Gravity trapping on a finite thickness domain wall: An analytic study

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We construct an explicit model of the gravity trapping domain-wall potential, where for the first time we can study explicitly the graviton wave function fluctuations for any thickness of domain wall. A concrete form of the potential depends on one parameter $0 \le x \le \frac{\pi}{2}$, which effectively parameterizes the thickness of the domain wall with specific limits $x \longrightarrow 0$ and $x \longrightarrow \frac{\pi}{2}$ corresponding to the thin and the thick wall, respectively. The analysis of continuum Kaluza-Klein fluctuations yields explicit expressions for both small and large Kaluza-Klein energy. We also derive specific explicit conditions in the regime $x > 1$, for which the fluctuation modes exhibit a resonance behavior, and which could sizably affect the modifications of the four-dimensional Newton's law at distances that typically are by 4 orders of magnitude larger than those relevant for Newton's law modifications of thin walls.

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I. INTRODUCTION

Bogomol'nyi-Prasad-Sommerfied (BPS) saturated supergravity domain walls [1], which interpolate between two BPS anti–de Sitter vacua, possess the feature that they can trap gravity [2]. The original infinitely thin, Z_2 symmetric domain-wall solution [3] in five dimensions can be treated explicitly to obtain the qualitative form of corrections to the four-dimensional Newton's law [2]. Subsequent studies extended further analysis to features of the finite thickness walls (see [9,10] and references therein) as well as numerous other subsequent studies of special models, primarily employing numerical analyses of fixed thickness solutions (for recent studies see e.g., [11,12] and references therein) [13].

The purpose of this paper is to advance the analysis of gravity trapping for domain walls in an explicit analytic model, where the graviton wave function fluctuations can be studied explicitly for any thickness of domain wall. The Schrödinger potential depends on one parameter $0 \le x \le \frac{\pi}{2}$, which effectively parameterizes the thickness of the domain wall. The two limits $x \longrightarrow 0$ and $x \longrightarrow \frac{\pi}{2}$ reproduce the infinitely thin and thick wall limits, respectively. This is the first analytic example where the graviton wave function can be obtained explicitly for any value of allowed x, both for small and large values of Kaluza-Klein masses of fluctuations. Intriguingly, we also find resonance behavior in the regime $x > 1$ and provide an explicit analytic study of resonances, which in turn can sizably modify the four-dimensional Newton's law at distances that are typically 4 orders of magnitude larger than those relevant for modifications in the thin wall setup.

The paper is organized in the following way. In Sec. II, we present the concrete form of gravity potential in the background of the BPS domain with a finite thickness, interpolating (in Z_2 invariant way) between two anti–de Sitter vacua with negative cosmological constant Λ. We discuss in detail matching conditions on the metric and the explicit form of the potential that depends only on one free parameter $0 \le x \le \frac{\pi}{2}$, effectively parameterizing the thickness of the wall. In Sec. III, we study graviton wave function fluctuations and, in particular, focus on the explicit form of the probability density at the center of the wall. In Sec. III A, we obtain analytic expressions for the probability density at the center of the wall both for small and large values of Kaluza-Klein masses. In Sec. III B, we analyze quantitatively the conditions under which the wave function exhibits resonances. In Sec. IV, we study the implications for the modification of Newton's law, and, in particular, obtain the analytic form of Newton's law modifications both in thin ($x < 1$) and thick ($x > 1$) regimes. Conclusions and proposals for further studies are presented in Sec. V.

II. FINITE THICKNESS DOMAIN-WALL MODEL

For the sake of simplicity, the model that we shall discuss is chosen to arise from a Z_2 symmetric finite thickness domain wall that interpolates between two BPS vacua with negative cosmological constant Λ in five dimensions [16]. Our motivation is to present an explicit, integrable model of the Schrödinger potential for graviton wave function modes, which would be explicitly parameterized by the thickness of the wall. Such a model would in turn allow for an explicit analysis of graviton fluctuation modes and the subsequent analysis of a modification of the four-dimensional Newton's law, thus providing a unifying, analytic approach, which addresses these effects as a function of the wall thickness.

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We choose the following form of the potential:

$$V(z) = -V_0; \qquad |z| \le \frac{d}{2}, \qquad V(z) = \frac{15}{4(z+\beta)^2}; \qquad |z| \ge \frac{d}{2}. \tag{1}$$

This potential enters the Schrödinger equation for the graviton wave function (for the derivation of the wave equation and other details see, e.g., [10]):

$$-\frac{d^2\psi_m(z)}{dz^2} + V(z)\psi_m(z) = m^2\psi_m(z), \tag{2}$$

where the graviton wave function $\bar{h}_{\mu\nu} = \eta_{\mu\nu}\psi(z) \times \exp(ik_\mu x^\mu)$ and $k_\mu k^\mu = m^2$ parameterizes the Kaluza-Klein energy associated with the four-dimensional momentum k_μ.

The potential $V(z)$ is related to the conformal factor $A(z)$ of the domain-wall metric

$$ds^2 = \exp[-A(z)]\Big(-dt^2 + dz^2 + \sum_{i=1}^{3} dx_i^2\Big) \tag{3}$$

through the following relation (see, e.g., [10]):

$$V(z) \equiv \frac{9}{16}\Big(\frac{dA(z)}{dz}\Big)^2 - \frac{3}{4}\frac{d^2A(z)}{dz^2}. \tag{4}$$

Along with the boundary conditions $A(0) = 0$, $A(0)' = 0$ and $A(z) \to 2\log(k|z|)$ as $|z| \to \infty$ the relation (4) determines for the potential (1) the following form of the metric:

$$A(z) = -\frac{4}{3}\log(\cos\sqrt{V_0}|z|); \qquad |z| \le \frac{d}{2}, \qquad A(z) = \log[k^2((|z| + \beta)^2]; \qquad |z| \ge \frac{d}{2}, \tag{5}$$

where $k = \sqrt{-\frac{\Lambda}{6}}$ (see, e.g., [14]) and the five-dimensional Planck constant M_{Planck_5} was set to 1.

Continuity of the metric and its derivative at the junction $|z| = \frac{d}{2}$ imposes the following two conditions among parameters V_0, d and β:

$$k\Big(\frac{d}{2} + \beta\Big) = \cos\Big(\frac{\sqrt{V_0}d}{2}\Big)^{-2/3}, \qquad \Big(\frac{d}{2} + \beta\Big)^{-1} = \frac{2}{3}\sqrt{V_0}\tan\Big(\frac{\sqrt{V_0}d}{2}\Big). \tag{6}$$

These two conditions can be viewed as fixing $\frac{d}{2} + \beta$ and V_0 in terms of *one free parameter*

$$x \equiv \frac{\sqrt{V_0}d}{2}. \tag{7}$$

The parameter x has a range $\{0, \frac{\pi}{2}\}$. The infinitely thin wall corresponds to the following parameter limit:

$$x \to 0; \qquad d \to 0, \qquad V_0 \to \infty, \qquad V_0 d = \frac{3}{\beta} = 3k, \tag{8}$$

while the infinitely thick wall corresponds to

$$x \to \frac{\pi}{2}; \qquad d \to \infty, \qquad V_0 \to 0, \qquad \sqrt{V_0}\beta \to -x, \qquad (\sqrt{V_0}\beta + x)^5\Big(\frac{k}{\sqrt{V_0}}\Big)^3 = \frac{9}{4}. \tag{9}$$

III. WAVE FUNCTION

The Schrödinger equation solution (2) has eigenvalues $m^2 \ge 0$, with $m = 0$ corresponding to the graviton bound state (see e.g., [10])

$$\psi_0(z) = N_0 e^{(-(3/4)A(z))}, \tag{10}$$

where the normalization constant N_0 is fixed by the following relationship:

$$\frac{k}{N_0^2} = \frac{2}{3}\sin(x)\cos(x)^{-5/3}(x + \sin(x)\cos(x)) + \cos(x)^{4/3}. \tag{11}$$

In the thin wall limit ($x \to 0$) and thick wall limit ($x \to \frac{\pi}{2}$) the normalization coefficient N_0 takes the following respective forms:

$$\frac{k}{N_0^2} \to 1, \quad \text{as} \quad x \to 0; \qquad \frac{k}{N_0^2} \to \frac{\pi}{3\cos(x)^{5/3}}, \quad \text{as} \quad x \to \frac{\pi}{2}. \tag{12}$$

The solution $\psi_m(z)$ of (2) in the continuum $m^2 > 0$ has the following form:

$$\psi_m(z) = A_m\cos(K|z|); \qquad |z| \le \frac{d}{2}, \qquad \psi_m(z) = N_m\sqrt{|z| + \beta}[a_m Y_2(m|z| + \beta) + b_m J_2(m|z| + \beta)]; \qquad |z| \ge \frac{d}{2}. \tag{13}$$

Here, $K \equiv \sqrt{m^2 + V_0}$ and the coefficients a_m and b_m satisfy $a_m^2 + b_m^2 = 1$. The normalization constant N_m is determined by employing a regulator $|z_c| \to \infty$ along with the asymptotic form of $\psi_m(z)$ as $|z| \to \infty$:

$$\psi_m(z) = N_m\sqrt{\frac{2}{\pi m}}\Big[a_m\sin\Big(m(|z| + \beta) - \frac{5\pi}{4}\Big) + b_m\cos\Big(m(|z| + \beta) - \frac{5\pi}{4}\Big)\Big]. \tag{14}$$

In the limit $|z_c| \to \infty$, the asymptotic wave function (14) ensures dominant contribution to the wave function probability, and thus $N_m = \sqrt{\frac{\pi m}{2 z_c}}$, which in the continuum limit becomes $N_m = \sqrt{\frac{m}{2}}$, i.e. z_c is replaced by π.

The matching of the wave function and its first derivative at $|z| = \frac{d}{2}$ determines the coefficients $a_m, b_m (a_m^2 + b_m^2 = 1)$ and A_m. In particular, the key expression in determining the deviations of Newton's law is the probability density of the wave function at the center of the wall

$$|\psi_m(0)|^2 = A_m^2 = \frac{1}{\pi^2} \frac{1}{(\tilde{J}_2 \tilde{S} - \tilde{J}_2' \tilde{C})^2 + (\tilde{Y}_2 \tilde{S} - \tilde{Y}_2' \tilde{C})^2}, \tag{15}$$

where

$$\tilde{C} = \cos(\tilde{y}_0), \qquad \tilde{S} = -K \sin(\tilde{y}_0), \tag{16}$$

and

$$\tilde{J}_2 \equiv \sqrt{\frac{y_0}{2}} J_2(y_0), \qquad \tilde{J}_2' \equiv \frac{d\tilde{J}_2(y_0)}{dy_0}, \qquad \tilde{Y}_2 \equiv \sqrt{\frac{y_0}{2}} Y_2(y_0), \qquad \tilde{Y}_2' \equiv \frac{d\tilde{Y}_2(y_0)}{dy_0}, \tag{17}$$

where J_2 and Y_2 are Bessel functions of order 2. The arguments y_0 and $\tilde{y}_0$ are defined as

$$y_0 = \frac{M}{\cos(x)^{2/3}}, \qquad \tilde{y}_0 = x\sqrt{1 + \frac{4M^2 \sin(x)^2}{9\cos(x)^{10/3}}}, \tag{18}$$

and coefficients M and K are defined as

$$M \equiv \frac{m}{k}, \qquad K \equiv \sqrt{1 + \frac{9\cos(x)^{10/3}}{4M^2 \sin(x)^2}}. \tag{19}$$

Note that the expression for A_m^2 (15) is a function of x and M, only. This allows us to fully explore the analytic behavior of the probability density A_m^2, which we shall do in the following subsections. In particular, in the thin wall limit ($x \to 0$) A_m^2 takes the form

$$A_m^2 = \frac{2}{\pi^2 M[J_1^2(M) + Y_1^2(M)]}. \tag{20}$$

For the sake of completeness, we also present the expression for the ratio

$$\frac{a_m}{b_m} = -\frac{\tilde{J}_2 \tilde{S} - \tilde{J}_2' \tilde{C}}{\tilde{Y}_2 \tilde{S} - \tilde{Y}_2' \tilde{C}} \tag{21}$$

and $b_m = 1/\sqrt{1 + (a_m/b_m)^2}$.

Note that for the thick wall $x > 1$, the expression for the probability density of the wave function varies significantly over the range of the wall. In this case, the probability density, averaged over the domain of the wall thickness, may be more appropriate:

$$\overline{|\psi_m|^2} = A_m^2 F_m; \qquad F_m = \frac{1}{2}\left(1 + \frac{\sin(2\tilde{y}_0)}{2\tilde{y}_0}\right). \tag{22}$$

Note that F_m is a mild function of x, and while $F_m \to 1$ as $x \to 0$, for $x \to \frac{\pi}{2}$, it approaches the asymptotic value $\frac{1}{2}$.

A. Small and large Kaluza-Klein energy expansion

The expression (15) can be readily expanded in different regimes of values of x and M parameters.

The expression (15) has a universal behavior

$$A_m^2 \to \frac{1}{\pi}, \quad \text{as} \quad M \to \infty. \tag{23}$$

On the other hand, one can also explicitly expand (15) in the limit of small M. We have done so up to the fourth order in M expansion

$$A_m^2 = M\alpha_0(1 + M^2\alpha_1) + \mathcal{O}(M^5), \quad \text{for } M \ll 1, \tag{24}$$

where coefficients α_0 and α_1 are the following explicit functions of x:

$$\alpha_0 = \frac{9\cos(x)^{10/3}}{2(\cos(x)^3 + 2x\sin(x) + 2\cos(x))^2}, \qquad \alpha_1 = \frac{p_1}{p_2}, \tag{25}$$

where

$$p_1 = \cos(x)\Bigg(8 - 34\cos(x)^2 - \cos(x)^4\left(1 + 54\log\left(\frac{2\cos(x)^{2/3}}{M\exp(\gamma)}\right)\right)\Bigg) + 2x\sin(x)(4 - 9\cos(x)^2 - 4\cos(x)^4), \tag{26}$$

$$p_2 = 18\cos(x)^{10/3}(\cos(x)^3 + 2x\sin(x) + 2\cos(x)). \tag{27}$$

The thin wall limit ($x \to 0$) produces the following values of α_0 and α_1:

$$\alpha_0 = \frac{1}{2}, \qquad \alpha_1 = -\frac{1}{2} + \log\left(\frac{M\exp(\gamma)}{2}\right), \quad \text{as} \quad x \to 0. \tag{28}$$

Note that (27) is a valid expansion for $x < \frac{\pi}{2}$; however, in the thick wall limit ($x \to \frac{\pi}{2}$), A_m has further suppressions for small values of M:

$$\alpha_0 \to \frac{9}{2\pi^2}\left(\frac{\pi}{2} - x\right)^{10/3}, \qquad \alpha_1 \to \frac{2}{9}\left(\frac{\pi}{2} - x\right)^{-10/3}, \quad \text{as} \quad x \to \frac{\pi}{2}. \tag{29}$$

and thus $A_m^2 \to \frac{1}{\pi^2} M^3$.

In Figs. 1 and 2 ,A_m^2 is plotted for a small and an intermediate value of x, respectively.

For the sake of completeness, we also give the expansion

$$\frac{a_m}{b_m} = M^2\beta_0(1 + M^2\beta_1) + \mathcal{O}(M^6), \quad \text{for } M \ll 1, \tag{30}$$

where

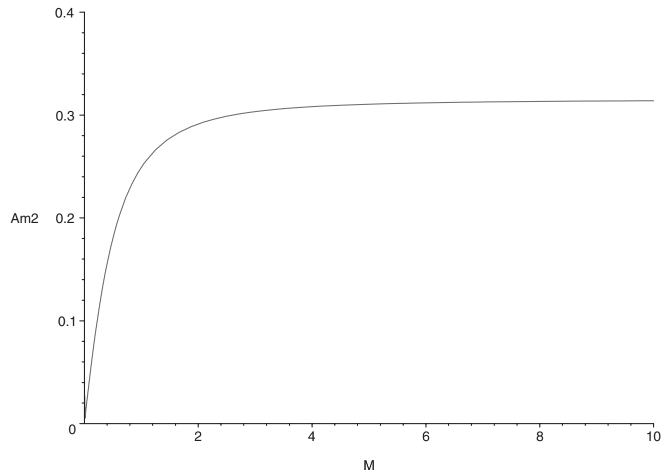

FIG. 1. Probability density A_m^2 plotted as a function of M for a small value of $x = 0.1$.

$$\beta_0 = \frac{3\pi \cos(x)^{5/3}}{4(\cos(x)^3 + 2x\sin(x) + 2\cos(x))}, \qquad \beta_1 = \frac{r_1}{r_2}, \tag{31}$$

where

$$\begin{aligned} r_1 &= 4x\sin(x)\cos(x)(2 - 11\cos(x)^2) \\ &\quad + \cos(x)^2(28 - 92\cos(x)^2 - 17\cos(x)^4) \\ &\quad - 4x^2\sin(x)^2(5 - 8\cos(x)^2) \\ &\quad - 108\cos(x)^6 \log\left(\frac{2\cos(x)^{2/3}}{M\exp(\gamma)}\right), \end{aligned} \tag{32}$$

$$r_2 = 72\cos(x)^{13/3}(\cos(x)^3 + 2x\sin(x) + 2\cos(x)), \tag{33}$$

whose thin and thick limits have the following form:

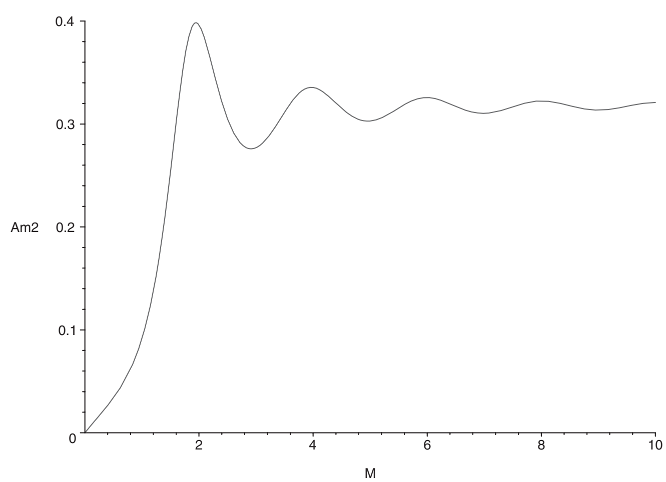

FIG. 2. Probability density A_m^2 plotted as a function of M for $x = 1$.

$$\begin{aligned} &\beta_0 = \frac{\pi}{4}, \\ &\beta_1 = -\frac{3}{8} + \frac{1}{2}\log\left(\frac{M\exp(\gamma)}{2}\right), \quad \text{as} \quad x \to 0; \\ &\beta_0 \to \frac{3}{4}\left(\frac{\pi}{2} - x\right)^{5/3}, \\ &\beta_1 \to -\frac{5\pi}{72}\left(\frac{\pi}{2} - x\right)^{-13/3}, \quad \text{as} \quad x \to \frac{\pi}{2}. \end{aligned} \tag{34}$$

B. Resonances

Another interesting behavior of A_m takes place in a particular window for the combination where parameter $x > 1$ and for small values of M:

$$y_0 = \frac{M}{\cos(x)^{2/3}} \ll 1, \qquad \tilde{y}_0 = x\sqrt{1 + \frac{4M^2\sin(x)^2}{9\cos(x)^{10/3}}} \gg 1, \tag{35}$$

or

$$\frac{M}{\cos(x)^{2/3}} \ll 1, \qquad \frac{M\tan(x)}{\cos(x)^{2/3}} \gg 1. \tag{36}$$

The probability density (15) exhibits a resonance behavior [17]. The resonance then appears when $\tilde{Y}_2\tilde{S} - \tilde{Y}_2'\tilde{C} \sim 0$, which for the regime (36) takes the form

$$\frac{\tilde{S}}{\tilde{C}} \sim -\frac{3}{2y_0}. \tag{37}$$

For the above expression, we have employed the small argument $y_0 \ll 1$ expansion of $\tilde{Y}_2 = -\frac{2\sqrt{2}}{\pi y_0^{3/2}} + \mathcal{O}(y_0^{1/2})$ and $\tilde{Y}_2' = \frac{3\sqrt{2}}{\pi y_0^{5/2}} + \mathcal{O}(y_0^{-1/2})$. Equation (37) is satisfied approximately when $\tilde{y}_0 \sim \frac{\pi}{2}(2n+1)$ ($n = 1, 2, \cdots$) or

$$M_n \sim 3\cos(x)^{5/3}\sqrt{n(n+1)}, \qquad n = 1, 2, \cdots. \tag{38}$$

Note that the above relation for the position of resonances becomes less valid as n increases.

When condition (37) is satisfied, the probability density (15) is highly peaked at

$$A_m^2 \sim \frac{18}{\pi^2 y_0^5} = \frac{18\cos(x)^{10/3}}{\pi^2 M_n^5} \gg 1. \tag{39}$$

To obtain the above expression, we set in (15) $\tilde{Y}_2\tilde{S} - \tilde{Y}_2'\tilde{C} \sim 0$ and employed the small y_0 argument expansion of $\tilde{J}_2 = \frac{(\sqrt{2}y_0^{5/2})}{16} + \mathcal{O}(y_0^{7/2})$ and $\tilde{J}_2' = \frac{5\sqrt{2}y_0^{3/2}}{32} + \mathcal{O}(y_0^{5/2})$, which yields [along with (37)] $\tilde{J}_2\tilde{S} - \tilde{J}_2'\tilde{C} \sim -\frac{y_0^{5/2}}{3\sqrt{2}}$. Note the sharp falloff of the density amplitude with the increase of the discretely valued M_n (38).

One can also estimate the approximate width ΔM_n of the resonances by determining the change in parameter M (by $\frac{1}{2}\Delta M_n$) when $\tilde{Y}_2\tilde{S} - \tilde{Y}_2'\tilde{C} \sim \tilde{J}_2\tilde{S} - \tilde{J}_2'\tilde{C} \sim -\frac{y_0^{5/2}}{3\sqrt{2}}$, using lo-

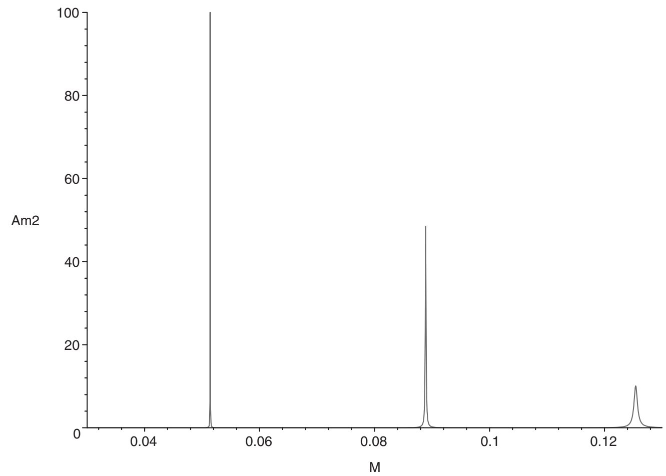

FIG. 3. Probability density A_m^2 plotted as a function of M for the value of $x = 1.5$. It exhibits distinct resonances.

cally linear approximation. This condition yields

$$\frac{\Delta M_n}{M_n} \sim \frac{\pi y_0^5}{9(\tilde{y}_0 - \frac{\pi^2}{4\tilde{y}_0})} = \frac{M_n^4}{3\cos(x)^{5/3}}\sqrt{1+\frac{1}{4n(n+1)}}. \quad (40)$$

Note that in this case, the estimate for $A_m^2 \Delta M_n$, which effectively parameterizes the integral of the sharply peaked density probability over its width, takes the form

$$A_m^2 \Delta M_n \sim \frac{2M_n}{\pi(\tilde{y}_0 - \frac{\pi^2}{4\tilde{y}_0})} = \frac{6\cos(x)^{5/3}}{\pi^2}\sqrt{1+\frac{1}{4n(n+1)}}, \quad (41)$$

and is thus close to being M_n independent. Of course, this is a valid approximation only for the first few resonances, whose width is still small, as we used only locally linear approximation for the quantity $\tilde{Y}_2\tilde{S} - \tilde{Y}_2'\tilde{C}$ near M_n.

The most distinct and pronounced resonances appear in the range $x \sim \{1.35, 1.55\}$. In Fig. 3, the density probability as a function of M is plotted for the regimes of x where the resonances are pronounced. We have checked explicitly that our analytic results for the location, the amplitude, and the width of resonances are in good agreement with numeric ones, as is evident from Table I.

TABLE I. The comparison of the numeric and analytic results for the location M, the amplitude A_m^2, and the width ΔM of the resonances for $x = 1.5$.

	n	1	2	3
numeric	M_n	0.051 41	0.088 89	0.125 43
analytic	M_n	0.051 33	0.088 91	0.125 74
numeric	A_m^2	680	50.07	10.12
analytic	A_m^2	749	48.06	8.50
numeric	ΔM_n	1.0710^{-5}	1.4710^{-4}	7.1610^{-4}
analytic	ΔM_n	1.0410^{-5}	1.5610^{-4}	8.7510^{-4}

IV. MODIFICATION OF NEWTON'S LAW

The analytic form of the probability density in turn allows us to study explicitly the deviation of Newton's law in the presence of continuum states. The (bound state) graviton wave function (10) determines the four-dimensional Newton's law, while continuum Kaluza-Klein fluctuation modes (13) are responsible for corrections to Newton's law. The four-dimensional gravitational potential between two pointlike particles with M_1 and M_2, a distance r apart, can be written in the form (see, [18])

$$V = G_N \frac{M_1 M_2}{r}(1+\delta), \quad (42)$$

where $G_N = M_{\mathrm{Planck}_5}^{-3} N_0^2$ and δ, the correction to Newton's law is of the form

$$\delta = \frac{1}{N_0^2}\int_{0^+}^{\infty} e^{-mr}\overline{|\psi_m|^2}dm. \quad (43)$$

Note that since N_0^2/k [Eq. (11)] is only a function of x, and $\overline{|\psi_m|^2}$ [Eq. (22)] is only a function of x and $M \equiv m/k$, the correction to Newton's law can be rewritten as an integral over M,

$$\delta = \frac{k}{N_0^2}\int_{0^+}^{\infty} e^{(-Mkr)}\overline{|\psi_m|^2}dM, \quad (44)$$

and is thus only a function of kr and x.

Note again that the averaged wave function density $\overline{|\psi_m|^2}$ [Eq. (22)] is of form $A_m^2 F_m$, where F_m is a mild function of x and M, with $F_m \to 1$ and $F_m \to 1/2$ as $x \to 0$ and $x \to \frac{\pi}{2}$, respectively. In the following, we shall therefore not address detailed wave function effects away from the center of the wall, but shall focus on the modification of Newton's law due to the wave function probability density near the center of the wall. Namely, our focus shall be on the A_m^2 factor and its effect on the structure of δ. However, further detailed studies of the nontrivial wave function profile would be of interest.

The amplitude A_m^2 [Eq. (15)] has a small M expansion of the form (24) and (27) and asymptotes to π^{-1} as $M \to \infty$ [Eq. (23)]. For $x \leq 1$, it is a reasonable approximation to split the integral (44) into two intervals $M = \{0, M_C\}$ and $M = \{M_C, \infty\}$, where for the form of A_m^2, the small M and large M expansion is used, respectively. A reasonable choice $M_C \sim (\pi\alpha_0)^{-1}$ produces an approximate result, which is in good agreement with the numerical one

$$\delta = \frac{k}{N_0^2}\Big\{\frac{\alpha_0}{(kr)^2}[1 - e^{(-M_C kr)}(M_C kr + 1)] + \frac{\alpha_0\alpha_1}{(kr)^4}[6 - e^{(-M_C kr)}((M_C kr)^3 + 3(M_C kr)^2 + 6M_C kr + 6)] + \frac{1}{\pi kr}e^{(-M_C kr)}\Big\}. \quad (45)$$

Again, note that for $r \gg (kM_C)^{-1}$, the leading correction is of the form

$$\delta = \frac{k}{N_0^2}\frac{\alpha_0}{(kr)^2}\left(1 + \frac{6\alpha_1}{(kr)^2}\right). \quad (46)$$

The intriguing possibility takes place for $x > 1$ where the resonance behavior takes place. Note that in this case, the resonances are highly peaked, and the integral over M can be approximated by a sum of expressions $kN_0^{-2}A_m^2\Delta M_n e^{-M_n kr}$ over n. Employing the analytic expressions from Sec. III B, we obtain the following correction to Newton's law:

$$\delta \sim \frac{2}{\pi}\sum_n \sqrt{1 + \frac{1}{4n(n+1)}}\, e^{-M_n kr}, \quad (47)$$

where M_n are locations of resonances (38), and the approximately constant prefactor is obtained by using the limit $x \to \frac{\pi}{2}$ in (11): $kN_0^{-2} \to \frac{\pi}{3}\cos(x)^{5/3}$ and (41): $(A_m^2\Delta M_n) \sim \frac{6}{\pi^2}\cos(x)^{5/3}\sqrt{1 + \frac{1}{4n(n+1)}}$. Note that the prefactor is approximately constant and of order 1.

M_n's (38) are typically in the range of 10^{-2}, and thus, for a choice of small k and range of distance $r \sim (M_n k)^{-1}$, the leading resonances can potentially contribute a sizable effect. In particular, when choosing $k \sim 10^6$ GeV, at distances $r \sim 10^{-4}$ GeV^{-1} the thick wall limit with resonances $M_n \sim 10^{-2}$ would produce an order 1 modification of Newton's law, while for the walls with $x \leq 1$, the corrections (45) and (46) would be negligible, i.e. $\mathcal{O}(10^{-4})$.

V. CONCLUSIONS

We have presented the first explicit model of a finite thickness domain wall, interpolating between Z_2 symmetric five-dimensional anti–de Sitter vacua, where the graviton wave function fluctuations can be studied explicitly for any thickness of the wall, parameterized by $x = \{0, \frac{\pi}{2}\}$. This allows us to explicitly determine the probability density of Kaluza-Klein fluctuations both for the small and large values of Kaluza-Klein energy. Notably, for $x > 1$ resonance behavior emerges, which is most pronounced in the range $x = \{1.35, 1.55\}$ and can be analyzed explicitly. For a specific range of Kaluza-Klein momenta (and anti–de Sitter cosmological constant Λ) this resonance behavior can significantly modify four-dimensional Newton's law by an effect of order 10^4 larger than that of thin walls.

While the concrete model is based on a specific, analytically solvable Schrödinger potential for the graviton fluctuation modes, we have not addressed the origin of the supergravity Lagrangian that would result in a BPS domain-wall solution whose metric leads to the proposed Schrödinger potential. Certainly, this is an important outstanding issue.

The analytic solvability of the model for graviton fluctuations lends itself to the further study of fermionic and spin-one fluctuation modes. In addition, while we chose to study fluctuation modes for codimension one (domain wall) configuration in five dimensions, extending the study to other dimensions is readily available and is relegated to future work.

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