

Clustering-Based Robot Navigation and Control

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Abstract—In robotics, it is essential to model and understand the topologies of configuration spaces in order to design provably correct motion planners. The common practice in motion planning for modelling configuration spaces requires either a global, explicit representation of a configuration space in terms of standard geometric and topological models, or an asymptotically dense collection of sample configurations connected by simple paths. In this short note, we present an overview of our recent results that utilize clustering for closing the gap between these two complementary approaches. Traditionally an unsupervised learning method, clustering offers automated tools to discover hidden intrinsic structures in generally complex-shaped and high-dimensional configuration spaces of robotic systems. We demonstrate some potential applications of such clustering tools to the problem of feedback motion planning and control. In particular, we briefly present our use of hierarchical clustering for provably correct, computationally efficient coordinated multirobot motion design, and we briefly describe how robot-centric Voronoi diagrams can be used for provably correct safe robot navigation in forest-like cluttered environments, and for provably correct collision-free coverage and congestion control of heterogeneous disk-shaped robots.

I. INTRODUCTION

With the increasing use of robots in our daily lives, from household applications [1] to elder/patient assistance [2] to self-driving vehicles [3], it has become even more crucial for autonomous robotics systems to be able to safely move in their workspaces in order to accomplish given tasks. Two commonly encountered approaches to tackle the safe robot navigation problem are *configuration space motion planning* and *sampling-based motion planning* [4], [5].

Once an explicit representation of a robot’s configuration space is obtained, a number of configuration space motion planners [4], [5], such as discrete planners, cell decomposition and roadmap methods, and feedback motion planners, can be used to safely steer the robot toward its target configuration, satisfying given task specifications. However, configuration spaces generally have complex shapes and are difficult, if not impossible, to explicitly describe in terms of standard geometric and topological models. Also, the complexity of motion planning is known to grow exponentially as configuration spaces grow in dimension [6]. These limitations therefore restrict the applicability of configuration space planners to low dimensional settings.

Alternatively, sampling-based methods [4], [5], such as probabilistic roadmaps, rapidly-exploring random trees, and their variants, resolve such limitations by producing (open-loop) navigation paths based on randomly sampled robot

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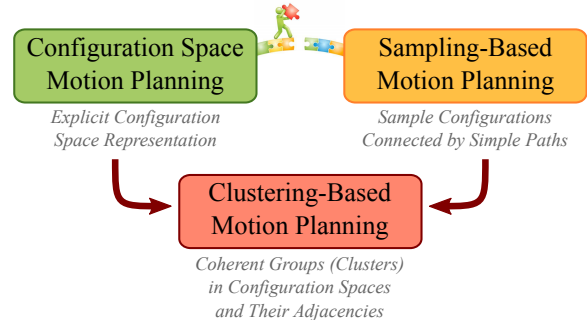


Fig. 1. Clustering-based motion planning: A new perspective for closing the gap in modelling configuration spaces between configuration space motion planning and sampling-based motion planning.

configurations and simple connectivity criteria. Although they require no explicit constructions of configuration spaces, sampling-based methods strongly rely on fast collision detectors, efficient nearest neighbor and graph search algorithms, effective sampling strategies (especially around narrow passages), and informative metric selection [7].

In brief, these two widely used motion planning methods fundamentally differ from each other in modelling configuration spaces: on one hand, we have methods based on global, explicit representations of configuration spaces; on the other hand, we have methods based on individual, sample configurations connected by simple paths. As a new alternative approach, we propose the use of *clustering* for closing the gap between these two complementary motion planning methods to combine their strengths in modelling configuration spaces (see Fig. 1).

Traditionally an unsupervised learning method, *clustering* offers automated tools to discover coherent groups (clusters) in configuration spaces to model their unknown global organizational structure (e.g., hierarchical clustering), and to determine collision-free local neighborhoods of robot configurations (e.g., partitional clustering) [8]. A unique strength of configuration space clustering over arbitrary configuration space partitioning, such as cellular decompositions, is that clustering not only yields a cover of a configuration space in terms of configuration clusters, but also relates each covering element to a *clustering model* (e.g., cluster hierarchies and finite set partitions) that deciphers the structural properties of the associated configuration cluster. Hence, on a more conceptual level, clustering can be viewed as a symbolic abstraction relating the continuous space of configurations to the (combinatorial) space of clustering models. We argue that the intrinsic local structures in configuration spaces that are identified by clustering can be exploited to design computationally efficient, provably correct motion planners.

II. OUR CONTRIBUTIONS

Inspired by the use of clustering for modelling global organizational structure, we introduce a novel abstraction for ensemble task encoding and control of multirobot systems in terms of hierarchical clustering that yields precise yet flexible organizational specifications at selectively multiple resolutions [9]. Based on this new abstraction and an explicit topological characterization of the associated configuration clusters, we construct a provably correct, computationally efficient hierarchical navigation framework [10], [11] for collision-free coordinated multirobot motion design toward a designated configuration via online sequential composition of hierarchy-preserving local controllers [9].

Inspired by the use of clustering for locality identification, we introduce a new, robot-centric application of Voronoi diagrams to identify a collision-free convex neighborhood of a robot moving in a cluttered environment, which turns out to be a simple but effective way of extracting the intrinsic local geometric structure of the configuration space around the robot's instantaneous position. Based on robot-centric Voronoi diagrams, we design a provably correct, collision-free coverage and congestion control algorithm for distributed mobile sensing applications of heterogeneous disk-shaped robots [12], and we introduce a sensor-based reactive navigation algorithm for exact navigation of a disk-shaped robot in forest-like cluttered environments [13].

The rest of the paper is organized to give an intuitive overview of these applications with specific references to the technical papers that present all the mathematical details. Section IV briefly describes and motivates the generic components of our hierarchical navigation framework for coordinated motion design. Section V gives an overview of robot-centric Voronoi diagrams and their applications to safe robot navigation and coverage control. Section VI concludes with a summary of our contributions.

III. RELATED LITERATURE

A commonly encountered approach in motion planning that is strongly related to clustering is *spatial decomposition* [4], [5]. For example, roadmap methods typically construct a global, one-dimensional graphical representation (skeleton) of an environment based on its Voronoi decomposition. Hierarchical decomposition methods, based on quadtrees and octrees, are also successfully applied for representing environments at multiple resolutions via adaptive cells. In particular, their recursive constructions yield computationally efficient solutions for robots operating in unknown and sparse environments [14]–[18]. In brief, spatial decomposition methods are generally employed to build efficient data structures that approximately model environments, independent of any specific robot configuration and model, whereas our intended use of clustering is to explicitly extract immanent local geometric and topological structures in configuration spaces around a given robot configuration.

Clustering has also played a key role in the design of scalable algorithms for motion planning and control of large groups of robots, because coordinated motion planning of

independent thick bodies in a compact space is known to be computationally difficult [19], [20]. Hierarchical coordination strategies that divide a large group of robots into small teams in order to limit coordination across robots have been shown to alleviate the combinatorial growth of complexity [21], [22]. Moreover, hierarchical discrete abstraction methods are successfully applied for scalable steering of a large number of robots as a unified group by controlling the group shape [23]. Group coordination via splitting and merging behaviours also creates effective strategies for obstacle avoidance [24], congestion control [25], shepherding [22], and area exploration [22]. Alternatively, we show that hierarchical clustering offers an interesting means of ensemble task encoding and control; especially, the ability to specify organizational structure in the precise but flexible terms that hierarchy permits enables us to specify group coordination behaviours at selectively multiple resolutions for safe multirobot navigation.

IV. COORDINATED ROBOT NAVIGATION VIA HIERARCHICAL CLUSTERING

Cooperative, coordinated action and sensing has been shown to promote efficiency, robustness, and flexibility in achieving complex tasks such as search and rescue, area exploration, surveillance and reconnaissance, and warehouse management [26]. While accomplishing such a diverse set of tasks, the spatial distribution of robotic agents is generally required to change in response to environmental stimulus. Moving from one spatial distribution to another is generally carried through rearrangements of robot groups (clusters) at different resolution corresponding to transitions between different cluster structures (hierarchies). This observation leads us to the notion of hierarchical clustering for modelling the spatial structural organization of multirobot systems.

Hierarchical clustering [8] offers a natural abstraction for ensemble task encoding and control in terms of precise yet flexible organizational specifications at different resolutions, by relating the continuous space of configurations to the combinatorial space of trees. This hierarchical abstraction intrinsically suggests a two-level navigation strategy for coordinated motion design: 1) At the low-level, perform finer adjustments on configurations by using hierarchy preserving vector fields [9]; and 2) At the high-level, resolve structural conflicts between configurations by using a discrete transition policy in tree space [27]. The connection between these two levels is established by an optimal selection of a portal configuration supporting two adjacent hierarchies [10], [28]. Accordingly, we propose a provably correct generic hierarchical navigation framework [11] for collision-free coordinated motion design toward any given destination via a sequence of hierarchy-preserving controllers [11], whose generic components and their relations are illustrated in Fig. 2.

For a choice of a hierarchical clustering algorithm, we demonstrate a computationally efficient instantiation of our hierarchical navigation framework for coordinated control of an arbitrary number of disk-shaped robots operating in an

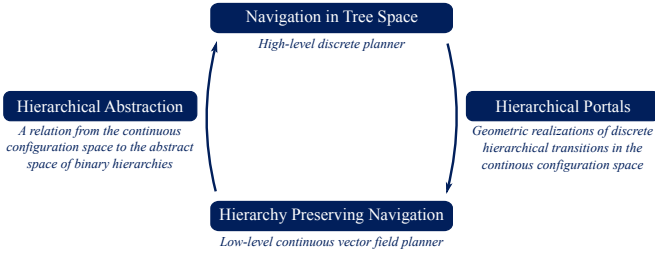


Fig. 2. Generic Components of Hierarchical Navigation Framework

ambient Euclidean space. Here, we reveal that the homotopy model of configurations sharing the same cluster hierarchy is a generalized torus. Accordingly, by taking advantage of the underlying topology, we introduce a recursively defined vector field for hierarchy-preserving navigation. To enable controllable switching between different cluster hierarchies (and so the associated local controllers), we also design a computationally efficient recursive algorithm for navigating through the space of cluster hierarchies of n robotic agents that is guaranteed to reach a desired hierarchy in $O(n^2)$ steps, each step costing $O(n)$ computations [27]. In Fig. 3, we present a sample trajectory of our hierarchical navigation planner for four disks moving on a plane, and the sequence of trees associated with deployed local controllers. It is useful to emphasize that although the number of potentially available local controllers for a group of n disks grows superexponentially with n , our hierarchical navigation planner automatically deploys at most $\frac{1}{2}(n-1)(n-2)$ local controllers [27], illustrating its computational efficiency.

V. ENCODING COLLISIONS VIA ROBOT-CENTRIC VORONOI DIAGRAMS

In motion planning, Voronoi diagrams are traditionally encountered in the design of roadmap methods [4], [5] for constructing unidimensional graphical representations of environments, independent of any specific robot configuration. We introduce a new, robot-centric application of Voronoi diagrams to encode robot collisions exactly by exploiting the local structure of configuration spaces around a robot configuration. This also enables us to determine a safe convex neighborhood of a robot configuration.

Motivated by recent interest in agile navigation in dense human crowds [29], or in natural forests, such as now negotiated by rapid flying [30] and legged [31] robots, we

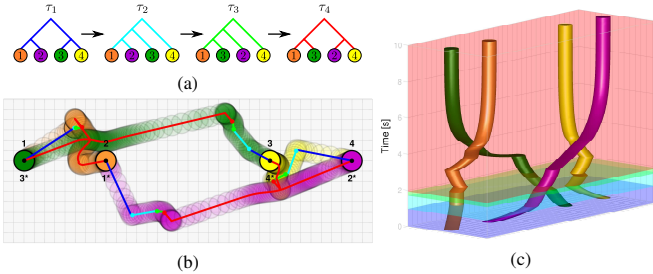


Fig. 3. Illustrative navigation trajectory of our hierarchical motion planner for four disks moving in a planar ambient space [11]: (a) Sequence of trees associated with deployed local controllers during the execution of the navigation planner, (b) Navigation paths, (c) Space-time curve of disks

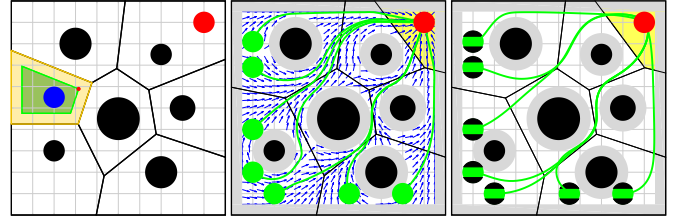


Fig. 4. Exact robot navigation using robot-centric generalized Voronoi diagrams [13]. The Voronoi cell (yellow) associated with the robot defines its obstacle free convex local neighborhood, and the continuous feedback motion towards the metric projection (red dot) of a given desired goal (red disk) onto the associated local free space (green polygon) asymptotically steers almost all robot configurations (green disks) to the goal, with no collisions along the way. Example trajectories of the feedback motion planner starting at a set of initial configurations (green disks) towards the goal location (red disk) for (middle) a fully actuated and (right) a differential drive robot.

propose a new reactive motion planner taking the form of a feedback law, relative to a fixed goal location, that can be computed using only local knowledge of the environment identified by the Voronoi cell around the robot [13], see Fig. 4. In particular, we show that the continuous feedback motion toward the metric projection of the desired goal onto the robot's convex Voronoi cell steers almost all robot configurations to the goal in environments cluttered with spherical obstacles, while avoiding collisions along the way. Having such a collision-free convex neighbourhood of the robot also enables us to simply extend these provable properties to the standard differential drive vehicle model.

In Figure 4, we demonstrate the motion pattern generated by our reactive motion planner in a cluttered environment. Here, it is important to observe that the resulting navigation trajectories and the boundary of the Voronoi diagram of the environment are significantly consistent. In other words, the robot balances its distance to all proximal obstacles while navigating toward its destination, which is a desired autonomous behaviour for many practical settings instead of grazing the obstacle boundary.

What is more, in distributed mobile sensing applications, Voronoi diagrams are often utilized for solving sensory task assignment and for modelling group heterogeneity in actuation, sensing, computation and energy sources [32]–[34]. In addition to these usages, we tailor Voronoi diagrams to encode collisions in a heterogeneous group of disk-shaped robots. Accordingly, based on standard coverage control of point robots [32]–[34], we propose a constrained coverage control law for heterogeneous disk-shaped robots that solves the combined sensory coverage and collision avoidance problem [12]. We further introduce a congestion management heuristic for unassigned robots to hasten the assigned robots' progress, while retaining the provable properties.

In Fig. 5, we present the resulting trajectories of our proposed coverage control algorithms for a sample distributed coverage task. As seen in Figure 5(left), the 2nd robot is initially not assigned to any region. Since our safe coverage control algorithms prevent self-collisions and collisions with the boundary of the environment, the 2nd robot stays unassigned for all future time; therefore, the 1st

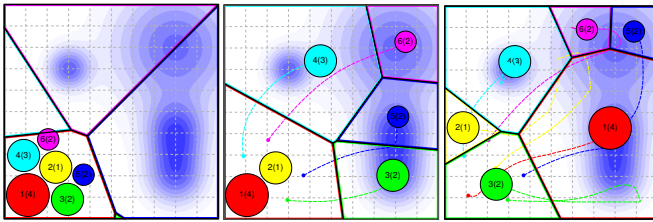


Fig. 5. Safe coverage control of heterogeneous disk-shaped robots with a heuristic management of unassigned robots [12]. (left) Initial configuration of heterogeneous robots, where the weight of sensory cell are shown in the parenthesis, and the resulting trajectories of (middle) our safe coverage control law, (right) our coverage control law with the congestion heuristic.

robot is blocked and it can not move to a better coverage location in Fig. 5(middle). Fortunately, while guaranteeing collision avoidance, our coverage control law with a congestion heuristic steers unassigned robots to improve assigned robots' progress, as illustrated in Fig. 5(right).

VI. CONCLUSION

In this paper, we give an overview of our use of clustering for modelling configuration spaces and for design of provably correct motion planners. This new philosophy for modelling configuration spaces, still in its infancy, yields promising results for closing the gap between standard configuration space and sampling-based motion planning approaches. To demonstrate some potential applications of clustering to feedback motion design, we present the use of hierarchical clustering for provably correct coordinated multirobot motion design [9]–[11], and we show how the robot-centric Voronoi diagrams can be used for provably correct safe robot navigation in cluttered environments [13], and for safe coverage and congestion control of heterogeneous mobile sensor networks [12]. We believe that these nontrivial applications of clustering to robot motion design only scratch the surface of its long-term potential.

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