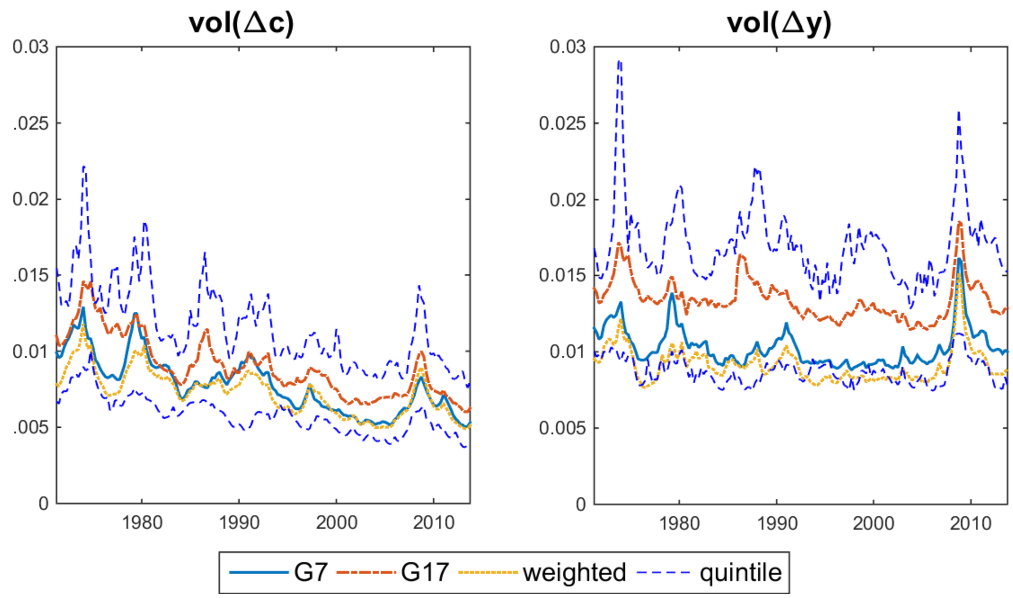



Figure 2.1: Macroeconomic Volatilities

This figure shows estimates of macroeconomic volatilities of real consumption and output growth. Volatilities, $e^{\sigma_t/2}$, are estimated at a country level according to equation (2.1). The G7 line shows the equally weighted cross-sectional average for G7 countries. "G17" reports the equally weighted average across all the G17 countries. "Weighted" reports the GDP-weighted average across G17 countries. Dashed lines show the first and fourth quantiles of the volatilities in the G17 cross section. Quarterly observations range from 1971:Q1 to 2013:Q4.

Consistent with the unconditional evidence in table 2.1, G17 countries have a larger average volatility level relative to the G7 group. In both country groups, our conditional estimates are subject to substantial and persistent fluctuations over time. More broadly, the time pattern of the estimated aggregate volatilities shares similar characteristics across G7 and G17 countries. These results suggest that our novel findings on international volatility comovements are quite general, as they apply to a large international cross section.

**Volatilities: comovements.** Uncertainty shocks appear to be modestly correlated across countries for both consumption and output. In table 2.1, we formally quantify this statement by reporting volatility correlations within and across countries. We find that the correlation structure of the volatilities mimics that of the levels.

Specifically, the cross-country correlation of endowment volatilities is about 0.30, a number close to the cross-country correlation of the levels of the growth rates. The cross-country correlation of consumption volatility is slightly higher than that of output volatility, once again consistent with that observed for the growth rates of the levels. Within each country, in contrast, the volatilities of consumption and output comove strongly with each other. Their correlation is 0.70, a figure similar to that of the consumption and output growth rates.

In our next step, we adopt a VAR approach to (i) better characterize the joint dynamics of both levels and volatilities, and (ii) quantify the pass-through of volatility shocks.

*2.2.3. Volatility Risk Pass-Through*

**Relative volatility shocks.** To evaluate the dynamic impact of shocks to relative volatility $(\sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}))$ across countries, we jointly estimate the following $N$ countries VAR(1):

$$\tilde{Y}_{t,i} = \tilde{\mu}_{Y,i} + \tilde{\Phi}\tilde{Y}_{t,i} + \tilde{\Sigma}\tilde{u}_{t,i}, \quad i = 1, 2, ..., N \tag{2.2}$$

where

$$\tilde{Y}_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix}, \tag{2.3}$$

where $\Delta y_i - \Delta y_{US}$, $\sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US})$, $\Delta c_i - \Delta c_{US}$, and $\Delta(NX/Y)_i - \Delta(NX/Y)_{US}$ denote the difference between country $i$ and the US in growth rates of endowments; the volatilities of consumption growth rates; the growth rates of consumption; and the net-export-to-output ratios, respectively. We note that $N$ is equal to 6 for G-7 countries and 16 for G-17 countries. In our appendix, we show that our key results are robust both to different specifications and estimation procedures, and to the choice of a global benchmark, rather than considering just the US. Furthermore, our results are virtually unchanged when we account for heterogenous exposure to a common global volatility process across countries.

Since we adopt the US as the baseline home country throughout our analysis, this specification allows us to focus on relative bilateral adjustments computed with respect to a common benchmark. To sharpen the system's identification, we assume that the fundamental persistence and volatility parameters $\tilde{\Phi}$ and $\tilde{\Sigma}$ are common across countries, whereas the intercepts $\tilde{\mu}_{Y,i}$ are allowed to be country specific. Under these assumptions, we can estimate the VAR parameters by pooling the demeaned data across countries. We estimate the system of VAR equations as Seemingly Unrelated Regressions.

Throughout this study, we take volatility shocks as primitive exogenous innovations. Consistent with this approach, we identify impulse responses through a lower diagonal Cholesky decomposition in which output volatility shocks are the most exogenous to the system, that is, they are ranked first. Using our estimated VAR, we can then trace the relative response of the macroeconomic variables to an increase in output volatility in the foreign country relative to the US.

In figure 2.2.2, we show the estimated impulse responses for the G7 countries to a relative volatility shock. In table 2.2.2, we report the contemporaneous responses of all the variables in the system to this type of shock. These numbers correspond to the entries in the first column of the matrix $\tilde{\Sigma}$ in equation (2.2). We perform this analysis for both the G7 and the remaining G17 countries (hereafter, the bottom-10 G17). Our empirical evidence highlights several important cross-sectional aspects of volatility shocks across countries.

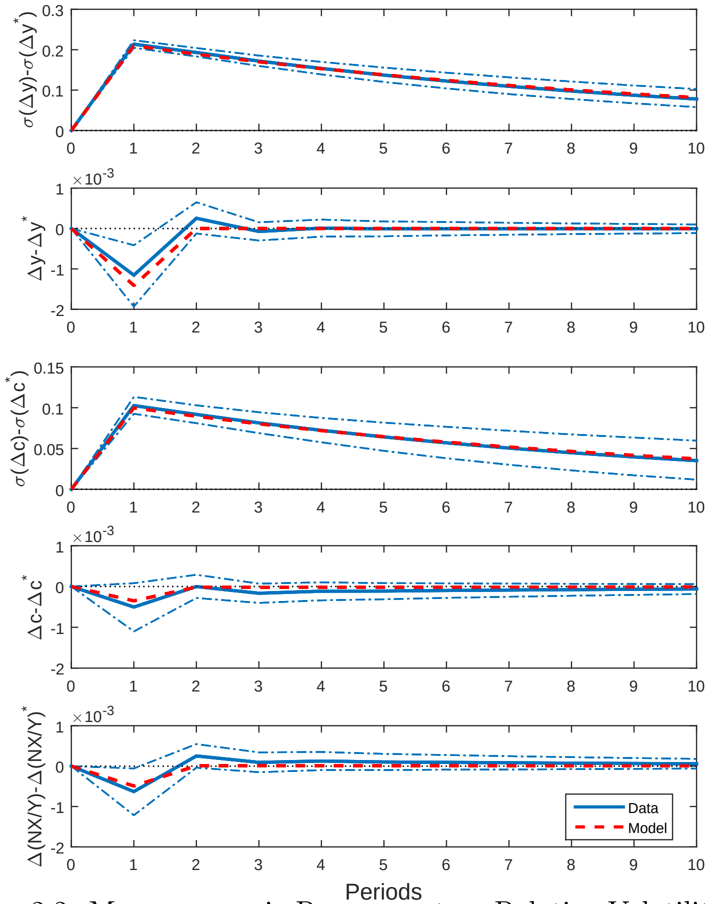



Figure 2.2: Macroeconomic Responses to a Relative Volatility Shock

This figure shows the estimates of the relative responses of the volatility and growth rate of output ($\Delta y$), the volatility and growth rate of consumption ($\Delta c$), and the change of net-export-to-output ratio ($\Delta NX/Y$) to a one-standard-deviation increase in the volatility of output in the foreign country relative to the US. Dashed (dotted) lines refer to the point estimates (95% credible interval) of the VAR(1) specified in equation (2.3). Solid lines show the output from our model under the benchmark quarterly calibration reported in table 2.3.1.

First, when country $i$ experiences an increase in its output volatility relative to the US, both its relative consumption and output growth rates fall. The estimated effects are large and almost always statistically significant. For example, in our G7 specification, foreign output growth falls by nearly half a percentage point relative to the US upon the realization of a one-standard-deviation relative volatility shock. These findings complement the one-

Table 2.2: Volatility Risk Pass-Through

Notes: Panel A shows the estimates of the contemporaneous responses ($\widetilde{\Sigma}_{1j}$) of the VAR(1) specified in equations (2.2)–(2.3) with respect to a shock to relative output volatility. Responses of output growth, consumption growth, and net-exports-to-output ratio are annualized, in percentages. Volatility pass-through is defined as in equation (2.4). Panel B reports pass-through measures based on the estimates of the VAR in equations (2.5)–(2.6) with respect to volatility shocks affecting either the US or the remaining countries. We report 95% credible intervals in brackets. Our quarterly data range from 1971:q1 to 2013:q4.

**Panel A: Contemporaneous adjustments to relative volatility shocks**

| $\sigma(\Delta y)$ | $\Delta y$ | $\sigma(\Delta c)$ | $\Delta c$ | $\Delta(NX/Y)$ | Pass-through |
|---|---|---|---|---|---|
| *US/G7 Countries:* | | | | | |
| 0.21 | -0.46 | 0.10 | -0.20 | -0.25 | 0.52 |
| [0.20; 0.22] | [-0.77; -0.17] | [0.09; 0.11] | [-0.44; 0.03] | [-0.49; -0.02] | [0.48; 0.56] |
| *US/Bottom-10 G17 Countries:* | | | | | |
| 0.21 | -0.57 | 0.08 | -0.16 | -0.39 | 0.61 |
| [0.21; 0.22] | [-0.95; -0.19] | [0.07; 0.09] | [-0.41; 0.09] | [-0.73; -0.06] | [0.56; 0.65] |

**Panel B: Pass-through and size**

| | Origin of Vol. Shock: | |
|---|---|---|
| | U.S. | Foreign Country |
| *US/G7 G17 Countries:* | 0.49 | 0.57 |
| | [0.43; 0.54] | [0.51; 0.63] |
| *US/Bottom-10 G17 Countries:* | 0.51 | 0.72 |
| | [0.45; 0.57] | [0.66; 0.78] |

country evidence in Bansal et al. (2014b) and Bloom (2009b) in showing that an increase in domestic volatility decreases real economic activity. For the same country group, the fall in the relative level of consumption growth is about 0.20%, that is, half of that of output. This mitigation happens through net imports, as the country with the highest volatility shock experiences a deterioration of its current account.

Second, upon the arrival of a relative increase in output volatility, the volatility of consumption increases as well. We find it convenient to explore this effect in greater detail by defining a volatility pass-through index as follows:

$$\text{Pass-through} \quad := \quad 1 - \frac{\partial(\sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}))}{\partial(\sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}))}. \tag{2.4}$$

Since our analysis is based on country pairs, this index is equal to zero if an increase in output volatility in one country results in a one-for-one increase in its own consumption volatility. If instead an output volatility shock results in an equally redistributed increase in consumption volatility across the two countries, the volatility pass-through is one.

In an economy with time-additive preferences defined over one good, perfect risk-sharing implies a pass-through of one, as consumption is equalized across all possible states and hence $\sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) = 0 \; \forall t$. Vice versa, in an endowment economy in which countries are subject to autarky, that is, they cannot trade, our index is equal to zero, as $C_{i,t} = Y_{i,t}$ $\forall i, t$ and hence $(\sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US})) = (\sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US})) \; \forall i, t$.

Our estimates suggest that the volatility pass-through is about 50% for G7 countries, meaning that if country $i$ receives a country-specific output volatility shock of one, its own consumption volatility goes up by just 0.50.[3] This index increases further to 60% when we focus on smaller countries, suggesting that the international sharing of volatility shocks is more relevant for this set of countries. In our theoretical investigation, we show that replicating these results in a model with time-additive preferences is a challenge.

The literature on economic uncertainty has investigated the possibility that volatility shocks may be endogenous, and proposed potential solutions to overcome this empirical issue (see for example Baker et al. (2016)). While the results discussed in this section are based on an identification strategy in which volatility shocks are assumed to be the exogenous to level shocks, we note that this assumption is not necessary for the calculation of the volatility pass-through index. Indeed the index can be computed for any identification scheme relying on the milder assumption that output volatility shocks are exogenous only to consumption volatility shocks. In the Appendix, we document the robustness of our results to some of these alternative VAR specifications.

**Country-specific volatility shocks.** The specification of the VAR in equation (2.3) is parsimonious, but it features three main shortfalls: (i) it does not provide information on

[3] According to our VAR specification in equations (2.2)–(2.3), we obtain $Pass-through = 1 - \widetilde{\Sigma}_{3,1}/\widetilde{\Sigma}_{1,1}$.

the size of country-level shocks; (ii) it is silent on the correlation of shocks across countries; and (iii) it is unable to detect potentially different responses depending on whether volatility shocks arise from big or small countries. The first two limitations are relevant for calibration reasons. The third shortcoming limits our understanding of volatility shock risk-sharing in the data.

To overcome these issues, we propose an extended VAR,

$$Y_{t,i} = \mu_{Y,i} + \Phi Y_{t,i} + \Sigma u_{t,i}, \tag{2.5}$$

in which we disentangle foreign and U.S. variables:

$$Y'_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_i) & \sigma_t(\Delta y_{US}) & \Delta y_i & \Delta y_{US} & \sigma_t(\Delta c_i) & \sigma_t(\Delta c_{US}) \end{bmatrix}. \tag{2.6}$$

As before, the persistence and scale matrices are common across countries, whereas the intercepts pick out country-specific differences in the means. For parsimony, we consider the smallest set of variables required for both calibration reasons and for the assessment of the volatility pass-through. As a result, we exclude both the change in net exports and the consumption growth rates from this VAR.

The estimation results used to guide our calibration are discussed in the next section and are reported in table 2.3.1. In panel B of table 2.2.2, we report the implied volatility pass-through due to either a one-standard-deviation increase in US output volatility or a one-standard-deviation in foreign output volatility for both the G7 and the bottom-10 G17 countries.[4] The additional insight provided by this estimation is that the pass-through is sensitive to the size distribution of the countries that we analyze.

[4]According to our VAR specification in equations (2.5)–(2.6), the pass-through for US-originated shocks is $Pass-through^{US} = 1 - (\widetilde{\Sigma}_{6,2} - \widetilde{\Sigma}_{5,2})/\widetilde{\Sigma}_{2,2}$. For non-US shocks, we adopt the same expression, but we change the order of variables in our VAR as follows:

$$Y'_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_{US}) & \sigma_t(\Delta y_i) & \Delta y_{US} & \Delta y_i & \sigma_t(\Delta c_{US}) & \sigma_t(\Delta c) \end{bmatrix}.$$

.

Specifically, when we focus on the G7 group, all countries tend to have a similar size and a pass-through in the common range (51%–54%), regardless of the origin of the volatility shock. In contrast, when we focus on the US versus the bottom-10 G17 countries, i.e., a cross section with more dispersion in size, the origin of the shock matters. We find that the volatility pass-through is larger if the volatility shock originates from the smaller economies.

According to our estimates, the bottom-10 G17 countries have a pass-through of 72% when they receive an adverse output volatility shock. When the US receives a volatility shock, in contrast, the pass-through to these smaller countries is just 51%, a number comparable to that estimated for the other G7 countries. All together, these results suggest a novel empirical finding: after a spike in endowment uncertainty, small countries mitigate their consumption volatility better than large countries.

**The volatility disconnect puzzle.** If agents have CRRA preferences and markets are complete, the scaled difference of consumption growth rates should equal the rate of depreciation of the exchange rate between the two countries' currencies:

$$\gamma \cdot (\Delta c_{h,t+1} - \Delta c_{f,t+1}) \quad = \quad \Delta e_{t+1}. \tag{2.7}$$

As a result, consumption growth rate differentials should be perfectly correlated with exchange rates. Starting with Backus and Smith (1993a), a vast literature has documented the empirical failure of this prediction (hereafter, the Backus and Smith puzzle). In the top part of table 2.2.2, we show that the Backus-Smith anomaly is present in our dataset as well.

Given our focus on the dynamics of volatility, we push our analysis one step further and study the implications for the conditional variance of consumption growth differentials and the conditional variance of exchange rate movements. Specifically, if we apply the conditi-

Table 2.3: Volatility Disconnect Puzzle

Notes: This table shows correlations between the level and conditional volatility of consumption growth differentials ($cd_t^i \equiv \Delta c_t^{US} - \Delta c_t^i$) and exchange rate growth ($\Delta e_t^{i|USD}$), respectively. In both cases, the US is considered the benchmark home country. Cumulative growth rates are denoted by '$\frown$'. 'G7 Avg.' ('G17 Avg.') refers to simple (both simple and GDP-weighted) averages of key moments for G7 (G17) countries. The rightmost two columns show the first and fourth quintiles of the moments of interest in the G17 cross-section. Consumption is seasonally adjusted, real, and per capita. Volatility estimates are based on the specification reported in equation (2.1). Quarterly observations are from the 1971:Q1–2013:Q4 sample.

| | G7 Avg. | G17 Avg. | | G17 Quintile | |
|---|---|---|---|---|---|
| | Simple | Simple | Weighted | $1^{st}$ | $4^{th}$ |
| *Levels Disconnect* | | | | | |
| $corr(\Delta cd_{t+1}, \Delta e_{t+1})$ | -0.14 | -0.11 | -0.13 | -0.19 | -0.04 |
| $corr(\Delta \widehat{cd}_{t+4}, \Delta \widehat{e}_{t+4})$ | -0.14 | -0.17 | -0.14 | -0.29 | -0.05 |
| *Volatility Disconnect* | | | | | |
| $corr(\sigma_t(\Delta cd_{t+1}), \sigma_t(\Delta e_{t+1}))$ | 0.20 | 0.21 | 0.20 | -0.01 | 0.42 |
| $corr(\sigma_t(\Delta \widehat{cd}_{t+4}), \sigma_t(\Delta \widehat{e}_{t+4}))$ | 0.27 | 0.25 | 0.26 | -0.02 | 0.52 |

onal variance operator to both sides of equation (2.7), we get

$$\gamma^2 \cdot Var_t\left(\Delta c_{h,t+1} - \Delta c_{f,t+1}\right) \quad = \quad Var_t\left(\Delta e_{t+1}\right). \tag{2.8}$$

Equivalently, the correlation between the conditional variance of consumption differentials and exchange rate movements should be equal to one.

As shown in the bottom portion of table 2.2.2, empirically this correlation is very modest. We call this novel empirical fact the *volatility disconnect puzzle*. To the best of our knowledge, we are the first ones to both document the existence of this empirical anomaly and address it in the context of a recursive risk-sharing equilibrium.[5]

To summarize, our evidence shows that output volatility shocks decrease relative output and consumption across countries and increase consumption volatility. In relative terms, the effects for the consumption growth rate are smaller than for output growth rates, and the consumption volatility response is larger if output volatility shocks originate in a lar-

[5] In untabulated tests, we estimated a similar level of disconnect also with the volatility of output differentials. These results are available upon request to the authors.

ger country. Equivalently, the pass-through from large to small countries is smaller than the pass-through from small to large countries. Furthermore, we find a strong disconnect between currency volatility and consumption differentials volatility. This empirical finding is an anomaly in the context of a frictionless risk-sharing model with CRRA preferences. In the next section, we develop an economic model that can explain and quantitatively replicate our volatility risk-sharing evidence.

## 2.3. Model

The economy consists of two countries, home ($h$) and foreign ($f$), and two goods, $X$ and $Y$. Agents' preferences are defined over consumption aggregates of the two goods as follows.

**Consumption aggregate.** Let $x_t^i$ and $y_t^i$ denote the consumption of good $X$ and good $Y$ in country $i \in \{h, f\}$ at date $t$. Let $\alpha \in (0, 1)$. The consumption aggregates in the home and foreign countries are

$$C_t^h = \left(x_t^h\right)^{\alpha} \left(y_t^h\right)^{1-\alpha} \quad \text{and} \quad C_t^f = \left(x_t^f\right)^{1-\alpha} \left(y_t^f\right)^{\alpha}, \tag{2.9}$$

respectively. The parameter $\alpha$ captures the degree of bias of the consumption of each representative agent. In what follows we assume that the home country is endowed with good $X$, while the foreign country is endowed with good $Y$. Following some of the international macrofinance articles surveyed by Lewis (2011), we assume that $\alpha$ is larger than 0.5. This allows us to build consumption home bias into the model.

**Preferences.** As in Epstein and Zin (1993), agents' preferences are recursive but not time separable:

$$U_t^i = \left[(1-\delta) \cdot \left(C_t^i\right)^{1-1/\psi} + \delta E_t \left[\left(U_{t+1}^i\right)^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \quad \forall i \in \{h, f\}. \tag{2.10}$$

The coefficients $\gamma$ and $\psi$ measure the relative risk aversion (RRA) and the IES, respectively.

In contrast to the constant RRA case, these preferences allow agents to be risk averse in future utility as well as future consumption. The extent of such utility risk aversion depends on the preference for early resolution of uncertainty, measured by $\gamma - 1/\psi > 0$. To better highlight this feature of the preferences, we focus on the ordinally equivalent transformation

$$V_t = \frac{U_t^{1-1/\psi}}{1-1/\psi}$$

and approximate it with respect to $\theta \equiv \frac{\gamma-1/\psi}{1-1/\psi}$ around $\theta_0 = 1$:

$$\begin{aligned} V_t &= (1-\delta)\frac{C_t^{1-1/\psi}}{1-1/\psi} + \delta E_t\left[V_{t+1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \qquad (2.11) \\ &\approx (1-\delta)\frac{C_t^{1-1/\psi}}{1-1/\psi} + \delta E_t\left[V_{t+1}\right] - \frac{\delta}{2}\frac{\theta}{E_t\left[V_{t+1}\right]}Var_t\left[V_{t+1}\right]. \end{aligned}$$

Note that the sign of $\left(\frac{\theta}{E_t[V_{t+1}]}\right)$ depends on the sign of $(\gamma - 1/\psi)$. When $\gamma = 1/\psi$, the agent is utility-risk neutral and preferences collapse to the standard time-additive case. When the agent prefers early resolution of uncertainty, that is, when $\gamma > 1/\psi$, the coefficient $\theta$ is positive: uncertainty about continuation utility reduces welfare and generates an incentive to trade off future expected utility, $E_t\left[V_{t+1}\right]$, for future utility risk, $Var_t\left[V_{t+1}\right]$.

This mean-variance trade-off is absent when agents have standard time-additive preferences, and it represents the most important element of our analysis, given our focus on the propagation of uncertainty shocks.

Since there is a one-to-one mapping between utility, $U_t^i$, and lifetime wealth, that is, the value of a perpetual claim to consumption, $W_{c,t}^i$,

$$U_t^i = \left[(1-\delta)(C_t^i + W_{c,t}^i)\right]^{\frac{1}{1-1/\psi}}, \quad \forall i \in \{h, f\}, \qquad (2.12)$$

the optimal risk-sharing scheme can also be interpreted in terms of the mean-variance trade-off of wealth. For this reason, in what follows we use the terms "wealth" and "continuation utility" interchangeably.

**Endowments.** We choose to endow each country with a stochastic supply of its most-preferred good. Endowments are specified in the spirit of Colacito and Croce (2013a), with the important difference of accounting also for time-varying risk:

$$\begin{aligned} \Delta \log X_t &= \mu_x + z_{1,t-1} + e^{\sigma_{x,t}/2}\sigma\varepsilon_{x,t} - ci_{t-1} \\ \Delta \log Y_t &= \mu_y + z_{2,t-1} + e^{\sigma_{y,t}/2}\sigma\varepsilon_{y,t} + ci_{t-1}, \end{aligned} \tag{2.13}$$

where the process $ci_t \equiv \tau \log(X_t/Y_t)$ with $\tau \in (0,1)$ introduces cointegration and guarantees the existence of the equilibrium, and the components $z_1$ and $z_2$ are highly persistent AR(1) processes,

$$z_{j,t} = \rho z_{j,t-1} + \sigma_z \varepsilon_{j,t}, \forall j \in \{1,2\}. \tag{2.14}$$

Throughout the paper, we refer to $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ as long-run shocks, due to their long-lasting impact on the growth rates of the two endowments. Similarly, we call $\varepsilon_{x,t}$ and $\varepsilon_{y,t}$ short-run shocks.

We focus on time-varying short-run risk, as captured by the following process:

$$\sigma_{j,t} = \rho_\sigma \sigma_{j,t-1} + \sigma_{sr}\varepsilon_{\sigma j,t}, \forall j \in \{x,y\}. \tag{2.15}$$

Shocks are jointly log-normal:

$$\xi_t \equiv \left[ \begin{array}{cccccc} \varepsilon_{1,t} & \varepsilon_{2,t} & \varepsilon_{x,t} & \varepsilon_{y,t} & \varepsilon_{\sigma 1,t} & \varepsilon_{\sigma 2,t} \end{array} \right] \sim \quad i.i.d.N(\mathbf{0}, \Sigma),$$

and the matrix $\Sigma$ is assumed to be block-diagonal to allow for cross-country correlation of shocks of the same type.

**Markets.** At each date, trade occurs in a complete set of one-period-ahead claims to state-contingent consumption. Financial and goods markets are assumed to be frictionless.

The budget constraints of the two agents can be written as

$$
\begin{aligned}
x_t^h + p_t y_t^h + \int_{\zeta^{t+1}} A_{t+1}^h \left(\zeta^{t+1}\right) Q_{t+1}(\zeta^{t+1}) &= A_t^h + X_t \\
x_t^f + p_t y_t^f + \int_{\zeta^{t+1}} A_{t+1}^f \left(\zeta^{t+1}\right) Q_{t+1}(\zeta^{t+1}) &= A_t^f + p_t Y_t,
\end{aligned}
\tag{2.16}
$$

where $p_t$ denotes the relative price of goods $X$ and $Y$ (the terms of trade), $A_t^i\left(\zeta^t\right)$ denotes country $i$'s claims to time $t$ consumption of good $X$, and $Q_{t+1}(\zeta^{t+1})$ gives the price of one unit of time $t+1$ consumption of good $X$ contingent on the realization of $\zeta^{t+1}$ at time $t+1$. In equilibrium, the market for international state-contingent claims clears, implying that $A_t^h + A_t^f = 0, \forall t$.

**Prices.** The stochastic discount factor in consumption aggregate units is

$$
M_{t+1}^i = \delta \left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}^{i1-\gamma}}{E_t\left[U_{t+1}^{i1-\gamma}\right]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{2.17}
$$

Since markets are assumed to be complete, the log growth rate of the real exchange rate is

$$
\Delta e_t = \log M_t^f - \log M_t^h \tag{2.18}
$$

and the relative price of the two goods is $p_t = \frac{(1-\alpha)x_t^h}{\alpha y_t^h}$.

**Allocations.** Under complete markets, we can compute efficient allocations by solving the associated Pareto problem. The planner attaches date 0 nonnegative Pareto weights $\mu^h = \mu$ and $\mu^f = 1-\mu$ to the consumers and chooses the sequence of allocations $\left\{x_t^h, x_t^f, y_t^h, y_t^f\right\}_{t=0}^{+\infty}$ to maximize

$$
\Lambda = \mu \cdot U_0^h + (1-\mu) \cdot U_0^f,
$$

subject to the following sequence of economy-wide feasibility constraints:

$$\begin{aligned} x_t^h + x_t^f &= X_t \\ y_t^h + y_t^f &= Y_t, \quad \forall t \geq 0, \end{aligned}$$

where the state-dependent notation is omitted for the sake of clarity. In characterizing the equilibrium, we follow Anderson (2005) and formulate the problem using the ratio of time-varying pseudo-Pareto weights, $S_t = \mu_t/(1-\mu_t)$, as an additional state variable. This technique enables us to take into account the nonseparability of the utility functions.

The first-order necessary conditions imply the following allocations:

$$\begin{aligned} x_t^h &= \alpha X_t \left[1 + \frac{(1-\alpha)(S_t-1)}{1-\alpha+\alpha S_t}\right], \qquad x_t^f = (1-\alpha)X_t \left[1 - \frac{\alpha(S_t-1)}{1-\alpha+\alpha S_t}\right] \\ y_t^h &= (1-\alpha)Y_t \left[1 + \frac{\alpha(S_t-1)}{\alpha+(1-\alpha)S_t}\right], \qquad y_t^f = \alpha Y_t \left[1 - \frac{(1-\alpha)(S_t-1)}{\alpha+(1-\alpha)S_t}\right], \end{aligned} \tag{2.19}$$

where

$$S_t = S_{t-1} \cdot \frac{M_t^h}{M_t^f} \cdot \left( \frac{C_t^h / C_{t-1}^h}{C_t^f / C_{t-1}^f} \right), \quad \forall t \geq 1 \tag{2.20}$$

and $S_0 = 1$, as we start the economy from an identical allocation of wealth and endowments. This is consistent with the ergodic distribution of the model, which implies that on average the two countries consume an identical share of world resources because of symmetry.

We make three remarks. First, $S_t$ is a key driver of the share of world consumption allocated to the home country, $SWC_t$,

$$SWC_t = \frac{x_t^h + p_t y_t^h}{X_t + p_t Y_t} = \frac{S_t}{1+S_t}. \tag{2.21}$$

The higher $S_t$ is, the larger is the home country. Second, as in Colacito and Croce (2013a), when the home country receives good news for the endowment of good $X$, there is a persistent reduction in the domestic share of world consumption. This countercyclical adjustment is consistent with equation (2.20): as good news for the supply of good $X$ relative to good $Y$ materializes, the home country experiences a drop in its marginal utility. Therefore, it is optimal to reallocate resources to the foreign country. In the decentralized economy, the home country optimally substitutes part of its current consumption with exports to its foreign trading partner. Third, $S_t$ introduces an endogenous time-varying volatility term into consumption growth, since allocations are nonlinear functions of this component. In section 2.4.3, we discuss the importance of this channel in the context of our explanation of the volatility disconnect anomaly.

### *2.3.1. Calibration and Solution Method*

We report our benchmark calibration in table 2.3.1. Panel A refers to parameters that have already been employed in this class of models and are standard in the literature (see, among others , Colacito and Croce (2011a), Colacito and Croce (2013a) and Bansal and Shaliastovich (2013)).

We set the intertemporal elasticity of substitution to 1.5, as in Colacito and Croce (2013a). Because of the presence of volatility risk, we can obtain a volatile stochastic discount factor with a risk aversion coefficient of 7, a value particularly conservative in this literature and applicable also to countries other than the US (Choi et al. (2014)).The subjective discount factor is chosen so as to keep the average annual risk-free rate close to 1% when possible.

The consumption home bias is set to 0.96, a number that falls in the middle of the range observed for our countries. For example, in our sample the US home bias is 0.95, as imports comprise an average of 5% of US consumption goods. (Erceg et al. (2008)). Balta and Delgado (2009)document a stronger consumption home bias for the European countries in our dataset and suggest a value of $\alpha = 0.97$. Setting $\lambda = 0.97$ would improve our quantitative results, as it would make our risk-sharing channel even more relevant. We prefer to work with $\alpha = 0.96$ in order to obtain conservative results.

Annualized average output growth is set to 2%, consistent with the empirical findings in

Table 2.4: Calibration

Notes: All parameters are calibrated at quarterly frequency. In panel B, the entries for the data are from the VAR specified in equations (2.5)–(2.6). Numbers in brackets denote the 95% credible intervals. Data are from the OECD dataset and refer to G-17 countries. The sample spans the post–Bretton Wood period, 1971:q1–2013:q4.

| Description | Parameter | Value |
|---|---|---|
| **Panel A: Standard Parameters** | | |
| Relative Risk Aversion | $\gamma$ | 7 |
| Intertemporal Elasticity of Substitution | $\psi$ | 1.50 |
| Subjective Discount Factor | $\delta^4$ | 0.98 |
| Degree of Home Bias | $\alpha$ | 0.96 |
| | | |
| Mean of Endowment Growth | $\mu \cdot 4$ | 2.00% |
| Short-Run Risk Volatility | $\sigma \cdot \sqrt{4}$ | 1.87% |
| Long-Run Risk Autocorrelation | $\rho^4$ | 0.953 |
| Relative Long-Run Risk Volatility | $\sigma_z/\sigma$ | 6.90% |
| | | |
| Cross-correlation of Short-Run Shocks | $\rho_X$ | 00.15 |
| Cross-correlation of Long-Run Shocks | $\rho_z$ | 00.92 |
| | | |
| **Panel B: Time-Varying Short-Run Risk** | | |
| Persistence of Short-Run Volatility | $\rho_\sigma$ | 0.90 |
| | | [0.89; 0.93] |
| Volatility of Short-Run Volatility | $\sigma_{sr}$ | 0.15 |
| | | [0.15; 0.16] |
| Cross-correlation of Short-Run Volatility | $\rho_{\sigma,\sigma^*}$ | 0.30 |
| | | [0.13; 0.45] |
| Short-Run Volatility Correlation with | $\rho_{\sigma,\Delta y}$ | -0.12 |
| Short-Run Shocks | | [-0.15; -0.05] |

table 2.1. Unconditional volatilities are calibrated to produce an unconditional output volatility of 1.90%, as in the data. The long-run components are calibrated in the spirit of the international long-run risk literature, as they are both highly persistent and correlated across countries (Colacito and Croce (2011, 2013)). Since we set $\sigma_z/\sigma = 0.07\%$, the implied consumption growth rate is almost *i.i.d.*, as in the data. Short-run output growth shocks, in contrast, are as poorly cross-country correlated as output growth in our dataset (see table 2.1).

In table 2.3.1, panel B, we report the parameters that govern the volatility process of short-run shocks, that is, the novel and most important element of our investigation. These parameters are calibrated to be consistent with our empirical results. Specifically, we pick values typically in the middle of the Bayesian 95% credible intervals of the VAR system specified in equations (2.5)–(2.6).

Consistent with our data, volatility shocks are as poorly correlated across countries as short-run growth shocks. We allow for negative within-country correlation between volatility and short-run growth shocks so that higher volatility is associated with economic slowdowns. Conditional volatilities are as persistent as in the data.

Given these parameters, we use perturbation methods to solve our system of equations. We compute an approximation of the third order of our policy functions using the dynare++ package. As documented in Colacito and Croce (2012), a third-order approximation is required to capture endogenous time-varying volatility due to the adjustments of the pseudo-Pareto weights. All variables included in our dynare++ code are expressed in log-units.

Both the calibration and the solution methods are standard in the literature. In what follows we discuss only the performance of our model for the dynamics of conditional volatilities, that is, the main objective of our investigation. For commonly targeted unconditional moments, we refer the reader to table A.2.3 in the appendix. For the sake of completeness, this table also shows the same moments for the case in which we abstract away from volatility shocks, and for the setting with CRRA preferences.

## 2.4. Main Results

In this section, we present the main results of our theoretical analysis. We start by describing the risk-sharing motives of both level and volatility shocks. To our knowledge, we are the first to connect recursive risk sharing to evidence on consumption volatility dynamics both within country and in the cross section of countries. We then assess the quantitative performance of our model by means of simulations and show that a frictionless recursive risk-sharing scheme can rationalize our empirical findings.

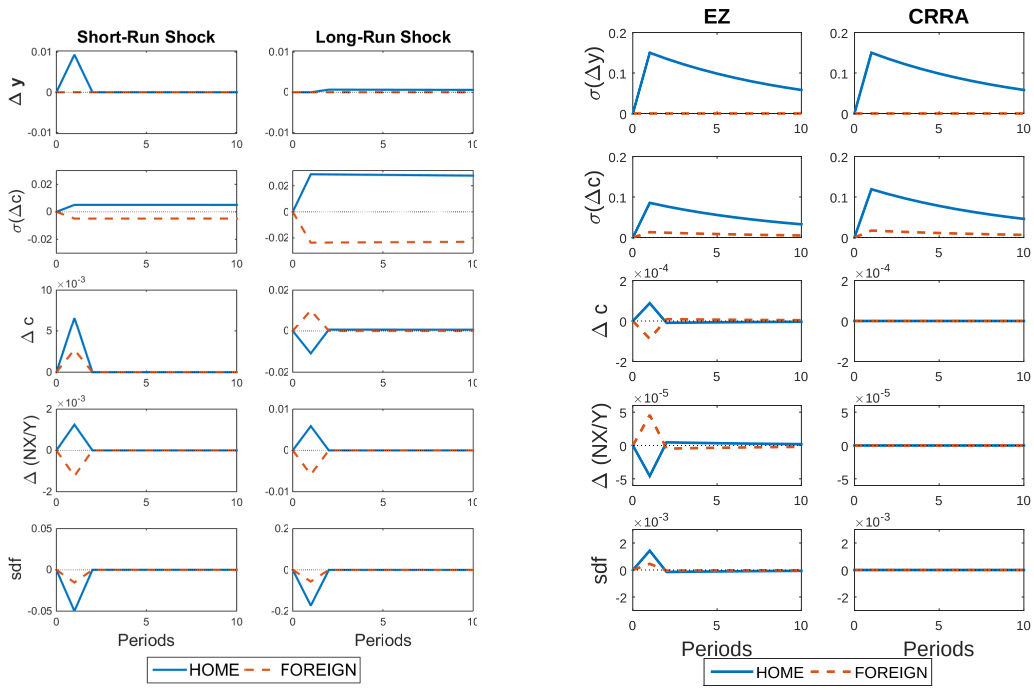



Figure 2.3: Impulse Responses

Panel (a) shows the percentage impulse response functions of output growth ($\Delta y$), consumption growth volatility ($\sigma(\Delta c)$), consumption growth ($\Delta c$), change of net-export–output ratio ($\Delta NX/Y$), and stochastic discount factors ($sdf$) to a shock to the home endowment for both the home country (solid line) and the foreign country (dashed line). Level shocks materialize only in the home country, and only at time 1. Shocks are not orthogonalized; we consider a positive $\sigma$ shock in the short-run, and a positive $\sigma_x$ shock for the long-run. In panel (b) we consider an endowment volatility shock which is orthogonalized within and across countries, i.e., it affects only the home country and it does not change the growth rate level. All parameters are calibrated to the quarterly values reported in Table 2.3.1.

### *2.4.1. Risk-Sharing Motives*

**Risk sharing of level shocks.** In figure 2.3, we report the response of the variables of interest to a short-run level shock (left panels) and to a long-run level shock (right panels) to the growth rate of the endowment of the home country. Note that on impact the short-run shock is sizeably larger than the long-run shock (figure 2.3, first row of panels). However, the long-run shock is highly persistent, and it ultimately affects the growth rate of the home endowment for a large number of periods.

Consistent with Colacito and Croce (2013a), the growth rates of consumption increase in both countries in response to a positive short-run shock, whereas they move in opposite directions in response to a positive long-run shock (figure 2.3, third row of panels). The asymmetric response of consumption growth rates to a long-run endowment shock is the result of the agents' extreme sensitivity to persistent news to the growth rates of their endowments.

When a shock of this nature materializes, the home country's marginal utility drops substantially (figure 2.3, bottom-right panel). To restore the equality of the marginal utilities of consumption across countries, an international redistribution of resources must take place. Specifically, the home country increases its exports, while the foreign country increases its imports (figure 2.3, fourth row of panels). Equivalently, the ratio of the pseudo-Pareto weights $S_t$ declines, as dictated by equation (2.20).

Since the long-run shock is a pure news shock, that is, a shock that results in a larger amount of home endowment only in future time periods, the international redistribution of resources takes place through a drop in home consumption and an increase in foreign consumption. As pointed out in Colacito and Croce (2013a), this immediate response of the consumption level simultaneously comes with an opposite swap of long-run consumption variance (as measured by $\sigma_t(U_{t+1})$). Specifically, the home country optimally reduces its current consumption share, $S_t/(1-S_t)$, in exchange for a reduction in $\sigma_t(U_{t+1})$ (see Figure 2.4, top-right panel). Consistent with equation (2.11), the reduction of long-term uncertainty improves welfare.

**The volatility frontiers.** Given our interest in the volatility pass-through and in the volatility disconnect anomaly, we pay particular attention to the response of the volatility of consumption growth rates, $\sigma_t(\Delta c^i_{t+1})$, to the three sources of risk that are present in the economy. Without loss of generality, we focus on the conditional volatility of the growth rate of consumption of the home country.

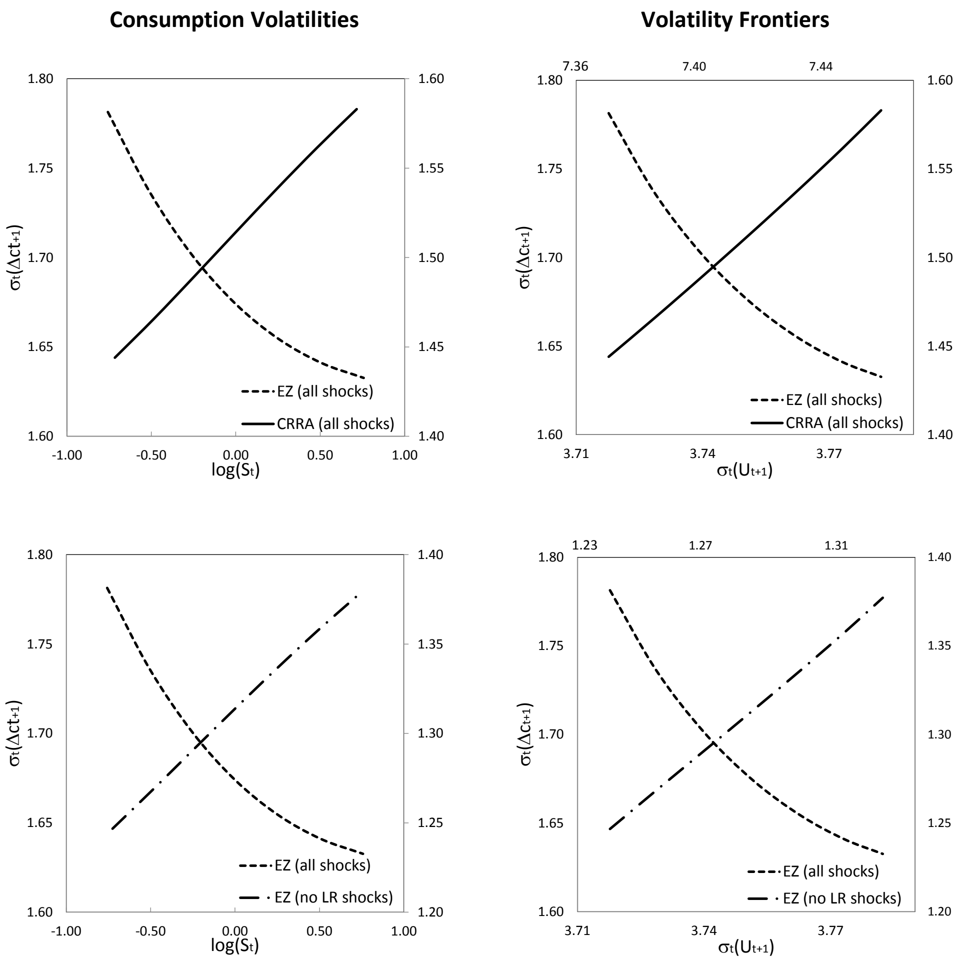



Figure 2.4: Variance Frontiers

The left panels report the the conditional volatility of the growth rate of consumption in the home country ($\sigma_t\left(\Delta c_{h,t+1}\right)$) as a function of the logarithm of the ratio of pseudo-Pareto weights ($s_t$). In the right panels we replace the logarithm of the ratio of pseudo-Pareto weights with the associated conditional volatility of the normalized continuation utility in the home country (i.e., $\sigma_t\left(U_{h,t+1}/(X_t^{\alpha}Y_t^{1-\alpha})\right)$). In the left panels, the left (right) axis reports the values for our benchmark (alternative) calibration. In the right panels, the values for our benchmark (alternative) calibration are reported on the left and bottom (top and right) axes. Across all cases, we keep both the exogenous long-run components and the exogenous volatility processes fixed at their unconditional mean.

The consumption growth in the home country can be expressed in terms of the primitive endowment processes and the share dynamics:

$$\Delta c_{t+1} = \Delta c_{t+1}^{aut} + f(S_{t+1}) - f(S_t), \tag{2.22}$$

where $\Delta c_{t+1}^{aut} := \alpha \Delta X_{t+1} + (1-\alpha)\Delta Y_{t+1}$ is the consumption growth rate that would prevail under financial autarky, and

$$f(S) := \log\left(\frac{\alpha}{1-\alpha}\right)^{2\alpha-1} + \log\left[\frac{S}{\left(1+\frac{\alpha}{1-\alpha}S\right)^{\alpha}\left(1+\frac{1-\alpha}{\alpha}S\right)^{1-\alpha}}\right]$$

captures the effects of relative size, as measured by $S_t$[6]. Note that under financial autarky there is no dynamic redistribution of resources across countries, that is, the reallocation effect is absent. Also, it can be easily shown that $f' > 0$, $f'' < 0$, $f''' > 0$, and that $\lim_{S\to\infty} f' = 0$. Equivalently, the $f$ function is increasing and concave in size, $S$, and it is relatively flatter (steeper) for larger (smaller) countries.

A first-order approximation of consumption growth at date $t+1$ about date $t$'s ratio of pseudo-Pareto weights yields

$$\begin{aligned}\sigma_t^2(\Delta c_{t+1}) \quad &\approx \quad \sigma_t^2(\Delta c_{t+1}^{aut}) + \left[f'(S_t)\right]^2 \sigma_t^2(S_{t+1}-S_t), \\ &\quad +2\cdot cov_t(\Delta c_{t+1}^{aut}, f'(S_t)\cdot(S_{t+1}-S_t)).\end{aligned} \tag{2.23}$$

The variance of consumption growth is thus driven by the variation in the fundamental endowment processes, variation in size, and the covariance between the two. To help illustrate the economic role of these channels, we consider two polar cases.

In the first case, assume that either there are no news shocks (i.e., $\sigma_\sigma = \sigma_z = 0$) or they are not priced (i.e., agents have CRRA preferences). Let the risk aversion coefficient be strictly greater than one, meaning that the risk-sharing motive is strong enough. The covariance term in equation (2.23) is negative for $S \in (0,\infty)$ because the size of the home country increases upon the arrival of a relative negative shock (see equation (2.20)). Furthermore, this negative covariance is greater than $[f'(S_t)]^2 \cdot \sigma_t^2(S_{t+1}-S_t)$ because the volatility of consumption under complete markets, $\sigma_t(\Delta c_{t+1})$, is smaller than the volatility of consumption

[6]This result is obtained from equations (2.9) and (2.19). See Cole and Obstfeld (1991) and Colacito and Croce (2013a) for the derivations.

under portfolio autarky, $\sigma_t(\Delta c_{t+1}^{aut})$. This is a common prediction of frictionless risk-sharing models (see, inter alia, Cole and Obstfeld (1991)).

As a result, with respect to only short-run shocks: (i) the reallocation channel *reduces* consumption growth volatility; (ii) this effect is stronger for smaller countries, since $f'' < 0$; and (iii) the consumption volatility frontier is *upward* sloping with respect to country size. These findings are consistent with the model of Hassan (2013), in which small countries feature a lower consumption volatility than large countries.

The second extreme case that we consider is the one in which there are only news shocks. By definition, pure news shocks realized at time $t+1$ do not change the level of $\Delta c_{t+1}^{aut}$. As a consequence, the covariance term in equation (2.23) is null. If news shocks are priced, they promote an international reallocation of resources at time $t+1$, implying that the conditional volatility of $S_{t+1}$ is strictly larger than zero. Equivalently, the reallocation channel *increases* the conditional volatility of consumption. Since $f'' < 0$, the intensity of this channel is stronger for small countries. Thus, the volatility of consumption growth inherits the properties of $f'(S)$ and the frontier is *downward* sloping in country size.

In our benchmark calibration with recursive preferences, news shocks are of first-order importance for the international reallocation of resources, which explains why our equilibrium consumption volatility frontier is downward sloping (figure 2.4, top-left panel). As a result, if a country experiences either a positive short-run shock or a positive long-run shock, its relative size decreases and the volatility of its consumption growth rate rises, whereas the opposite holds for the other country (figure 2.3, second row).

Since the recursive risk-sharing mechanism of this economy is characterized by the agents' willingness to trade off size for a smoother future consumption profile, $\sigma_t(U_{t+1})$, savings are dynamically adjusted to achieve long-run consumption smoothing at the cost of increasing short-run consumption volatility, $\sigma_t(\Delta c_{t+1})$ (figure 2.4, top-right panel).

Consistent with our analysis of equation (2.23), this trade-off is absent when we use CRRA preferences, as news shocks are not priced and hence the consumption volatility frontier is upward sloping (figure 2.4, top panels, solid lines). This trade-off also disappears when we retain recursive preferences but remove long-run shocks. In this case, short-run level shocks dominate and the consumption volatility frontier is again upward sloping (figure 2.4, bottom panels), implying that long-run consumption smoothing coexists with smoother short-run consumption profiles.

A second important insight from figures 2.3 and 2.4 is that the absolute change in volatility for the country that is affected by a positive level shock (i.e., negative adjustment in its pseudo-Pareto weight) is larger than the absolute change in the volatility for the other country. Equivalently, the short-run consumption volatility frontier is convex, due to the convexity of $f'$ (recall that $f''' > 0$). As a result, the variation of the share of world consumption produces greater variability in the consumption aggregate of smaller countries, an important feature that allows us to explain our evidence on the volatility pass-through.

**Risk sharing of vol shocks.** Figure 2.3 shows the response of our main set of variables of interest to a volatility shock in the home country. For comparability, we report the responses from both our benchmark model and a model with standard time-additive CRRA preferences.

We first point out that the responses of consumption, net exports, and stochastic discount factors in the model with EZ preferences are the mirror image of those obtained for a positive long-run endowment shock, since a positive volatility shock is a negative news shock.

Second, we note that the relative response of the volatilities of consumption growth rates in the two countries differs across the two preference specifications. With CRRA preferences, volatility news shocks are not directly priced and hence marginal utilities do not move. There is no reallocation of resources across countries, and as a result the increase in volatility of the domestic endowment is almost entirely absorbed by domestic consumption. According to our definition of volatility pass-through, in this situation our index takes on a value close

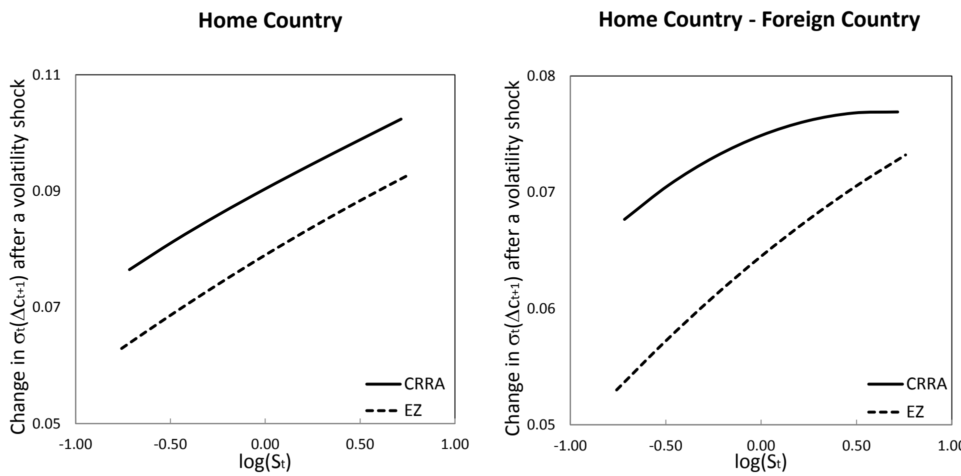



Figure 2.5: Response to an Adverse Volatility Shock to Good $X$.
The left panel reports the change in the conditional volatilities of consumption growth in the home country after an adverse shock to the volatility of the good $X$. The right panel reports the change in the cross-country difference of conditional volatility of consumption growth for the same shock. In each panel, the dashed (solid) line refers to the case of EZ (CRRA) preferences. Across all cases, we keep all other exogenous state variables fixed at their unconditional mean.

to zero.

In the specification with EZ preferences, the risk-sharing motive that we described in the previous section partially offsets the increased amount of volatility in the economy. As depicted in the left panel of figure 2.5, upon the arrival of an adverse volatility shock, the short-run consumption volatility frontier of the home country shifts upward by a lesser extent than under CRRA preferences. Since our agents are averse to conditional variance, their trade is arranged to reduce the time variation of their own conditional volatilities. Furthermore, these countries tend to keep their volatilities aligned to each other. As shown in the right panel of figure 2.5, the cross-country difference of the conditional volatilities increases by a smaller amount under EZ preferences than in the CRRA setting. We address this finding in further detail in our discussion of our volatility pass-through results below.

### 2.4.2. Volatility Comovements and Pass-through

**Unconditional comovements.** We use simulations to quantify the ability of the model to reproduce our empirical findings for the dynamics of volatility. We find it useful to consider first a setting with standard CRRA preferences, as in this case shocks do not produce any sizeable endogenous reallocation of volatility from one country to the other. Equivalently, the pseudo-Pareto weights are almost constant (Colacito and Croce (2013a)), similarly to the autarky scenario in Cole and Obstfeld (1991). In this situation, the correlation between consumption and output volatility within each country is almost perfect, as the volatility of consumption moves one-to-one with the volatility of the output growth rate.

Table 2.5: Comovements and Pass-Through

Notes: In panel A, we report correlations between the conditional volatility ($\sigma_t$) of consumption and output growth within and across countries. Conditional volatilities are obtained by estimating equation (2.1) country by country. The data refer to G-17 countries and are described in section 2.2.1. Panel B reports estimated pass-through coefficients (see equation (2.4)) with respect to both domestic (US) and foreign volatility shocks for both the G7 and bottom-10 G17 countries. SWC denotes the share of world consumption, $S/(1+S)$, keeping the US as home country. For each country, we compute the moments of interest over the post–Bretton Wood period, 1971:Q1–2013:Q4. For each moment, we report first and fourth cross-country quintiles. The entries from the model are obtained from 100 repetitions of small samples. Our benchmark quarterly calibration is reported in table 2.3.1.

**Panel A: Unconditional Moments**

| | Avg. | Quintiles $[1^{st}; 4^{th}]$ | Bench-mark | No TVV ($\sigma_\sigma = 0$) | CRRA ($\gamma = 7$) |
|---|---|---|---|---|---|
| $corr(\sigma_t(\Delta c_{t+1}), \sigma_t(\Delta y_{t+1})$ | 0.65 | [0.26; 0.80] | 0.88 | – | 0.98 |
| $corr(\sigma_t(\Delta c_{t+1}), \sigma_t(\Delta c^*_{t+1}))$ | 0.45 | [0.35; 0.66] | 0.35 | -0.93 | 0.50 |

**Panel B: Pass-through and size**

| | SWC | US vol shock | Foreign vol shock |
|---|---|---|---|
| *US/G7 Countries:* | | | |
| Data | [0.44; 0.51] | [0.43; 0.54] | [0.51; 0.63] |
| Model (EZ) | 0.50 | 0.53 | 0.53 |
| Model (CRRA) | 0.50 | 0.30 | 0.30 |
| *US/Bottom-10 G17 Countries:* | | | |
| Data | [0.72; 0.77] | [0.45; 0.57] | [0.66; 0.78] |
| Model (EZ) | 0.72 | 0.39 | 0.70 |
| Model (CRRA) | 0.72 | 0.38 | 0.37 |

The international correlation of the consumption volatilities is 50%, a number slightly higher than that observed in the data. Recall that the exogenous international correlation of the output volatility shocks is set to 30%. Since the pseudo-Pareto weights are almost fixed under CRRA, the log-consumption bundles in each country are a constant weighted average of the two goods, and hence their volatilities are more correlated than those of the two underlying endowment processes.

In contrast to the CRRA case, our model with recursive preferences is able to produce a less-than-perfect contemporaneous correlation between output and consumption volatility. This result is driven by the fact that level shocks are an important endogenous driver of consumption volatility independently of our exogenous output volatility shocks. As shown in table 5, without volatility shocks, the cross-country correlation of the consumption profiles would be almost perfectly negative. This channel counterbalances the tendency for consumption profiles to be more correlated than output. At the equilibrium, our model produces a final correlation of 35%, a figure that is well within the confidence region of our cross section of countries.

**Pass-through.** Overall, unconditional comovements do not allow us to discriminate between the CRRA and EZ settings, as both models produce results that lie within the empirical ranges. This conclusion changes when we focus on conditional responses and, in particular, on our pass-through index. When we compare countries of similar size, that is, the US versus the remaining G7 countries, only the model with EZ preferences generates a pass-through of 50%, as in the data. This result is particularly relevant because it is obtained with a simultaneous response of the current account that replicates that observed in the data, as shown in figure 2.2.2. Under CRRA preferences, however, the pass-through is very limited, as volatility news shocks are not an independent determinant of risk-sharing motives.

Furthermore, when we alter the relative consumption share in the model and set the home country consumption to be about three times larger than that of the foreign country, as in the comparison between the US and the bottom-10 G17 countries, our model can replicate the asymmetry documented in the data. Specifically, our model predicts that when a volatility shock hits a big country, the pass-through is limited. Vice versa, small countries can better share shocks to their endowment volatility as documented by their higher pass-through. This result follows directly from the convexity of the short-run volatility frontier depicted in the top-left panel of figure 2.4 for the case of EZ preferences.

In the CRRA case, however, the relative country size does not play any major role in determining the extent of volatility pass-through, a result that is at odds with the data. In the next subsection, we show that a model with CRRA preferences fails also to explain the volatility disconnect in our international data set.

*2.4.3. Risk Sharing and the Volatility Disconnect Anomaly*

In table 2.4.3, we compare our empirical findings on the disconnect between exchange rates and consumption differentials to our simulation results. In the top panel, we show that our benchmark model is able to replicate the slightly negative correlation between consumption growth differentials and exchange rate movements observed in the data over both at a quarterly and an annual horizon. As in the model with constant volatility (Colacito and Croce (2013a), news shocks are sufficient to break the perfect correlation of the consumption differentials and the exchange rate. Consistent with the observation in Backus and Smith (1993a), under CRRA preferences this correlation is counterfactually high.

Most importantly, with CRRA preferences also the correlation between the conditional variance of consumption differentials and exchange rate movements is one (bottom portion of table 2.4.3, rightmost column). This outcome is at odds with the findings of our empirical investigation and can be explained by looking at the right panels of figure 2.6: regardless of whether a volatility shock, a short-run shock, or a long-run shock hits the economy, the two sets of volatilities are always characterized by a perfect degree of comovement.

In our benchmark model, the behavior of the volatilities instead depends on the nature of

Table 2.6: Volatility Disconnect Anomaly and Risk Sharing

Notes: This table reports key moments for real consumption growth differentials ($\Delta cd = \Delta c - \Delta c^*$) and exchange rate growth ($\Delta e$). Foreign variables are marked by '*'; cumulative growth rates are denoted by '$\widehat{\ }$'. Conditional log-volatilities are denoted by $\sigma_t$. The empirical moments are obtained by estimating equation (2.1) country by country, as detailed in section 2.2.2. The data refer to G-17 countries and are described in section 2.2.1. For each country, we compute the moments of interest over the post–Bretton Wood period, 1971:Q1–2013:Q4, as detailed in section 2.2.1. For each moment, we report (i) its GDP-weighted average across countries; and (ii) its first and fourth cross-country quintiles. The entries from the model are obtained from 100 repetitions of small samples. Our benchmark quarterly calibration is reported in table 2.3.1.

| | G-17 Data | | Model | | |
|---|---|---|---|---|---|
| | Aver. | Quintiles $[1^{st}; 4^{th}]$ | Bench-mark | No TVV ($\sigma_\sigma = 0$) | CRRA ($\gamma = 7$) |
| *Levels Disconnect* | | | | | |
| $corr(\Delta cd_{t+1}, \Delta e_{t+1})$ | -0.13 | [ -0.19; -0.04 ] | -0.25 | -0.27 | 1.00 |
| $corr(\Delta \widehat{cd}_{t+4}, \Delta \widehat{e}_{t+4})$ | -0.14 | [ -0.29; -0.05 ] | -0.21 | -0.24 | 1.00 |
| *Volatility Disconnect* | | | | | |
| $corr(\sigma_t(\Delta cd_{t+1}), \sigma_t(\Delta e_{t+1}))$ | 0.20 | [ -0.01; 0.42 ] | 0.56 | -0.75 | 1.00 |
| $corr(\sigma_t(\Delta \widehat{cd}_{t+4}), \sigma_t(\Delta \widehat{e}_{t+4}))$ | 0.26 | [ -0.02; 0.52 ] | 0.47 | -0.75 | 1.00 |

the shock hitting the economy (figure 2.6, left panels). Specifically, a volatility shock in the home country is still characterized by a positive comovement between exchange rates and differential of consumption growth rates. This is because the two countries share the risk associated with an increase in macroeconomic uncertainty, as explained in the previous section.

We note that short-run shocks are irrelevant in this context, as they result in a negligible response of the two volatilities, since investors' marginal utilities are not particularly sensitive to this type of shock (figure 2.6, middle-left panel). In contrast, a long-run shock to the home country generates a significant negative comovement between the two volatilities and lowers their unconditional correlation (figure 2.6, bottom-left panel). Over an annual horizon, this channel enables our model to produce a correlation well within our empirical range (table 2.4.3, bottom two lines).

To explain the origin of this negative comovement, it is useful to decompose the variance

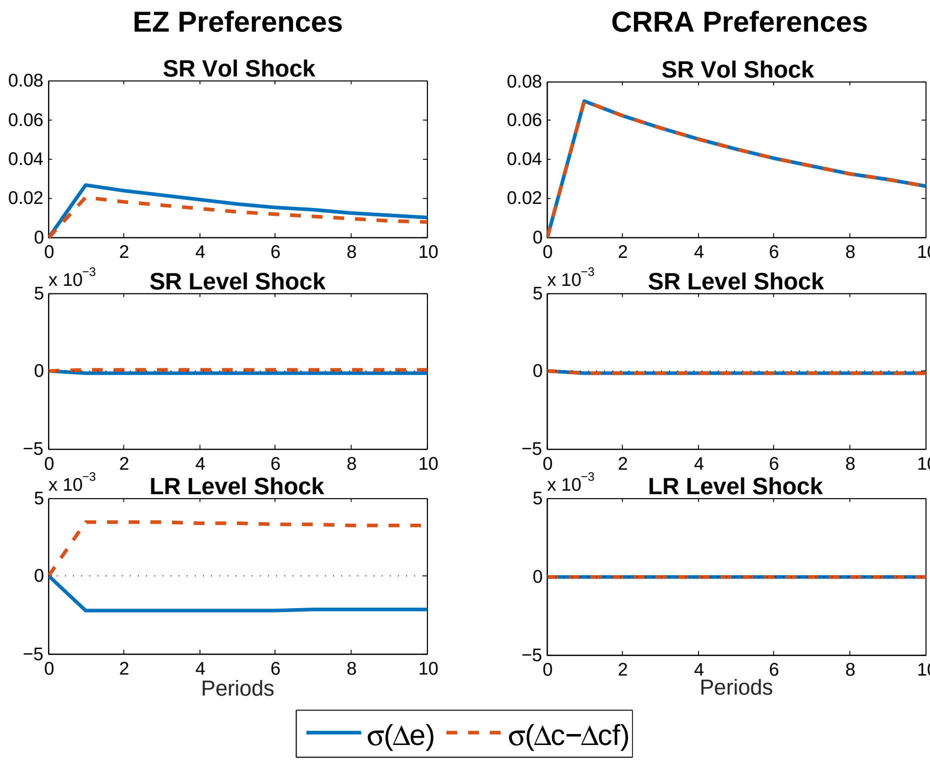



Figure 2.6: Impulse Response Functions and Volatility Disconnect
This figure shows the percentage response of the volatility of consumption growth differentials (dashed line) and exchange rate growth rate volatility (thick line) to a volatility shock in the home country (top panels), a short-run shock in the home country (middle panels), and a long-run shock in the home country (bottom panels). The left (right) panels report the response functions for our benchmark model with EZ (CRRA) preferences.

of the consumption differential growth rate into its subcomponents:

$$\begin{aligned} Var_t(\Delta c_{t+1} - \Delta c^*_{t+1}) &= Var_t(\Delta c_{t+1}) + Var_t(\Delta c^*_{t+1}) \\ &\quad -2 \cdot \sqrt{Var_t(\Delta c_{t+1}) \cdot Var_t(\Delta c^*_{t+1})} \cdot corr(\Delta c_{t+1}, \Delta c_{t+1}). \end{aligned} \tag{2.24}$$

At the equilibrium, the conditional correlation of consumption growth rates is almost time invariant.[7] As a result, the dynamics of the variance of consumption differentials is mostly

[7]This correlation is driven by the positive comovement between the short-run shock of a country and the adjustment in the share of consumption of the other country. In equilibrium, this correlation increases

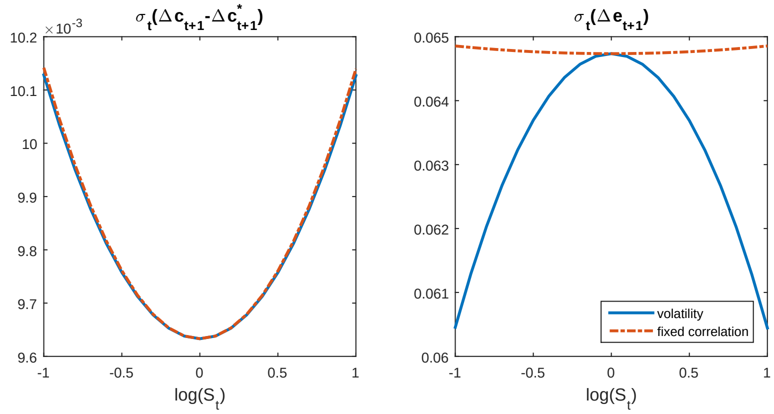



Figure 2.7: Conditional Volatilities Disconnect

The left panel plots the conditional volatility of the difference between the growth rate of consumption in the home and foreign countries, $\sigma_t\left(\Delta c_{t+1} - \Delta c^*_{t+1}\right)$. The right panel depicts the conditional volatility of the growth rate of the exchange rate, $\sigma_t\left(\Delta e_{t+1}\right)$. Both volatilities are plotted against the logarithm of the ratio of the pseudo-Pareto weights, $S_t$. Across all cases, both the exogenous long-run components and the exogenous volatility processes are fixed at their unconditional mean. In each panel, the solid line refers to the conditional volatility obtained at the equilibrium, whereas the dashed line refers to the conditional volatility obtained by holding the correlations fixed at their unconditional mean in equations (2.24)–(2.25).

determined by the sum of the variances of the consumption growth rates across countries, as depicted in the left panel of figure 2.7.

Because of the convexity of the short-run volatility frontier (Figure 2.4, top-left panel, dashed line), the sum of the variances of the growth rates of consumption is increasing in wealth inequality, that is, it is U-shaped with respect to the log-ratio of the Pareto weights (figure 2.7, left panel). As a result, starting from an equal distribution of wealth, $\sigma_t(\Delta c_{t+1} - \Delta c^*_{t+1})$ increases upon the arrival of a long-run shock (figure 2.6, bottom-left panel).

Given our assumption of complete markets, the variance of the exchange rate growth can

modestly in wealth inequality.

be decomposed as follows:

$$\begin{aligned} Var_t\left(\Delta e_{t+1}\right) &= Var_t(\Delta m_{t+1} - \Delta m^*_{t+1}) = Var_t(\Delta m_{t+1}) + Var_t(\Delta m^*_{t+1}) \\ &\quad -2\sqrt{Var_t(\Delta m_{t+1}) \cdot Var_t(\Delta m^*_{t+1})} \cdot corr_t(\Delta m_{t+1}, \Delta m_{t+1}). \end{aligned} \quad (2.25)$$

In a model with long-run growth news, most of the volatility of the stochastic discount rates is driven by the continuation utilities. As shown in the top-right panel of figure 2.4, the utility variance frontier is linear, meaning that the drop in the conditional volatility of the utility of one country is almost entirely offset by the increase in volatility of the other country. As a result, $Var_t(\Delta m_{t+1}) + Var_t(\Delta m^*_{t+1})$ is close to being time invariant and the conditional volatility of the exchange rate is mostly explained by the endogenous time variation in the correlation of the stochastic discount factors (figure 2.7, right panel).

An example can help clarify the dynamics of this adjustment. When a country receives good news for the long run, its utility increases immediately, reflecting the total discounted impact of the news. The other country benefits from the international redistribution of resources, which determines an increase in its share of consumption. Given the persistent nature of the consumption shares, the other country also experiences an increase in the present value of its consumption and, thus, its utility. As a consequence, the extent of comovement of the continuation utilities (and of the stochastic discount factors in general) increases.

This effect is more pronounced when wealth inequality is more extreme, because the concavity of the utility function enhances risk-sharing opportunities. Given the higher correlation of the stochastic discount factors associated with wealth inequality, the exchange rate volatility has an inverse U-shape with respect to the log-ratio of the Pareto weights (see the right panel of figure 2.7). Equivalently, starting from an equal distribution of wealth, the impulse response of the exchange rate volatility is negative, in sharp contrast to the response of the volatility of the consumption differentials.

We conclude our analysis by noting that without volatility shocks (e.g., the case considered in Colacito and Croce (2013a)), the volatility disconnect generated by our model would be counterfactually large. This is because of the dominance of the endogenous response of volatilities to long-run shocks, which results in a negative correlation. We regard the ability of our benchmark model to match the correlation between the volatilities of consumption growth differentials and exchange rate fluctuations as an important finding that further highlights the relevance of volatility shocks in enhancing our understanding of the dynamics of international asset pricing and quantities.

## 2.5. Conclusion

In this paper, we construct a measure of bilateral volatility pass-through and we use it to document the sizeable extent of international propagation of output volatility shocks. Furthermore, we provide novel empirical evidence regarding the disconnect between the volatility of consumption differentials and the volatility of exchange rates. We show that these findings constitute a puzzle from the standpoint of a frictionless model with CRRA preferences. We then develop a frictionless general equilibrium model featuring long-run growth news shocks, volatility shocks, and two countries populated by agents with recursive preferences and demonstrate that our model can replicate these empirical findings.

Future developments should focus on extending this setting to international real business cycle models in an effort to better understand the role of international investment flows and international frictions in the origination and international propagation of volatility shocks. The investigation of the roles of trading frictions, portfolio composition, and market incompleteness are other promising directions for future research.

# CHAPTER 3 : Volatility, Intermediaries, and Exchange Rates

(with Xiang Fang)

## 3.1. Introduction

Exchange rates are puzzling. They are disconnected from economic fundamentals, especially relative consumption growth rate, which is in sharp contrast to implications of most international macro models (Backus and Smith, 1993b). High-interest-rate currencies do not tend to depreciate as the uncovered interest parity suggests, and sometimes they even appreciate (Hansen and Hodrick, 1980, Fama, 1984). As a result, excess returns of currency investment can be predicted by interest rate differentials. Finally, it is hard to obtain correct exchange rate volatility in standard international macro models (Chari et al., 2002).

In the foreign exchange (FX) market, more than 85% of turnovers have financial institutions involved, according to the BIS triennial survey. Even after excluding the amount of high-frequency dealer trading, financial institutions still play as the major participant in the FX market. Furthermore, in 21 OECD countries [1], the BIS reporting banks account for about half of the countries' total external claims, and more than 40% of total external liabilities on average. As emphasized by the recent margin-based asset pricing literature (for example, Brunnermeier and Pedersen, 2009, Garleanu and Pedersen, 2011), a large number of financial institutions take leverage, thus being subject to funding shocks. These shocks are tightly linked to the portfolio volatility faced by institutions, as Adrian and Shin (2014) shows that the Value-at-risk (VaR) rule is a good description of intermediaries' balance sheet adjustments. This paper studies the exchange rate behavior in an open economy model featuring levered intermediaries as marginal investors with fluctuating VaR leverage constraint. Our model can resolve the three exchange rate puzzles: generating the violation of uncovered interest rate parity, disconnect between exchange rate and consumption growth differential, and closer exchange rate volatility to data.

[1] Details will be shown in section 2.

Throughout the paper, we refer to “US” as the home country, and “UK” as the foreign country. Both US and UK have a continuum of homogeneous households and intermediaries. Households in both countries only have access to a risk-free money market account in local intermediaries. Local intermediaries combine money market deposits and their own net wealth to invest in a local risky asset, namely “stock”, and a single international bond. The international bond is denominated in US dollars (US composite good) [2]. Both intermediaries face a value-at-risk induced leverage constraint, such that the size of the balance sheet (stock and international bond position) cannot exceed a fraction of their market value (Gertler and Karadi, 2011). Consequently, intermediaries in each country value their net worth and deposits differently, depending on how tight the constraints are. The constraint tightness varies with intermediary balance sheet volatility. For given amount of net worth, intermediaries are allowed to take less leverage if they face with higher portfolio volatility. Different tightness of constraints creates a wedge between the marginal value of net worth for intermediaries in the two countries, thus driving exchange rates fluctuations.

Suppose the two countries start from their initial state with no international bond positions. When there is a positive volatility shock in the US economy, local stock market volatility increases, and US intermediaries take less leverage. Intermediaries are impeded from investing in the risky asset regardless of the desirable investment opportunities, thus their valuation of one dollar invested in the US stock increases. US intermediaries take short international bond positions to finance its purchase of home stock. Due to the imperfect risk sharing across intermediaries in the two countries, the marginal value of one dollar for US intermediaries is higher than that of one British pound for UK intermediaries, thus the US dollar is expected to depreciate in the next period.

We show that the calibrated model can resolve the three exchange rate puzzles. In our model, the balance sheet condition, and intermediary book leverage, enter into the pricing kernel. Expected exchange rate change and interest rate differential are linked through

[2] The denomination of international bond does not change the result qualitatively.

the balance sheet channel: when US volatility gets larger, balance sheet constraint gets tighter in the US, so that its interest rate gets lower but US composite good is expected to depreciate. The usual consumption-based Euler equation is broken down, so that exchange rates are not necessarily linked to consumption growth rate differential. The extra shocks make exchange rates more volatile than in the standard international macro models.

We examine the empirical implications of our model. First, the growth rate of US total outstanding financial commercial paper has predictive power over the exchange rate of various currencies vis-a-vis the dollar. The larger amount of commercial paper predicts a lower exchange rate growth in the next period. The predictability is preserved after controlling for several credit demand indicators. This finding is in line with Adrian et al. (2015). We also provide further evidence to link exchange rate to foreign exchange market volatility, as a proxy of financial market volatility in the model. Since US intermediaries are more globalized than intermediaries in other countries, exchange rate volatility affects US intermediaries the most. We find that average annual realized volatility of major currencies in the world has predictive power on exchange rate change for various major currencies vis-a-vis US dollar. The higher global foreign exchange realized volatility predicts an appreciation of foreign currencies and a depreciation of the dollar. Thirdly, we run the classic Fama regression of bilateral currency returns on interest rate differential after controlling for US outstanding financial commercial paper. We find the UIP coefficient becomes insignificant after controlling for commercial paper, indicating our mechanism potentially helps resolve the forward premium puzzle empirically.

**Related Literature**

There are two main strands of literature related to this paper: exchange rate literature and financial market frictions literature.

In the recent three decades, many studies have attempted to resolve these exchange rate puzzles. Following the consumption-based asset pricing literature, Verdelhan (2010b) provides a habit-based model of exchange rates consistent with the anomalies above. Bansal and Shaliastovich (2013) explains bond market and foreign exchange market altogether in a long run risk setting with stochastic volatility. Colacito and Croce (2011b), Colacito and Croce (2013b) use a long-run risk framework to account for various international asset pricing anomalies, including the comovement of global stock returns, exchange rate volatility, disconnect between consumption growth and exchange rate change, and forward premium puzzle. Farhi and Gabaix (2016) gives a rare disaster explanation of exchange rate anomalies. There are also various attempts to explain these puzzles with incomplete financial market, including Corsetti et al. (2008) for the Backus Smith puzzle, Favilukis et al. (2015) for the forward premium puzzle, etc. However, Lustig and Verdelhan (2016) shows that standard international macro models with only financial market incompleteness cannot resolve the three puzzles simultaneously.

Bai and Ríos-Rull (2015) matches both business cycle moments and Backus-Smith correlation with a model of demand shock and search friction. Bacchetta and Van Wincoop (2010) resolves the forward premium puzzle with infrequent portfolio decisions. Burnside et al. (2011) explains the forward premium puzzle with investor overconfidence.

The second strand of literature considers the role of financial market frictions in macro dynamics and asset prices, in both closed and open economies. The seminal work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) build insightful yet tractable frameworks to study the macroeconomic effect of financial frictions. Mendoza (2010) shows that real shocks can be amplified by financial frictions, leading to sudden stops and financial crisis. Jermann and Quadrini (2012) emphasizes the importance of financial shocks in understanding both the macro economy and firms' capital structure. Gertler and Karadi (2011) provide models suitable for analyzing the effect of unconventional policies. Li (2013) studies asset prices in this framework. As for the asset pricing literature, Brunnermeier and Pedersen (2009), Garleanu and Pedersen (2011) are examples of margin-based asset pricing models. Financial frictions have also been shown to be important in open economies. Dedola et al. (2013) builds an open economy version of Gertler and Karadi (2011) and studies the transmission of financial shocks. Kim (2015) further extends it into a two-good model to explain Backus-Smith puzzle. Our paper also explains the forward premium puzzle and exchange rate volatility puzzle.

Our paper is closely related to Gabaix and Maggiori (2015b). They provide a simple, flexible, and rich framework with imperfect intermediation in international financial market. Exchange rates are determined jointly by real import export choice and imperfect intermediation. Our paper is different from theirs (GM hereafter) in several aspects. First of all, we highlight the link between stochastic volatility and intermediary balance sheet condition fluctuations through VaR. Second, the direction of capital flows in GM is independent of the intermediation and financial market imperfectness only alters the quantity of flows. The role of costly intermediation is to impede consumption risk sharing. In our model, idiosyncratic volatility shocks creates international capital flows, which can be viewed as sharing risks associated with intermediation. Interest rates and exchange rates are jointly determined by idiosyncratic balance sheet conditions in respective countries. Third, our paper resolves the exchange rate puzzles in a quantitative manner. Lastly, we provide supportive evidence on the mechanism.

The rest of the paper is organized as follows. Section 3.2 lays out some institutional features of the foreign exchange market to show the relevance of financial institutions in exchange rate determination. Section 3.3 presents the model and section 3.4 shows how this model can qualitatively resolve the three exchange rate puzzles. In section 3.5 we report the quantitative performance of the model. Empirical implications of the model are tested in section 3.6 and section 3.7 concludes.

## 3.2. The Relevance of Financial Institutions

The goal of this section is to sketch the basic structure of the foreign exchange market and show the relevance of financial institutions as the marginal pricer in the foreign exchange market. [3]

The foreign exchange market is the largest financial market in the world, with daily trading volume exceeding five trillion dollars in 2013, according to the BIS triennial survey. The foreign exchange market is a two-tier market, including the inter-dealer market and dealer-customer market. Most of the inter-dealer transactions are making the market at very high frequencies. According to Bjønnes and Rime (2005), the half-life of inventory for dealers is between 1 to 30 minutes and dealers usually end the day with small amount of inventory. These high frequency transactions are not our considerations. However, according to Sager and Taylor (2006), dealers also take speculative positions in propriety trading, with horizons from one day to three months. These longer horizon speculation should be included in our model.

The second tier is the dealer-customer market, in which transactions take place between dealers and customers. Main categories of customers include financial customers, corporate customers[4], and retail customers[5]. Financial customers can be divided into two groups: real money investors and levered investors. Real money investors include mutual funds, pensions funds, endowments, and so on, which do not take much leverage and adjust less frequently on their portfolios. Levered investors include non-dealer commercial banks, hedge funds and commodity trading advisors, and so on. They take high leverages and actively adjust their portfolios. Apart from these financial customers, dealers also take speculative positions in propriety trading, with horizons from one day to three months (Sager and Taylor, 2006).

Levered investors account for a substantial portion of turnovers in the FX market. Table 3.7 shows the fraction of FX turnovers by different entities from 1998 to 2016, reported by the BIS triennial survey. Turnovers associated with nondealer financial institutions account for

[3] We describe the common features of the market across time. Admittedly, there has been tremendous changes in many aspects of the foreign exchange market in the recent three decades, including the use of electronic trading systems, the increase of foreign exchange transactions between financial institutions, etc. For more institution details of the foreign exchange market, see Osler (2008) and King et al. (2011).

[4] Corporate customers trade for real purposes, such as production, investment, and dividend payout. The size of corporate transactions is small relative to financial transactions.

[5] Retail customers, accounting for a very small fraction, are not studied in this paper.

51% in 2016. Starting from 2013, the BIS triennial survey makes a detailed split of nondealer financial institutions into nonreporting banks (24%, 22%[6]), institutional investors (11%, 16%), hedge funds and PTFs (11%, 8%), official sector (1%, 1%), and other institutions (6%, 4%). Among these institutions, nonreporting banks, hedge funds and PTFs, and part of institutional investors take leverage. Meanwhile, nonfinancial transactions account for no more than 20% of all turnovers, and it has been decreasing in the recent decades. These facts motivate us to focus on the behavior of levered institutions to study exchange rates.

Speculative positions are constrained for reasons such as regulation, risk management and avoidance of excess risk taking for each trader. Banks in different countries are subject to Basel regulatory capital adequacy framework. FX market participants also face market discipline in balance sheet management, usually in the form of value-at-risk (VaR) constraint (Sager and Taylor, 2006). Usually, position limits are imposed on traders to avoid individual excess risk taking (Osler, 2008). These practices shed light on the role of balance sheet conditions in exchange rate determination.

Aggregate banking data also show the important role of banks in holding cross border claims and liabilities. Figure 3.1 plots the time series weighted average of the ratio of banking claims (liabilities) over total claims (liabilities) from 1977 to 2014. The sample includes 22 OECD countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, South Korea, Spain, Sweden, Switzerland, UK, and US. Each country's total external claims (liabilities) are obtained from Lane and Milesi-Ferretti (2007), while each country's cross border banking claims (liabilities) are from BIS locational statistics [7]. We use a country's total claims (liabilities) share as weight. In the late 1970s and early 1980s, banks account for about half of external claims and 40 percent of external liabilities. This number declined substantially in late 1990s, to 40 percent (claims) and 30 percent (liabilities) at the trough, possibly due to the global stock market boom. This number rebounced back quickly in the

[6] Numbers in 2013 and 2016, respectively.

[7] BIS Locational statistics is residence based, including all activities by banks operating in the country.

2000s, until the global financial crisis in 2007.

In this section, we provide two pieces of evidence to support the preeminent role financial institutions (banks) play in the international financial market: (1) a substantial portion of turnovers in the FX market are by financial institutions facing balance sheet constraint; (2) BIS reporting banks hold about half of countries' external claims and liabilities. These facts motivate us to focus on the behaviors of financial institutions to study exchange rates.

## 3.3. Model

There are two countries in the economy, US and UK, each endowed with a unit measure of households and a Lucas tree. Fruits from the two trees differ. In both countries, each household sends out a manager to operate the intermediary it owns. Households deposit in local intermediaries. Intermediaries combine deposits and their own net worth to invest in risky assets, including the local Lucas tree and an international bond. Intermediation is imperfect, since the intermediaries in each bank face a leverage constraint, whose tightness is determined by the portfolio volatility faced by the bank through value-at-risk constraint. Every period, a fixed fraction of intermediaries exit the market and rebate back their net worth to their owners, while the same measure of new intermediaries is set up with some initial funds to keep the measure of intermediaries stationary. The structure of the economy is similar to Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).

We describe the behavior of households and intermediaries in detail in the following subsections.

### *3.3.1. Households*

Households in US and UK are endowed with a Lucas tree with different fruits, X for US and Y for UK. They follow cointegrated processes:

$$logX_{t+1} - logX_t = \mu + \tau(logY_t - logX_t) + \sigma_{X,t+1}\epsilon_{X,t+1}$$

$$logY_{t+1} - logY_t = \mu - \tau(logY_t - logX_t) + \sigma_{Y,t+1}\epsilon_{Y,t+1} \tag{3.1}$$

Volatilities are stochastic, following:

$$\sigma^2_{X,t+1} = (1-\rho_\sigma)\bar{\sigma}^2 + \rho_\sigma\sigma^2_{Xt} + \eta_{X,t+1}, \sigma^2_{Y,t+1} = (1-\rho_\sigma)\bar{\sigma}^2 + \rho_\sigma\sigma^2_{Yt} + \eta_{Y,t+1} \tag{3.2}$$

The two goods aggregate into a composite consumption for agents in both countries. The aggregator takes the form of constant elasticity of substitution:

$$C = [(1-\alpha)C_X^{\frac{\sigma-1}{\sigma}} + \alpha C_Y^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}, C^* = [(1-\alpha)C_Y^{*\frac{\sigma-1}{\sigma}} + \alpha C_X^{*\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

$C_X, C_Y$ are consumption of X and Y for US households, while variables with an asterisk refer to the UK counterpart. Households in US and UK put different weights on X and Y with consumption home bias, captured by $\alpha$, typically smaller than 0.5. $\sigma$ is the price elasticity of substitution between X and Y. The composite good of US households is in dollars (as numeraire), while the composite good of UK households is in pounds. The price of pounds over dollars is defined to be the real exchange rate $Q_t$. An increase in $Q$ means an appreciation of the UK pound and a depreciation of the US dollar.

In every period, given composite consumption of $C, C^*$ and prices $P_X, P_Y$, households choose how much X and Y to consume. US households solve the intratemporal optimization problem:

$$\min_{C_X, C_Y} P_X C_X + P_Y C_Y$$

$$s.t.: C = [(1-\alpha)C_X^{\frac{\sigma-1}{\sigma}} + \alpha C_Y^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

The allocation between X and Y are solved as:

$$C_X = \frac{C(\frac{P_X}{P_Y}\frac{\alpha}{1-\alpha})^{-\sigma}}{P_Y + P_X(\frac{P_X}{P_Y}\frac{\alpha}{1-\alpha})^{-\sigma}}, C_Y = \frac{C}{P_Y + P_X(\frac{P_X}{P_Y}\frac{\alpha}{1-\alpha})^{-\sigma}} \tag{3.3}$$

Similarly, for UK households:

$$C_X^* = \frac{C^*(\frac{P_X}{P_Y}\frac{\alpha}{1-\alpha})^{-\sigma}Q}{P_Y + P_X(\frac{P_X}{P_Y}\frac{\alpha}{1-\alpha})^{-\sigma}}, C_Y^* = \frac{C^*Q}{P_Y + P_X(\frac{P_X}{P_Y}\frac{\alpha}{1-\alpha})^{-\sigma}} \tag{3.4}$$

The prices UK households face in UK pound are $\frac{P_X}{Q}, \frac{P_Y}{Q}$.

All households have identical Constant Relative Risk Aversion (CRRA) preference over their composite goods with risk aversion $\gamma$. A fraction $\alpha_l$ of the Lucas tree goes to the households as labor income, while the remaining are capitalized in the equity market and paid out as dividends. Households do not hold equities directly. Empirical evidence supports the stock market limited participation hypothesis Vissing-Jørgensen (2002). [8] Moreover, we can view this assumption as an extreme case of infinitely large efficiency loss for households to trade risky assets. Intermediaries have comparative advantage in investment expertise and risk capacity, which is indeed the role of financial intermediaries in our economy. This assumption can relaxed to a less extreme situation in which households pay a cost to trade the risky assets, despite the additional complexity of the model.

Households solve a standard intertemporal optimization problem:

$$\max_{C_t, D_{t+1}} E\sum_{t=0}^{\infty} \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

$$s.t. : C_t + D_t = \alpha_l P_{Xt} X_t + R_{t-1} D_{t-1} + \Pi_t$$

$D_t$ is the deposit by households into intermediaries at time t, while $D_{t-1}R_{t-1}$ is the repayment from intermediaries of principal and interest. $\Pi_t$ is the net lump-sum payout from the intermediaries that exit the market, which will be specified later. Euler equations hold for households in both countries:

$$E_t\beta(\frac{C_{t+1}}{C_t})^{-\gamma} R_t = 1, \quad E_t\beta(\frac{C_{t+1}^*}{C_t^*})^{-\gamma} R_t^* = 1 \tag{3.5}$$

[8] Alternatively, we can assume that households hold a fixed amount of risky asset without trading (Chien et al., 2012).

*3.3.2. Intermediaries*

Each intermediary is owned by a household, and faces a portfolio choice problem on how much deposit to take, how many home stock shares and international bonds to purchase. We exclude the holding of foreign stocks by intermediaries. In most countries' portfolios, domestic stocks account for a much larger share than their market cap share (home equity bias, French and Poterba, 1991). Technically, by making this assumption, we avoid indeterminacy of asset holdings at the deterministic steady state when assets are perfect substitutes (Devereux and Sutherland, 2011).

Intermediation is imperfect with a leverage constraint on intermediaries in both countries.

$$V_t \geq \theta_t (Z_t s_t + d_{It}), \quad V_t^* \geq \theta_t^* (Z_t^* s_t^* + d_{It}^*) \tag{3.6}$$

$V_t, V_t^*$ are the market value of an intermediary in US and UK. $Z_t, Z_t^*$ are the prices of the Lucas trees in both countries, $s_t, s_t^*$ are the holding shares, and $d_{It}$ is the holding of the international bond by US intermediaries. The international bond is denominated in US composite goods (dollars). [9] When $d_{It} < 0$, US intermediaries are effectively borrowing from UK intermediaries to purchase US stocks. $\theta_t$ and $\theta_t^*$ measures the tightness of leverage constraint faced by intermediaries, which are linked to the portfolio volatility of intermediaries. We interpret them as "value-at-risk" (VaR) constraints, expressed as:

$$\theta_t = \sqrt{a^2 + c \times var_t R_{t+1}^P}, \theta_t^* = \sqrt{a^2 + c \times var_t R_{t+1}^{P*}}$$

$R_{t+1}^P$ is the portfolio return of US intermediaries, while the UK counterpart is $R_{t+1}^{P*}$, given as follows:

$$R_{t+1}^P = \frac{Z_t S_t}{Z_t s_t + d_{It}} R_{S,t+1} + \frac{d_{It}}{Z_t s_t + d_{It}} R_{It} \equiv \omega_t R_{S,t+1} + (1 - \omega_t) R_{It}$$

[9] The denomination of the international bond does not alter the intuition of the mechanism, but changes the risk sharing scheme between two countries.

$$R^{P*}_{t+1} = \frac{Z^*_t s^*_t}{Z^*_t s^*_t + d^*_{It}} R^*_{S,t+1} + \frac{d^*_{It}}{Z^*_t s^*_t + d^*_{It}} \frac{Q_t R_{It}}{Q_{t+1}} \equiv \omega^*_t R^*_{S,t+1} + (1-\omega^*_t) \frac{Q_t R_{It}}{Q_{t+1}}$$

$\omega_t, \omega^*_t$ are the portfolio weight allocated to local equities by US and UK intermediaries. $R_{S,t+1}$ and $R^*_{S,t+1}$ are returns on investing in local equities, $R_{It}$ is the return on the international bond in dollars. Only UK intermediaries face exchange rate risk.

The constraint captures the feature of VaR, which is microfounded with optimal contracting framework by Adrian and Shin (2014). Creditors require higher equity value of the intermediaries if their portfolio volatility is higher, in which case they are more likely to default. This market discipline is captured by $c \times var_t R^P_{t+1}$. $a$ captures leverage restrictions caused by time-invariant frictions. Furthermore, borrowing and lending between intermediaries across border are settled before repaying the households so that they are prone to default. Therefore, the VaR constraint is imposed on the sum of local risky assets and international position, even if it is a short position. We can easily extend the constraint to include different risk weight for local risky assets and international position.

In every period, each intermediary has probability $1-p$ to exit the market, so that intermediaries will not accumulate enough net worth to grow out of the leverage constraint. Upon exit, intermediaries rebate all their net worth to the households, and the managers return to their households. In the meantime, these households each will send out new managers and establish new intermediaries with some startup funds to initiate their operation. Each new intermediary is endowed with $\xi$ fraction of the average net worth of current incumbents.

The value function of a representative intermediary can be written recursively:

$$V(s_t, d_t, d_{It}) = \max_{s_{t+1}, d_{t+1}, d_{I,t+1}} E_t \beta (\frac{C_{t+1}}{C_t})^{-\gamma} [(1-p) N_{t+1} + pV(s_{t+1}, d_{t+1}, d_{I,t+1})]$$

$$s.t.: n_{t+1} = Z_{t+1} s_{t+1} + d_{I,t+1} - d_{t+1}$$

$$V_t \geq \theta_t (Z_t s_t + d_{It})$$

$V(s_t, d_t, d_{It})$ is the value of the intermediary at the end of period t, with the holding of domestic tree, international bond and deposit as state variables. $\beta(\frac{C_{t+1}}{C_t})^{-\gamma}$ is the stochastic discount factor of households. If the intermediary exits, it pays out its net worth $n_{t+1}$. If not, it continues to operate and choose the holding of assets and liabilities, with value function $V_{t+1}(s_{t+1}, d_{t+1}, d_{I,t+1})$. The first constraint is the balance sheet identity, with left-hand side the intermediary's net worth and right-hand side the intermediary's assets net of liabilities. The second constraint is the leverage constraint as discussed before. The dynamics of net worth for a single intermediary is given by:

$$n_{t+1} = R_{S,t+1} Z_t s_t + R_{It} d_{It} - R_t d_t$$

where $R_{S,t+1} = \frac{Z_{t+1}+(1-\alpha_l)P_{Xt}X_t}{Z_t}$ is the return to holding domestic equities. Following Gertler and Karadi (2011), we guess the value function is linear in all three state variables, which will be verified later. Suppose it has the following form:

$$V_t = \nu_{St} Z_t s_t + \nu_{It} d_{It} - \nu_t d_t$$

Assigning $\lambda_t$ and $\psi_t$ to be the Lagrangian multipliers to the two restrictions, we can obtain the first order conditions:

$$1 - p + p\nu_{St} + \lambda_t - \psi_t \theta_t = 0$$

$$1 - p + p\nu_{It} + \lambda_t - \psi_t \theta_t = 0$$

$$1 - p + p\nu_t + \lambda_t = 0$$

From these three first order conditions, we have our first key result:

$$\nu_{St} = \nu_{It} \geq \nu_t \tag{3.7}$$

When the leverage constraint does not bind, all three of them are equal. This is the case where the marginal benefit of investing in both risky assets is the same as the marginal cost of taking deposits. When the leverage constraint binds, the marginal benefit of investing in home stock and international bond are still identical, both larger than the marginal cost of taking deposits. The key determinant of whether the leverage constraint binds or not is the net worth of the intermediaries. If they have ample net worth, they will exhaust investment opportunities before they hit the constraint. In the remaining part of the paper, I will use the notation $\nu_{St}$ whenever $\nu_{It}$ shows up.

We plug back the value function into the Bellman equation, and derive expressions for these time-varying coefficients on the state variables:

$$\nu_{St} = E_t\beta(\frac{C_{t+1}}{C_t})^{-\gamma}(1-p+p\theta_{t+1}\phi_{t+1})R_{S,t+1} = E_t\beta(\frac{C_{t+1}}{C_t})^{-\gamma}(1-p+p\theta_t\phi_t)R_{It} \tag{3.8}$$

$$\nu_t = E_t\beta(\frac{C_{t+1}}{C_t})^{-\gamma}(1-p+p\theta_{t+1}\phi_{t+1})R_t \tag{3.9}$$

$\phi_t$ is the aggregate book leverage of US intermediaries, [10] defined to be:

$$\phi_t \equiv \frac{Z_tS_t + D_{It}}{N_t}$$

UK intermediaries face the same problem. The only difference is with the international bond. US intermediaries do not face exchange rate risk in trading international bond, while UK intermediaries do. The pricing equation of the international bond for UK intermediaries is:

$$\nu^*_{St} = E_t\beta(\frac{C^*_{t+1}}{C^*_t})^{-\gamma}(1-p+p\theta^*_t\phi^*_t)R_{It}\frac{Q_{t+1}}{Q_t} \tag{3.10}$$

[10] More precisely, $\phi_t$ is the ratio of US intermediaries' risky position over net worth. In our simulated economy, the properties of $\phi_t$ barely change if we define $\phi_t = \frac{Z_tS_t+D_{It}I(D_{It}\geq 0)}{N_t}$, where $I$ is an indicator function.

*3.3.3. Aggregation*

We already specify the problem as well as the optimality conditions of any single intermediary. The linearity of the model simplifies aggregation. Since we have a representative intermediary, each intermediary has the same optimality conditions and makes the same choice. We can directly replace individual variables $n_t, s_t, d_t, d_{It}$ and their foreign counterparts $n_t^*, s_t^*, d_t^*, d_{It}^*$ with the aggregate variables $N_t, S_t, D_t, D_{It}, N_t^*, S_t^*, D_t^*, D_{It}^*$ in the flow of funds constraints, leverage constraints, and optimality conditions.

The net worth dynamics in aggregate is different from the one for a single intermediary, due to the entry and exit. The aggregate dynamics is given by:

$$N_{t+1} = (1 - p + \xi)(R_{S,t+1} Z_t S_t - R_t D_t + R_{It} D_{It}) \tag{3.11}$$

$$N_{t+1}^* = (1 - p + \xi)(R_{S,t+1}^* Z_t^* S_t^* - R_t^* D_t^* + R_{It} D_{It}^* \frac{Q_t}{Q_{t+1}}) \tag{3.12}$$

*3.3.4. Equilibrium*

Lastly, we have market clearing conditions for good markets and asset markets.

$$C_{Xt} + C_{Xt}^* = X_t, \quad C_{Yt} + C_{Yt}^* = Y_t, \quad S_t = S_t^* = 1, \quad D_{It} + D_{It}^* Q_t = 0 \tag{3.13}$$

A competitive equilibrium consists of a sequence of allocations$\{C_t, C_t^*, D_t, D_t^*, N_t, N_t^*, S_t, S_t^*, D_{It}, D_{It}^*, \phi_t, \phi_t^*\}$, a sequence of prices $\{R_t, R_t^*, P_{Xt}, P_{Yt}, Z_t, Z_t^*, Q_t, R_{It}, R_t^P, R_t^{*P}\}$, a sequence of volatility $\{\theta_t, \theta_t^*, var_t R_{t+1}^P, var_t R_{t+1}^{*P}\}$, and a sequence of intermediary valuation $\{\nu_{St}, \nu_t, \nu_{St}^*, \nu_t^*\}$ such that:

(i) Households in both countries solve their optimization problem;

(ii) Intermediaries in both countries solve their constrained optimization problem;

(iii) Good markets (X and Y) clear;

(iv) Asset markets (deposits, stocks, and international bond) clear.

*3.3.5. Asset Prices*

In this model, intermediaries play the central role in pricing all the assets. The asset pricing equations can be written as:

$$E_t \Omega_{t,t+1} R_{S,t+1} = E_t \Omega_{t,t+1} R_{It} = \nu_{St} \tag{3.14}$$

$$E_t \Omega_{t,t+1} R_t = \nu_t \tag{3.15}$$

where:

$$\Omega_{t,t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\gamma}(1 - p + p\theta_{t+1}\phi_{t+1})$$

The stochastic discount factor that prices equities and the international bond $\frac{\Omega_{t,t+1}}{\nu_{St}}$ is different from the one that prices deposits $\frac{\Omega_{t,t+1}}{\nu_t}$. The difference depends on the wedge between the marginal benefit of investment and the marginal cost of taking deposits, which essentially relies on the tightness of the leverage constraint.

The stochastic discount factor has three components: consumption growth, the product of leverage $\phi_t$ and VaR $\theta_t$, and the marginal value of net worth or the marginal cost of taking deposits.

One dollar to the intermediary will be distributed with probability $1-p$, and will remain in the intermediary with probability $p$. When leverage constraint binds, equation (6) becomes:

$$V_t = \theta_t \phi_t N_t$$

The dollar remaining in the intermediary generates market value of $\theta_t \phi_t$. In other words, $\theta_t \phi_t$ measures the conversion rate of one unit of net worth to market value of the intermediary.

## 3.4. Exchange Rate Puzzles

In this section, we review the three exchange rate puzzles in the literature, and analyze how our model helps resolve these puzzles.

### *3.4.1. Backus-Smith Puzzle*

Denote the US stochastic discount factor (SDF) to be $M_{t,t+1}$, and the UK SDF $M^*_{t,t+1}$. Consider the return of US risk-free bond $R_t$, the following two equations hold:

$$E_t M_{t,t+1} R_t = E_t M^*_{t,t+1} R_t \frac{Q_t}{Q_{t+1}} = 1$$

$Q_t$ is the price of UK goods in terms of US goods, consistent with the notation in section 3.3. If the financial market is complete, then the equation holds state by state. Therefore:

$$\Delta q_{t+1} = m^*_{t,t+1} - m_{t,t+1}$$

Lower-case letters are natural logarithms of variables. If we assume a constant relative risk aversion utility function, following the convention of business cycle analysis, then we obtain:

$$\Delta q_{t+1} = \gamma(\Delta c_{t+1} - \Delta c^*_{t+1})$$

Exchange rate change is proportional to consumption growth differential. Even when the financial market is incomplete, such as in the models of Heathcote and Perri (2002) and Chari et al. (2002), the correlation between $\Delta q_{t+1}$ and $\Delta c_{t+1} - \Delta c^*_{t+1}$ is still close to 1. This is inconsistent with the weak correlation between US and a number of developed economies (For example, the correlation is -0.21 between US and UK nominal exchange rate change and nominal consumption growth rate differential). This puzzle is named after Backus and Smith (1993b).

In our model, we have augmented Euler equations for intermediaries in both countries:

$$E_t\beta(\frac{C_{t+1}}{C_t})^{-\gamma}\frac{(1-p+p\theta_{t+1}\phi_{t+1})}{\nu_{St}} = E_t\beta(\frac{C^*_{t+1}}{C^*_t})^{-\gamma}\frac{(1-p+p\theta^*_{t+1}\phi^*_{t+1})}{\nu^*_{St}}\frac{Q_{t+1}}{Q_t}$$

Following the logic in standard international macro models, in which the correlation between exchange rate change and relative log stochastic discount factors are close to 1, we have:

$$corr(\Delta q_{t+1}, -\gamma(\Delta c_{t+1} - \Delta c^*_{t+1}) + \log(\frac{1-p+p\theta_{t+1}\phi_{t+1}}{1-p+p\theta^*_{t+1}\phi^*_{t+1}}) + \log\frac{\nu_{St}}{\nu^*_{St}}) \approx 1$$

Exchange rate change is correlated to consumption growth differential plus two extra terms: one capturing the relative financial conditions and leverage, the other measuring the relative marginal value of the investment. The latter two terms dominate so that consumption growth differential seems to be disconnected with exchange rate change.

The response of consumption and exchange rate to endowment and volatility shocks also helps us understand the disconnect.[11] Upon an endowment shock in the US, US consumption increases relatively, while the US dollar depreciates. When the US experiences a positive volatility shock, US consumption increases relatively as well, but the US dollar appreciates. The two forces at play offset with each other and generate the disconnect phenomenon.

*3.4.2. Forward Premium Puzzle*

*3.4.3. Forward Premium Puzzle*

Uncovered Interest Rate Parity suggests that when US has a higher interest rate than UK, we would expect the US dollar to depreciate in the next period, so that investing in US and UK deliver the same payoffs in expectation. However, this parity condition is rejected by data. We regress a US investor's currency excess return of UK pounds on interest rate differential between UK and US Treasury bills, and obtain a regression coefficient of 1.4,

[11] In Section 3.5, we will discuss the impulse response functions to both endowment and volatility shocks in detail. IRFs are shown in Figure 3.1.

being significantly different from 0. For many other countries, the coefficients are all larger than 1. This result shows that the currency with the higher interest rate tends to further appreciate. This puzzle is also called the "forward premium puzzle".

In our model, volatility shocks mainly explain the puzzle through altering intermediaries' balance sheet condition. When the US experiences a positive volatility shock, the US interest rate is lower. If we log-normally approximate household Euler equation, we have:

$$\log R_t \approx -\log \beta + \gamma E_t(\log C_{t+1} - \log C_t) - \frac{1}{2}\gamma(\gamma - 1)var_t(\log C_{t+1} - \log C_t)$$

There are two forces that drive down US interest rate upon a positive US volatility shock. First of all, the variance term $\frac{1}{2}\gamma(\gamma-1)var_t(\log C_{t+1} - \log C_t)$ is larger and interest rate falls. Secondly, the increased volatility tightens the constraint of US intermediaries. US households consume more than usual, thus lowering the expected consumption growth $\gamma E_t(\log C_{t+1} - \log C_t)$. The second force also makes UK households to consume less, and increases UK interest rate. The interest rate spread between UK and US widens.

As for exchange rates, increased US volatility mainly tightens US intermediaries' constraint, since each country's intermediaries mainly hold stocks in their own country. The wedge between investing and borrowing becomes larger. Expected exchange rate change in the next period will make investing in home tree and borrowing from abroad equivalent. When US intermediaries have a larger wedge, the US dollar is expected to depreciate.

We can also understand the forward premium puzzle from the intermediaries' perspective. From the Euler equations for international bond for both intermediaries:

$$E_t \Delta q_{t+1} \approx E_t[(\log \Omega^*_{t+1} - \log \nu^*_{St}) - (\log \Omega_{t+1} - \log \nu_{St})]$$

If we ignore the higher than first order moments, the expected exchange rate can be reduced to:

$$E_t \Delta q_{t+1} \approx r_t - r_t^* + (\nu_{St} - \nu_t) - (\nu_{St}^* - \nu_t^*) + \frac{1}{2} var_t log \Omega_{t+1}^* - \frac{1}{2} var_t log \Omega_{t+1}$$

when the US experiences a positive volatility shock, $r_t < r_t^*$. Since US intermediaries are more constrained in taking leverage, the wedge is larger for US, $\nu_{St} - \nu_t > \nu_{St}^* - \nu_t^*$. If the wedges do not exist, we have uncovered interest rate parity. However, in our model, the wedge dominates interest rate difference in driving exchange rates.

There is abundant literature about the relationship between stochastic volatility and forward premium puzzle. Bekaert (1996) and Bansal (1997) show that in a complete market setting with affine linear stochastic discount factors, stochastic volatility is necessary to violate UIP. Bansal and Shaliastovich (2013) also attributes time variation in currency premia to volatility fluctuations. Our model is different from theirs. The channel volatility affects exchange rates in our model is through affecting the behavior of intermediaries, inducing time-varying leverage constraint. Time-varying consumption risk also exists, but is very weak, as is well known since Mehra and Prescott (1985). Therefore, we are proposing a new mechanism to link volatility to exchange rates, in complementary with the vast existing literature.

### *3.4.4. Exchange Rate Volatility Puzzle*

Most international macro models with incomplete financial market cannot generate volatile exchange rates as in the data. Our model also falls into the category. We will show in the quantitative section that exchange rate volatility in our model is close to data for two reasons. First, the leverage constraint of intermediaries amplify the fundamental volatilities into financial market volatility. Second, volatility shocks make the tightness of leverage constraint time-varying, to generate even more volatile exchange rates.

### *3.4.5. Forward Premium Puzzle and Carry Trade*

In the exchange rates literature, there are two similar but distinct literature: forward premium puzzle and carry trade. Forward premium puzzle indicates that the regression slope coefficient of currency return on interest rate differential is positive and even greater than 1, while carry trade refers to the persistent interest rate differential across countries without offsetting exchange rate movement. This paper focuses on the former puzzle, with all fluctuations of interest rates and exchange rates in our model being conditional. In our model, two countries are symmetric except for the currency denomination of international bond. On the other hand, explanations of carry trade requires explicitly modelling of cross-country heterogeneity that accounts for heterogenous exposure to some global risk factor, as shown in Lustig et al. (2011b), Hassan and Mano (2017), etc.

## 3.5. Quantitative Results

### *3.5.1. Calibration*

We calibrate the model at quarterly frequency. Benchmark parameter values are reported in Table 3.2. Three parameters commonly used in macro models are chosen ad hoc. Time discount factor, labor income share, and risk aversion are set to be 0.995, 0.67, and 2, respectively. Following Colacito and Croce (2013b), we have the degree of consumption home bias $\alpha$ to be as small as 0.03, and we impose a weak cointegration relationship between the two endowment processes with the error correction parameter $\tau$ to be 0.0005. The elasticity of substitution between the two goods is 0.9, following estimates of various studies, including Stockman and Tesar (1995) and Heathcote and Perri (2002).

We set up a growth economy to capture the effect of low-frequency component on asset prices, following the convention of macro asset pricing literature, but we assume the average growth rate to be 0 in order to match the low interest rate level. The tension between consumption growth and risk-free rate is a long-standing puzzle (risk-free rate puzzle, Weil, 1989). This paper does not attempt to provide any new insight on the resolution of the risk-free rate puzzle. Volatility and cross-country correlation of endowment match the data counterpart in the US (US-UK for correlation) from 1973 to 2015.

The stochastic volatility processes are the main driving forces of our model. Volatility is a persistent variable, so we calibrate its persistence to be 0.98. The mean of endowment growth volatility is calibrated to the data counterpart from 1973 to 2015, and the volatility of volatility shock is assumed to be half of the mean of endowment growth volatility. The two parameters determining the relationship between VaR and volatility, $a, c$ are to match the mean and volatility of intermediary leverage. We follow the definition by Krishnamurthy and Vissing-Jorgensen (2015) of the "financial sector" of all institutions supplying short-term debt. [12]Quarterly assets and liabilities data for each type of financial institution are from Flow of Funds, published by the Federal Reserve Board. Payout rate $1 - p$ and initial funds for new intermediaries $\xi$ are set to be 0.87 and 0.03, to make the constraint binding in most cases and keep the net payout rate at 10% of net worth of the existing intermediaries.

We make two normalizations in the benchmark case. We assume the fluctuations of both shocks are independent across countries, and volatility shocks and endowment shocks are orthogonal to each other. With zero correlated endowment process, our model produces cross-country correlation of output and consumption growth similar to the data, due to the correlation of goods price. In our model, it is the idiosyncratic volatility that moves the relative tightness of leverage constraints, which in turn leads to cross-border borrowing and lending. Fluctuations in the common component of volatility do not affect the incentive for intermediaries to borrow or lend internationally, so they are abstracted from the model.

### *3.5.2. Impulse Response Functions*

Figures 3.1 and 3.2 report the impulse responses of different US and UK variables to a one-standard-deviation US positive endowment shock and volatility shock.

[12]These institutions include US-Chartered Depository Institutions, Foreign Banking Offices in US, Banks in US-Affiliated Areas, Credit Unions, Money Market Mutual Funds, Issuers of Asset-Backed Securities, Finance Companies, Mortgage Real Estate Investment Trusts, Security Brokers and Dealers, Holding Companies and Funding Corporations.

When US has a one standard deviation positive endowment shock, the dividend payment of the Lucas tree increases for US investors and consumption growth increases in the US. The price of the US tree increases and the net worth of US intermediaries are strengthened. As a result, US intermediaries can borrow more, which further pushes up the price of the tree. The amplification mechanism explains the relative larger asset price change with respect to the change in the dividend. The marginal value of investing in the tree and the marginal cost of borrowing both decline, as does the wedge between the two. This can be reflected in the return on the international bond. In this scenario US intermediaries are lending to the UK intermediaries. The shock in the US is transmitted to UK with higher consumption growth rate in UK as well as a higher price of UK goods. The price of the UK tree also increases, so that all intermediary variables in UK move in the same direction as in the US. The synchronization of the intermediary balance sheet has been studied in Dedola et al. (2013). Due to more supply of the US good X, the price of X decreases and the US dollar depreciates. The leverage of intermediaries decreases, because the strengthening of net worth dominates the expansion of balance sheet in determining leverage. Interest rate changes in the US and UK in this case are intuitively ambiguous. The higher endowment creates more supply of deposit, while the loosening of leverage constraint generates more demand. The direction of interest rate movement depends on which force dominates. Under our parameterization, the US interest rate increases, and the UK interest rate decreases.

The more interesting channel in our model can be seen in the impulse responses to a one standard deviation volatility shock. The volatility shock is amplified through the same feedback loop between net worth, borrowing and asset price illustrated previously. The greater volatility in the financial market will tighten intermediaries' leverage constraint, which forces them to sell part of their asset to satisfy the constraint. The same feedback loop makes the price of the US tree decline further. Since the US intermediaries borrow less, US households' consumption increases, and US interest rate decreases. The marginal benefit of investing in the Lucas tree as well as the marginal cost of borrowing both increase, and so does their difference, reflected in the increase of international bond return. US intermediaries are willing to borrow from UK intermediaries so that some of resources are transferred from UK to US. The price of the UK tree also decreases and all variables of UK intermediaries move in the same direction as US intermediaries. The US dollar is expected to depreciate.

*3.5.3. Quantitative Results*

Table 3.3 presents the quantitative results of the model.

Panel A lists country-specific moments in the model, as well as US data counterparts of these moments. Output volatility, leverage mean, leverage standard deviation, and risk-free rate in the US are targets for calibrating the mean and volatility of the two exogenous processes. Financial sector dividend yield is the moment used to calibrate the exit probability or payout ratio. The quarterly risk-free deposit rate is about 0.4% in the model, higher than 0.23% in the data from 1973 to 2015. However, if we exclude the period after the financial crisis, in which the real interest rate is known to be negative, the average interest rate will be as high as about 0.5%. Therefore, we target the interest rate to be 0.4%. Quarterly stock return in the model is 1.6%, close to 1.4% in the data. The model does not generate enough volatility in stock return. The standard deviation of stock return in the model is only 1.6%, about 20% of 8.5% in the data.

Panel B shows the correlation and autocorrelation moments. Though the correlation between two endowment shocks is zero, the correlations between output growth and consumption growth across the two countries are close to their data counterpart. This is because of the correlation of their prices. In our model, consumption growth is more correlated than output growth, being inconsistent with the data (Backus et al., 1992).

Panel C shows that our model can resolve the three exchange rate puzzles quantitatively. Exchange rate volatility is 3.8% per quarter, close to 5.2% in the data, being higher than most international macro models. Consumption growth differential is disconnected with exchange rate change, due to the augmented Euler equation shown in section 3.4.1. Finally, the UIP coefficient is larger than 1. Though there is a bit of overshooting in UIP coefficient, the model can explain why low-interest-rate currencies tend to further depreciate instead of appreciating.

## 3.6. Empirical Implications

Our model sheds light on the determination of exchange rate by the fluctuating balance sheet constraint originated from time-varying financial market volatility. We implement two empirical tests on our implications. First, we show that the year-over-year growth rate of US aggregate financial commercial paper has predictive power in explaining bilateral exchange rates between the dollar and major foreign currencies. Aggregate US financial commercial paper serves as a measure of the constraint in the US. Further, we control for different indicators of credit demand and find the predictive power remains. Second, we find that global realized exchange rate volatility in the past 12 months also has predictive power in bilateral exchange rates. Third, we find that after controlling for outstanding US financial commercial paper, the Fama regression coefficient turns insignificant.

### *3.6.1. Data*

#### 3.6.1.1. Exchange Rates

We have 12 countries in our sample, including Australia, Belgium, Canada, Denmark, France, Germany, Italy, Japan, New Zealand, Switzerland, Sweden, and the UK. We calculate the log difference in nominal exchange rates in each currency with respect to US for both monthly and quarterly frequency, from 1991 to 2015.

#### 3.6.1.2. Measuring Financial Conditions

Commercial papers are one of the major forms of external funding used by financial institutions in the US, as illustrated by Adrian and Shin (2010). We measure US financial conditions by both (one-sided log detrended) level and growth rate of US outstanding commercial paper for financial business (commercial paper for short), published by Flow of

Funds. Our data span from January 1991 to December 2015 at monthly frequency. The measurement is in line with Adrian et al. (2015). Further, we control for several variables that represent investment opportunities, including investment-capital ratio, stock dividend yield, stock market return, and output growth. The control variables are from National Income and Product Accounts and CRSP.

### *3.6.2. Predictive Regression with US Commercial Paper*

In Table 3.4, we report results for univariate predictive regression of bilateral exchange rates on year-over-year US financial commercial paper growth.[13] Standard errors are calculated using Hansen and Hodrick (1980). US financial commercial paper as a measure of US balance sheet constraint, contains predictive information of bilateral exchange rates of US against all major currencies at monthly, quarterly and annual frequencies. Higher commercial paper (looser constraint) predicts a depreciation of foreign currencies in the next period, which is consistent with our model prediction. For robustness, we control for four variables that are regarded as indicators of credit demand, including investment-capital ratio, dividend yield, stock market return, and GDP growth in the US, to eliminate concerns that aggregate commercial paper growth is caused by higher demand of credit. Table 3.5 shows the results for monthly predictive regressions.[14]

### *3.6.3. Predictive Regression with Global Foreign Exchange Volatility*

Our model attributes the balance sheet condition fluctuations to time-varying financial market volatility. A natural candidate of volatility measure is exchange rate volatility [15]. As US intermediaries are larger, more open and active in the FX market, as reported by BIS, they are more exposed to exchange rate risk. Global exchange rate volatility is calculated

[13]If we use monthly, quarter-over-quarter growth rate or one-sided detrended US financial commercial paper, the result is qualitatively identical. These results are available upon request.

[14]For sake of space, we do not report the regression with controls for quarterly and annual regressions. Results are robust, and available upon request.

[15]Our model implies stock market volatility should be an ideal measure. In practice, banks and levered institutions mainly hold assets such as long term bonds, various derivatives, and mortgages, etc, instead of stocks. The "stock" or "Lucas tree" in our model is only a shortcut of local risky security. Therefore we turn to exchange rate volatility.

as the simple average of realized volatility for each currency pair with US dollar in the past 12 month. [16] Table 3.6 reports results for predictive regressions of bilateral exchange rate and global exchange rate volatility at monthly, quarterly, and annual frequencies. An increase in global FX volatility predicts an appreciation of foreign currencies in the next period. The empirical finding is consistent with our model implications.

### *3.6.4. UIP Revisit*

Finally, we rerun the Fama regression after controlling for commercial paper in the US. Table 7 shows the regression results. Panel A shows the results for standard Fama regression for the time period 1992 to 2015 at quarterly frequency in country pairs. For almost all countries, the coefficient is greater than 1. Results are substantially different in panel B after controlling for commercial papers. The coefficient turns insignificant for all countries except Japan. Australia, Canada, Denmark, and UK's coefficients become negative. Italy and Sweden even have a significantly negative coefficient. The contrast implies that commercial paper contains the information that explains both interest rate and exchange rate dynamics. While the amount of commercial paper is tightly associated with intermediary lending and financial constraint tightness, these results provide empirical support for our explanations of the forward premium puzzle.

## 3.7. Conclusion

This paper builds a two-country endowment economy model with financial intermediaries to study the effect of financial intermediary behaviors on exchange rate dynamics. Intermediaries are subject to fluctuating leverage constraints induced by time-varying financial market volatility. We show that the fluctuations in balance sheet conditions can potentially resolve three exchange rate puzzles: the Backus-Smith puzzle, the forward premium puzzle, and the volatility puzzle. We also find empirical evidence consistent with our model.

[16] If we take the first principal component, we get similar results.

Table 3.1: Foreign Exchange Turnovers by Counterparties
Units are in percentage point. Data source: BIS triennial survey for specific years.

| | 1998 | 2001 | 2004 | 2007 | 2010 | 2013 |
|---|---|---|---|---|---|---|
| Reporting dealers | 63 | 59 | 53 | 43 | 38.9 | 39 |
| Nonfinancial customers | 20 | 13 | 13 | 17 | 13.4 | 9 |
| Other financial institutions | 17 | 28 | 33 | 40 | 47.7 | 53 |

Table 3.2: Calibration

| Variable | Notation | Value | Target |
|---|---|---|---|
| Discount factor | $\beta$ | 0.995 | Standard |
| Home bias | $\alpha$ | 0.03 | Colacito-Croce 2013 |
| Nontradable share | $\alpha_l$ | 0.67 | Standard |
| Elasticity | $\sigma$ | 0.9 | Heathcote-Perri 2002 |
| Risk aversion | $\gamma$ | 2 | Standard |
| Mean X | $\mu_x$ | 0 | Risk Free Rate |
| Mean volatility | $\bar{\sigma}$ | 0.006 | Output volatility |
| Volatility persistence | $\rho_\sigma$ | 0.98 | Standard |
| Error Correction | $\tau$ | 0.0005 | Colacito-Croce 2013 |
| X,Y correlation | $\rho_{XY}$ | 0 | Output corrleation |
| Volatility correlation | $\rho_{\eta_X,\eta_Y}$ | 0 | Normalized |
| Exit rate | p | 0.87 | Payout rate |
| Initial intermediary fund | $\xi$ | 0.03 | Payout rate |
| VaR constant | a | 0.36 | Mean of leverage |
| VaR slope | c | 150 | Volatility of leverage |

Table 3.3: Quantitative Results

| Panel A: Country specific moments | | | |
|---|---|---|---|
| | Home | Foreign | Data(US) |
| Output growth std | 0.0046 | 0.0057 | 0.0052 |
| Consumption growth std | 0.0042 | 0.0061 | 0.0051 |
| Leverage mean | 9.35 | 9.28 | 9.24 |
| Leverage std | 1.46 | 2.85 | 1.60 |
| Risk free rate | 0.005 | 0.005 | 0.0023 |
| Equity return | 0.016 | 0.016 | 0.014 |
| Equity return std | 0.016 | 0.023 | 0.085 |
| **Panel B: (Auto)correlation moments** | | | |
| Output growth | | 0.37 | 0.34 |
| Consumption growth | | 0.48 | 0.30 |
| **Panel C: Exchange rate moments** | | | |
| | | Model | Data(US/UK) |
| FX rate change sd | | 0.038 | 0.052 |
| UIP coefficient | | 3.64 | 1.40 |
| Backus Smith correlation | | 0.11 | -0.21 |

Table 3.4: Exchange Rates and US Financial Commercial Paper

We report results for the predictive regression of bilateral exchange rate on US aggregate financial commercial paper year-over-year growth rate at monthly, quarterly and annual frequency. Data covers from 1991 to 2015. The t-statistics are reported in the parenthese, based on heteroscedasticity and autocorrelation consistent standard errors. * significant at 10% confidence level, ** significant at 5% confidence level.

| | AUD | BEF | CAD | CHF | DEU | FRF | GBP | ITY | JPN | NZD | NOK | SWE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Monthly | -0.05** | -0.02 | -0.01 | -0.02 | -0.02 | -0.02 | -0.05** | -0.03* | -0.02 | -0.02 | -0.02 | -0.01 |
| | (-3.25) | (-1.54) | (-1.05) | (-1.50) | (-1.27) | (-1.56) | (-2.91) | (-1.84) | (-1.50) | (-1.56) | (-1.30) | (-1.00) |
| $R^2$ | 0.04 | 0.01 | 0.00 | 0.01 | 0.00 | 0.01 | 0.04 | 0.01 | 0.00 | 0.01 | 0.00 | 0.00 |
| Quarterly | -0.13** | -0.06* | -0.05* | -0.06 | -0.05 | -0.06 | -0.14** | -0.09** | -0.07** | -0.06 | -0.05 | -0.04 |
| | (-3.10) | (-1.64) | (-1.70) | (-1.58) | (-1.33) | (-1.64) | (-3.05) | (-2.30) | (-2.17) | (-1.63) | (-1.28) | (-1.15) |
| $R^2$ | 0.08 | 0.02 | 0.02 | 0.02 | 0.01 | 0.02 | 0.10 | 0.03 | 0.03 | 0.02 | 0.01 | 0.01 |
| Annual | -0.36** | -0.14 | -0.16** | -0.13 | -0.09 | -0.14 | -0.44** | -0.24** | -0.21** | -0.14 | -0.10 | -0.05 |
| | (-2.97) | (-1.24) | (-2.02) | (-1.17) | (-0.82) | (-1.23) | (-3.62) | (-2.46) | (-2.55) | (-1.22) | (-0.62) | (-0.68) |
| $R^2$ | 0.15 | 0.03 | 0.07 | 0.03 | 0.01 | 0.03 | 0.20 | 0.06 | 0.08 | 0.03 | 0.01 | 0.00 |

Table 3.5: Exchange Rates and US Financial Commercial Paper with Controls

We report results for the predictive regression of bilateral exchange rate on US aggregate financial commercial paper year-over-year growth rate at monthly frequency. We add additional controls of investment capital ratio, dividend yield, US stock return and US GDP growth into the predictive regression. Control variables data are from Amit Goyal's website. For the sake of space, we only include the results for monthly regressions. Data covers 1991 to 2015. The t-statistics are reported in the parenthese, based on heteroscedasticity and autocorrelation consistent standard errors. * significant at 10% confidence level, ** significant at 5% confidence level.

| | AUD | BEF | CAD | CHF | DEU | FRF | GBP | ITY | JPN | NZD | NOK | SWE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CP | -0.05** | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.05** | -0.03* | -0.02 | -0.02 | -0.02 | -0.01 |
| | (-3.20) | (-1.48) | (-1.16) | (-1.46) | (-1.20) | (-1.51) | (-3.01) | (-1.81) | (-1.47) | (-1.50) | (-1.34) | (-1.22) |
| I/K | -0.04 | -0.06 | 0.02 | -0.06 | -0.06 | -0.06 | 0.01 | -0.04 | -0.07 | -0.06 | -0.02 | 0.08** |
| | (-0.55) | (-1.25) | (0.53) | (-1.13) | (-1.24) | (-1.22) | (0.20) | (-0.74) | (-1.24) | (-1.27) | (-0.47) | (1.98) |
| $R^2$ | 0.04 | 0.01 | 0.00 | 0.01 | 0.01 | 0.01 | 0.04 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 |
| CP | -0.05** | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.05** | -0.04** | -0.02 | -0.02 | -0.02 | -0.02 |
| | (-3.29) | (-1.38) | (-1.42) | (-1.37) | (-1.61) | (-1.41) | (-2.95) | (-2.06) | (-1.43) | (-1.40) | (-1.24) | (-1.46) |
| DP | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | -0.01 |
| | (-0.17) | (0.25) | (-1.02) | (0.22) | (-0.74) | (0.25) | (0.13) | (-0.68) | (0.09) | (0.25) | (0.18) | (-1.06) |
| $R^2$ | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| CP | -0.05** | -0.02 | -0.01 | -0.02 | -0.02 | -0.02 | -0.05** | -0.03* | -0.02 | -0.02 | -0.02 | -0.01 |
| | (-3.21) | (-1.41) | (-1.07) | (-1.39) | (-1.14) | (-1.44) | (-2.94) | (-1.80) | (-1.40) | (-1.43) | (-1.28) | (-1.16) |
| Ret | -0.03 | -0.07 | 0.02 | -0.06 | -0.07 | -0.07 | 0.00 | -0.04 | -0.06 | -0.07 | -0.03 | 0.07* |
| | (-0.49) | (-1.36) | (0.42) | (-1.24) | (-1.38) | (-1.32) | (0.02) | (-0.77) | (-1.20) | (-1.37) | (-0.52) | (1.72) |
| $R^2$ | 0.04 | 0.01 | 0.00 | 0.01 | 0.01 | 0.01 | 0.04 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 |
| CP | -0.06** | -0.02 | -0.02 | -0.02 | -0.01 | -0.02 | -0.06** | -0.04** | -0.02 | -0.02 | -0.03* | -0.02 |
| | (-3.32) | (-1.59) | (-1.29) | (-1.52) | (-1.00) | (-1.58) | (-3.25) | (-2.18) | (-1.53) | (-1.58) | (-1.90) | (-1.61) |
| $\Delta GDP$ | 0.09 | 0.04 | 0.05 | 0.03 | -0.06 | 0.03 | 0.11 | 0.12 | 0.01 | 0.03 | 0.17 | 0.15 |
| | (0.45) | (0.24) | (0.42) | (0.16) | (-0.32) | (0.17) | (0.49) | (0.71) | (0.08) | (0.18) | (1.20) | (1.35) |
| $R^2$ | 0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 |

Table 3.6: Exchange Rates and Global Realized FX Volatility

We report results for the predictive regression of bilateral exchange rate on global FX volatility. Data covers from 1980 to 2015. We use daily exchange rate data to calculate the realized volatility of exchange rate change in the past 12 months in each currency pair reported in the table, and take the simple average as measure of global FX volatility. Predictive horizons include 1 month, 1 quarter and 1 year ahead.The t-statistics are reported in the parenthese, based on heteroscedasticity and autocorrelation consistent standard errors. * significant at 10% confidence level, ** significant at 5% confidence level.

| | AUD | BEF | CAD | CHF | DEU | FRF | GBP | ITY | JPN | NZD | NOK | SWE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Monthly | 0.02** | 0.02* | 0.01 | 0.02** | 0.02** | 0.02** | 0.01 | 0.02** | 0.02** | 0.02* | 0.02** | 0.02* |
| | (1.99) | (1.90) | (1.17) | (2.11) | (2.00) | (1.95) | (1.04) | (2.16) | (2.19) | (1.93) | (2.34) | (1.90) |
| $R^2$ | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| Quarterly | 0.06** | 0.04* | 0.03 | 0.05** | 0.05* | 0.05* | 0.02 | 0.05** | 0.04* | 0.07** | 0.05** | 0.05** |
| | (2.11) | (1.82) | (1.61) | (2.19) | (1.94) | (1.90) | (1.04) | (2.12) | (1.72) | (2.26) | (2.30) | (2.01) |
| $R^2$ | 0.03 | 0.02 | 0.02 | 0.03 | 0.02 | 0.02 | 0.01 | 0.03 | 0.02 | 0.05 | 0.03 | 0.02 |
| Annual | 0.25** | 0.18* | 0.13** | 0.19** | 0.17 | 0.17* | 0.11** | 0.18* | 0.17** | 0.32** | 0.22** | 0.21** |
| | (4.25) | (1.69) | (3.37) | (2.54) | (1.62) | (1.71) | (1.97) | (1.76) | (2.21) | (5.97) | (3.53) | (3.76) |
| $R^2$ | 0.16 | 0.07 | 0.12 | 0.10 | 0.07 | 0.07 | 0.04 | 0.07 | 0.07 | 0.20 | 0.14 | 0.09 |

Figure 3.1: Impulse Response Functions to a Positive US Endowment Shock

This figure reports the impulse responses to a positive one-standard-deviation US endowment shock. Variables include interest rate (R), equity price (Z), equity return ($R_S$), consumption growth (dc), marginal value of investment for intermediaries ($\nu_s$), marginal cost of taking deposit for intermediaries ($\nu$) and leverage ($\phi$). Impulse responses of both US and UK variables are shown in the same figure. Also, the responses of exchange rate (Q) and US position on international bond ($D_I$, being positive means US is borrowing from UK) are reported.

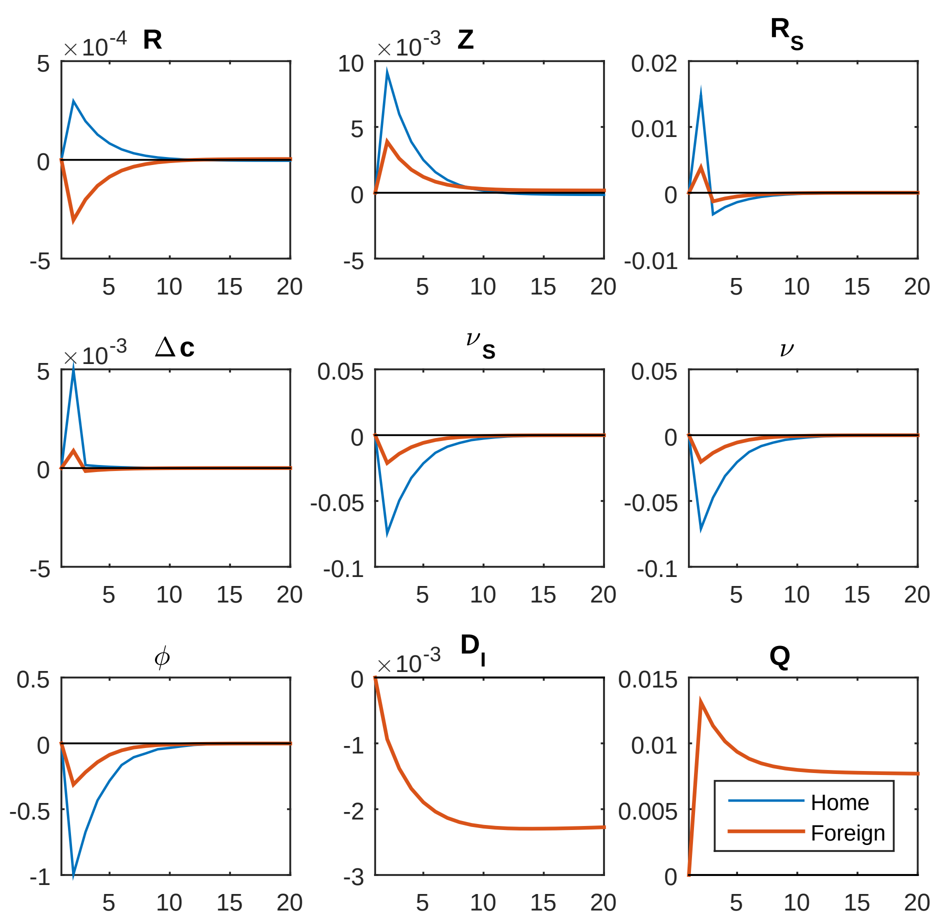
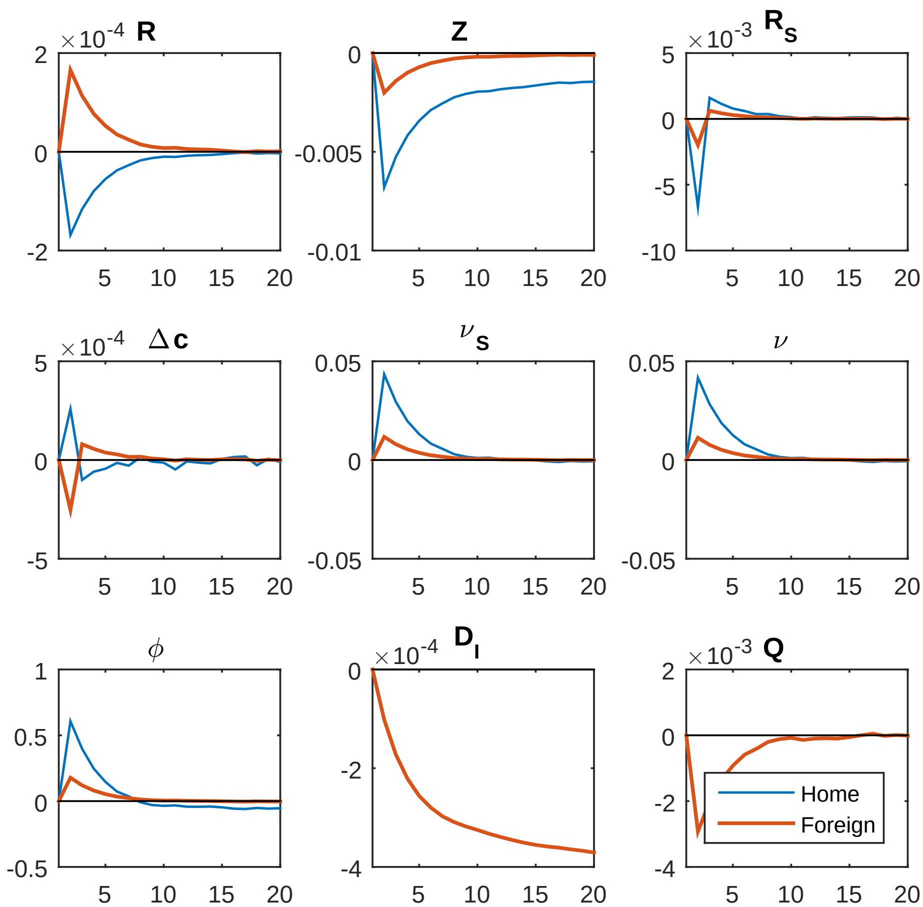


Figure 3.2: Impulse Response Functions to a Positive US Volatility Shock
This table reports the impulse responses to a positive one-standard-deviation US volatility shock. Variables include interest rate (R), equity price (Z), equity return ($R_S$), consumption growth (dc), marginal value of investment for intermediaries ($\nu_S$), marginal cost of taking deposit for intermediaries ($\nu$) and leverage ($\phi$). Impulse responses of both US and UK variables are shown in the same figure. Also, the responses of exchange rate (Q) and US position on international bond ($D_I$, being positive means US is borrowing from UK) are reported.



# APPENDIX

## A.1. Appendix for Chapter 1

### *A.1.1. Data*

The appendix details the data source.

The government debt data are from Dallas Fed, Flow of Funds, and George Hall. Dallas Fed reports the monthly level of par and market values of gross, non-marketable and net debt from 1942 to 2014. Flow of Funds database reports debt held by the Federal Reserve System and rest of the world. George Hall kindly provides debt data from 1926 to 1941.

The stock return and predictors are from Amit Goyal's website. The stock return is the return on S&P 500 index from CRSP. The risk-free rate is the 3-month T-bill. The price-dividend ratio (pd) is the difference between the log of prices and the log of dividends. The dividend yield (dy) is the difference between the log of dividends and the log of lagged prices. The price-earning ratio (pe) is the difference between log of prices and log of earnings. The dividend-earning ratio (d/e) is the difference between log of dividends and log of earnings. Stock return volatility (svar) is the sum of squared daily returns on S&P 500. The book-to-market ratio (bm) is the ratio of book value to market value for the Dow Jones Industrial Average. Net equity expansion (ntis) is the ratio of twelve-month moving sums of net issues by NYSE-listed stocks divided by the total market capitalization of NYSE stocks. Treasury bill rate (tbl) is the 3-month treasury bill rate. Long-term yield (lty) is the long-term government bond yield. Long-term return (ltr) is the long-term government bond return. Term spread (tms) is the difference between the long-term yield on government bonds and the T-bill. Default yield spread (dfy) is the difference between BAA- and AAA- rated corporate bond yields. Inflation (infl) is the CPI inflation. Investment-capital ratio (ik) is the ratio of aggregate (private nonresidential fixed) investment to aggregate capital for the whole economy. Consumption-wealth ratio (cay) is the error correction term calculated from the co-integration of consumption, income, and wealth. Two predictors are constructed separately. GDP gap (gap) is the difference between actual GDP and potential GDP over potential GDP. Potential GDP is from CBO. Government investment-capital ratio (gik) is the ratio of aggregate government investment to aggregate government capital.

Investment-grade and high-yield corporate bond return indices are from Barclay's. Gilchrist and Zakrajŝek (2012) spread is from Simon Gilchrist's website. Moody's Aaa, Aa, A, Baa bond yields are from Bloomberg. 30-year treasury bond yield is from CRSP. General collateral repo rate (Repo) combines two series. It is the banker's acceptance rate from Fred before 1991 and 3-month general collateral repo rate from Bloomberg after 1991. Certificate of Deposits rate (CD) and AA commercial paper rate are from Fred. 1-month and 3-month nominal risk-free rates are from CRSP. Expected inflation in Livingston survey is from Philly Fed. The average return on government debt is from George Hall.

The data on macroeconomic and fiscal variables are from NIPA. In constructing the fiscal uncertainty measure, I use a large panel of quarterly macroeconomic series from FRED-QD database (McCracken and Ng, 2016). I include all the series that are available from 1948 in Group 1: NIPA, Group 2: Industrial Production, Group 3: Employment and Unemployment, Group 5: Inventories, Orders, and Sales, Group 6: Prices, Group 7: Earnings and Productivity, and Group 9: Money and Credit. I exclude other groups about financial markets. These result in 132 macroeconomics series. ID shows the series number in the database. TC denotes the data transformation: (1) no transformation; (2) $\Delta x_t$; (3) $\Delta^2 x_t$; (4)$log(x_t)$;(5)$\Delta log(x_t)$;(6)$\Delta^2 log(x_t)$. (7)$\Delta(x_t/x_{t-1}) - 1$). The transformation follows McCracken and Ng (2016). I augment the database with 37 fiscal-policy-related series from NIPA Table 3.2, 3.3, 3.9.5, and 3.10.5. These variables cover the major components of government receipts, consumption, investment, and transfer in the federal and state and local level. I take the ratio between the fiscal variables and nominal GDP and take the log difference.

| Fiscal Policy Variables | Federal | State and Local |
|---|---|---|
| Current receipts | 3.2 | 3.3 |
| Current tax receipts | 3.2 | 3.3 |
| Personal current taxes | 3.2 | 3.3 |
| Taxes on production and imports | 3.2 | 3.3 |
| Taxes on corporate income | 3.2 | 3.3 |
| Contributions for government social insurance | 3.2 | |
| Current transfer receipts | 3.2 | |
| Capital transfer receipts | 3.2 | 3.3 |
| Current expenditures | 3.2 | 3.3 |
| Consumption expenditures | 3.2 | 3.3 |
| Compensation of general government employees | 3.10.5 | 3.10.5 |
| Consumption of general government fixed capital | 3.10.5 | 3.10.5 |
| Durable goods | 3.10.5 | |
| Nondurable goods | 3.10.5 | |
| Services | 3.10.5 | |
| Current transfer payments | 3.2 | 3.3 |
| Interest payments | 3.2 | 3.3 |
| Subsidies | 3.2 | |
| Gross investment | 3.9.5 | 3.9.5 |
| Structures | 3.9.5 | |
| Equipment | 3.9.5 | |
| Intellectual property products | 3.9.5 | |
| National defense consumption expenditures and gross investment | 3.9.5 | |
| Nondefense consumption expenditures and gross investment | 3.9.5 | |

| ID | TCFRED | MNEMONIC | DESCRIPTION |
|---|---|---|---|
| | | | 1. NIPA |
| 1 | 5 | GDPC96 | Real Gross Domestic Product, 3 Decimal |
| 2 | 5 | PCECC96 | Real Personal Consumption Expenditures |
| 3 | 5 | PCDG | Personal Consumption Expenditures: Durable Goods |
| 4 | 5 | PCESV | Personal Consumption Expenditures: Services |
| 5 | 5 | PCND | Personal Consumption Expenditures: Nondurable Goods |
| 6 | 5 | GPDIC96 | Real Gross Private Domestic Investment, 3 decimal |
| 7 | 5 | FPI | Fixed Private Investment |
| 8 | 5 | Y033RC1Q027SBEA | Gross Private Domestic Investment: Fixed Investment: Nonresidential: Equipment |
| 9 | 5 | PNFI | Private Nonresidential Fixed Investment |
| 10 | 5 | PRFI | Private Residential Fixed Investment |
| 11 | 1 | A014RE1Q156NBEA | Shares of gross domestic product: Gross private domestic investment: Change in private inventories |
| 16 | 5 | EXPGSC96 | Real Exports of Goods and Services, 3 Decimal |
| 17 | 5 | IMPGSC96 | Real Imports of Goods and Services, 3 Decimal |
| 18 | 5 | DPIC96 | Real Disposable Personal Income |
| 19 | 5 | OUTNFB | Nonfarm Business Sector: Real Output |
| 20 | 5 | OUTBS | Business Sector: Real Output |
| 19 | 42 | B020RE1Q156NBEA | Shares of gross domestic product: Exports of goods and services |
| 19 | 52 | B021RE1Q156NBEA | Shares of gross domestic product: Imports of goods and services |
| | | | 2. Industrial Production |
| 22 | 5 | INDPRO | Industrial Production Index |
| 23 | 5 | IPFINAL | Industrial Production: Final Products (Market Group) |
| 24 | 5 | IPCONGD | Industrial Production: Consumer Goods |
| 25 | 5 | IPMAT | Industrial Production: Materials |
| 26 | 5 | IPDMAT | Industrial Production: Durable Materials |
| 28 | 5 | IPDCONGD | Industrial Production: Durable Consumer Goods |
| 29 | 5 | IPB51110SQ | Industrial Production: Durable Goods: Automotive products |
| 30 | 5 | IPNCONGD | Industrial Production: Nondurable Consumer Goods |
| 31 | 5 | IPBUSEQ | Industrial Production: Business Equipment |
| 34 | 1 | CUMFNS | Capacity Utilization: Manufacturing (SIC) |
| 19 | 85 | IPMANSICS | Industrial Production: Manufacturing (SIC) |
| 20 | 11 | NAPMPI | ISM Manufacturing: Production Index |
| 20 | 51 | NAPM | ISM Manufacturing: PMI Composite Index |

| ID | TC | FRED MNEMONIC | DESCRIPTION |
|---|---|---|---|
| | | | 3. Employment and Unemployment |
| 35 | 5 | PAYEMS | All Employees: Total Nonfarm Payrolls |
| 36 | 5 | USPRIV | All Employees: Total Private Industries |
| 37 | 5 | MANEMP | All Employees: Manufacturing |
| 38 | 5 | SRVPRD | All Employees: Service-Providing Industries |
| 39 | 5 | USGOOD | All Employees: Goods-Producing Industries |
| 40 | 5 | DMANEMP | All Employees: Durable goods |
| 41 | 5 | NDMANEMP | All Employees: Nondurable goods |
| 42 | 5 | USCONS | All Employees: Construction |
| 43 | 5 | USEHS | All Employees: Education and Health Services |
| 44 | 5 | USFIRE | All Employees: Financial Activities |
| 45 | 5 | USINFO | All Employees: Information Services |
| 46 | 5 | USPBS | All Employees: Professional and Business Services |
| 47 | 5 | USLAH | All Employees: Leisure and Hospitality |
| 48 | 5 | USSERV | All Employees: Other Services |
| 49 | 5 | USMINE | All Employees: Mining and logging |
| 50 | 5 | USTPU | All Employees: Trade, Transportation and Utilities |
| 51 | 5 | USGOVT | All Employees: Government |
| 52 | 5 | USTRADE | All Employees: Retail Trade |
| 53 | 5 | USWTRADE | All Employees: Wholesale Trade |
| 54 | 5 | CES9091000001 | All Employees: Government: Federal |
| 57 | 5 | CE16OV | Civilian Employment |
| 58 | 2 | CIVPART | Civilian Labor Force Participation Rate |
| 59 | 2 | UNRATE | Civilian Unemployment Rate |
| 60 | 2 | LNS13008397 | Of Total Unemployed, Percent Unemployed Less than 5 Weeks |
| 61 | 2 | LNS13025703 | Of Total Unemployed, Percent Unemployed 27 Weeks and over |
| 62 | 2 | LNS14000012 | Unemployment Rate: 16 to 19 years |
| 63 | 2 | LNS14000025 | Unemployment Rate: 20 years and over, Men |
| 64 | 2 | LNS14000026 | Unemployment Rate: 20 years and over, Women |
| 65 | 5 | UEMPLT5 | Number of Civilians Unemployed for Less Than 5 Weeks |
| 66 | 5 | UEMP5TO14 | Number of Civilians Unemployed for 5 to 14 Weeks |
| 67 | 5 | UEMP15T26 | Number of Civilians Unemployed for 15 to 26 Weeks |
| 68 | 5 | UEMP27OV | Number of Civilians Unemployed for 27 Weeks and Over |
| 74 | 5 | HOABS | Business Sector: Hours of All Persons |
| 76 | 5 | HOANBS | Nonfarm Business Sector: Hours of All Persons |
| 77 | 1 | AWHMAN | Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing |
| 202 | 2 | UEMPMEAN | Average (Mean) Duration of Unemployment |
| 203 | 2 | CES0600000007 | Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing |
| 204 | 1 | NAPMEI | ISM Manufacturing: Employment Index |
| | | | 5: Inventories, Orders, and Sales |
| 94 | 1 | NAPMSDI | ISM Manufacturing: Supplier Deliveries Index |
| 206 | 1 | NAPMNOI | ISM Manufacturing: New Orders Index |
| 207 | 1 | NAPMII | ISM Manufacturing: Inventories Index |

| ID | TCFRED | MNEMONIC | DESCRIPTION |
|---|---|---|---|
| | | **6. Prices** | |
| 96 | 6 | PCECTPI | Personal Consumption Expenditures: Chain-type Price Index |
| 98 | 6 | GDPCTPI | Gross Domestic Product: Chain-type Price Index |
| 99 | 6 | GPDICTPI | Gross Private Domestic Investment: Chain-type Price Index |
| 100 | 6 | IPDBS | Business Sector: Implicit Price Deflator |
| 101 | 6 | DGDSRG3Q086SBEA | PCE: Goods (chain-type price index) |
| 102 | 6 | DDURRG3Q086SBEA | PCE: Durable goods (chain-type price index) |
| 103 | 6 | DSERRG3Q086SBEA | PCE: Services (chain-type price index) |
| 104 | 6 | DNDGRG3Q086SBEA | PCE: Nondurable goods (chain-type price index) |
| 105 | 6 | DHCERG3Q086SBEA | PCE: Services: Household consumption expenditures (chain-type price index) |
| 106 | 6 | DMOTRG3Q086SBEA | PCE: Durable goods: Motor vehicles and parts (chain-type price index) |
| 107 | 6 | DFDHRG3Q086SBEA | PCE: Durable goods: Furnishings and durable household equipment (chain-type price index) |
| 108 | 6 | DREQRG3Q086SBEA | PCE: Durable goods: Recreational goods and vehicles (chain-type price index) |
| 109 | 6 | DODGRG3Q086SBEA | PCE: Durable goods: Other durable goods (chain-type price index) |
| 110 | 6 | DFXARG3Q086SBEA | PCE: Nondurable goods: Food and beverages purchased for off-premises consumption (chain-type price index) |
| 111 | 6 | DCLORG3Q086SBEA | PCE: Nondurable goods: Clothing and footwear (chain-type price index) |
| 112 | 6 | DGOERG3Q086SBEA | PCE: Nondurable goods: Gasoline and other energy goods (chain-type price index) |
| 113 | 6 | DONGRG3Q086SBEA | PCE: Nondurable goods: Other nondurable goods (chain-type price index) |
| 114 | 6 | DHUTRG3Q086SBEA | PCE: Services: Housing and utilities (chain-type price index) |
| 115 | 6 | DHLCRG3Q086SBEA | PCE: Services: Health care (chain-type price index) |
| 116 | 6 | DTRSRG3Q086SBEA | PCE: Transportation services (chain-type price index) |
| 117 | 6 | DRCARG3Q086SBEA | PCE: Recreation services (chain-type price index) |
| 118 | 6 | DFSARG3Q086SBEA | PCE: Services: Food services and accommodations (chain-type price index) |
| 119 | 6 | DIFSRG3Q086SBEA | PCE: Financial services and insurance (chain-type price index) |
| 120 | 6 | DOTSRG3Q086SBEA | PCE: Other services (chain-type price index) |
| 121 | 6 | CPIAUCSL | Consumer Price Index for All Urban Consumers: All Items |
| 123 | 6 | PPIFGS | Producer Price Index by Commodity for Finished Goods |
| 124 | 6 | PPIACO | Producer Price Index for All Commodities |
| 125 | 6 | PPIFCG | Producer Price Index by Commodity for Finished Consumer Goods |
| 126 | 6 | PPIFCF | Producer Price Index by Commodity for Finished Consumer Foods |
| 127 | 6 | PPIIDC | Producer Price Index by Commodity Industrial Commodities |
| 128 | 6 | PPIITM | Producer Price Index by Commodity Intermediate Materials: Supplies and Components |
| 129 | 1 | NAPMPRI | ISM Manufacturing: Prices Index |
| 131 | 5 | WPU0561 | Producer Price Index by Commodity for Fuels and Related Products and Power: Crude Petroleum (Domestic Production) |
| 214 | 6 | PPICRM | Producer Price Index by Commodity for Crude Materials for Further Processing |
| 215 | 6 | PPICMM | Producer Price Index by Commodity Metals and metal products: Primary nonferrous metals |
| 216 | 6 | CPIAPPSL | Consumer Price Index for All Urban Consumers: Apparel |
| 217 | 6 | CPITRNSL | Consumer Price Index for All Urban Consumers: Transportation |
| 218 | 6 | CPIMEDSL | Consumer Price Index for All Urban Consumers: Medical Care |
| 220 | 6 | CUUR0000SAD | Consumer Price Index for All Urban Consumers: Durables |
| 222 | 6 | CPIULFSL | Consumer Price Index for All Urban Consumers: All Items Less Food |
| 223 | 6 | CUUR0000SA0L2 | Consumer Price Index for All Urban Consumers: All items less shelter |

| ID | TCFRED MNEMONIC | DESCRIPTION |
|---|---|---|
| | | 7. Earnings and Productivity |
| 1345 | CES2000000008 | Average Hourly Earnings of Production and Nonsupervisory Employees: Construction |
| 1355 | CES3000000008 | Average Hourly Earnings of Production and Nonsupervisory Employees: Manufacturing |
| 1375 | COMPRNFB | Nonfarm Business Sector: Real Compensation Per Hour |
| 1385 | RCPHBS | Business Sector: Real Compensation Per Hour |
| 1405 | OPHNFB | Nonfarm Business Sector: Real Output Per Hour of All Persons |
| 1415 | OPHPBS | Business Sector: Real Output Per Hour of All Persons |
| 1425 | ULCBS | Business Sector: Unit Labor Cost |
| 1445 | ULCNFB | Nonfarm Business Sector: Unit Labor Cost |
| 1455 | UNLPNBS | Nonfarm Business Sector: Unit Nonlabor Payments |
| 2256 | CES0600000008 | Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing |
| | | 9. Money and Credit |
| 1625 | AMBSLREAL | Real St. Louis Adjusted Monetary Base |
| 1675 | BUSLOANS | Commercial and Industrial Loans, All Commercial Banks |
| 1685 | CONSUMER | Consumer Loans at All Commercial Banks |
| 1695 | NONREVSL | Total Nonrevolving Credit Owned and Securitized, Outstanding |
| 1705 | REALLN | Real Estate Loans, All Commercial Banks |
| 1725 | TOTALSL | Total Consumer Credit Owned and Securitized, Outstanding |
| 2266 | DTCOLNVHFNM | Consumer Motor Vehicle Loans Owned by Finance Companies, Outstanding |
| 2276 | DTCTHFNM | Total Consumer Loans and Leases Owned and Securitized by Finance Companies, Outstanding |
| 2286 | INVEST | Securities in Bank Credit at All Commercial Banks |

### *A.1.2. Time-varying Fiscal Uncertainty in a Pareto Problem*

I show that higher debt-to-GDP causes higher uncertainty in future tax policy in a simple setting where a benevolent government wants to smooth tax.

#### A.1.2.1. Taxation

Potential GDP is $A_t$ without tax distortion. Assume tax rate $\tau_t$ leads to distortion $A_t f(\tau_t)$ and results in actually GDP $Y_t = A_t(1 - f(\tau_t))$. The distortion function has the property that $0 < f < 1$, $f' > 0$, $f'' > 0$ . The total tax receipt is $T_t = A_t(1 - f(\tau_t))\tau_t$. Laffer curve, which plots tax rate $\tau_t$ against tax receipt $T_t$, is concave, and has two roots at 0 and 1. Under zero tax rate,there is no tax receipt and no distortion $f(\tau_t) = 0$. Under 100% tax rate, there is 100% distortion $f(\tau_t) = 1$ and tax receipt is also zero. The slope

$$\frac{dT_t}{d\tau_t} = A_t(1 - f(\tau_t)'\tau_t - \tau_t)$$

is positive until tax rate reaches the peak $\frac{dT_t}{d\tau_t}|_{\tau=\bar{\tau}} = 0$, $\bar{\tau} \in (0,1)$, and then become negative. This can also be seen from the second derivative.

$$\frac{dT_t^2}{d\tau_t^2} = -A_t(f(\tau_t)''\tau_t + f(\tau_t)' + 1) < 0$$

I follow the common assumption that the tax rate is always below $\bar{\tau}$, since otherwise the high tax is pure distortion.

A.1.2.2. Pareto Problem

Discount rate r is fixed and given. The benevolent government wants to maximize the output. It has to finance the exogenous spending need either through taxation or government, given the initial debt level $B_0$.

$$max_{\tau_t} E_0 \Sigma_{t=1}^{\infty}(1+r)^{-t} A_t(1 - f(\tau_t))$$

s.t.

$$B_0 = E_0 \Sigma_{t=1}^{\infty}[(1+r)^{-t}(T_t - G_t)]$$

The first order condition $f(\tau_t)' - \lambda(1 - f(\tau_t)'\tau_t) = 0$ leads to the classic tax smoothing results that the government wants to keep the tax rate constant.

Given A, G at the steady state , the optimal steady state tax rate$\tau_{ss}$ will solve

$$A(1 - f(\tau))\tau = T = rB_0 + G$$

A.1.2.3. Government Debt and Fiscal Uncertainty

We first consider a spending shock. If spending increases in period 1, $G_1 = (1+\epsilon)G$. The optimal tax rate$\tau$ will solve

$$A(1-f(\tau))\tau = T = rB_0 + G + \frac{r}{1+r}\epsilon G$$

Tax smoothing occurs. Tax receipt increases by a fraction $\frac{r}{1+r}$ of the increase of spending.

Fiscal uncertainty is measured by the conditional volatility of tax rate, which depends on the impulse response of $\tau_t$ to spending shock $\epsilon$.

$$\frac{\partial \tau_t}{\partial \epsilon} = \frac{\partial \tau_t}{\partial T_t}\frac{\partial T_t}{\partial \epsilon}$$

The sensitivity of tax receipt on spending is fixed $\frac{\partial T_t}{\partial \epsilon}$. However, in order to collect the same amount of tax receipt, the tax rate change depends on the Laffer curve.

$$\frac{\partial \tau_t}{\partial \epsilon} = \frac{\partial \tau_t}{\partial T_t}\frac{\partial T_t}{\partial \epsilon}$$

Specifically, when initial debt is high, the initial tax rate is closer to the Laffer curve peak. Therefore, the government need to move tax rate a lot more to collect enough tax receipt than the case when debt is low.

$$\frac{\partial}{\partial B_0}\frac{\partial \tau_t}{\partial T_t}|_{\tau_t=\tau_{ss}} = \frac{\partial^2 \tau_t}{\partial T_t^2}\frac{\partial \tau_{ss}}{\partial B_0} > 0$$

We have similar effect when GDP shock hits the economy. If potential GDP decreases in period 1, $A_1 = (1-\epsilon)A$. The optimal tax rate$\tau$ will solve

$$A(1-f(\tau))\tau = T = \frac{1+r}{1+r-r\epsilon}(rB_0 + G)$$

In recession, the tax rate has to increase. Following the same logic, the response to a GDP shock increases with $B_0$.

$$\frac{\partial}{\partial B_0}\frac{\partial \tau_t}{\partial T_t}|_{\tau_t=\tau_{ss}} = \frac{\partial^2 \tau_t}{\partial T_t^2}\frac{\partial \tau_{ss}}{\partial B_0} > 0$$

Therefore, high debt $B_0$ increases the sensitivity of tax rate to fundamental shocks. This generates fiscal uncertainty even though the government is optimally making fiscal policy.

## A.2. Appendix for Chapter 2

### *A.2.1. Volatility Estimation*

We use an auxiliary mixture sampler to estimate the model specified in (2.1) and extract latent volatility components, following Kim, Shephard, and Chib (1998). Specifically, we rewrite the observation equation,

$$log((z_t - \mu - \rho z_{t-1})^2) = \sigma_t + log(\eta_t^2). \tag{A.1}$$

The distribution of $log(\eta_t^2)$ can be well approximated by a mixture of Gaussian distributions:

$$p(log(\eta_t^2)) = \Sigma_{i=1}^n \pi_i \varphi(\eta_t; \mu_{,\eta i}, \sigma_{\eta,i}^2), \tag{A.2}$$

where $\varphi$ is the probability density function of a Gaussian distribution with mean $\mu_{\eta,i}$ and standard deviation $\sigma_{\eta,i}$. In the Markov Chain Monte Carlo procedure, $s_t \in [1,\ T]$ is drawn to indicate one Gaussian distribution to sample $log(\eta_t^2)$. Conditioning on $s_t$, the model is in Gaussian linear state-space form, and a standard forward-filtering, backward-sampling scheme can be applied. The algorithm thus takes the form:

1. Initialize $\mu,\ \rho,\ \mu_\sigma,\ \nu,\ \sigma_\omega,\ s_t$

2. Sample $\sigma_t$ from $p(\sigma_t|z, \mu,\ \rho,\ \mu_\sigma,\ \nu,\ \sigma_\omega,\ s_t, z)$

3. Sample $s_t$ from $p(s_t = i) \propto \pi_i \varphi(log((z_t - \mu - \rho z_{t-1})^2); \sigma_t + \mu_{\eta,i}, \sigma_{\eta,i}^2)$

4. Sample $\mu,\ \rho,\ \mu_\sigma,\ \nu,\ \sigma_\omega$ from $p(\mu,\ \rho,\ \mu_\sigma,\ \nu,\ \sigma_\omega|\sigma_t,\ z)$

5. Repeat 2–4 until convergence

In our empirical implementation the priors are very loose: $\mu \sim N(0.005, 2)$, $\rho \sim N(0.3, 100)$, $\mu_\sigma \sim$

$N(-10, 20), \nu \sim N(0.9, 0.5)$, and $\sigma_\omega \sim IG(2, 0.5)$. We sample 20,000 times and discard the first 5,000. The posterior mean of $\sigma_t$ is the volatility used in the empirical analysis.

### *A.2.2. Robustness of Empirical Results*

In this section, we verify that our key empirical evidence on the volatility risk sharing is quite robust to three modifications of our benchmark analysis. First of all, we show that our VAR results are not specific to the US by substituting the US with a global aggregate. Specifically, we replace US variables with cross-sectional averages of the corresponding variables across G17 countries ('Global Benchmark' case).

Second, we assess our VAR results by looking at the US against the remaining G7 countries, assuming that they all share the same parameters in equation (2.1) ('US/Pooled G7' case). Third, we run our benchmark empirical estimation adopting a 2-lag VAR specification.

As shown in table A.2.2, our main empirical results are quite robust to all the specifications. Relative consumption and output generally decline and consumption volatility increases due to volatility shocks, and the effects on consumption are smaller than on output. The relative magnitudes of the effects are quite stable across the specifications. For example, the measures of the volatility pass-through are in 0.5-0.6 range, consistent with our benchmark estimates.

In table A.2.2, we provide further robustness checks for our main results. For parsimony, we report the estimates of the volatility pass-through based on the relative volatility shock, the US volatility shock, and the difference between the Foreign and US volatility shock. The confidence interval for the latter allows us to assess a statistical significance of the size effect in volatility pass through.

In Panel A, we show that our results remain unchanged when we let the volatility processes be the 'least primitive' processes in the context of the Cholesky decomposition, by replacing equations (2.3) and (2.6) with

$$\tilde{Y}_{i,t} = \begin{bmatrix} \Delta y_i - \Delta y_{US} \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \\ \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \end{bmatrix}, \tag{A.3}$$

Table A1: Robustness of Pass-Through Results

Notes: Panel A shows the estimates of the contemporaneous responses ($\widetilde{\Sigma}_{1j}$) of the VAR specified in equations (2.2)–(2.3) with respect to a shock to relative output volatility. Responses of output growth, consumption growth, and the net-exports-to-output ratio are annualized, in percentages. Volatility pass-through is defined as in equation (2.4). "Global Benchmark" is defined as the average of the corresponding series across all countries. In "Pooled G7" specification, we estimate macroeconomic volatility assuming that the volatility parameters are the same across G7 countries except the US. "VAR(2)" is the 2-lag VAR with US/G7 countries. Panel B reports pass-through measures based on the estimates of the VAR in equations (2.5)–(2.6) with respect to volatility shocks affecting either the benchmark or the remaining countries. We report 95% credible intervals in brackets. Our quarterly data range from 1971:q1 to 2013:q4.

**Panel A: Contemporaneous adjustments to relative volatility shocks**

| $\sigma(\Delta y)$ | $\Delta y$ | $\sigma(\Delta c)$ | $\Delta c$ | $\Delta(NX/Y)$ | Pass-through |
|---|---|---|---|---|---|
| *Global Benchmark/G7 Countries:* | | | | | |
| 0.14 | -0.27 | 0.07 | -0.10 | -0.16 | 0.54 |
| [0.14; 0.15] | [-0.51; -0.03] | [0.06; 0.07] | [-0.28;0.08] | [-0.36; 0.03] | [0.49; 0.58] |
| *US/Pooled G7:* | | | | | |
| 0.19 | -0.52 | 0.09 | -0.26 | -0.26 | 0.53 |
| [0.19; 0.20] | [-0.83; -0.23] | [0.08; 0.10] | [-0.50;-0.02] | [-0.49; -0.03] | [0.49; 0.56] |
| *VAR(2) Model:* | | | | | |
| 0.21 | -0.41 | 0.09 | -0.11 | -0.29 | 0.59 |
| [0.20; 0.21] | [-0.71; -0.11] | [0.08; 0.09] | [-0.34; 0.13] | [-0.53; -0.06] | [0.55; 0.62] |

**Panel B: Pass-through and size**

| | Origin of Vol. Shock: | |
|---|---|---|
| | U.S. | Foreign Country |
| Global Benchmark/G7 Countries: | 0.59 | 0.53 |
| | [0.45; 0.72] | [0.48; 0.58] |
| *US/Pooled G7:* | 0.47 | 0.64 |
| | [0.43; 0.52] | [0.58; 0.70] |
| *VAR(2) Model:* | 0.55 | 0.63 |
| | [0.50; 0.60] | [0.58; 0.68] |

and

$$Y'_{i,t} = \Big[\Delta y_i \quad \Delta y_{US} \quad \sigma_t(\Delta y_i) \quad \sigma_t(\Delta y_{US}) \quad \sigma_t(\Delta c_i) \quad \sigma_t(\Delta c_{US})\Big], \tag{A.4}$$

respectively. We have tested other orders as well, with similar results. We omit them for the sake of brevity.

In Panel B, we augment our benchmark VAR specifications by introducing a global volatility component as follows:

$$\tilde{Y}_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_{global}) \\ \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix}, \tag{A.5}$$

$$Y'_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_{global}) & \sigma_t(\Delta y_i) & \sigma_t(\Delta y_{US}) & \Delta y_i & \Delta y_{US} & \sigma_t(\Delta c_i) & \sigma_t(\Delta c_{US}) \end{bmatrix}. \tag{A.6}$$

We consider both an average measure computed across our 17 countries, and a measure based on the first principle component (henceforth "PC") of country volatilities from our cross-section.

Since our VAR specifications impose that the parameters of interest must be the same across countries, this estimation procedure implicitly assumes that all countries have the same exposure to the global volatility component. In Panel C, we relax this assumption by reporting our results when we relax this assumption by using the idiosyncratic, rather than total, output volatilities in equations (2.3) and (2.6)

$$\tilde{Y}_{i,t} = \begin{bmatrix} \widehat{\sigma}_t(\Delta y_i) - \widehat{\sigma}_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix}, \tag{A.7}$$

and

$$Y'_{i,t} = \begin{bmatrix} \widehat{\sigma}_t(\Delta y_i) & \widehat{\sigma}_t(\Delta y_{US}) & \Delta y_i & \Delta y_{US} & \sigma_t(\Delta c_i) & \sigma_t(\Delta c_{US}), \end{bmatrix} \tag{A.8}$$

where the idiosyncratic volatility for country $i$, $\widehat{\sigma}_t(\Delta y_i)$ is the residual of the following regression:

$$\sigma_t(\Delta y_i) = \overline{\sigma}^i + \beta^i_\sigma \sigma_t(\Delta y_{global}) + \widehat{\sigma}_t(\Delta y_i). \tag{A.9}$$

All of these specifications confirm our two main findings: (i) among large countries, the pass-through

Table A2: Robustness of Pass-Through Results (II)

Notes: The second column refers to pass-through measures obtained from variations of the VAR specified in equations (2.2)–(2.3) with respect to a shock to relative output volatility. The rightmost two columns report pass-through measures based on the modified estimates of the VAR in equations (2.5)–(2.6) with respect to volatility shocks affecting either the US or the remaining countries. The last column reports the difference in pass-through with respect to the US. In Panel A, variables are sorted as in equations (A.3)–(A.4). In Panel B, we add a global vol measure to our VAR, as specified in equations (A.5)–(A.6). We consider both a cross-country average of volatility and a measure obtained through a principal components approach (PC). In Panel C, we focus on the VAR specified in equations (A.7)–(A.8) with country-specific volatility processes estimated as in equations (A.9). We report 95% credible intervals in brackets. Our quarterly data range from 1971:q1 to 2013:q4.

| | Relative Vol Shock: | Origin of Vol Shock: | |
|---|---|---|---|
| | | US | Foreign-US (Diff.) |
| **Panel A: Different Cholesky Order** | | | |
| *US/G7* | 0.52 | 0.49 | 0.08 |
| | [0.47; 0.56] | [0.43; 0.54] | [0.00; 0.16] |
| *US/Bottom-10 G17* | 0.61 | 0.50 | 0.22 |
| | [0.56; 0.66] | [0.44; 0.57] | [0.13; 0.31] |
| **Panel B: Controlling for Global Vol** | | | |
| *US/G7 (Average)* | 0.52 | 0.51 | 0.03 |
| | [0.48; 0.57] | [0.45; 0.57] | [-0.06; 0.12] |
| *US/Bottom-10 G17 (Average)* | 0.61 | 0.52 | 0.18 |
| | [0.57; 0.66] | [0.45; 0.59] | [0.08; 0.27] |
| *US/G7 (PC)* | 0.53 | 0.53 | 0.00 |
| | [0.49; 0.57] | [0.48; 0.59] | [-0.08; 0.09] |
| *US/Bottom-10 G17 (PC)* | 0.63 | 0.58 | 0.10 |
| | [0.59; 0.68] | [0.52; 0.65] | [0.00; 0.19] |
| **Panel C: Heterogenous Exposure to Global Vol** | | | |
| *US/G7 (Average)* | 0.56 | 0.55 | 0.02 |
| | [0.51; 0.60] | [0.50; 0.61] | [-0.08; 0.11] |
| *US/Bottom-10 G17 (Average)* | 0.66 | 0.59 | 0.14 |
| | [0.61; 0.70] | [0.52; 0.65] | [0.05; 0.24] |
| *US/G7 (PC)* | 0.54 | 0.53 | 0.02 |
| | [0.49; 0.58] | [0.47; 0.59] | [-0.07; 0.11] |
| *US/Bottom-10 G17 (PC)* | 0.63 | 0.59 | 0.08 |
| | [0.58; 0.68] | [0.52; 0.65] | [0.00; 0.18] |

is about 0.50; and (ii) the pass-through is economically and statistically larger for smaller countries.

### *A.2.3. Standard Moments from the Model*

In table A.2.3, we focus on unconditional moments typically targeted in the international finance

Table A3: Standard Unconditional Moments

Notes: This table reports key moments for real consumption ($C$), output ($Y$), the exchange rate ($E$), the risk-free rates ($R^f$), the net-export-to-output ratio ($NX/Y$), and the stochastic discount factor ($M$). Small letters refer to log-units; changes are denoted by '$\Delta$'; foreign variables are marked by '$^*$'. We denote expectation, standard deviation, correlation, and first order auto-correlation by $E$, $\sigma$, $corr$, and $ACF1$, respectively. The data refer to G-17 countries and are described in section 2.2.1. For each country, we compute the moments of interest over the post–Bretton Wood period, 1971:Q1–2013:Q4, as detailed in section 2.2.1. For each moment, we report (i) its GDP-weighted average across countries; and (ii) its first and fourth cross-country quintiles. The entries from the model are obtained from 100 repetitions of small samples. Our benchmark quarterly calibration is reported in table 2.3.1.

| | G-17 Data | | Model | | |
|---|---|---|---|---|---|
| | Avg. | Quintiles [ $1^{st}$; $4^{th}$ ] | Bench-mark | No TVV ($\sigma_\sigma = 0$) | CRRA ($\gamma = 7$) |
| $corr(\Delta c, \Delta c^*)$ | 0.25 | [0.13; 0.33] | 0.38 | 0.37 | 0.74 |
| $\sigma(\Delta c)(\%)$ | 1.67 | [1.34; 2.47] | 1.85 | 1.82 | 1.64 |
| $\sigma(\Delta c)/\sigma(\Delta y)$ | 0.88 | [0.57; 0.82] | 0.93 | 0.94 | 0.83 |
| $ACF1(\Delta c)$ | 0.17 | [-0.16; 0.31] | 0.06 | 0.07 | 0.08 |
| | | | | | |
| $\sigma(M)/E(M)(\%)$ | – | – | 47.86 | 47.85 | 11.49 |
| $\sigma(\Delta e)(\%)$ | 10.50 | [10.2; 11.4] | 12.80 | 12.65 | 8.31 |
| | | | | | |
| $E(r^f)(\%)$ | 1.35 | [1.44; 2.41] | 2.17 | 2.19 | 14.91 |
| $\sigma(r^f)(\%)$ | 1.79 | [1.61; 2.27] | 0.33 | 0.33 | 3.47 |
| $corr(r^f, r^{f*})$ | 0.51 | [0.37; 0.56] | 0.91 | 0.92 | 0.98 |
| $\sigma(\Delta(NX/Y))/\sigma(\Delta y)$ | 0.70 | [0.67; 0.97] | 0.32 | 0.32 | 0.16 |

literature. Our benchmark calibration conforms well with our data, both with and without volatility shocks. The adoption of CRRA preferences generates well-known puzzles: (i) the market price of risk is excessively low; (ii) the risk-free rate is too high; and (iii) international trade is modest. In our model the net exports are not as volatile as in our G17 dataset, but they are twice as volatile compared to the CRRA case.

# BIBLIOGRAPHY

Tobias Adrian and Hyun Song Shin. The changing nature of financial intermediation and the financial crisis of 2007–2009. *Annual Review of Economics*, 2:603–618, 2010.

Tobias Adrian and Hyun Song Shin. Procyclical leverage and value-at-risk. *Review of Financial Studies*, 27(2):373–403, 2014.

Tobias Adrian, Erkko Etula, and Hyun Song Shin. Risk appetite and exchange rates. 2015.

E. Anderson. The dynamics of risk-sensitive allocations. *Journal of Economic Theory*, 125(2):93–150, 2005.

Martin M Andreasen, Jesús Fernández-Villaverde, and Juan Rubio-Ramírez. The pruned state-space system for non-linear dsge models: Theory and empirical applications. working paper, National Bureau of Economic Research, 2013.

Andrew Ang and Geert Bekaert. Stock return predictability: Is it there? *Review of Financial studies*, 20(3):651–707, 2007.

Andrew Ang, Geert Bekaert, and Min Wei. The term structure of real rates and expected inflation. *The Journal of Finance*, 63(2):797–849, 2008.

Philippe Bacchetta and Eric Van Wincoop. Infrequent portfolio decisions: A solution to the forward discount puzzle. *The American Economic Review*, 100(3):870–904, 2010.

David K. Backus and Gregor Smith. Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, 35:297–316, 1993a.

David K Backus and Gregor W Smith. Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, 35(3):297–316, 1993b.

David K Backus, Patrick J Kehoe, and Finn E Kydland. International real business cycles. *Journal of political Economy*, pages 745–775, 1992.

David K Backus, Patrick J Kehoe, and Finn E Kydland. Dynamics of the Trade Balance and the Terms of Trade: The J-Curve? *American Economic Review*, 84(1):84–103, March 1994.

David K. Backus, Federico Gavazzoni, Christopher Telmer, and Stanley E. Zin. Monetary policy and the uncovered interest parity puzzle. Working paper, NBER, July 2010.

Yan Bai and José-Víctor Ríos-Rull. Demand shocks and open economy puzzles. *The American Economic Review*, 105(5):644–649, 2015.

Scott R Baker, Nicholas Bloom, and Steven J Davis. Measuring economic policy uncertainty. Working paper, National Bureau of Economic Research, 2015.

Scott R. Baker, Nicholas Bloom, and Steven J. Davis. Measuring economic policy uncertainty. *Quarterly Journal of Economics*, page forthcoming, 2016.

Narcissa Balta and Juan Delgado. Home bias and market integration in the EU. *CESifo Economic Studies*, 55(1):110–44, 2009.

Ravi Bansal. An exploration of the forward premium puzzle in currency markets. *Review of Financial Studies*, 10(2):369–403, 1997.

Ravi Bansal and Wilbur John Coleman. A monetary explanation of the equity premium, term premium, and risk-free rate puzzles. *Journal of Political Economy*, pages 1135–1171, 1996.

Ravi Bansal and Ivan Shaliastovich. A long-run risks explanation of predictability puzzles in bond and currency markets. *Review of Financial Studies*, 26(1):1–33, 2013.

Ravi Bansal and Amir Yaron. Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance*, 59(4):1481–1509, 2004.

Ravi Bansal, Dana Kiku, Ivan Shaliastovich, and Amir Yaron. Volatility, the macroeconomy, and asset prices. *The Journal of Finance*, 69(6):2471–2511, 2014a.

Ravi Bansal, Dana Kiku, Ivan Shaliastovich, and Amir Yaron. Volatility, the Macroeconomy, and Asset Prices. *Journal of Finance*, 69(6):2471–2511, December 2014b.

Robert J Barro. Are government bonds net wealth? *Journal of Political Economy*, 82(6): 1095–1117, 1974.

Susanto Basu and Brent Bundick. Uncertainty shocks in a model of effective demand. Working paper, NBER, 2012.

Geert Bekaert. The time variation of risk and return in foreign exchange markets: A general equilibrium perspective. *Review of Financial Studies*, 9(2):427–470, 1996.

Frederico Belo and Jianfeng Yu. Government investment and the stock market. *Journal of Monetary Economics*, 60(3):325–339, 2013.

Kimberly A. Berg and Nelson C. Mark. Global macro risks in currency excess returns. *Working Paper*, page Bank of Canada and University of Notre Dame, 2016.

Ben Bernanke and Mark Gertler. Agency costs, net worth, and business fluctuations. *The American Economic Review*, pages 14–31, 1989.

Antje Berndt, Hanno Lustig, and Şevin Yeltekin. How does the us government finance fiscal shocks? *American Economic Journal: Macroeconomics*, 4(1):69–104, 2012.

Huixin Bi, Eric M Leeper, and Campbell Leith. Uncertain fiscal consolidations. *The Economic Journal*, 123(566):F31–F63, 2013.

Geir Høidal Bjønnes and Dagfinn Rime. Dealer behavior and trading systems in foreign exchange markets. *Journal of Financial Economics*, 75(3):571–605, 2005.

Nicholas Bloom. The impact of uncertainty shocks. *Econometrica*, 77(3):623–685, 2009a.

Nick Bloom. The impact of uncertainty shocks. *Econometrica*, 77:623–89, 2009b.

Nick Bloom, Stephen Bond, and John Van Reenen. Uncertainty and investment dynamics. *Review of Economic Studies*, 74:391–415, 2007.

Nick Bloom, Max Floetotto, Nir Jaimovich, Itay Saporta, and Stephen Ter. Really uncertain business cycle. *Working Paper*, 2016.

Henning Bohn. The sustainability of fiscal policy in the united states. *CESifo Working Paper Series*, 2005.

Benjamin Born and Johannes Pfeifer. Policy risk and the business cycle. *Journal of Monetary Economics*, 68:68 – 85, 2014.

Markus K Brunnermeier and Lasse Heje Pedersen. Market liquidity and funding liquidity. *Review of Financial studies*, 22(6):2201–2238, 2009.

Craig Burnside, Bing Han, David Hirshleifer, and Tracy Yue Wang. Investor overconfidence and the forward premium puzzle. *The Review of Economic Studies*, 78(2):523–558, 2011.

John Y. Campbell and John H. Cochrane. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107:205–251, 1999.

John Y Campbell and Robert J Shiller. The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies*, 1(3):195–228, 1988.

John Y Campbell and Samuel B Thompson. Predicting excess stock returns out of sample: Can anything beat the historical average? *Review of Financial Studies*, 21(4):1509–1531, 2008.

John Y Campbell and Motohiro Yogo. Efficient tests of stock return predictability. *Journal of Financial Economics*, 81(1):27–60, 2006.

Varadarajan V Chari, Patrick J Kehoe, and Ellen R McGrattan. Can sticky price models generate volatile and persistent real exchange rates? *The Review of Economic Studies*, 69(3):533–563, 2002.

Mikhail Chernov, Jeremy J Graveline, and Irina Zviadadze. Crash risk in currency returns. Working paper, 2014.

YiLi Chien, Harold Cole, and Hanno Lustig. Is the volatility of the market price of risk due to intermittent portfolio rebalancing? *American Economic Review*, 102(6):2859–96, May 2012.

Horag Choi, Steven Lugauer, and Nelson C. Mark. Precautionary saving of chinese and us households. *Working Paper*, page Monash University and University of Notre Dame, 2014.

John H Cochrane. Production-based asset pricing and the link between stock returns and economic fluctuations. *The Journal of Finance*, 46(1):209–237, 1991.

John H Cochrane. Presidential address: Discount rates. *The Journal of Finance*, 66(4): 1047–1108, 2011.

Timothy Cogley and Thomas J. Sargent. Drifts and volatilities: monetary policies and outcomes in the post-WWII US. *Review of Economic Dynamics*, 8(2):262–302, 2005.

Riccardo Colacito. Six anomalies looking for a model. a consumption based explanation of international finance puzzles. Working paper, University of North Carolina, Chapel Hill, 2008.

Riccardo Colacito and Mariano M. Croce. Risks for the long run and the real exchange rate. *Journal of Political Economy*, 119(1):153–81, 2011a.

Riccardo Colacito and Mariano M. Croce. Recursive allocations and wealth distribution with multiple goods. Working paper, University of North Carolina, Chapel Hill, 2012.

Riccardo Colacito and Mariano M. Croce. International asset pricing with recursive preferences. *Journal of Finance*, 68(6):2651–86, 2013a.

Riccardo Colacito and Mariano M Croce. International asset pricing with recursive preferences. *The Journal of Finance*, 68(6):2651–2686, 2013b.

Riccardo Colacito and Mariano Massimiliano Croce. Risks for the long-run and the real exchange rate. *Journal of Political Economy*, 119(1), 2011b.

Harold L. Cole and Maurice Obstfeld. Commodity trade and international risk sharing : How much do financial markets matter? *Journal of Monetary Economics*, 28(1):3–24, August 1991.

Diego Comin and Mark Gertler. Medium-term business cycles. *The American Economic Review*, 96(3):523–551, 2006.

Ilan Cooper and Richard Priestley. Time-varying risk premiums and the output gap. *Review of Financial Studies*, 22(7):2801–2833, 2009.

Giancarlo Corsetti, Luca Dedola, and Sylvain Leduc. International risk sharing and the transmission of productivity shocks. *The Review of Economic Studies*, 75(2):443–473, 2008.

Pasquale Della Cortet, Lucio Sarno, and Ilias Tsiakas. An Economic Evaluation of Empirical Exchange Rate Models. *Review of Financial Studies*, 22(9):2491–530, 2009.

Mariano Massimiliano Croce. Long-run productivity risk: A new hope for production-based asset pricing? *Journal of Monetary Economics*, 66:13–31, 2014.

Mariano Massimiliano Croce, Howard Kung, Thien T Nguyen, and Lukas Schmid. Fiscal policies and asset prices. *Review of Financial Studies*, 25(9):2635–2672, 2012.

Massimiliano Mariano Croce, Thien Nguyen, Steve McGregor Raymond, and Lukas Schmid. Government debt and the returns to innovation. Working paper, 2016.

Luca Dedola, Peter Karadi, and Giovanni Lombardo. Global implications of national unconventional policies. *Journal of Monetary Economics*, 60(1):66–85, 2013.

Pablo D'Erasmo, Enrique G Mendoza, and Jing Zhang. What is a sustainable public debt? In *Handbook of Macroeconomics*, volume 2B. Elsevier, 2016.

Michael B Devereux and Alan Sutherland. Country portfolios in open economy macromodels. *Journal of the European Economic Association*, 9(2):337–369, 2011.

Simeon Djankov, Tim Ganser, Caralee McLiesh, Rita Ramalho, and Andrei Shleifer. The effect of corporate taxes on investment and entrepreneurship. *American Economic Journal: Macroeconomics*, 2(3):31–64, 2010.

Itamar Drechsler, Alexi Savov, and Philipp Schnabl. A model of monetary policy and risk premia. Working paper, National Bureau of Economic Research, 2014.

Douglas W Elmendorf and N Gregory Mankiw. Government debt. In *Handbook of Macroeconomics*, volume 1, pages 1615–1669. Elsevier, 1999.

Eric M Engen and R Glenn Hubbard. Federal government debt and interest rates. In *NBER Macroeconomics Annual 2004, Volume 19*, pages 83–160. MIT Press, 2005.

Larry G. Epstein and Stanley Zin. Substitution, risk aversion and the temporal behavior of consumption and asset returns: An empirical analysis. *Journal of Political Economy*, 99:263–86, 1991.

Larry G Epstein and Stanley E Zin. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica: Journal of the Econometric Society*, pages 937–969, 1989.

Christopher Erceg, Luca Guerrieri, and Christopher Gust. Trade Adjustment and the Composition of Trade. *Journal of Economic Dynamics and Control*, 32(8):2622–50, 2008.

Eugene F Fama. Forward and spot exchange rates. *Journal of monetary economics*, 14(3): 319–338, 1984.

Eugene F Fama and Kenneth R French. Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics*, 25(1):23–49, 1989.

E. Farhi, S. Fraiberger, X. Gabaix, R. Ranciere, and A. Verdelhan. Crash risk in currency markets. Working paper, 2015.

Emmanuel Farhi and Xavier Gabaix. Rare disasters and exchange rates. Working paper, Harvard and NYU, 2008.

Emmanuel Farhi and Xavier Gabaix. Rare disasters and exchange rates. *Quarterly Journal of Economics*, 131(1):1–52, 2016. Lead Article.

Jack Favilukis, Lorenzo Garlappi, and Sajjad Neamati. The carry trade and uncovered interest parity when markets are incomplete. *Available at SSRN*, 2015.

Jesus Fernandez-Villaverde, Pablo Guerron-Quintana, Juan F. Rubio-Ramirez, and Martin Uribe. Risk matters: The real effects of volatility shocks. *American Economic Review*, 101(6):2530–61, 2011.

Jesús Fernández-Villaverde, Pablo Guerrón-Quintana, Keith Kuester, and Juan Rubio-Ramírez. Fiscal volatility shocks and economic activity. *The American Economic Review*, 105(11):3352–3384, 2015.

Alessandra Fogli and Fabrizio Perri. Macroeconomic volatility and external imbalances. *Journal of Monetary Economics*, 69:1–15, 2015. ISSN 0304-3932.

Kenneth R French and James H Poterba. Investor diversification and international equity markets. *The American Economic Review*, 81(2):222–226, 1991.

X. Gabaix and M. Maggiori. International liquidity and exchange rate dynamics. *Quarterly Journal of Economics*, 130(3):1369–420, 2015a.

Xavier Gabaix and Matteo Maggiori. International liquidity and exchange rate dynamics. *Quarterly Journal of Economics*, 130(3), 2015b.

Kenneth D. Garbade. The first debt ceiling crisis. Staff Reports 783, Federal Reserve Bank of New York, 2016.

Nicolae Garleanu and Lasse Heje Pedersen. Margin-based asset pricing and deviations from the law of one price. *Review of Financial Studies*, 24(6):1980–2022, 2011.

Federico Gavazzoni, B. Sambalaibat, and Chris Telmer. Currency risk and pricing kernel volatility. Working paper, 2013.

Norman Gemmell, Richard Kneller, and Ismael Sanz. The timing and persistence of fiscal policy impacts on growth: evidence from oecd countries. *The Economic Journal*, 121 (550):F33–F58, 2011.

Mark Gertler and Peter Karadi. A model of unconventional monetary policy. *Journal of monetary Economics*, 58(1):17–34, 2011.

Mark Gertler and Nobuhiro Kiyotaki. Financial intermediation and credit policy in business cycle analysis. *Handbook of monetary economics*, 3(3):547–599, 2010.

Simon Gilchrist and Egon Zakrajšek. Credit spreads and business cycle fluctuations. *The American Economic Review*, 102(4):1692–1720, 2012.

Simon Gilchrist, Jae Sim, and Egon Zakrajsek. Uncertainty, financial frictions and investment dynamics. Working paper, NBER, 2014.

Francisco Gomes, Alexander Michaelides, and Valery Polkovnichenko. Fiscal policy in an incomplete markets economy. *Review of Financial Studies*, 26(2):531–566, 2013.

Gary B Gorton and Guillermo Ordonez. The supply and demand for safe assets. Working paper, National Bureau of Economic Research, 2013.

Francois Gourio, Michael Siemer, and Adrien Verdelhan. Uncertainty and international capital flows. Working paper, 2014.

John Graham, Mark T Leary, and Michael R Roberts. How does government borrowing affect corporate financing and investment? Working paper, National Bureau of Economic Research, 2014.

Robin Greenwood and Dimitri Vayanos. Bond supply and excess bond returns. *Review of Financial Studies*, 27(3):663–713, 2014.

Robin Greenwood, Samuel G Hanson, and Jeremy C Stein. A comparative-advantage approach to government debt maturity. *The Journal of Finance*, 70(4):1683–1722, 2015.

George J Hall and Thomas J Sargent. Interest rate risk and other determinants of post-wwii us government debt/gdp dynamics. *American Economic Journal: Macroeconomics*, 3(3): 192–214, 2011.

George J. Hall and Thomas J. Sargent. A history of u.s. debt limits. Working Paper 21799, National Bureau of Economic Research, 2015.

Lars Peter Hansen and Robert J Hodrick. Forward exchange rates as optimal predictors of future spot rates: An econometric analysis. *The Journal of Political Economy*, pages 829–853, 1980.

Tarek Hassan. Country size, currency unions, and international asset returns. *The Journal of Finance*, 68:2269–308, 2013.

Tarek A Hassan and Rui C Mano. Forward and spot exchange rates in a multi-currency world. 2017.

Zhiguo He and Wei Xiong. Rollover risk and credit risk. *The Journal of Finance*, 67(2): 391–430, 2012.

Jonathan Heathcote and Fabrizio Perri. Financial autarky and international business cycles. *Journal of Monetary Economics*, 49(3):601–627, 2002.

C. Heyerdahl-Larsen. Asset Prices and Real Exchange Rates with Deep Habits. *Review of Financial Studies*, 27(11):3280–317, 2015.

Robert J Hodrick. Dividend yields and expected stock returns: Alternative procedures for inference and measurement. *Review of Financial studies*, 5(3):357–386, 1992.

Urban Jermann and Vincenzo Quadrini. Macroeconomic effects of financial shocks. *American Economic Review*, 102(1):238–71, 2012.

Urban J Jermann. Asset pricing in production economies. *Journal of Monetary Economics*, 41(2):257–275, 1998.

Larry Jones, Rodolfo Manuelli, Henry Siu, and Ennio Stacchetti. Fluctuations in convex models of endogenous growth I: Growth effects. *Review of Economic Dynamics*, 8:780–804, 2005.

Kyle Jurado, Sydney C. Ludvigson, and Serena Ng. Measuring uncertainty. *American Economic Review*, 105(3):1177–216, 2015a.

Kyle Jurado, Sydney C Ludvigson, and Serena Ng. Measuring uncertainty. *The American Economic Review*, 105(3):1177–1216, 2015b.

Alejandro Justiniano and Giorgio E. Primiceri. The time-varying volatility of macroeconomic fluctuations. *American Economic Review*, 98(3):604–41, 2008.

Nam Jong Kim. Intermediary-determined exchange rates. *mimeo, Carnegie Mellon University*, 2015.

S. Kim, N. Shephard, and S. Chib. Stochastic volatility: Likelihood inference and comparison with ARCH models. *Review of Economic Studies*, 65(3):361–93, 1998.

Michael R King, Carol L Osler, and Dagfinn Rime. Foreign exchange market structure, players and evolution. *Norges Bank working paper*, 2011.

Nobuhiro Kiyotaki and John Moore. Credit chains. *Journal of Political Economy*, 105(21): 211–248, 1997.

R. Kollmann. Essays on international business cycles. *University of Chicago*, 1991.

R. Kollmann. International business cycles and risk sharing with uncertainty shocks and recursive preferences. *Journal of Economic Dynamics and Control*, page forthcoming, 2016.

Smitu P Kothari and Jay Shanken. Book-to-market, dividend yield, and expected market returns: A time-series analysis. *Journal of Financial Economics*, 44(2):169–203, 1997.

Arvind Krishnamurthy and Annette Vissing-Jorgensen. The aggregate demand for treasury debt. *Journal of Political Economy*, 120(2):233–267, 2012.

Arvind Krishnamurthy and Annette Vissing-Jorgensen. The impact of treasury supply on financial sector lending and stability. *Journal of Financial Economics*, 118(3):571–600, 2015.

Howard Kung. Macroeconomic linkages between monetary policy and the term structure of interest rates. *Journal of Financial Economics*, 115(1):42–57, 2015.

Howard Kung and Lukas Schmid. Innovation, growth, and asset prices. *The Journal of Finance*, 70(3):1001–1037, 2015.

Philip R Lane and Gian Maria Milesi-Ferretti. The external wealth of nations mark ii: Revised and extended estimates of foreign assets and liabilities, 1970–2004. *Journal of international Economics*, 73(2):223–250, 2007.

Thomas Laubach. New evidence on the interest rate effects of budget deficits and debt. *Journal of the European Economic Association*, 7(4):858–885, 2009.

Martin Lettau and Sydney Ludvigson. Consumption, aggregate wealth, and expected stock returns. *Journal of Finance*, 56(3):815–849, 2001.

Martin Lettau, Sydney Ludvigson, and Jessica Wachter. The declining equity premium: What role does macroeconomic risk play? *Review of Financial Studies*, 21(4):1653–87, 2008.

Karen Lewis. Global Asset Pricing. *Annual Reviews of Financial Economics*, 3(435-66), 2011.

Kai Li. Asset pricing with a financial sector. *mimeo, HKUST*, 2013.

Wendy C.Y. Li and Bronwyn H. Hall. Depreciation of business R&D capital. Working paper, National Bureau of Economic Research, July 2016.

Francis A Longstaff, Sanjay Mithal, and Eric Neis. Corporate yield spreads: Default risk or liquidity? new evidence from the credit default swap market. *The Journal of Finance*, 60(5):2213–2253, 2005.

Hanno Lustig and Adrien Verdelhan. Does incomplete spanning in international financial markets help to explain exchange rates? *NBER Working Paper*, 2016.

Hanno Lustig, Nikolai Roussanov, and Adrien Verdelhan. Common risk factors in currency markets. *Review of Financial Studies*, 24:3731–77, 2011a.

Hanno Lustig, Nikolai Roussanov, and Adrien Verdelhan. Common risk factors in currency markets. *The Review of Financial Studies*, 24(11), 2011b.

Matteo Maggiori. Financial intermediation, international risk sharing, and reserve currencies. Working paper, Harvard University, 2011.

Margaret McConnell and Gabriel Perez Quiros. Output fluctuations in the united states: What has changed since the early 1980s? *American Economic Review*, 90(5):1464–76, 2000.

Michael W McCracken and Serena Ng. Fred-md: A monthly database for macroeconomic research. *Journal of Business & Economic Statistics*, 34(4):574–589, 2016.

Rajnish Mehra and Edward C Prescott. The equity premium: A puzzle. *Journal of monetary Economics*, 15(2):145–161, 1985.

Enrique G Mendoza. Sudden stops, financial crises, and leverage. *The American Economic Review*, 100(5):1941–1966, 2010.

Philippe Mueller, Andreas Stathopoulos, and Andrea Vedolin. International correlation risk. Working paper, 2015.

Stefan Nagel. The liquidity premium of near-money assets. Working paper, National Bureau of Economic Research, 2014.

Emi Nakamura, Dimitriy Sergeyev, and Jon Steinsson. Growth-rate and uncertainty shocks in consumption: Cross-country evidence. Working paper, 2012.

Dennis Novy and Alan M. Taylor. Trade and uncertainty. Working paper., 2014.

Maurice Obstfeld. The global capital market: Benefactor or menace? *Journal of Economic Perspectives*, 12:9–30, 1998.

Carol L Osler. Foreign exchange microstructure. *Encyclopedia of Complexity and System Science*, 2008.

L'uboš Pástor and Pietro Veronesi. Political uncertainty and risk premia. *Journal of Financial Economics*, 110(3):520–545, 2013.

Lubos Pastor and Pietro Veronesi. Political cycles and stock returns. Technical report, National Bureau of Economic Research, 2017.

Anna Pavlova and Roberto Rigobon. Asset prices and exchange rates. *Review of Financial Studies*, 20:1139–1181, 2007.

Anna Pavlova and Roberto Rigobon. An asset-pricing view of external adjustment. *Journal of International Economics*, 80:144–156, 2010.

Anna Pavlova and Roberto Rigobon. International macro-finance. *Handbook of Safeguarding Global Financial Stability: Political, Social, Cultural, and Economic Theories and Models*, 2:169–176, 2013.

Giorgio E. Primiceri. Time varying structural vector autoregressions and monetary policy. *Review of Economic Dynamics*, 72:821–52, 2005.

Dennis P. Quinn. The correlates of change in international financial regulation. *American Political Science Review*, 91(3):531–551, 1997.

Dennis P. Quinn and Hans-Joachim Voth. A century of global equity markets correlations. *American Economic Review*, 98:535–40, 2008.

Gary Ramey and Valery Ramey. Cross-country evidence on the link between volatility and growth. *American Economic Review*, 85:1138–51, 1995.

Robert Ready, Nic Roussanov, and Colin Ward. Commodity trade and the carry trade: A tale of two countries. Working paper, the Wharton School, 2012.

Paul Romer. Endogenous technological change. *Journal of Political Economy*, 98(5):2, 1990.

Michael J Sager and Mark P Taylor. Under the microscope: the structure of the foreign exchange market. *International Journal of Finance & Economics*, 11(1):81–95, 2006.

Pedro Santa-Clara and Rossen Valkanov. The presidential puzzle: Political cycles and the stock market. *The Journal of Finance*, 58(5):1841–1872, 2003.

Frank Schorfheide, Dongho Song, and Amir Yaron. Identifying long-run risks: A bayesian mixed-frequency approach. Working paper, National Bureau of Economic Research, 2014.

Clemens Sialm. Tax changes and asset pricing. *The American Economic Review*, 99(4): 1356–1383, 2009.

Zheng Song, Kjetil Storesletten, and Fabrizio Zilibotti. Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt. *Econometrica*, 80 (6):2785–2803, 2012.

Robert F Stambaugh. Predictive regressions. *Journal of Financial Economics*, 54(3):375–421, 1999.

Andreas Stathopoulos. Asset prices and risk sharing in open economies. Working paper, University of Washington, 2012.

James Stock and Mark Watson. Has the business cycle changed and why. In M. Gertler and K. Rogoff, editors, *NBER Macroeconomics Annual: 2002*. MIT Press, Cambridge, MA, 2002.

Alan C. Stockman and Linda L. Tesar. Tastes and technology in a two-country model of the business cycle: Explaining international comovements. *The American Economic Review*, 85(1):168–185, 1995.

Alan M. Taylor. A century of current account dynamics. *Journal of International Money and Finance*, 21:725–48, 2002.

Adrien Verdelhan. A habit-based explanation of the exchange rate risk premium. *The Journal of Finance*, 65(1):123–45, 2010a.

Adrien Verdelhan. A habit-based explanation of the exchange rate risk premium. *The Journal of Finance*, 65(1):123–146, 2010b.

Annette Vissing-Jørgensen. Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy*, 110(4), 2002.

Jessica A Wachter. Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance*, 68(3):987–1035, 2013.

Philippe Weil. The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics*, 24(3):401–421, 1989.

Ivo Welch and Amit Goyal. A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies*, 21(4):1455–1508, 2008.

Irina Zviadadze. Term structure of consumption risk premia in the cross section of currency returns. Working paper, Stockholm School of Economics, 2015.