PHYSICAL REVIEW C **87**, 014305 (2013)

Unraveling the structure of ^{13}Be

H. T. Fortune
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

(Received 14 September 2012; revised manuscript received 9 December 2012; published 7 January 2013)

Using a simple model for low-lying positive-parity resonances in ^{13}Be as ^{10}Be x $(sd)^3$ and $^{12}\mathrm{Be}_{1p}$ x (sd), I find that the lowest $5/2^+$ state is predominantly $(sd)^3$. I give predictions for several additional states.

DOI: 10.1103/PhysRevC.87.014305 PACS number(s): 21.10.Dr, 21.10.Hw, 21.60.Cs, 27.20.+n

I. INTRODUCTION AND BRIEF HISTORY

Uncertainty and confusion abound, both experimentally and theoretically, concerning the low-lying states of ^{13}Be. Several experiments have reported an s-wave state just above the $^{12}\mathrm{Be} + n$ threshold, but others disagree. Another common feature of many experiments is one or two d-wave resonances at energies of 2.0 to 2.9 MeV. One experiment reported a p-wave resonance at 0.51 MeV

Poskanzer and coworkers [1] detected residues from 5.3-GeV proton bombardment of a uranium target and concluded that ^{13}Be was "most probably particle unstable". Later [2], with 5.5-GeV protons and the addition of time-of-flight information, they reported the instability as firm—a fact later confirmed by Artukh *et al.* [3].

Several experiments have attempted to populate the low-lying resonances of ^{13}Be. Aleksandrov *et al.* [4], with the ^{14}C(^{7}Li, ^{8}B) reaction, saw a state at 1.8(5) MeV.

Ostrowski *et al.* [5] produced ^{13}Be with the reaction ^{13}C(^{14}C,^{14}O)^{13}Be at $E_{\mathrm{Lab}} = 337$ MeV. The lowest state they observed was particle unstable with respect to one neutron emission by 2.01 MeV, and had a width of 0.3(2) MeV, supporting an assignment of $J^\pi = 5/2^+$ or $1/2^-$, but excluding $J^\pi = 1/2^+$.

Korsheninnikov *et al.* [6] investigated ^{13}Be with the ^{12}Be(d,p) reaction, in reverse kinematics, using a ^{12}Be beam incident on a CD$_2$ target. They saw a state at $E_n = 2$ MeV, in agreement with Ref. [5], and three other tentative states at (5), (7), and (10) MeV, none of which were consistent with earlier work. Background from C in the target prevented extraction of reliable information at low energy.

Von Oertzen *et al.* [7] used the ^{13}C(^{14}C,^{14}O) reaction to populate a state at 2.01(5) MeV, which they identified as $d_{5/2}$; and another at $E_n = 5.13$ MeV, which they suggested might have the structure $2^+ \times d_{5/2}$.

Belozyorov *et al.* [8] used the ^{14}C(^{11}B,^{12}N) reaction at 190 MeV and reported six possible states, including three at $E_n = 0.80(9)$, 2.02(10), and 2.90(16) MeV. The second one was by far the strongest state they observed, with a width of about 1 MeV.

Thoenessen *et al.* [9] bombarded a ^{9}Be target with 80A MeV ^{18}O and detected $n + {}^{12}$Be at 0 degrees. They suggested the existence of an s-wave state at an unbound energy of <200 keV. However, they stated that they could be observing the n decay of an excited state of ^{13}Be to a particle-bound excited state of ^{12}Be. They mentioned the possibility of $5/2^+ \rightarrow 2^+$, but stated that they would need a 2^+ branching ratio of 75% to explain their data. Their experiment was sensitive only to very low relative $n + {}^{12}$Be energies.

Marques *et al.* [10] used breakup of ^{14}Be on C and Pb targets and observed $^{12}\mathrm{Be} + n + n$ coincidences. From a measurement of finite delay between the two neutrons, they concluded that the process was sequential through low-lying resonances in ^{13}Be.

Simon *et al.* [11] reported two resonances, the lower of which they identified as p wave. They tentatively identified the higher one with the d state seen by others [4,5].

Simon *et al.* [12] observed a low-energy structure, which they identified as an s-wave resonance, not a virtual s state. They also reported a d state at 2.0 MeV, and another structure at 3.04 MeV. They presented a plot of $E(d) - E(s)$ for ^{17}O, ^{15}C, and ^{13}Be, similar to one I will exploit below.

Lecouey [13] analyzed the ^{13}Be to $^{12}\mathrm{Be} + n$ breakup spectrum and found a 700-keV resonance that he identified as $\ell = 0$. He fitted it with a Breit-Wigner shape, with a width of 1.3 MeV. No explanation was given for the mechanism of an s-wave neutron resonance. He also observed a d state at 2 MeV.

Christian *et al.* [14] detected fragments from a 60A MeV ^{48}Ca beam, and observed sequential neutron decay. Their study was limited to small E_n.

Al Falou *et al.* [15] postulated that the very narrow structure at threshold in the $^{12}\mathrm{Be} + n$ relative energy spectrum observed in some experiments arises instead from the $2n$ decay of ^{14}Be(2^+). They discount earlier reports of a strong virtual s-wave state in ^{13}Be.

Kondo *et al.* [16] investigated $^{13}\mathrm{Be} \rightarrow {}^{12}\mathrm{Be} + n$, in anti-coincidence with ^{12}Be γ rays. They observed a resonance at 0.51(1) MeV, with a width of 0.45(3) MeV, which they interpreted as p wave, and a d resonance at 2.39(5) MeV, with a width of 2.4(2) MeV, or two d states at 2.0 and 2.9 MeV. We suggested [17] the p-wave resonance was too wide by about a factor of two, and suggested two possibilities [17,18] for this additional width.

Calculations cannot agree on the J, or even the parity, of the lowest expected states in ^{13}Be. Several researchers have treated ^{13}Be as $^{12}\mathrm{Be} + n$, and ^{14}Be as $^{12}\mathrm{Be} + n + n$, and have used the known $2n$ separation energy of ^{14}Be (g.s.) to deduce properties of the low-lying resonances of ^{13}Be. Bertsch and Esbensen [19] found that they could reproduce S_{2n} with a $d_{5/2}$ state at 2.4 MeV and an $s_{1/2}$ state just above threshold.

Thompson and Zhukov [20] found they needed $E_d = 1.3$ or 1.0 MeV, together with an s state just above threshold.

However, their calculations for ^{12}Be (g.s.) produced a wave function that was about 25% $(sd)^2$ and 75% p shell, considerably different from the one generally accepted today [21]. Furthermore, their model for ^{14}Be treated ^{12}Be (g.s.) as pure p shell.

Labiche *et al.* [22] concluded that in order to fit the known $2n$ separation energy of ^{14}Be (g.s.) and to have a d state near 2 MeV in ^{13}Be, the ground state (g.s.) of ^{13}Be should not be $1/2^+$, but rather it was necessary to have a $1/2^-$ resonance near 0.3 MeV as the g.s. of ^{13}Be.

Tarutina *et al.* [23] found they could fit ^{13,14}Be simultaneously if they assumed that ^{12}Be has a very large quadrupole deformation, $\beta > 0.8$.

Earlier, Descouvemont [24] had used a microscopic cluster model to compute the low-energy structure of ^{13}Be. He concluded that the g.s. was $1/2^+$ and that it should be slightly bound. He produced a d state just above 2 MeV, with a width of 0.35 MeV. He later amended the s state's position to "very close to threshold" [25].

Pacheco and Vinh Mau [26] concluded the $s_{1/2}$, $p_{1/2}$ ordering in ^{12,13}Be was the same as in ^{11}Be, as did Blanchon *et al.* [27]. Hamomoto [28] suggested the two lowest states in ^{13}Be might both be $1/2^+$.

Quite recently, Kanada-En'yo [29] did calculations for ^{13}Be using the method of antisymmetrized molecular dynamics (AMD), combined with variation after spin-parity projection (VAP). She also computed the low-lying states of ^{13}Be in a $^{12}\mathrm{Be}+n$ model, using pure $2\,\hbar\omega$ for ^{12}Be (g.s.) and pure $0\,\hbar\omega$ for the excited 0^+ state. Predictions of the various approaches were considerably different. A pertinent summary is presented in Table I, where I list the J^π and dominant structure of the first two predicted states. Thus, her g.s. is either $3/2^+$, $5/2^+$, or $1/2^-$, and the first-excited state is either $1/2^-$, $1/2^+$, or $5/2^+$. None of those calculations produce $1/2^+$ as the lowest state. In the $^{12}\mathrm{Be}+n$ model, the lowest state is at $E_n = 1.2$ MeV, whereas in VAP(1), it is at about 4 MeV.

II. CALCULATIONS AND RESULTS

Positive-parity states can be of two types: $^{12}\mathrm{Be}_{1p} \times sp$ ($0\ \hbar\omega$) or $^{10}\mathrm{Be}_{1p} \times (sd)^3$ ($2\ \hbar\omega$), where sp is either $2s_{1/2}$ or $1d_{5/2}$ single particle. The lowest negative-parity state is almost certainly $1/2^-$ with configuration $^{11}\mathrm{Be}(1/2^-) \times (sd)^2$ ($1\,\hbar\omega$). Higher negative-parity states of the same configuration will also exist. As there is no confusion about their structure, I do not address negative-parity states further. We have previously made two estimates [17,30] of a few of the lowest positive-parity states. Both suggested $1/2^+$ as the lowest positive-parity resonance. But, one calculation [17] put the $(sd)^3$ $5/2^+$ somewhat below the sp one, while the other [30] had the sp state lower. Here, I try to settle that question.

TABLE II. Input energies (MeV) from core + $1n$ nuclei.

Core	E_n (g.s.)	E_x		E_n	
		$1/2^+$	$5/2^+$	$1/2^+$	$5/2^+$
^{16}O	– 4.144	0.871	0	−3.273	−4.144
^{14}C	−1.218	0	0.740	−1.218	−0.478
^{10}Be	– 0.503	0	1.778	−0.503	+1.275

I ignore the $d_{3/2}$ orbital throughout. With that restriction, within the $(sd)^3$ space, there are three $5/2^+$ states—linear combinations of the three configurations d^3, $d^2{}_2s$, and $ds^2{}_0$, where s stands for $2s_{1/2}$ and d for $1d_{5/2}$. I have calculated energies and wave functions for these three $5/2^+$ states in three nuclei ^{19}O, ^{17}C, and ^{13}Be, in the spirit of Lawson [31], assuming a configuration of $(sd)^3$ coupled to the ground states of ^{16}O, ^{14}C, and ^{10}Be, respectively. Relevant input energies are listed in Table II. I use "local" single-particle energies (SPEs) from the lowest $1/2^+$ and $5/2^+$ states in the core + 1n nuclei, and global two-body matrix elements (2BMEs) from an earlier fit involving ^{18}O [32]. The resulting $3n$ energies are absolute, and I then convert them into excitation energies, using known $3n$ energies of the relevant ground states. Results are listed in Table III.

Wave functions of the $5/2^+$ states turn out to be very different in the three nuclei because of the large differences in $1/2^+$–$5/2^+$ spacing. In ^{19}O, the resulting calculated excitation energy for the lowest $5/2^+$ state is 103 keV, compared to 0 for the g.s. of that nucleus. Its wave function is primarily d^3, whereas in ^{13}Be it is mostly $ds^2{}_0$. In ^{17}C, the two configurations are of comparable magnitude [33]. As can be seen from the table, the calculations work surprisingly well for the energies in ^{19}O and ^{17}C, perhaps an indication that the result for ^{13}Be will be reasonably robust. The prediction of 1.79 MeV above threshold in ^{13}Be clearly indicates that this $5/2^+$ state is very likely to be the lowest $5/2^+$ state [i.e., the $(sd)^3$ $5/2^+$ state is below the sp one]. All other $(sd)^3$ states are calculated to lie significantly higher.

I turn now to the $1/2^+$ and $5/2^+$ sp states. Because the $1/2^+$ state is considerably below the $5/2^+$ in ^{11}Be, we should expect the same ordering in ^{13}Be, but not necessarily the same spacing. Between ^{17}O and ^{15}C, the $s_{1/2}$–$d_{5/2}$ spacing changes drastically—from +0.87 MeV in ^{17}O to −0.74 MeV in ^{15}C. I noticed that a continuation of a linear trend would place the $1/2^+$ state about 2.35 MeV below the sp $5/2^+$ in ^{13}Be.

TABLE I. Results of Ref. [29] for first two states of ^{13}Be.

Procedure[a]	Ground state		First-excited state	
	J^π	Configuration	J^π	Configuration
AMD + VAP(1)	$3/2^+$	$2\,\hbar\omega$	$1/2^-$	$1\,\hbar\omega$
AMD + VAP(2)	$5/2^+$	$0\,\hbar\omega$	$1/2^+$	$0\,\hbar\omega$
$^{12}\mathrm{Be}+n$	$1/2^-$	$1\,\hbar\omega$	$5/2^+$	$0\,\hbar\omega$

[a] See text and Ref. [29].

TABLE III. Results (MeV) for first $5/2^+$ states in core + $3n$ nuclei.

Final	E_{3n} (g.s.)	E_{3n} (calc)	E_x (calc)	E_x (exp)
^{19}O	−16.14	−16.04	0.103	0
^{17}C	−6.20	−5.814	0.386	0.33
^{13}Be	−3.673	−1.882	1.79	2.0

(These three nuclei all have $N = 9$.) This behavior is indicated in Fig. 1. Simon *et al.* [12] had also presented such a plot. In ^{9}Be and ^{11}Be the $1/2^+$ is below $5/2^+$ in both nuclei, and the spacing changes by a modest amount between the two nuclei. For the $Z = 4$ nuclei, extending an assumed linear trend to ^{13}Be (also in Fig. 1) suggests a spacing of about 2.2 MeV for that nucleus, reasonably close to the expectation from the $N = 9$ nuclei. This procedure is certainly nowhere close to a fundamental approach, but I expect it to be approximately correct.

Thus, if the $1/2^+$ state is at or above threshold, we expect the $5/2^+$ sp state to be above 2.2–2.35 MeV, compared to the earlier prediction of 1.79 MeV for the lowest $(sd)^3$ $5/2^+$ state. If the $1/2^+$ state is further above threshold, then so should be the $5/2^+$ sp state. One possibility for the location of the $1/2^+$ state is within the 0.51-MeV structure mentioned earlier. In that case, the $5/2^+$ sp state would be near 2.8 MeV, close to the position of a possible second d-wave resonance in some experiments [16]. If the $1/2^+$ state is at 0.7 MeV [13], the $5/2^+$ sp state would be near 3 MeV.

Therefore, I consider the outcome of the present exercise to be a prediction that the lowest $5/2^+$ resonance in ^{13}Be has dominant $(sd)^3$ structure, and the predominantly sp $5/2^+$ will lie higher. This was the ordering suggested in Ref. [17], where we estimated the energy spacing and mixing of the first two $5/2^+$ states. The emphasis there was on the neutron decays of the *second* $5/2^+$ state. Here, I have computed the absolute energy of the *lowest* $(sd)^3$ $5/2^+$ state in three nuclei ^{19}O, ^{17}C, and ^{13}Be.

The other $(sd)^3$ states with $J^\pi = 1/2^+$–$9/2^+$ will lie considerably higher—the lowest is $1/2^+$, about 1.3 MeV above the first $5/2^+$, with the first $3/2^+$ state about 0.4 MeV above that. In all three nuclei, the $(sd)^3$ $3/2^+$ states are linear combinations of the two configurations d^3 and $d^2{}_2s$. The absence of a ds^2 component in the $3/2^+$ states causes the wider separation between $5/2^+$ and $3/2^+$ in my model of ^{13}Be. Wave-function components for the lowest $5/2^+$ and $3/2^+$ states in the three nuclei are listed in Table IV. In all three nuclei, the $1/2^+$ states are pure d^2s.

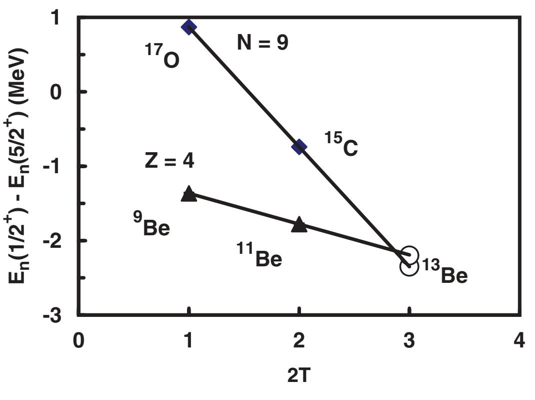

FIG. 1. Energy differences between $1/2^+$ and $5/2^+$ states in $N = 9$ and $Z = 4$ nuclei, plotted vs $2T$, where T is isospin.

TABLE IV. Configuration amplitudes for first $5/2^+$ and $3/2^+$ states in relevant nuclei.

J^π	Nucleus	d^3	d^2s	ds^2
$5/2^+$	^{19}O	0.945	0.046	0.325
	^{17}C	0.731	0.018	0.682
	^{13}Be	0.464	0.077	0.881
$3/2^+$	^{19}O	0.913	0.407	0.000
	^{17}C	0.746	0.666	0.000
	^{13}Be	0.566	0.825	0.000

The primary purpose of the present exercise was to estimate the absolute energy in ^{13}Be of the lowest $(sd)^3$ $5/2^+$ state, and thus decide the question of the likely dominant configuration of the known d state at $E_n = 2.0$ MeV. Good agreement with absolute energies of $5/2^+$ states in ^{19}O and ^{17}C (Table III) provides some confidence in the ^{13}Be calculation.

Less useful will be the results for $3/2^+$ and $1/2^+$ states. Mixing with neglected configurations (e.g., $d_{3/2}$) is likely to be more important than for $5/2^+$. Values for these J^π in ^{19}O and ^{17}C are listed in Table V. We note that the calculated $3/2^+$ energy in ^{19}O is too high by about 0.6 MeV. In Lawson's treatment [31], the calculated excitation energy was 0.61 MeV. Even in a full $(sd)^3$ shell-model calculation, the $3/2^+$ state is still too high—at $E_x = 0.30$ MeV. The agreement is even worse in ^{17}C, where the g.s. is $3/2^+$. The lowest $3/2^+$ state is probably lowered through a combination of processes, including the $d_{3/2}$ orbital, deformation, mixing with $(sd)^5$ $(1p)^{-2}$ states, etc.—all of which have been neglected here. For this reason, I make no definite predictions for the $(sd)^3$ $3/2^+$ and $1/2^+$ energies in ^{13}Be, even though I do give the results of the calculation. As noted in Table I, the AMD + VAP(1) calculations of Ref. [29] predict $3/2^+$ for the g.s. In all three calculations Ref. [29] has the sp $5/2^+$ state below the $(sd)^3$ one.

Still higher than the first sp and $(sd)^3$ states will be the $5/2^+$ and $3/2^+$ states resulting from the structure $^{12}\mathrm{Be}_{1p}(2^+) \times 2s_{1/2}$. And, beginning about 2.3 MeV above them will be a multiplet with $J^\pi = 1/2^+$–$9/2^+$ from the configuration

TABLE V. Energies (MeV) of $1/2^+$ and first $3/2^+$ states relative to first $5/2^+$ state.

Nucleus	J^π	$E(J^\pi)$–$E(5/2^+_1)$	
		Calculated	Experimental
^{19}O	$3/2^+$	0.70	0.10
	$1/2^+$	1.88	1.47
^{17}C	$3/2^+$	0.98	−0.33
	$1/2^+$	1.02	−0.11

TABLE VI. Energies (relative to $^{12}\mathrm{Be}+n$) of first few predicted positive-parity states in ^{13}Be.

Configuration	J$^\pi$	E_n (MeV)
^{10}Be (g.s.) × $(sd)^3{}_J$	$1/2^+$	3.12
	$3/2^+$	3.54
	$5/2^+$	1.79
$^{12}\mathrm{Be}_{1p}$ (g.s.) × $2s_{1/2}$	$1/2^+$	E_s
$^{12}\mathrm{Be}_{1p}$ (g.s.) × $1d_{5/2}$	$5/2^+$	$E_s+2.3$

$^{12}\mathrm{Be}_{1p}$ (2^+) × $d_{5/2}$. Also present should be $(sd)^3$ states built on the 2^+ of ^{10}Be. The estimated energies of the first few of these states are listed in Table V. Above the first few states, the mixing among the various configurations will probably be appreciable, resulting in reasonably complicated wave functions, and moving the energies around somewhat. So I offer no further details for them.

III. $5/2^+$ WIDTH IN ^{13}Be and ^{13}F

Ironically, even with a $5/2^+$ state that is predominantly $(sd)^3$, its spectroscopic factor to ^{12}Be (g.s.) is quite large because of the large $(sd)^2$ component in the latter. With a pure $(sd)^3$ $5/2^+$ state and our favorite ^{12}Be (g.s.) wave function [21], the value of S is 0.62. With the $5/2^+$ mixing estimated in Ref. [17], S is 0.94. For a $d_{5/2}$ neutron at $E_n = 2.0$ MeV, the single-particle width is 0.49(5) MeV, implying an expected width of 0.30(3) MeV (pure) or 0.46(5) MeV (mixed) with the S above. Experimental widths reported for this state range from 0.3(2) MeV [5] to 1.0 MeV [8] to 2.42(2) MeV [16].

When we computed the properties of the $5/2^+$ state in ^{13}F [30], using mirror correspondence, we used $^{12}\mathrm{Be}_{1p}$ (g.s.) × $d_{5/2}$ as the dominant configuration. With the current results, the predicted energy in ^{13}F changes only slightly, but the new calculated width is very different—0.96(16) MeV (pure) or 1.3(2) MeV (mixed) rather than 0.3 to 0.4 MeV [30].

IV. SUMMARY

I have calculated the energies in ^{13}Be of states whose structure is ^{10}Be × $(sd)^3$. I find that the prediction of the absolute energy for the lowest $5/2^+$ resonance of this configuration is $E_n = 1.79$ MeV. I also have estimated the $s_{1/2}$–$d_{5/2}$ energy difference for $^{12}\mathrm{Be}_{1p}$ × (sd) to be about 2.3 MeV. The conclusion is that the first $5/2^+$ state near $E_n = 2$ MeV is predominantly $(sd)^3$. This result greatly increases the width predicted for the mirror of this state in ^{13}F [30].

[1] A. M. Poskanzer, S. W. Cosper, E. K. Hyde, and J. Cerny, Phys. Rev. Lett. **17**, 1271 (1966).
[2] A. M. Poskanzer, G. W. Butler, E. K. Hyde, J. Cerny, D. A. Landis, and F. S. Goulding, Phys. Lett. B **27**, 414 (1968).
[3] A. G. Artukh, V. V. Avdeichikov, J. Er, G. F. Gridnev, V. L. Mikheev, V. V. Volkov, and J. Wilczyński, Phys. Lett. B **33**, 407 (1970).
[4] D. V. Aleksandrov, E. A. Ganza, Yu. A. Glukhov, V. I. Dukhanov, I. B. Mazurov, B. G. Novatsky, A. A. Ogloblin, D. N. Stepanov, V. V. Paramonov, and A. G. Trunov, Yad. Fiz. **37**, 797 (1983); Sov. J. Nucl. Phys. **37**, 474 (1983).
[5] A. N. Ostrowski *et al.*, Z. Phys. A **343**, 489 (1992).
[6] A. A. Korsheninnikov, E. Yu. Nikolskii, T. Kobayashi, D. V. Aleksandrov, M. Fujimaki, H. Kumagai, A. A. Ogloblin, A. Ozawa, I. Tanihata, Y. Watanabe, and K. Yoshida, Phys. Lett. B **343**, 53 (1995).
[7] W. von Oertzen *et al.*, Nucl. Phys. A **588**, c129 (1995).
[8] A. V. Belozyorov *et al.*, Nucl. Phys. A **636**, 419 (1998).
[9] M. Thoennessen, S. Yokoyama, and P. G. Hansen, Phys. Rev. C **63**, 014308 (2000).
[10] F. M. Marqués *et al.*, Phys. Rev. C **64**, 061301 (2001).
[11] H. Simon *et al.*, Nucl. Phys. A **734**, 323 (2004).
[12] H. Simon *et al.*, Nucl. Phys. A **791**, 267 (2007).
[13] J. L. Lecoey, Few Body Systems **34**, 21 (2004).
[14] G. Christian *et al.*, Nucl. Phys. A **801**, 101 (2008).
[15] H. Al Falou, A. Leprince, and N. A. Orr, arXiv:1004.3233v1.
[16] Y. Kondo *et al.*, Phys. Lett. B **690**, 245 (2010).
[17] H. T. Fortune and R. Sherr, Phys. Rev. C **82**, 064302 (2010).
[18] H. T. Fortune, Nucl. Instrum. Methods Phys. Res. Sect. A **681**, 7 (2012).
[19] G. F. Bertsch and H. Esbensen, Ann. Phys. (NY) **209**, 327 (1991).
[20] I. J. Thompson and M. V. Zhukov, Phys. Rev. C **53**, 708 (1996).
[21] H. T. Fortune and R. Sherr, Phys. Rev. C **85**, 051303 (2012).
[22] M. Labiche, F. M. Marqués, O. Sorlin, and N. Vinh Mau, Phys. Rev. C **60**, 027303 (1999).
[23] T. Tarutina, I. J. Thompson, and J. A. Tostevin, Nucl. Phys. A **733**, 53 (2004).
[24] P. Descouvemont, Phys. Lett. B **331**, 271 (1994).
[25] P. Descouvemont, Phys. Rev. C **52**, 704 (1995).
[26] J. C. Pacheco and N. Vinh Mau, Phys. Rev. C **65**, 044004 (2002).
[27] G. Blanchon, N. Vinh Mau, A. Bonaccorso, M. Dupuis, and N. Pillet, Phys. Rev. C **82**, 034313 (2010).
[28] Ikuko Hamamoto, Phys. Rev. C **77**, 054311 (2008).
[29] Yoshiko Kanada-En'yo, Phys. Rev. C **85**, 044320 (2012).
[30] H. T. Fortune and R. Sherr, Phys. Rev. C **86**, 034301 (2012).
[31] R. D. Lawson, in *Theory of the Nuclear Shell Model* (Clarendon Press, Oxford, 1980), p. 63ff.
[32] R. L. Lawson, F. J. D. Serduke, and H. T. Fortune, Phys. Rev. C **14**, 1245 (1976).
[33] H. T. Fortune, Phys. Lett. B **703**, 71 (2011).