ESSAYS ON DYNAMIC MACROECONOMICS

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ABSTRACT

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This dissertation consists of two chapters that study substantive and methodological issues in business cycle research.

In the first chapter, I study a business cycle model where agents learn about the state of the economy through accumulating capital. During recessions, agents invest less, and this generates noisier estimates of macroeconomic conditions and an increase in uncertainty. The endogenous increase in aggregate uncertainty further reduces economic activity, which in turn leads to more uncertainty, and so on. Thus, through changes in uncertainty, learning gives rise to a multiplier effect that amplifies business cycles. I calibrate the model to measure the size of this uncertainty multiplier and find that it is large. Moreover, the model quantitatively replicates the VAR relationship between output and uncertainty.

In the second chapter, I evaluate the common practice of estimating dynamic stochastic general equilibrium (DSGE) models using seasonally adjusted data.[1] The simulation experiment shows that the practice leads to sizeable distortions in estimated parameters. This is because the effects of seasonality, which are magnified by the model's capital accumulation and labor market frictions, are not restricted to the so-called seasonal frequencies but instead are propagated across the entire frequency

[1]This chapter is published in the *Journal of Econometrics*.

domain.

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Chapter 1

The Uncertainty Multiplier and Business Cycles

1.1 Introduction

A central criticism of modern business cycle theories is that they require an unrealistic magnitude of primitive shocks to fit the data. For example, plain vanilla real business cycle models need to invoke sizable technological regress in order to explain recessions. Motivated by this challenge, a rapidly growing literature argues that shocks to uncertainty are a significant driver of business cycle dynamics—see, for example, Bloom (2009), Fernández-Villaverde et al. (2011), Gourio (2012), and Christiano et al. (2012). However, this literature also faces an important criticism similar to that of the standard theories. While various proxies of uncertainty rise during almost all recessions,[1] exogenous events that significantly increase the volatil-

[1] See, for example, Bloom et al. (2012).

ity of the economy seem to be rare. This observation suggests that fluctuations in uncertainty may be, at least partially, endogenous. The distinction is crucial. For example, if uncertainty is an equilibrium object, policy experiments that treat uncertainty as exogenous are subject to the Lucas critique.

In this paper, I present a quantitative business cycle model where the level of economic activity influences the level of aggregate uncertainty. The endogenous movement in uncertainty, in turn, affects the level of economic activity. I demonstrate that this two-way feedback between economic activity and uncertainty is important for understanding business cycles.

The model builds on a standard equilibrium business cycle framework with several real and nominal rigidities (Christiano et al. 2005). I introduce information frictions by subjecting the economy to aggregate shocks that agents cannot directly observe, namely, shocks to the marginal efficiency of investment and shocks to the depreciation rate of capital. Because the former are persistent while the latter are transitory, what matters for agents' optimal decision is the evolution of the efficiency of investment. Agents use the path of capital stock and investment to form their estimates in a Bayesian manner.[2] However, the capital stock is not perfectly revealing about the unobservable shocks because it is subject to a non-invertibility problem: Agents cannot tell whether an unexpectedly high realization of capital stock is due to a high efficiency of investment or to a low depreciation rate of capital.

In the model, the level of investment endogenously determines the informativeness

[2] In the model, all information necessary for optimal learning is contained in the path of capital stock and investment. While agents have access to other endogenous variables, including prices, they do not reveal additional information about the unobservable shocks.

of the capital stock about the shocks to the efficiency of investment. When agents invest less, their estimates are imprecise because the level of capital stock is largely determined by the realization of the depreciation shock. Conversely, when they invest more, their estimates are accurate because the current capital mostly reflects shocks to the efficiency of investment. Thus, aggregate uncertainty becomes endogenously countercyclical over the business cycle.

The countercyclical uncertainty gives rise to a novel multiplier effect that amplifies business cycles. Imagine that the economy is hit by a negative shock that lowers investment (for example, an exogenous tightening of monetary policy). Since agents learn less about the current period shock to the efficiency of investment, uncertainty increases. This, in turn, further reduces investment and other economic activity because of households' precautionary motive and countercyclical movements in markups. The opposite channel works when the economy is hit by a positive shock. I call this amplification mechanism the *uncertainty multiplier*.

To measure the size of the uncertainty multiplier, I perform numerical simulations. Since my purpose is to investigate the direct effect of changes in uncertainty, I go beyond linear approximations and use a third-order perturbation method to solve the model. In a standard first-order approximation, changes in uncertainty play no role, since the decision rules of agents are forced to follow a certainty equivalence principle. In the second-order approximation, changes in uncertainty only appear in the decision rules as cross-product terms with other state variables. Only in the third-order approximation do changes in uncertainty show up as an independent term.

The model is calibrated to match the business cycle properties of the postwar U.S. quarterly data. An interesting challenge I face is that the choice of the variance parameters has important effects on the strength of learning dynamics. More specifically, when the variance of the depreciation shock is too small compared to that of the shock to the efficiency of investment, the capital stock is almost perfectly revealing about the shock to the efficiency of investment. Conversely, when the depreciation shock is too large, the capital stock is uninformative and little learning takes place. In both cases, fluctuations in aggregate uncertainty are negligible. To ensure that agents face a realistic amount of information frictions, I pin down the variance parameters so that the model replicates the properties of survey data on macroeconomic forecasts.

The uncertainty multiplier is large. In particular, under the benchmark calibration the standard deviation of output is amplified by 33%. Other real variables, such as investment and hours, are also amplified by a similar amount. The results are due to two main features of the model. First, in my model changes in uncertainty generate positive comovements among real variables. Second, the uncertainty process is volatile and persistent because it is tied to the movement of investment.

Finally, I provide an external validation of my theory by showing that it quantitatively replicates the VAR impulse response of the survey measure of uncertainty. In particular, it can account for the negative relationship between output and uncertainty and it also reproduces gradual responses of the two variables. This is because in the model uncertainty is inversely related to investment, which exhibits hump-shaped dynamics, and this uncertainty in turn induces gradual adjustments by households. I conclude that the uncertainty multiplier could be a key force that transforms relatively small shocks into larger business cycles.

The rest of the paper is organized as follows. In the next section, I describe my contributions with respect to the existing literature. In Section 3, I present the model. In Section 4, I discuss its solution and calibration. In Section 5, I present the results. In Section 6, I provide evidence of my theory from survey data. Finally, Section 7 concludes with some directions for future research.

1.2 Connections to the Literature

This paper is related to several strands of the literature. First, it is related to a growing literature on uncertainty shocks. A leading example is a paper by Bloom (2009), who shows that an exogenous increase in the volatility of firm-level productivity reduces output through a "wait-and-see" effect due to investment irreversibility. Fernández-Villaverde et al. (2011) show that volatility shocks to real interest rates generate sizable contractions in an otherwise standard small open economy model. Other examples include Arellano et al. (2012), Basu and Bundick (2011), Christiano et al. (2012), Fernández-Villaverde et al. (2012), Gilchrist et al. (2010), Gourio (2012), Ilut and Schneider (2011), and Schaal (2012). I show that time-varying uncertainty could be an important amplification (rather than an impulse) mechanism of the business cycle. As stated in the Introduction, this distinction is important because now uncertainty is an equilibrium object.

Recently, some authors have argued that changes in uncertainty have negligible effects given small and transient fluctuations in observed realized volatility (Bachmann and Bayer 2012, Born and Pfeifer 2012, and Chugh 2012). The problem of this approach is that the realized volatility may not accurately reflect the actual uncertainty that agents face. In fact, in my model, uncertainty features a large and persistent fluctuation that is not linked with movements in the realized volatility of macro variables.[3] As a result, unlike in these papers, changes in uncertainty have sizable effects.

Several papers attempt to explain the countercyclical firm-level volatility through conventional first-moment shocks. For example, in Bachmann and Moscarini (2011), recessions induce firms to price-experiment, which in turn raises the cross-sectional dispersion of price changes. See also D'Erasmo and Boedo (2012), Kehrig (2011), and Tian (2012). An important distinction between my paper and theirs is that, while their models endogenously deliver ex-post volatility, mine delivers ex-ante uncertainty. This is why in my model uncertainty is not merely a by-product of agents' response to first-moment shocks, but rather an important factor that affects real allocations.

The main mechanism of this paper builds on a literature on asymmetric learning, for example, Veldkamp (2005), Nieuwerburgh and Veldkamp (2006), Ordoñez (2012), and Görtz and Tsoukalas (forthcoming). They argue that the time-varying speed of learning about the macroeconomic conditions could explain the asymmetries in growth rates over the business cycle. When the economy passes the peak of a boom, agents are able to precisely detect the slowdown, leading to an abrupt crash. At

[3] Ilut and Schneider (2011) also propose a business cycle model where changes in uncertainty are not followed by changes in realized volatility by assuming ambiguity-averse preferences.

the end of the recession, agents' estimates about the extent of recovery are noisy, slowing reactions and delaying booms. My contribution is to explore the direct effects of endogenous fluctuations in uncertainty that shift the levels of macro variables. Recessions are deeper because high uncertainty leads precautionary households to cut consumption. Booms are stronger for the opposite reason. This channel has been overlooked in the previous literature.

Finally, this paper joins a long tradition in macroeconomics by considering the role of imperfect information and expectations in shaping business cycle dynamics. Recent contributions include Angeletos and La'o (forthcoming), Barsky and Sims (2012), Beaudry and Portier (2004), Eusepi and Preston (2011), Lorenzoni (2009), Jaimovich and Rebelo (2009), and Schmitt-Grohe and Uribe (2012). These papers emphasize changes in the *mean* of agents' subjective estimates about fundamentals. The current paper, instead, demonstrates the importance of changes in the *variance* of estimates about fundamentals.

1.3 The Model

I embed a learning problem into the capital accumulation process of a standard monetary business cycle framework (Christiano et al. 2005, Justiniano et al. 2010, and Smets and Wouters 2007). This framework is a natural laboratory for my quantitative investigation, since it has now become the foundation of applied research in both academic and government institutions.

In the first subsection, I describe the information frictions. In the second subsection, I present the standard part of the model.

1.3.1 Learning and Endogenously Countercyclical Uncertainty

I divide the presentation of the information frictions into several parts. First, I describe the setup. Second, I express the learning process as a Kalman filtering problem. Third, I present a simple example that illustrates the key properties of the filtering problem. Finally, I rewrite the capital accumulation process from the perspective of the agents. This clarifies the impact of changes in uncertainty on the agents' decision making.

Setup

The law of motion for capital, K_t, is subject to two types of structural disturbances:

$$K_t = (1 - \delta_t)K_{t-1} + \mu_t I_{t-1}.$$

The depreciation shock, δ_t, follows

$$\delta_t = \delta - \epsilon_{\delta,t},$$

where $\epsilon_{\delta,t}$ is i.i.d. distributed from a normal distribution with mean zero and variance σ_δ^2. The investment shock, μ_t, determines the marginal efficiency of investment. I assume that μ_t follows the stochastic process

$$\mu_t = g_{t-1} + (1-\rho_\mu)\mu + \rho_\mu \mu_{t-1} + \epsilon_{\mu,t},$$

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t},$$

where $\epsilon_{\mu,t}$ and $\epsilon_{g,t}$ are i.i.d. distributed from a normal distribution with mean zero and variance σ_μ^2 and σ_g^2, respectively. The growth shock, g_t, controls the growth rate of μ_t.[4] Agents cannot directly observe the current or previous values of δ_t, μ_t, and g_t. This informational assumption gives rise to a non-invertibility problem: Agents cannot tell whether an unexpectedly high realization of capital stock is due to a high efficiency of investment or to a low depreciation rate of capital. As a result, they face a signal-extraction problem in forecasting the evolution of the shocks. Agents use all available information, including the path of capital stock, to form their estimates.

A literal interpretation of the depreciation shock is that it represents an exogenous change in the physical depreciation rate of capital. However, as in Gourio (2012), Gertler and Karadi (2011), and Liu et al. (2011), a broader interpretation is possible. For example, it can represent an economic obsolescence of capital. Alternatively, reallocation of capital may be subject to temporary frictions and could show up as a change in the "quality" of aggregate capital.

The investment shock was originally proposed by Greenwood et al. (1988). In a medium-scale DSGE model similar to the one employed in this paper, Justiniano et al. (2010) have found that the shock is the most important driver of the U.S. business cycle. In general, there are two ways to think about the investment shocks. The first

[4]The growth shock is not strictly necessary for the theoretical results of the paper. However, as I show below the shock helps match some of the survey data moments.

interpretation is that they represent disturbances that affect the transformation of consumption goods into investment goods. The second interpretation is that they are shocks that affect the transformation of investment goods into installed capital. In this paper, I adopt the second interpretation.[5] Then the investment shock can be thought of as a disturbance to the intermediation ability of the financial system. An important implication of this interpretation is that, unlike Fisher (2006), the investment shock does not affect the price of investment goods relative to consumption goods. Thus, agents cannot back out the shocks by observing the price.

As summarized in Figure 1.1, the timing of events is as follows: At the end of period $t-1$, agents choose their investment level I_{t-1} given the current capital level K_{t-1} and their estimates about the unobservable state. Then, at the beginning of period t, unobservable shocks are realized. Finally, after observing the level of new capital K_t, agents update the estimates.

The Kalman Filtering Problem

Agents update their estimates about μ_t and g_t in an optimal (Bayesian) manner. The learning process can be expressed as a Kalman filtering problem:

$$\begin{bmatrix} \mu_t \\ g_t \end{bmatrix} = \begin{bmatrix} (1-\rho_\mu)\mu \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_\mu & 1 \\ 0 & \rho_g \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ g_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\mu,t} \\ \epsilon_{g,t} \end{bmatrix}, \tag{1.1}$$

$$K_t - (1-\delta)K_{t-1} = [I_{t-1} \quad 0] \begin{bmatrix} \mu_t \\ g_t \end{bmatrix} + K_{t-1}\epsilon_{\delta,t}. \tag{1.2}$$

[5] See Justiniano et al. (2011) for supportive evidence based on a DSGE model estimation.

Equation (1.1) is the state equation that characterizes the evolution of the unobservable state. Equation (1.2) is the measurement equation that describes the observables as a linear function of the underlying state. I point out two things regarding the measurement equation. First, $\epsilon_{\delta,t}$ serves as a measurement error in the filtering system. Second, unlike standard time-invariant systems, the coefficient matrices are time-varying.[6]

The key property of the system is that the signal-to-noise ratio is procyclical, which follows from the fact that $\frac{I_{t-1}}{K_{t-1}}$ is procyclical. The flip side implication of this property is that *uncertainty is countercyclical*. Denote Σ_t as the error-covariance matrix of the unobservable states,

$$\Sigma_t = \begin{bmatrix} \mathrm{Var}_t(\mu_t - \tilde{\mu}_t) & \mathrm{Cov}_t(\mu_t - \tilde{\mu}_t, g_t - \tilde{g}_t) \\ \cdots & \mathrm{Var}_t(g_t - \tilde{g}_t) \end{bmatrix},$$

then the elements of Σ_t are decreasing in $\frac{I_{t-1}}{K_{t-1}}$. Intuitively, when agents invest less, their estimates about the efficiency of investment are imprecise because the level of capital stock is largely determined by the realization of the depreciation shock. Conversely, their estimates are accurate when they invest more because the current capital mostly reflects shocks to the efficiency of investment.

[6] As in Veldkamp (2005) and Nieuwerburgh and Veldkamp (2006), I rule out active experimentation for computational reasons. Cogley et al. (2007) have shown, in the context of U.S. monetary policy making, that the two approaches (learning with and without experimentation) produce very similar decision rules.

Understanding Why Uncertainty Is Countercyclical

I explain how a procyclical signal-to-noise ratio leads to countercyclical uncertainty by going through a simpler example. In particular, assume that there is no growth shock.[7] Then the filtering problem reduces to

$$\mu_t = (1-\rho_\mu)\mu + \rho_\mu \mu_{t-1} + \epsilon_{\mu,t}, \tag{1.3}$$

$$y_t = I_{t-1}\mu_t + K_{t-1}\epsilon_{\delta,t}, \tag{1.4}$$

where (1.3) is the state equation and (1.4) is the measurement equation. I define $y_t \equiv K_t - (1-\delta)K_{t-1}$. In period $t-1$, agents enter with the mean estimate $\tilde{\mu}_{t-1}$ and its associated error variance $\Sigma_{t-1} \equiv \text{Var}_{t-1}(\mu_{t-1} - \tilde{\mu}_{t-1})$. Then, the period $t-1$ prediction of μ_t and its associated error variance is given by

$$\tilde{\mu}_{t|t-1} = (1-\rho_\mu)\mu + \rho_\mu \tilde{\mu}_{t-1}$$

$$\Sigma_{t|t-1} = \rho_\mu^2 \Sigma_{t-1} + \sigma_\mu^2$$

After observing the outcome y_t, they update their estimates according to

$$\tilde{\mu}_t = \tilde{\mu}_{t|t-1} + Gain_t(y_t - I_{t-1}\tilde{\mu}_{t|t-1}),$$

[7] In the Appendix, I provide a full derivation with the growth shock.

where $Gain_t$ is the Kalman gain and is given by

$$Gain_t = \underbrace{\frac{I_{t-1}^2 \Sigma_{t|t-1}}{I_{t-1}^2 \Sigma_{t|t-1} + K_{t-1}^2 \sigma_\delta^2}}_{\text{Informativeness of observation}} \cdot \underbrace{\frac{1}{I_{t-1}}}_{\text{Adjustment}} .$$

The first term represents the informativeness of observation y_t and is given by the variance of the signal divided by the total variance (the variance of the signal and noise). The term is increasing in $\frac{I_{t-1}}{K_{t-1}}$. The second term is the scale adjustment term reflecting the fact that μ_t is multiplied by I_{t-1} in the observation.

The error variance associated with $\tilde{\mu}_t$ is given by

$$\begin{aligned}\Sigma_t &= (1 - Gain_t I_{t-1}) \Sigma_{t|t-1} \\ &= \underbrace{\frac{K_{t-1}^2 \sigma_\delta^2}{I_{t-1}^2 \Sigma_{t|t-1} + K_{t-1}^2 \sigma_\delta^2}}_{\text{Un-informativeness of observation}} \cdot \Sigma_{t|t-1}.\end{aligned}$$

The first line says that the error shrinks as we learn more from the observation; the error is decreasing in the size of the Kalman gain. The second line says that the error variance is increasing in the un-informativeness of observation (the variance of noise divided by the total variance). Since the un-informativeness term is decreasing in $\frac{I_{t-1}}{K_{t-1}}$, Σ_t is decreasing in $\frac{I_{t-1}}{K_{t-1}}$. Since investment is much more volatile than capital, $\frac{I_{t-1}}{K_{t-1}}$ moves almost proportionally to I_{t-1}. Thus, less investment leads to more uncertainty.

Implications of Time-Varying Uncertainty From the Perspective of the Agents

How do changes in uncertainty about the current efficiency of investment affect agents' decision making? The key insight here is that, because shocks to the efficiency of investment are persistent, uncertainty about the current state translates into uncertainty about the future realization of capital.

To see this, it is useful to rewrite the capital accumulation equation from the perspective of the agent at period $t-1$:

$$K_t = (1-\delta_t)K_{t-1} + (\tilde{\mu}_{t|t-1} + u_t)I_{t-1},$$

where $\tilde{\mu}_{t|t-1}$ is the mean forecast of μ_t at time $t-1$ and u_t is normally distributed with mean zero and variance $\sigma^2_{u,t}$. The innovation u_t takes into account not only the exogenous innovation to μ_t, but also its estimation error:

$$\begin{aligned} u_t &= \mu_t - \tilde{\mu}_{t|t-1} \\ &= (g_{t-1} - \tilde{g}_{t-1}) + \rho_\mu(\mu_{t-1} - \tilde{\mu}_{t-1}) + \epsilon_{\mu,t}, \end{aligned}$$

and hence its volatility is given by

$$\sigma^2_{u,t} = \rho^2_\mu \Sigma^{11}_{t-1} + 2\rho_\mu \Sigma^{12}_{t-1} + \Sigma^{22}_{t-1} + \sigma^2_\mu.$$

Thus, the fluctuation in uncertainty shows up as a fluctuation in volatility of the innovation to the marginal efficiency of investment. Moreover, this fluctuation in volatility is persistent to the extent that investment is persistent.

1.3.2 Standard Part of the Model

I now describe other components of the model. The economy is composed of the final-goods sector, intermediate-goods sector, household sector, employment sector, and a central bank. I start by describing the production side of the economy.

The Final-Goods Sector

In each period t, the final goods, Y_t, are produced by a perfectly competitive representative firm that combines a continuum of intermediate goods, indexed by $j \in [0, 1]$, with technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\theta_p - 1}{\theta_p}} \, dj \right]^{\frac{\theta_p}{\theta_p - 1}}.$$

$Y_{j,t}$ denotes the time t input of intermediate good j and θ_p controls the price elasticity of demand for each intermediate good. The demand function for good j is

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_p} Y_t,$$

where P_t and $P_{j,t}$ denote the price of the final good and intermediate good j, respectively. Finally, P_t is related to $P_{j,t}$ via the relationship

$$P_t = \left[\int_0^1 P_{j,t}^{1-\theta_p} dj \right]^{\frac{1}{1-\theta_p}}.$$

The Intermediate-Goods Sector

The intermediate-goods sector is monopolistically competitive. In period t, each firm j rents $K_{j,t}$ units of capital stock from the household sector and buys $H_{j,t}$ units of aggregate labor input from the employment sector to produce intermediate good j using technology

$$Y_{j,t} = z_t K_{j,t}^{\alpha} H_{j,t}^{1-\alpha}.$$

z_t is the level of total factor productivity that follows

$$z_t = (1 - \rho_z) z + \rho_z z_{t-1} + \epsilon_{z,t},$$

where $\epsilon_{z,t}$ is i.i.d. distributed from a normal distribution with mean zero and variance σ_z^2.

Firms face a Calvo-type price-setting friction: In each period t, a firm can reoptimize its intermediate-goods price with probability $(1 - \xi_p)$. Firms that cannot reoptimize index their price according to the steady-state inflation rate, π.

The Household Sector

There is a continuum of households, indexed by $i \in [0, 1]$. In each period, household i chooses consumption C_t, investment I_t, bond purchases B_t, and nominal wage $W_{i,t}$ to maximize utility:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s d_{t+s} \left[\frac{(C_{t+s} - bC_{t+s-1})^{1-\sigma}}{1-\sigma} - \frac{H_{i,t+s}^{1+\eta}}{1+\eta} \right],$$

where β is a discount factor, σ is a risk-aversion coefficient, b represents consumption habit, η controls (the inverse of) the Frisch labor supply elasticity, and $H_{i,t}$ is the number of hours worked. d_t is a preference shock that follows

$$d_t = (1 - \rho_d)d + \rho_d d_{t-1} + \epsilon_{d,t},$$

where $\epsilon_{d,t}$ is i.i.d. distributed from a normal distribution with mean zero and variance σ_d^2.

The household's budget constraint is

$$P_t C_t + P_t I_t + B_t \leq W_{i,t} H_{i,t} + R_t^k K_t + R_{t-1} B_{t-1} + D_t + A_{i,t},$$

where R_t^k is the rental rate of capital, K_t is the stock of capital, R_{t-1} is the gross nominal interest rate from period $t-1$ to t, and D_t is the combined profit of all the intermediate-goods firms distributed equally to each household. I assume that households buy securities, whose payoffs are contingent on whether it can reoptimize its wage.[8] $A_{i,t}$ denotes the net cash inflow from participating in state-contingent security markets at time t.

As in Christiano et al. (2005), I add an investment adjustment cost to the capital

[8]The existence of state-contingent securities ensures that households are homogeneous with respect to consumption and asset holdings, even though they are heterogeneous with respect to the wage rate and hours because of the idiosyncratic nature of the timing of wage reoptimization. See Christiano et al. (2005).

accumulation equation described above:

$$K_t = (1-\delta_t)K_{t-1} + \mu_t\left(1 - S\left(\frac{I_{t-1}}{I_{t-2}}\right)\right)I_{t-1},$$

where

$$S\left(\frac{I_{t-1}}{I_{t-2}}\right) = \frac{\kappa}{2}\left(\frac{I_{t-1}}{I_{t-2}} - 1\right)^2,$$

with $\kappa > 0$. Other components of the capital accumulation, like the stochastic process of shocks or the informational structure, are exactly the same as described in the previous section.

The Employment Sector and Wage Setting

In each period t, a perfectly competitive representative employment agency hires labor from households to produce an aggregate labor service, H_t, using technology

$$H_t = \left[\int_0^1 H_{i,t}^{\frac{\theta_w - 1}{\theta_w}} di\right]^{\frac{\theta_w}{\theta_w - 1}},$$

where $H_{i,t}$ denotes the time t input of labor service from household i and θ_w controls the price elasticity of demand for each household's labor service. The agency sells the aggregated labor input to the intermediate firms for a nominal price of W_t per unit. The demand function for the labor service of household i is

$$H_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} H_t,$$

where $W_{i,t}$ denotes the nominal wage rate of the labor service of household i. W_t is related to $W_{i,t}$ via the relationship

$$W_t = \left[\int_0^1 W_{i,t}^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}}.$$

Households face a Calvo-type wage-setting friction: In each period t, a household can reoptimize its nominal wage with probability $(1 - \xi_w)$. Households that cannot reoptimize index their wage according to the steady-state inflation rate, π.

The Central Bank, Resource Constraint, and Equilibrium

The central bank sets the nominal interest rate according to a Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_R} \left\{ \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\phi_Y} \right\}^{1-\rho_R} \exp(\epsilon_{R,t}),$$

where R is the steady-state level of the nominal interest rate, ρ_R is the persistence of the rule, and ϕ_π and ϕ_Y are the size of the policy response to the deviation of inflation and output growth from their steady states, respectively. $\epsilon_{R,t}$ is a monetary policy shock and is i.i.d. distributed from a normal distribution with mean zero and variance σ_R^2.

Finally, the aggregate resource constraint is $C_t + I_t = Y_t$. I employ a standard sequential market equilibrium concept and hence its formal definition is omitted.

1.4 Model Solution and Calibration

I follow Fernández-Villaverde et al. (2011) and solve the model using a third-order perturbation method around its deterministic steady state.[9] I use perturbation because the model has many state variables and it is the only method that delivers an accurate solution in a reasonable amount of time (Aruoba et al. 2006). The third-order approximation is necessary because my purpose is to analyze the direct impact of endogenous changes in aggregate uncertainty. In a standard first-order approximation, changes in uncertainty play no role since the decision rules of agents are forced to follow a certainty equivalence principle. In the second-order approximation, changes in uncertainty only appear in the decision rules as cross-product terms with other state variables. Only in the third-order approximation do changes in uncertainty show up as an independent term.

The parameterization of the model is done in two steps. In the first step, I fix several parameter values following micro evidence or estimates found in other papers. In the second step, I choose values of the remaining parameters by matching the simulated moments of the model to the data. The first step reduces the number of parameters to be calibrated and thus sharpens the exercise in the second step.

The discount factor, β, is set so that the model steady-state interest rate implied by the Euler equation matches that of the data. The capital share is set to 0.3. $\delta = 0.02$ implies an annual depreciation rate of 8%. The elasticity of goods demand $\theta_p = 21$ and labor demand $\theta_w = 21$ are consistent with previous estimates, for example, Altig et al. (2011).

[9] The computation is carried out with Dynare (http://www.dynare.org/).

The coefficient of risk aversion is $\sigma = 2$ and the habit persistence parameter is set to $b = 0.65$. The latter value is taken from the estimates in Christiano et al. (2005). As emphasized in Boldrin et al. (2001), a strong habit persistence parameter helps to account for various asset pricing puzzles. Chetty et al. (2011) suggest a Frisch elasticity of labor supply of 0.5 for a macro model that does not distinguish between intensive and extensive margins. This leads to $\eta = 2$.

The Calvo price and wage parameters imply an average duration of one year. As found in Smets and Wouters (2007) and Justiniano et al. (2010), prices and wages need to be sufficiently sticky in order to account for the inflation and wage dynamics in the data. Turning to the monetary policy parameter, I match the steady-state inflation rate to its historical mean. The Taylor rule coefficients feature inertia with a strong response to inflation and a weak response to output growth (Levin et al. 2006, Smets and Wouters 2007, and Justiniano et al. 2010).

I also shut off shocks other than those to the capital accumulation equation, i.e., I set $\sigma_z = \sigma_d = \sigma_R = 0$. This is motivated by the recent evidence that disturbances to capital accumulation are a major driving force of business cycles (Justiniano et al. 2010 and Liu et al. 2011). It also reduces some parameters and makes the presentation of the main mechanism as transparent as possible.

To determine the values of other parameters, I choose them so that the moments simulated from the model matches the selected moments in the data.[10] There are 6

[10] To simulate the model, I use the pruning procedure as described in Kim et al. (2008) and Den Haan and De Wind (2012). I compute a total of 200 replications of 250 period simulations. I throw away the initial 50 periods. For each sample I compute the business cycle moments and then take medians across 200 replications. I checked that the results are not driven by explosive behavior.

parameters to calibrate: $\{\kappa, \rho_\mu, \rho_g, \sigma_\mu, \sigma_g, \sigma_\delta\}$. I target the following 6 data moments:

- Macroeconomic variables:

  Standard deviations of output and investment.

  Correlations of investment with respect to output.

  Autocorrelation of output.

- Forecast errors from the Survey of Professional Forecasters:

  1st-order autocorrelation and mean size of forecast errors on nominal GDP growth.

Table 1.1 summarizes the resulting parameter values.

The calibration of the standard deviation of the depreciation shock σ_δ needs further discussion. The parameter is important because it determines the strength of information frictions. With too small σ_δ, the learning problem becomes trivial. With too large σ_δ, agents learn little about the aggregate state. Thus in both cases, changes in the level of investment have a negligible effect on the level of uncertainty. I discipline the choice of σ_δ by using statistics on forecast errors in the Survey of Professional Forecasters data.[11] The first row in Table 1.2 reports statistical properties of the one-quarter-ahead median forecast errors on nominal GDP growth rate.[12] The second column shows that the forecast errors of GDP growth are positively autocorrelated. The third column shows the mean size of forecast errors (i.e., forecast

[11] A similar calibration strategy has been used in, for example, Eusepi and Preston (2011) and Görtz and Tsoukalas (forthcoming).

[12] I choose the nominal GDP growth rate because this is the longest forecast series available from the survey. Also, the forecasts do not appear to be biased because the time-series average of the forecast errors is very close to zero.

precision). I also report the model predictions of the forecast errors for various values of σ_δ.[13] First note that for all values of σ_δ reported, the autocorrelations are positive. This is due to the relatively high persistence parameter of the investment growth shock, ρ_g. The forecast errors are autocorrelated because agents only gradually realize the change in growth rate in response to an innovation to g_t. As σ_δ increases, the autocorrelation decreases because of the additional noise in the filtering problem. On the other hand, the size of the error increases with σ_δ simply because the information friction becomes more severe. I choose $100\sigma_\delta = 0.015$, which matches both the autocorrelation and the size well.

As a preliminary diagnosis of the model's performance, I compare the business cycle moments from the data and the model in Table 1.3. The model matches the data reasonably well, even for moments that are not explicitly targeted.

1.5 Results

In this section, I present the results. First, by comparing impulse responses and business cycle moments, I show that the uncertainty multiplier is large. Second, I examine the sensitivity of the size of the multiplier to different parameter values for the shock processes. Third, I highlight the role of real and nominal rigidities by shutting each component one-by-one. Finally, I decompose the multiplier into the direct effect and the feedback effect.

[13] For the computation of the numbers reported in this Table, I only change the value of σ_δ and fix other parameters at the benchmark calibration reported in Table 1.1.

1.5.1 The Uncertainty Multiplier Is Large

I divide the presentation of the main results into two parts. First, I use impulse responses to explain the basic mechanism of the uncertainty multiplier. Second, I compute business cycle moments and measure the size of the multiplier.

Impulse Response Analysis

Before examining the impulse responses, I need to consider how to measure the effects of endogenous changes in uncertainty. One potential way is to compare the baseline model with a version of the model without any information friction (i.e., agents know the true value of the shocks). However, this approach is problematic since it confounds the effects of changes in the *variance* of the agents' estimates (which is the main focus of the paper) with the effects of changes in the *mean* of the estimates. Therefore, I consider a version of the model where the variance of the estimates is held constant but agents still face information frictions. This way, I can precisely quantify the contribution of fluctuations in uncertainty to business cycle dynamics.

I examine the impulse responses to a negative one-standard-deviation investment shock. Recall that from the perspective of the agent at the end of period $t-1$, the capital accumulation equation can be rewritten as follows:

$$K_t = (1-\delta_t)K_{t-1} + (\tilde{\mu}_{t|t-1} + u_t)I_{t-1},$$

where u_t is normally distributed with mean zero and variance $\sigma^2_{u,t}$. In the baseline model featuring the uncertainty multiplier, $\sigma_{u,t}^2$ is given by

$$\sigma_{u,t}^2 = \rho_\mu^2 \Sigma_{t-1}^{11} + 2\rho_\mu \Sigma_{t-1}^{12} + \Sigma_{t-1}^{22} + \sigma_\mu^2.$$

I shut down the uncertainty multiplier by fixing expectations over $\sigma_{u,t}^2$ at its steady-state level:

$$\sigma_{u,t}^2 = \rho_\mu^2 \Sigma_{ss}^{11} + 2\rho_\mu \Sigma_{ss}^{12} + \Sigma_{ss}^{22} + \sigma_\mu^2,$$

where Σ_{ss}^{11}, Σ_{ss}^{12}, and Σ_{ss}^{22} are the steady-state levels of Σ_t^{11}, Σ_t^{12}, and Σ_t^{22}. Figure 1.2 shows the actual and perceived levels of unobservable shocks (investment shock μ_t and growth shock g_t). Note that they are identical between the two simulations.[14] This means that differences in the dynamics of the endogenous variables are only due to a difference in agents' perception of ex-ante uncertainty.

Figure 1.3 shows that the output decline is substantially deeper when the uncertainty multiplier is present. This is because, as shown in Figure 1.4, in the baseline model agents perceive an increase in uncertainty (increase in $\sigma_{u,t}$) due to a decline in investment. This increase in uncertainty contributes to the additional drop in output compared to the case where the uncertainty multiplier is turned off ($\sigma_{u,t}$ is held constant). Figure 1.4 also shows that the declines in other real variables are amplified by a similar amount. However, for nominal variables like inflation and the interest rate, the amplification is negligible.

The uncertainty multiplier amplifies the contraction in economic activity for the

[14] Strictly speaking, agents' perceived level of unobservable shocks could be moderately different between the two simulations because the signal-to-noise ratio changes due to the feedback effect. It turns out, however, that this channel is negligible in the impulse response shown here.

following reasons. Due to the precautionary motive, an increase in uncertainty induces households to consume less and save more. However, on the saving side, the physical capital becomes a worse hedge for aggregate shocks because the return on capital is subject to more uncertainty. On net, this risk-aversion channel dominates and investment falls as well.

Why, then, the working hours fall? On the one hand, the fall in consumption induces a desire for households to supply more labor. On the other hand, since aggregate demand is lower, firms demand less labor for a given wage. Since wages are sticky, wages cannot adjust to accommodate more labor and thus equilibrium hours fall. Since prices are sticky, firms increase their price markups and this leads to a further decline in hours. The overall outcome is that output drops substantially.

It is important to stress that in my model, an increase in uncertainty generates a simultaneous fall in output, investment, consumption, and hours. In standard real business cycle models, an increase in uncertainty reduces consumption but also induces a "precautionary labor supply" (Basu and Bundick 2011). As a result, contrary to the data, consumption and hours move in opposite directions. With nominal rigidities, the business cycle comovement is restored through countercyclical movements in markups.

Business Cycle Moments

I measure the size of the uncertainty multiplier by computing the business cycle moments with and without the multiplier. Figure 1.5 plots the sample path of output from numerical simulations. The uncertainty multiplier amplifies both booms and recessions because uncertainty decreases during booms and increases during recessions. To quantify the magnitude of the amplification, Table 1.4 compares the standard deviations of output and other variables. The uncertainty multiplier is large. In particular, the standard deviation of output is 1.33 times larger with the multiplier.[15] Other real variables like investment and hours are amplified by a similar amount. Consistent with the findings from the impulse response analysis, for inflation and the interest rate the amplification is negligible. Finally, Table 1.5 shows that, for a reasonable range of parameterization of the standard deviation of the depreciation shock, σ_δ, the uncertainty multiplier is sizable. For example, consider $100\sigma_\delta = 0.050$. While this parameterization implies that the autocorrelation is too low and the forecast errors are too large, the uncertainty multiplier for output is 1.20.

I conclude this subsection by pointing out that the amplification results reported above are likely to be conservative lower bounds. Including features such as non-convex adjustment costs (Bloom 2009) or financial frictions (Gilchrist et al. 2010) in the model would further increase the size of the uncertainty multiplier.

1.5.2 Changing the Parameters of the Shock Processes

I consider the effects of changing the parameters of the shock processes from the benchmark calibration. The exercise provides additional insights regarding determinants of the size of the uncertainty multiplier.

[15]The baseline numbers are derived from the HP-filtered ($\lambda = 1600$) moments. The results are not sensitive to the choice of the detrending method. For example, when I use linearly detrended moments, the uncertainty multiplier for output is 1.40.

Table 1.6 reports the uncertainty multiplier for output under different parameterizations of the standard deviation of the investment shock σ_μ and the growth shock σ_g. I change the ratio of the standard deviations, σ_μ/σ_g, from the benchmark calibration ($\sigma_\mu/\sigma_g = 0.55$) while keeping the standard deviation of output constant. The multiplier is increasing in the relative size of the growth shock. Intuitively, agents respond more to changes in uncertainty about the expected trend growth than to those about the fluctuation around the trend. The uncertainty multiplier is also increasing in the absolute size of the shocks. This can be seen in Table 1.7, where I scale the standard deviations of shocks (σ_μ, σ_g, and σ_δ) proportionally from the benchmark calibration. The reason is that the fluctuation in uncertainty becomes more important to agents' decision making as the volatility of shocks becomes larger.

The results in this subsection have an interesting implication for emerging market economies. As shown in Aguiar and Gopinath (2007), these economies feature more volatile business cycles that could be well characterized by fluctuations in expected growth rates.[16] This suggests that the uncertainty multiplier may be much larger in emerging markets than in the U.S.

1.5.3 The Role of Real and Nominal Rigidities

The benchmark model features several real and nominal rigidities that are absent in a plain vanilla real business cycle model. Table 1.8 reports the uncertainty multiplier for output under various combinations of frictions.

I highlight three observations. First, nominal rigidities are crucial for generating

[16] See also Boz et al. (2011), who extend Aguiar and Gopinath (2007)'s analysis by incorporating a learning problem.

sizable multipliers. The output uncertainty multiplier is 1.06 without sticky prices and 1.01 without sticky wages. This point connects to Basu and Bundick (2011) and Fernández-Villaverde et al. (2012), who argue that countercyclical markups due to nominal rigidities are important in accounting for the quantitative effects of changes in uncertainty. Second, although less important than nominal rigidities, frictions on the real side of the economy also matter. The real rigidities magnify households' response to changes in uncertainty because they make future adjustments in consumption and investment more costly. Third, there are interactions among each set of rigidities. For example, while both real rigidities only and nominal rigidities only economies produce negligible output amplification (1.00 and 1.04, respectively), when the full set of rigidities is present, the amplification is significant (1.33).

The findings are related to Bloom (2009), who shows that the firm-level irreversibilities are essential in analyzing the effects of changes in uncertainty. The results in this subsection indicate that abstracting from a realistic amount of rigidities may result in an underestimation of the size of the uncertainty multiplier.

1.5.4 The Role of the Feedback Effect

An interesting property of my model is that it features a two-way feedback mechanism between uncertainty and economic activity. Amplification of investment due to uncertainty leads to even more amplification, because uncertainty itself is also amplified by the amplification of investment. I assess the quantitative importance of this channel by computing the uncertainty multiplier when the feedback effect is turned off.[17]

In particular, I assume that the agents perceive $\sigma^2_{u,t}$ to evolve as follows:

$$\sigma^2_{u,t} = \rho^2_\mu \tilde{\Sigma}^{11}_{t-1} + 2\rho_\mu \tilde{\Sigma}^{12}_{t-1} + \tilde{\Sigma}^{22}_{t-1} + \sigma^2_\mu,$$

where $\tilde{\Sigma}_{t-1}$ is derived from the decision rules of investment when the uncertainty multiplier is turned off (i.e., $\sigma^2_{u,t} = \sigma^2_{u,ss}$). Table 1.9 reports the uncertainty multiplier with and without the feedback effect. Without feedback, the uncertainty multiplier is slightly smaller than the baseline. The feedback effect can account for about $((33 - 30)/33 \approx)$ 10% of the total output amplification.

1.6 Survey Data Evidence

In this section, I use a unique survey data that directly measures subjective uncertainty and argue that the model is consistent with the data. In particular, I show that the model quantitatively replicates the VAR relationship between output and uncertainty.[18]

Since uncertainty is an ex-ante concept, its measurement using ex-post realized data is inherently difficult. Probabilistic forecasts reported in the Survey of Professional Forecasters are unique in yielding numeric values on ex-ante uncertainty for a sufficiently long period of time. This survey asks each forecaster for a subjective probability density of the annual percentage change in real GDP. Following the stan-

[17] I thank Toshi Mukoyama for suggesting this exercise.

[18] Bloom (2009) and Gourio (2012) conduct similar exercises.

dard in the literature (Zarnowitz and Lambros 1987 and D'Amico and Orphanides 2008), I take the average across the standard deviations of those probability densities for each forecaster and use it as a measure of uncertainty.[19] While the survey data starts from 1968:Q4, concerns regarding data consistency and missing data force me to conduct the analysis using the data during 1986:Q2–2011:Q4.[20] Finally, since the survey asks for the percentage change in GDP between the previous and current calender year, there is a seasonality in the forecast horizons. For example, in the first quarter, it is a 4-quarter-ahead-forecast. In the second quarter, it is a 3-quarter-ahead-forecast. I eliminate this seasonality by applying the Tramo-Seats filter.[21]

I characterize the relationship between real GDP and uncertainty with a generalized impulse response analysis (Pesaran and Lambros 1998) from a bivariate VAR with four lags. The generalized impulse response is appealing in this context because, in contrast to a standard recursive VAR, the results are invariant to the ordering of variables.[22] Both variables are logged and HP-filtered with $\lambda = 1600$. I emphasize that the purpose of this exercise is to look for a statistical relationship between output and uncertainty. Hence, no causal inference is drawn from the impulse responses.

Figure 1.6 shows that, in the data, an increase in uncertainty is associated with

[19] The survey asks each forecaster to place probabilities in bins spanning a wide range of outcomes for the percentage change in real GDP. To compute the individual standard deviations, I fit a normal distribution to the individual probabilities. For more details, see D'Amico and Orphanides (2008). I have also tried other methods and obtained similar results.

[20] Nevertheless I conducted the analysis using the whole sample period and found similar results.

[21] Since the survey response between the current and the following year is also available, it is possible to construct uncertainty data with different forecast horizons. I have conducted the analysis with different forecast horizons and found similar results.

[22] Nevertheless I also tried a recursive VAR and obtained similar results.

a decline in output that reaches a trough after five quarters. On the other hand, an increase in output is associated with a decline in uncertainty. Hence the VAR responses indicate a clear negative relationship between output and uncertainty. The figure also shows that running a VAR on the artificial data from the model generates impulse responses that are quantitatively in line with the actual data.[23] In the model, the negative relationship between output and uncertainty is due to the endogenous movement in uncertainty and its feedback to real economic activity. Note that the model replicates well the gradual responses of the two variables. This is because uncertainty is driven by investment, which exhibits hump-shaped dynamics, and this uncertainty in turn induces gradual adjustments by households.

1.7 Conclusion

Much learning about macroeconomic conditions seems to occur through actually undertaking economic activity. This paper formalized the idea in an equilibrium business cycle framework and explored its quantitative implications. Recessions are times of high uncertainty because agents invest less and hence learn less about the state of the economy. The endogenous fluctuations in aggregate uncertainty interact with rigidities and amplify business cycles.

Because the level of learning is tied to the level of investment, changes in uncertainty are large and persistent. As a result, the uncertainty multiplier is sizable. Under the benchmark calibration, it amplifies the standard deviation of output by

[23] In the model, I define uncertainty as the standard deviation of the density forecast (conditional on the agents' information sets) of the annual percentage change in output: $Std_{t+s|t}(\Delta Y_{t+s})$. The forecast horizon is chosen in a way consistent with the survey data.

33%. Other real variables, such as investment and hours, are also amplified by a similar amount. Thus, the uncertainty multiplier could be a key force that transforms relatively small shocks into larger business cycles.

My framework opens the door to a set of exciting questions. First, in this paper changes in uncertainty propagate through households' precautionary motive and countercyclical markups. It would be useful to explore alternative channels through which endogenous uncertainty amplifies business cycles. One such example would be financial frictions. Second, we need to know how we should conduct monetary policy under fluctuating uncertainty. Could we set policy in a way that reduces the economy's response to changes in uncertainty? Does it matter whether uncertainty is exogenous or endogenous? I plan to address these issues in future research.

1.8 Tables and Figures

Table 1.1: Parameters and targets

	Description	Value	Comments/Targets
Technology and preference			
β	Discount factor	0.9948	Historical mean of interest rate
θ_p	Goods demand elasticity	21	5% price markup (Altig et al. 2011)
θ_w	Labor demand elasticity	21	5% wage markup (Altig et al. 2011)
α	Capital share	0.3	Standard choice
δ	Depreciation rate	0.02	8% annual depreciation
σ	Risk aversion	2	Standard choice
η	Inverse Frisch elasticity	2	Frisch elasticity = 0.5 (Chetty et al. 2011)
b	Habit persistence	0.65	Christiano et al. (2005)
κ	Investment adj. cost	0.3	*Calibrated*
ξ_p	Calvo price	0.75	Duration of price 4 quarters
ξ_w	Calvo wage	0.75	Duration of wage 4 quarters
Monetary policy			
π	SS inflation rate	1.0095	Historical mean of inflation rate
ρ_R	Taylor rule smoothing	0.9	Standard choice
ϕ_π	Taylor rule inflation	2	Standard choice
ϕ_Y	Taylor rule output growth	0.1	Standard choice
Shock process			
ρ_μ	Investment level	0.9	*Calibrated*
ρ_g	Investment growth	0.86	*Calibrated*
$100\sigma_\mu$	Investment level	0.43	*Calibrated*
$100\sigma_g$	Investment growth	0.775	*Calibrated*
$100\sigma_\delta$	Depreciation	0.015	*Calibrated*

Table 1.2: Identification of σ_δ from survey data moments

	$Corr(FE_t^{1Q}, FE_{t-1}^{1Q})$	$Mean(\|FE_t^{1Q}\|)$
Data	0.17	0.55
Model		
$100\sigma_\delta = 0.002$	0.36	0.37
$100\sigma_\delta = 0.005$	0.31	0.40
$100\sigma_\delta = 0.010$	0.24	0.51
$100\sigma_\delta = 0.015$	0.19	0.63
$100\sigma_\delta = 0.025$	0.16	0.81
$100\sigma_\delta = 0.050$	0.12	1.15
$100\sigma_\delta = 0.075$	0.10	1.37

Notes: The forecast errors are multiplied by 100 to express them in percentage terms. The data statistics are calculated using the final data vintage. As a robustness check, I calculated the statistics using alternative data vintages and found that they are similar. For example, $(Corr(FE_t^{1Q}, FE_{t-1}^{1Q}), Mean(|FE_t^{1Q}|))$ for the first, the third, and the fifth vintages are (0.23, 0.47), (0.18, 0.52), and (0.20, 0.54), respectively.

Table 1.3: Business cycle moments

	Std.	$Corr(Y_t, X_t)$	AR(1)
Data			
Output	1.61	1.00	0.87
Investment	6.31	0.94	0.87
Consumption	0.93	0.84	0.87
Hours	1.99	0.88	0.92
Real wage	0.84	0.07	0.76
Inflation	0.29	0.18	0.48
Interest rate	0.41	0.34	0.75
Model			
Output	1.60	1.00	0.88
Investment	6.22	0.92	0.90
Consumption	0.71	0.61	0.85
Hours	2.42	0.99	0.85
Real wage	0.82	-0.15	0.89
Inflation	0.51	0.37	0.68
Interest rate	0.30	0.00	0.93

Notes: Both data and model moments are in logs, HP-filtered ($\lambda = 1600$), and multiplied by 100 to express them in percentage terms.

Table 1.4: The uncertainty multiplier is large

	Amplification $\sigma_{\text{With multiplier}}/\sigma_{\text{Without multiplier}}$
Output	1.33
Investment	1.30
Consumption	1.19
Hours	1.34
Real wage	1.14
Inflation	1.04
Interest rate	1.03

Notes: Both data and model moments are in logs and HP-filtered ($\lambda = 1600$).

Table 1.5: The uncertainty multiplier for different values of σ_δ

	$Corr(FE_t^{1Q}, FE_{t-1}^{1Q})$	$Mean(\|FE_t^{1Q}\|)$	Output amplification
Data	0.17	0.55	
Model			
$100\sigma_\delta = 0.002$	0.36	0.37	1.03
$100\sigma_\delta = 0.005$	0.31	0.40	1.14
$100\sigma_\delta = 0.010$	0.24	0.51	1.23
$100\sigma_\delta = 0.015$	0.19	0.63	1.33
$100\sigma_\delta = 0.025$	0.16	0.81	1.38
$100\sigma_\delta = 0.050$	0.12	1.15	1.20
$100\sigma_\delta = 0.075$	0.10	1.37	1.16

Notes: Both data and model moments are in logs and HP-filtered ($\lambda = 1600$). The forecast errors are multiplied by 100 to express them in percentage terms.

Table 1.6: The uncertainty multiplier is increasing in the relative size of the growth shock

	$Corr(FE_t^{1Q}, FE_{t-1}^{1Q})$	$Mean(\|FE_t^{1Q}\|)$	Output amplification
Data	0.17	0.55	
Model			
$\sigma_\mu/\sigma_g = 1.00$	0.11	0.76	1.23
$\sigma_\mu/\sigma_g = 0.80$	0.16	0.69	1.31
$\sigma_\mu/\sigma_g = 0.55$	0.19	0.63	1.33
$\sigma_\mu/\sigma_g = 0.30$	0.24	0.57	1.34
$\sigma_\mu/\sigma_g = 0.00$	0.24	0.55	1.36

Notes: Both data and model moments are in logs and HP-filtered ($\lambda = 1600$).

Table 1.7: The uncertainty multiplier is increasing in the size of shocks

	Output standard dev.	Output amplification
Data	1.61	
Model		
$(\sigma_\mu, \sigma_g, \sigma_\delta) \times 0.85$	1.09	1.21
$(\sigma_\mu, \sigma_g, \sigma_\delta) \times 0.95$	1.37	1.28
$(\sigma_\mu, \sigma_g, \sigma_\delta) \times 1.00$	1.60	1.33
$(\sigma_\mu, \sigma_g, \sigma_\delta) \times 1.05$	1.82	1.38
$(\sigma_\mu, \sigma_g, \sigma_\delta) \times 1.15$	2.65	1.45

Notes: Both data and model moments are in logs and HP-filtered ($\lambda = 1600$).

Table 1.8: The role of real and nominal rigidities

Consump. habit	Investment adj. cost	Sticky price	Sticky wage	Output amplification
✓	✓	✓	✓	1.33
✓	✓		✓	1.06
✓	✓	✓		1.01
✓	✓			1.00
	✓	✓	✓	1.23
✓		✓	✓	1.07
		✓	✓	1.04
				1.00

Notes: Both data and model moments are in logs and HP-filtered ($\lambda = 1600$). For each specification, I scale $(\sigma_\mu, \sigma_g, \sigma_\delta)$ proportionally to generate the standard deviation of output as in the benchmark specification ($\sigma_Y = 1.60$).

Table 1.9: The uncertainty multiplier with and without the feedback effect

	Amplification $\sigma_{\text{With multiplier}} / \sigma_{\text{Without multiplier}}$	Amplification without feedback $\sigma_{\text{No feedback}} / \sigma_{\text{Without multiplier}}$
Output	1.33	1.30
Investment	1.30	1.27
Consumption	1.19	1.17
Hours	1.34	1.30
Real wage	1.14	1.12
Inflation	1.04	1.03
Interest rate	1.03	1.03

Notes: Both data and model moments are in logs and HP-filtered ($\lambda = 1600$).

Figure 1.1: Timing of events

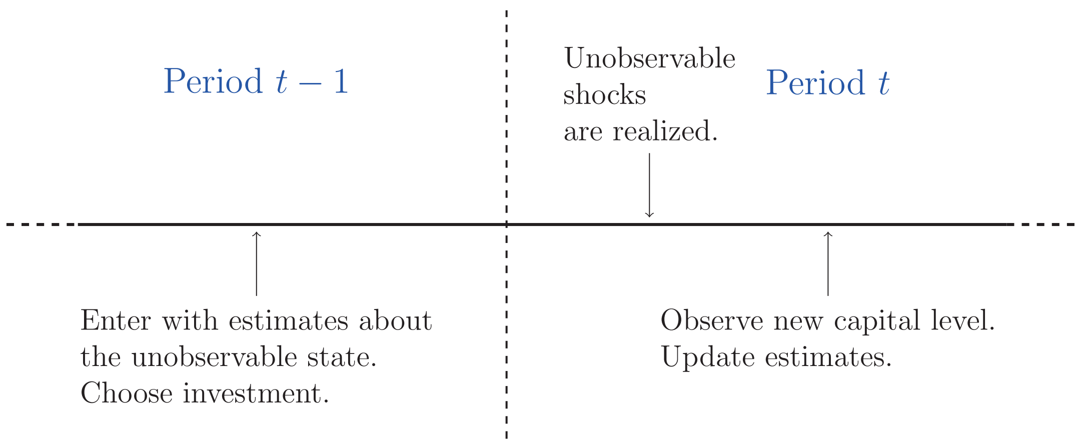


Figure 1.2: The actual or perceived levels of unobservable shocks are identical between the two simulations

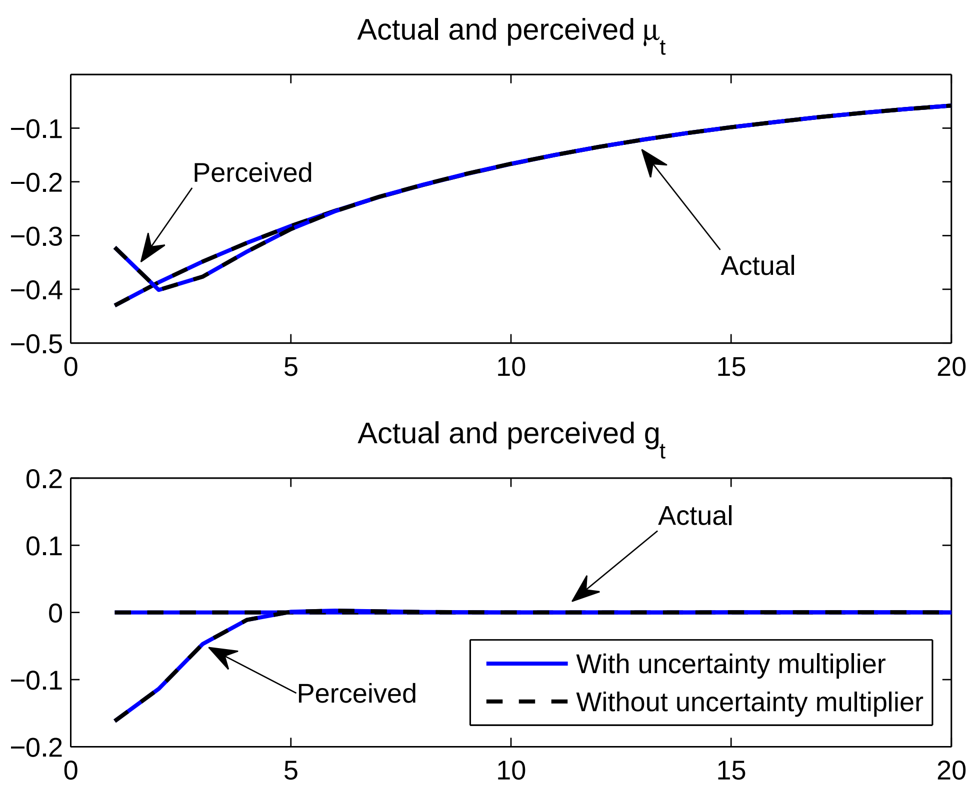
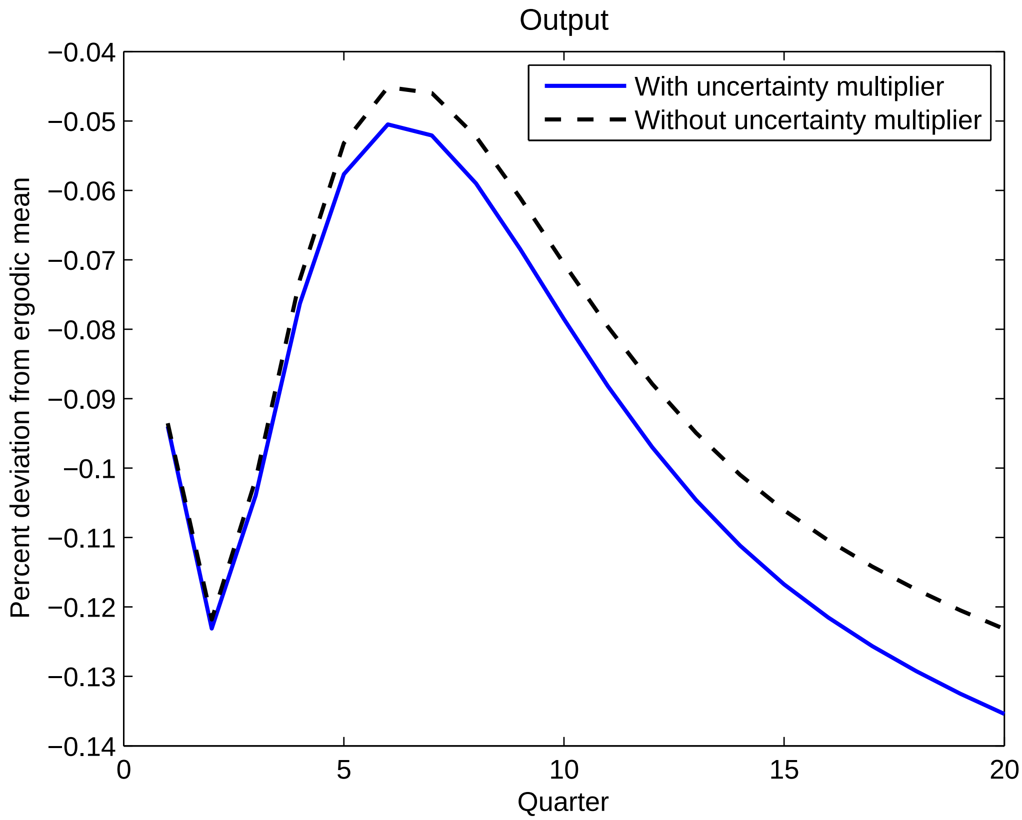
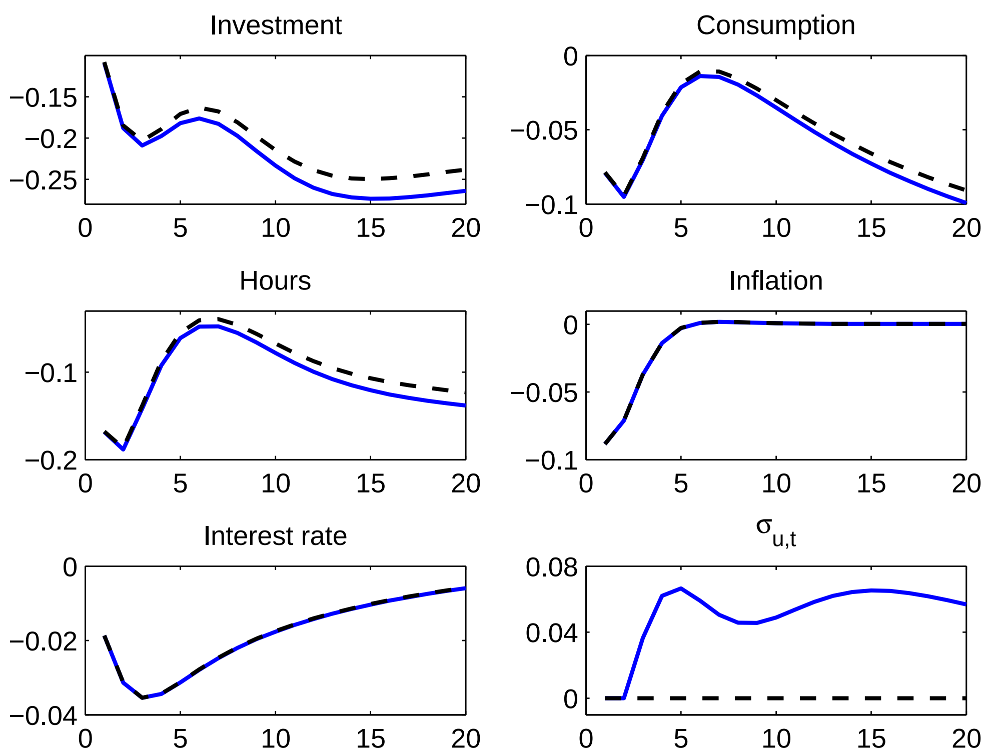


Figure 1.3: The uncertainty multiplier amplifies output response



Notes: Since third-order approximations move the ergodic distribution of endogenous variables away from the steady state (Fernández-Villaverde et al. 2011), I report the impulse responses in terms of percent deviation from the ergodic mean.

Figure 1.4: Responses of other real variables are also amplified



Notes: See the notes for Figure 1.3.

Figure 1.5: The uncertainty multiplier amplifies business cycles

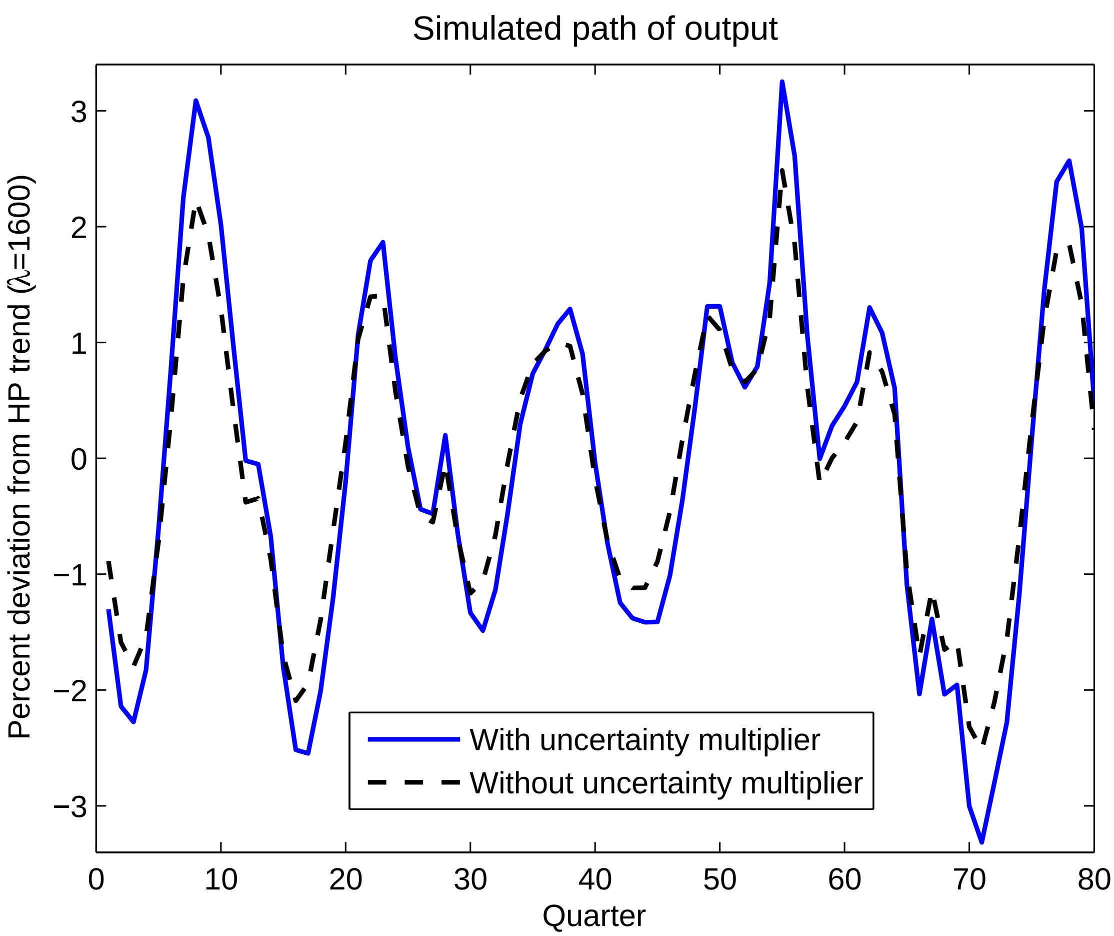
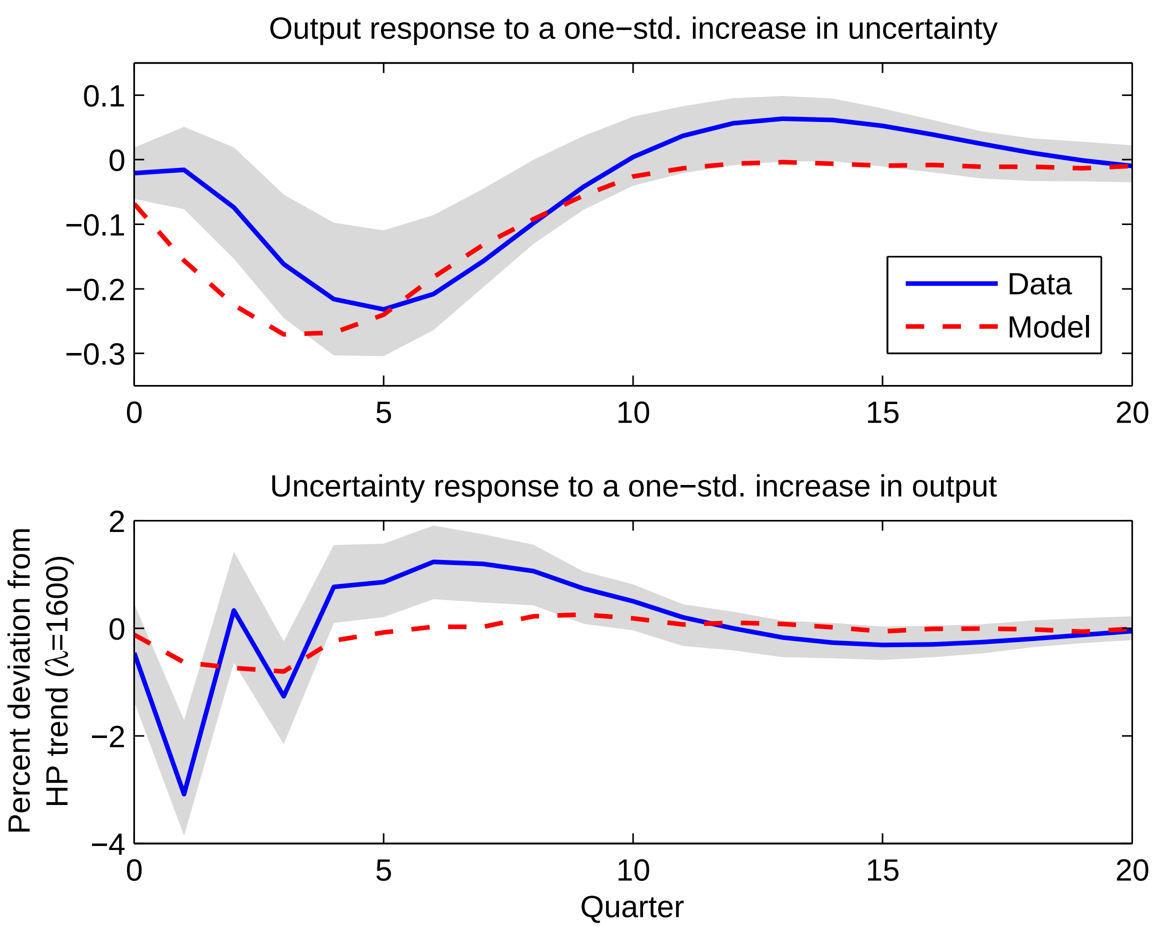


Figure 1.6: Impulse responses in a bivariate VAR



Notes: The shaded area represents ± one-standard-deviation bootstrap confidence bands.

1.9 Appendix

1.9.1 Data Source

The data set spans the period 1969Q1 to 2011Q4.[24] Whenever the data set is provided in monthly frequencies, I simply take the average to transform it into quarterly frequencies.

Data from the National Income and Product Accounts are downloaded from the Bureau of Economic Analysis website. Nominal GDP, nominal consumption (defined as the sum of personal consumption expenditures on nondurables and services), and nominal investment (defined as the sum of gross private domestic investment and personal consumption expenditures on durables) are divided by the civilian noninstitutional population,[25] downloaded from the Bureau of Labor Statistics (BLS hereafter) website, to convert the variables into per capita terms. I then divide them by the GDP deflator to convert them into real terms.

Working hours are measured by nonfarm business hours (available on the BLS website) divided by the population. Real wages are measured by hourly compensation in nonfarm business sectors (available on the BLS website) divided by the GDP deflator. Inflation rates are measured by changes in the GDP deflator. I use the effective federal funds rates (downloaded from the Federal Reserve Board website) to measure the nominal interest rates.

To compute the forecast error statistics, I use the median forecast of nominal

[24] I pick this starting date because the Survey of Professional Forecasters began around that time.

[25] Since raw population data display occasional breaks due to changes in population controls, I use an HP-filtered ($\lambda = 1600$) trend instead.

GDP growth rate, downloaded from the FRB Philadelphia website. The one-period-ahead forecast error is defined as the one-period-ahead nominal GDP growth rate forecast minus the realized nominal GDP growth rate.

1.9.2 Countercyclical Uncertainty: Full Derivation

I restate agents' Kalman-filtering problem below:

$$\begin{bmatrix} \mu_t \\ g_t \end{bmatrix} = \begin{bmatrix} (1-\rho_\mu)\mu \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_\mu & 1 \\ 0 & \rho_g \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ g_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\mu,t} \\ \epsilon_{g,t} \end{bmatrix},$$

$$K_t - (1-\delta)K_{t-1} = [I_{t-1} \quad 0] \begin{bmatrix} \mu_t \\ g_t \end{bmatrix} + K_{t-1}\epsilon_{\delta,t}.$$

At the end of period $t-1$, agents forecast the values of $\{\mu_t, g_t\}$:

$$\tilde{\mu}_{t|t-1} = (1-\rho_\mu)\mu + \rho_\mu \tilde{\mu}_{t-1} + \tilde{g}_{t-1},$$

$$\tilde{g}_{t|t-1} = \rho_g g_{t-1}.$$

The elements of the associated forecasting error covariance matrix, $\Sigma_{t|t-1}$, are

$$\Sigma^{11}_{t|t-1} = \rho_\mu^2 \Sigma^{11}_{t-1} + 2\rho_\mu \Sigma^{12}_{t-1} + \Sigma^{22}_{t-1} + \sigma_\mu^2,$$

$$\Sigma^{12}_{t|t-1} = \rho_\mu \rho_g \Sigma^{12}_{t-1} + \rho_g \Sigma^{22}_{t-1},$$

$$\Sigma^{21}_{t|t-1} = \Sigma^{12}_{t|t-1},$$

$$\Sigma^{22}_{t|t-1} = \rho_g^2 \Sigma^{22}_{t-1} + \sigma_g^2.$$

After observing period t realization of capital, K_t, agents update their belief according to

$$\tilde{\mu}_t = \tilde{\mu}_{t|t-1} + \frac{I_{t-1}\Sigma^{11}_{t|t-1}}{I^2_{t-1}\Sigma^{11}_{t|t-1} + K^2_{t-1}\sigma^2_\delta} \cdot \{K_t - (1-\delta)K_{t-1} - I_{t-1}\tilde{\mu}_{t-1}\},$$
$$\tilde{g}_t = \tilde{g}_{t|t-1} + \frac{I_{t-1}\Sigma^{12}_{t|t-1}}{I^2_{t-1}\Sigma^{11}_{t|t-1} + K^2_{t-1}\sigma^2_\delta} \cdot \{K_t - (1-\delta)K_{t-1} - I_{t-1}\tilde{\mu}_{t-1}\}.$$

The elements of the forecasting error covariance matrix are given by

$$\Sigma^{11}_t = \left[1 - \frac{I^2_{t-1}\Sigma^{11}_{t|t-1}}{I^2_{t-1}\Sigma^{11}_{t|t-1} + K^2_{t-1}\sigma^2_\delta}\right]\Sigma^{11}_{t|t-1},$$
$$\Sigma^{12}_t = \left[1 - \frac{I^2_{t-1}\Sigma^{11}_{t|t-1}}{I^2_{t-1}\Sigma^{11}_{t|t-1} + K^2_{t-1}\sigma^2_\delta}\right]\Sigma^{12}_{t|t-1},$$
$$\Sigma^{21}_t = \Sigma^{12}_t,$$
$$\Sigma^{22}_t = \Sigma^{22}_{t|t-1} - \frac{I^2_{t-1}(\Sigma^{12}_{t|t-1})^2}{I^2_{t-1}\Sigma^{11}_{t|t-1} + K^2_{t-1}\sigma^2_\delta}.$$

Thus, the elements of Σ_t are decreasing in $\frac{I_{t-1}}{K_{t-1}}$.

Chapter 2

Estimating DSGE Models Using Seasonally Adjusted and Unadjusted Data

2.1 Introduction

Most aggregate time series display large seasonal fluctuations. As Barsky and Miron (1989) show, seasonal fluctuations account for a substantial fraction of total variations in quantity variables, such as GDP, investment, and hours worked. Nevertheless, the common practice among economists when estimating dynamic stochastic general equilibrium (DSGE) models is to simply ignore seasonality and use seasonally adjusted data. The practice implicitly assumes that seasonal adjustments can decompose data into seasonal and nonseasonal components, and values of interesting parameters can be recovered correctly. However, modern dynamic economic theory dictates that seasonality interacts with other endogenous variables in a complex and possibly nonlinear manner.[1] Hence seasonal adjustments based on arbitrary identifying restrictions would necessarily lead to distorted inference. An important question for macroeconomists is whether those distortions are quantitatively relevant.

In this paper, I develop a general equilibrium business cycle model that can account for broad features of U.S. seasonal and nonseasonal fluctuations. Building on recent contributions (e.g., Christiano et al., 2005; Smets and Wouters, 2007; Justiniano et al., 2010), the model incorporates a host of real and nominal frictions and various types of shocks. The model is also subject to seasonal variations in technology and preference. Endogenous responses by agents to those seasonal variations allow the model to reproduce the seasonality observed in the U.S. aggregate data. I then simulate artificial data from the parameterized model in order to analyze the effects of estimating DSGE models using seasonally adjusted data.

A hypothetical econometrician uses the seasonally adjusted data to estimate an aseasonal counterpart of the baseline model using Bayesian methods. I find that the estimated parameters differ substantially from their true values. In contrast, when estimated with seasonally unadjusted data, most parameters are very precisely estimated. The result is crucial, because it suggests that the conventional practice of estimating DSGE models may lead to severely biased inference and that policy experiments based on the estimated parameters could be misleading.

Given the significance of the finding, I devote considerable effort to studying the

[1]For an ellegant exposition of this issue, see Ghysels (1988).

reasons for this distortion. Importantly, the distortions cannot be mitigated by constructing alternative seasonal adjustment filters, as they still arise in large sample environments with "ideal" filters. Using frequency domain tools, I show that the effects of seasonality are not confined to the so-called seasonal frequencies but instead are propagated across other nonseasonal frequencies. In particular, the effects are noticeable at higher frequencies and act in ways that raise spectral power in those regions. The intuition is relatively straightforward: Since seasonality induces agents to reallocate their resources across seasons within a year, the effects of seasonality are noticeable at higher frequencies. Moreover, because of seasonality, agents have different responses across seasons to the same shocks, and this additional source of volatility raises spectral power. I show that two key frictions in the model—the investment adjustment cost and the nominal wage rigidity—magnify the nonlinear interactions of seasonality and endogenous variables and make the propagation of the seasonal components quantitatively relevant. As a result, standard seasonal adjustment procedures that try to dampen spectral power only near seasonal frequencies are not effective, and the estimated parameters have to adjust in order to compensate for the discrepancy of spectra between seasonal and aseasonal versions of the model. I also provide some evidence suggesting that frictions that generate large distortions are not limited to those I assumed in the baseline model but include other general classes of capital accumulation and labor market frictions as well.

The present paper builds on several important contributions from the previous literature. Sims (1993) and Hansen and Sargent (1993) forcefully defend the common practice of estimating DSGE models using seasonally adjusted data. Their argument is based on two observations. First, directly modelling seasonality may lead to large distortions if the mechanism generating seasonality is misspecified. Second, in most examples they consider, using seasonally adjusted data leads to fairly accurate estimates. My contribution with respect to their papers is to show that, in a state-of-the-art DSGE model that is parameterized to match certain features of U.S. business cycle fluctuations, the second observation does not hold. I also deal with concerns about model misspecifications in more detail later in the paper. This paper is also related to Christiano and Todd (2002). There are two main departures from their study. First, they focus on the effects of seasonal adjustment on business cycle statistics. I consider the effects on a likelihood-based inference. Since a likelihood function contains all information from cross-equation restrictions imposed by dynamic economic theory, implications of seasonality may be quite different from those based on arbitrary sets of moments. Moreover, since it has now become a widely accepted approach to estimate DSGE model parameters using formal econometric methods, I believe this is a relevant application for many researchers. Second, they use a standard real business cycle model to answer their question at hand. My model introduces additional frictions and propagation structures (e.g., habit persistence, capital utilization, nominal rigidities, etc.) into their model. As I will show, some of the new added features in my model are the key driving force of my results.

The rest of the paper is organized as follows. The next section constructs a DSGE model with seasonality. Section 3 sets up the main experiment. Section 4 reports the results and shows that seasonal adjustments lead to sizeable distortions in parameter estimates. Section 5 identifies reasons for the distortions. Section 6 proposes a practical procedure that helps researchers decide whether or not to include seasonality in their models when potential model misspecifications are of concern. Finally, Section 7 concludes.

2.2 The Seasonal DSGE Model

The baseline seasonal model builds on a medium-scale DSGE model with a number of real and nominal frictions, along the lines of Christiano et al. (2005), Smets and Wouters (2007), and Justiniano et al. (2010). Following the previous literature on the subject (e.g., Chatterjee and Ravikumar, 1992; Braun and Evans, 1995; Liu, 2000), seasonality originates from deterministic shifts in technology and preference. Variations in technology could represent, for example, seasonal fluctuations in weather. Variations in preferences could represent expenditures due to several kinds of social events, such as Christmas. Presumably modelling seasonality in such a way that it originates from deeper structures of the economy would strenghten the case for using seasonally unadjusted data. However, the question I would like to ask in this paper is whether even a seemingly innocuous, simple mechanism for seasonality would generate large distortions through the *endogenous* responses to seasonality by optimizing agents.

The economy is composed of the final-goods sector, intermediate-goods sector, household sector, employment sector, and a government. I will begin by describing the production side of the economy.

2.2.1 The Final-Goods Sector

In each period t, the final goods, Y_t, are produced by a perfectly competitive representative firm that combines a continuum of intermediate goods, indexed by $j \in [0,1]$, with technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\theta_p - 1}{\theta_p}} dj\right]^{\frac{\theta_p}{\theta_p - 1}}.$$

Here, $Y_{j,t}$ denotes the time t input of intermediate good j and θ_p controls the price elasticity of demand for each intermediate good. The demand function for good j is

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_p} Y_t,$$

where P_t and $P_{j,t}$ denote the price of the final good and intermediate good j, respectively. Finally, P_t is related to $P_{j,t}$ via the relationship

$$P_t = \left[\int_0^1 P_{j,t}^{1-\theta_p} dj\right]^{\frac{1}{1-\theta_p}}.$$

2.2.2 The Intermediate-Goods Sector

The intermediate-goods sector is monopolistically competitive. In period t, each firm j buys $K_{j,t}$ units of capital service from the household sector and $H_{j,t}$ units of aggregate labor input from the employment sector to produce intermediate good j using technology

$$Y_{j,t} = z_t K_{j,t}^{\alpha} (X_t H_{j,t})^{1-\alpha},$$

where z_t is the neutral technology shock at time t. z_t follows the law of motion

$$\ln\left(\frac{z_t}{z_q}\right) = \rho_z \ln\left(\frac{z_{t-1}}{z_{q-1}}\right) + \epsilon_{z,t}, \quad \epsilon_{z,t} \sim N(0, \sigma_z^2),$$

where z_q is the steady-state level of z_t in season q. α is the capital share in the production function and X_t is a deterministic technological process that grows at rate γ.

In period t, the firm can reoptimize its intermediate-goods price with probability $(1-\xi_p)$. Firms that cannot reoptimize index their price according to the following: $P_{j,t} = \pi_{t-1}^{\chi_p}\pi^{1-\chi_p}P_{j,t-1}$, where π_{t-1} is the inflation rate in period $t-1$, π is the steady-state inflation rate (which is different from the steady-state level of the inflation rate in season q, π_q), and $\chi_p \in [0,1]$ is a parameter that controls the degree of indexation to past inflation.

2.2.3 The Household Sector

There is a continuum of households, indexed by $i \in [0,1]$. In each period, household i chooses consumption C_t, investment I_t, bond purchases B_t, and nominal wage $W_{i,t}$ to maximize utility given by the following:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \left[\tau_{t+s} \ln(C_{t+s} - bC_{t+s-1}) - \varphi \frac{H_{i,t+s}^{1+\eta}}{1+\eta}\right],$$

where β is a discount factor, b represents consumption habit, η controls (the inverse of) the Frisch labor supply elasticity, and $H_{i,t}$ is the number of hours worked by i. φ is a scale factor that determines hours worked in the steady state. I normalize

$\varphi = 1$. τ_t is the preference shock that follows the process:

$$\ln\left(\frac{\tau_t}{\tau_q}\right) = \rho_\tau \ln\left(\frac{\tau_{t-1}}{\tau_{q-1}}\right) + \epsilon_{\tau,t}, \quad \epsilon_{\tau,t} \sim N(0, \sigma_\tau^2),$$

where τ_q is the steady-state level of τ_t in season q.

The household's budget constraint is

$$P_t C_t + P_t I_t + B_t \leq W_{i,t} H_{i,t} + R_t^k u_t K_{t-1}^p + R_{t-1} B_{t-1} + D_t + A_{i,t} + T_t.$$

where R_t^k is the rental rate of capital, u_t is the utilization rate of capital, K_{t-1}^p is the stock of physical capital, R_{t-1} is the gross nominal interest rate from period $t-1$ to t, D_t is the combined profit of all the intermediate-goods firms distributed equally to each household, and T_t are lump-sum transfers from the government. I assume that households buy securities, whose payoffs are contingent on whether it can reoptimize its wage.[2] $A_{i,t}$ denotes the net cash inflow from participating in state-contingent security markets at time t.

Capital utilization transforms physical capital into capital services according to

$$K_t = u_t K_{t-1}^p.$$

[2]The existence of state-contingent securities ensures that households are homogeneous with respect to consumption and asset holdings, even though they are heterogeneous with respect to the wage rate and hours because of the idiosyncratic nature of the timing of wage reoptimization. See Christiano et al. (2005).

The physical capital stock evolves according to the following law of motion:

$$K_t^p = (1 - \delta(u_t))K_{t-1}^p + \mu_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t.$$

Following Greenwood et al. (1988), I assume that increasing the intensity of capital utilization speeds up the rate of depreciation $\delta(u_t)$. As in Schmitt-Grohe and Uribe (2012), I adopt a quadratic formulation for the function δ:

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2,$$

with $\delta_0, \delta_1, \delta_2 > 0$. The function S captures the notion of adjustment costs in investment, as proposed in Christiano et al. (2005). I adopt the following specification for S:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2,$$

with $\kappa > 0$. Finally, μ_t is the investment technology shock that follows the process:

$$\ln\left(\frac{\mu_t}{\mu_q}\right) = \rho_\mu \ln\left(\frac{\mu_{t-1}}{\mu_{q-1}}\right) + \epsilon_{\mu,t}, \quad \epsilon_{\mu,t} \sim N(0, \sigma_\mu^2),$$

where μ_q is the steady-state level of μ_t in season q.

2.2.4 The Employment Sector and Wage Setting

In each period t, a perfectly competitive representative employment agency hires labor from a continuum of households, indexed by $i \in [0, 1]$, to produce an aggregate labor service, H_t, using technology

$$H_t = \left[\int_0^1 H_{i,t}^{\frac{\theta_w - 1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w - 1}},$$

where $H_{i,t}$ denotes the time t input of labor service from household i and θ_w controls the price elasticity of demand for each household's labor service. The agency sells the aggregated labor input to the intermediate firms for a nominal price of W_t per unit. The demand function for the labor service of household i is

$$H_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\theta_w} H_t,$$

where $W_{i,t}$ denotes the nominal wage rate of the labor service of household i. W_t is related to $W_{i,t}$ via the relationship

$$W_t = \left[\int_0^1 W_{i,t}^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}}.$$

In each period t, a household faces a probability $(1-\xi_w)$ of being able to reoptimize its nominal wage. Households that cannot reoptimize index their wage according to the following: $W_{i,t} = \gamma \pi_{t-1}^{\chi_w} \pi^{1-\chi_w} W_{i,t-1}$, where $\chi_w \in [0, 1]$ is a parameter that controls the degree of indexation to past inflation.

2.2.5 The Government and Resource Constraint

The central bank follows a Taylor-type reaction function:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left\{ \left(\frac{\pi_t}{\pi_t^\star}\right)^{\phi_\pi} \left(\frac{Y_t/Y_{t-1}}{Y_q/Y_{q-1}}\right)^{\phi_Y} \right\}^{1-\rho_R} e^{\epsilon_{R,t}}, \quad \epsilon_{R,t} \sim N(0, \sigma_R^2).$$

where R is the steady-state level of the nominal interest rate, ρ_R is the persistence of the rule, and ϕ_π and ϕ_Y are the size of the policy response to the deviation of inflation and output growth from their targets, respectively. Y_t/Y_{t-1} is the growth rate of output in period t and Y_q/Y_{q-1} is the steady-state growth rate of output in season q. $\epsilon_{R,t}$ is an exogenous shock to the interest rate rule. $\pi_t^\star$ is the central bank's inflation target, which evolves according to

$$\ln\left(\frac{\pi_t^\star}{\pi_q}\right) = \rho_\pi \ln\left(\frac{\pi_{t-1}^\star}{\pi_{q-1}}\right) + \epsilon_{\pi,t}, \quad \epsilon_{\pi,t} \sim N(0, \sigma_\pi^2),$$

where π_q is the steady-state inflation rate in season q.

The aggregate resource constraint is $C_t + I_t + G_t = Y_t$. G_t is the amount of government spending, which is determined as a time-varying fraction of output

$$G_t = g_t Y_t,$$

and g_t follows the process

$$\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \epsilon_{g,t}, \quad \epsilon_{g,t} \sim N(0, \sigma_g^2),$$

where g is the steady-state ratio of government spending to output. Finally, the government balances the budget constraint every period given by

$$G_t = -T_t.$$

2.2.6 Solution Method

The choice of the solution method is very important. There are currently two major methods for solving DSGE models with seasonality. The first method is the one used in Chatterjee and Ravikumar (1992). We log-linearize the seasonal steady state around the balanced growth path and log-linearize the equilibrium conditions around the log-linearized seasonal steady state (CR method). The seasonal steady state is a periodic perfect foresight path that satisfies equilibrium conditions without uncertainty for each quarter. A more accurate alternative is the one used in Braun and Evans (1995). We directly solve for the seasonal steady state using a nonlinear solution method and log-linearize the equilbrium conditions around the exact seasonal steady state (BE method).

As is well known, a solution to a linear rational expectations system can be cast in a state-space representation. The state-space representation could form a basis of the Kalman filtering algorithm in building a likelihood for the estimation. The transition equation that characterizes the evolution of endogenous variables[3] is

$$\widehat{\mathbf{s}}_{t,q} = X_q(\theta)\widehat{\mathbf{s}}_{t-1,q-1} + Y_q(\theta)\epsilon_t,$$

where $\widehat{\mathbf{s}}_{t,q}$ is a vector that collects $\widehat{s}_{t,q} = \ln(s_{t,q}/s_q)$, which is the log-deviation of a variable $s_{t,q}$ in time t at quarter q from its seasonal steady state s_q. $X_q(\theta)$ and $Y_q(\theta)$ are the coefficient matrices that depend on a vector of the structural parameters θ, and ϵ_t is a vector of exogenous shocks. The CR method delivers a solution that restricts $X_q(\theta) = X(\theta)$ and $Y_q(\theta) = Y(\theta)$ for all quarters $q = 1, \ldots, 4$. The BE method delivers a solution that allows $X_q(\theta)$ and $Y_q(\theta)$ to take different values across different quarters.

Now consider a seasonal adjustment procedure that substracts the seasonal steady states from the data.[4] In this case we have $\widehat{\mathbf{s}}_t^{SA} = \widehat{\mathbf{s}}_{t,q}$, where $\widehat{\mathbf{s}}_t^{SA}$ is a vector that collects the log-deviations of the seasonally adjusted variables from their steady states. Suppose that an econometrician fits an aseasonal DSGE model to the seasonally adjusted data $\widehat{\mathbf{s}}_t^{SA}$. Observe that the CR method delivers consistent estimates. The BE method, on the other hand, may deliver important distortions, since the econometrician is fitting a model with constant $X(\theta)$ and $Y(\theta)$ to a data generating process where $X_q(\theta)$ and $Y_q(\theta)$ are periodically varying. Since my purpose is to quantify those distortions, I choose to work with the BE method.

[3]For simplicity, I assume that all endogenous variables are observable to an econometrician (i.e., the coefficient matrix of the observation equation is an identity matrix with no measurement error). All of the discussion below extends to the more general case where some variables are latent.

[4]Note that any reasonable seasonal adjustment filter can accomplish this task.

2.3 The Experiment

I ask whether using seasonally adjusted data leads to large distortions by estimating model parameters with simulated data from the seasonal DSGE model. First, I need to assign some values to the structural parameters and establish that the model is able to match certain features of U.S. seasonal and nonseasonal fluctuations.

2.3.1 Parameterization

There are three sets of parameters. The first set of parameters are those that characterize technology, preferences, and the central bank policy in the model and do not vary over quarters. The second set of parameters are those that vary across quarters. The first and the second sets of parameters jointly determine the seasonal steady state. The third set of parameters are those that characterize the stochastic shock processes.

The first set of parameters are reported in Panel A in Table 2.1. The parameters are picked around the values typically calibrated or estimated in the literature. The only parameter that deserves further attention is the parameter that controls κ (investment adjustment cost). The value ($\kappa = 1$) is slightly smaller than the values usually found in the literature. I assign this value because for larger adjustment costs, I had to assume implausibly large seasonal shifts in μ (investment technology) to match the seasonal pattern of investment observed in the U.S. data.

The second set of parameters are reported in Table 2.2. I allow the steady-state values of z (neutral technology), τ (preference), and μ (investment technology) to vary across quarters. Using a numerical minimization routine and a nonlinear equation solver, I calibrate the values so that the seasonal patterns of output, investment, and hours worked in the model match those in the data. Note that the average value of each parameter over quarters is ensured to be unity. Table 2.3 compares seasonal patterns in the data and in the model, given the assigned values of the first and second set of parameters. The model fit is very good. In particular, the model correctly predicts that seasonality is small in nominal variables such as the growth rate of real wages and the inflation rate.[5] There are two reasons for this success. First, prices and wages are assumed to be sufficiently sticky. This makes prices and wages less responsive to seasonal shifts in z, τ, and μ. In fact, if I lower the Calvo price and wage parameters (which implies less price and wage stickiness), I find that the seasonal steady states of the growth rate of wages and inflation become considerably more volatile over seasons. Second, seasonal shifts in τ (preference) effectively dampen the seasonal fluctuations in the real interest rate. To understand this, consider the intertemporal Euler equation as in Liu (2000):

$$1 = \beta \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{R_t}{\pi_{t+1}} \right) \right],$$

where λ_t denotes the marginal utility of consumption in period t. Suppose for a moment $b = 0$ and $\tau_t = 1$ for all quarters. Then $\lambda_{t+1}/\lambda_t = C_t/C_{t+1}$. Given the strong seasonality in consumption observed in the U.S. data, the real interest rate also has to exhibit strong seasonality in order to cancel out shifts in λ_{t+1}/λ_t. Seasonal fluctuations in τ perform a role of seasonal adjustment in λ_{t+1}/λ_t so that the interest

[5] I fix the steady state of the nominal interest rate to be constant across quarters.

rate becomes relatively stable across seasons.[6]

Finally, the third set of parameters are reported in Panel B of Table 2.1. The parameters are chosen so that second moments of the model resemble those of the data. In the Appendix, by comparing moments I show that overall the model is successful in replicating business cycle features of U.S. aggregate data. I conclude that the model serves as an empirically credible data generating mechanism for exploring the effects of estimating DSGE models using seasonally adjusted data.

2.3.2 Estimation Using Simulated Data

Given the parameterization described above, I simulate 200 observations of artificial data sets (after throwing away the initial 100 periods). I employ a Bayesian procedure. The likelihood is calculated based on the following vector of observables:

$$[\Delta \ln Y_t, \Delta \ln C_t, \Delta \ln H_t, \Delta \ln(W_t/P_t), \ln \pi_t, \ln R_t],$$

where Δ is the first-difference operator. I conduct two different estimation experiments. In the first experiment, I estimate the baseline seasonal DSGE model using seasonally unadjusted data. In the second experiment, I estimate the aseasonal ver-

[6] The first-order condition for each household's labor supply indicates that the marginal utility of consumption is also connected to movements in real wages and hours. While strong seasonality in hours observed in the data may suggest that wages also have to display strong seasonality in order to compensate for the weak seasonality in the marginal utility, this is not necessarily the case in our environment. In fact, the optimal wage for households adjusting their individual wages is relatively constant across seasons, since wages are sticky and hence households care about the influence of their current wage choice on their labor supply not only in the current quarter but also in future quarters. In other words, wage-setting policies that respond to seasonal movements of hours only in the current quarter are sub-optimal.

sion of the baseline DSGE model using seasonally adjusted data. Specifically, during estimation I impose $z_q = 1$, $\tau_q = 1$, and $\mu_q = 1$ for all quarters $q = 1, \ldots, 4$. All data except for interest rates are seasonally adjusted using the X-12-Arima filter.[7]

During estimation, I fix g, δ_0, and δ_2 to the true value, since they are difficult to identify. I also fix γ, π, and β, since most information for those parameters is contained in the levels of data and should not be affected much by the seasonal adjustment. Finally, I fix the steady-state price and wage markup, since I ran into some numerical difficulties when exploring the posterior distributions.[8] I assume flat priors for all the parameters, subject to some loose boundary constraints. As pointed out in, e.g., Fernández-Villaverde and Rubio-Ramírez (2008), with flat priors the posterior is proportional to the likelihood function. Thus the mode of the posterior can be interpreted as the parameter estimates of a maximum likelihood exercise.

2.4 Results

The estimates of the posterior distributions based on 200,000 draws from a random-walk Metropolis-Hastings algorithm are presented in Table 2.4. There are three things to observe. First, using seasonally unadjusted data delivers estimates that are quite close to the true values. This is not surprising, as I am estimating a correctly specified model using unfiltered data. Second, using X-12-Arima-filtered data

[7] X-12-Arima is a software package developed by the U.S. Census Bureau and is the official seasonal adjustment procedure of the U.S. government. The seasonal adjustment is conducted using software called "Demetra," which is provided by Eurostat.

[8] More specifically, the problem arises when I estimate the model using seasonally adjusted data. I also re-estimated the model by fixing the price and wage markup at several other values and found that the qualitative features of the results are unaffected.

delivers distorted estimates compared to using unadjusted data. The distortions are pronounced in some of the key structural parameters, such as α (capital share), η (inverse of the Frisch labor supply elasticity), κ (investment adjustment cost), ξ_w (Calvo wage parameter), and ϕ_π (Taylor rule coefficient on inflation). Third, the standard deviations of posterior distributions are smaller when seasonally unadjusted data are used. As discussed in Barsky and Miron (1989), seasonal fluctuations provide additional identifying restrictions that are not present in nonseasonal fluctuations, and hence I am able to obtain sharper estimates.

In Figure 2.1, I report the log-likelihood profiles for a selected set of parameters given seasonally unadjusted data (solid lines) and X-12-Arima-filtered data (dashed lines).[9] I move each structural parameter around its calibrated value in each panel while fixing other parameters at their calibrated values. To facilitate comparison, I show the true value for each parameter in a vertical line. Information drawn from Figure 2.1 is similar to that drawn from Table 2.4. The seasonally unadjusted likelihood peaks around the true parameter values, while the seasonally adjusted likelihood delivers considerable biases for many structural parameters. Directions of the biases are similar to those reported in Table 2.4.

Some readers may think that my results are sensitive to the way I seasonally adjust the simulated data. To ensure the robustness of the results against different choices of seasonal adjustment filters, I seasonally adjust the data using two alternative methods. First, I seasonally adjust using the Tramo-Seats filter.[10] Second,

[9] The log-likelihood profiles for other parameters are given in the Appendix.

[10] Tramo-Seats is a time series analysis package constructed from signal extraction principles and used extensively at the European Central Bank and Eurostat. The Bank of Spain's website (`http://www.bde.es/servicio/software/econome.htm`) provides a detailed explanation of the

I seasonally adjust by directly using the DSGE model.[11] To understand the second procedure, recall that the law of motion for endogenous variables is given by

$$\widehat{\mathbf{s}}_{t,q} = X_q(\theta)\widehat{\mathbf{s}}_{t-1,q-1} + Y_q(\theta)\epsilon_t,$$

where $\widehat{\mathbf{s}}_{t,q}$ is a vector that collects $\widehat{s}_{t,q} = \ln(s_{t,q}/s_q)$. This can be rewritten as $s_{t,q} = s_q \exp(\widehat{s}_{t,q})$. To seasonally adjust $s_{t,q}$, replace a seasonal steady state from the seasonal model, s_q, with a steady state from the aseasonal model, s: $s_{t,q} = s \exp(\widehat{s}_{t,q})$. Thus, the procedure can be thought of as regressing the data on seasonal dummies, but in a way consistent with the DSGE model. I simply call this the "DSGE-based" seasonal adjustment. As I show in the Appendix, for both methods, the posterior estimates are very similar compared to when X-12-Arima-filtered data are used.

I argue that these distortions in parameter estimates are important for economic inference because they (1) alter the transmission mechanism of shocks, (2) affect the business cycle statistics generated by the model, and (3) bias the results of policy analysis.

To illustrate the first point, I compare the impulse responses based on seasonally adjusted and unadjusted estimates. Comparing impulse responses is tricky here, since when seasonality is present the transmission of shocks differs considerably when the quarter in which the shock hits are different. Thus I consider the following comparison.

1. From the seasonal DSGE model, I draw parameters from the posterior distri-

procedure.

[11]I thank the Associate Editor for the suggestion.

bution of seasonally unadjusted estimates and compute impulse responses. To compute impulse responses, I first compute four versions of impulse responses, each differing with respect to the quarter in which the shock hits. I then take the average of the four responses. The resulting response could be thought of as a "seasonally adjusted" impulse response (i.e., impulse response without conditioning on a season) of the seasonal model.

2. From the aseasonal DSGE model, I draw parameters from the posterior distribution of X-12-Arima-filtered estimates and compute impulse responses.

I note that the "seasonally adjusted" responses generated from the seasonal model are almost identical to the responses generated from the aseasonal model when the same parameter values are used. Comparing the two versions of impulse responses, I can ask whether the impulse responses using the seasonally adjusted estimates can successfully predict the average response across quarters. I plot mean posterior impulse responses and their 90% point-wise intervals of a neutral technology shock and a monetary policy shock in Figures 2.2 and 2.3, respectively. Observe that the true responses generated from the seasonal DSGE model are very close to the mean responses of the seasonally unadjusted estimates. The responses are qualitatively similar between the seasonally adjusted and unadjusted estimates. For example, an exogenous improvement in technology robustly delivers hump-shaped increases in output and investment, persistent increases in consumption and real wages, and immediate declines in hours worked and inflation. An exogenous decrease in the interest rate leads to moderate but persistent increases in output, consumption, investment, hours, and inflation. Note, however, that there are also some important quantitative differences. For example, the seasonally adjusted estimates considerably understate output, consumption, and investment responses to an improvement in technology. Interestingly, the responses of hours worked are precisely matched. They also understate output, consumption, and investment responses to an expansionary monetary policy shock, but again the responses to hours worked are precisely matched. On the other hand, inflation responses to a monetary policy shock are overstated.

To substantiate the second point, I compare the second moments generated from two sources. To generate a first set of moments, I simulate data from the seasonal model with parameters fixed at the posterior means of seasonally unadjusted estimates and then seasonally adjust the data using the X-12-Arima filter. To generate a second set of moments, I simulate data from the aseasonal model with parameters fixed at the means of the X-12-Arima-filtered estimates. In Table 2.5, I compare those two sets of moments, together with the moments generated from the seasonal model under the true parameters.[12] Columns under the label "Percent standard deviation" in Table 2.5 show that the seasonally adjusted estimates considerably understate the standard deviation of output growth and overstate the standard deviation of hours growth, both by about 0.10. Moreover, they predict only about half the volatility of investment growth. Columns under the label "Corr. with output growth" in Table 2.5 show that correlations with output growth are in general understated. For example, using seasonally adjusted estimates, consumption growth correlation is less than half of what is predicted using the true parameters or seasonally unadjusted estimates. In evaluating those differences in moments, it is important to note that

[12] Fernández-Villaverde and Rubio-Ramírez (2005) document that moments generated from linear and nonlinear likelihood estimates are considerably different.

when the second set of moments are generated from the aseasonal model using seasonally unadjusted estimates (instead of seasonally adjusted estimates), the two sets of moments are almost identical and close to the true moments.

Finally I show that bias in point estimates translates into bias in policy analysis. I consider the following counterfactual policy experiment.[13] I compute the percent standard deviations of output growth and inflation when I increase the inflation response coefficient in the Taylor rule from the benchmark value ($\phi_\pi = 1.7$), both for the seasonal model using seasonally adjusted estimates and the aseasonal model using X-12-Arima-filtered estimates. Again, simulated data from the seasonal model are adjusted using the X-12-Arima filter. The results are shown in Table 2.6. The seasonally unadjusted estimates correctly predict the size of changes in output growth and inflation volatilities in response to the increase in ϕ_π. The seasonally adjusted estimates correctly predict the changes in inflation volatility. However, they understate the magnitude of the increase in output growth volatility. While both true parameters and seasonally unadjusted estimates predict that the standard deviation of output growth increases by about 30% compared to the benchmark case when $\phi_\pi = 10$, the seasonally adjusted estimates predict that it increases by only about 10%.

The results presented so far are important for applied macroeconomics research. They suggest that the conventional practice of estimating DSGE models using seasonally adjusted data may lead to biased inference, and hence policy experiments

[13] For other work on policy experiments in misspecified DSGE models, see Chang et al. (2011) and Cogley and Yagihashi (2010).

based on the estimated parameters could be misleading.[14]

2.5 Inspecting the Sources of Distortions

Why does estimating the DSGE model using seasonally adjusted data create sizeable distortions, as reported in the previous section? In the first subsection, I show that the main reason for the distortions is that the effects of seasonality are not restricted to the seasonal frequencies, but instead are propagated across the entire frequency domain. In the second subsection, I argue that the capital accumulation and labor market frictions in the model amplify nonlinear interactions between seasonality and endogenous variables and make the distortions quantitatively relevant.

2.5.1 Evidence From the Frequency Domain

Before turning to a detailed investigation, first it would be useful to take a look at what the standard seasonal adjustment methods do to the data. In Figure 2.4, I plot the sample periodogram of the simulated data (seasonally unadjusted, X-12-Arima-filtered, Tramo-Seats-filtered, and DSGE-based-filtered data) used in the previous section.[15] First, the spectra of seasonally unadjusted data have spikes at seasonal frequencies ($\omega = \pi$ and, in particular, $\frac{\pi}{2}$). Second, the seasonal adjustment

[14] I also conducted experiments replacing $\Delta \ln I_t$ with $\Delta \ln C_t$ as observables. In this case, using seasonally adjusted data still leads to substantially distorted estimates. However, using seasonally unadjusted data, the parameters controlling the government shock process (ρ_g and $100\sigma_g$) are imprecisely estimated due to a weak identification problem. For this reason, I focus on results that use $\Delta \ln C_t$ as observables for the rest of the paper.

[15] The periodogram is smoothed by taking the equally weighted average of periodograms on 7 frequencies at and in the neighborhood of each frequency $\omega_j = 2\pi j/T, j = 0, 1, \ldots, T-1$.

procedures eliminate those seasonal spikes but leave the spectral densities at other frequencies unaffected. These observations suggest that the distortions found in the previous section are due to the fact that the seasonal adjustment procedures fail to completely eliminate the effects of seasonality because seasonality also influences the spectral densities at other nonseasonal frequencies as well. For the rest of this section, I will formalize this argument by using a set of tools developed by previous authors.

In Figure 2.5, for the seasonal and aseasonal DSGE models, I plot the log spectrum of the variables used in the estimation.[16] For both models the parameters are fixed at the values (reported in Tables 2.1 and 2.2) used to generate data for the experiment in the previous sections. The log spectrum of the seasonal model is shown in thick solid lines, and the log spectrum of the aseasonal model is shown in thick dashed lines. Observe that for output growth and hours growth, there are considerable discrepancies between the spectra of the seasonal and aseasonal model at the entire frequency domain. In particular, the discrepancies are noticeable at high frequencies (frequencies above $\omega = \frac{\pi}{2}$). In those regions, the seasonal model has more spectral power. On the other hand, for nominal variables such as wage growth, inflation rates, and interest rates, the discrepancies are small and confined to the seasonal freqencies ($\omega = \frac{\pi}{2}, \pi$).[17]

The intuition behind these discrepancies is relatively straightforward. Since seasonality induces agents to reallocate their resources across seasons within a year, the

[16]As in Hansen and Sargent (1993), I use the formula of Tiao and Grupe (1980) to compute the spectral densities of the seasonal model. The formula provides an expression for the mean-adjusted periodic process, without conditioning on a season of the year.

[17]The coherence shows a similar pattern of discrepancies. I omit the figures to conserve space.

discrepancies of spectra are noticeable at higher frequencies. Moreover, because of seasonality, agents have different responses across seasons to the same shocks, and this additional source of volatility raises the spectral power of the seasonal model.

Sims (1993) and Hansen and Sargent (1993) recommend using seasonally adjusted data in estimating rational expectations models. Their recommendation is based on two arguments. First, directly modelling seasonality may lead to large distortions, if the mechanism generating seasonality is misspecified. Second, since the effects of seasonality are likely to be confined to seasonal frequencies, dampening those seasonal frequencies by seasonally adjusting the data and trying to fit aseasonal models to the nonseasonal frequencies leads to fairly accurate estimates. In my model, the second argument does not hold. The effects of seasonality propagate across the entire frequency domain, and hence trying to fit the nonseasonal frequencies using the aseasonal version of the model leads to substantial distortions in parameter estimates.[18]

I consider the implications of the discrepancies of spectral densities by using a frequency domain approximation for the probability limits of misspecified maximum likelihood estimators developed by Hansen and Sargent (1993). The frequency domain approximation is useful for two reasons. First, it allows me to isolate the effects of discrepancies of spectral densities from other factors that potentially bias estimates (e.g., seasonal adjustment filters or weak identification due to small samples). Second, it allows me to take a closer look at which particular frequencies are responsible for the bias.

[18] Ghysels (1988) presents a simple production market model demonstrating this phenomenon. Also see Cogley (2001), Canova (2009), and Canova and Ferrroni (2011) for a related point concerning the interactions of trend and cyclical components in DSGE models.

Hansen and Sargent (1993) show that the maximum likelihood estimator of a parameter vector θ converges almost surely to the minimizer of the following formula:

$$A(\theta) = A_1(\theta) + A_2(\theta) + A_3(\theta), \tag{2.1}$$

where

$$\begin{aligned}
A_1(\theta) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \ln\det G(\omega;\theta)d\omega, \\
A_2(\theta) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \text{trace}[G(\omega;\theta)^{-1}F(\omega)]d\omega, \\
A_3(\theta) &= [\mu - \mu(\theta)]'G(0;\theta)^{-1}[\mu - \mu(\theta)].
\end{aligned}$$

μ is the population mean of a stationary process and $F(\omega)$ is the spectral density function at frequency ω. $\mu(\theta)$ and $G(\omega;\theta)$ are the model-based mean and spectral density function. $A_1(\theta)$ captures the variance of the model-based one-step forecast errors. $A_2(\theta)$ and $A_3(\theta)$ measure the distance between data and model-based spectral densities and means, respectively. In implementing expression (2.1), it is useful to approximate the integrals in $A_1(\theta)$ and $A_2(\theta)$ by Riemann sums:

$$\begin{aligned}
\widehat{A}_1(\theta) &= \frac{1}{T}\sum_{j=0}^{T-1} \ln\det G(\omega_j;\theta), \\
\widehat{A}_2(\theta) &= \frac{1}{T}\sum_{j=0}^{T-1} \text{trace}[G(\omega_j;\theta)^{-1}F(\omega_j)],
\end{aligned}$$

where $\omega_j = 2\pi j/T, j = 0, 1, \ldots, T-1$.

The column labeled "Baseline" in Table 2.7 reports the probability limits of maximum likelihood estimates when seasonally adjusted data are used. To compute the probability limits, I apply the Riemann sum approximation of formula (2.1), with $T = 200$ and $G(\omega;\theta)$ generated from the aseasonal DSGE model and $F(\omega)$ generated from the seasonal DSGE model. Zero weight is assigned to frequencies at and near the seasonal frequencies[19] and deterministic seasonal means are removed by dropping $A_3(\theta)$ from formula (2.1). This allows me to mimic an "ideal" seasonal adjustment procedure in the frequency domain.

The biases in parameter estimates are similar to those reported in Table 2.4, although their magnitudes are slightly smaller. In Figure 2.5, the spectral density of the asymptotic maximum likelihood estimates is plotted in solid lines. In order to achieve better fit in output and hours growth, the likelihood estimator tries to shift spectral power from low to high frequencies by distorting the parameter estimates.[20]

Another useful measure to examine is a version of the likelihood-ratio statistic developed in Christiano and Vigfusson (2003):

$$\lambda = 2[A(\theta_{true}) - A(\theta^*)],$$

where θ_{true} is a vector of parameters fixed at their true values and θ^* is a vector of parameters fixed at their estimated values (in this case asymptotic maximum

[19] I impose zero weight to the 9 frequencies at and in the neighborhood of $\omega = \pi/2$, and also $\omega = \pi$ and the 4 next lower frequencies. The results are robust to the choice of the number of frequencies assigned zero weight.

[20] As pointed out in Cogley (2001), it is difficult to develop intuition of a direction of the bias in a particular parameter when all parameters are allowed to adjust simultaneously. This is because sometimes the partial effects of parameter adjustments interact in ways that counteract one another. For example, while the upward bias in α (capital share) and the downward bias in η (inverse of the Frisch labor supply elasticity) act in ways that raise the spectral power of output and hours growth, the upward bias in κ (investment adjustment cost) dampens it.

likelihood estimates). Define

$$\lambda(\omega) = \ln\det G(\omega;\theta_{true}) - \ln\det G(\omega;\theta^*) + \operatorname{trace}[(G(\omega;\theta_{true})^{-1} - G(\omega;\theta^*)^{-1})F(\omega)],$$

so that

$$\lambda = \lambda(0) + 2\sum_{j=1}^{T/2-1}\lambda(\omega_j) + \lambda(\pi).$$

The cumulative likelihood ratio is defined as

$$\Lambda(\omega) = \lambda(0) + 2\sum_{\omega_j\le\omega}\lambda(\omega_j), \qquad 0 < \omega < \pi,$$
$$\Lambda(0) = \lambda(0),$$
$$\Lambda(\pi) = \lambda.$$

If bias of the estimated parameters is due to discrepancies of seasonal and aseasonal spectra in some specific frequency region, we should see a sharp increase in $\Lambda(\omega)$. Figure 2.6 shows that there is a sharp increase at medium and high frequencies.[21] On the other hand, there is a mild decrease in the ratio at low frequencies. This observation confirms that the likelihood estimator is distorting the estimated parameters in order to acheive a better fit at higher frequencies.

2.5.2 The Role of Frictions

My model features a number of real and nominal frictions. The frictions magnify the nonlinear interactions between seasonality and endogenous variables, which in turn

[21]Note that since I omit the seasonal frequencies and their neighborhood during computation of the probability limits, the cumulative likelihood ratio is flat in that region.

leads to larger discrepancies of spectra between the seasonal and aseasonal DSGE models. Thus, to understand the source of bias, it is crucial to know the quantitative role of each friction in the model. To this end, I turn off each friction of the model, recompute the probability limits, and compare the resulting biases of the estimates with the baseline model. I identify two key frictions—the investment adjustment cost and the nominal wage rigidity—which play important roles.

In the column in Table 2.7 labeled "No inv. adj.," I report the probability limits of the maxmimum likelihood estimator when the investment adjustment cost is turned off ($\kappa = 0$). Since the magnitude of the adjustment cost does affect the seasonal steady states, I recalibrate seasonal shifts in neutral and investment technology and preference in order to match the data. I also adjust the parameters characterizing stochastic shock processes so that the model without the adjustment cost generates realistic second moments.[22] The estimated parameters come closer to the true values compared to those reported in the column labeled "Baseline." In particular, for some key parameters, including α (capital share), η (inverse of the Frisch labor supply elasticity), ξ_w (Calvo wage parameter), and ϕ_π (Taylor rule coefficient on inflation), distortions disappear almost completely. In the column labeled "No wage rig.," I report the probability limits when the wage rigidity is (almost) turned off ($\xi_w = 0.01, \chi_w = 0$). Again, I recalibrate the seasonal shifts in technology and preference and readjust the parameters characterizing the stochastic shock processes.

[22]Without adjustment, the volatilities of output and hours growth become extremely large. Moreover the spectra of those variables reach their peak at the highest frequency, which is the opposite of what we see in the data (Granger, 1966). As pointed out by Christiano and Todd (2002), since the weight assigned in the approximation criterion to the spectra is proportional to the level of the corresponding empirical estimates (expression 2.1), even a very small discrepancy in the spectrum at higher frequencies creates implausibly large parameter biases.

Similar to the case when the investment adjustment cost is turned off, the distortions are quite small (except for the persistence parameter of government spending, ρ_g, which is considerably understated). I have also examined model specifications where habit persistence is turned off ($b = 0$), capital utilization is turned off ($\delta_2 = 1000$), price rigidity is turned off ($\xi_p = 0.01, \chi_p = 0$), price and wage indexation is turned off ($\chi_p = 0, \chi_w = 0$), and the Taylor rule responding to a deviation of output (rather than output growth) from the steady state. None of these alternative specifications delivered precise estimates.[23] I view these as evidence showing that the investment adjustment cost and the nominal wage rigidity are the key frictions responsible for creating distortions.

Given the finding, readers might guess that other forms of capital accumulation or labor market frictions may contribute to creating distortions as well. This is indeed the case. To formalize the argument, I consider two alternative model specifications where (a) the investment adjustment cost is replaced with a capital adjustment cost and (b) the sticky wage assumption is replaced with a labor adjustment cost, and see whether the seasonal adjustment creates distortions.

For the capital adjustment cost, consider

$$K_t^p = (1 - \delta(u_t))K_{t-1}^p + \mu_t\left(I_t - S\left(\frac{K_t^p}{K_{t-1}^p}\right)K_{t-1}^p\right),$$

where for the functional form for S, I assume

$$S\left(\frac{K_t^p}{K_{t-1}^p}\right) = \frac{\kappa_K}{2}\left(\frac{K_t^p}{K_{t-1}^p} - \gamma\right)^2.$$

[23]Details of the results are available from the author upon request.

Similar specifications for the capital adjustment cost were used, for example, in Bernanke et al. (1999) and Chari et al. (2000). I set $\kappa_K = 24$ so that the moments generated from the capital adjustment cost model are similar to those generated from the baseline model. The column in Table 2.7 labeled "Capital adj." reports the probability limits of the maximum likelihood estimator. As in the baseline model, the capital adjustment cost model delivers sizeable distortions, although their directions and magnitudes are somewhat different. For example, α (capital share), κ (capital adjustment cost), and $100\sigma_\mu$ (volatility parameter of the investment technology shock process) are overstated. Also ρ_z and $100\sigma_z$ (persistence and volatility parameters of the neutral technology shock process) are understated.

To investigate the role of the labor adjustment cost, I simply add a quadratic disutility term into the household's utility function:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \left[\tau_{t+s} \ln(C_{t+s} - bC_{t+s-1}) - \varphi \frac{H_{i,t+s}^{1+\eta}}{1+\eta} - \frac{\kappa_H}{2} \left(\frac{H_{i,t+s}}{H_{i,t+s-1}} - 1 \right)^2 \right].$$

I impose $\kappa_H = 0.8$ and recalibrate the seasonal shifts in technology and preference and readjust the parameters characterizing the stochastic shock processes. The column in Table 2.7 labeled "Labor adj." reports the probability limits of the maximum likelihood estimator. The labor adjustment cost model delivers considerable biases. For example, κ (investment adjustment cost), χ_p (price indexation), and ϕ_π and ϕ_Y (Taylor rule coefficients on inflation and output growth) are understated. ρ_g and $100\sigma_g$ (parameters characterizing the stochastic process of government spending shocks) are also imprecise. The results of the capital adjustment cost model and the labor adjustment cost model suggest that frictions that create distortions may not be limited to those I assumed in the baseline model.

2.6 Practical Considerations

So far I have argued that in current DSGE models, distortions due to misspecification arising from ignoring seasonality could be potentially large. However, this claim is based on an experiment in a considerably restricted setting. In particular, I have assumed that an econometrician has complete knowledge about the structure of the economy and the mechanism generating seasonality. In practice, such knowledge is not fully available. A researcher who ignores seasonality could be even worse off if she introduces a grossly misspecified mechanism of seasonality (Sims 1993 and Hansen and Sargent 1993). Thus, researchers face an important trade-off on whether to explicitely model seasonality or not. In this section I propose a simple procedure that helps researchers in making this decision, and I demonstrate how to use it.

A key component of the proposed procedure is to allow a coherent comparison between seasonal and aseasonal DSGE models. Let $\mathbf{y}_{t,q}$ be a vector of time-series data in time t at quarter q. Then the mapping of data from the model is,

$$\ln \mathbf{y}_{t,q} = \widehat{\mathbf{s}}_{t,q} + \ln \mathbf{s}_q, \tag{2.2}$$

for the seasonal model and

$$\ln \mathbf{y}_{t,q} = \widehat{\mathbf{s}}_t + \ln \mathbf{s} + \ln \mathbf{k}_q, \tag{2.3}$$

for the aseasonal model. Here $\mathbf{k}_q$ is a vector of seasonal dummies that is meant to capture seasonal variations in the data that cannot be explained by the aseasonal model. The orthogonal decomposition between $\widehat{\mathbf{s}}_t$ and $\mathbf{k}_q$ is consistent with a standard practice of seasonal adjustment. $\mathbf{k}_q$ is jointly estimated with structural parameters of the aseasonal model. Then a researcher can evaluate the fit across specifications by comparing the marginal likelihoods.[24]

I apply the mappings (2.2) and (2.3) to the simulated data used in the main experiment. The goal of the exercise is to demonstrate the usefulness of the approach for determining whether or not to explicitely model seasonality when there is potential danger of misspecification. I consider three examples of misspecification. The first example is the misspecification arising from ignoring seasonality (i.e., misspecification arising from using the aseasonal model), which has been the main focus of this paper. The second is the misspecification arising from the structure of the economy that is not directly related to the mechanism generating seasonality. In particular, I assume that a researcher thinks that the central bank responds to the output gap but not to output growth:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left\{ \left(\frac{\pi_t}{\pi_t^\star}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_q}\right)^{\phi_Y} \right\}^{1-\rho_R} e^{\epsilon_{R,t}}, \quad \epsilon_{R,t} \sim N(0, \sigma_R^2).$$

In the third example, the mechanism generating seasonality is misspecified. I assume that a researcher thinks that the seasonality in preference originates from shifts in the habit term b_q, but not τ_q. To compare the biases across specifications in a

[24] In this respect, the proposed procedure resembles the one-step approach of trend estimation in Ferroni (2011).

systematic way, as in Ferroni (2011) I consider the following quadratic loss function that measures overall distortions:

$$QL = (\overline{\theta} - \theta_{true})\Sigma_\theta(\overline{\theta} - \theta_{true})',$$

where $\overline{\theta} = 1/N \sum_{i=1}^{N} \theta_i$ and $\Sigma_\theta = 1/N \sum_{l=1}^{L} (\overline{\theta} - \theta_i)(\overline{\theta} - \theta_i)'$. Thus a larger value of quadratic loss implies larger bias.[25]

Table 2.8 presents the results under various model specifications. Two things emerge. First, when seasonality is explicitely modelled, parameter biases are likely to be modest even when other parts of the model are misspecified. In contrast, when seasonality is not modelled, the biases are large. This suggests that misspecification arising from ignoring seasonality is practically important in potentially misspecified models. For example, when both the Taylor rule and the mechanism for seasonality is misspecified, the quadratic loss is 0.0209. This is less than half compared to when seasonality is not modelled (0.1238 and 0.0791).[26] Second, although a smaller marginal likelihood does not necessarily imply larger parameter biases, it appears to be a relatively good indicator for a measure of biases. Other forms of misspecifications not considered here may imply substantially larger biases. Nevertheless, a researcher can diagnose the presence of misspecification by comparing marginal likelihoods.

[25] In the vectors θ and θ_{true}, I only include the structural parameters that are common between the seasonal and aseasonal models.

[26] When X-12-Arima-filtered data are used (Table 2.4), the quadratic loss is 0.0669.

2.7 Conclusion

Conventional wisdom among economists is that seasonal adjustments represent an innocuous data filtering that allows econometricians to focus on the estimation of objects of interest with little distortion. In this paper, I have challenged that view. Using a state-of-the-art DSGE model that can match salient features of U.S. seasonal and nonseasonal fluctuations, I showed that estimation using seasonally adjusted data leads to important distortions. The problem cannot be mitigated by constructing alternative seasonal adjustment filters, as the distortions still arise in large sample environments with "ideal" filters. This is because the effects of seasonality, which are magnified by several frictions built into the model, are propagated across the entire frequency domain.

One limitation of the analysis in this paper is that I have focused my attention on a full-information likelihood approach. Since the main reason for the distortions is that agents have different responses to shocks across seasons, we may be able to obtain better estimates by using only moments that do not condition on a season. For example, as I mentioned in Section 4, since "seasonally adjusted" impulse responses from the seasonal model and impulse responses from the aseasonal model are almost identical when the same parameter values are used, it seems reasonable to perform indirect inference by matching impulse responses of seasonally adjusted data and the aseasonal model. A systematic investigation of this idea is left for future research.

2.8 Tables and Figures

Table 2.1: Parameters that are fixed across quarters

Parameter	Description	Value
Panel A: Technology, preference, policy		
g	SS government spending	0.19
δ_0	SS depreciation rate	0.025
δ_2	Curvature of utilization cost	0.1
γ	SS technology growth	1.003
π	SS inflation rate	1.011
β	Discount factor	0.998
$\frac{\theta_p}{\theta_p-1}-1$	SS price markup	0.1
$\frac{\theta_w}{\theta_w-1}-1$	SS wage markup	0.1
α	Capital share	0.3
b	Habit persistence	0.7
η	Inverse Frisch elasticity	2
κ	Investment adjustment cost	1
ξ_p	Calvo price	0.6
ξ_w	Calvo wage	0.6
χ_p	Price indexation	0.3
χ_w	Wage indexation	0.3
ρ_R	Taylor rule smoothing	0.7
ϕ_π	Taylor rule inflation	1.7
ϕ_Y	Taylor rule output	0.2
Panel B: Shock process		
ρ_z	Neutral technology	0.95
ρ_τ	Preference	0.95
ρ_μ	Investment technology	0.95
ρ_π	Inflaton target	0.95
ρ_g	Government spending	0.95
$100\sigma_z$	Neutral technology	0.9
$100\sigma_\tau$	Preference	1.7
$100\sigma_\mu$	Investment technology	1.4
$100\sigma_\pi$	Inflation target	0.1
$100\sigma_R$	Monetary policy	0.1
$100\sigma_g$	Government spending	1

Table 2.2: Parameters that vary across quarters

Parameter	Description	Q1	Q2	Q3	Q4	Average
z_q	Neutral technology	0.99	1.00	0.99	1.02	1.00
τ_q	Preference	0.85	1.04	1.01	1.10	1.00
μ_q	Investment technology	0.88	1.06	0.98	1.08	1.00

Table 2.3: Seasonal patterns

	Data				Model			
Series	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Output*	-6.37	3.08	0.61	2.69	-6.37	3.08	0.61	2.69
Consumption	-6.50	2.27	0.09	4.14	-5.59	2.07	0.58	2.94
Investment*	-8.12	5.30	0.68	2.14	-8.11	5.30	0.68	2.14
Hours*	-3.83	3.00	1.59	-0.59	-3.87	2.97	1.55	-0.64
Wage growth	-0.45	-0.53	0.36	0.61	-0.53	0.17	0.06	0.30
Inflation rate	0.09	0.23	-0.14	-0.18	0.40	0.01	-0.06	-0.35
Interest rate	-0.03	0.04	0.01	-0.03	0.00	0.00	0.00	0.00

Notes: The table reports percent changes of variables from the previous quarter, taken from sample averages in the data and seasonal steady states in the model. Variables with * indicate those used as calibration targets.

Table 2.4: Posterior estimates

Parameter	Description	True	Unadjusted	X-12
α	Capital share	0.3	0.29	0.50
			(0.0120)	(0.0431)
b	Habit persistence	0.7	0.73	0.71
			(0.0134)	(0.0270)
η	Inverse Frisch elasticity	2	2.01	1.01
			(0.1287)	(0.1646)
κ	Investment adjustment cost	1	1.03	1.49
			(0.0760)	(0.1905)
ξ_p	Calvo price	0.6	0.60	0.57
			(0.0024)	(0.0084)
ξ_w	Calvo wage	0.6	0.59	0.49
			(0.0061)	(0.0321)
χ_p	Price indexation	0.3	0.30	0.33
			(0.0080)	(0.0311)
χ_w	Wage indexation	0.3	0.30	0.35
			(0.0112)	(0.0409)
ρ_R	Taylor rule smoothing	0.7	0.71	0.72
			(0.0178)	(0.0302)
ϕ_π	Taylor rule inflation	1.7	1.69	2.11
			(0.1398)	(0.3506)
ϕ_Y	Taylor rule output	0.2	0.22	0.34
			(0.0499)	(0.1047)
ρ_z	Neutral technology	0.95	0.95	0.95
			(0.0020)	(0.0059)
ρ_τ	Preference	0.95	0.94	0.97
			(0.0091)	(0.0076)
ρ_μ	Investment technology	0.95	0.96	0.92
			(0.0103)	(0.0222)
ρ_π	Inflation target	0.95	0.96	0.95
			(0.0066)	(0.0105)
ρ_g	Government spending	0.95	0.95	0.62
			(0.0109)	(0.1210)

(Table continues on the next page.)

Table 2.4: Posterior estimates (continued)

Parameter	Description	True	Unadjusted	X-12
$100\sigma_z$	Neutral technology	0.9	0.90	0.78
			(0.0452)	(0.0397)
$100\sigma_\tau$	Preference	1.7	1.72	1.52
			(0.1010)	(0.1441)
$100\sigma_\mu$	Investment technology	1.4	1.68	1.41
			(0.2194)	(0.1878)
$100\sigma_\pi$	Inflation target	0.1	0.09	0.11
			(0.0120)	(0.0168)
$100\sigma_R$	Monetary policy	0.1	0.10	0.12
			(0.0054)	(0.0072)
$100\sigma_g$	Government spending	1	0.90	1.48
			(0.0472)	(0.1551)
$\tilde{z}_1$	Neutral technology Q1	0.97	0.97	–
			(0.0005)	
$\tilde{z}_2$	Neutral technology Q2	0.97	0.97	–
			(0.0003)	
$\tilde{z}_3$	Neutral technology Q3	0.97	0.97	–
			(0.0001)	
$\tilde{\tau}_1$	Preference Q1	0.77	0.74	–
			(0.0126)	
$\tilde{\tau}_2$	Preference Q2	0.95	0.94	–
			(0.0023)	
$\tilde{\tau}_3$	Preference Q3	0.92	0.92	–
			(0.0037)	
$\tilde{\mu}_1$	Investment technology Q1	0.81	0.80	–
			(0.0121)	
$\tilde{\mu}_2$	Investment technology Q2	0.98	0.97	–
			(0.0034)	
$\tilde{\mu}_3$	Investment technology Q3	0.91	0.91	–
			(0.0069)	

Notes: The table reports the MCMC estimates of posterior means. Standard deviations are reported in parentheses. The following reparameterizations are used: $\tilde{z}_q = z_q/z_4$, $\tilde{\tau}_q = \tau_q/\tau_4$, and $\tilde{\mu}_q = \mu_q/\mu_4$ for $q = 1, 2, 3$.

Table 2.5: Business cycle statistics: seasonally adjusted vs. unadjusted estimates

	Percent standard deviation			Corr. with output growth		
Series	True	Unadjusted	X-12	True	Unadjusted	X-12
Output growth	0.96	0.94	0.83	–	–	–
Consumption growth	0.57	0.57	0.54	0.38	0.31	0.12
Investment growth	2.78	2.87	1.48	0.89	0.88	0.84
Hours growth	0.99	0.98	1.10	0.54	0.53	0.48
Wage growth	0.41	0.41	0.39	0.77	0.77	0.60
Inflation rate	0.73	0.78	0.70	-0.22	-0.21	-0.21
Interest rate	0.69	0.77	0.69	-0.18	-0.15	-0.16

Notes: True moments are calculated by applying the X-12-Arima filter to the simulated data from the seasonal model, where the parameters are fixed at their true values. Similarly, seasonally unadjusted moments are calculated by applying the X-12-Arima filter to the simulated data from the seasonal model, where the parameters are fixed at the posterior means of seasonally unadjusted estimates. X-12-Arima moments are calculated using simulated data from the aseasonal model, where the parameters are fixed at the posterior means of X-12-Arima-filtered estimates. I did not apply any seasonal adjustment filter to the simulated data for X-12-Arima moments. All simulations are based on 100 replications of artificial time-series of length 200.

Table 2.6: Percent standard deviation of variables under alternative values of ϕ_π: seasonally adjusted vs. unadjusted estimates

	ϕ_π				
Series	1.7	2.5	5.0	7.5	10.0
Output growth					
True	0.96	1.00	1.12	1.21	1.27
	[1.00]	[1.04]	[1.17]	[1.26]	[1.32]
Unadjusted	0.95	0.97	1.11	1.20	1.26
	[1.00]	[1.03]	[1.17]	[1.26]	[1.33]
X-12	0.86	0.86	0.91	0.95	0.97
	[1.00]	[1.00]	[1.06]	[1.10]	[1.12]
Inflation					
True	0.73	0.53	0.41	0.36	0.34
	[1.00]	[0.73]	[0.56]	[0.50]	[0.47]
Unadjusted	0.77	0.55	0.42	0.37	0.35
	[1.00]	[0.72]	[0.55]	[0.49]	[0.46]
X-12	0.84	0.62	0.45	0.42	0.39
	[1.00]	[0.74]	[0.54]	[0.50]	[0.46]

Notes: Other monetary policy paramters are set to the true benchmark values ($\rho_R = 0.7, \phi_Y = 0.2$). The numbers in square brackets indicate the ratios relative to the benchmark case ($\phi_\pi = 1.7$). For simulation details, see the footnote of Table 2.5.

Table 2.7: Probability limits of the likelihood estimator when seasonally adjusted data are used

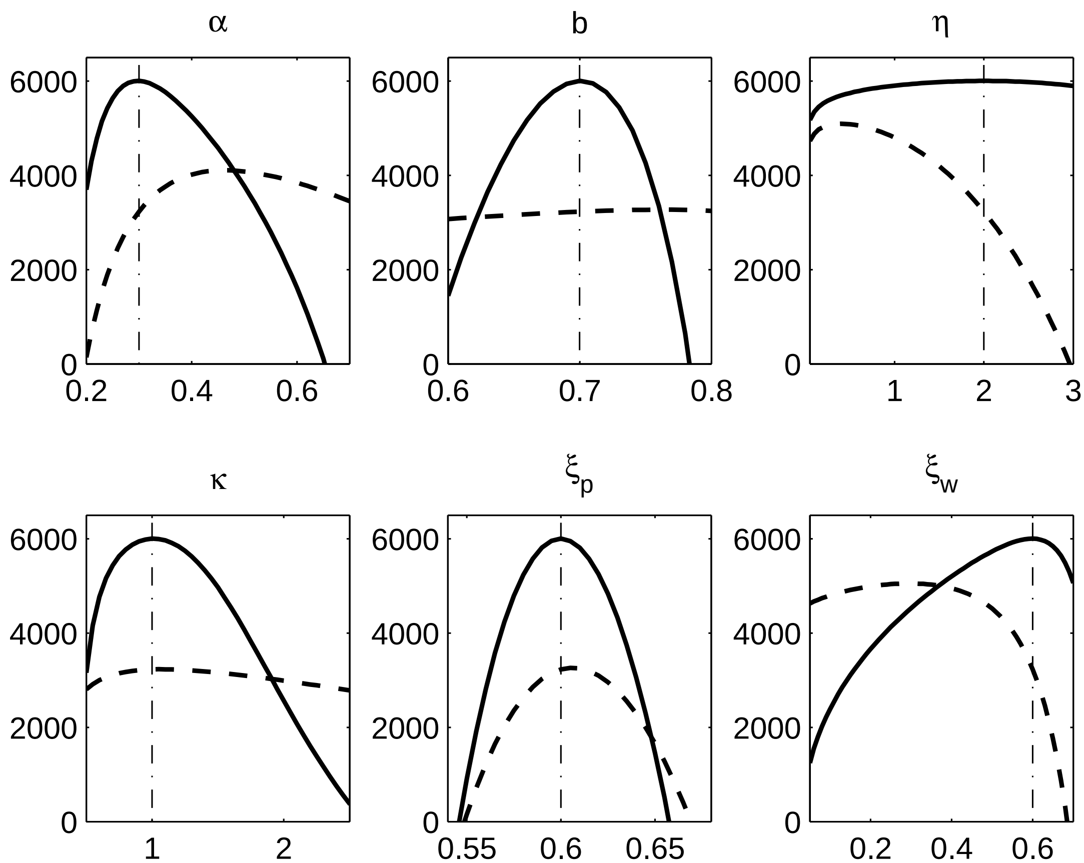
		Baseline		No inv. adj.		No wage rig.		Capital adj.		Labor adj.	
Parameter	Description	True	Plim	True	Plim	True	Plim	True	Plim	True	Plim
α	Capital share	0.3	0.46	0.3	0.31	0.3	0.27	0.3	0.39	0.3	0.26
b	Habit persistence	0.7	0.67	0.7	0.70	0.7	0.71	0.7	0.68	0.7	0.70
η	Inverse Frisch elasticity	2	1.24	2	2.15	2	1.99	2	2.07	2	1.94
κ	Investment adjustment cost	1	1.25	–	–	1	0.89	–	–	1	0.77
κ_K	Capital adjustment cost	–	–	–	–	–	–	24	43.45	–	–
κ_H	Labor adjustment cost	–	–	–	–	–	–	–	–	0.8	0.80
ξ_p	Calvo price	0.6	0.57	0.6	0.60	0.6	0.60	0.6	0.58	0.6	0.63
ξ_w	Calvo wage	0.6	0.54	0.6	0.60	–	–	0.6	0.60	–	–
χ_p	Price indexation	0.3	0.31	0.3	0.29	0.3	0.29	0.3	0.32	0.3	0.19
χ_w	Wage indexation	0.3	0.32	0.3	0.31	–	–	0.3	0.32	–	–
ρ_R	Taylor rule smoothing	0.7	0.71	0.7	0.70	0.7	0.70	0.7	0.72	0.7	0.66
ϕ_π	Taylor rule inflation	1.7	1.81	1.7	1.72	1.7	1.72	1.7	1.77	1.7	1.45
ϕ_Y	Taylor rule output	0.2	0.22	0.2	0.20	0.2	0.21	0.2	0.21	0.2	0.10
ρ_z	Neutral technology	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.46	0.95	0.95
ρ_τ	Preference	0.95	0.96	0.95	0.95	0.95	0.95	0.95	0.96	0.95	0.94
ρ_μ	Investment technology	0.95	0.92	0.95	0.79	0.95	0.96	0.95	0.86	0.95	0.96
ρ_π	Inflation target	0.95	0.94	0.95	0.96	0.95	0.95	0.95	0.95	0.95	0.96
ρ_g	Government spending	0.95	0.86	0.95	0.92	0.95	0.42	0.95	0.92	0.95	0.28
$100\sigma_z$	Neutral technology	0.9	0.87	0.4	0.40	1.3	1.32	0.9	0.18	1.2	1.22
$100\sigma_\tau$	Preference	1.7	1.51	2	2.03	1.6	1.70	1.7	1.71	1.2	1.23
$100\sigma_\mu$	Investment technology	1.4	1.32	0.1	0.05	1.4	1.69	1.4	9.78	2	2.37
$100\sigma_\pi$	Inflation target	0.1	0.10	0.01	0.01	0.08	0.09	0.1	0.11	0.06	0.08
$100\sigma_R$	Monetary policy	0.1	0.11	0.01	0.01	0.1	0.10	0.1	0.10	0.1	0.10
$100\sigma_g$	Government spending	1	1.94	1	1.68	1	1.38	1	1.39	1	1.36

Table 2.8: Comparison across alternative model specifications

Source of misspecification				
Seasonality not modelled	Misspecified Taylor rule	Misspecified seasonality	Marginal likelihood	Quadratic loss
			5824.4	0.0039
	✓		5802.3	0.0157
		✓	5788.4	0.0135
	✓	✓	5761.8	0.0209
✓			5259.2	0.1238
✓	✓		5226.4	0.0791

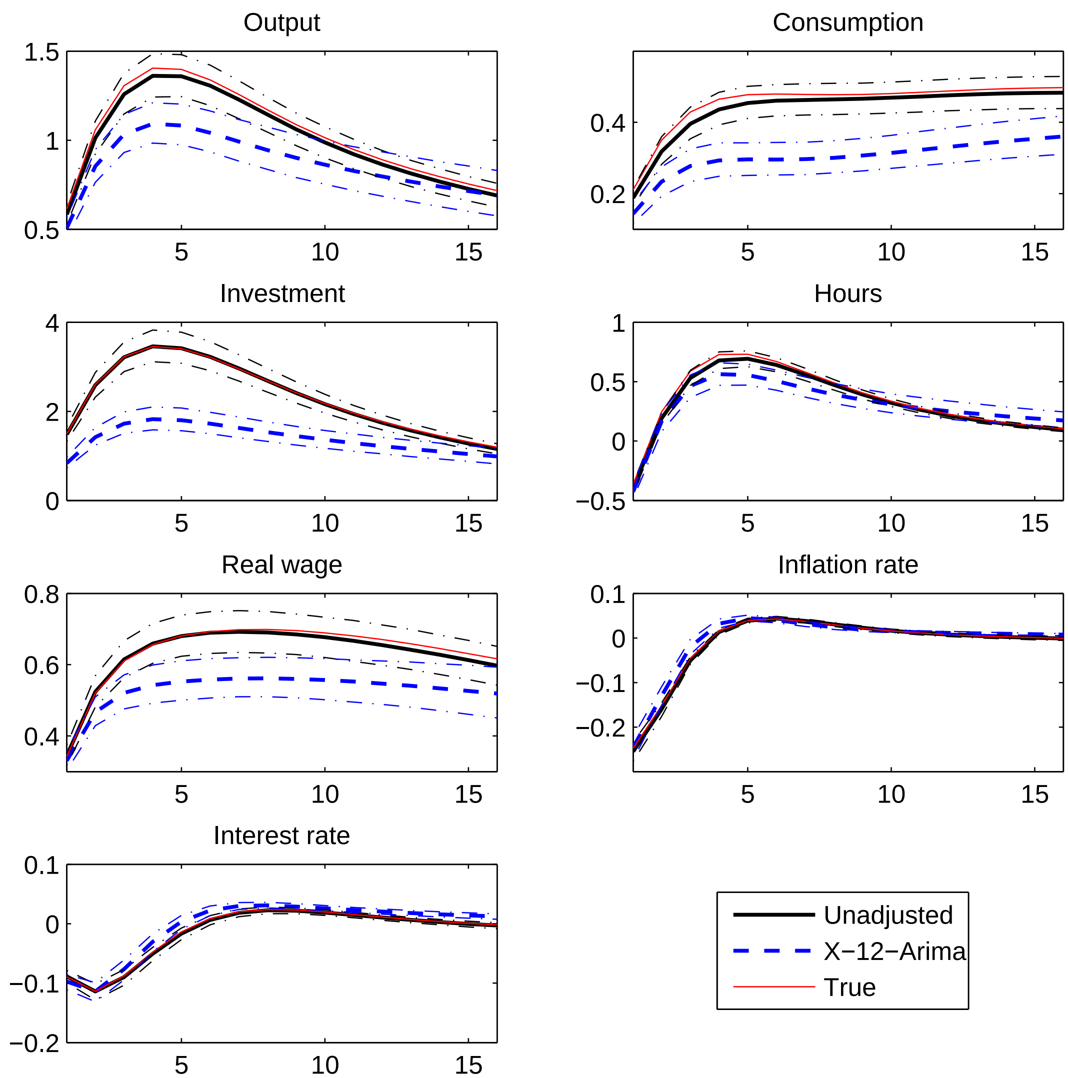
Notes: The marginal likelihoods are calculated based on the modified harmonic mean estimator by Geweke (1999). For the truncation value I use $p = 0.5$. Other values deliver similar results.

Figure 2.1: Likelihood profiles: seasonally adjusted vs. unadjusted data



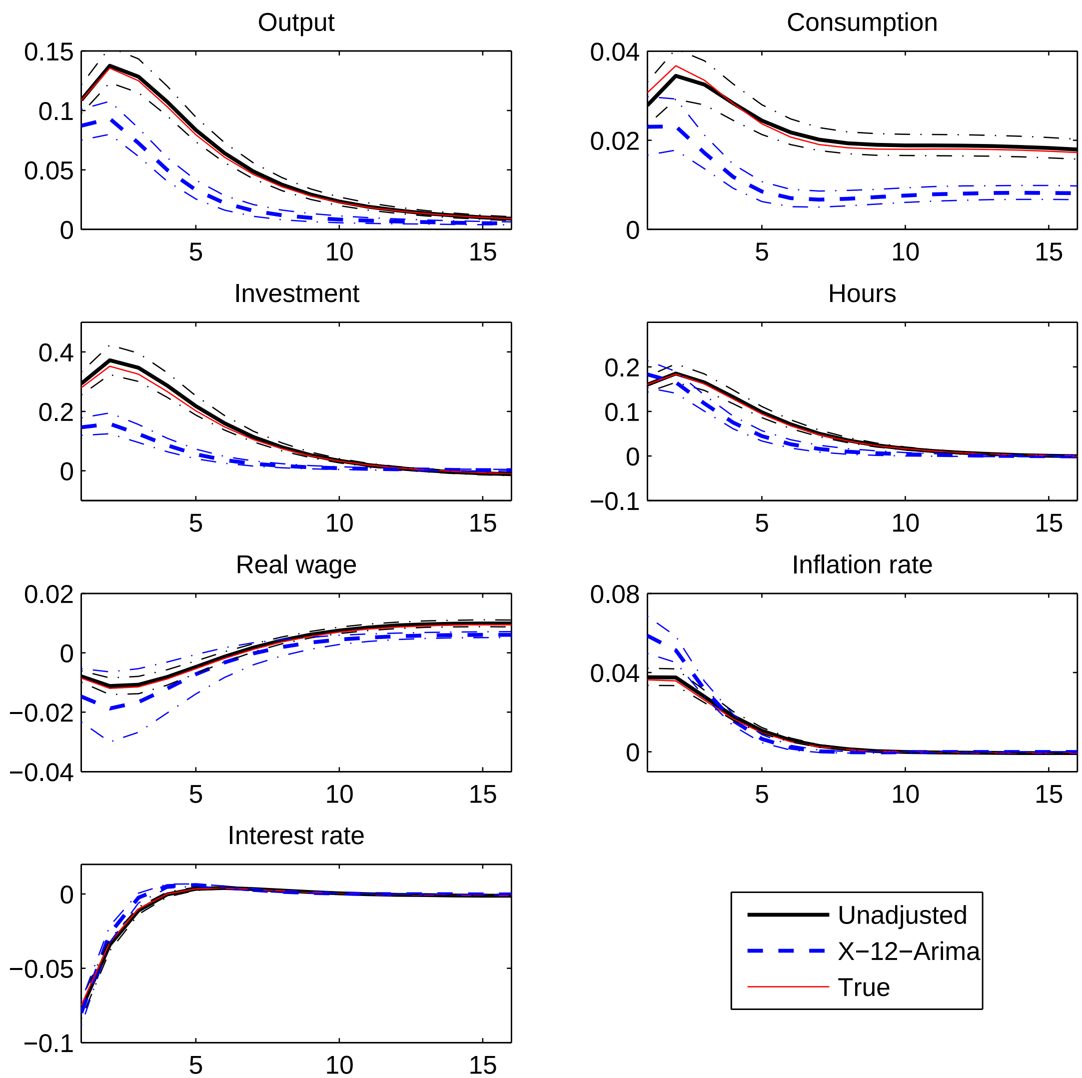
Notes: The figure plots likelihood profiles for seasonally unadjusted data (solid lines) and X-12-Arima-filtered data (dashed lines). Vertical lines signify true values.

Figure 2.2: Impulse response: neutral technology shock



Notes: Each panel describes the percentage-point response to a one-standard-deviation shock. Computations are based on 1,000 draws from the posterior distributions.

Figure 2.3: Impulse response: monetary policy shock



Notes: Each panel describes the percentage-point response to a one-standard-deviation shock. Computations are based on 1,000 draws from the posterior distributions.

Figure 2.4: Log spectrum of the simulated data (output growth)

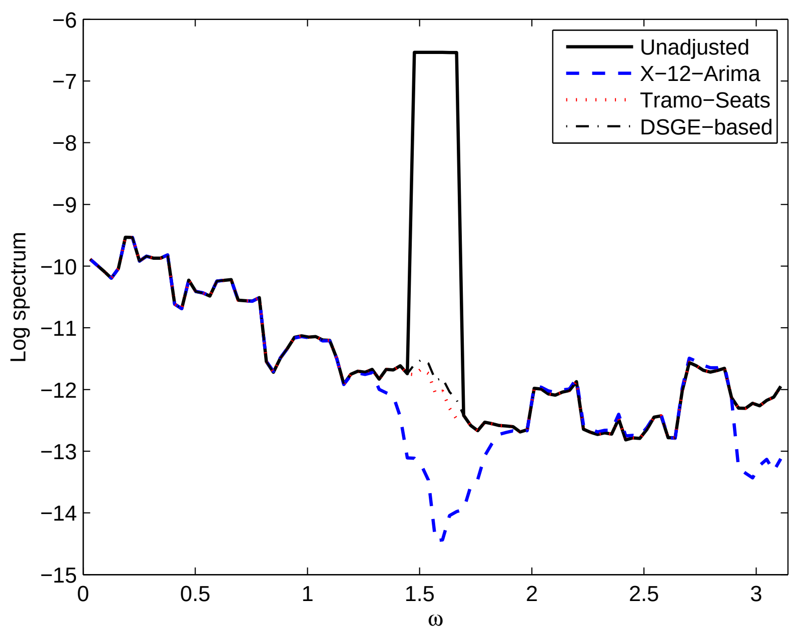
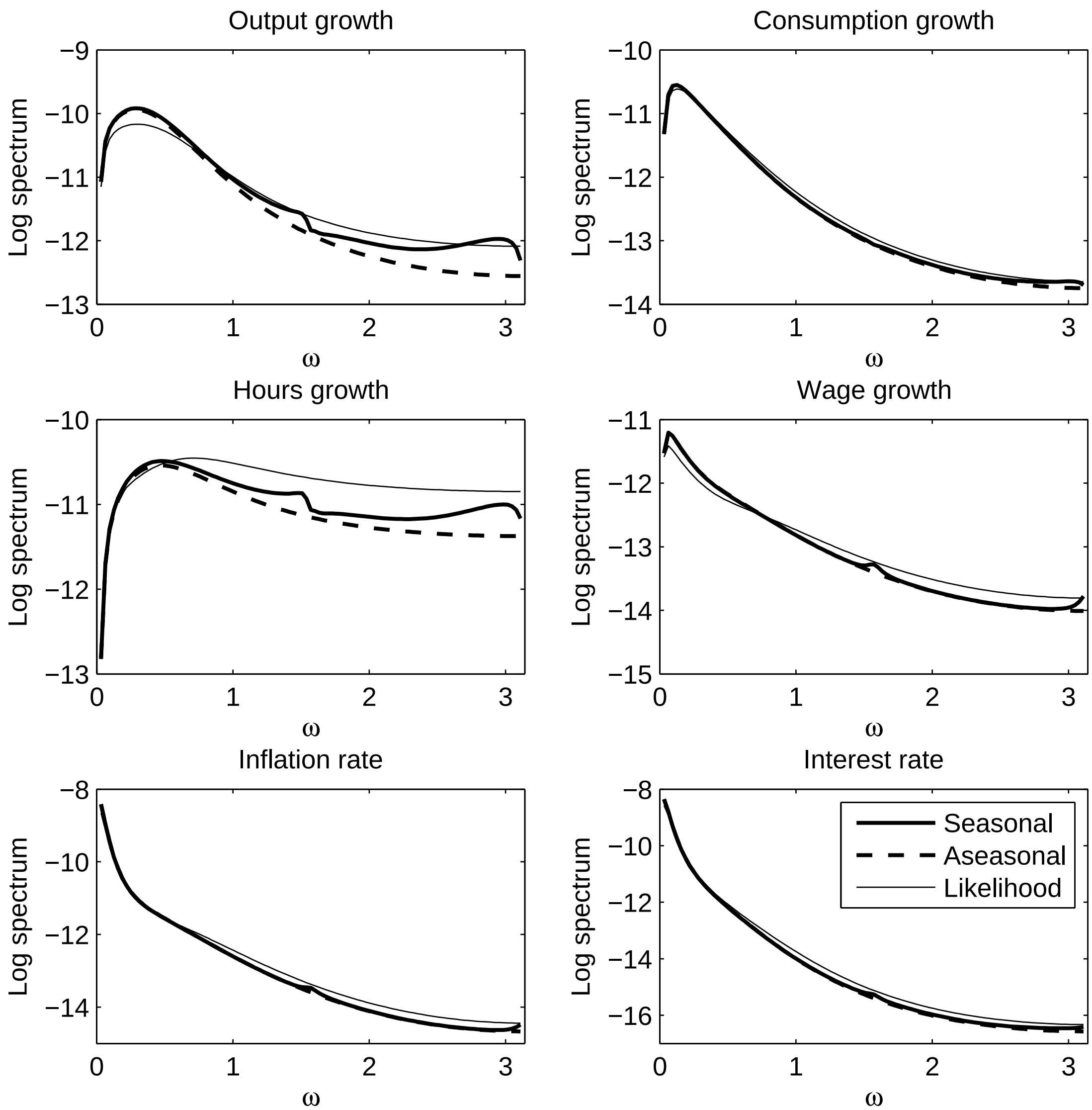
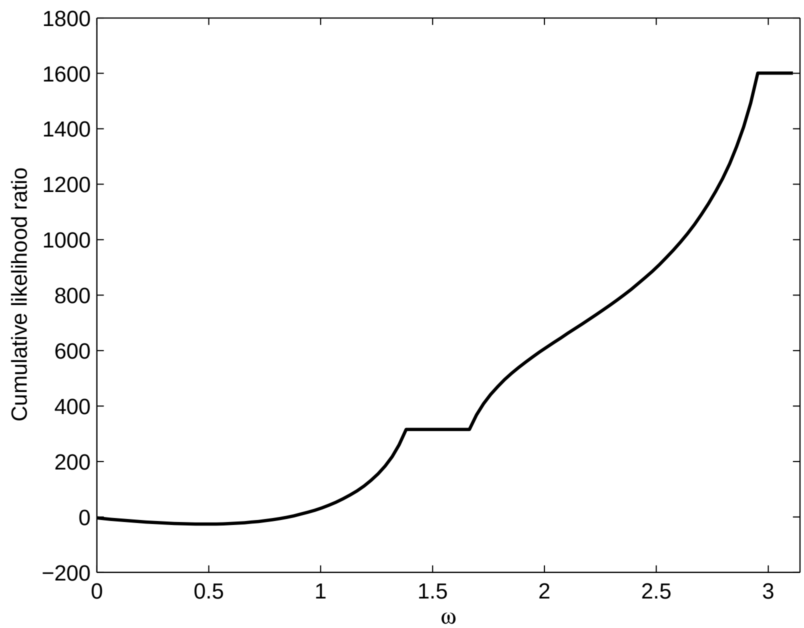


Figure 2.5: Log spectrum of the seasonal and aseasonal model



Notes: Thick solid and dashed lines plot spectra of seasonal and aseasonal model whose parameters are fixed at the values used to generate data in the main experiment. Thin solid lines plot spectra of the aseasonal model whose parameters are fixed at the asymptotic maximum likelihood estimates when seasonally adjusted data are used.

Figure 2.6: Cumulative likelihood ratio



2.9 Appendix

2.9.1 Data Source

I report the sources of data used in Table 3 in the main text and Table 2.9 in this Appendix. The data set spans the period 1965Q1 to 2004Q4. Whenever the data set is provided in monthly frequencies, I simply take the average to transform it into quarterly frequencies. Both seasonally adjusted and unadjusted National Income and Product Accounts data are downloaded from the Bureau of Economic Analysis website. Nominal GDP, nominal consumption (defined as the sum of personal consumption expenditures on nondurables and services), and nominal investment (defined as the sum of gross private domestic investment and personal consumption expenditure on durables) are divided by the civilian noninstitutional population, downloaded from the Bureau of Labor Statistics (BLS hereafter) website, to convert the variables into per capita terms. To convert the variables into real terms, I divide them by the consumer price index for all urban consumers, available on the BLS website.[27] Working hours are measured by aggregate weekly hours in total private industries (available on the BLS website) divided by the population. Real wages are measured by average hourly earnings of production workers in total private industries (available on the BLS website), divided by the CPI. Inflation rates are measured by changes in the CPI. I use the effective federal funds rates (downloaded from the Federal Reserve Board website) to measure the nominal interest rates.

[27] I use the CPI for measuring price level since the seasonally unadjusted GDP deflator is not available.

2.9.2 Solving the Seasonal DSGE Model

The equilibrium of the baseline seasonal DSGE model in the main text is characterized by the equilibrium conditions below (B.1). To solve for the equilbrium, I log-linearize the equilibrium conditions (B.3) around the seasonal steady states (B.2). After carefully stacking the log-linearized equilibrium conditions so that they are consistent with the seasonal orderings, I can obtain the law of motion of the economy using a standard solution method of linear rational expectations models. A few words on notation: variables with a tilde denote detrended variables ($\widetilde{A}_t = A_t/X_t$) and variables with a hat denote log deviations of variables from their seasonal steady states ($\widehat{B}_t = \ln(B_t/B_q)$).

Equilbrium Conditions

$$\frac{\widetilde{K}_t}{H_t} = \frac{\alpha}{1-\alpha}\left(\frac{\widetilde{w}_t}{r_t^k}\right) \tag{B.1.1}$$

$$mc_t = \frac{1}{z_t\alpha^\alpha(1-\alpha)^{1-\alpha}}(\widetilde{w}_t)^{1-\alpha}(r_t^k)^\alpha \tag{B.1.2}$$

$$p_t^* = \frac{\theta_p}{\theta_p - 1}\left(\frac{P_t^n}{P_t^d}\right) \tag{B.1.3}$$

$$P_t^n = \widetilde{\lambda}_t mc_t \widetilde{Y}_t + \xi_p \beta \mathrm{E}_t \left(\frac{\pi_t^{\chi_p}\pi^{1-\chi_p}}{\pi_{t+1}}\right)^{-\theta_p} P_{t+1}^n \tag{B.1.4}$$

$$P_t^d = \widetilde{\lambda}_t \widetilde{Y}_t + \xi_p \beta \mathrm{E}_t \left(\frac{\pi_t^{\chi_p}\pi^{1-\chi_p}}{\pi_{t+1}}\right)^{1-\theta_p} P_{t+1}^d \tag{B.1.5}$$

$$1 = (1-\xi_p)(p_t^*)^{1-\theta_p} + \xi_p \left(\frac{\pi_{t-1}^{\chi_p}\pi^{1-\chi_p}}{\pi_t}\right)^{1-\theta_p} \tag{B.1.6}$$

$$\widetilde{Y}_t = (\widetilde{p}_t)^{\theta_p} z_t \widetilde{K}_t^\alpha H_t^{1-\alpha} \tag{B.1.7}$$

$$(\widetilde{p}_t)^{-\theta_p} =(1-\xi_p)(p_t^*)^{-\theta_p} + \xi_p\left(\frac{\pi_{t-1}^{\chi_p}\pi^{1-\chi_p}}{\pi_t}\right)^{-\theta_p} \tag{B.1.8}$$

$$\widetilde{\lambda}_t =\frac{\gamma\tau_t}{\gamma\widetilde{C}_t - b\widetilde{C}_{t-1}} - \beta b \mathrm{E}_t\left(\frac{\tau_{t+1}}{\gamma\widetilde{C}_{t+1} - b\widetilde{C}_t}\right) \tag{B.1.9}$$

$$1 =\beta\mathrm{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{R_t}{\gamma\pi_{t+1}}\right)\right] \tag{B.1.10}$$

$$\begin{aligned}\widetilde{\lambda}_t =&\widetilde{\psi}_t\mu_t\left[1 - S\left(\frac{\gamma\widetilde{I}_t}{\widetilde{I}_{t-1}}\right) - S'\left(\frac{\gamma\widetilde{I}_t}{\widetilde{I}_{t-1}}\right)\frac{\gamma\widetilde{I}_t}{\widetilde{I}_{t-1}}\right] \\ &+ \beta\mathrm{E}_t\left[\frac{\widetilde{\psi}_{t+1}}{\gamma}\mu_{t+1}S'\left(\frac{\gamma\widetilde{I}_{t+1}}{\widetilde{I}_t}\right)\left(\frac{\gamma\widetilde{I}_{t+1}}{\widetilde{I}_t}\right)^2\right]\end{aligned} \tag{B.1.11}$$

$$\gamma\widetilde{\psi}_t =\beta\mathrm{E}_t[\widetilde{\lambda}_{t+1}r_{t+1}^k u_{t+1} + \widetilde{\psi}_{t+1}(1-\delta(u_{t+1}))] \tag{B.1.12}$$

$$\widetilde{\lambda}_t r_t^k =\widetilde{\psi}_t\delta'(u_t) \tag{B.1.13}$$

$$\widetilde{K}_t =u_t\widetilde{K}_{t-1}^p \tag{B.1.14}$$

$$\gamma\widetilde{K}_t^p =(1-\delta(u_t))\widetilde{K}_{t-1}^p + \mu_t\left(1 - S\left(\frac{\gamma\widetilde{I}_t}{\widetilde{I}_{t-1}}\right)\right)\widetilde{I}_t \tag{B.1.15}$$

$$f_t^1 =f_t^2 \tag{B.1.16}$$

$$f_t^1 =(\widetilde{w}_t^*)^{1-\theta_w}\widetilde{\lambda}_t H_t\widetilde{w}_t + \xi_w\beta\mathrm{E}_t\left(\frac{\pi_t^{\chi_w}\pi^{1-\chi_w}\widetilde{w}_t^*}{\widetilde{\pi}_{t+1}^w\widetilde{w}_{t+1}^*}\right)^{1-\theta_w} f_{t+1}^1 \tag{B.1.17}$$

$$f_t^2 =\frac{\theta_w}{\theta_w - 1}(\widetilde{w}_t^*)^{-\theta_w(1+\eta)}\varphi H_t^{1+\eta} + \xi_w\beta\mathrm{E}_t\left(\frac{\pi_t^w\pi^{1-\chi_w}\widetilde{w}_t^*}{\widetilde{\pi}_{t+1}^w\widetilde{w}_{t+1}^*}\right)^{-\theta_w(1+\eta)} f_{t+1}^2 \tag{B.1.18}$$

$$1 =(1-\xi_w)(\widetilde{w}_t^*)^{1-\theta_w} + \xi_w\left(\frac{\pi_{t-1}^{\chi_w}\pi^{1-\chi_w}}{\widetilde{\pi}_t^w}\right)^{1-\theta_w} \tag{B.1.19}$$

$$\widetilde{\pi}_t^w =\frac{\pi_t\widetilde{w}_t}{\widetilde{w}_{t-1}} \tag{B.1.20}$$

$$\frac{R_t}{R} =\left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left\{\left(\frac{\pi_t}{\pi_t^\star}\right)^{\phi_\pi}\left(\frac{\widetilde{Y}_t/\widetilde{Y}_{t-1}}{\widetilde{Y}_q/\widetilde{Y}_{q-1}}\right)^{\phi_Y}\right\}^{1-\rho_R} e^{\epsilon_{R,t}} \tag{B.1.21}$$

$$\widetilde{Y}_t =\widetilde{C}_t + \widetilde{I}_t + g_t\widetilde{Y}_t \tag{B.1.22}$$

$$\ln\left(\frac{z_t}{z_q}\right) =\rho_z\ln\left(\frac{z_{t-1}}{z_{q-1}}\right) + \epsilon_{z,t} \tag{B.1.23}$$

$$\ln\left(\frac{\tau_t}{\tau_q}\right) = \rho_\tau \ln\left(\frac{\tau_{t-1}}{\tau_{q-1}}\right) + \epsilon_{\tau,t} \tag{B.1.24}$$

$$\ln\left(\frac{\mu_t}{\mu_q}\right) = \rho_\mu \ln\left(\frac{\mu_{t-1}}{\mu_{q-1}}\right) + \epsilon_{\mu,t} \tag{B.1.25}$$

$$\ln\left(\frac{\pi^\star_t}{\pi_q}\right) = \rho_\pi \ln\left(\frac{\pi^\star_{t-1}}{\pi_{q-1}}\right) + \epsilon_{\pi,t} \tag{B.1.26}$$

$$\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \epsilon_{g,t} \tag{B.1.27}$$

Seasonal Steady States

$$\frac{\widetilde{K}_q}{H_q} = \frac{\alpha}{1-\alpha}\left(\frac{\widetilde{w}_q}{r^k_q}\right) \tag{B.2.1}$$

$$mc_q = \frac{1}{z_q \alpha^\alpha (1-\alpha)^{1-\alpha}} (\widetilde{w}_q)^{1-\alpha} (r^k_q)^\alpha \tag{B.2.2}$$

$$p^*_q = \frac{\theta_p}{\theta_p - 1}\left(\frac{P^n_q}{P^d_q}\right) \tag{B.2.3}$$

$$P^n_q = \widetilde{\lambda}_q mc_q \widetilde{Y}_q + \xi_p \beta \left(\frac{\pi_q^{\chi_p} \pi^{1-\chi_p}}{\pi_{q+1}}\right)^{-\theta_p} P^n_{q+1} \tag{B.2.4}$$

$$P^d_q = \widetilde{\lambda}_q \widetilde{Y}_q + \xi_p \beta \left(\frac{\pi_q^{\chi_p} \pi^{1-\chi_p}}{\pi_{q+1}}\right)^{1-\theta_p} P^d_{q+1} \tag{B.2.5}$$

$$1 = (1-\xi_p)(p^*_q)^{1-\theta_p} + \xi_p \left(\frac{\pi_{q-1}^{\chi_p} \pi^{1-\chi_p}}{\pi_q}\right)^{1-\theta_p} \tag{B.2.6}$$

$$\widetilde{Y}_q = (\widetilde{p}_q)^{\theta_p} z_q \widetilde{K}_q^\alpha H_q^{1-\alpha} \tag{B.2.7}$$

$$(\widetilde{p}_q)^{-\theta_p} = (1-\xi_p)(p^*_q)^{-\theta_p} + \xi_p \left(\frac{\pi_{q-1}^{\chi_p} \pi^{1-\chi_p}}{\pi_q}\right)^{-\theta_p} \tag{B.2.8}$$

$$\widetilde{\lambda}_q = \frac{\gamma \tau_q}{\gamma \widetilde{C}_q - b\widetilde{C}_{q-1}} - \beta b \left(\frac{\tau_{q+1}}{\gamma \widetilde{C}_{q+1} - b\widetilde{C}_q}\right) \tag{B.2.9}$$

$$1 = \beta\left[\frac{\lambda_{q+1}}{\lambda_q}\left(\frac{R}{\gamma \pi_{q+1}}\right)\right] \tag{B.2.10}$$

$$\widetilde{\lambda}_q = \widetilde{\psi}_q \mu_q \left[1 - S\left(\frac{\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}}\right) - S'\left(\frac{\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\frac{\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}}\right] + \beta\left[\frac{\widetilde{\psi}_{q+1}}{\gamma}\mu_{q+1} S'\left(\frac{\gamma \widetilde{I}_{q+1}}{\widetilde{I}_q}\right)\left(\frac{\gamma \widetilde{I}_{q+1}}{\widetilde{I}_q}\right)^2\right] \tag{B.2.11}$$

$$\gamma \widetilde{\psi}_q = \beta[\widetilde{\lambda}_{q+1} r^k_{q+1} u_{q+1} + \widetilde{\psi}_{q+1}(1 - \delta(u_{q+1}))] \tag{B.2.12}$$

$$\widetilde{\lambda}_q r^k_q = \widetilde{\psi}_q \delta'(u_q) \tag{B.2.13}$$

$$\widetilde{K}_q = u_q \widetilde{K}^p_{q-1} \tag{B.2.14}$$

$$\gamma \widetilde{K}^p_q = (1 - \delta(u_q))\widetilde{K}^p_{q-1} + \mu_q \left(1 - S\left(\frac{\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\right)\widetilde{I}_q \tag{B.2.15}$$

$$f^1_q = f^2_q \tag{B.2.16}$$

$$f^1_q = (\widetilde{w}^*_q)^{1-\theta_w}\widetilde{\lambda}_q H_q \widetilde{w}_q + \xi_w \beta \left(\frac{\pi_q^{\chi_w}\pi^{1-\chi_w}\widetilde{w}^*_q}{\widetilde{\pi}^w_{q+1}\widetilde{w}^*_{q+1}}\right)^{1-\theta_w} f^1_{q+1} \tag{B.2.17}$$

$$f^2_q = \frac{\theta_w}{\theta_w - 1}(\widetilde{w}^*_q)^{-\theta_w(1+\eta)}\varphi H_q^{1+\eta} + \xi_w \beta \left(\frac{\pi_q^{w}\pi^{1-\chi_w}\widetilde{w}^*_q}{\widetilde{\pi}^w_{q+1}\widetilde{w}^*_{q+1}}\right)^{-\theta_w(1+\eta)} f^2_{q+1} \tag{B.2.18}$$

$$1 = (1 - \xi_w)(\widetilde{w}^*_q)^{1-\theta_w} + \xi_w \left(\frac{\pi_{q-1}^{\chi_w}\pi^{1-\chi_w}}{\widetilde{\pi}^w_q}\right)^{1-\theta_w} \tag{B.2.19}$$

$$\widetilde{\pi}^w_q = \frac{\pi_q \widetilde{w}_q}{\widetilde{w}_{q-1}} \tag{B.2.20}$$

$$\widetilde{Y}_q = \widetilde{C}_q + \widetilde{I}_q + g\widetilde{Y}_q \tag{B.2.21}$$

Log-linearized Equilibrium Conditions

$$\widehat{\widetilde{K}}_t - \widehat{H}_t = \widehat{\widetilde{w}}_t - \widehat{r}^k_t \tag{B.3.1}$$

$$\widehat{mc}_t = -\widehat{z}_t + (1 - \alpha)\widehat{\widetilde{w}}_t + \alpha \widehat{r}^k_t \tag{B.3.2}$$

$$\widehat{p}_t^* = \widehat{P}_t^n - \widehat{P}_t^d \tag{B.3.3}$$

$$\begin{aligned} P_q^n \widehat{P}_t^n = & \widetilde{\lambda}_q mc_q \widetilde{Y}_q (\widehat{\widetilde{\lambda}}_t + \widehat{mc}_t + \widehat{\widetilde{Y}}_t) \\ & + \xi_p \beta \left(\frac{\pi_q^{\chi_p} \pi^{1-\chi_p}}{\pi_{q+1}} \right)^{-\theta_p} P_{q+1}^n [\theta_p (\mathrm{E}_t \widehat{\pi}_{t+1} - \chi_p \widehat{\pi}_t) + \mathrm{E}_t \widehat{P}_{t+1}^n] \end{aligned} \tag{B.3.4}$$

$$\begin{aligned} P_q^d \widehat{P}_t^d = & \widetilde{\lambda}_q \widetilde{Y}_q (\widehat{\widetilde{\lambda}}_t + \widehat{\widetilde{Y}}_t) \\ & + \xi_p \beta \left(\frac{\pi_q^{\chi_p} \pi^{1-\chi_p}}{\pi_{q+1}} \right)^{1-\theta_p} P_{q+1}^d [(\theta_p - 1)(\mathrm{E}_t \widehat{\pi}_{t+1} - \chi_p \widehat{\pi}_t) + \mathrm{E}_t \widehat{P}_{t+1}^d] \end{aligned} \tag{B.3.5}$$

$$0 = (1-\xi_p)(p_q^*)^{1-\theta_p} \widehat{p}_t^* + \xi_p \left(\frac{\pi_{q-1}^{\chi_p} \pi^{1-\chi_p}}{\pi_q} \right)^{1-\theta_p} (\chi_p \widehat{\pi}_{t-1} - \widehat{\pi}_t) \tag{B.3.6}$$

$$\widehat{\widetilde{Y}}_t = \theta_p \widehat{\widetilde{p}}_t + \widehat{z}_t + \alpha \widehat{\widetilde{K}}_t + (1-\alpha) \widehat{\widetilde{H}}_t \tag{B.3.7}$$

$$(\widetilde{p}_q)^{-\theta_p} \widehat{\widetilde{p}}_t = (1-\xi_p)(p_q^*)^{-\theta_p} \widehat{p}_t^* + \xi_p \left(\frac{\pi_{q-1}^{\chi_p} \pi^{1-\chi_p}}{\pi_q} \right)^{-\theta_p} (\chi_p \widehat{\pi}_{t-1} - \widehat{\pi}_t) \tag{B.3.8}$$

$$\begin{aligned} \widetilde{\lambda}_q \widehat{\widetilde{\lambda}}_t = & \frac{\gamma b \tau_q}{(\gamma \widetilde{C}_q - b \widetilde{C}_{q-1})^2} \widetilde{C}_{q-1} \widehat{\widetilde{C}}_{t-1} - \left[\frac{\gamma^2 \tau_q}{(\gamma \widetilde{C}_q - b \widetilde{C}_{q-1})^2} + \frac{\beta b^2 \tau_{q+1}}{(\gamma \widetilde{C}_{q+1} - b \widetilde{C}_q)^2} \right] \widetilde{C}_q \widehat{\widetilde{C}}_t \\ & + \frac{\gamma \beta b \tau_{q+1}}{(\gamma \widetilde{C}_{q+1} - b \widetilde{C}_q)^2} \widetilde{C}_{q+1} \mathrm{E}_t \widehat{\widetilde{C}}_{t+1} \\ & + \frac{\gamma \tau_q}{\gamma \widetilde{C}_q - b \widetilde{C}_{q-1}} \widehat{\tau}_t - \frac{\beta b \tau_{q+1}}{\gamma \widetilde{C}_{q+1} - b \widetilde{C}_q} \mathrm{E}_t \widehat{\tau}_{t+1} \end{aligned} \tag{B.3.9}$$

$$0 = \mathrm{E}_t \widehat{\widetilde{\lambda}}_{t+1} + \widehat{R}_t - \widehat{\widetilde{\lambda}}_t - \mathrm{E}_t \widehat{\pi}_{t+1} \tag{B.3.10}$$

$$\begin{aligned} \widetilde{\lambda}_q \widehat{\widetilde{\lambda}}_t = & \widetilde{\psi}_q \mu_q \kappa \left(\frac{\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}} \right) \left(\frac{3\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}} - 2\gamma \right) \widehat{\widetilde{I}}_{t-1} \\ & + \widetilde{\psi}_q \mu_q \left[1 - \frac{\kappa}{2} \left(\frac{\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}} - \gamma \right)^2 - \kappa \left(\frac{\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}} - \gamma \right) \left(\frac{\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}} \right) \right] (\widehat{\widetilde{\psi}}_t + \widehat{\mu}_t) \\ & - \left[\widetilde{\psi}_q \mu_q \kappa \left(\frac{\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}} \right) \left(\frac{3\gamma \widetilde{I}_q}{\widetilde{I}_{q-1}} - 2\gamma \right) \right. \\ & \left. + \frac{\beta}{\gamma} \widetilde{\psi}_{q+1} \mu_{q+1} \kappa \left(\frac{\gamma \widetilde{I}_{q+1}}{\widetilde{I}_q} \right)^2 \left(\frac{3\gamma \widetilde{I}_{q+1}}{\widetilde{I}_q} - 2\gamma \right) \right] \widehat{\widetilde{I}}_t \end{aligned}$$

$$
\begin{aligned}
&+\frac{\beta}{\gamma}\widetilde{\psi}_{q+1}\mu_{q+1}\kappa\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}\right)^2\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}-\gamma\right)\mathrm{E}_t\widehat{\widetilde{\psi}}_{t+1} \\
&+\frac{\beta}{\gamma}\widetilde{\psi}_{q+1}\widetilde{\mu}_{q+1}\kappa\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}\right)^2\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}-\gamma\right)\mathrm{E}_t\widehat{\mu}_{t+1} \\
&+\frac{\beta}{\gamma}\widetilde{\psi}_{q+1}\mu_{q+1}\kappa\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}\right)^2\left(\frac{3\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}-2\gamma\right)\mathrm{E}_t\widehat{\widetilde{I}}_{t+1}
\end{aligned}
\tag{B.3.11}
$$

$$
\begin{aligned}
\gamma\widetilde{\psi}_q\widehat{\widetilde{\psi}}_t =&\beta\widetilde{\lambda}_{q+1}r^k_{q+1}u_{q+1}\mathrm{E}_t\widehat{\widetilde{\lambda}}_{t+1} \\
&+\beta\widetilde{\lambda}_{q+1}r^k_{q+1}u_{q+1}\mathrm{E}_t\widehat{\widetilde{r}}^k_{t+1} \\
&+\beta\widetilde{\psi}_{q+1}\left[1-\delta_0-\delta_1(u_{q+1}-1)-\frac{\delta_2}{2}(u_{q+1}-1)^2\right]\mathrm{E}_t\widehat{\widetilde{\psi}}_{t+1}
\end{aligned}
\tag{B.3.12}
$$

$$
0=-\widetilde{\lambda}_q r^k_q(\widehat{\widetilde{\lambda}}_t+\widehat{r}^k_t)+\widetilde{\psi}_q[(\delta_1+\delta_2(u_q-1))\widehat{\widetilde{\psi}}_t+\delta_2 u_q\widehat{u}_t] \tag{B.3.13}
$$

$$
\widehat{\widetilde{K}}_t=\widehat{u}_t+\widehat{\widetilde{K}}^p_{t-1} \tag{B.3.14}
$$

$$
\begin{aligned}
\gamma\widetilde{K}^p_q\widehat{\widetilde{K}}^p_t =&\left[1-\delta_0-\delta_1(u_q-1)-\frac{\delta_2}{2}(u_q-1)^2\right]\widetilde{K}^p_{q-1}\widehat{\widetilde{K}}^p_{t-1} \\
&+\kappa\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}-\gamma\right)\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\mu_q\widetilde{I}_q\widehat{\widetilde{I}}_{t-1} \\
&-(\delta_1+\delta_2(u_q-1))u_q\widetilde{K}^p_{q-1}\widehat{u}_t+\left(1-\frac{\kappa}{2}\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}-\gamma\right)^2\right)\mu_q\widetilde{I}_q\widehat{\mu}_t \\
&+\kappa\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}-\gamma\right)\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\mu_q\widetilde{I}_q\widehat{\widetilde{I}}_{t-1}
\end{aligned}
\tag{B.3.15}
$$

$$
\widehat{f}^1_t=\widehat{f}^2_t \tag{B.3.16}
$$

$$
\begin{aligned}
f^1_q\widehat{f}^1_t =&\left[(\widetilde{w}^*_q)^{1-\theta_w}\widetilde{\lambda}_q H_q\widetilde{w}_q+\xi_w\beta\left(\frac{\pi_q^{\chi_w}\pi^{1-\chi_w}\widetilde{w}^*_q}{\widetilde{\pi}^w_{q+1}\widetilde{w}^*_{q+1}}\right)^{1-\theta_w}f^1_{q+1}\right](1-\theta_w)\widehat{\widetilde{w}}^*_t \\
&+(\widetilde{w}^*_q)^{1-\theta_w}\widetilde{\lambda}_q H_q\widetilde{w}_q(\widehat{\widetilde{\lambda}}_t+\widehat{H}_t+\widehat{\widetilde{w}}_t) \\
&+\xi_w\beta\left(\frac{\pi_q^{\chi_w}\pi^{1-\chi_w}\widetilde{w}^*_q}{\widetilde{\pi}^w_{q+1}\widetilde{w}^*_{q+1}}\right)^{1-\theta_w}f^1_{q+1} \\
&\times[(1-\theta_w)(\chi_w\widehat{\pi}_t-\mathrm{E}_t\widehat{\widetilde{\pi}}^w_{t+1}-\mathrm{E}_t\widehat{\widetilde{w}}^*_{t+1})+\mathrm{E}_t\widehat{f}^1_{t+1}]
\end{aligned}
\tag{B.3.17}
$$

$$f_q^2 \widehat{f}_t^2 = - \left[\frac{\theta_w}{\theta_w - 1} (\widetilde{w}_q^*)^{-\theta_w(1+\eta)} \varphi H_q^{1+\eta} + \xi_w \beta \left(\frac{\pi_q^{\chi_w} \pi^{1-\chi_w} \widetilde{w}_q^*}{\widetilde{\pi}_{q+1}^w \widetilde{w}_{q+1}^*} \right)^{-\theta_w(1+\eta)} f_{q+1}^2 \right]$$
$$\times \theta_w (1+\eta) \widehat{\widetilde{w}}_t^*$$
$$+ \frac{\theta_w}{\theta_w - 1} (\widetilde{w}_q^*)^{-\theta_w(1+\eta)} \varphi H_q^{1+\eta} (1+\eta) \widehat{H}_t$$
$$+ \xi_w \beta \left(\frac{\pi_q^{\chi_w} \pi^{1-\chi_w} \widetilde{w}_q^*}{\widetilde{\pi}_{q+1}^w \widetilde{w}_{q+1}^*} \right)^{-\theta_w(1+\eta)} f_{q+1}^2$$
$$\times [-\theta_w (1+\eta)(\chi_w \widehat{\pi}_t - \mathrm{E}_t \widehat{\widetilde{\pi}}_{t+1}^w - \mathrm{E}_t \widehat{\widetilde{w}}_{t+1}^*) + \mathrm{E}_t \widehat{f}_{t+1}^2] \quad \text{(B.3.18)}$$
$$0 = (1 - \xi_w)(\widetilde{w}_q^*)^{1-\theta_w} \widehat{\widetilde{w}}_t^* + \xi_w \left(\frac{\pi_{q-1}^{\chi_w} \pi^{1-\chi_w}}{\widetilde{\pi}_q^w} \right)^{1-\theta_w} (\chi_w \widehat{\pi}_{t-1} - \widehat{\widetilde{\pi}}_t^w) \quad \text{(B.3.19)}$$
$$\widehat{\pi}_t^w = \widehat{\pi}_t + \widehat{\widetilde{w}}_t - \widehat{\widetilde{w}}_{t-1} \quad \text{(B.3.20)}$$
$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1 - \rho_R)[\phi_\pi (\widehat{\pi}_t - \widehat{\pi}_t^*) + \phi_Y (\widehat{\widetilde{Y}}_t - \widehat{\widetilde{Y}}_{t-1})] + \epsilon_{R,t} \quad \text{(B.3.21)}$$
$$(1-g)\widetilde{Y}_q \widehat{\widetilde{Y}}_t = \widetilde{C} \widehat{\widetilde{C}}_t + \widetilde{I} \widehat{\widetilde{I}}_t + g \widetilde{Y}_q \widehat{g}_t \quad \text{(B.3.22)}$$
$$\widehat{z}_t = \rho_z \widehat{z}_{t-1} + \epsilon_{z,t} \quad \text{(B.3.23)}$$
$$\widehat{\tau}_t = \rho_\tau \widehat{\tau}_{t-1} + \epsilon_{\tau,t} \quad \text{(B.3.24)}$$
$$\widehat{\mu}_t = \rho_\mu \widehat{\mu}_{t-1} + \epsilon_{\mu,t} \quad \text{(B.3.25)}$$
$$\widehat{\pi}_t^\star = \rho_\pi \widehat{\pi}_{t-1}^\star + \epsilon_{\pi,t} \quad \text{(B.3.26)}$$
$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \epsilon_{g,t} \quad \text{(B.3.27)}$$

2.9.3 Additional Tables and Figures

Table 2.9 compares the business cycle statistics in the data and in the baseline DSGE model (Section 3 in the main text). The model tends to underpredict the volatility of both investment and wage growth and overstate the volatility of hours growth. The model also tends to underpredict the correlation of consumption growth with respect to output growth and overpredict the correlation of both investment and wage growth with respect to output growth. Nevertheless, overall the model is successful in replicating salient features of the U.S. aggregate data.

Table 2.11 reports posterior estimates of the main experiment using Tramo-Seats-filtered and DSGE-based-filtered data (Section 4 in the main text). The Tramo-Seats filter and the DSGE-based filter deliver similar biases compared to the X-12-Arima filter. (See, for example, the biases in some of the key structural parameters, α, η, and κ.) As in Section 6 in the main text, I consider the quadratic loss function by Ferroni (2011) that measures overall distortions. For the four estimation experiments considered,

1. Seasonally unadjusted data: $QL = 0.0039$
2. X-12-Arima-filtered data: $QL = 0.0669$
3. Tramo-Seats-filtered data: $QL = 0.0455$
4. DSGE-based-filtered data: $QL = 0.0418$

The DSGE-based-filtered data deliver considerable biases, although the magnitude is slightly smaller than that of the Tramo-Seats filter. I also note that all other results in Section 4 (impulse responses, business cycle statistics, and policy analysis) are robust to the choice of a seasonal adjustment filter. Figure 2.7 plots the log-likelihood profiles for the model parameters given seasonally adjusted and unadjusted data.

Tables 2.12, 2.13, and 2.14 report some model statistics of the alternative models considered in Section 5 in the main text. I briefly comment on each alternative model.

1. No investment adjustment cost model: The calibrated seasonal shifts in investment technology are almost constant across seasons (Table 2.12).[28] Also, the model significantly underpredicts the volatilities of real wage growth, the inflation rate, and the interest rate (Table 2.14).

2. No wage rigidity model: The model overpredicts seasonality and volatility in real wage growth (Tables 2.13 and 2.14).

3. Capital adjustment cost model: The model does a good job of replicating data moments, although it slightly underpredicts the volatilities of investment and wage growth (Table 2.14).

4. Labor adjustment cost model: The model significantly overpredicts seasonality and volatility in real wage growth (Tables 2.13 and 2.14). Also, it underpredicts the correlation of seasonally adjusted hours and wage growth with respect to output growth (Table 2.14).

[28]Some readers may think that the distortions in the estimated parameters in the baseline model are driven by the seasonal shifts in investment technology. To address this issue, I recomputed the probability limits for the baseline model but this time fixed the steady-state investment technology level constant across seasons. The result remained basically unchanged.

Table 2.9: Business cycle statistics

	Percent standard deviation		Corr. with output growth	
Series	Data	Model	Data	Model
Output growth	0.95	0.96	–	–
Consumption growth	0.59	0.57	0.63	0.38
Investment growth	3.21	2.77	0.66	0.89
Hours growth	0.86	0.99	0.63	0.54
Wage growth	0.55	0.41	0.56	0.77
Inflation rate	0.75	0.73	-0.48	-0.22
Interest rate	0.76	0.69	-0.34	-0.18

Notes: Moments are calculated by applying the X-12-Arima filter to the simulated data from the seasonal model, where the parameters are fixed at their true values. All simulations are based on 100 replications of artificial time-series of length 200.

Table 2.10: Posterior estimates

Parameter	Description	True	Unadjusted	T-S	DSGE-based
α	Capital share	0.3	0.29	0.58	0.53
			(0.0120)	(0.0325)	(0.0361)
b	Habit persistence	0.7	0.73	0.66	0.67
			(0.0134)	(0.0255)	(0.0270)
η	Inverse Frisch elasticity	2	2.01	0.96	1.06
			(0.1287)	(0.1443)	(0.1569)
κ	Investment adjustment cost	1	1.03	1.44	1.55
			(0.0760)	(0.1681)	(0.1936)
ξ_p	Calvo price	0.6	0.60	0.56	0.56
			(0.0024)	(0.0073)	(0.0075)
ξ_w	Calvo wage	0.6	0.59	0.56	0.52
			(0.0061)	(0.0187)	(0.0230)
χ_p	Price indexation	0.3	0.30	0.34	0.31
			(0.0080)	(0.0322)	(0.0303)
χ_w	Wage indexation	0.3	0.30	0.33	0.35
			(0.0112)	(0.0309)	(0.0309)
ρ_R	Taylor rule smoothing	0.7	0.71	0.71	0.71
			(0.0178)	(0.0254)	(0.0230)
ϕ_π	Taylor rule inflation	1.7	1.69	1.97	1.85
			(0.1398)	(0.2413)	(0.2012)
ϕ_Y	Taylor rule output	0.2	0.22	0.29	0.26
			(0.0499)	(0.0732)	(0.0664)
ρ_z	Neutral technology	0.95	0.95	0.95	0.95
			(0.0020)	(0.0054)	(0.0056)
ρ_τ	Preference	0.95	0.94	0.97	0.97
			(0.0091)	(0.0050)	(0.0068)
ρ_μ	Investment technology	0.95	0.96	0.88	0.90
			(0.0103)	(0.0207)	(0.0210)
ρ_π	Inflation target	0.95	0.96	0.95	0.95
			(0.0066)	(0.0099)	(0.0103)
ρ_g	Government spending	0.95	0.95	0.27	0.52
			(0.0109)	(0.0903)	(0.1152)

(Table continues on the next page.)

Table 2.11: Posterior estimates (continued)

Parameter	Description	True	Unadjusted	T-S	DSGE-based
$100\sigma_z$	Neutral technology	0.9	0.90	0.83	0.84
			(0.0452)	(0.0416)	(0.0422)
$100\sigma_\tau$	Preference	1.7	1.72	1.47	1.54
			(0.1010)	(0.1232)	(0.1356)
$100\sigma_\mu$	Investment technology	1.4	1.68	1.11	1.18
			(0.2194)	(0.1144)	(0.1348)
$100\sigma_\pi$	Inflation target	0.1	0.09	0.11	0.10
			(0.0120)	(0.0142)	(0.0142)
$100\sigma_R$	Monetary policy	0.1	0.10	0.12	0.11
			(0.0054)	(0.0065)	(0.0064)
$100\sigma_g$	Government spending	1	0.90	1.30	1.46
			(0.0472)	(0.0954)	(0.1464)
$\tilde{z}_1$	Neutral technology Q1	0.97	0.97	–	–
			(0.0005)		
$\tilde{z}_2$	Neutral technology Q2	0.97	0.97	–	–
			(0.0003)		
$\tilde{z}_3$	Neutral technology Q3	0.97	0.97	–	–
			(0.0001)		
$\tilde{\tau}_1$	Preference Q1	0.77	0.74	–	–
			(0.0126)		
$\tilde{\tau}_2$	Preference Q2	0.95	0.94	–	–
			(0.0023)		
$\tilde{\tau}_3$	Preference Q3	0.92	0.92	–	–
			(0.0037)		
$\tilde{\mu}_1$	Investment technology Q1	0.81	0.80	–	–
			(0.0121)		
$\tilde{\mu}_2$	Investment technology Q2	0.98	0.97	–	–
			(0.0034)		
$\tilde{\mu}_3$	Investment technology Q3	0.91	0.91	–	–
			(0.0069)		

Notes: The table reports the MCMC estimates of posterior means. Standard deviations are reported in parentheses. The following reparameterizations are used: $\tilde{z}_q = z_q/z_4$, $\tilde{\tau}_q = \tau_q/\tau_4$, and $\tilde{\mu}_q = \mu_q/\mu_4$ for $q = 1, 2, 3$.

Table 2.12: Parameters that vary across quarters: alternative models

Parameter	Description	Q1	Q2	Q3	Q4	Average
Panel A: No investment adjustment cost						
z	Neutral technology	0.99	1.00	0.99	1.02	1.00
τ	Preference	0.85	1.04	1.01	1.10	1.00
μ	Investment technology	1.00	1.00	1.00	1.00	1.00
Panel B: No wage rigidity						
z	Neutral technology	0.99	1.00	0.99	1.02	1.00
τ	Preference	0.84	1.04	1.01	1.10	1.00
μ	Investment technology	0.87	1.06	0.99	1.08	1.00
Panel C: Capital adjustment cost						
z	Neutral technology	0.99	1.00	0.99	1.02	1.00
τ	Preference	0.85	1.04	1.01	1.10	1.00
μ	Investment technology	0.94	1.00	1.01	1.04	1.00
Panel D: Labor adjustment cost						
z	Neutral technology	1.00	1.00	0.99	1.02	1.00
τ	Preference	0.84	1.04	1.02	1.10	1.00
μ	Investment technology	0.87	1.05	0.99	1.09	1.00

Table 2.13: Seasonal patterns: alternative models

Series	Q1	Q2	Q3	Q4
Panel A: No investment adjustment cost				
Output*	-6.32	3.18	0.59	2.55
Consumption	-5.46	2.11	0.51	2.85
Investment*	-8.20	5.55	0.76	1.89
Hours*	-3.92	3.05	1.60	-0.73
Wage growth	-0.51	0.16	0.05	0.30
Inflation rate	0.39	0.02	-0.05	-0.35
Interest rate	0.00	0.00	0.00	0.00
Panel B: No wage rigidity				
Output*	-6.37	3.08	0.61	2.69
Consumption	-5.58	2.07	0.58	2.94
Investment*	-8.12	5.30	0.68	2.14
Hours*	-3.87	2.96	1.55	-0.64
Wage growth	-4.11	7.50	-1.13	-2.26
Inflation rate	-0.02	0.66	0.16	-0.80
Interest rate	0.00	0.00	0.00	0.00
Panel C: Capital adjustment cost				
Output*	-6.38	3.08	0.61	2.69
Consumption	-5.59	2.07	0.58	2.94
Investment*	-8.12	5.31	0.68	2.14
Hours*	-3.87	2.95	1.55	-0.64
Wage growth	-0.53	0.17	0.06	0.30
Inflation rate	0.40	0.01	-0.06	-0.35
Interest rate	0.00	0.00	0.00	0.00
Panel D: Labor adjustment cost				
Output*	-6.37	3.08	0.61	2.69
Consumption	-5.59	2.07	0.58	2.94
Investment*	-8.11	5.30	0.68	2.14
Hours*	-3.87	2.96	1.55	-0.64
Wage growth	-13.95	26.45	-9.59	-2.92
Inflation rate	-0.95	2.04	0.30	-1.39
Interest rate	0.00	0.00	0.00	0.00

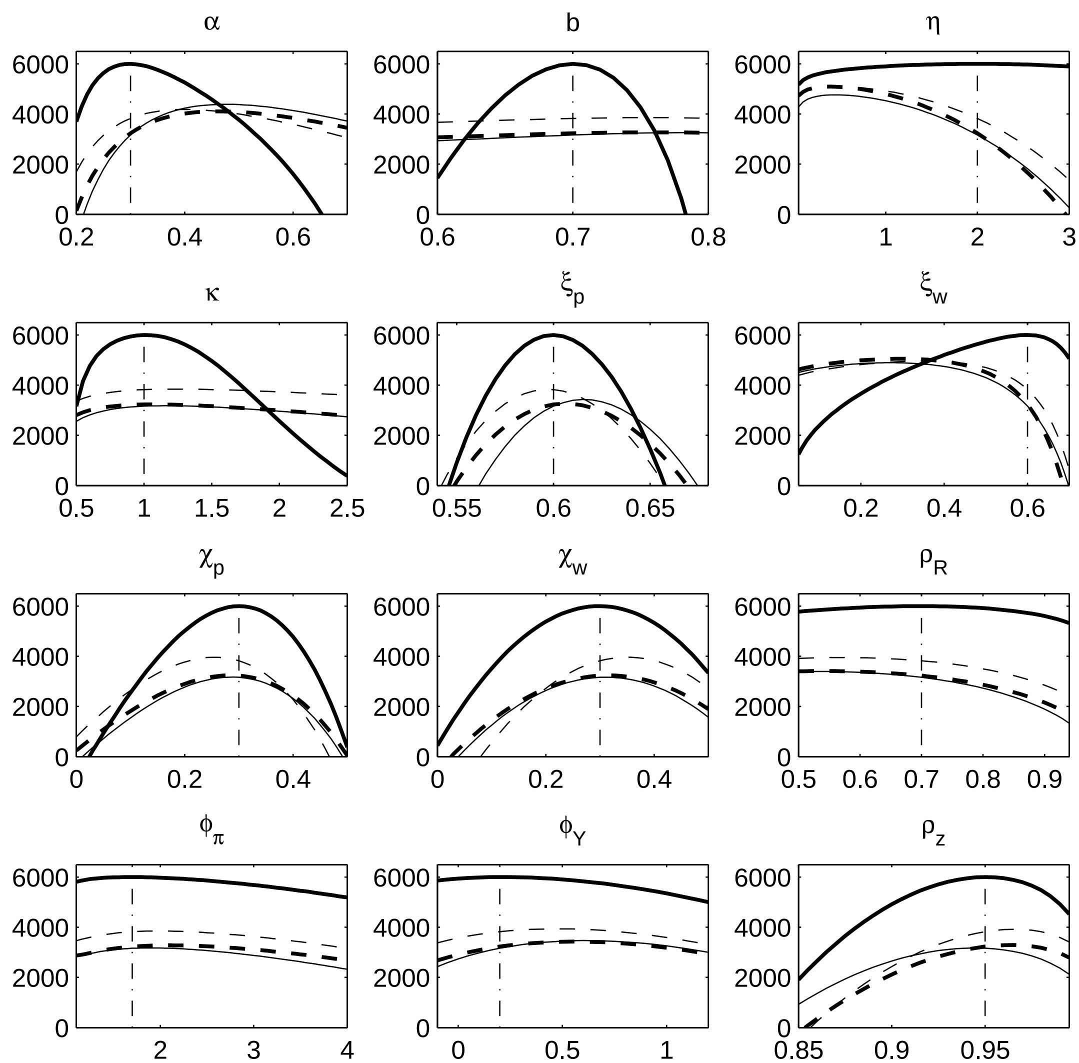
Notes: The table reports percent changes of variables from the previous quarter, taken from seasonal steady states in the model. Variables with * indicate those used as calibration targets.

Table 2.14: Business cycle statistics: alternative models

Series	Percent standard deviation	Corr. with output growth
Panel A: No investment adjustment cost		
Output growth	0.91	–
Consumption growth	0.55	0.23
Investment growth	2.90	0.89
Hours growth	0.86	0.92
Wage growth	0.17	0.58
Inflation rate	0.13	-0.18
Interest rate	0.18	0.18
Panel B: No wage rigidity		
Output growth	0.98	–
Consumption growth	0.58	0.46
Investment growth	2.71	0.89
Hours growth	0.97	0.09
Wage growth	1.05	0.60
Inflation rate	0.73	-0.42
Interest rate	0.64	-0.32
Panel C: Capital adjustment cost		
Output growth	0.94	–
Consumption growth	0.58	0.73
Investment growth	2.13	0.90
Hours growth	0.74	0.42
Wage growth	0.43	0.79
Inflation rate	0.73	-0.39
Interest rate	0.67	-0.20
Panel D: Labor adjustment cost		
Output growth	0.96	–
Consumption growth	0.57	0.28
Investment growth	2.92	0.88
Hours growth	0.83	0.15
Wage growth	2.06	0.24
Inflation rate	0.72	-0.38
Interest rate	0.70	-0.26

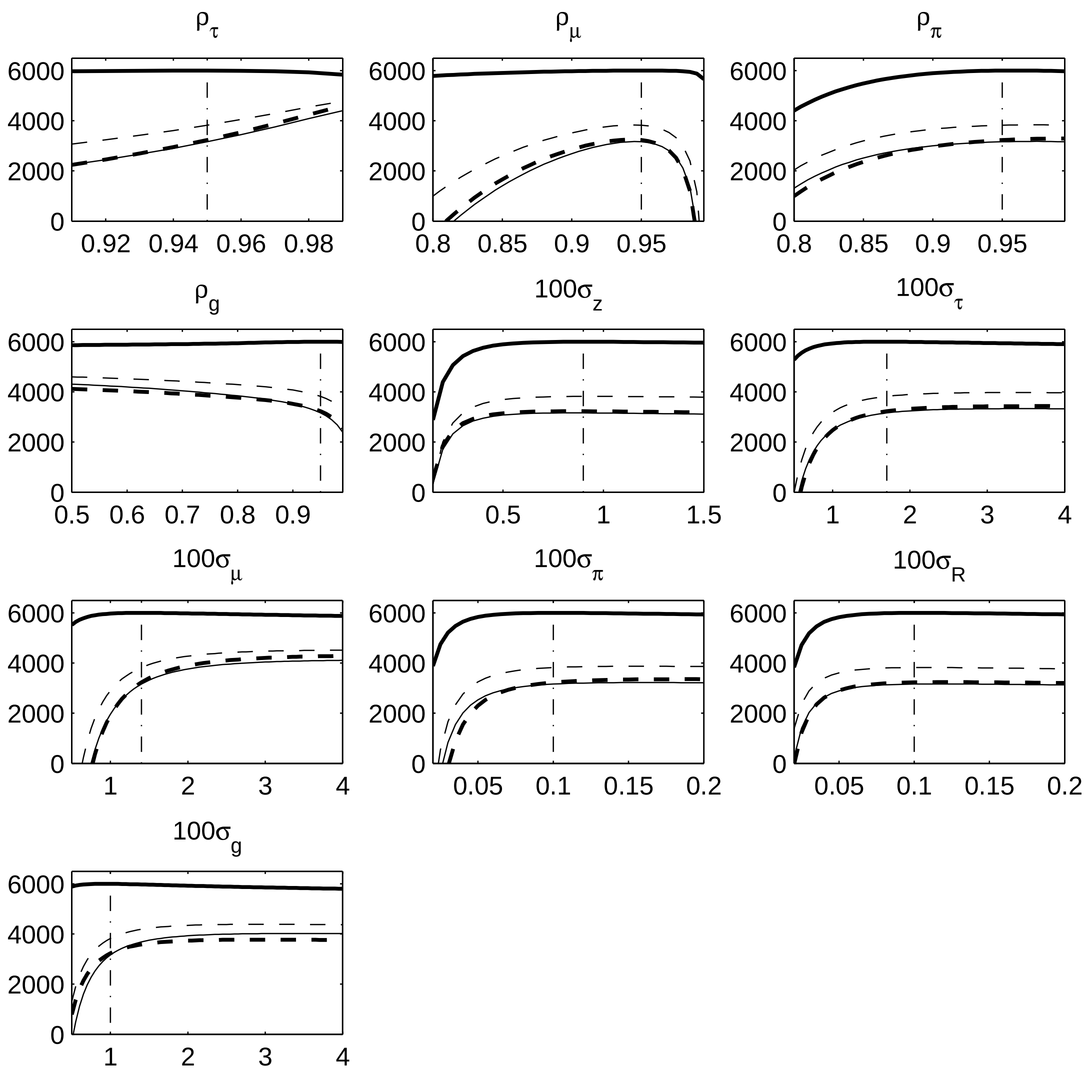
Notes: Moments are calculated by applying the X-12-Arima filter to the simulated data from the seasonal model, where the parameters are fixed at their true values. All simulations are based on 100 replications of artificial time-series of length 200.

Figure 2.7: Likelihood profiles: seasonally adjusted vs. unadjusted data



(Figure continues on the next page.)

Figure 2.8: Likelihood profiles: seasonally adjusted vs. unadjusted data (continued)



Notes: The figure plots likelihood profiles for seasonally unadjusted data (thick solid lines), X-12-Arima-filtered data (thick dashed lines), Tramo-Seats-filtered data (solid lines), and DSGE-based-filtered data (dashed lines). Vertical lines signify true values.

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