SPECIFICATION OF THE JOY OF GIVING: INSIGHTS FROM ALTRUISM

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Abstract—This paper analyzes the joy of giving bequest motive in which the utility obtained from leaving a bequest depends only on the size of the bequest. It exploits the fact that this formulation can be interpreted as a reduced form of an altruistic bequest motive to derive a relation between the value of the altruism parameter and the value of the joy of giving parameter. Using previous discussions of an a priori range of plausible values for the altruism parameter we then derive plausible restrictions on the joy of giving parameter. We demonstrate that this parameter may well be orders of magnitude larger than assumed in the existing literature.

Bequest motives by individual consumers have important implications for the behavior of financial markets, the macroeconomic impacts of fiscal policies and the intergenerational transmission of inequality in the distribution of wealth. At least four reasons for the existence of bequests have been discussed in the literature: (1) bequests may be the unintentional by-product of precautionary savings and a stochastic date of death in the absence of an annuity market (Abel (1985)); (2) the prospect of bequests is used by parents to induce children to behave as desired by the parents (Bernheim, Shleifer, and Summers (1985)); (3) bequests may arise from intergenerational altruism, that is, consumers obtain utility from their heirs' utility as well as from their own consumption (Barro (1974)); and (4) bequests may arise from the "joy of giving," that is, consumers leave bequests simply because they obtain utility directly from the bequest (Yaari (1964)).

For some theoretical and empirical analyses of the issues affected by voluntary intergenerational transfers, the reason for the bequest motive is critical. For example, the validity of the Ricardian Equivalence Theorem and the implied inefficacy of fiscal policy depends crucially on an altruistic motive rather than a joy of giving motive. For many other purposes, however, the reason for the bequest motive is not crucial. Many economists have used the joy of giving model, either in the belief that it captures the true reason for bequests, or more likely, because it is a tractable "reduced form" representation of altruistic preferences. This model has been used by Yaari (1965), Hakansson (1969), Fischer (1973), and Richard (1975) to examine the joint demand for life insurance and risky assets; Blinder (1974) included a joy of giving bequest motive among the mechanisms creating inequality in the distribution of income and wealth; Seidman (1983) analyzed consumption, inheritance, wage and capital income taxes in a life cycle growth model extended to include joy of giving bequests; and Hubbard (1984), Friedman and Warshawsky (1985) and Abel (1986) discussed the implications of imperfections in private and public annuity markets for savings behavior and capital accumulation in a joy of giving framework.

In most applications of altruism and joy of giving, the bequest motive is parameterized by a small number of parameters. Economic theory provides substantial guidance on the admissible, or at least plausible, values of the parameters in the simple formulations of the altruism model and these implications have been discussed by Drazen (1978) and Weil (1987). However, there has evidently been no systematic discussion of the range of appropriate parameter values for simple formulations of the joy of giving model, despite the popularity of this formulation in simulation work. Indeed, in discussing the appropriate value of the joy of giving parameter, Blinder (1974) states that "there is little intuition that can be brought to bear here" (p. 95).

This paper explores the implications of economic theory for the appropriate range of parameter values for a popular specification of the joy of giving motive. Our strategy is to assume that the bequest is actually motivated by altruism and then to express the parameter of a joy of giving bequest motive in terms of the altruism parameter. A striking result of this analysis is that the joy of giving parameter could be orders of magnitude larger than the values that appear in the simulation literature (Fischer (1973), Blinder (1974), Seidman (1983), Hubbard (1984)). A related finding is that the apparently large joy of giving parameters found by Friedman and Warshawsky (1985) correspond to a quite modest degree of altruism.

I. A Model of Individual Behavior

Consider a family in which each consumer lives for L periods and in which N periods elapse between the birth of successive generations. Suppose that each con-

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sumer has one child and that bequests from parent to child are made at the beginning of the child's life. Let I^j be the inheritance received by a generation j consumer at the beginning of his life, let Y^j be the present value of labor income of the generation j consumer and let $c_i^j, i = 1, \ldots, L$ be the consumption of a generation j consumer when he is age i. Letting R be the (gross) rate of return on wealth, the lifetime budget constraint is

$$Y^j + I^j = \sum_{i=1}^{L} R^{-(i-1)} c_i^j + R^{-N} I^{j+1}. \tag{1}$$

It will be convenient to define H^j as the present value of the human wealth of the generation j consumer and all of his descendents

$$H^j = \sum_{k=0}^{\infty} (R^{-N})^k Y^{j+k}. \tag{2}$$

Next, let W^j denote the total wealth, human plus non-human, as of the beginning of the generation j consumer's life,

$$W^j = I^j + H^j. \tag{3}$$

Finally, let B^j denote the bequest left by a generation j consumer and observe that $I^{j+1} = B^j$. Therefore, equation (3) implies that

$$W^{j+1} = B^j + H^{j+1}. \tag{4}$$

Suppose that the utility function is time-separable and displays altruism. Let V^j denote the utility of the generation j consumer and suppose that

$$V^j = \max\left\{ \sum_{i=1}^{L} \beta^{i-1} u(c_i^j) + \beta^N \alpha V^{j+1} \right\} \tag{5}$$

where $u' > 0$, $u'' < 0$, β captures time preference ($0 < \beta < 1$) and $\alpha > 0$ indicates the strength of the bequest motive. The maximization in (5) is subject to (1) and to the solvency condition $\lim_{j \to 0} R^{-Nj} W^j \geq 0$.

In order for the maximand in (5) to be finite, the weight on the heir's utility, $\beta^N \alpha$, must lie between 0 and 1. This restriction does *not* require α to be less than or equal to 1. To help interpret the value of α, we will define the term "full altruism" to mean that in every period in which both the generation j consumer and the generation $j+1$ consumer are alive, the optimal allocation of family consumption is for the parent and child to have equal consumption ($c_{N+i}^j = c_i^{j+1}$, $i = 1, \ldots, L - N$).[1] Under the utility function in (5), full altruism corresponds to $\alpha = 1$.[2,3]

If all generations in an infinitely-lived altruistic family have the same utility function, then the utility of the generation j consumer is a function of the total wealth at birth $V^j = V(W^j)$. Hence equation (5) may be written as

$$V(W^j) = \max\left\{ \sum_{i=1}^{L} \beta^{i-1} u(c_i^j) + \beta^N \alpha V(W^{j+1}) \right\}. \tag{6}$$

Recalling that $W^{j+1} = B^j + H^{j+1}$, equation (6) has the appearance of a "joy of giving" bequest motive. Strictly speaking, it is not a joy of giving bequest motive because the function $V(\ )$ cannot be specified independently; it is the solution to a functional equation. Below we solve this functional equation and express the parameter of the joy of giving specification in terms of the altruism parameter α.[4]

We begin by characterizing the solution to the maximization problem on the right-hand side of (6). The first-order conditions are

$$u'(c_1^j) = (R\beta)^{i-1} u'(c_i^j), \qquad i = 2, \ldots, L \tag{7a}$$

$$u'(c_1^j) = (R\beta)^N \alpha V'(W^{j+1}) = (R\beta)^N \alpha u'(c_1^{j+1}) \tag{7b}$$

where the second equality in (7b) follows from the envelope theorem. A steady state is characterized by $c_i^j = c_i^{j+1}$, $i = 1, \ldots, L$ and $W^j = W^{j+1}$. It follows from (7b) that $\alpha(R\beta)^N = 1$ in the steady state.

II. The Implied Weight of the Joy of Giving Bequest Motive

In this section we present the function $V^j = V(W^j)$ under the assumption that $u(c)$ has the isoelastic form

$$u(c) = \frac{1}{1-\sigma}[c^{1-\sigma}]; \ \sigma > 0. \tag{8}$$

It can be verified that under isoelastic utility, the solution to the functional equation in (6) is[5]

$$V(W) = \phi \cdot \frac{1}{1-\sigma} W^{1-\sigma} \tag{9a}$$

where

$$\phi = \left\{ \Gamma / \left[1 - R^{-N} (\alpha \beta^N R^N)^{1/\sigma} \right] \right\}^{\sigma} \tag{9b}$$

and

$$\Gamma \equiv \sum_{i=1}^{L} \left[R^{(1/\sigma)-1} \beta^{1/\sigma} \right]^{i-1}. \tag{9c}$$

[1] Meade (1968) defined a similar concept called "perfect altruism."

[2] For more general specifications of the utility from one's own consumption, there may not exist any value of α for which the utility function displays full altruism.

[3] To verify that full altruism corresponds to $\alpha = 1$, observe that for $i = 1, \ldots, L - N$, $u'(c_{N+i}^j) = (R\beta)^{-(N+i-1)} u'(c_1^j) = (R\beta)^{-(i-1)} \alpha u'(c_1^{j+1}) = \alpha u'(c_i^{j+1})$ where the first and third equalities follow from (7a) and the second equality follows from (7b) below. Therefore, $c_{N+1}^j = c_i^{j+1}$ if and only if $\alpha = 1$.

[4] Blinder (1974, pp. 37–39) also calculates the value of the joy of giving parameter implied by altruism but this calculation is restricted to the case of full altruism ($\alpha = 1$).

[5] See Abel and Warshawsky (1987) for details.

TABLE 1.—IMPLIED WEIGHTS ON JOY OF GIVING FUNCTION AND ASSUMED DEGREE OF ALTRUISM

			λ			
$\beta^{-1}-1$	R	α	(σ = 0.5)	(σ = 1)	(σ = 2)	(σ = 4)
0.04	1.06	0.56	1.14	4.96	100.99	43,076
0.04	1.04	1.00	1.80	10.49	356.76	412,807
0.02	1.06	0.32	2.01	7.47	142.29	58,940
0.02	1.04	0.56	2.86	15.80	524.71	600,161
0.02	1.02	1.00	4.91	43.70	3,459.06	21,673,136
0.01	1.06	0.23	2.91	9.57	172.13	69,611
0.01	1.04	0.42	3.93	20.24	649.94	732,042
0.01	1.02	0.74	6.38	55.96	4,398.76	27,466,003
0.01	1.01	1.00	9.84	130.53	22,964.63	710,820,614

Source: Calculations based on equation (11) with N = 30, L = 60.

Using equations (4) and (9a, b, c) we rewrite the utility function in (6) as

$$V(W^j) = \left\{ \sum_{i=1}^{L} \beta^{i-1}(c_i^j)^{1-\sigma} + \lambda(B^j + H^{j+1})^{1-\sigma} \right\} \Big/ (1-\sigma) \tag{10a}$$

where

$$\lambda = R^{-N}\left\{\Gamma \Big/ \left[\left(\alpha\beta^N R^N\right)^{-1/\sigma} - R^{-N}\right]\right\}^{\sigma}. \tag{10b}$$

Equation (10a) expresses the utility of the generation j consumer as a function of his own consumption c_i^j, $i = 1, \ldots, L$ and the bequest he makes, B^j. This equation is equivalent to a joy of giving formulation. Treating the exogenous human wealth term H^{j+1} as a parameter, the joy of giving function is a member of the HARA class of utility functions. In the absence of human wealth ($H^j \equiv 0$), this function has the frequently-used isoelastic form.

We have defined λ so that, in the absence of human capital, it is comparable to the bequest weight $\hat{b}_t$ in Fischer (1973). In the steady state, $\alpha(R\beta)^N = 1$, so that (10b) implies

$$\lambda = R^{-N}\{\Gamma/[1 - R^{-N}]\}^{\sigma} \quad \text{in the steady state.} \tag{11}$$

Table 1 presents the values of λ and α corresponding to various rates of time preference and steady state interest rates. The last four columns of each row reveal that λ is an increasing function of the coefficient of relative risk aversion σ. Even when σ is as low as 2, the value of λ can be orders of magnitude larger than the values assumed by previous authors. For example, in four sets of his simulations, Fischer (1973) used a rate of time preference of 0.04 (actually $\beta = 0.96$), a net interest rate of 0.06, and a coefficient of relative risk aversion of 2.0. Although he used a time-varying weight on the bequest motive, this weight was roughly equal to 1 (it was between 0.42 and 1.20).[6] The first row of table 1 indicates that for $\sigma = 0.5$ a value of λ around 1 is consistent with $\alpha = 0.56$ but for $\sigma = 2$, a value of λ around 100 is required to be consistent with $\alpha = 0.56$ in the steady state.

III. Estimates of Altruism

Table 1 shows the implied joy of giving parameter consistent with a given degree of altruism. We can also address the inverse question: given a time preference discount factor β, a gross rate of return R and a joy of giving parameter λ, what is the implied value of the altruism parameter α? In this section we provide a general solution to this question. Then we apply this solution to calculate the values of the altruism parameter implied by the values of the joy of giving parameter estimated by Friedman and Warshawsky (1985).

We begin by observing that in terms of consumer behavior, it is marginal utility rather than the utility *per se* which is important. In the altruistic formulation in (10a) the marginal utility of leaving a bequest is

$$\frac{\partial V^j}{\partial B^j} = \lambda(B^j + H^{j+1})^{-\sigma}. \tag{12}$$

Using (4) and the fact that $B^j = I^{j+1}$, we may rewrite (12) as

$$\frac{\partial V^j}{\partial B^j} = \lambda\left(\frac{I^{j+1}}{W^{j+1}}\right)^{\sigma}(B^j)^{-\sigma}. \tag{13}$$

Now consider a joy of giving bequest motive. Under the commonly used isoelastic form $\lambda^*(B^j)^{1-\sigma}/(1-\sigma)$, the marginal utility of a bequest is

$$\frac{\partial V^j}{\partial B^j} = \lambda^*(B^j)^{-\sigma} \tag{14}$$

where λ^* is the weight on the bequest motive. In order

[6] Blinder (1974), Seidman (1983) and Hubbard (1984) assumed similarly small values for the joy of giving parameter in their simulations.

to calibrate λ^* so that the calculated marginal utility in (14) would equal the marginal utility in (13), we equate the right-hand sides of (13) and (14) to obtain

$$\lambda^* = \lambda\left(\frac{I^{j+1}}{W^{j+1}}\right)^{\sigma} = R^{-N}\left\{\left(I^{j+1}/W^{j+1}\right)\Gamma \Big/ \left[\left(\alpha\beta^N R^N\right)^{-1/\sigma} - R^{-N}\right]\right\}^{\sigma} \quad (15)$$

The second equality in (15) follows from (10b). The adjustment factor $(I^{j+1}/W^{j+1})^{\sigma}$ in (15) depends on the bequest B^j. However, since the goal of this adjustment is merely to choose an appropriate magnitude for λ^* in empirical and simulation work, some proxies for I^{j+1}/W^{j+1} may be used such as the population average ratio of inheritances to total wealth, or a particular family's historical average value of this ratio. Note that in the presence of human wealth, $I^{j+1} < W^{j+1}$ so that $\lambda^* < \lambda$ where λ is given by (10b). Equivalently, the altruism parameter α corresponding to a particular value of λ^* is larger than the α corresponding to the same value of λ in the model without human wealth. We can, using (15), calculate the value of α corresponding to a given value of λ^* as

$$\alpha = (\beta R)^{-N}\left\{R^{-N} + \left(I^{j+1}/W^{j+1}\right)\left(R^N\lambda^*\right)^{-1/\sigma}\Gamma\right\}^{-\sigma} \quad (16)$$

Equation (16) can be used to interpret the joy of giving parameters estimated by Friedman and Warshawsky (1985). Using empirically observed annuity prices and a life cycle model of saving and portfolio behavior, they concluded that an intentional bequest motive must be present in order to explain the observed small degree of participation in annuity markets. They also derived the minimum values for the joy of giving parameter that would eliminate purchases of individual annuities under various assumptions about the gross interest rate, R, the proportion of Social Security and pensions in the average retired individual's portfolio, S, the degree of risk aversion and the degree to which annuity prices exceed the actuarially fair prices. Their results, which are reproduced in the top panel of table 2, might explain the failure of most consumers to buy annuities as the consequence of apparently strong bequest motives.

An alternative measure of the strength of the bequest motive is the implied value of the altruism parameter α. The bottom panel of table 2 reports the calculated values of α using (16) with $N = 30$, $L = 60$, $\beta = (1.01)^{-1}$ and $R = 1.01$ and 1.04. Since Social Security income is not bequeathable, Social Security wealth is appropriately treated as human wealth rather than as a tangible asset. For the ratio of tangible property wealth to total wealth, I/W, we use $1 - S$, where S is the share of Social Security and pension wealth in total wealth reported in the top panel of table 2. Finally, the values of λ^* are taken from the top panel of table 2. The picture which emerges from the bottom panel of table 2 is quite different from that in the top panel. In all cases the degree of the implied altruism parameter is quite small.[7] Thus, a weak altruistic bequest motive will be sufficient to eliminate the purchase of private annuities.

TABLE 2.—ESTIMATES OF BEQUEST MOTIVE PARAMETER λ^*, FROM FRIEDMAN AND WARSHAWSKY (1985)

	$S = 0.4$	$S = 0.5$	$S = 0.6$
$R = 1.01$			
$\sigma = 2$	18	9	4
$\sigma = 3$	169	58	18
$\sigma = 4$	1488	343	74
$R = 1.04$			
$\sigma = 2$	10	5	3
$\sigma = 3$	66	24	7
$\sigma = 4$	419	105	22

IMPLIED VALUES OF ALTRUISM PARAMETER α

	$I/W = 0.6$	$I/W = 0.5$	$I/W = 0.4$
$R = 1.01$			
$\sigma = 2$	0.026	0.019	0.014
$\sigma = 3$	0.007	0.005	0.003
$\sigma = 4$	0.002	0.001	0.001
$R = 1.04$			
$\sigma = 2$	0.031	0.023	0.022
$\sigma = 3$	0.013	0.009	0.005
$\sigma = 4$	0.005	0.003	0.002

Source: Top Panel—Friedman and Warshawsky (1985), table 9; $\beta = (1.01)^{-1}$.
Bottom Panel—Equation (16) with $\beta = (1.01)^{-1}$, $N = 30$, $L = 60$, λ^* from Top Panel with $I/W = 1 - S$.

IV. Conclusions

This note analyzes the joy of giving bequest motive in which the utility obtained from leaving a bequest depends only on the size of the bequest. It exploits the fact that this formulation can be interpreted as a reduced form of an altruistic bequest motive to derive a relation between the value of the altruism parameter and the value of the joy of giving parameter. We demonstrate that the joy of giving parameter may well be orders of magnitude larger than assumed in the existing

[7] In assessing these small values of α it must be kept in mind that the Friedman and Warshawsky calculations produced a lower bound on the strength of the bequest motive. Additionally, the present value of human wealth of future generations has been ignored. The bequest motives may, therefore, be substantially larger than the implied lower bounds presented in table 2.

literature. In addition, existing large empirical estimates of the joy of giving parameter are shown to be consistent with a weak altruistic bequest motive.

Despite its analytic tractability, there has been some reluctance to use the joy of giving formulation even in analyses where only a generic bequest motive is necessary. This reluctance may owe to the difficulty of making reasonable assumptions about, and in empirical work and simulation models reasonable interpretations of, the joy of giving parameter. In removing this difficulty, this paper takes an important step in interpreting empirical work and simulation results that are directed at understanding actual economic phenomena related to bequests.

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NONPARAMETRIC ANALYSIS IN PARAMETRIC ESTIMATION: AN APPLICATION TO TRANSLOG DEMAND SYSTEMS

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Abstract—We examine whether the use of nonparametric analysis can provide information that improves the performance of the translog utility function. We evaluate the performance of the translog by checking to see if parameter estimates are consistent with monotonicity and convexity of the indifference surfaces at each sample point. We found that the indirect translog performs better when applied to data sets found by nonparametric analysis to be consistent with utility maximization. The performance of the direct translog was generally poor.

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I. Introduction

A fundamental problem associated with empirical demand studies is the concept of the Hicks representative consumer and utility maximization (Phlips (1983)). In other words, can the data be rationalized by *any* well-behaved utility function?[1] Swofford and Whitney

[1] Earlier demand studies used functional forms which satisfied the theoretical restrictions implied by the theory of demand but were themselves highly restrictive. For example, the linear expenditure system meets all theoretical restrictions for a system of demand equations but imposes additive utility. For a discussion of this and other functional forms for demand systems, see Intriligator (1978).