

Consistent description of ^{11}Be and ^{12}Be and of the $^{11}\text{Be}(d,p)^{12}\text{Be}$ reaction

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Simple wave functions for $^{11,12}\text{Be}$ have been around for a long time. They have been tested against many independent processes involving (and properties of) these nuclei. All are consistent, except $^{11}\text{Be}(d,p)$, where the discrepancy is a 4.7σ effect for the 2^+ state and 15σ for the ground state. Here, we propose a resolution to this dilemma.

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From the earliest days, an understanding of the structure of the $1/2^+$ ground state (g.s.) of ^{11}Be centered on a dominant configuration of an $s_{1/2}$ neutron from the next major shell coupled to a p shell ^{10}Be (g.s.). Very early, Auton [1] investigated the $^{10}\text{Be}(d,p)^{11}\text{Be}$ reaction, at a bombarding energy of 12.0 MeV, and extracted a spectroscopic factor of $S = 0.73(6)$. The following decades saw a rash of theoretical papers [2–11] concerning this state, espousing widely different spectroscopic factors—from 0.55 [2] to 0.93 [4]. Winfield *et al.* [12] summarized many of these calculations. A later $^{10}\text{Be}(d,p)$ investigation at 25 MeV [13] gave $S = 0.77$. Reanalysis of these two sets of data also produced a wide range of S 's, some quite small—0.36 or 0.44 in Ref. [14] and 0.5 in Ref. [15]. Barker [16] quotes a “published” value of 0.19(2), and cites a conference proceeding [17]. Aumann *et al.* [18] studied neutron knockout from ^{11}Be and concluded that its g.s. is dominated by the $s_{1/2}$ single-particle component with a small $2^+ \times d_{5/2}$ admixture. Many of the various experimental spectroscopic factors were summarized by Palit [19]. For the mirror ^{11}N , two values were extremely small—about 0.2 [20] and 0.1–0.2 [21]. These wildly different values of S had the potential to do great harm to our (supposed) understanding of nuclear structure. Barker [16], for one, treated them on an equal footing with results from more conventional approaches. Keeley *et al.* [22] used a continuum discretized coupled channels approach to compute angular distributions for both sets of $^{10}\text{Be}(d,p)$ data. With a g.s. spectroscopic factor of 0.93 [4], their computed forward-angle cross sections at 12 MeV are about 1.22 times the experimental ones, implying $S \sim 0.76$.

Quite recently, there appeared a very careful experiment and a detailed analysis of the $^{10}\text{Be}(d,p)^{11}\text{Be}$ reaction at several energies [23]. The average of their S 's was 0.71(5), very close to Auton's value. Some of this excellent agreement may be thought to be fortuitous, because the uncertainty in Auton's $S = 0.73(6)$ does not include the uncertainty in absolute cross-section scale arising from normalizing $^{10}\text{Be} + d$ elastic scattering to calculations of an optical model. Nevertheless, this important spectroscopic factor now appears to have been reliably determined as 0.71(5). It is also consistent with the values of Refs. [13,22]. If ^{11}Be (g.s.) had not been dominated by a single-particle component, simple wave functions for ^{12}Be would not have been possible.

If we write

$$^{11}\text{Be}(\text{g.s.}) = a(0^+ \times 2s_{1/2}) + b(2^+ \times 1d_{5/2}) + c(\text{other}), \quad (1)$$

then we have $a^2 = S$ from above. Here, 0^+ and 2^+ refer to states of a p shell ^{10}Be . We leave “other” unspecified for now, but it probably contains some component of three sd -shell neutrons coupled to ^8Be , and perhaps some $1s_{1/2}$ hole strength [11], plus coupling to other excited states of ^{10}Be . Many estimates of b^2 are in the range 0.12–0.20—both experimentally and theoretically.

We turn now to ^{12}Be . The states being considered are the 0^+ g.s., the 2^+ state at $E_x = 2.1$ MeV, and the excited 0^+ at 2.24 MeV. Barker [24] was the first to write down explicit wave functions for the first two 0^+ states, with a mixture of p -shell and an $(sd)^2$ intruder component. [The notation $(sd)^2$ represents two nucleons in the sd shell.] Since then, it has become standard practice to consider the wave functions of these two 0^+ states as predominantly orthogonal linear combinations of an $(sd)^2$ amplitude and a p -shell component. This description did not apply to Barker's wave functions, either in normalization of the $(sd)^2$ part, or the relative phases. His g.s. wave function had 62% $(sd)^2$, but his excited state had even more; and the majority of his s^2 strength was in the excited state. A combination of experiments and calculations, including $^{10}\text{Be}(t,p)$ [25,26], β decay of ^{12}Be (g.s.) [27], Coulomb energy of ^{12}O (g.s.) [28], and neutron removal from ^{12}Be [29], all resulted in a consensus of about 68% $(sd)^2$ and 32% p shell for the g.s. The Coulomb energy calculation [28] estimated the s^2 component to be 53(3)%, leaving 15% for d^2 .

And there the situation stood for a long time, until an investigation of the $^{11}\text{Be}(d,p)$ reaction [30] gave conflicting results. They used inverse kinematics, with an ^{11}Be beam incident on a target of C^2H_2 . In that experiment, spectroscopic factors for the g.s. and first-excited 2^+ state were claimed to be only about 40% and 20%, respectively, of their expected values. And S for the excited 0^+ state was about three times as large as expected. In that experiment, the 2^+ and excited 0^+ states were not resolved, so an incorrect estimate of their relative contributions to the doublet peak could explain part of

TABLE I. Various processes involving $^{11,12}\text{Be}$ and tests of simple wave functions.

Process	Year(s)	Reference(s)	Consistent?
$^{10}\text{Be}(d,p)$	1970	[1]	Yes
$^{10}\text{Be}(d,p)$	1979	[13]	Yes
$^{10}\text{Be}(t,p)$	1978, 1994	[25,26]	Yes
β decay	1997	[27]	Yes
^{12}O mass excess	1999	[28]	Yes
n removal	2000	[29]	Yes
$^{14}\text{C}(p,t)$	1976, 2006	[32,33]	Yes
$^{14}\text{O}(p,t)$	2009, 2011	[31,34,35]	Yes
$^{11}\text{Be}(d,p)$	2010	[30]	No
$B(E2)$'s	2007,2009, 2012	[36–38]	Yes
$^{10}\text{Be}(d,p)$	2012	[23]	Yes
GT from ^{12}B	2012	[39]	Yes

the discrepancy. However, the authors stated that even if all of the doublet cross section corresponded to the 2^+ state, $S(2^+)$ would be only 0.29—still very far from the theoretical value of ~ 0.5 . With their uncertainties, $S(0^{+'})$ was only 1.8σ from zero, but $S(2^+)$ was 4.7σ from one theoretical value of 0.52 [31], and $S(\text{g.s.})$ was 15σ from the prediction of our model. In the meantime, our wave functions have been shown [31,32] to be consistent with results of the reactions $^{14}\text{C}(p,t)$ [33] and $^{14}\text{O}(p,t)$ [34,35]. Very recently, it was demonstrated [36] that they also gave reasonable agreement with measured $B(E2)$ strengths [37,38] of the two 0^+ states. Thus, the $^{11}\text{Be}(d,p)$ results were the only experimental quantities that were inconsistent with our wave functions. The situation is summarized in Table I.

A very recent measurement [39] of the Gamow-Teller (GT) strengths of the two 0^+ states from the 1^+ g.s. of ^{12}B was made using the reaction $^{12}\text{B}(^7\text{Li},^7\text{Be})$ in inverse kinematics. This experiment is the first to directly measure the p -shell component of the excited 0^+ state. Other investigations had inferred it from orthogonality with the g.s., or through destructive interference in (t,p) and $B(E2)$. These new results have clearly indicated that the commonly accepted wave functions are approximately correct: Their intensities of 0.25(5) and 0.60(5) for the p -shell component of the g.s. and excited 0^+ state, respectively, are to be compared to our 0.32 and 0.68. Barker's wave functions gave 0.38 and 0.23 for these two numbers. As this reference points out, their results are not at all consistent with the claims from $^{11}\text{Be}(d,p)$ [30]. So, there is no longer any doubt that something is wrong with the results of Ref. [30].

Here, we propose a two-pronged solution to the problem. For present purposes, we assume nothing is wrong with the distorted-wave (DW) calculations in Ref. [30], but we note that a factor of N in the experimental cross sections is exactly equivalent to a factor $1/N$ in the DW calculations. We think the absolute cross sections are too small by a factor of 2, or more, and the $2^+/0^{+'}$ separation is incorrect. Most shell-model calculations (summarized in Refs. [30,31]) estimate the 2^+ spectroscopic factor to be 0.41–0.55, and $S(\text{g.s.})$ should be

about $1.06 \times S[^{10}\text{Be}(d,p) \ ^{11}\text{Be}(\text{g.s.})] = 0.75(7)$. Recently, we wrote the 2^+ wave function as [36]

$$^{12}\text{Be}(2^+) = A [0^+ \times (sd)_2^2] + B \text{ } p \text{ shell} + C [2^+ \times (sd)_0^2].$$

The third term had been ignored earlier, because it has no direct one-step direct route in (t,p) , but an analysis [36] of the $B(E2)$ strengths [37,38] connecting the two 0^+ states to the 2^+ demonstrated its presence. One estimate of the mixing was $A^2 = 0.66$, $B^2 = 0.15$, and $C^2 = 0.19$, with the competition between B^2 and C^2 not well determined.

With this third term in the 2^+ wave function, the second term in the ^{11}Be g.s. [Eq. (1) above] can contribute to $S(2^+)$, through the amplitude $d_{5/2} \rightarrow (sd)_0^2$ (which contains a d^2 part). This would cause a slight increase in the predicted S , except for the fact that the presence of this term produces a slight decrease in the dominant term, through renormalization. The resulting change is small.

From information presented in Ref. [30], the relationship between S 's for the two members of the doublet is $S(2^+) + 0.26 S(0^{+'}) = 0.29$, where we have suppressed the uncertainties for clarity of presentation. It would be a simple matter to reinsert them later. With a normalization factor of N in the experimental cross sections, we have $S(\text{g.s.}) = 0.28N$, and $S(2^+) + 0.26S(0^{+'}) = 0.29N$. If we take $S(\text{g.s.})$ to be 0.68 [1σ below the expected value of 0.75(7)], then the cross-section renormalization required to bring the results of Ref. [30] into agreement is $N = 2.4$, with some uncertainty. If we then use the relationship established in that reference between $S(2^+)$ and $S(0^{+'})$, and we further assume that $S(0^{+'})/S(\text{g.s.}) = 0.32/0.68$ from the commonly accepted wave functions, then we can derive $S(2^+) = 0.62$, reasonably close to the earlier prediction of 0.52 mentioned above.

An experiment that cleanly separates the doublet members can easily determine the individual relative spectroscopic factors. To answer the normalization question, a new experiment should measure the d content of the target, not the C content as was done in Ref. [30]. We strongly encourage another look at the $^{11}\text{Be}(d,p)$ reaction.

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