

SHAPING AND SIGNALING MATHEMATICS:
EXAMINING CASES OF BEGINNING MIDDLE SCHOOL MATHEMATICS TEACHERS'
INSTRUCTIONAL DEVELOPMENT

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A DISSERTATION

in

Education

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2016

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Enakshi Bose

To my family:

Mummy and Baba;

Meena, Colin, Lucy and Brian,

and my sweetheart Mike and my darling Amaia

ACKNOWLEDGMENT

Writing and defending my dissertation, completing graduate school, concluding my student status and shifting towards lifelong inquiry and learning all bear the mark of milestones – distinct moments in time for which *before* and *after* connote a life-changing moment. My time in graduate school was marked additionally by two other life-changing moments. I respectfully and lovingly remember my father, Nirmal Kumar Bose, and my father-in-law, Michael Paul Smith: two extraordinary inspired and inspiring educators. How I wish you were here. On every walk I take and every journey I make, I carry you in my heart and in my mind.

The paradox of pursuing a doctorate became clear to me in the time I wrote my dissertation. While so much of the work was individual – analyzing data, reading articles, and of course writing and revising – completing the analysis and earning this degree very much required sustained support from my community of mentors, peers, colleagues, friends, and family. To this community I extend heartfelt gratitude and appreciation for extending hands to hold, shoulders to lean upon, and ears to listen.

To my advisor, Janine Remillard: Thank you for your patience, your guidance, and your open-mindedness. You allowed me room to muddle through and grow and ultimately find my voice – and I am so thankful that you worked with me even as life and geography moved me further away from the graduate school cocoon at GSE. Through working with you I have developed a deeper appreciation for the complex nature of both teaching and research – and I come to appreciate “writing to think” as a necessary companion to “thinking before writing.” And thank you for teaching me about teaching teachers!

To my committee members, Laura Desimone and Caroline Ebby: Thank you both for your patience, your flexibility, and the thoughtful feedback you have offered throughout this process. Laura, thank you for the opportunity to work on the Induction Study research team. Your comments and perspective often have forced me to step back, to clarify, to focus – and as such

have been invaluable. Caroline, your willingness to meet or talk through ideas was a lifeline when I was swimming in too many ideas – thank you for helping keep me afloat, especially in the early days when I was going in circles with analysis decisions.

To my professional community: This dissertation uses data collected as part of the Assessment of Induction and Mentoring (AIM) project, conducted jointly between 2006 and 2012 by researchers at the University of Pennsylvania and Vanderbilt University. The research was supported by the National Science Foundation under grant number 0554434. Any opinions, findings, and conclusions or recommendations expressed in this document are mine and do not necessarily reflect the views of the National Science Foundation. In addition, this research would not have been possible without the participants in the AIM study. I am grateful to the beginning middle school mathematics teachers for valuing this work enough to contribute to it. I am also grateful to my colleagues on the AIM research team, Dr. Kristin McGraner, Dr. Eric Hochberg, and Dr. Katherine Taylor Haynes. I am thankful for the guidance of the education researchers with whom I have had the privilege to study and work and learn throughout graduate school: Dr. Melissa Boston, Dr. Sarah Bush, Dr. Erin Henrick, and Dr. Kara Jackson. Finally, focusing back on the GSE community, I appreciate the support from Dr. Veronica Aplenc, Betty Deane, and Ramon Monras-Sender in navigating the dissertation process.

To my dissertation buddies: I extend heartfelt gratitude for the friends and peers who shaped my graduate school experience so meaningfully – and who brought new meaning to the phrases “use a lifeline” and “phone a friend”: Dr. Eric Hochberg (my compatriot in dissertation and life adventures), Dr. Michelle Milstein (my fellow distance-learner), and Dr. Jackie Van Schooneveld (my fellow teacher educator extraordinaire!). And thank you to Vivian Lim, with whom I raced to this final finish line – writing boot camp buddies to the end!

To my students, past, present, and future: Thank you to the children from P.S. 98 M and Bank Street's School for Children, and the preservice teachers at Bellarmine University for reminding me always why I teach.

To my friends and support network in my new home: Diana, Lisa, Suzanne, and my Supper Club gals, thank you for reminding me that writing a dissertation is not a pedestrian effort, and that the struggles were part of the course. Also, thank you to the wonderful crew of babysitters through the years who reassured me that my child was fine while my head was in a cloud – Cassidy, Allison, Courtney, Katie, Kia, Sara Jane, and Mary Alice – and a special shout out to Lolo, who became part of our extended family and thank goodness for that!

To my family by marriage: Grandpa and Grandma, I carry your kindness, encouragement, and steadfastness in my heart, and Mom, thank you (and Dad) for the continued interest and encouragement.

To my niece and nephew: Lucy and Brian, the many times I got to play “games” with you helped me attend to young children's mathematical thinking in ways that I hadn't before. My peers in my early grad school classes heard all about your thinking!

To my sister Meena: You continue to set the bar high, and yet also always look back for me and extend a hand to help or hold.

To my mother: Being told I am like you is one of the best compliments I receive. You are my model for continued, lifelong reading and learning and persevering, even in the face of change or adversity.

To my darling Amaia: Thank you for reminding me daily that without compassion and kindness and love, the road feels too long, and the days too dark. May you always reach for the stars – I love you to the moon and back, and would go to all the planets and all the stars and all the galaxies with you. Throughout life, when you begin a “mission” may you have the confidence and perseverance to complete it. Thank you for all the celebrating you organized as Mommy

finished her dissertation! Did you know that if I traced your face on a piece of paper, I would get a heart?!

To my dear Mike: Thank you for helping me navigate life and its ups and downs – and for reminding me, when I needed it, that I only needed to write for 20 minutes and then I could stop... Se hace camino al andar. Together we walk on our path, with its twists and curves, every day, and one day we will walk El Camino de Santiago! We're a great team.

ABSTRACT

SHAPING AND SIGNALING MATHEMATICS:

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Enakshi Bose

Janine Remillard

How learners understand content is interwoven with the practices in which they engage. Classroom experiences of how students engage with mathematical ideas and problems shape the mathematics that is learned (Boaler, 2002; Franke, Kazemi, & Battey, 2007), affecting the mathematical learning opportunities and the ways in which learners may view the subject and their own knowledge and capability. Consequently, teaching mathematics necessitates attention and sensitivity both to content and to students, and it involves managing dilemmas while maintaining productive relationships (Lampert, 2001; Potari & Jaworski, 2002; Brodie, 2010). For novice teachers navigating multiple demands and expectations, the period of teacher induction (the first years of a teaching career) marks a unique time of teacher learning, when new teachers try, take up, modify, and discard instructional practices, based on perceived effectiveness. The induction years are a time of rehearsal, formation, and evolution of teaching practice.

This dissertation presents a close study of instruction over time to illuminate the ways that normative practices may shape mathematical learning opportunities and signal messages about mathematics. The study examined the instructional practice of six novice middle school mathematics teachers teaching in a district with multiple ongoing initiatives to support mathematics instruction with an emphasis on rich tasks and discourse and new teachers' learning.

Applying an instrumental case study approach, the study used observation and interview data, analyzed with a grounded theory approach, to answer the research questions. The analysis illuminated multiple strands of normative practices that, when interwoven, composed instruction and shaped mathematical learning opportunities in either capped or promising ways. Over time these patterns tended to take hold, with certain practices amplified, supported by both contextual and individual factors.

In attending to the nature and qualities of instruction of novice teachers in the induction years, the study bridges math education and teacher education to provide insights into how teachers' actions shape what it means to *do* math in classrooms, what those actions signal about the discipline and what it means to *know* math, and what opportunities exist to support teacher capacity around teaching mathematics in a connected and relevant way.

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CHAPTER 1: INTRODUCTION

Descriptions of what it means to learn and know mathematics in curriculum frameworks or policy documents often focus on the content, breadth, depth and progression or trajectory of learners' mathematical understanding. How learners understand that content, however, is interwoven with the practices in which they engage. This view – that the classroom experiences of how students engage with mathematical ideas and problems matter to the mathematics that is learned – is supported by research (Boaler, 2002; Franke, Kazemi, & Battey, 2007) and is endorsed in policy documents including, most recently, the Common Core Standards for Mathematical Practice (Common Core State Standards Initiative [CCSSI], 2010). *Knowing* mathematics requires developing capacity with mathematical practices, such as representation, generalization, and justification (RAND Mathematics Study Panel, 2003). As a result, *learning* mathematics requires engagement from both teachers and students in forms of mathematical argumentation, such that the legitimacy of a strategy or solution is determined by students assessing the mathematics and making a case for how the approach to the problem works rather than relying on a single expert authority, such as the teacher or the textbook, to adjudicate correctness (Lampert, 1990; Amit & Fried, 2005). Experiencing and coming to know mathematics as useful, connected, relevant, and flexible – to students in their world, within the discipline, and in relation to other disciplines – is woven into the learning of the subject, beyond a scope and sequence of concepts and skills. In other words, developing *mathematical proficiency* (Kilpatrick, Swafford, & Findell, 2001) – or what is necessary for anyone to learn mathematics successfully – involves not only conceptual understanding, procedural fluency, and strategic competence, but also adaptive reasoning and a productive disposition, or the “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy” (Kilpatrick et al., 2001, p. 5). How students engage in mathematical work

matters to what is learned or accessible and to what learners believe and know they can do, and as such, the nature and qualities of classroom instruction matter (Schoenfeld, 2014).

Including the development of mathematical practices, experiences, and engagement as goals for learning in addition to concepts and skills has significant implications for the work of teaching. Teachers are responsible for much more than delivering or transmitting mathematics content to students. Teaching mathematics necessitates an attention and sensitivity both to *content* – the big ideas or structures and the connections between them, and the ways of approaching problem scenarios and explaining strategies and solutions – and to *students* – for example, the understandings they bring, and the ways they are making sense of the mathematics (Ball & Forzani, 2010; see also Ball & Bass, 2000; Cohen & Ball, 1999). The roles and responsibilities of the teacher in planning and enacting instruction blossom to encompass selecting mathematical tasks, introducing tools, representations, and alternative methods, asking questions and fostering productive discourse, establishing an environment and social culture in the class in which students can construct understanding, and systematically reflecting upon and analyzing the mathematical work in the class (Ball & Forzani, 2011; Ball & Forzani, 2010; National Council of Teachers of Mathematics [NCTM], 2014; see also NCTM, 1991; Hiebert et al., 1997). These dimensions of the work of teachers are inextricably intertwined – they interweave and complement each other, and are integral to fostering learning environments that promote and value mathematical sense-making.

Teaching is challenging work. The work of teaching involves managing multiple problems or dilemmas with respect to instruction while maintaining productive relationships with both students and content (Lampert, 2001; Potari & Jaworski, 2002; Brodie, 2010). Schoenfeld (2004) explains the demands this work places on the teacher:

Teaching in the ways envisioned by the authors of the reform documents is hard. It calls for both knowledge and flexibility on the part of the teacher, who must provide support for students as they engage in mathematical sense making. This

means knowing the mathematics well, having a sense of when to let students explore and when to tell them what they need to know, and knowing how to nudge them in productive directions. (p.272)

For teachers, developing the capacity to move flexibly between their own understandings, knowledge of content, knowledge of students, and knowledge of teaching and learning is paramount. Multiple studies have illustrated, however, that developing this knowledge and reasoning is demanding (Ball & Forzani, 2010; Grossman et al., 2009; see also Heaton, 2000; Schifter & Fosnot, 1993) and may require an accompanying shift in beliefs about both the nature of mathematics and the process of learning mathematics. The relationship between teacher beliefs and knowledge (teacher characteristics) and instructional practice is not linear. As Lampert (1985) notes, teachers must deal with uncertainty and navigate dilemmas arising in practice. Their responses –actions or inactions, intentional or not – may address particular issues or concerns at the expense or detriment of others (Brodie, 2010). Teachers may enact an amalgamation of beliefs and practices that are competing or inconsistent (Cohen, 1990; Herbel-Eisenmann, Lubienski, & Id-Deen., 2006). Teachers' instructional development or change happens over time, and in different ways for teachers at varying points in their careers (Drake, 2002), raising questions about which roles and configurations of supports and resources for teachers, particularly at the local (school and district) levels (Lappan, 1997), are productive at specific stages in teachers' career and learning trajectories.

For novice teachers, who are navigating multiple demands and expectations, the period of teacher induction, at the beginning of the career, mark a unique time in the teacher learning trajectory. Closely following formal teacher education, augmented by mentoring and professional development opportunities at school and district levels, this is a time when new teachers try and test instructional practices, which are either taken up, altered, or discarded, based on perceived effectiveness (Grossman et al., 2009; Feiman-Nemser, 1983). Beginning teachers are trying to figure out how to teach in their content area while also developing skills that may seem decidedly

non-mathematical in nature: managing behavior, for example, or forging relationships with students. During the early years of their careers, new teachers are developing both their instructional and pedagogical persona (in other words, who they are as teachers) and their pedagogical repertoire (or what they do as teachers) (Sowder, 2007), all within the context of the specific setting in which they work and the particular groups of students they teach. The induction years are as much a time of rehearsal, formation, and evolution of teaching persona as they are an initiation into the teaching profession. Novice teachers are negotiating real-time challenges and tensions as they develop their practice and learn from that practice. A recent and ongoing direction in teacher education on instructional practices that are “high leverage” (Ball & Forzani, 2010) aims to identify instructional practices that have the potential to be generative, to interrupt traditional or unexamined notions and lead to continued growth for teachers’ knowledge and practice. These high-leverage practices include tasks that may seem general, such as launching a task in class or checking on students’ understanding, but do have content-specific attributes. For example, a mathematical discussion calls for different reasoning than a social studies one and thus requires different questioning moves and probes on the part of the teacher to shape student learning opportunities, engagement, and participation. Learning and enacting these generative high-leverage practices with changing groups of students is complex and ongoing. As Heaton (2000) described, “Teaching mathematics for understanding is not something that is ever completely learned. One can get better at it but the ‘it’, the teaching, is forever under construction” (p. 141).

This dissertation presents a close study of instructional practice over time to examine the relationship between patterns in instruction, mathematical learning opportunities, and novice teachers developing their repertoire of practice. The analysis focused on the normative practices set by teachers through their words and actions as a window into both the learning experiences and opportunities accessible to students and the development of teachers’ practice. Carpenter and

Lehrer's (1999) describe normative practices in relation to classroom mathematical learning experiences and environments:

Norms in a particular class determine how students and the teacher are expected to act or respond to a particular situation. Normative practices form the basis for the way tasks and tools are used for learning, and they govern the nature of the arguments that students and teachers use to justify mathematical conjectures and conclusions. These norms can be manifest through overt expectations or through more subtle messages that can permeate the classroom environments. (pp. 25-26)

The analysis also examined personal and structural or contextual factors that influence beginning teachers' developing pedagogical repertoire. In attending to the nature and qualities of instruction of novice teachers in the induction phase of their careers, the study bridges math education and teacher education to help understand

- a) How teachers' actions or practices *shape* what it means to *do* math in classrooms – how enacted normative practices shape the mathematical activity and discourse, for example;
- b) What those enacted normative practices *signal* about the discipline and what it means to *know* math as conveyed through the learning environment and opportunities – in other words, connecting how students *do* the work of math to what they learn about mathematical practices and considering implications for developing learners' efficacy (and other dimensions of a productive disposition towards doing mathematics); and
- c) What openings might exist to build teacher capacity around teaching mathematics in a connected and relevant way.

Shaping and Signaling Mathematics: An Overview of the Study

Questions about the kinds of normative practices fostered and emphasized through instruction necessitate a close examination of instruction; they cannot be answered through a study of lesson plans or reflection alone, as they are shaped through the interactions in the moment between a teacher, a group of students, and the specific content materials through which

the teacher and students engage with mathematics. Within the set of interactions that comprise episodes of classroom instruction, teachers take on a large and important role because, by nature of their position as teacher, they make decisions that mediate how students interact with the content, the materials, and each other. In short, as teachers engage in different kinds of interactions between students and materials, they shape the kinds of opportunities to learn both mathematics content and practices. In turn, the normative practices emphasized in the class signal both meanings of what doing mathematics encompasses – for example, whether it is connected to students’ lives, or whether it consists of static rules or offers a dynamic way of understanding, or both – and how the teacher and students might see their own capability or efficacy in relation to the mathematical work they do (Boaler, 2002; Franke et al., 2007; Battey, 2013).

The interactions – with both students and materials – that teachers engage in through their role in the classroom both shape what it means to do math in the classroom and signal what it means to know math. Attending to teacher interactions explicitly connected to content – in other words, those actions that guide the development of mathematical ideas – is one area of interest. Classrooms, however, are complex systems in which multiple problems, challenges, and dilemmas are worked through simultaneously. It is important to consider how other teacher actions – even those that have social or other non-mathematical purposes – connect to or influence the mathematical learning opportunities, the students’ access to different means of engagement and environments, and the kinds of practices valued or emphasized.

A study of teacher practice through an examination of instructional interactions could yield different results depending on the point in a teacher’s career because a teacher’s knowledge and experience shape practice and are shaped by practice. Central to this study is the question of how novice teachers’ instruction develops. Teacher induction marks a unique time point for the development of practice: proximity to formal teacher education, multiple inputs from colleagues and administrators at school and district levels, and the development of a pedagogical persona

through the day to day work of teaching and the responsibilities for managing and guiding a community of students. Novice teachers are responsible both for teaching content and also for developing the social culture of the classroom. The normative practices they enact, both explicitly and implied, guide how the students engage with the teacher, with each other, and with the content.

The following research questions frame this study of novice middle school mathematics' teachers' instruction during induction:

(1) What is the nature of a beginning middle school mathematics teacher's instruction in the first year? What changes and what stays the same in year 2?

How does the teacher guide the development of mathematical ideas?

How does the teacher develop the social culture of the classroom?

(2) How does the teacher's instruction shape the kinds of opportunities to learn mathematics? What does the instruction signal about what it means to know and do math?

(3) What are the personal and contextual factors that influence novice teachers' instruction and changes in instruction?

Applying an instrumental case study approach (Stake, 2005) and drawing on data from a longitudinal study of novice teachers' experiences in the first two years of teaching, I examined recorded episodes of classroom instruction from three time points – the fall and spring of the teachers' first year teaching and the spring of their second year – to identify patterns of interactions and how they shaped what it meant to do mathematics in those classroom (in other words, identifying and analyzing which normative practices were enacted). I then considered how the normative practices shaped mathematical learning opportunities and signaled messages relevant to how attributes of mathematics learners' productive dispositions (Kilpatrick et al., 2001) were conveyed and supported. Finally, I drew on interview data with the teachers, their

administrators, and mentors and colleagues to understand and explain the emerging story lines of change. As such, the analysis presented here is as much a close examination of instruction as it is a study of beginning middle school mathematics teachers' instructional development over time. In considering the longitudinal dimension of the development of teacher practice, findings from this study will contribute not only an understanding of the challenges new mathematics teachers may encounter as they learn to teach, but also the openings and possibilities for learning to teach math in ways that support mathematically productive practices and dispositions.

Scholarly and Practical Significance

This study contributes to ongoing conversations in mathematics education research about the relationship between instruction and the mathematics that students have the access or opportunity to learn. The "opportunity to learn" perspective, simply stated, is that students learn what they have the most opportunity to learn (Hiebert & Grouws, 2007). Here, this would suggest that what students come to learn as mathematical practices is what they experience; for example if much of the classroom mathematical activity focuses on practicing skills and computations, students may form an impression of math as a static rule-based discipline. Alternatively, in a classroom in which explaining and justifying strategies and solutions are normative practices, students may hold different understandings of where authority for knowing math and adjudicating accuracy and validity resides.

The study's focus on novice teachers' experiences and practice will also contribute to research on teacher learning. Teachers have learned the content they are teaching (as students and through teacher education), but to teach they also must know mathematics in a manner specific to the work of teaching (Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Ball, Thames, & Phelps, 2008). In developing their practice, teachers draw on pedagogical reasoning (Wilson, Shulman, & Richert, 1987; Shulman, 1987) as they move from their own (more expert or established) knowledge and understanding of a concept to those representations, examples, and

associations likely to engage and initiate understanding on the parts of the learners, the students. This knowledge *for* practice is developed in part *through* practice, as teachers take into account the students they are teaching, the particulars of the context, and the curriculum and materials used (Shulman, 1986, 1987).

Brodie (2004) contends that “particular teaching practices constrain and constitute what counts as knowledge. Thus not only does practice require certain knowledge from teachers, it also creates what counts as knowledge for teachers, and of course for their learners” (p. 71). The basis for my questions about beginning teachers’ instructional development is a conceptualization of the dialogic and dynamic relationship between knowledge, learning, and practice. For teachers, learning happens through practice, in conjunction with practice, and the context of practice is constantly changing (for example students change with each class and year, new curricula are adopted, and so on). In this way, learning teaching entails “changing participation in changing practices” (Lave, 1996, p. 161). Brodie (2004) frames how and what the teacher learns during instruction as an ongoing, iterative process:

Teachers’ mathematical knowledge and their mathematics teaching practices are mutually constitutive; that is each one shapes, creates, and constrains the other, while remaining distinct analytical objects. Thus we cannot say that particular kinds of teacher knowledge will help create particular teaching practices, because this alerts us to only one side of the relationship. Particular kinds of teaching practices will also constrain the knowledge that teachers use, value, and develop, for themselves as well as for learners. (p. 72)

There is a formative element of instructional experiences for teachers, through which particular practices may be reinforced, adapted, or set aside; for example, they may base future practice on prior perceived successes or difficulties. In this way particular practices and representations of mathematics may be reinforced, as teachers develop their “wisdom of practice” (Shulman, 1987, p. 11). Findings from this study may contribute to the identification of practices that are “high-leverage” (Ball & Forzani, 2010; Ball & Forzani, 2011), or that are generative in that they suggest openings for teachers’ continued growth through their practice.

Organization of the Dissertation

This dissertation presents an analysis of novice middle school mathematics teachers' instruction closely examined at three distinct time points during the first two years of the teachers' careers to understand (a) the normative practices, both content-related and non-content related, that emerged through instruction, (b) the ways those practices shaped the mathematical learning opportunities and presented signals about what it meant to do and know mathematics, and (c) both the personal and contextual factors that mattered to how the novice teachers developed their teaching practice, their pedagogical repertoire. In Chapter 2, I describe the theoretical and conceptual frameworks underpinning this analysis and through a review of the literature note how this analysis contributes to and extends existing conversations in mathematics education research. In Chapter 3, I explain the research design and methodological decisions that shaped this analysis, and I introduce the participating teachers in the study. I present findings in the following chapters. Drawing on analysis of videotaped observations, in Chapter 4, I describe the normative practices that made up the teachers' instructional practice, and then I analyze how those practices affected mathematical learning opportunities and messages or signals about knowing and doing mathematics. Next, in Chapter 5, I move from a close examination of practice to an analysis of change, or lack of change, in practices over time. To understand these trajectories of change I present findings from an analysis of interviews with the teachers, their administrators, and their extended support network of mentors and colleagues. Finally in Chapter 6, I consider how these findings contribute to the knowledge base of both teaching and learning mathematics and also learning teaching (the cross-section of mathematics education focused on improving instruction and teacher education developing through practice), and consider directions for further study.

CHAPTER 2: GUIDING FRAMEWORKS & LITERATURE REVIEW

This dissertation is a close study of novice teachers' instructional practice over time to illuminate the relationship between normative practices and patterns in instruction and mathematical learning opportunities, framed within the development of novice beginning teachers' practice *through* practice. In attending to the nature and qualities of instruction at this particular time in teachers' careers, the study bridges math education and teacher education to help understand how teachers' actions, whether content-related or not, may *shape* what it means to do math in classrooms and what those actions might *signal* about what it means to know, learn, and teach math.

In this chapter I first identify the theoretical underpinnings to this analysis of instruction as a means to understanding both mathematical learning opportunities and teacher in-service learning through classroom instruction. I then distinguish the many ways *practice* is defined and used in education research in relation to teaching, mathematical learning, and learning teaching and articulate how I conceptualize these practice constructs in this study. Next, I introduce a conceptual framework to explain both how I will examine episodes of instruction and how that analysis will provide insights into normative practices, mathematics learning opportunities, and teacher learning. Following this is a review of the existing literature of how instruction (and the teacher actions that comprise instruction) matter to mathematics learning opportunities and how teachers develop their practice and learn to teach mathematics. I consider what we know and identify gaps in our understanding and questions for further study, including the research questions guiding this study.

Theoretical Framework

Assumptions about the nature of learning and teaching form the foundation for this analysis. Learning is both individual and social, or shared. Classroom teaching is complex and relational work, spanning multiple terrains and relationships between teacher, students, and

content (Lampert, 2001). Interactions between teacher, student, and materials essentially compose instruction (Cohen & Ball, 1999). Hiebert et al.'s (1997) framework identifying key dimensions of instruction that foster sense making and understanding provides a springboard for analyzing episodes of instruction. And finally, teaching is deliberate and cumulative work and, as with other learning endeavors, instructional experiences form a basis for future actions. As such, examining classroom teaching practice is important to understand possible effects or outcomes of instruction and opportunities for teacher learning. Pedagogies and practices matter because they shape the mathematical learning opportunities and experiences in which the learners engage and signal what is involved in knowing and doing mathematics (Franke et al., 2007).

Assumptions about learning

Central to this study are assumptions about the nature of learning, both for students and for teachers. Drawing on socioconstructivist theory (Cobb, Yackel, & Wood, 1992), I maintain that what students learn is intrinsically connected with how it is taught and experienced. Mathematics learning is a process of both individual construction of concepts in the mind and social enculturation into the shared practices of the community and broader society. As Cobb (1994) observes, “The critical issue is then not *whether* students are constructing, but the *nature* or *quality* of those socially and culturally situated constructions” (p. 4). Different approaches to school mathematics produce different experiences for learners, and shape learner identities as doers and knowers of mathematics (Boaler, 2002; Boaler & Greeno, 2000). Learning is social and shared (Franke et al., 2007, p. 228). This holds for both students and teacher learners. Teacher learning opportunities about how to guide classroom discussions to develop mathematical concepts, for example, or how to respond to student questions to clarify misconceptions and treat errors as sites for learning, abound in real-time instructional experiences, which serve as a frame for the development of teachers’ pedagogical repertoire and persona or identity.

Assumptions about teaching and instruction

A guiding assumption about the work of classroom teaching is that it is complex, “relational work” (Lampert, 2001) between teachers, students, and content. This is not to say that teaching cannot be understood or examined or generalized; however, it is necessary to study teaching in a way that allows us to see how the different domains –the social and the intellectual or academic – are integrated and managed in practice. To understand classroom teaching and its complexity, Lampert (2001) proposes an analytic framework of the many relationships and “problem spaces” that comprise the scope of teaching. These include the relationships between teacher and students (T – S), between teacher and content (T – C), between students and content (S – C), and between teacher and students around content (T – S|C). Figure 1 (adapted version of figure from Lampert, 2001, p. 33) illustrates this framework.

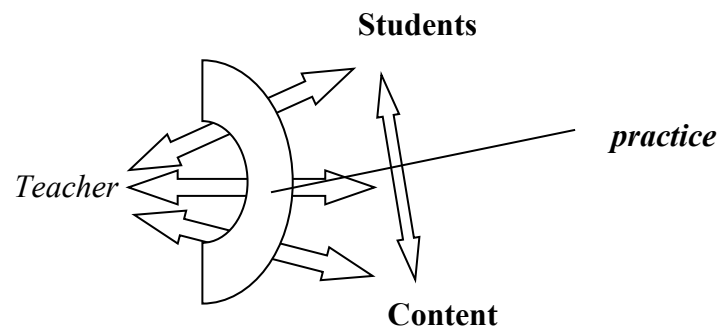


Figure 1. Relationships and “problem spaces” between Teacher, Students, and Content that make up instructional practice (adapted from Lampert, 2001, p. 33).

In this framework, the practice of teaching plays out along different trajectories or vectors (T – S, T – C, and T – S|C), but these are not separate or distinct areas. Teaching actions play out simultaneously and in the moment, as teachers address multiple questions or problems of practice during teaching. For example, with respect to a singular episode of instruction, a seemingly brief moment in a series of instructional moments, Lampert raises the following questions:

What would be [the] most mathematically appropriate thing to do? What would move the lesson along? What would bring more students into the discussion? ... [These questions]

are not the thought experiments of the academic. They had to be answered with action. (2001, p. 19)

And frequently these actions may occur at the same time. Teaching actions may cross multiple spaces, and may not always align with the distinct goals within a single “problem space.” For example, to maintain the pace of the classroom (a T – S move or a T – S|C), a teacher may elect to spend less time on a specific problem, which may alter the academic or intellectual goal or potential of the task (T – C). This framework highlights the complexity both of the work of teaching and the study of this work.

While the practice of teaching is comprised of many processes, including planning and reflection, that may take place beyond scheduled class times, for this analysis, I focus on those practices of teaching embedded in or visible through instruction. Instruction is multi-dimensional (see Franke et al., 2007; also see Hiebert et al., 1997), and it is more than the sum of its parts. To study instruction both as a whole and as a composite of many different pieces, I drew upon Cohen and Ball’s (1999) framework of instruction as composed of interactions between three elements: teacher, students, and resources (the material representation of content). In this framework, instruction is dynamic. Any given element of instruction shapes the nature, potential, and quality of what is taught and learned through the way it interacts with and influences other elements. Instructional interactions include not only what teachers say and do, but also what students say and do, and how both teacher and students interpret and make use of material resources (such as curricula). However, teachers are a particularly compelling focal point because of their role mediating the material resources and organizing the learning goals and activities of the students. Framing instruction as composed of these interrelated parts extends the discussion of improving instruction to one of improving instructional capacity in a more dynamic way, recognizing that a change along one interaction vector will necessarily influence the others. Figure 2 illustrates

Lampert's (2001) model of the relational spaces of teaching mapped onto this framework of instructional interactions.

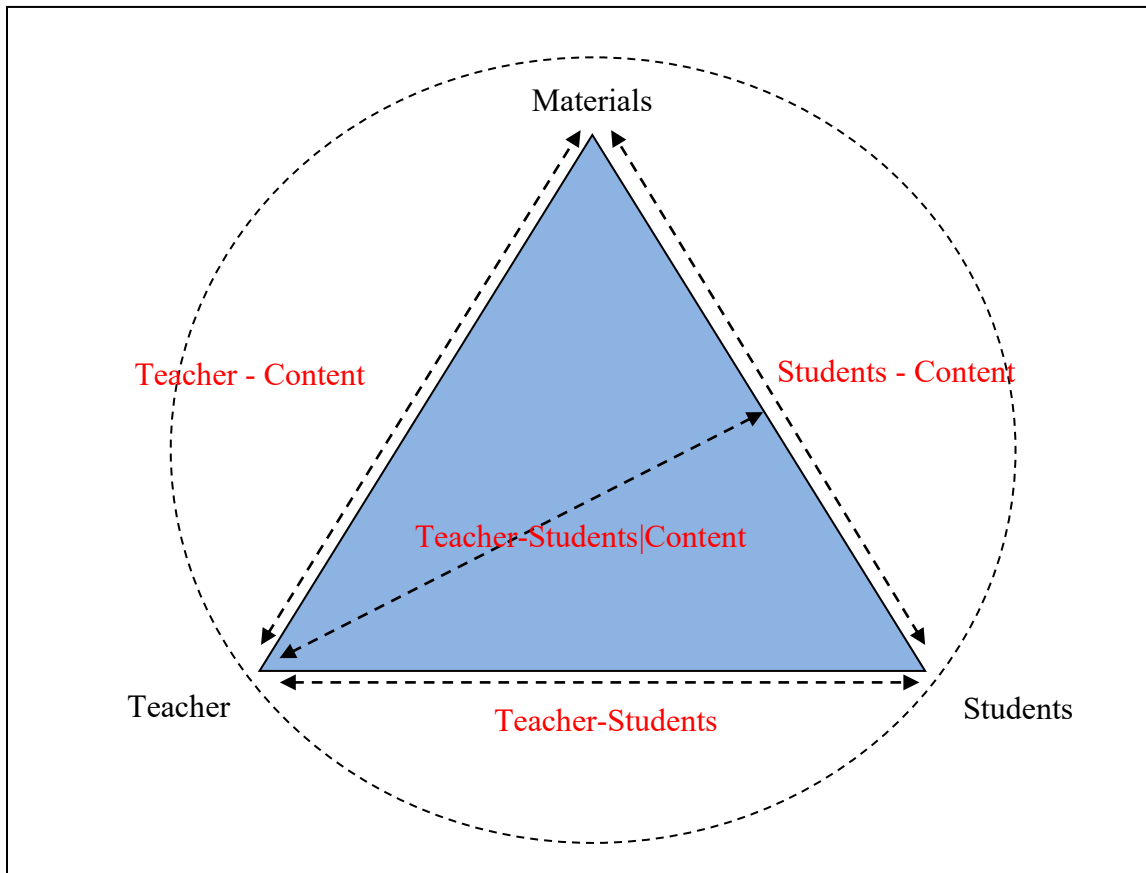


Figure 2. Relationships within teaching framework (Lampert, 2001) mapped on to the Instructional Triangle (Instruction composed of Interactions) (Cohen & Ball, 2001).

In these models of instruction and teaching, the mathematical learning opportunities are not confined to what is written in the materials used. Instead, mathematical learning opportunities – messages about what it means to do math, to know math, and to learn math – are embedded in the practices in which students engage. Normative practices (Carpenter & Lehrer, 1999) structure the ways in which the teacher and the students interact with each other and with the content. Instantiated through interactions, both through explicit routines and implicit expectations, normative practices form a bulwark for both social and content-specific aspects of the classroom learning environment. *How* mathematics is experienced in the classroom matters, and not simply

as a pedagogical method, but as the way students – novice mathematics learners – come to engage with mathematical work and in mathematical thinking and sense-making. Boaler and Greeno (2000) describe how attention to participation may direct one’s perspective on instruction:

We consider knowing and understanding mathematics as aspects of participation in social practices, particularly discourse practices, in which people engage in sense-making and problem solving using mathematical representations, concepts, and methods as resources. ... An important implication of this idea is that students’ learning of mathematics can be considered a *trajectory of participation* in the practices of mathematical discourse and thinking. This view goes beyond recognizing that social practices provide a context for learning mathematics – instead, according to this view, participation in social practices is what learning mathematics is. (p. 172)

For this analysis I studied instructional interactions to understand the normative practices they fostered and reinforced and, in turn, how those normative practices shaped the mathematical learning opportunities accessible to students.

Assumption about learning the work of teaching

How teachers learn to teach is a process that takes place over time, with multiple inputs, beginning with teachers’ “apprenticeship of observation” (Lortie, 1975), or their experiences as learners, and continuing through professional education and, importantly, through practice, as teachers adopt and adapt shared practices of their professional community and distill a personal list of “what works” and “what matters.” Drawing on Lampert’s (2010) work, I frame the process as *learning teaching* rather than *learning to teach* “because the infinitive form can suggest that the action is to occur in the future, after something is learned, while the form *teaching* allows us to hold out the possibility that learning also occurs while doing the work” (p. 21).

Induction refers to the first teaching situation beginning teachers encounter. Feiman-Nemser (1983) notes that in specifically identifying this span of time in the career trajectory of teachers, the term *induction* signals “the fact that the first year of teaching has a character of its own, that it is different from what has gone before and likely to influence what is to come” (p. 157). During induction, teachers adjust to the reality of the work of teaching and the nature and

volume of interactions with students and content in new ways (Sowder, 2007). Kagan (1992) suggests that this induction period constitutes a developmental stage for novice teachers, during which they learn about their students (and how students learn and interact), they develop routines to integrate classroom management and instruction, and they reconstruct their image of themselves as teachers. Grossman (1992) critiques this model, however, as erroneously overlooking the role of content, observing, “For better or worse, classroom management and instruction are eternally married. How teachers manage classrooms enables or constrains the possibilities of teaching, classroom discourse, and student learning” (p. 174). As teachers establish relationships with their students, they create the space for mathematical work to take place. Accepting this conceptualization of the intertwining of management and procedural development with content instruction, induction serves as a time when novice teachers try out approaches to content and develop an orientation towards mathematics and mathematics instruction that emerges from their experiences with what seemed to “work” or “not work” with students – their emergent case knowledge, or “wisdom of practice” (Shulman, 1987).

Teaching is “deliberate work” (Franke et al., 2007, p. 228), as teachers manage and negotiate challenges and dilemmas of content and of relationships between students and content. It is worth examining instruction closely to learn from both planned actions and those that arise in-the-moment, in response to real contexts, real students, and real problems of practice. Indeed, studying the experience of teaching and identifying key (high-leverage) practices that result in student learning is a fertile area for scholarship connecting research to practice (Ball & Forzani, 2010; Ball & Forzani, 2011).

Defining Practices

The term *practice* has multiple meanings and usages in education research. This dissertation explores how different kinds of *practice* intersect in novice middle school mathematics teachers’ classrooms over time. In this section I will define three distinct but

complementary constructs of *practice* as observed in instructional episodes: the *normative practices* that emerge through the enactment phase of instruction (when teachers and students are interacting around content, in the classroom setting), the *mathematical practices* and learning opportunities that are possible and accessible to learners through particular normative practices, and the *practice* and pedagogical repertoire that novice teachers learn through the work of teaching. These different forms of practice are interrelated so that the whole picture of teaching and learning that develops through instruction is more than the sum of the parts. By focusing on practice in these varied forms, this analysis – a close scrutiny of instructional episodes over time - will illuminate both the mathematical learning opportunities accessible to students and the instructional repertoire novice teachers develop. By focusing on the practices of novice teachers, this work complements recent work in teacher education to identify ways to prepare teachers to teach in more intellectually ambitious ways (Lampert, Franke, Kazemi, Ghouseini, Turrou, Beasley, Cunard, & Crowe, 2013) but in this study, the teachers’ “rehearsals” of practice took place in their own classrooms, with real students, in real-time, over the teachers’ first two years of teaching.

Instructional Practice and Normative Practices

If instruction is composed of interactions between teachers, student, and content, a composite of what happens in different relational or “problem spaces” in a classroom, it clearly is a complex activity to study. Hiebert and colleagues (1997) identify features or dimensions of instruction in classrooms in which mathematical sense-making and understanding are emphasized, and these serve as a useful framework for analyzing and improving instruction. They identify five features of instruction: the nature of the learning task, the role of the teacher, the social culture of the class, the kinds of mathematical tools available, and the accessibility of mathematics for every student (Hiebert et al., 1997). Of course, these features are interconnected. A task that is mathematically worthwhile, that has multiple entry points and opportunities for

many solution strategies, is more likely to generate a richer discussion in which connections between representations and generalizations may be possible. Similarly, a classroom in which there is an expectation (or a norm) that students share and question each other's approaches may be one in which more students have access to rich mathematical content. Since the teacher holds a unique place in the class because of her/his professional and pedagogical position and mediates interactions between content and students, the role of the teacher is a critical feature to study. The teacher's role may be examined in two distinct categories of responsibilities: guiding the development of the mathematical ideas and activities and developing the social culture of the classroom. How the teacher navigates these two responsibilities is communicated through interactions, both content-related or not, and intentional or explicit or implicit. For this analysis, I focus on the normative practices that develop through instructional interactions within these two distinct roles of the teacher, to understand how math learning opportunities are shaped and how themes about mathematics (what it means to do mathematics, the role and responsibility of teacher and learners) are signaled through instruction. The normative practices affect both task and discussion during enactment, both of which are integral to mathematical sense-making.

Together, teachers and students establish patterns or ways for interacting and working in the social space of the classroom. These *normative practices* set up the shared expectations and responsibilities – the norms – of the classroom. While certain normative practices connect in unambiguous ways to content and the mathematical learning experiences (for example, how and when the teacher provides information), normative practices that set up the social or management norms also matter to the mathematical learning opportunities accessible to students. Wood (1998) articulates the role of norms in classroom instruction and learning opportunities:

The social norms underlie the patterns and routines that become established in the classroom and that enable the students and teacher to interact harmoniously. The patterns become the *hidden regularities* that guide the actions of the participants in the classrooms. As such, they become the taken-for-granted ways of interactions that constitute the culture of the classroom. (p.170)

The patterns in interaction may create or constrict opportunities for communication. For example, regularly prompting students to support an answer with an explanation or to ask each other questions during a whole class discussion (e.g., Does anyone have a question for Lydia?) may be considered social norms as specific patterns of behavior. Social norms may apply to any subject and as such may not be unique to mathematics classrooms (expecting explanations may be a norm in other disciplines), but they still affect the nature of the mathematical activity because they encompass the “regularities” or patterns guiding discussions, shaping the learning environment as opportunities for participation and access. Making up the social culture of the classroom, normative practices emerge through routine and may be verbal or nonverbal, explicitly cultivated or the unintentional result of repeated actions, and mathematical or non-content related. Yackel and Cobb (1996) suggest a distinction between social norms, which while affecting the classroom learning environment need not be specific to mathematical activity, and sociomathematical norms: “The understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm” (p. 461). Sociomathematical norms include “what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant (p. 461). These different types of norms are related, and perhaps social norms may be considered a precursor to developing sociomathematical norms: for example, before a class community could develop consensus over what count as mathematical distinct solution strategies, learners would need experiences sharing strategies and seeking connections between representations. Even with the absence of articulated sociomathematical norms, however, social norms may shape mathematical learning opportunities as they reinforce particular storylines of participations and of ways of doing math, for both students and teachers (Franke et al., 2007).

Who talks or shows work, how s/he shows and explains work, how others ask questions, how the teacher typically solicits participation or responds to questions or student strategies and solutions – these interactions are all connected to classroom norms and affect the many features of instruction, and relatedly, the opportunities for learning and sense-making. Furthermore, seemingly non-content interactions may mediate access to mathematics. For example, acknowledging student contributions, framing students’ mathematical ability, and addressing behavior may influence which students have opportunities to access the mathematical learning opportunities possible through class discussions (Battey, 2013). Interactions aimed at establishing relationships and rapport with students, at connecting classroom mathematical activity to students’ interests and experiences, for example, may include more students in mathematical sense-making (Franke et al., 2007). Non-content specific interactions, like those structuring expectations for behavior or for setting the social climate, still affect the mathematical work in the classroom because they make content-related discussions possible (Ottmar et al., 2015).

For this work, *normative practices* refer to those patterns or regularities in interactions that are inherent in the role of the teacher: to guide the development of the mathematical activity and to develop the social culture of the classroom. The *normative practices* stemming from the teacher’s moves or actions shape the classroom norms, the routines, regularities, and shared expectations of how the teacher and students interact with one another and with the content. It is important to note that normative practices that guide the development of the mathematical activity are not necessarily sociomathematical; they may also address social interactions around the work of the class. Through structuring participation or engagement with mathematical tasks or tools, normative practices shape the learning opportunities accessible to students and act as signals of what doing and knowing mathematics entails. As such these normative practices initiated by the teacher may support or constrain access to mathematical practices and proficiencies, constructs developed further in the next section.

Mathematical Practices & Proficiencies

In current educational discourses in policy, practice, and research, goals for learning mathematics are framed not only as developing fluency with sets of topics and concepts but as fostering ability to engage in *mathematical practices*, or “the way(s) in which people approach, think about, and work with mathematical tools and ideas” (RAND Mathematics Study Panel, 2003, p. 32). Learning mathematics in school has come to mean more than developing ease and competence with mathematical content and skills (such as computing or using well-known formula and algorithms). Knowing mathematics involves more than knowing *what* or knowing *how to*; it involves knowing *why* and *when to* – making and justifying strategic decisions about methods and approaches when confronted with novel problems. Table 1 illustrates the convergence around the ideas of mathematical practices, from the initial Process Standards outlined in the NCTM *Standards* (1989, 1991, 2000) documents to the research-based framework of *mathematical proficiencies* (Kilpatrick et al., 2001) to the recently published Common Core State Standards (CCSSI, 2010). Taken as a set, these documents codify what learning to think mathematically entails: the development of both a “mathematical point of view – valuing the processes of mathematization and abstraction and having the predilection to apply them” and “competence with the tools of the trade, and using those tools in the service of a goal of understanding structure – mathematical sense-making” (Schoenfeld, 1992, p. 335).

Table 1.

Mathematics Learning Goals and the Emergence of Mathematical Practices in Standards Documents and Research Reports

Process Standards (NCTM, 1989, 2000)	Mathematical Proficiencies (Kilpatrick et al., 2001)	Standards for Mathematical Practice (CCSSI, 2010)
Problem Solving Reasoning and Proof Communication Connections Representations	Conceptual Understanding - <i>comprehension of mathematical concepts, operations, and relations</i> Procedural Fluency - <i>skill in carrying out procedures flexibly, accurately, efficiently, and appropriately</i> Strategic Competence - <i>ability to formulate, represent, and solve mathematical problems</i> Adaptive Reasoning - <i>capacity for logical thought, reflection, explanation, and justification</i> Productive Disposition - <i>habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy</i>	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Three important definitions or clarifications surface from this attention to mathematical practices, to ways of doing and knowing mathematics. First, mathematical practices, or those way that learners engage with tasks and problems when thinking mathematically, include using multiple representations and symbolic notation efficiently, justifying mathematical claims, seeking and describing patterns and making mathematical generalizations, and reconciling alternative explanations and arguments (RAND Mathematics Study Panel, 2003). This attention to mathematical practices permeates the learning goals across the standards documents through verbs and phrases such as *construct*, *model*, and *express regularity* (suggesting what learners should *do*) and the repeated use of words like *reasoning*, *explanation* and *strategic* (suggesting

how learners might be thinking). Second, in moving from a conception of mathematical learning as the transmission and receipt of existing facts, skills, and concepts, to partaking in mathematical activity, epistemological questions arise over where authority for adjudicating logic and correctness reside. Through practices of reasoning and justification, authority potentially shifts to learners, rather than being the sole province of the teacher or text (Lampert, 1990). This has implications for learner perceptions of self-efficacy, as learners potentially are able to justify their strategies and solutions based on the soundness of the mathematics and not the external evaluation of the teacher. Third, there is attention to the role and place of mathematical knowing outside of school, particularly through the concept of “productive disposition” – the “habitual inclination to see mathematics as sensible, useful, and worthwhile” (Kilpatrick et al., 2001, p. 5) – and the importance of persevering with problems. This evokes a sense of mathematics as a relevant, connected, dynamic, growing discipline, as opposed to one that is codified, stagnant, archaic, and disconnected from other disciplines and the world.

Normative practices – those interactions that compose the internal regularities in how teachers and students engage with content and with each other – matter to the ways in which mathematical practices and proficiencies are developed in the classroom. Simply stated, “The ways norms are shaped influence which students learn, what they learn, and how they learn it” (Franke et al., 2007, p. 238). Normative practices set the stage for a learning environment in which mathematical sense-making and understanding are prioritized. Hiebert and colleagues (1997) identify four attributes of classrooms that foster a healthy social culture in which students and teacher engage in mathematical sense-making: 1) discussions are about methods and ideas, with the aim of making them more powerful, connected, and efficient; 2) students choose their own methods and share them with others, thus distributing the responsibility for learning and the authority for knowing; 3) mistakes are sites for learning; and 4) correctness is determined by the logic of the mathematics, thus engaging students in methods of argumentation. Connecting the

social climate of the classroom to the development of mathematical practices and proficiencies is straightforward: a classroom in which students choose and share methods becomes one in which learners may be using multiple representations, making connections between alternative strategies, and possibly making generalizations. As such, learners are developing strategic competence and reasoning, and possibly a sense of self-efficacy as their math understanding helps them assess the validity of different approaches.

For this work *mathematical practices* refer to an emergent analytical category that includes both the presence of mathematical thinking practices like representation, connections, and reasoning in instruction and also the potential to develop a productive disposition towards mathematics. A productive disposition towards doing mathematics is related both to the math learning opportunities accessible to students (developing perceptions of self-efficacy, of being able to do particular mathematical practices) and also what is signaled about what it means to do and know mathematics – the ways in which it is “sensible, useful, ... worthwhile,” and relevant to students. In this analysis I considered what kinds of mathematical learning opportunities were shaped by the normative practices and social culture of the classroom, and how.

Teaching Practice & Learning Teaching

With respect to teaching, the term *practice* is used broadly and, in some instances, interchangeably, to connote what is happening during instruction (a teacher’s daily practice), what should be happening during the work of teaching (developing productive or “best” practices), how it is happening (through practice), and as a synonym for the teaching profession (entering practice or becoming a practitioner). Lampert (2010) explicates four different conceptions of practice and their relationship to teacher learning and learning teaching. Lampert emphasizes learning teaching rather than learning to teach “because the infinitive form can suggest that the action is to occur in the future, after something is learned, while the form *teaching* allows us to hold out the possibility that learning also occurs while doing the work”

(p.21). Understanding the learning that happens through the work of teaching is relevant at all points in the teaching career trajectory, but the induction stage in the first years of teaching are of particular interest both because of its chronological proximity to formal (preservice) teacher education and its formative nature for how teachers teach moving forward (as they evaluate, for example, instructional ideas or methods which are effective and therefore worth repeating in years to come).

The term practice can take on multiple meanings with respect to learning teaching. Lampert (2010) identifies four conceptions, each with different implications for analysis and research on instruction, the work of teaching, and teacher learning. These include

- a) Practice in contrast to theory – a dichotomous contrast of “the process of actively carrying out an idea as distinct from the process of having an idea” (p.23)
- b) Teaching as a collection of practices – “things that people do, constantly and habitually” (p. 25), or strategies and routines to achieve particular goals
- c) Practice as rehearsal for future performance – in this sense teaching practice is something learned from experience; and,
- d) The practice of teaching – if teaching is conceived of as a profession, much like law or medicine, then learning the practice of teaching is “about more than acquiring skills or best practices. It involves adopting the identity of a teacher, being accepted as a teacher, and taking on the common values, language, and tools of teaching” (p. 29)

This study’s analysis of patterns of interactions speaks to the latter three constructs: the development of teaching as a collection of practices, the rehearsal in the first two years of teaching in order to develop pedagogical expertise, and the assumption of the professional identity of a teacher, replete with a knowledge base and repertoire of instructional moves.

Through the experience of classroom instruction, teachers learn what works, for whom, and when – a practical or experiential personal body of knowledge to reference. With respect to

teaching mathematics with an attention to sense-making, this experiential knowledge becomes even more important because decisions on how and when to pursue explanations, to solicit participation, to make connections between representations, and so forth are dynamic, interwoven with content and students in moment-to-moment interactions. Franke and colleagues (2007) explain how normative practices, whether social or sociomathematical, develop teachers' own knowledge and instructional practice:

The kind of sociomathematical norms that teachers attempt to develop in the classroom will reflect how they are able to make the mathematics central as they attend to the sharing, make sense of a justification, or decide about the sophistication of a solution. Thus, teachers' learning opportunities are shaped by the sociomathematical norms that are developed in the classroom. If only social norms are developed, such as sharing solutions so that ideas are on the table, and the mathematical differences among strategies are not consistently part of the conversation, teachers do not have to listen for them, focus on them, or learn from them. (p. 242)

Through this analysis of novice teachers' instruction over time, in which I sought to understand how normative practices shaped mathematical learning opportunities and signaled distinct messages about what knowing and doing mathematics entailed, there is an additional opportunity to connect to teacher education and understand how teachers, especially novice teachers, may be learning their practice through practice.

Research Questions & Conceptual Framework

Up to this point, drawing on assumptions about the relationship between learning and teaching in classroom settings for both students and teachers, I have tried to establish a need for examining teachers' instructional practice more closely in order to understand the role of classroom normative practices, as structured by teachers, in the shaping of the mathematical learning opportunities and the crafting of signals or messages about what it means to do and know mathematics. I construe mathematical learning opportunities more broadly than what is there in print, in the materials and tasks learners do, to include the kinds of practices with which learners have access to engage and the potential messages about why doing school mathematics is

worthwhile and who can do and know mathematics – in other words, features of a productive disposition towards learning and doing mathematics. In this section I introduce the research questions guiding this study and the conceptual framework that shapes the analytical lens I used to study instruction.

Research Questions

The following research questions guided this analysis of novice middle school mathematics teachers' instruction over the first two years of teaching.

(1) What is the nature of a beginning middle school mathematics teacher's instruction in the first year? What changes and what stays the same in year 2?

How does the teacher guide the development of mathematical ideas?

How does the teacher develop the social culture of the classroom?

(2) How does the teacher's instruction shape the kinds of opportunities to learn mathematics?

What does the instruction signal about what it means to know and do math?

(3) What are the personal and contextual factors that influence novice teachers' instruction and changes in instruction?

For this study I organize normative practices along two key roles of the teacher: guiding the development of mathematical ideas and developing the social culture of the classroom.

Developing the social culture includes interactions that teachers make to establish relationships and rapport with students; while not strictly oriented towards content, I contend that these actions nonetheless shape the mathematical learning opportunities and the signals or messages about why and who do and know mathematics. In examining teachers' instruction over time, I consider factors, both personal and contextual, that may influence teachers' practice in order to understand both resources and constraints on how teachers develop their teaching. Personal factors may include orienting beliefs about mathematics, mathematics teaching, and teaching in general, and perceptions about the context (school and district), the students, and the curriculum. Contextual

factors include curricular guides, school and district structures, and opportunities to receive and implement feedback on practice from colleagues and mentors.

Conceptual Framework

Lampert (2001) explains “In order to go beyond compelling narratives toward a better understanding of the practice of teaching, we need a representation of the multiple levels of teaching action as they occur in different social relationships over time to accomplish multiple goals simultaneously” (p.28). I endeavor to do this, to understand the role of normative practices, both content-related or not, explicit or implied, planned or unintentional, on the mathematical activity in which teachers and students engage. Normative practices shape learning opportunities in part by influencing the participation structures within the classroom: who talks, about what, when, and how. The conceptual framework, shown in Figure 3, represents how different normative practices developed within classroom instruction between teachers and students come to shape mathematical learning opportunities and signals of what it means to know and do mathematics. Here I explain the dimensions of this framework and map the research questions to it.

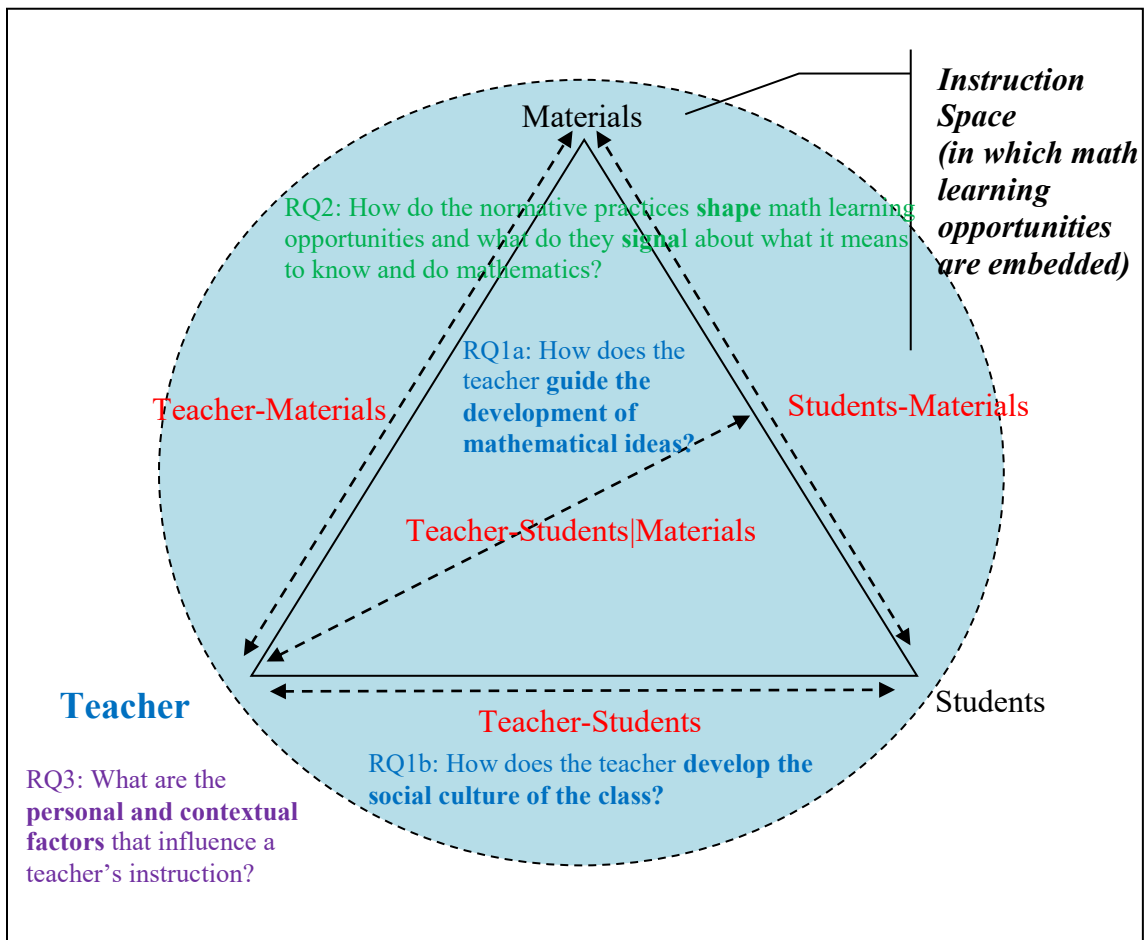


Figure 3. Conceptual Framework guiding this analysis, with a focus on the interactions encompassed in the teacher’s role in instruction.

Lampert’s (2001) “problem spaces” and relations and Cohen and Ball’s (1999) instruction-as-composed-of-interactions perspectives provide the underpinnings of the conceptual framework. For this analysis of instruction I focus on the actions and interactions in which the teacher is involved by nature of her/his position as the pedagogical guide or authority in the classroom; within a classroom setting the teacher, by her job description alone, mediates how students engage with content. The major roles of the teacher (Hiebert et al., 1997) include guiding the development of the content (the mathematical ideas, skills, and concepts) and developing the social culture of the classroom community (cultivating both social and sociomathematical norms,

and establishing relationships and rapport, both with individual students and with the class as a whole). The roles offer a way to group the many different actions teachers make in the course of a single lesson. These actions, in overlapping occurrences within lessons, form the normative practices that matter to the students' math experience and learning opportunities (Research Questions 1 and 2). How the teacher guides the development of mathematical ideas traverses both the teacher-materials "problem space" and the teacher-students|materials "space" (including interactions with students about content, and decisions about content based on what students are doing or understanding). How the teacher develops the social culture of the classroom primarily traverses the teacher-student space in that those interactions may have to do with who participates and which behaviors or ways of interacting are supported or curbed and how.

I contend that how the teacher guides the development of mathematical ideas is not the only way through which interactions shape and signal what it means to do mathematics in this classroom. The other actions the teacher takes, while perhaps not directly content-related, shape what it means to do math in the class – the contours of participation in the mathematical practices in this classroom (Research Question 2). The moves that teachers make that may not seem directly related to mathematics still influence student access to particular mathematical practices and proficiencies by shaping the learning opportunities in which the students engage and the ways students may perceive their self-efficacy. Interactions along the T-S vector speak to the broader classroom climate and conditions conducive or prohibitive to learning opportunities that emphasize mathematical practices of justification, explanation, representation, and communication, and that develop a productive disposition towards learning mathematics.

Finally, with the focus on the teacher in her pivotal role in instruction, it is necessary to acknowledge that teachers hold orienting ideas about what it means to learn mathematics, about what effective instruction looks like, and about what matters in the course of their daily work (for example, supporting students by cultivating relationships and knowing student interests).

Teachers also are affected by factors in their context that shape their work. For example, curriculum guides and pacing maps may shape how teachers interact with curriculum materials and plan lessons, along the teacher-materials vector, and school or district initiatives about student conduct may influence decisions along the teacher-student vector. In the induction phase of teaching, teachers are establishing patterns that may become routine, part of their pedagogical repertoire, based on how successful or effective they seem, but with the multiple competing demands on teachers, which “successes” are prioritized and which routines are repeated may not necessarily be those that support the development of mathematical practices. For example, if management of student behavior is a priority, interactions to support a participation structure in which there is a lot of student talk and discussion may not happen. Research Question 3 considers possible explanatory factors, both personal and contextual, that affect teachers’ decisions and interactions. This conceptual framework suggests how normative practices, emerging through teacher interactions, may shape mathematical learning opportunities and signal what it means to know and do mathematics, which may have implications for the development of students’ productive dispositions towards doing math. The framework guides the analytical lens of studying instruction closely. Applied to an analysis of instruction over time, findings about normative practices may provide a window to the development of teachers’ practice and pedagogical repertoire.

Literature Review

In their review of research on the effects of classroom teaching on student learning, Hiebert and Grouws (2007) identify two key features of teaching that promote conceptual understanding and the development of connections between mathematical facts, procedures, and ideas. First, teachers and students attend explicitly to content and treat connections in explicit, public ways; and second, students struggle with important mathematics. Here, the word “struggle” does not mean students engage in work that is too hard, but that challenge and

(cognitive) perplexity are necessary for cognitive growth. For instruction to share both these features – to be connected in explicit and public ways and to involve productive struggle – task (the mathematical work), discourse (how the mathematical ideas, strategies, and questions are discussed), and normative practices (how students individually and as a class participate and work with the mathematics) are necessarily interwoven, even as they may be studied separately.

Key teaching practices supporting these features that promote conceptual understanding include the selection and implementation of worthwhile mathematical tasks, the cultivation of productive discourse, and the establishment of a social culture in which students not only know not only when and how to participate but also have a shared understanding of what counts as a legitimate contribution (NCTM, 2014; Schoenfeld, 2014; see also Kilpatrick et al., 2001; Hiebert et al., 1997). Teachers themselves, however, must navigate and reconstruct their own position as pedagogical authority and mathematical authority in the classroom. Cobb and colleagues (1992) explain this quandary:

... there is still a necessary power imbalance between the teacher and students in that the teacher is the only member of the classroom community who can assess which of the students' constructions constitute a productive base for further learning. One facet of the teacher's active and demanding role is therefore to facilitate mathematical discussions between students while at the same time acting as a participant who can legitimize certain aspects of their mathematical activity and sanction others. (p. 102)

This “stepping in and out” of discussion (Rittenhouse, 1998), of guiding the mathematical activity while still sharing the work on inquiry with students is messy and complicated; the teacher must manage “the tension between the group's pursuit of shared mathematical understanding and individuals' conflicting contributions” (O'Connor, 2001, p. 180; see also Brodie, 2010).

How teachers negotiate these concurrent dilemmas may be understood more clearly by examining instruction in context. In this section, I consider the contributions of research on mathematics teaching and learning to understanding of the relationship between dimensions of instruction, such as task and discourse, and the mathematics experienced (the “what” is learned)

and mathematical experience (the “how” it is learned). I then review how analyses of classroom interactions and discourse may provide further insights into understanding the relationship between instructional moves and the ways opportunities to learn and engage in mathematical practices. Finally, I consider what we know about teachers’ learning to teach math, and specific recent efforts to support preservice and novice teachers.

Research on Mathematics Instruction

Instruction and pedagogy matter to the mathematics students learn because instructional interactions shape the mathematical practices in which students engage. An “opportunity to learn” claim further orients this perspective: that students learn what they have the most opportunity to learn (Hiebert & Grouws, 2007). Opportunity to learn is not solely a function or product of teaching; it is not just what is intended to be taught and learned. Opportunity to learn is affected by the conditions of instruction, including the knowledge students bring to the instructional encounter, the expectations of engagement, the nature and purpose of tasks, and so forth. Teachers play an influential role, as their teaching moves – the kinds of tasks they pose, the questions they ask, the expectations they set – all affect the nature of the opportunities students have to engage in mathematical activity and thinking. In this section I review research in mathematics education on dimensions of instruction framed as critical for fostering mathematical sense-making, the role of the teacher, and factors that mediate what happens within each dimension. These dimensions include tasks, discourse and mathematical talk, and normative practices.

Dimensions of Instruction

Task. Mathematical tasks are the “classroom activities, the purpose of which is to focus students’ attention on a particular mathematical idea” (Stein, Remillard, & Smith, 2007, p. 346). Core features of tasks that encourage mathematical sense-making include making the mathematics problematic (cognitive challenge), connecting with students existing knowledge, and

leaving behind mathematical “residue” – those understandings, of both the structure of mathematics and strategies or methods for solving problems, that students take with them (Hiebert et al., 1997). Stein, Grover, and Henningsen (1996) propose a framework for categorizing mathematical tasks based on the kind of thinking they elicit. The *cognitive demand* of a task refers to the kind of thinking processes entailed in solving the task. Tasks that place high-level cognitive demands on students are ones which require that students engage actively in investigation and justification (“doing mathematics”) or connect procedures meaningfully to underlying concepts (“procedures with connections”). Tasks that place a lower level of cognitive demand on students are ones which encourage students to use procedures in ways that are not explicitly connected to meaning (“procedures without connections”), or that primarily emphasize memorization or the rote reproduction of previously learned facts (“memorization”). Considering *task features* is also important: does the task lend itself to multiple representations? Are there multiple solution strategies? Does it demand explanation or justification?

These characteristics of mathematical tasks are not static; the Mathematical Task Framework (Stein et al., 1996) traces mathematical activities from their potential, through their implementation and enactment, thus illuminating what students have an opportunity to learn. In attending to tasks as enacted, the framework highlights those teacher actions that may either maintain the cognitive demand of tasks (such as scaffolding or sustaining pressure for explanation and meaning) or cause the decline of cognitive demand (for example, reducing task complexity by suggesting use of procedures without connections to meaning) (Henningsen & Stein, 1997). The Mathematical Task Framework offers a means to consider not just the intended task, but the one enacted by teachers and students during instruction: “When employing the construct of mathematical task, however, one needs to be constantly vigilant about the possibility that the tasks with which students actually engage may or may not be the same task that the teacher announced at the outset” (Stein et al., 1996, p. 462).

Given evidence that solving tasks of high cognitive demand has a positive impact on students' conceptual understanding (Stein & Lane, 1996) and that such tasks afford important learning opportunities for all students (Zohar & Dori, 2003), researchers continue to study how to support and maintain the enactment of cognitively demanding tasks. For example, how the task is set up is a critical time because during that time, teachers can alter the cognitive demand by changing expectations or providing a clarification that narrows or funnels the task by emphasizing one particular strategy (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). Teachers' enactment of cognitively demanding tasks is related both to teachers' mathematical knowledge for teaching and conceptions of teaching and learning mathematics (Wilhelm, 2014). Wilhelm's analysis of the relationship between teachers' knowledge, conceptions, and practice suggests, perhaps not surprisingly, that teachers who posed high-level tasks but described visions of math teaching in which clear, direct instruction is pivotal tended to enact tasks in ways that decreased the cognitive demand of the task. However, teachers who stated productive views of how to support struggling students were more likely to maintain the cognitive demand; it would be useful to the field to understand how these teachers enacted supports for the struggling students that also supported the enactment of the tasks.

Discourse, or Mathematical Talk. Most simply stated, discourse in instruction refers to how teachers and students talk with one another in the context of the classroom. This mathematical talk is integral to learning; Lerman (2001) writes, "Learning mathematics or learning to think mathematically is learning to speak mathematically" (p. 107). Classroom discourse is the means for students to engage in mathematical practices of communication, explanation, reasoning, and justification. For teachers, it also provides a window into students' thinking, questions, and developing (mis)understandings. The role and responsibilities of the teacher in orchestrating and supporting classroom discourse have been enumerated in many ways. Rittenhouse (1998) ascribes two roles to the teacher as s/he steps in and out of discussions – that

of *participant*, modeling requests and questions, and that of *commentator* talking about the mathematical talk and supporting the class establishment of what constitutes an acceptable and complete solution and how to put forward the mathematical merits of strategies. Fraivillig, Murphy, and Fuson (1999) articulate a framework of teaching strategies to advance learners' mathematical thinking: the domains of activity are *eliciting*, *supporting*, and *extending* mathematical thinking. Eliciting moves include encouraging elaboration and clarification of strategies, while reminding students of background knowledge or similar problems are supporting moves. Extending moves deepen the mathematical thinking: for example, considering the interrelationships among concepts, or promoting use of more efficient solution methods. Similarly, Walshaw and Anthony (2008) distinguish distinct roles the teacher enacts with respect to classroom discourse. These include drawing out specific mathematical ideas, fine-tuning the mathematical language and conventions, and shaping mathematical argumentation. These many ways of describing the role of the teacher in classroom discourse share common features, namely that through discourse moves, teachers negotiate understanding between students and the discipline (language, conventions), and support students in taking up mathematical practices.

The teacher orchestrates classroom discourse when she or he poses questions, asks for clarification and elaboration, addresses errors, and decides whether or not to provide information, model strategies, or pursue in greater depth an idea from what students suggest during a discussion. The decisions to pursue these kinds of actions are not straightforward; they take place in the moment, as teachers may be juggling competing or conflicting demands, such as pursuing a particular mathematical idea, addressing misconceptions, or monitoring students' participation. For instance, determining what to *tell* and how and when poses a dilemma for teachers (Chazan & Ball, 1999; Lobato, Clarke, & Ellis, 2005; Smith, 1996). Smith (1996) raises questions about teachers' sense of efficacy, and authority, when traditional modes of "telling" – such as stating facts and demonstrating procedures – are discouraged. Chazan and Ball (1999) argue for

reconsidering purposes of telling – such as to introduce mathematical language or representations that focus, build upon, or redirect ideas. Lobato, Clark, and Ellis (2005) propose reformulating “telling” actions as interactions that respect the constructed nature of knowledge: *initiating* actions stimulate students’ mathematical understandings by introducing new ideas into the classroom conversation, while *eliciting* actions help the teacher to ascertain how students are interpreting the information. How, when, and whether teachers address errors raises another dilemma. While it may seem counter-intuitive to hear that a teacher leaves an incorrect student contribution unaddressed, O’Connor (2001) proposes that the place of errors within the lesson matters. O’Connor observes a teacher correcting students’ unclear references during a review of an idea that has been discussed extensively, in order to give learners the opportunity to solidify their knowledge. In such summative exercises, correctness and accuracy is key; O’Connor raises the question of whether corrections are necessary, even appropriate, during exploratory talk. Deciding when and how to ask questions or press for clarification or elaboration also is fraught with competing priorities for teachers. Brodie’s (2010) examination of “press” moves during instruction identifies the dilemmas two teachers encountered as they made decisions, in the moment, of whether to take up or ignore learner contributions, and whether or not to press for meaning. These were not singular issues the teachers encountered; rather, they occurred while the teachers were also considering how to support individual learners while also maintaining a climate in which many students could engage. In promoting productive discourse, teachers are dilemma managers (Lampert, 1985), making decisions, both explicit and unintentional, that shape what it means to do and learn mathematics in their classrooms.

Normative Practices. Together, teachers and students establish the norms for doing and learning mathematics; these normative practices make up the shared expectations and perspectives of what counts as mathematical work in the classroom (Franke, Kazemi, & Battey, 2007; see also Carpenter & Lehrer, 1999). Hiebert and colleagues (1997) propose four guidelines

for fostering a healthy social culture in which students and teacher engage in mathematical sense-making: 1) discussions are about methods and ideas, towards making them more powerful, connected, and efficient; 2) students choose their own methods and share them with others, thus distributing the responsibility for learning and the authority for knowing; 3) mistakes are sites for learning; and 4) correctness is determined by the logic of mathematics, thus engaging students in methods of argumentation. These norms, however, could be adapted to other subjects or disciplines; Yackel and Cobb (1996) distinguish between these social norms and sociomathematical norms: “The understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm” (p. 461). Sociomathematical norms include “what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant” (p. 461). Establishing what counts as acceptable explanation and justification again raises questions of authority in the classroom. Examining normative practices offers a window on the extent to which teachers and students share authority (Amit & Fried, 2005; Hoffman, Bregfoyle, & Dressler, 2009; Hufferd-Ackles, Fuson, & Sherin, 2004).

In attending to the role of normative practices, this study contributes to an ongoing research about how instructional interactions – including non-content interactions – may mediate students’ access to different mathematical learning opportunities (Battey, 2013; Stockero & Van Zoest, 2013). Battey (2013) explores how relational interactions, defined as “a communicative action between two people that conveys meaning,” ranging from nonverbal cues to a quick comment to an extended exchange, enable or constrain students’ access to quality mathematics. Relational interactions such as addressing behavior, framing mathematics ability, acknowledging student contributions, and attending to culture and language mediated access to mathematics, thus affecting students’ learning opportunities. Ottmar, Rimm-Kaufman, Larsen, and Berry (2015)

study a similar issue from a different perspective, examining how a specific approach to developing and cultivating certain social norms was related to the enactment of standards-based mathematics teaching practices that promote engagement with mathematical practices. Cooper (2014) also focuses on how teaching interactions and practices – content-oriented (such as academic rigor) or more general (such as lively teaching) – individually and collectively facilitate opportunities for student engagement. Wood (2013) demonstrates how even within a single lesson, minor changes in context may cause dramatic shifts, so that at one point a student might be engaged in an identity that undermines learning and at another moment be engaged in one that supports a more productive disposition. The analysis presented here contributes to this body of research by highlighting the role of teachers’ content-related *and* non-content interactions in fostering norms that may facilitate (or constrain) mathematical learning opportunities for students.

Current Approaches to Researching Mathematics Teaching and Learning

In the previous section I drew on mathematics education literature to define and characterize dimensions of instruction: task, discourse, and norms. Here, I outline key ideas emerging from recent approaches to researching mathematics teaching and learning that focus on discourse or interactions related to “talk” during instructional episodes. Researchers have utilized tools and methods from sociolinguistics to identify, to describe, and to explain talk moves that support or hinder mathematical meaning-making in classrooms. Here, I first define teacher discourse moves and then examine findings from discourse analysis at both the word/phrase and interaction levels.

In the *Professional Standards for Teaching Mathematics* (NCTM, 1991), discourse is defined expansively:

The discourse of a classroom – the ways of representing, thinking, talking, agreeing and disagreeing – is central to what students learn about mathematics as a domain of human inquiry with characteristic ways of knowing. Discourse is

both the way ideas are exchanged and what the ideas entail: Who talks? About what? In what ways? What do people write, what do they record and why? What questions are important? How do ideas change? Whose ideas and ways of thinking are valued? Who determines when to end a discussion? The discourse is shaped by the tasks in which students engage and the nature of the learning environment; it also influences them. (p. 34)

Discourse may be categorized as discourse for content learning purposes or discourse for social purposes (Herbel-Eisenmann & Cirillo, 2009; see also Cazden, 2001). For the proposed study, I draw upon Cirillo's (2008) definition of discourse moves: "actions taken by the teacher that influence the discourse in the classroom. These actions may or may not be deliberate and may have unintended consequences potentially at odds with teacher's purpose" (p. 43). Deliberate discourse moves may take on the form of challenges, probes, press for elaboration or clarification, hints, directions, and so forth (Krussel, Edwards, & Springer, 2004), but both intentional and unintentional moves shape the mathematical experience (how content is learned) and the mathematics in which learners engage (what is learned). The consequences of discourse moves may be immediate – such as shifting the nature of the discourse, lowering the cognitive demand of a task, or honing focus on a particular idea or strategy. Discourse moves may also have long-term consequences that accumulate, such as the establishment of norms of argumentation or justification.

When investigating the nature, form, and consequences of discourse moves, the grain-size of the construct under analysis may vary, from word- or phrase-length constructs to interaction-length constructs and practices. For example, pronoun use and language choice may highlight relationships of authority and positioning in the classroom: Who is setting the task? Who is doing the work? Who is adjudicating correctness? For example, Herbel-Eisenmann (2009) illustrates how language choice can position teacher, students, and texts (curriculum materials) and index authority during a lesson. Herbel-Eisenmann suggests that one teacher's use of "they" in reference to the authors of curriculum materials allowed her to position herself as an

interpreter, aligned with the classroom community and distant from the authority of the textbook. In other situations, phrases like “they want us to,” in which the “they” again refers to curriculum materials, may position the textbook as the authoritative expert.

At the interaction level, discourse moves such as *revoicing* and *questioning patterns* shape the mathematical work. *Revoicing* – the reuttering or restating of another person’s speech – may take on the form of repetition, expansion, or rephrasing. When a teacher revoices a student contribution, that discourse move “serves to clarify or amplify an idea and allows the teacher to substitute mathematical vocabulary for everyday words or to redirect the conversation” (Franke et al., 2007, p. 233). Consequences of revoicing include recognizing, reorganizing, or possibly diminishing particular stances or ideas (O’Connor, 2001; Forman, Larreamendy-Joerns, Stein, & Brown, 1998). Questioning patterns (Wood, 1998; see also Herbel-Eisenmann & Bregfoyle, 2005; Piccolo et al., 2008) also provide information about which mathematical ideas are positioned prominently and where authority resides in the classroom. Wood (1998) identifies *funneling* and *focusing* patterns of questioning. In a *funneling* pattern, the teacher is the primary authority, asking pointed questions, doing the cognitive work of making connections as she or he pursues clarification from students. Wood suggests that in these scenarios, students need not understand the mathematics they describe; rather, from the pattern of questioning, they can figure out what the teacher wants to hear and say it. In a *focusing* pattern, the teacher shares authority with the students, directing conversation back to the student to explain her/his thinking. Through analysis of discourse moves, we may trace both the nature and content of the mathematical work and the nature of mathematical authority (for example, privileging teacher, privileging text, shared) in the classroom.

Research on Learning to Teach Mathematics while “Learning Teaching”

From an instruction as interaction perspective, teacher actions related to task, discourse, and normative practices can influence the mathematical learning opportunities accessible to

students. In this study I focus on the instructional development of novice middle school mathematics teachers during their first two years of teaching. In this section, I introduce findings from research in teacher education to frame what we know and believe about “learning teaching,” particularly through experience and practice.

Learning to teach mathematics in a manner that encourages meaning making is challenging work, for beginning teachers and experienced teachers alike. Extensive research has conceptualized and explored the different ways teachers must come to know mathematics in order to teach effectively (Ball & Bass, 2000; Ball et al., 2008; Hill, Rowan, & Ball, 2005). Sherin (2002) suggests two types of teacher learning occur during the act of teaching: the modification of existing content knowledge during instruction, as teachers must apply their knowledge flexibly, and the development of new knowledge – of the mathematical domain, of curriculum, and of student learning. Curriculum materials may also play a role in teacher learning. Investigating what teachers learn from successive enactments of curricula, Choppin (2009, 2011) identifies *learned adaptations*, or teachers’ empirically developed knowledge of how curricula develops in practice. Framing the learning trajectory of novice teachers within dimensions of practice evokes a situative perspective on how teachers learn mathematics content and mathematics-specific pedagogy and also develop a sense of professional identity as a mathematics teacher (Peressini, Borko, Romagnano, Knuth, & Willis, 2004).

Learning to teach math in a way that emphasizes talk, shared authority, argumentation, and other mathematical practices has been compared to “relearning” to dance (Heaton, 2000) in that it often requires that teachers teach in a way different from how they learned and experienced mathematics instruction, and it calls for some “invention” and flexibility or improvising as the teachers and students work (or dance) together around mathematical tasks. It makes sense that this kind of teaching might require that teachers shift or hold a different stance or perception of their role. Even when there is evidence of shifting perceptions and beliefs about teaching,

enacting those practices can result in mixed results. For example, Herbel-Eisenmann, Lubienski, and Id-Deen (2006) find that while *global* changes, or shifts in beliefs, values, and practices, as conveyed through practice or through changes in the ways teachers talk about teaching, may support an orientation towards certain teaching practices, *local* changes or contextual factors can attenuate those teaching practices; for example, student expectations or parental or administrative pressure affected how a teacher enacted a curricula despite her alignment at a *global* level with reform-oriented beliefs and practices.

Recognizing that changes in teacher practice do not happen automatically or easily despite supportive orientations towards specific practices or the availability of cognitively demanding tasks and curricula, recent efforts in teacher education have focused on supporting the development of teacher practice through intentional focus on identifying, exploring, and *practicing* practices (Grossman, Compton, Igra, Ronfeldt, Shahan, and Williamson, 2009) or *high-leverage* practices (Ball & Forzani, 2010). Lampert, Franke, Kazemi, Ghouseini, Turrou, Beasley, Cunard, and Crowe (2013) examine how teacher education may be a site for rehearsal, for novice teachers and teacher educators to work together to learn, study, and practice interactions like eliciting and responding to students' performance. Similarly, Kisa and Stein (2015) describe working with teachers to develop intentionally the skill of *noticing*, to support teacher reasoning and thinking about students' thinking, work, and interactions with each other and with content. What is noteworthy of these different efforts is the attention to developing teacher practices intentionally, through practice or rehearsal or sustained engagement, and also the focus on interactions in classes as a window into student (and teacher) learning experiences and opportunities to learn. What distinguishes these practices as high-leverage or promising is that they have the potential to be generative, to disrupt traditional notions and lead to continued growth.

For novice teachers, the work of teaching *in* the classroom – the engagement with content and students – is but one arena for learning teaching. Through resources and supports such as mentoring (Wang and Odell, 2002), collegial interactions (Little, 1982), and professional development (Desimone, Porter, Garet Yoon, & Birman, 2002; Penuel, Fishman, Yamaguchi, & Gallagher, 2007), novice teachers develop understandings not only about content and students, but also about their own instruction. Desimone and colleagues (2002) find that professional development is more effective in changing teachers’ classroom practice when teachers participated collectively (for instance, from the same school, department, or grade) and engaged in active learning opportunities (for example, receiving feedback on their own teaching, or examining student work). Little (1982) identifies school features that support “learning on the job”: these include norms of collegiality, a range of professional interactions including talk about instruction and shared preparation, and a common language. The influence of these kinds of external resources on the developing classroom practice of beginning teachers is fluid and dynamic – intertwined with the teachers’ own beliefs, perceptions, and knowledge of content, students, and the role and responsibilities of the position. In short, *learning teaching* happens both in and out of the classroom, overlapping the many facets of teachers’ work.

Summary

In this chapter I presented the rationale and framework for articulating how normative practices shape both the mathematical work in which students engage and also the “messages” signaled about what it means to do and know mathematics and who knows or is capable of knowing. This study investigates normative practices that beginning middle school mathematics teachers enact, corresponding to and extending existing research both in mathematics education and in teacher education. In the next chapter, I describe the research design, introduce the participating teachers, and explain the analytical lens I developed from the conceptual framework introduced in this chapter.

CHAPTER 3: RESEARCH DESIGN & METHODOLOGY

For this analysis I studied the role normative practices played in shaping mathematical learning opportunities and signaling messages about what it meant to know and do mathematics in novice middle school mathematics teachers' instruction. I then examined changes in practice over the teachers' first two years of teaching. I applied a grounded theory approach (Glaser & Strauss, 1967): through iterations and ongoing systematic review of preliminary findings from jointly coding and analyzing, I refined both the conceptual framework and the emergent analytical categories of normative practices, their features and dimensions, and their role shaping and signaling mathematics. I applied multi-tiered coding procedures (Strauss & Corbin, 1990) and progressively organized findings in both time-ordered and conceptually-ordered matrices (Miles & Huberman, 1994) to understand the role and development of normative practices in the novice middle school mathematics teachers' classrooms. Throughout, I wrote analytic memos (Maxwell, 2005) to organize findings and questions, both by individual teacher and by theme. The findings from this analysis speak to ongoing conversations in mathematics education about the nature and quality of instruction and how to study and characterize instructional practice. Findings also speak to questions in teacher education with respect to how "learning teaching" happens during the induction phase of teachers' careers, particularly if teachers' instructional practice is framed as rehearsal of future practice (Lampert, 2010).

In this chapter I first review the research questions that framed this study and present an overview of the research design of this study. Next, I briefly describe the larger study through which the data were collected. I then explain the rationale underlying the selection of teacher cases for this analysis and describe the teachers whose instruction I studied. Finally, I provide an overview of the data analyzed and describe the methods of analysis.

Research Design

Research Questions and Overview of Process

The following research questions guided this analysis of practice.

(1) What is the nature of a beginning middle school mathematics teacher's instruction in the first year? What changes and what stays the same over the first two years?

How does the teacher guide the development of mathematical ideas?

How does the teacher develop the social culture of the classroom?

(2) How does the teacher's instruction shape the kinds of opportunities to learn mathematics?

What does the instruction signal about what it means to know and do math?

(3) What are the personal and contextual factors that influence novice teachers' instruction and changes in instruction?

To answer these questions, I examined patterns of practice that emerged during episodes of classroom instruction. To identify normative practices, I considered both the prevalence of both content-related and non-content practices, such as soliciting student strategies or solutions or responding to behavior, and also features of certain kinds of practices that mattered to the mathematical learning experience but may not have happened with predictable regularity, such as the kinds of questions posed or how errors were addressed. Then, after analyzing groups of normative practices separately, I re-considered them as an interwoven composite of practices that shaped the mathematical learning opportunities by, for example, advancing or promoting connections between ideas. Finally, I analyzed data from interviews with both teachers and their school administrators and supporting mentors and colleagues to understand both the contextual structures, such as the curriculum or resources for teacher learning, and the orienting ideas about mathematics teaching and learning that the teachers brought to their pedagogy and their practice. Through iterative analysis processes, findings from the analysis informed the next round of data

analysis and coding. Studying these factors shed light on why certain norms or patterns in practice surfaced and whether they changed over the teachers' first two years of teaching.

Research Design: Connecting Questions, Evidence, & Analysis

To address these questions of how normative practices may shape mathematical learning opportunities and communicate, both explicitly and implicitly, signals or messages about what it means to know and do math, I structured a *multiple case study* (Stake, 2005) of 6 new teachers in a single district in which there were structures in place at local and state levels to support mathematics teaching and new teachers. Stake (2005) describes a *multiple case study*:

A number of cases may be studied jointly in order to investigate a phenomenon, population, or general condition. I call this *multiple case study* or *collective case study*. It is instrumental study extended to several cases. Individual cases in the collection may or may not be known in advance to manifest some common characteristic. ... They are chosen because it is believed it is believed that understanding them will lead to better understanding, and perhaps better theorizing, about a still larger collection of cases. (pp. 445-446)

In an *instrumental case study*, the case is examined “mainly to provide insight into an issue or to redraw a generalization. ... It facilitates our understanding of something else” (Stake, 2005, p. 445). In this analysis, I focused on cases of beginning middle school mathematics teachers during their induction into the profession in order to understand their instructional practices and development, the mathematics they enacted and communicated, and the factors that contributed to their instruction, including their beliefs and knowledge as well as their perceptions of challenges, school contexts and expectations, and also resources and support structures.

As stated before, I applied a grounded theory approach (Glaser & Strauss, 1967) to analyzing both the observations and the interviews. To identify normative practices, I began with categories informed by theoretical frameworks and empirical work: in the initial round of analysis

I looked for examples of norms that emerged from the teacher's dual roles of guiding the development of the content and cultivating the social culture in the class. Once I had identified patterns in occurrences and types of interactions, I used analytic memos (Maxwell, 2005) and matrix displays (Miles & Huberman, 1994) to organize and review these patterns, so that I could then revise the conceptual and analytical framework. (For example, I identified an emergent category of normative practices around participation and responding to students that complemented but was distinct from the primary roles of the teacher.) I periodically reassessed the data, looking for repeating or missing concepts. This constant comparative method (Strauss & Corbin, 1990) of identifying organizing concepts – grouping, comparing, and contrasting them helped me identify commonalities and differences in normative practices as well as consistencies and inconsistencies that further informed my analysis, theory-building, and findings. Given this interwoven, iterative process between phases of data analysis, while I present the methodological process for analyzing observations and interviews in this chapter, in the findings chapters (Chapters 4 and 5) I also discuss how the data analysis informed coding and grouping decisions in a recursive way. With multiple sources of data – observations at different time points, artifacts, findings from other assessments of instructional quality, and interviews, I was able to triangulate the information to understand and explain the findings. Table 2 provides an overview of the data or evidence analyzed and the methods of analysis to answer each research question.

Table 2.

Research Matrix – Questions, Data, and Methods of Analysis

Research Question	Data or Evidence	Methods of Analysis
(1) What is the nature of a beginning middle school mathematics teacher’s instruction in the first year? What changes and what stays the same over the first two years?	Recorded videos of instruction in a class over 2 consecutive days, at 3 time points during a teacher’s first 2 years of teaching: Fall Year 1 (FY1), Spring Year 1 (SY1), and Spring Year 2 (SY2)	A grounded theory approach (Glaser & Strauss, 1967), using the constant comparative method (Strauss & Corbin, 1990) to explicitly, systematically and iteratively code and analyze patterns in interactions, as observed in recorded videos of instruction; codes and counts organized in matrix displays and analytic memos to reveal and identify patterns and groupings
(2) How does the teacher’s instruction shape the kinds of opportunities to learn mathematics? What does the instruction signal about what it means to know and do math?	Related classroom artifacts (when available)	
(3) What are the personal and contextual factors that influence novice teachers’ instruction and changes in instruction?	Interviews with teachers and principals at 3 time points (FY1, SY1, SY2), and with mentors and/or supporting colleagues at 2 time points (SY1, SY2)	Code interview transcripts by theme (beliefs, perceptions), then use conceptually-ordered matrices to identify themes within and across teacher cases.

Data Collection: The Teacher Induction Study

For this analysis I studied data collected as part of a five-year longitudinal study of new middle school mathematics teachers’ induction and professional development experiences. The induction study was designed to examine the relationships between new teacher supports, such as mentoring, induction, and formal and informal professional development, teacher knowledge, instructional practice, and student achievement. The study included 66 teacher participants from 11 school districts of varying characteristics in the South and Mid-Atlantic regions in the U.S. School districts varied in geography, size, and population served.

Data collection took place between 2007 and 2011 (4 academic years). Three successive cohorts of novice 7th or 8th grade middle school mathematics teachers entered the study during their first year of teaching and, provided they remained enrolled in the study, comparable data were collected for their first two or three years in the profession. The study used multiple methods to gather evidence about the new teachers' induction and mentoring experiences, school context, content knowledge, instructional quality, and student performance. Data sources included surveys and semi-structured interviews about their backgrounds, professional learning experiences, and work contexts; a content knowledge for teaching assessment of Mathematical Knowledge for Teaching (MKT; Hill, Rowan, & Ball, 2005); and videotaped classroom observations in the fall and spring of their first year and in the spring of their second year (and third year for the all but the third cohort of teachers), which were then coded using the Instructional Quality Assessment (IQA; Boston & Wolf, 2006; more on this in Data Analysis section). Teachers, principals, and mentors (identified by teachers, including those formally assigned as well as other colleagues who were informal sources of support) were interviewed each year.

Research Setting Focus for this Analysis

The larger induction study took place in 11 districts across 4 states in the South and the Mid-Atlantic regions of the United States. For this analysis, however, I focused on teachers working in one district because of several compelling features that composed an “infrastructure” of support for both mathematics teaching and new teachers. (I use the term “infrastructure” in the same manner as Hopkins, Lowenhaupt, and Sweet (2015, p. 411) to describe “the resources that support school leaders and teachers in providing high-quality instruction”.) At the time of the study's data collection, the district had embarked on multiple instructional improvement initiatives for mathematics education. Furthermore, the district was located in a state in which there had been substantial education reform efforts in the past decade, including, as part of the

credentialing process, a structured system to support teacher induction. In concentrating on this district and the context it provided for beginning teachers, I applied a *purposeful selection* strategy (Maxwell, 2005): “This is a strategy in which particular settings, persons, or activities are selected deliberately in order to provide information that can’t be gotten as well from other choices.” In the following paragraphs I describe the particular features, at both the state and local level, of the district that made it a compelling site to study questions about beginning teachers’ instruction and experiences. Then, in the next section, I explain the rationale underlying the case selection of teachers for this close analysis of practice.

Spanning urban, suburban, and more rural areas within a single county, the school district served over 100,000 students and was racially and socioeconomically diverse. Of the 150 schools across the district, there were 23 middle schools. The middle schools included both typical neighborhood schools (serving a student population based on geographic residence) and magnet schools or programs to which students either applied or were accepted by lottery; these programs included two single-sex learning environments (an all-boys school and an all-girls school). Within such a large district there was a range of school achievement, with a third of middle schools designated as “persistently low-achieving” by the state department of education. As a result, the district was engaged in many instructional improvement initiatives, with the mathematics efforts supported by a major corporation foundation grant. At the time of the study, at the middle school level, the mathematics instruction initiatives included the adoption of an inquiry-oriented, standards-based curricula for grades 6 through 8, *Connected Mathematics 2* (CMP2), and extended professional development related to the curriculum for teachers. The CMP2 (<http://www.phschool.com/cmp2/>) curriculum was designed to support reasoning, communication, and representation in mathematics through thematically and conceptually focused explorations of mathematically rich problems. Also, beginning in the 2010-2011 academic year, the district began using an algebra-preparedness curricula, *College Preparatory*

Mathematics: Algebra Connections (CPM) for 8th grade, to support the transition to Algebra in high school. District-wide resources for teachers included unit-aligned professional development sessions organized by district personnel, a cohort “study group” (with teachers receiving in-school coverage to attend meetings) for each thematic unit, support to attend weeklong CMP2 summer institute at Michigan State University, and the assignment of math specialists to specific schools to support instruction in collaboration with school personnel.

State-wide efforts to support teacher development also shaped the context in which new teachers in the district were inducted into the profession. Over the past two decades, through consistent legislative efforts the state had made commitments to improving student learning through system-wide changes, including standards, assessment, and teacher education. One area receiving particular attention was professional development. To complete their state teaching credentials, all teachers with less than two years of experience were required to participate in the internship program. Certified teachers typically completed the internship during their first year in the classroom; for those teachers who were alternatively certified (not entering the profession with a degree in teacher education), participation was delayed typically to their second year of teaching so that it would follow completion of at least some teacher education coursework. Through the mandated participation in a teacher internship (TI) program, beginning teachers were assigned a three-member committee, consisting of an experienced mentor teacher, an administrator, and a teacher educator affiliated with a teacher education program in the state. All three committee members were required to observe teachers three times and then meet and provide feedback as a team. The bulk of the internship was guided by the mentor teacher, who met with the new teacher at least 30 hours outside of school hours, in addition to providing in-class support and observations. The beginning teacher submitted a portfolio, for which major tasks were organized into three cycles spanning the school year. The internship components were aligned to the state teaching standards. They focused on classroom teaching (developing,

enacting, and reflecting upon lesson plans), analysis of student learning, professional responsibilities, and the development of an instructional unit.

In sum, I chose to focus on the instructional practice and development of teachers working in this specific district because of the multi-faceted “infrastructure” in place to support both mathematics sense-making during instruction and also the practice of beginning teachers. Through the recommended launch-explore-summarize instructional sequence the CMP2 curriculum as written created openings for student explanation and reasoning. The induction study requested that observed lessons include some work time and class discussion; within this district, with the CMP2 curricular focus, these components were expected to be part of a typical day’s instruction. With its coherent vision of curriculum implementation and multiple resources to support that vision across the district as well as at school and individual teacher levels, the district stood out in its potential to help new teachers teach with an inquiry-oriented curriculum. Finally, given the state mandates for teacher induction, new teachers were likely to have access to multiple mentors and resources in addition to informal or collegial networks within their schools.

Teacher Case Selection for this Analysis

The induction study was designed to capture beginning middle school mathematics teachers’ experiences, particularly with respect to induction and mentoring activities, and to examine the relationship between these experiences, instructional quality, and student achievement. Focusing on a subset of the data collected, the analysis presented in this dissertation attended to the normative practices novice mathematics teachers cultivated in their classrooms over time, the relationship between those enacted norms and the mathematical learning opportunities available to students, and the ways in which these practices became part of the teachers’ pedagogical repertoire and how they were influenced by the teachers’ orienting views and mindset and their contexts. While curriculum materials and teacher knowledge or educational background do play a role in instructional interactions (they are components of the elements in

the instructional triangle), this analysis presented in this dissertation was not set up as a study of curriculum use, or of the bearing of teacher certification status or educational background on instructional practice. That said, features such as the curriculum, teachers' background experiences and knowledge, and school status (particularly as struggling) all influenced the novice teachers' experiences as they entered the profession. Therefore, to select teacher cases to study, I again used a *purposeful selection* (Maxwell, 2005) strategy. In doing so, I hoped to capture both representativeness (typicality of experiences across beginning teachers in the same district) and heterogeneity or range of experience, and, when appropriate, to establish comparisons (confirming and disconfirming cases) through cross-case analysis.

Both 7th and 8th grade teachers were recruited to participate in the larger induction study, as those grades were typically middle school grades. (In some districts 6th grade was still an elementary grade, though in the focus district for this analysis, 6th grade marked the entrance to middle school.) For this analysis I focused on Cohort 2 (entering teaching in the 2008-2009 school year) and Cohort 3 (entering teaching in the 2009-2010 school year) teachers who were teaching 8th grade, for multiple reasons. First, in the district, the CMP2 curriculum was rolled out on a yearly basis, so that in middle schools the 8th grades were the last to receive the new curriculum. By the time the teachers in this study entered their classrooms, most of their students, unless they were new to the district, had had some prior experience with the curriculum and its launch/ exploration/ summary sequence. The specific inquiry approach suggested by CMP2 was not new to students. Second, because the eighth grade was the last grade to transition to the new curriculum, the infrastructure support at the school and district levels had been developed. In this study, the Cohort 2 teachers began teaching during the initial year of using the CMP2 curriculum for 8th grade, and Cohort 3 teachers were part of the second year of using CMP2. In the spring of the 2010-2011 school year the district began implementing algebra preparedness curriculum

CPM; as such the eighth grade teachers also navigated expectations to use curricula materials with different approaches and content.

Of the Cohort 2 and 3 teachers who began teaching in eighth grade, I focused on the six for whom the data sets were complete (all observations, interviews, and surveys collected for teachers, principals, and mentors and colleagues). To study changes in teacher practice over time, I opted to not examine data from teachers who withdrew from the study after one year or less. I attended to similarities and differences in the teachers' characteristics, including educational background, certification status and pathways to teaching, and the contexts in which the teachers worked, including those whose teaching assignments were "stable" (same grade, same school, across all years in the study) as well as those who switched or were reassigned grades. Table 3 provides a description of the teacher participants.

Table 3

Description of Teacher Participants' Backgrounds

Teacher (Cohort)	Educational / Professional Background	Certification Status (Assigned State Internship Year)	Teaching Assignment (Grade, Curriculum)	School Status	Other Notes
Faith (Cohort 2)	Education Major (Middle School Math, Language Arts)	Certified (Year 1)	Yr1: 8 th Yr2: 6 th CMP2, CPM (Yr3)	School identified as persistently low-performing in 2011	Attended CMP2 training at MSU Reassigned to 6 th grade in Year 2
Marie (Cohort 2)	Education Major (Middle School Math, Language Arts)	Certified (Year 1)	All years: 8 th grade CMP2, CPM (Yr3)	School identified as low-performing, but no sanctions	Attended CMP2 training at MSU
Becca (Cohort 3)	Physics	Alternative (Year 2)	All years: 8 th grade CMP2, CPM	School restructured to single-sex learning environment	Taught single-sex classes (all girls)
Kevin (Cohort 3)	Communication	Alternative (Year 2/3 Jan-Dec)	All years: 8 th grade CMP2, CPM (Yr2)	School identified as persistently low-performing in 2011	
Kyra (Cohort 3)	Business	Alternative (Year 2)	All years: 8 th grade CMP2, CPM	School identified as persistently low-performing in 2011	
Noah (Cohort 3)	Business	Alternative (Year 2)	Yr1: 8 th Yr2: 6 th CMP2	School restructured to single-sex learning environment, identified as persistently low-performing in 2011	Taught single-sex class (boys) Attended CMP2 training at MSU Reassigned to 6 th grade in Year 2

Researcher Role and Access

As a member of the larger induction study's research team, I participated in recruiting, observing, and interviewing teacher participants in the focus district. For teachers in Cohorts 2 and 3, I recruited them at new teacher orientations and through follow-up school visits. I was the primary data collector for many of these teachers, in that I frequently videotaped the observations and conducted the interviews for each year of data collection. As a result, I came to know the teachers and developed rapport over time, even if there were many months in between data collection activities.

In designing the proposed study, I considered possible bias rooted in this familiarity and rapport. To avoid selecting teachers based on personal affinity or informal interpretations of their experiences, I constructed a list of teacher and school characteristics. These included educational background, certification status, school status, and degree of stability or change in each teacher's assignment. I also mapped the teacher participants on to the conceptual framework to note unique or compelling cases. As one aspect of this analysis was to understand teachers' orienting views towards their work, as relayed through perceptions of challenges, supports, and resources, I considered which teachers had switched grades or curricula or had attended extensive professional development related to the CMP2 curriculum, and those whose experiences did not include these activities. Across each category there were at least 2 teachers with shared characteristics, supporting a "replication" logic (Yin, 2003) for the instrumental case study (see Table 4 for more detail).

Table 4.

“Replication” logic (Yin, 2003) of Teacher Cases

Teacher or School Characteristics	Teacher Cases
Educational Background: Completed an undergraduate teacher education program	Faith, Marie
Alternative Certification Status in Year 1	Becca, Kevin, Kyra, Noah
School Status: Single-Sex Magnet Schoolwide Program	Becca (all-girls), Noah (all-boys)
Assigned to 8 th Grade for Years 1 and 2	Marie, Becca, Kevin, Kyra
Reassigned to a different grade (6 th grade) in Year 2	Faith, Noah
Schools identified as persistently low-performing	Faith, Kevin, Kyra, Noah
Professional Development: Teacher received funding to attend out-of-state week-long summer CMP2 institute	Faith, Kyra, Marie, Noah

Data Analysis

Sources of Data

The goals of this study were twofold. First, I sought to identify the normative practices that emerged in novice middle school mathematics teachers’ classrooms and understand how they shaped the mathematical learning opportunities. Second, I examined the ways in which beginning middle school mathematics teachers’ instruction developed over time, by looking both within the classroom as well as at structures that comprised the contexts in which the teachers worked and at the teachers’ orienting views towards teaching mathematics. For the analysis, I drew on the observation and interview data from the induction study.

Observation Data. In the first year of the induction study, teacher participants were observed for two consecutive days in the fall and in the spring. In the second and third years, if available, teacher participants were observed for two consecutive days in the spring. All

observations were video recorded. No requests were made for specific content in lessons; however, when possible, teachers were asked that observed lessons include both times when students were working on a mathematical task and time for students to discuss their work in a whole-class setting. As noted before, in the focus district, these requests for work time and discussion aligned with the intended curricular emphasis teachers were expected to follow. However, observations were scheduled at the teacher's convenience, typically to avoid coinciding with testing or other interruptions. As a result, in the focus district for this analysis, while the recorded observations captured typical instruction, in that the teachers did not alter their plans to create special lessons for observation, the lessons did not always showcase worktime or discussion. Additionally, because of these efforts to accommodate the teachers' schedules and priorities, and also due to changes in materials and pacing calendars from year to year, the observations did not necessarily capture the same content or lesson each year or across teachers; it was a coincidence if the same lesson was taught in two or more teachers' classrooms. Appendix A provides an overview of each observed lesson topic. For the purposes of this analysis, the breadth in topics – from algebraic concepts like slope to geometry to mathematical modeling – supported a focus on normative practices because while the content varied, the ways in which teachers guided the development of ideas or developed the social culture of the class served as the overarching framework through which the students engaged with and experienced mathematics. Both algebraic topics and geometry topics could be taught in a rule-based way, or in a way that emphasized students co-constructing meaning.

In addition to the recorded observations themselves, the teacher induction study used the Instructional Quality Assessment (IQA; Boston & Wolf, 2006) to measure two indicators of high-quality instruction in mathematics lessons: the rigor of the lesson activities and the nature of the class discourse. The Academic Rigor indicator is based on the premise that students'

opportunities to learn are determined by the types of instructional tasks used and the implementation of these tasks during lessons. In particular it focuses on the cognitive demand of tasks, as written and as enacted. It consists of three rubrics: Task Potential, Task Implementation, and Discussion. The other five rubrics provide information on the discourse by focusing on the Accountable Talk in the lesson, or the teacher and student talk moves that respond to and further develop what others in the group have said. The five Accountable Talk rubrics include Participation, Teacher Linking, Student Linking, Teacher Press (posing extending questions) and Student Providing. With respect to the Academic Rigor, ratings of 3 or 4 corresponded to tasks implemented in a way that supported or maintained a high-level cognitive demand; these included tasks that supported applying procedures in a connected way (connected to representations or to other strategies) or exploring and understanding mathematical relationships and scenarios. Ratings of 1 or 2 suggested tasks implemented at a lower level, emphasizing the application of a specific rule or formula, for example. With respect to the Discussion and Talk rubrics, a rating of a 1 or 2 typically corresponded to brief-or one-word answers, or procedural descriptions or demonstrations of steps for solving a problem but not an extended discussion of strategies or ideas. Ratings of 3 or 4 indicated discussion of more than one strategy or representation for solving the task, with explanations including how and/or why those were used. In the induction study, these rubrics were applied to the “main task,” which was defined as the task or activity taking up the majority of the class time. The discussion rubrics applied only to whole class discussions *after* work time. For each observed lesson, two coders from the induction study research team assessed the lesson using the eight rubrics. The ratings for each of the six teachers’ observed lessons in Year 1 (Fall and Spring) and Year 2 (Spring only) may be found in Appendix B. In the analysis of observations I used these ratings both to confirm and to raise questions (or suggest alternate explanations) about the normative practices identified.

Interview Data. In their first year of the induction study, teacher participants were interviewed twice, once in the fall, once in the spring. In the second and third years, if available, they were interviewed in the spring only. The semi-structured interviews probed at a number of issues related to the work and nature of teaching during induction and to mathematics instruction specifically. Topics included challenges, sources of support, interactions with school personnel (teams, administration, leadership), mentor relationships, perspectives on high quality mathematics instruction and its dimensions, and professional development.

Principals were interviewed on the same calendar as teachers (twice in Year 1, once in Years 2 and 3, if applicable). Each year, teachers were asked to identify mentors, both formally assigned and “informal” (typically colleagues); these mentors were interviewed once, unless they were identified as sources of support for multiple years. In semi-structured interviews, both principals and mentors were asked about their perceptions of the teacher’s instruction, of high quality mathematics instruction, and of the resources and sources of support available to the beginning teachers.

Affordances and Constraints of the Data Sources. The induction study was designed to capture beginning middle school mathematics teachers’ experiences, particularly with respect to induction and mentoring activities, and to examine the relationship between these experiences, teaching practice, and student achievement. The purpose of the observations was not to connect instructional planning, enactment, and reflection; pre- and post-observation interviews were not conducted, so there was not an opportunity to ask teachers about specific events in observed lessons. Therefore, I could not make assertions about teacher goals or intentions with respect to specific lessons. In this analysis, however, the purpose of attending to teacher interactions was to illuminate the relationship between normative practices and mathematical learning opportunities

and to examine teachers' practice over time; both these questions could be answered with the data available.

Observation Analysis

The purpose of this analysis was to identify patterns of normative practices. Given that interactions arise within the social space of the classroom, this analysis responded to the need identified in the literature “to consider the relationship between particular classroom practices and opportunities for students to engage” (Franke et al., 2007, p. 226). Normative practices made up the regularity of classroom interactions, so the analytical task was twofold: to identify norms and their features or characteristics, and to establish them as regularities or patterns within the instructional practice. Here I provide an overview of how I strived to do this with the analysis of recorded observations; as noted earlier, given the iterative, grounded nature of this analysis, methodological decisions that accompanied or were driven by findings are described with those findings and evidence in Chapters 4 and 5.

I analyzed the recorded observations in a multi-staged iterative process. I initially viewed a subset of observations and took field notes in free form, taking notes on two groups of questions. First, how could dimensions of instruction like the task and the discourse be described? What were the “activity structures” (Herbel-Eisenmann & Cirillo, 2009), such as going over homework, conducting class business, providing information through a lecture/guided lesson, or launching an investigation. I recorded the duration of these different activity structures and other features, such as how the students worked and who spoke. Second, I listed norms that seemed to develop, categorized in the dual roles/responsibilities of the teacher to guide the development of mathematical ideas and to develop the social culture of the classroom. I noted how ideas were recorded, made public and assessed, evaluated, critiqued, or extended. I recorded these notes on memos I called Observation Overview Sheets, and used other observation artifacts to note the

tasks or problems on which students worked. From this initial field note process I generated a list of normative practices in each of the two role categories as well as some norms that were freestanding (not assigned to a group).

In the second stage of observation analysis, I applied an explicit and systematic lens to coding for normative practices, and I used different matrices both to take notes while coding and to organize those codes (Miles & Huberman, 1994). I focused on interactions that were public and audible to the whole class. There were not always summary discussions, so I opted to attend to this public talk throughout the entire lesson; these interactions contributed to the establishment of the classroom culture because they framed not only how the students did the mathematical work, but also how they interacted with the teacher and with each other. For this iteration of the observation analysis, I watched and coded for norms, recording my notes on a time-ordered matrix, with time codes and activity structures in rows and normative practices in columns (see Appendix C for the Observation Analysis Coding Guide, and Appendix D for a template of the field notes matrix). I tallied when different interactions occurred, noting time points and transcribing illustrative examples as needed. I noted, for example, when the teacher posed questions or provided information or stated a command (“Open to page ...”), when students spoke and what they said, and when a mathematical convention or vocabulary term was introduced. I also noted when the teacher addressed behavior, or acknowledge effort, or encouraged students to participate. Drawing on work describing different kinds of relational interactions (Battey, 2013), I distinguished between social culture moves that were “coaching” or encouraging in nature (for example, praise, both specific or vague) or appreciation for a student contribution from those that referenced math ability or know-how. Codes for norms were not mutually exclusive; this process allowed for overlapping codes. For example, this is how I later

distinguished whether language or conventions were introduced as part of a procedural lesson or through clarification or articulation of new ideas based on student work.

In the next stage of the analysis process, I organized the field notes from the time-ordered matrices onto a new display, an event listing (Miles & Huberman, 1994). I counted the frequency of certain moves or norms during different activity structures (see Appendix E for an example of this Evidence Table). I used these matrix displays for each observation at each time point to explore and describe patterns in normative practices both by time point (What kinds of norms did I observe in Fall Year 1, for example?) and by teacher (Which normative practices changed? Which recurred or amplified? Did certain practices attenuate others?). I then used both analytic memos and descriptive tables or matrices to organize and group the different norms observed; Through this iterative process of watching the videos, coding public interactions, and analyzing and grouping those codes, I developed portraits of what it meant to do math in these classrooms and to what extent there were changes over time. I used memos, detailed narratives, and video excerpts to record and revisit ideas for emergent patterns and questions or inconsistencies. Throughout this process, I was mindful of plausible alternative groupings or new categories, and hewed closely to transcripts and video excerpts to name normative practices. I noted when practices seemed to fall in more than one category and considered whether the groupings of practices implied a revision to the conceptual framework. Through this iterative process and systematic review, the category “responding to students and structuring participation” emerged as distinct group of interactions, rather than a subset of the other groupings.

More detailed descriptions of the specific analysis processes and revisions are embedded in the findings chapters, reflecting the iterative nature of this analysis and the methodological decisions made through constant comparisons. For example, it was through the coding process that I became aware of distinctions between normative practices and how to establish their

“regularity”. While some normative practices occurred with a regularity that could be counted (for example, it was reasonable to anticipate teachers regularly addressing behavior, especially in the first semester of teaching), other interactions would not necessarily occur in *every* lesson (for instance, there was no guarantee the class would address mistakes in a given lesson), so in those instances rich description of the normative practices and interactions was more appropriate.

Finally, to compare and contrast the portraits of instruction I composed through the analysis of normative practices, I also analyzed ratings of the observations using the IQA rubrics for Academic Rigor and Accountable Talk. In this way I confirmed points of alignment between my analysis of instruction and the IQA, and I also identified inconsistencies or areas to which the normative practice analysis would contribute additional information or insight.

Interview Analysis

The analysis of interviews served two purposes. First, I analyzed the interviews to identify explanatory factors, both contextual and personal, that could have affected the normative practices observed in each teacher’s class. Second, the interview analysis served as a method of triangulation, to reconcile and raise questions about the patterns in practices and change I observed with the teachers’ perspectives of their practice over time and also those of their support network (including principals, mentors, and colleagues).

For the interview analysis, I coded teacher (and their related network of principal, mentor, and colleagues) interview transcripts using the Atlas.ti software program for challenges, successes, resources, beliefs and perceptions about mathematics teaching, and perceptions of the teachers’ instruction over time. (Please see Appendix F for the Interview Analysis Coding Guide.) Following this coding, I organized the coded excerpts into case-ordered matrices (Miles & Huberman, 1994) to identify patterns and themes among contrasting and similar cases. I then studied the themes in relation to the practices observed, determining factors that supported or

added nuance to the findings about the teachers' developing practice. Throughout this process I regularly relied on analytical memos (Maxwell, 2005) to record notes, questions, and preliminary findings for further study.

Summary

In this chapter I have presented both methodological decisions and the rationale for them. Consistent with the iterative nature of a grounded theory approach, with each round of analysis I considered the implications of the findings for the framework and the questions raised, and accordingly adjusted and refined codes, categories, and the framework. As these decisions occurred in concert with analysis, I explain them further, as necessary, in Chapters 4 and 5, in which I present findings about the normative practices that comprised these teachers' instruction and the story lines of change over time.

CHAPTER 4: NORMATIVE PRACTICES, SHAPING & SIGNALING MATH

Classroom normative practices, or norms, are those “regularities in classroom social interactions that constitute the grammar of classroom life” (Franke et al., 2007, p. 238). Parallel to how grammar affects how one makes meaning of and from words and sentences and communicates ideas in a manner consistent with language disciplines, norms influence the nature of the learning opportunities in the classroom by shaping how participants make sense of and engage in both the social and the intellectual activity. In mathematics classrooms, normative practices are present in the in-the-moment interactions between a teacher, the students, and the specific content materials through which the teacher and students engage with mathematics. Norms may be intentionally cultivated – for example, establishing that answers should be supported with explanation or justification – but they also exist within the unstated but implied assumptions and expectations teachers and students have of each other and themselves. During classroom instruction, teachers play a critical role because, by nature of their position as the pedagogical authority, they make decisions that mediate how students interact with the content, the materials, and each other. As teachers engage in different kinds of interactions between students and materials, they shape the kinds of opportunities to learn both mathematics content and practices. The normative practices also serve as signals or messages of what it means to know and do math in school, and are interwoven into other dimensions of instruction such as the task (norms of how to work on problems influence the level of cognitive demand during curriculum enactment, for example) and the discourse (who talks, and when, about what, and how). For example, are meanings co-constructed? To what extent is math connected to students’ lives, and how? Does it consist of static rules to be applied? Is there a more dynamic way of making sense of mathematical problems, so that questions both are generated and answered by students? Who determines correctness and validity? Who participates, and when, and about what? Normative

practices contribute to the development of students' *productive dispositions* (Kilpatrick et al., 2001) by conveying why the mathematics activity is useful or relevant and who is able to learn and do mathematics. The latter message is integral to conceptions of self-efficacy around mathematics work.

For this analysis, using data from videotaped observations of instruction during novice teachers' first and second years of teaching, I examined the normative practices – intentionally cultivated or not, mathematical or social – that made up the instruction and shaped the mathematical learning opportunities. I also considered what the norms evoked or signaled about what it meant to do mathematics in these classrooms. The findings presented in this chapter are based on a close and thorough analysis of the teachers' instruction in the fall – the first semester – of their first year of teaching. In the next chapter, after applying a similar analytical lens to observations from the spring of the teachers' first and second years of teaching, I present findings of change, or lack of change, in normative practices over time.

Inside the Classroom: An Overview of Findings from Fall Year 1 Analysis

Before introducing the normative practices that emerged from close examination of novice middle school math teachers' instruction, I first present excerpts of instruction from three teachers' classrooms, to show contrasts in how the teachers enacted their instruction, to set instructional practice within the teachers' classroom contexts, and to introduce emergent analytic categories. These cases serve as windows into how normative practices connected to mathematical learning opportunities and the implications for the development of students' dispositions towards the subject and their views of their own self-efficacy. The vignette of Faith's instruction illustrates the ways the teacher, through norms about how and when students worked on a mathematical task, limited student engagement with the task and portrayed math as a rule-bound subject for which she as the teacher was the guide providing the rules to the students. The

vignette of Marie's instruction provides a contrast; even though both observed lessons addressed the same investigation and content, in Marie's class the norms she cultivated support student engagement with the content so that students co-construct the meaning. Finally, the vignette of Kevin's instruction features how seemingly non-content related interactions – those geared towards fostering a positive social culture, such as acknowledging and encouraging student contributions – also supported mathematical practices of reasoning, communication, and connections.

The Case of Faith

As students entered the class from lunch, the teacher, Faith, asked them to begin their warm up, the Problem of the Day, which was displayed on a projector: naming coordinate pairs for four points on a coordinate grid. Faith repeated instructions over and over, reminding students frequently about the notebook page they should be working on, the expectations for no talking, the need to keep these warm ups in a certain order for an upcoming math binder check. After seven minutes, she reviewed the warm up, calling on four different students to name each of four coordinate pairs. When one student mixed up the order of the pairs, Faith immediately corrected her and restated the answer for the class. Finally, fifteen minutes after class had started, Faith introduced the main activity for the day:

On p. 10, this is going to be our next activity. All right. What they want us to do is they want us to construct squares, all right? They want us to use these points to create specific parks. Ok. Because we talked a little bit yesterday about locations and the use of maps. Well this one, the whole purpose of doing this is we're going to be constructing different figures on this grid. All right? But before we can do that we need to talk about certain properties of figures. So we're going to talk about squares, rectangles, right triangles, and then parallelograms today. All right. But this is the activity. This is our goal. We're striving to get to this activity. ... All right, so if we take a look at this down below it tells us that we need to suppose that there is a park and it needs to be a square, what kind of coordinates would we use? K, and then they want us to construct a non-square rectangle, a right triangle, and a parallelogram. So what we need to do first though is we need to think exactly about the characteristics and the properties that each of those figures have. So we're going to be taking notes over those. So make sure that you have your pencils still out. Once you get this sheet you need to make sure you put your name on it. (Teacher

passed out papers.) All right, so I will be writing the notes as we go along so my paper just looks like yours right now until we start filling it in. (Faith, Fall Year 1, Day 1)

The notetaking lasted more than 30 minutes and was interrupted by a 10-minute bathroom break.

Almost an hour after the students had first entered the classroom, they began working on the activity: drawing a square “park” given coordinates for two sets of vertices. As written, the curriculum task “Planning Parks” (a CMP2 activity from the book *Looking for Pythagoras*) included both work time and discussion and a class summary after that work time. For this activity, there were two ways to draw the square, using the two given points to form one side or border. After three or four minutes of quiet independent work time, Faith called the class back together and demonstrated how to draw one possible square. She then hinted to students to use a line of reflection to find the other square. Before asking the students to resume working, she asked everyone to stand for a stretch break, during which the students took turns counting by 1s or 2s. Whenever a student said 10, she or he had to sit down, and the counting recommenced at 1. This skip counting exercise continued for five minutes until there were only 2 students left standing. Finally, after their counting face-off, Faith gave the last student standing a free homework pass and then marked on the projector the coordinates that she noticed one student, Kiera, had written for the second square park. Without further discussion, Faith instructed the other students to copy this solution down in their notes. Class ended 80 minutes after it had started. Throughout the work time, and for that matter, the entire class, Faith alternated between reminding and exhorting students to do more work, to be prepared, to talk less, to pay attention; with each repeated direction her tone grew more frustrated, her volume slightly elevated.

The Case of Marie

On the second day Marie’s class was observed, the students also were working on the CMP2 investigation “Planning Parks.” Given that Descartes Park had one border with vertices (1, 1) and (4, 2), students were asked, “Suppose the park is to be a square. What could the

coordinates of the other two vertices be?" The text prompted students to find both possible squares. The following excerpt depicts what transpired in the 8 minutes the class worked on this question, which they began promptly after their warm up work time and review, about 12 minutes into the class. (Marie, Fall Year 1, Day 2)

Teacher Marie: What are the labels - what are the coordinates of the park that they give us? Blake? (1, 1), and (4, 2). So I'm marking it on my thing so you have it. (*Marie marked the coordinates on the projected image.*) Now part A - make sure you're just in the grid that has A on it. Suppose the park is to be a square? What could the coordinates of the other two vertices be? Experiment. Experiment around. See if you can figure out how to make a square. How do you make a square using these two coordinates that they already started? A square means that each of your side lengths has to be equal. So use your pencil. Mark lightly if you need to. There's more than one answer. So you need to make a square. This, right, here, this is one edge. Each edge has to be that same length 'cause it's a square.

The students worked in their table groups while Marie moved around the classroom, observing what students were doing and answering individual questions. After about 3 minutes of work time, Marie began a discussion with the whole class while students continued to work.

Teacher Marie: All right, Darien, will you, will you use a different color in there and see if you can find a square. Cause this one's a little hard. It would be easy if it were along the same line, it would be easy if it were along the same line or a road (*She gestured to a grid line, parallel to the x-axis*), like we were talking yesterday. It's not. ...

Darien: (0, 4) and (3, 5).

Teacher: Connect those please. You can use purple. All right, does this look like a square?

Class (more than a few students): Yeah.

Teacher: Perfect. This is a great answer. This isn't the only answer. Is there another answer? 'Cause remember, we're using this segment (*Marie outlined the line segment connecting the original vertices, (1, 1) and (4, 2)*). So in other words this segment has to be one of the edges of our square. (*Marie called on another student, Lakiya.*) Do you want to see if you can find another square in it? (*Then she spoke to the whole class in an upbeat tone, while Lakiya draws.*) When you're finding these shapes, it's going to take a little bit of work. You're going to have to experiment and um you're going to have to do some erasing. Good thing it's math class and we've got pencils!

Lakiya drew a square with sides that were 3 units long, but her proposed square was flush with the grid lines, with coordinates (1,0), (4, 0), (1, 3), and (4, 3). This square did not meet the criteria outlined in the problem because it did not use the segment with the given coordinates as a side. Instead, that segment intersected the proposed square. (See Figure 4.)

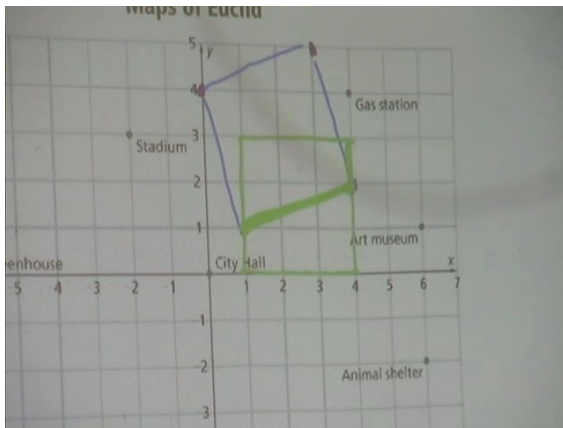


Figure 4. Vignette from Marie’s class– Screen Shot: A student’s proposed solutions to square problem from “Planning Parks” investigation.

After Lakiya finished drawing, Marie assessed her proposed square:

Teacher: Really? OK, that’s a square, I agree. But Lakiya, that line, that I made darker, has to be one of the edges of it, of our square.

Lakiya murmured something inaudible, erased the square and moved to sit down. Marie continued,

Teacher: Um, okay, who has, who can try a different one? Go ahead, Shelby.

While Shelby drew her different square, the teacher continued to walk around the class, answering questions and making suggestions to students as they work. After another two minutes, Marie drew the class’s attention to Shelby’s proposed square:

Teacher: All right, perfect. All right, thanks for staying focused. How many of you - show of hands - how many of you got one of these 2 squares? Good, nice job. If you didn’t that’s okay, that’s totally okay. This is going to take a little bit of imagination for you to find them because they are not very easily identifiable, they’re not jumping out of the page at you. But here are two squares. See how we got those?

Marie paused, and then, with inflection in her tone, repeated that question with emphasis.

Teacher: How *did* we get those? 'Cause we were given these two points. Darien, how did you find your square?

Darien: I just measured how far the blocks were and then I tried to - I measured that line right there and then ... I just did that on the other side.

Teacher: What did you measure with? Because you don't have a ruler on your table? How'd you measure it?

Darien: My pencil.

Teacher: Your pencil. OK. Marley?

Marley: I counted 3 squares.

Teacher: Interesting. Interesting. So what you're saying, Marley, is that you counted these 3 squares. (*Marie pointed to the three squares that bounded the given line segment.*) And then you're like, well, that's how long it is and it's going from the diagonal to the diagonal, right? Diagonally across those squares. So how did you use that to help you find the answer, Marley?

Marley's words were not audible on the recording, but she gestured with her arms, moving her hands in a diagonal way.

Teacher: Perfect. perfect. Each line has to be this length. Each line has to go diagonally across three squares, right, cause that's the length of this line. So, she's like, well, if I went diagonally across these squares it would be all the way up here. If I used these three - Want me to draw these out, what I'm saying? She's talking about these 3 squares, right here, and then she used this same point, right here, and used these three squares, and then went diagonally across those. (See Figure 5.) OK? And then she used these 3. Each time she moved, she went diagonally across 3 squares. Does that make sense?

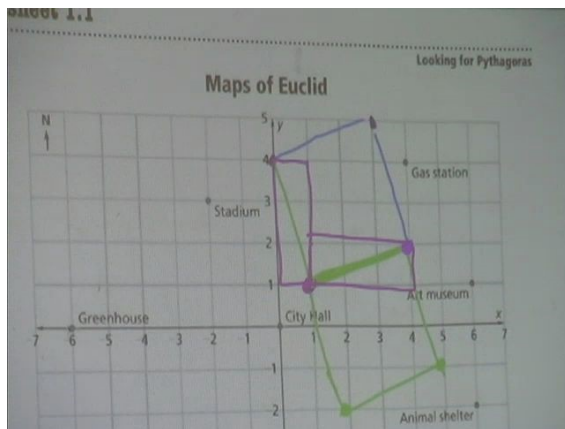


Figure 5. Vignette from Marie’s class – Screen Shot: Proposed solutions to square problem from “Planning Parks” investigation, with teacher’s demonstration of how to use diagonals to determine equal side lengths.

After this, Marie posed a similar problem to the class: draw another two square parks given a different set of vertices and length for one side. The class reviewed the idea of using the number of squares that the given line segment crossed in order to make sure each side is the same length. Then they moved on to the next parts of the investigation: finding coordinates to create a non-square rectangular park, a triangular park, and a park in the shape of a parallelogram. This work took up the remainder of the hour-long class.

The Case of Kevin

As students entered the class, Kevin chatted with them, until, after a minute or two, he counted down: “5, 4, 3, 2, 1, and zip. And zip.” He then asked students to begin work on the warm up, which reviewed concepts and ideas they would use during the lesson. Students were asked to find the areas for given figures drawn on grid paper. (See Figure 6.)

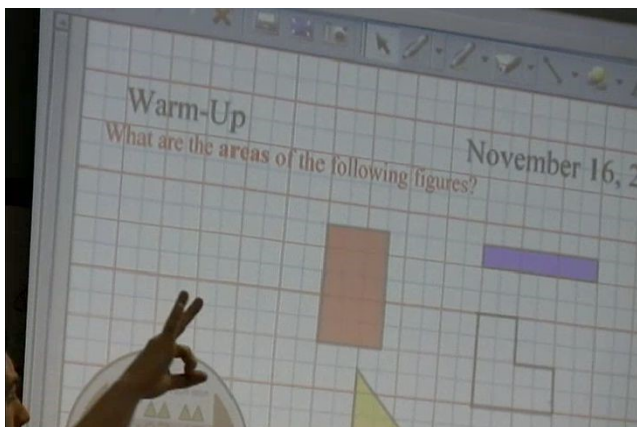


Figure 6. Vignette from Kevin’s class – Screen Shot: Warm Up Task “What are the areas of the following figures?”

During the 7 minutes of work time, Kevin walked around the classroom, frequently praising on-task behaviors he wanted to encourage, and reminding students that he appreciated their effort, that he believed they would have a great class, and so on. Even when speaking to individual students, he made these comments loudly, audibly, so that the entire class heard; these comments were intended to be public. They served as reminders of the expectations Kevin wanted to reinforce for students’ work and behavior.

When the warm up work time was over (determined by a timer displayed on the projection screen), Kevin commented, “As far as I’m concerned, every single person in here did come in and work on their warm up. Every single person came in here and worked on their warm up. That’s a 10% today for everybody, so far. I appreciate that.” He then began the review of the warm up task. (Kevin, Fall Year 1, Day 1)

Teacher Kevin: Let’s go over this. Someone raise their hand, anybody, and tell me the shape, or tell me the color - tell me the color of the shape and what the area of that shape is. Save the yellow one [*the triangle*], let’s not do the yellow one just yet. Ronnie, pick one.

Ronnie: The, uh, red one. [*the wide rectangle*]

Teacher: OK. What is the length? What is the length of the red one?

Ronnie: 5.

Teacher: What's the width?

Ronnie: 3. 15 units squared.

Teacher: What is it?

Ronnie: 15.

Teacher: 15 units squared. 15 units squared. 15 units squared. You're absolutely right, great job, Ronnie. Units squared, because we're talking about actual physical space. So they're square units. Remember how we talked about the tiles in here, how many squares are there? Well, in this classroom, if that were a classroom, there would be 15 units squared. Someone pick another shape, you have the choice, save the yellow one for me, save the yellow one for me. Shia, pick one for me.

Shia: Um, the blue one. [*the long rectangle*]

Teacher: The blue one. What's the area of the blue shape?

Shia: 6

Teacher: What strategy did you use for that?

Shia: I did length times width. I did – I counted what was going on the y-axis and I counted what was on the x-axis and I times it by each other.

Teacher: Great. So you did what times what?

Shia: 1 times 6.

Teacher: Which equals?

Shia: 6.

Teacher: 6 units squared. Fantastic, Shia. Very good work. Now, someone raise their hands if you had another strategy. Raise your hand if you had another strategy for the blue one here. Raise your hand if you just counted. Did anyone just count the 6 blocks? Good. That's a –

Student, calling out: It's just like 1 times 6, so that automatically (*voice faded*) ...

Teacher: That's a perfect strategy, especially for those really small shapes. Why not count 'em? You don't have to do a whole computation for 1 times 6. Shia? Couldn't you have just counted those 6?

Shia agrees: Um hum.

Teacher: Good. OK, who can do the white one [*the irregular shape*] for me? Miles, what did you get for the white one? Think about it, this white shape right here.

Miles' response was barely audible so Kevin paraphrased what he said.

Teacher Kevin: OK, I like how you are using the words length and width but it's a little bit different than that. So I like how you used those words though. Derek? What's the area?

Derek: 12.

Teacher: It's 12. But here's the deal. What if the width is 4 and the length is 4? What's 4 times 4?

The class murmurs: 16.

Teacher: 16. Is that 16?

Class: No.

Teacher: No. Oooh, so – (*He walked around looking at students' work, then stopped to listen to one student explain.*) Fretinisha, that's a fantastic strategy. I wish you would have raised your hand but it's a fantastic strategy. (*Kevin moved back towards the projector.*) She said, "I made this into a whole block. I did 4 times 4. Then I subtracted out that little piece." Who has another strategy? Ronnie?

Ronnie: ... 'cause there was 4 in each box.

Teacher: Yep! Yeah, great. Let me draw on there so you guys can see what I'm talking about, what Ronnie's talking about. Ronnie said, 'Just break 'em down into 3 four-boxes.' Here's a box of 4, here's a box of 4, and here's a box of 4. So Derek, again, can you tell me what you said the area was?

Derek: 12.

Teacher: 12 units squared (emphasized). Fantastic. OK, let's bring out our big guns, our ultimate thinking hats, all right? What is the area – gotta raise your hand – of the yellow triangle? Megan?

Megan: 6.

Teacher: It's close to 6, it's not 6. Derek?

Derek: 8.

Teacher: 8. 'kay. Raise your hand if you got 8. Be proud of that. Do not feel bad if you didn't get that because this is brand new. Shhhh. I'll wait. I'll wait on you guys. Thank you. It's 8. Derek, what strategy did you use to get 8 for this?

Derek: I found the whole squares.

Teacher: How many whole squares did you find?

Derek: 6.

Teacher: 6. So he did – Megan, this might have been what you did, he counted 1, 2, 3, 4, 5, 6 whole squares. But what did you have left over? Shh. I know if you found it but give Derek the respect, please.

Derek: I put the halves together.

Teacher: OK, so you decided that this half can go right here and this half can go right there. And what was your total area? 8. 8. That's a great way of doing it. But what if, what if your triangle is as big as like a house. Do you have time to count every single block? No. So hold on, yeah, what's your strategy?

Male student, unidentified: You – like, you multiply the ones first and then you (*his voice faded and was not audible on the recording*).

Teacher: OK, that's a close strategy, it's a great strategy, why not?

Derek: Uh, length times width divided by 2.

Teacher: Beautiful, that's a fantastic – you read my mind, Derek, gold star today. Look up here. Everyone needs to know how to do this, for a couple reasons. We'll get to those reasons in a minute but right now let's watch. If I created an entire square right here, if I just created an entire square, what would the area be of the square I created? Somebody think about it? 4 times 4. What's the area of that square? Raise your hand, raise your hand. Miles?

Miles: (*said something inaudible, then*) 16.

Teacher: 16. Yes, good, great. I like how you corrected yourself. What's half of 16, everybody?

Class: 8.

Teacher: What's half of 16, everybody?

Class: 8.

Teacher: 8. Half of 16 is 8. What was the area of this triangle? 8. So if I have a square and I divide it like this into a triangle, what's that triangle always going to be in comparison to that square? What's the area of this triangle going to be in comparison to that square? Raise your hand if you think you know. Shia? Half. Half. For every single triangle that I draw inside a square, as long as it fills it up, that's going to be half. So let's do a little test, see if you can figure this one out. What if I draw this one? (*Kevin drew a*

right triangle with height and base 3 blocks long.) What's the area of that? Well, the whole block is going to be what, be what, Beau? 9. And then what did you say the answer was.

Beau: 4 and a half.

Teacher: 4 and a half, absolutely. Because it's half of that whole square. Does that make sense? Does that make sense? Great. Let's stop there.

Kevin moved the class on to the main lesson topic of the day, finding areas of irregular figures.

The class used the different strategies discussed during the warm up to decompose the irregular figures into known figures for which they could find areas easily, either by counting or making familiar squares, rectangles, and right triangles.

Reflecting on the Vignettes: A Preliminary Discussion of Findings

I share these excerpts from Faith, Marie, and Kevin's classes not only to illustrate the range of normative practices that comprised how the class community – the teacher and the students – engaged with mathematics, but also to provide a window into the connections between the composite of these classroom norms and how they shaped mathematical learning opportunities and signaled messages about knowing and doing math. For example, when and how the teachers chose to provide relevant information differed, both in timing and duration (when in class, and for how long) and in format (as notes, as hints, or through discussion). Faith focused on providing relevant information before allowing students to work on problems. Through detailed notes, definitions and conventions, she emphasized that precision and accuracy matter in mathematics. She repeatedly addressed behaviors she deemed disruptive. She controlled the activity in the classroom – what the students did, what they wrote, how long the class spent on notes and work time, and which students' solutions were presented to the entire class. Compared to the lessons of the other two teachers, the pace in Faith's class was very slow, with minimal time for students to work on problems on their own. The break in the rote activity – the skip-counting stretch break – loosely involved math, but it was not grade-level appropriate math

content for eighth graders, and even though it was only a break, it took up more than 5 precious minutes of instructional time. The implicit messages about math were clear: in this class, math work was directed by an unidentified “they” (the curriculum) and then guided by the teacher. With respect to developing efficacy, since extensive notes preceded any work on the task, the students were not allowed adequate time to work on the mathematical task or to draw on any ideas or prior knowledge they may have had to make sense of the problems.

When presented together, the vignette of Faith’s instruction provides a stark contrast with how Marie and Kevin’s classes engaged with their respective math tasks. In some ways, Marie’s instruction shared features with Faith’s. Marie also paced the students and typically brought the class back together to discuss one problem before moving on to the next. What stood out, however, was how the students presented their own strategies, which Marie then articulated for further emphasis and discussion. Mistakes happened, but they did not detract from accuracy or precision; Lakiya’s incorrect square provided an opportunity for Marie to clarify the conditions of the problem. Marie also emphasized sense-making when she asked students to explain their strategies. She did focus attention on strategies that could be useful for upcoming problems; for example, she took time to demonstrate how Marley used the grid squares to “measure” the length of the side because this was an informal strategy for confirming that line segments or sides were equal. There was, however, space for others to articulate their approaches, like Darien’s use of his “pencil” as a nonstandard measuring tool. In this excerpt, as students worked on constructing the squares, they also practiced explaining, communicating reasoning, and connecting different approaches. As they engaged in these mathematical practices with their own work and their own ideas, their opportunity to develop a sense of efficacy with respect to doing math was markedly different from that of the students in Faith’s classroom. And, what took more than an hour of

class time in Faith's lesson took less than 10 minutes in Marie's class; not only did the students in Marie's class work on math differently, but they also worked on more kinds of math problems.

In Kevin's instruction there was a similar emphasis on students' sharing their strategies, though one difference was in how this took place. In Marie's classroom the students actually demonstrated their approaches; literally, they wielded the projector pen. In Kevin's classroom, students may have stated their approaches but he was the one to "show" on the projector, or to clarify or explain to the rest of the class. Both Marie and Kevin articulated student ideas to make sure the entire class heard them; both teachers made students' math work and efforts public.

Through Kevin's instruction, an additional layer of norms emerged as distinct and noteworthy. Through deliberate statements that acknowledged behaviors he wanted to encourage and recognized student contributions and effort, Kevin developed a positive tone in his classroom. He treated students as valued participants in the class community, and while the repetition in his speech mirrored that of Faith's, he typically repeated praise, encouragement, and positive acknowledgements. This was noticeably different from repeated reminders to *not* do certain things and created a space in which student effort clearly was expected, was recognized and was valued. These contrasting cases illustrate how even seemingly non-mathematical moves or practices, such as interactions in which teachers give directions, instructions, or other classroom management tasks, may support or constrain the ways in which students work with mathematics and also how they may see their knowledge and skill as rendering them able – or not – to make sense of and solve mathematical problems.

In this remainder of this chapter I describe the nature of the six novice teachers' instruction in the fall of their first year of teaching by examining the normative practices in which the teachers engaged in their two primary roles guiding the development of mathematical ideas and developing the social culture of the classroom. As I attended to the class norms and patterns

in instruction, another group of practices emerged as increasingly relevant to the shaping and signaling of mathematics: how the teacher responded to students and the ways the teacher structured participation. Then, I analyzed how these normative practices shaped the mathematical learning opportunities and served as signals or messages about knowing and doing mathematics that could have impact on the development of students' productive dispositions. Through these portraits of instruction, I establish how normative practices shaped the development of mathematical practices and proficiencies. In the following chapter I present findings from an extended analysis of instruction over time, during which I applied the same analytical lens to instruction to understand which practices took hold or changed over the induction period of teaching for these teachers. In that chapter I also explore the role of contextual factors, including the nature of the curriculum tasks and other structural resources and constraints, and also teacher beliefs and perceptions of the challenges and responsibilities of teaching mathematics to their specific groups of middle school students.

The Nature of Instruction: Identifying Normative Practices

Before presenting findings, it is helpful to review how as an observer I identified and named normative practices in a classroom. While watching recorded observations, the analytical question I asked was not, "Did such and such a practice happen, ever?" but "What kinds of interactions occurred regularly in the class? How often did these kinds of interactions or practices happen? To what extent? To what end? And, which kinds of interactions made up the 'grammar' of the class?" To answer some of these questions, counts of interactions were appropriate and necessary; I then used those counts to determine a degree of predominance or prevalence of a normative practice as compared to other practices or interactions. I considered, "Did this kind of interaction happen routinely or frequently, occasionally, rarely, or not at all?" In this analysis, for the last degree of prevalence, "not observed" was more accurate since I could make claims only

from the lessons observed, which provided snapshots of instruction. For example, as I considered how the teachers developed social culture, I attended to how often and how many discipline or management interactions occurred, or how often and how many encouraging or dismissive statements were made. The frequency of different kinds of interactions became the evidence I used to support assertions about patterns in instruction that set the tone of the class.

In order to answer other questions about the teachers' instruction, such as how the teacher provided relevant information about conventions or addressed errors, the counts themselves became less significant when compared to specific features and the range of interactions. For example, in a given lesson a teacher may have introduced only two or three vocabulary terms or conventions, thus rendering the "routine" or "occasional" prevalence descriptors moot, but *how* she or he provided this information spoke to a different category of normative practices. For example, were definitions provided only by the teacher or the text? Were examples and counterexamples included? Did the teacher solicit ideas or background knowledge from students, or make connections to previously taught material? To answer these kinds of questions, rich description offered a window into the nature of practices in the classroom that were normative because they constituted the "grammar" of math instruction. With these distinctions in mind, in this chapter I endeavor to capture the normative practices that made up each teachers' instruction by evoking both a sense of prevalence of some interactions and rich description of the features and range of other interactions.

As I analyzed each recorded episode of the teachers' instruction, I noted the occurrence of specific moves or interactions by the teacher that were directed or audible to the entire class, to all students – in other words, that were in the public domain. I attended to two categories of interactions aligned with my conceptual framework: those interactions that guided the development of mathematical ideas, and those that contributed to the development of the social

culture of the class. Within the guiding category, I noted when teachers gave procedural directions – from non-mathematical ones about pacing, group work expectations, section and page numbers, and so forth, to more mathematical “how to” modeling or reviewing of steps to applying formulas or focusing on specific approaches. I noted when teachers revoiced what students said, and whether that public marking of student utterances was low-level – repeating or recapping exactly what students said – or potentially more high-level – probing the responses, for example, for clarification or reasoning, or connecting to representations or other strategies. I also noted specific references to mathematical language or conventions, and I flagged when mistakes or errors happened and to what extent and by whom they were addressed or clarified. Within the developing social culture category, I recorded when and how the teacher addressed behavior or management issues, both those that were negative or disruptive to the lesson and also those that were positive exemplars of behaviors the teacher wanted to recognize and reinforce. I also marked when the teacher acknowledged student effort, contributions, or mathematical ability. I identified interactions that emphasized expectations for work in the classroom, whether general or specific to mathematics. Finally, I tracked who participated, how, and when, and whether there were connections to either the real-world, student lives or interests, or other applications. (For more information about codes and categories, please see Chapter 3 and/or the Appendix.)

In the following sections I present findings from this analysis, with Tables 5 through 13 providing overviews and summaries of each teachers’ instruction along these roles. As I studied how these teachers guided the development of mathematical ideas and developed the social culture of the class, a third category of normative practices emerged as relevant both to these responsibilities and also to the mathematical learning opportunities available and accessible to students: how the teachers responded to students and structured opportunities for participation. Table 14 and Figure 10 present overviews of this analysis. After this, in the next part of this

chapter, I describe findings from a cross-case analysis for which I looked across the set of teachers and the normative practices that made up their instruction in their first semester teaching and suggest implications of these patterns in instruction for mathematical learning opportunities and messages conveyed about what it meant to know and do mathematics.

Guiding the Development of Mathematical Ideas

Guiding the development of mathematical ideas clearly is a primary responsibility of a mathematics teacher. Of course, there is a vast range of ways teachers may go about doing this; beyond a simplistic dichotomy of “teacher-directed” and “discovery- or exploration-oriented” there are many amalgams of interactions between teacher and students around content. Norms that guide the *development* of the mathematical ideas are not necessarily sociomathematical (Yackel & Cobb, 1996) because they may not be specifically mathematical in nature (for example, a norm about what is accepted as a mathematically elegant or efficient solution) but they still matter to the mathematical work of the class. For example, the norm of a teacher asking students to provide explanations of their answers is a social norm because it is not a pattern specific to mathematics; in other disciplines students are also expected to provide reasoning with answers. However, that normative practice of asking for explanations shapes the mathematical learning opportunities because when students are expected to provide rationales or justification for their answers, their engagement in mathematical sense-making is different compared to contexts in which applying and recounting steps of a formula suffices as reasoning. The former potentially raises the cognitive demand of the enacted task, while the latter limits or caps the cognitive demand. Through this analysis, I identified the following category of patterns in normative practices that guide the development of mathematical ideas in these teachers’ classrooms: a) non-mathematical procedural or pedagogical actions to manage the work of the class; b) how and when the teacher provided relevant information, the form of that information,

and the allocation of class time for that purpose; c) how and when the teacher posed questions, the nature of those questions, and the explicit or implicit mathematical emphases; and d) how and when the class addressed errors. Tables 5 through 10 provides an overview of each teachers' instruction through this lens of normative practices that guide the development of mathematical ideas.

As noted previously, I applied different approaches, depending on the question and kind of norm, to determine patterns in interactions (“regularities” of the class) from two recorded episodes of instruction from two consecutive days. When trying to establish how frequently a teacher addressed non-content issues, counts were helpful to distinguish the degree of prevalence of specific practices. For these kinds of patterns, I used the following scale to assess frequency: *routinely* indicated an interaction that happened more than 3 times in two or more activity structures; *occasionally* signified an interaction or practice that happened twice in 1 or more activity structures; *rarely* indicated that action that happened once, in 1 or 2 activity structures; and, if a practice did not happen during the observed lesson, I described it as *not observed*. For other patterns of practice, a qualitative rich or “thick” descriptive approach was more helpful because while the given practice may have happened only a few times, it was still relevant to the mathematical work of the class. For example, there may not have been errors in every lesson, but if one did occur, what happened? Who noticed it? Who corrected it, and how? Based on observations of two consecutive lessons in the fall of the first year of novice middle school mathematics teachers’ careers, in Tables 5 through 10 I present both the prevalence of certain interactions and description of other interactions, all which guided the development of mathematical ideas. In the following pages, I describe features or dimensions of guiding norms and highlight themes across groups of teachers, using examples to illustrate both recurring norms and also unique practices or exceptions.

Table 5

Faith's Instruction: Normative Practices Guiding the Development of Mathematical Ideas

	Presence/nature of non-math procedural or pedagogical interactions	When and how did the teacher provide relevant information? (e.g., notes, vocabulary, conventions, hints, other methods)	What were the mathematical emphases? (precision, conventions, procedures, sense-making)	How were errors addressed?
<i>Features</i>	<i>How often? When? About what?</i>	<i>About what? When? Use of class time</i>	<i>Were questions asked of students low level (brief answers sufficient) or high level (explaining, extending)? Did students ask questions? Implicit or explicit emphases</i>	<i>Who identified? (How) Were they addressed? • By students or teacher? Did errors go unnoticed?</i>
Teacher				
Faith	Routinely repeated directions or instructions Throughout class Focused on <i>what to do</i> (page to read, notes to write, “you need” or “I need you to”) and <i>what not to do</i> (no talking)	Provided notes, vocabulary, guided examples first, before work time (precursor to work time) Provided answers: after 1-2 minutes of work time, showed what to do or provided an answer and then gave students time to copy Most of class time spent on notetaking or teacher-directed review or guided demonstration of solving problems	Most questions required brief answers only, which teacher then repeated; even with a student solution, teacher shared/made public, no student explanation expected or elicited If students asked questions, only on individual basis to teacher during work time; not public Emphasis on definitions and guided examples preceding student work time	Teacher made mistakes which were not identified or corrected: • Misstated definition of parallel as two lines of the same length • Misstated that a right triangle has “at least one right angle” • Despite emphasis on definitions and characteristics of polygons, misidentified student suggestion as a rectangle, when it was a parallelogram; asked class to record incorrect answer

Table 6

Marie’s Instruction: Normative Practices Guiding the Development of Mathematical Ideas

	Presence/nature of non-math procedural or pedagogical interactions	When and how did the teacher provide relevant information? (e.g., notes, vocabulary, conventions, hints, other methods)	What were the mathematical emphases? (precision, conventions, procedures, sense-making)	How were errors addressed?
<i>Features</i>	<i>How often? When? About what?</i>	<i>About what? When? Use of class time</i>	<i>Were questions asked of students low level (brief answers sufficient) or high level (explaining, extending)? Did students ask questions? Implicit or explicit emphases</i>	<i>Who identified? (How) Were they addressed? • By students or teacher? Did errors go unnoticed?</i>
Teacher				
Marie	Occasionally repeated directions or instructions During work time to individuals, redirected attention during class discussion Focus of these interactions mostly on work expectations and pacing (“This shouldn’t take very long”)	Provided conventions, language, or reminders during discussions, in response to student answers, or during work time (routinely sought information from students and then repeated or emphasized) Incorporated students’ ideas, work, strategies, solutions into progression of lesson Class time spent approximately in equal parts on launching task and work time (independent, in pairs), followed by 3-8 minute discussion	(routinely) Asked questions to extend student-provided solutions, including “Can you repeat” what another student said, “Let me know if you disagree,” or “Explain in a different way” If students asked questions, they did so during work time; during discussion, teacher invited students to ask questions of each other Provided real-world examples/context; Emphasis on connecting to students’ interests or experiences	When students made errors, teacher addressed in different ways: <ul style="list-style-type: none">• During warm up she asked, “Does anyone have a different answer? Why?” (does not explicitly state as incorrect)• Pointed out how a proposed solution did not meet the constraints of the problem (for example, that the given points needed to form one side of a square); explicitly stated that mistakes happen and are part of the process of doing math work

Table 7

Becca’s Instruction: Normative Practices Guiding the Development of Mathematical Ideas

	Presence/nature of non-math procedural or pedagogical interactions	When and how did the teacher provide relevant information? (e.g., notes, vocabulary, conventions, hints, other methods)	What were the mathematical emphases? (precision, conventions, procedures, sense-making)	How were errors addressed?
<i>Features</i>	<i>How often? When? About what?</i>	<i>About what? When? Use of class time</i>	<i>Were questions asked of students low level (brief answers sufficient) or high level (explaining, extending)? Did students ask questions? Implicit or explicit emphases</i>	<i>Who identified? (How) Were they addressed? • By students or teacher? Did errors go unnoticed?</i>
Teacher				
Becca	Routinely repeated directions or instructions Mostly during work times Focus on volume, work expectations (pay attention, get going), how to take/keep notes and binders, pacing, and also what <i>not</i> to do (not talking)	Provided notes, formulas, and guided examples first, as precursor to work time Showed short videos, which presented representations and demonstrated steps of applying formulas, but few connections made between those representations and the formulas after viewing Class time divided into two main activity structures: guided lesson emphasizing procedures, and independent quiet work time	Most questions only required brief answers or choral responses, which teacher then recapped/revoiced If students asked questions, only on individual basis to teacher during work time; not public Emphasis on substituting different values into formulas and applying demonstrated procedures; guided examples preceded work time	When reviewing warm up and main task problems, teacher only reviewed steps of a problem if there was an error (“Let’s look at this”) in student answer; teacher identified errors, demonstrated procedure and provided correction for students to copy

Table 8

Kevin’s Instruction: Normative Practices Guiding the Development of Mathematical Ideas

	Presence/nature of non-math procedural or pedagogical interactions	When and how did the teacher provide relevant information? (e.g., notes, vocabulary, conventions, hints, other methods)	What were the mathematical emphases? (precision, conventions, procedures, sense-making)	How were errors addressed?
<i>Features</i>	<i>How often? When? About what?</i>	<i>About what? When? Use of class time</i>	<i>Were questions asked of students low level (brief answers sufficient) or high level (explaining, extending)? Did students ask questions? Implicit or explicit emphases</i>	<i>Who identified? (How) Were they addressed? • By students or teacher? Did errors go unnoticed?</i>
Teacher				
Kevin	Routinely repeated directions and reminders Throughout class Focus varied: about work expectations (volume, participation), pacing; redirecting reminders interspersed with positive comments	Elicited and suggested alternate methods during discussion after work time; provided hints during work time Reviewed vocabulary and conventions and elicited prior knowledge during warm up and lesson launch; connected to task Class time divided approximately in equal parts on launching task, providing work time – both independent and in pairs – and discussing solutions	Posed questions to elicit both answers and strategies or student thinking; frequently recapped what students said; asked students to explain, comment, or extend work, accepting <i>how</i> answers (“what I did”) Students asked questions during work time; teacher decided whether to make those questions public (with hint or clarification) Emphasis: explaining <i>how</i> students solved problems (and occasionally <i>why</i> approaches worked); condensing strategies for a “toolkit”	When student made mistake, teacher identified (“It’s close to 6, it’s not 6”), elicited answers or explanations from other students, recapped once a correct answer was suggested, and then suggested where the error might have happened (e.g., not counting partial squares).

Table 9

Kyra's Instruction: Normative Practices Guiding the Development of Mathematical Ideas

	Presence/nature of non-math procedural or pedagogical interactions	When and how did the teacher provide relevant information? (e.g., notes, vocabulary, conventions, hints, other methods)	What were the mathematical emphases? (precision, conventions, procedures, sense-making)	How were errors addressed?
<i>Features</i>	<i>How often? When? About what?</i>	<i>About what? When? Use of class time</i>	<i>Were questions asked of students low level (brief answers sufficient) or high level (explaining, extending)? Did students ask questions? Implicit or explicit emphases</i>	<i>Who identified? (How) Were they addressed? • By students or teacher? Did errors go unnoticed?</i>
Teacher				
Kyra	Rarely repeated directions Focus was managerial: who would read, how and when to take notes; Rarely addressed talking or behavior because class was very quiet)	Provided notes, vocabulary, and guided examples first, before work time Structured class talk to review steps of formulas as relevant (for example, calculating surface area); demonstrated the procedure students were expected/ encouraged to use; gave students time to record notes or answers Class divided into 2 primary activity structures: guided lesson directed by teacher, and independent quiet work time	During guided lesson, as working through problems with students, asked, "What do we notice?" Brief answers sufficed (or no response) and questions did not require extended discussion; teacher recapped Few questions posed by students. When suggesting examples and counterexamples of geometric elements, twice students suggested representations Emphasis on precision and applying procedures; guided examples preceded work time	Teacher reviewed solution only when there was an error in student response; teacher provided correction (in this case incomplete because class ended)

Table 10

Noah's Instruction: Normative Practices Guiding the Development of Mathematical Ideas

	Presence/nature of non-math procedural or pedagogical interactions	When and how did the teacher provide relevant information? (e.g., notes, vocabulary, conventions, hints, other methods)	What were the mathematical emphases? (precision, conventions, procedures, sense-making)	How were errors addressed?
<i>Features</i>	<i>How often? When? About what?</i>	<i>About what? When? Use of class time</i>	<i>Were questions asked of students low level (brief answers sufficient) or high level (explaining, extending)? Did students ask questions? Implicit or explicit emphases</i>	<i>Who identified? (How) Were they addressed? • By students or teacher? Did errors go unnoticed?</i>
Teacher				
Noah	Routinely repeated directions or instructions Throughout class Focus on what to do (page to read, notes to write, procedures for using calculators) and what <i>not</i> to do (no talking, stay seated, stay in class); frequent management interruptions	Scaffolded tasks: decomposed problems or investigations into single questions, provided work time, then reviewed answer before moving to next question (very structured format) Majority of class time spent in this scaffolded guided lesson format, with frequent interruptions of disruptive behavior or management issues	Posed questions that required brief or choral answers; Attempted to elicit student descriptions of work or strategy, but often interrupted by misbehavior; Attempted to ask questions to elicit connections: "Describe the pattern of change"; again brief answers sufficed and management issues prevented further discussion If students asked questions, only on individual basis to teacher. Emphasis on making sense of problem through scaffolded structure with pace set by teacher	When student suggested an incorrect answer, teacher responded, "Let's check this" and referred students back to information of the problem; encouraged students to check and identify corrections When students identified an error the teacher made setting up the units on an axis of a graph, the teacher acknowledged, thanked, corrected, and then moved on

Presence/nature of non-math procedural or pedagogical interactions. The first column, non-mathematical procedural or pedagogical interactions refers to those interactions that speak to the management aspects of the class work (for example, what the students will work on, from page to problem numbers, and how the students will work, from independently to pairs to small groups). These played an important, and perhaps underestimated role in how the teachers guided the development of mathematical ideas and concepts. These interactions included directives, like, “Turn to page 55 and read,” references to pacing (“This shouldn’t take very long”), and instructions on how students should work individually, with partners, or in groups. These interactions may have happened in conjunction with behavior or management issues, but were distinct; I describe how behavior was addressed in the following section on normative practices that undergird “Developing the Social Culture.” Here, I focus on these non-mathematical procedural interactions that fell under the teacher’s responsibility to be the pedagogical authority in the classroom who establishes how the students worked, on what, for how long.

How these interactions affected the mathematical work of the classroom varied inversely to their presence and frequency: when these kinds of interactions happened less often, without repetition, other interactions around math content happened and had the potential to be more substantive. Plainly speaking, in a classroom in which the teacher did not spend extended time repeating directions to the entire class, more “air” time was available for substantive discourse about content – though that was not guaranteed. In this analysis, however, for the most part, this inverse relationship seemed to hold true. Both Marie and Kyra engaged in these kinds of instructions occasionally or rarely, typically without repetition; there was little lag time in between when they set a direction for the class activity and when the students followed along. How the remaining talk time was allocated to math content differed, however, demonstrating that a smooth-running classroom in which non-math procedural interactions were minimal did not

guarantee that significant amounts of class time were allocated to discussion. (This will be explored further later in this section.)

In the other teachers' classrooms, the sheer repetition of these general pedagogical statements and instructive commands stood out: Becca, Faith, and Noah routinely repeated even simple instructions four, five, even six times. Transitions to and between activities took up a lot of time. Becca made more of these statements during the quiet work time, routinely redirecting students to the math activity. When these kinds of interactions were repeated again and again, the consequences were twofold: less time to talk about the math task, and, more generally, a focus or emphasis on compliance with the teacher's directions. There was an exception: in Kevin's case, repetition did not limit math-focused discussions. This was due as much to the presence of other intentional moves around the mathematics as it was to his tone. Typically upbeat and energetic, his routine reminders about basic directions (page number, volume during work time) were interspersed with positive comments about the students' effort, work, contributions; the constant hum his repeated reminders created was notable in its enthusiasm and energy. (I examine this idea of the class tone or energy set by the teacher further in the next section, when I present how the teacher develops the social culture of the classroom.) In general, it seemed that when these kinds of interactions were repeated, the teachers were devoting significant effort to shepherding students to work on math or engage with content at all.

When and How Did the Teacher Provide Relevant Information? What Mathematical Emphases Guided the Questioning and Task Engagement? The next two categories (columns) of normative practices relate to how the teacher guided the development of mathematical ideas – when and how the teacher provided relevant information, and the mathematical emphases as conveyed through questions and the sequence of discussion – and overlap in key ways. In Faith, Becca, and Kyra's classrooms, the following practices were routine: a) either the majority or a plurality of the class time was spent in the lesson introduction; b) during that introduction, the

teachers presented definitions, vocabulary, formulas or procedures, and guided examples; c) notetaking was the primary activity in which students engaged; d) quiet independent work time followed these lessons; and e) there was little to no time allotted for discussion of the problems or investigations on which students worked. In the public domain (what was audible to the entire class), the teachers posed all the questions to the students, and brief or choral responses sufficed. Only in one instance did instruction waver from these patterns: as Kyra reviewed definitions of geometric elements, she solicited real-world examples and counterexamples, but even here, she determined which student responses became part of the class notes or record, and the notetaking preceded any work time with actual problems. The implicit emphasis or message was that before students could work on a task, they needed information – in the form of vocabulary, conventions, formulas, and examples – provided by the teacher. Time to engage with mathematical problems, investigations, or tasks, was capped by the teacher-directed or guided lesson, which took precedence over work time. Occasionally the teachers stated this explicitly. For example, in the vignette of instruction presented earlier, Faith explained that while the goal was to get to the activity, the notes on properties of shapes had to come first. Similarly, Kyra stated, “OK, this is not the most fun-filled lesson that we’ve done this year but it’s one that’s important before we move into our next book, that we have certain concepts down and understood” (Fall Year 1, Day 1). This normative practice of the teacher providing notes and information first, before the students did any mathematical work, in order *for* the students to be able to do any mathematical work, evoked a sense of instruction as primarily a transmission of knowledge imparted from the teacher to the students. This pattern of interactions suggested a perception of learners as not capable of figuring a problem or task out without information and possibly a demonstration presented by the teacher first; learners’ efficacy is thus managed by the teacher.

In contrast, Kevin and Marie’s instruction diverged from Faith, Becca, and Kyra’s, both with respect to how, when and what they provided to students as relevant information for the

mathematical task and with what they emphasized through the kinds of questions they asked and when they posed them. While both teachers still attended to vocabulary and other disciplinary conventions, they regularly used student ideas, strategies, and answers to introduce and emphasize this information. They incorporated student ideas into the progression of the lesson, and as such, teaching about conventions did not precede student work on a task but instead was interwoven with that work. They both structured the class so that before work time, the task was launched or introduced, but not with an extended lecture or review of concepts, or a guided example, though through the warm up activity and discussion they may have introduced key ideas or elicited prior knowledge related to the topic. (This use of the warm up contrasted with that of the other teachers: in Becca and Kyra's classes, the warm up problems were distinct from the lesson topic and reviewed specific skills, and in Faith and Noah's classes, the warm up tasks may have addressed concepts relevant to the main lesson, but those connections were not emphasized. In fact, because the transition between warm up and main task was often extended, typically due to behavior disruptions and lag time in students complying with procedural directions, the thread between warm up and main task was dropped and not picked up again.) In addition, a discussion time regularly followed student work time; talk about math takes place both before and after students have work time. And, after work time, students were expected at a minimum to present their answers with at least a representation or explanation of *how* they arrived at that answer ("showing their work"), with the potential for questions further extending the explanation to *why* a particular approach worked. Marie often posed questions that suggested connections between ideas, emphasizing to students that they should attend not only to what she, the teacher, was saying, but also to the strategies or comments their peers put forward. She asked questions like, "Can you repeat that?" or "Explain it another way" to redirect student attention to the math, and "Let me know if you disagree," and "You should be comparing these" to direct attention to specific students' work. Kevin led discussions with a similar emphasis on drawing students'

attention to what their peers were suggesting. When wrapping up the discussion after students find areas for irregular polygons, Kevin asked students to identify different strategies they could use, explaining that he would like the students to develop a “kind of an arsenal of tools to be able to figure these [problems] out” (Fall Year 1 Day 1). He cited students and their work when listing possible strategies: “We learned strategies from Ronnie, Derek, um, Malik, Miles, Roman, Fretinisha told us some strategies. We learned all kinds of strategies.” A normative routine in both Marie and Kevin’s classrooms was that student work was interwoven into the lesson’s progression, and talk about the math work and topics permeated the different activity structures in the lesson, from the warm up to the launch to the discussions following work.

Noah’s instruction stood out as an exception, illustrating how the lines demarcating these categories were nuanced. Noah implemented the CMP2 investigations using a scaffolded approach, posing one question or task to students, allowing them a few minutes of work time, then reviewing that question before moving on to the next. This approach was very structured, and he often provided detailed instructions like how to set up a table (what will be the columns and rows) or how to label axes of a graph. However, he gave students time to complete whatever mini-tasks or steps he set up, and he expected students to take part in making the connections between these mini-tasks; these were not guided examples and he was not demonstrating steps of applying formulas or procedures. He posed questions like, “Is this linear?” and “Describe the pattern of change” for an investigation about mathematical modeling. These attempts to engage students in making sense of the math work they were doing, however, were mostly incomplete. When the students were talking about math, the potential for connecting ideas was there, but consistently disruptive behaviors interrupted the flow of the lesson and sent the teacher and class off topic. These student behavior disruptions, which the teacher had to address because they were not insignificant (students were getting up to yell, to move desks, to walk out of the classroom, and so on), invariably forestalled any follow up discussion of a math question by the class, even

when the teacher tried to repeat the question, with determined effort. The lag time between the questions and the time for a class discussion was protracted. Again, this effect of management issues is revisited at length in the following section on how teachers develop the social culture of their classes.

Treatment of Errors. Thus far I have focused on the routine or regular practices – the norms – of how the teachers guided the development of mathematical ideas. I turn now to mistakes and how the teachers addressed them. Taken alone these interactions may seem singular; after all, mistakes happen in the moment and there is no guarantee that one will happen in a given class around a particular topic. How the teachers responded or addressed mistakes, however, may be interpreted as evidence confirming other normative practices; or, put another way, when mistakes happened, how the teacher addressed them often overlapped or resembled co-existing norms about providing information or emphasizing precision or sense-making. In Becca and Kyra’s classes, when reviewing answers to either warm up problems or task problems, the teachers reviewed *how* to solve a problem only if there was a mistake in a suggested answer. In keeping with the other teacher-directed routines, they evaluated whether an answer is incorrect and then they showed the correct way to solve the problem (correcting a substitution in a formula, for example). In contrast, in Noah’s class, when a student identified a mistake, a simple error in how Noah labeled units along an axis, Noah quickly made the change, acknowledged the contribution with a “Thank you,” and moved on to the question. This was consistent with his tendency to keep the class moving through discrete tasks and with his focus on student exerting effort or attention to the task (rather than just repeating a step he showed). Later, when a student proposed an expression that was not correct, Noah repeatedly asked, “Let’s check this” and urged students to check if the information provided in the problem fit the expression. Noah attempted to use the error as another entry point for student engagement. Similarly, as illustrated in the vignette of instruction, when a student (Lakiya) proposed an incorrect solution, Marie pointed out

how the solution did not meet the criteria of the problem, but then moved on to solicit other possibilities, acknowledging later that this task was not easy or straightforward but was still within reach of the students' understanding. Effectively she treated the mistake as part of the mathematical conversation and continued to use student responses in the lesson's progression; a mistake did not shut down a conversation or result in a showing or telling of an answer.

Thus far I have presented two possible approaches to mistakes: teachers correcting them and showing or providing the correct answer, and teachers engaging in a dialogue with students about either the mistake or the problem itself, integrating the mistake into the class discussion. Another possibility, unfortunately, was that a mistake was unidentified and unaddressed; this happened several times in an egregious way during one of Faith's lesson. During the second observation, Faith's class was working on "Planning Parks". The task was to draw various polygons (square, non-square rectangle, parallelogram) on a coordinate grid given two vertices that make up a side. In the previous class Faith spent the majority of class time reviewing and emphasizing properties of these polygons, and in this lesson, much of the class time was spent on guided examples. Forty minutes into the day's class, Faith asked students to work on their own to find a non-square rectangle. After a few minutes, she put the coordinates for one student's response on the smart board, to emphasize that the question was asking for a non-square rectangle. She says the coordinates the student suggested make a square, and that even though a square *is* a rectangle, the problem is asking for a non-square rectangle. However, as illustrated in Figure 7, the student's proposed solution was not a square because the side lengths are not equal. It was a parallelogram, but, to determine that it was a rectangle, the class would need to confirm that all angles were 90 degrees.

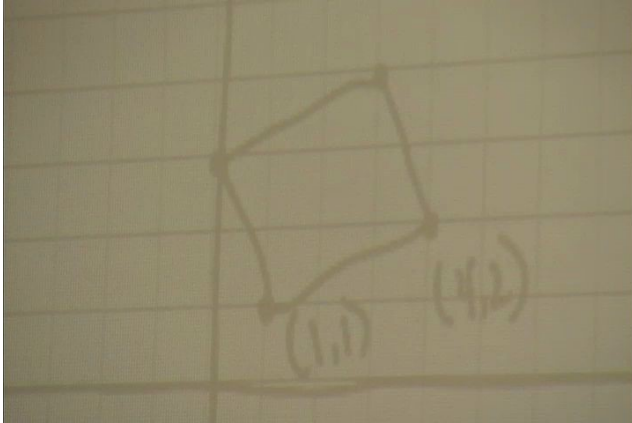


Figure 7. Error – Screen Shot: Student’s proposed rectangle, which Faith mistakenly identifies as a square.

Faith directed students to continue trying to find coordinate to make a rectangle. After 2 or 3 more minutes of work time, during which she was walking around looking at student work, Faith decided to show Elizabeth’s solution on the projector (see Figure 8).

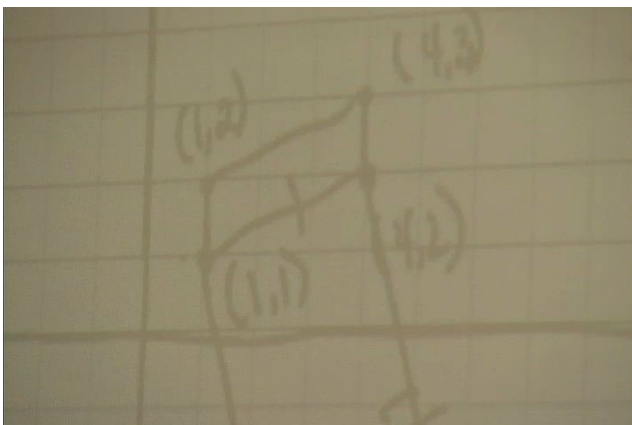


Figure 8. Error – Screen Shot: Elizabeth’s proposed “rectangle” (actually a parallelogram) with coordinates (1,1), (4,2), (1, 2), and (4, 3).

The given coordinates were (1,1) and (4, 2). Elizabeth proposed as a rectangular park one with two other vertices at points (1, 2) and (4, 3). After showing this, Faith hedged on whether this was correct:

So she has (1,2) and then she has (4,3). ... This is, this isn’t one of the examples that they actually had in the book, okay, but because it’s slanted a little bit, I’m going to have to look into that and see. But for now you guys can go ahead and put that one. I’m not, I’m not really for sure if that works. K. I’m going to have to look into that one.

Faith accurately questioned that shape, but allowed and even encouraged students to record it as a rectangle without checking it with the characteristics of a rectangle that the class had reviewed as a group. Such a check may have presented an opportunity to discuss the differences between rectangles and the broader class of parallelograms. Then, in keeping with her tendency to show students answers, she continued,

But the book has (1,1), (4,2), and instead of (4,3) they use (4,1). So if they use (4,1) that would actually be down here and they would connect it this way instead. (See Figure 9) I know that's a little bit messy but they would have it like this.

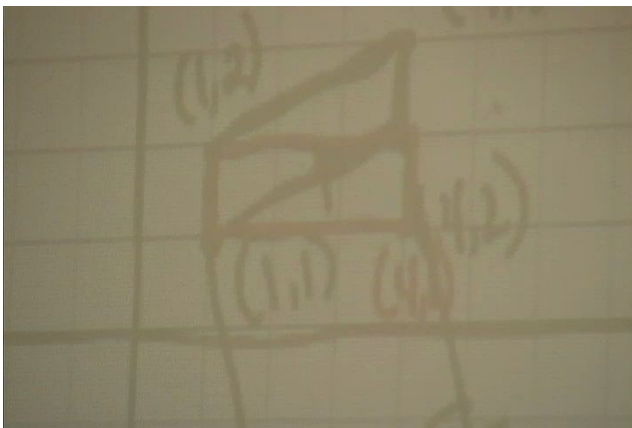


Figure 9. Error – Screen Shot: Teacher Faith’s presentation of solution from the textbook: a rectangle with coordinates (1,1), (4, 1), (4, 2), and (1, 2).

After emphasizing definitions, and relatedly, the importance of accuracy and precision in mathematics, Faith made mistakes in identifying shapes and whether they met the criteria of the problem and then provided students with a correct answer from the text without re-examining the other shapes. She did not solicit student questions or reasoning about the shapes. These actions were consistent with the other normative practices in her instruction to guide the development of mathematical ideas by showing students what to do. The normative practices that were routine in Faith’s instruction – an emphasis on teacher-guided lessons and a de-emphasis on student talk beyond brief responses – were not sufficient to counter or address these errors.

Summary of Norms Guiding the Development of Mathematical Ideas. Through interactions with content, with students, and with students around content, teachers established normative practices in their role of guiding the development of mathematical ideas. These normative practices, these routines, may have been stated explicitly and emphatically; or, they may have been unstated but implied, or even the unintended result of other practices. These varied normative practices shaped the learning opportunities in the class in part through the ways they affected other dimensions of instruction like task: for example, when the majority of class time was devoted to notetaking or guided examples, opportunities for students to make sense or meaning of problems, to experiment with alternate methods, or to communicate reasoning were limited or capped. A single norm in isolation did not dictate the learning opportunities; normative practices do not have neat boundaries, and instruction is a complex, multi-faceted composite of different practices. As such, it was helpful to look for patterns in which norms developed in combination with others, and then to consider emphases: Was sense-making an emphasis? In which classrooms? Was accuracy or precision an emphasis? In Table 11 I reorganize the guiding norms approximately into two groups: norms that evoked mathematics as rule-bound and mathematics teaching as teacher-directed, -guided, and -facilitated, and norms that evoked mathematics as a site for meaning and sense-making co-constructed by teachers with students. I use shading to indicate groupings of norms that were observed in concert. For example, in Becca's classroom, a routine of providing notes prior to work time was accompanied, not surprisingly, by an emphasis on knowing definitions and applying formulas. In Marie and Kevin's classrooms, the discussion of strategies throughout the class allowed for connections to other representations or strategies. The column indicating the presence of routine repetitions of non-mathematical procedural directions shows how these non-content interactions occurred, often amplifying an existing tendency towards a transmission model of teaching and learning, as in the cases of Faith, Becca, and Noah. In Kevin's case, however, the nature of the directions and non-

mathematical comments was more positively-oriented, towards praising and reinforcing students' work habits.

Table 11

Fall Year 1 Snapshot of Norms Guiding the Development of Mathematical Ideas

(√ indicates consistent presence in the teacher’s instruction; ~ indicates attempts or incomplete/unrealized efforts)

	Routinely repeated non-math procedural directions	Routinely provided vocabulary, conventions, procedures, formulas as lesson/launch, before work time	Mathematical emphasis on knowing definitions, applying formulas	For most questions, brief or choral responses sufficient (including listing of procedural steps)	Errors or mistakes were identified by teacher, who then demonstrated correction	Interspersed language and conventions with student work, discussion of strategies	Mathematical emphasis on making sense of problem or connections	Questions had potential for higher-level extensions, representations, explanations	Errors or mistakes part of class discussion, with teacher soliciting corrections or alternate methods
Faith	√	√	√	√					
Marie						√	√	√	√
Becca	√	√	√	√	√				
Kevin	√					√	√	√	√
Kyra		√	√	√	~				
Noah	√			√			~	~	~

Later in this chapter I re-examine these groupings – of normative practices and of teachers around specific practices – and weigh the ways in which these normative practices, in conjunction with others, shaped mathematical learning opportunities and signaled messages about what it meant to know, learn, and do mathematics. Did an emphasis on precision necessarily mean that procedures and applications of formulas made up the bulk of the mathematical tasks in which students engage? Did an emphasis on students sharing their work result in mathematical practices like generalization? Now, though, I return to the extended analysis of teachers’

instruction to consider the normative practices that emerged through studying what teachers did as part of their other professional role in the classroom: developing the social culture of the class. To what extent and in what ways did these norms affect the mathematical work of the class?

Developing the Social Culture of the Class

The other primary responsibility of teachers is to develop the social culture of the classroom. Developing the social culture involves establishing and maintaining expectations for how the class will work. If students are sharing answers or strategies, what are other students expected to do? During work time, what does independent work look and sound like? Partner work? Group work? What does “come to class prepared” look like? These are all features of the social culture, and they may be developed explicitly, or with implicit messages. For example, without always saying, “I expect you to be quiet when another student is speaking,” a teacher may imply this with a quiet statement of the student’s name, or with a glance, or with a move towards the student, increasing physical proximity. How teachers address behavior is a key feature of the social culture, and often is referred to as “classroom management”. However, norms to develop the social culture include more than corrective, redirecting, or even complimentary (praising) reactions to behavior. Through their words and tone, teachers convey messages about what they believe is possible for students to do and what they value or appreciate. Praising a student for completing her work, for example, sends a message that the teacher values that student’s effort. Publicly marking or acknowledging a student’s contribution to a discussion may send additional messages: in general, that student talk is valued in the class community, and perhaps more specifically, that the individual student made a noteworthy comment. At a minimum, marking the student’s contribution with praise or recognition functions as positive reinforcement of that kind of student talk. Praise and acknowledgement for what a student does may recognize behavior, achievement, or attempts and effort. This is distinct but related to encouragement, when the emphasis may be on motivating students, but not necessarily in

response to student or class actions. Encouraging comments may be as concise as “Let’s have a great day today.” They evoke a spirit of coaching (or cheerleading). And of course, what matters is not just what is said, but how: what is the tone? Is it energetic? Enthusiastic? Frustrated? Hostile? Volume matters: a calm voice sets a different tone than a loud or screechy one, or an angry or plaintive one.

In this analysis, practices to develop the social culture included those actions to manage behavior and communicate expectations, to encourage, and to acknowledge student effort, contributions, and even mathematical ability. As I reviewed the different teachers’ lessons, I paid attention to the occurrence and prevalence of interactions that spoke to developing the social culture. I tracked those interactions in which teachers addressed behavior, both positively (praising) and negatively (correcting or disciplining). Noting when teachers acknowledged student contributions, I distinguished between those more general comments about effort (e.g., “Good job!” or “Thank you”) and those which either marked a contribution to the whole-class mathematical discourse around content or reflected on the students’ math ability. I noted encouraging or “coaching” comments as a different category. As before, I used the following scale to assess frequency or prevalence of practices from the interactions that were public to the entire class: *routinely* indicated an interaction that happened more than 3 times in two or more activity structures; *occasionally* signified an interaction or practice that happened twice in 1 or more activity structures; *rarely* indicated that action that happened once, in 1 or 2 activity structures; and, if a practice did not happen, I described it as *not observed* and note it as “--” on the table. Finally, I noted when teachers seemed to empathize with students, based on their perceptions of student feelings; as this was not an interaction I anticipated seeing regularly, I noted these with description. Table 12 provides an overview of each teachers’ instruction through this lens.

Table 12

Normative Practices Developing the Social Culture of the Classroom

Teacher	Addressed behavior	Acknowledged Effort “Good Job”	Acknowledged Contribution “That’s a great idea”	Acknowledged Math Ability	Encouraged (General)	Encouraged (Math-specific)
Faith	Routinely Corrective	Occasionally	--	--	--	--
Marie	Occasionally Redirect	Routinely	Occasionally	--	--	Occasionally
Becca	Routinely Corrective/ Redirect	Routinely	Rarely	--	--	--
Kevin	Routinely Praise, Redirect	Routinely	Routinely	Occasionally	Routinely	Occasionally
Kyra	Rarely	--	Rarely	--	--	--
Noah	Routinely Corrective, also Praise	Routinely	Occasionally	Rarely	Routinely	--

When studying the distribution of interactions to develop the social culture, both similarities and stark differences between teachers' practice were apparent. The absence of positive interactions stood out in Faith's practice. Indeed, during both observed lessons, she repeatedly, and often in a frustrated tone, chastised students for talking, for not paying attention, for not being prepared (with pencils or note sheets). Becca and Kyra's instruction was similar in that the prevalence of different kinds of social culture moves was limited. In Becca's classroom, she routinely addressed behavior infractions like talking by redirecting students calmly but insistently: "I need you to ..." and "You need to be ..." were common phrases of reminders. While she did praise efforts, most student responses were brief or choral, and the "Good job" commendation was similarly brief and perfunctory. Kyra rarely addressed behavior in a public way: for the most part in her classroom students complied with her instructions, and she made any redirecting comments quietly to an individual student if necessary. Acknowledgement of ideas was infrequent, in large part because she rarely solicited students' ideas to guide the development of mathematical ideas; in the infrequent instances when a student volunteered an idea, it was followed up by the teacher and the class in a limited or cursory way. For example, when answering a question about distinguishing between a line and line segment, Kyra had the following exchange with her student Courtney:

Teacher Kyra: OK, so when we think of a graph, when we're graphing a line, or graphing 2 points, what do we usually do to signify that it's a line, that it's continuous? We just put the arrow. So there are some situations where we draw a line, um, we just - uh when we're doing coordinates, we're graphing, we just usually put the arrow that symbolizes that the line goes forever, we don't just keep writing up the page, right? So that's why we use some of those things. Courtney, did you have another point of view?

Student Courtney: I agree and disagree. 'Cause I did it on a line because it says at some point we die but another generation is born so hopefully it's never ending.

Teacher: OK. That's a unique perspective. Ok, so Courtney used it in terms of as an individual you're born and then you die, so that's a line segment, right, but as far as life and reproduction, you would hope that it would just continue?

Courtney: Yes.

Teacher: So, that's just a whole totally unique way of thinking of it, which I probably wouldn't have. So did everybody get that point? Kind of? It's different, but I can see how that might be a possibility. OK.

In these three teachers' classrooms, the moves to establish social culture are either infrequent or corrective in tone.

In contrast, Kevin's and Marie's instruction was comprised of prevalently positive interactions. Marie's instruction illustrated how nonverbal cues could set social culture as much as verbal ones. When she did address negative behaviors, for example, it was with simple redirecting statements ("Stay with me!") made in an energetic tone. These did not interrupt the flow of the mathematics work. She routinely acknowledged effort, but only occasionally verbalized recognition of students' contributions to the class discussion; however, she consistently created so many opportunities for students to participate that it became clear their contributions were both valued and expected because of the time she allocated for students to share work and strategies. (I explore this further in the next section on Responding to Students and Participation Structures.) Meanwhile, Kevin's interactions stood out through the sheer magnitude of positivity that pervaded the lessons. Almost as a running commentary during discussions and work time, he commended students for their work, he praised on-task behaviors like being prepared, he solicited student ideas and then commended those contributions to the class discussion, he repeated encouraging remarks like, "Let's have a good day" and "I want you to do well." His tone was consistently upbeat; his typical strategy when redirecting students' from negative behaviors like talking out of turn was to say, "I'll wait." And then he paused and waited. His positivity was not limited to interactions with the whole class; he recognized individual students, regularly praising them by name for their effort, their contributions, and even their mathematical ability: "Butler is showing a lot of effort today. He should be proud of that. You should!" (Fall Year 1 Day 2). Kevin's enthusiasm and warmth – and concerted efforts to recognize positive behaviors promptly and respond to negative behaviors calmly – offered a sharp

contrast to Faith's frustration, just as Marie's inviting of student ideas was a stark contrast to the brief and choral exchanges between teacher and students in Becca's and Kyra's classrooms.

Once again, within this set of teachers, Noah's instruction stood out in that interactions developing social culture were mixed. The frequency with which he addresses negative behaviors surpassed a "routine" descriptor; within a given lesson, most if not all questions posed about math were interrupted by numerous behavioral infractions that could not be ignored. Both he and his administrators handled crisis moments during the class, when students were threatening to argue, to fight, or to leave. (On both days his lessons were observed, the principal entered the classroom and loudly reprimanded the students for misbehaving, threatening consequences like detention or suspension.) And yet, after these crisis moments passed, and the talk refocused on math, Noah persisted in making concerted efforts to acknowledge student work, effort, and contributions. One of his strategies with respect to discipline was to positively reinforce appropriate student behaviors. However, within a given lesson he grappled with an onslaught of behavioral disruptions and infractions that affected how much math work and discussion the class completed.

Interactions that suggest empathy or commiseration formed a unique, emergent category. They did not happen in every lesson, but they stood out when they did. When Faith and Becca commiserated with how they perceived students were feeling, they seemed to be acknowledging tacitly that the students would rather not be in math class, doing math work. For example, near the end of a lesson in which notetaking and guided examples took up the majority of class time, Faith sighed heavily and said in a resigned voice, "Soon as we get through this example I'll give you guys a different activity to do. So just listen, watch. K?" (Day 2). When several students were talking, Becca pleaded jokingly, "Shh, girls, I need you to be *so* into math right now" (Day 1). It seemed as if these teachers – who most of the time were redirecting or correcting misbehavior –

had found a way to align with their students against a common foe: the math work or activity itself.

A different way of empathizing involved acknowledging the difficulty or challenge of the mathematics and casting students as able to take on that challenge. Kyra and Kevin were more specific as they attempted to relate to what they perceived as student frustration with the mathematical task. After modeling several examples of how to calculate surface area for a prism, Kyra noted,

Now that was a lot of steps, wasn't it? But when we're dealing with geometry, again, that's what we have to do in order to find the surface area. We break it down into different formulas once we find the area of those, the surface area, then we can combine them and find the total surface area. ... So some of you may feel that it's cumbersome, that it's having to go through so many different steps, but it's, um, it gets easier. Just like linear equations, we felt like they were kind of difficult at first, and now we've been doing them for a while so we've kind of gotten practice. The same thing happens with geometry. I can't say that I'm the best when it comes to geometry, but I know that filling in formulas, I'm able to substitute, and do the math. (Day 2)

Beginning a lesson on finding areas of irregular shapes, Kevin confided to the class, "Some of you guys, you get it right off the bat. That's awesome. But some of us, we have to think mathematically. It does for me. I go slow sometimes" (Day 1). While what he meant by "think mathematically" was not explained further, his emphasis was that some learners needed more time with problems, and that was fine. Kyra's emphasis was more specific: that with extended practice students would become more adept with problems that seemed challenging at first, just as she did.

The norms to develop the social culture of the classroom may seem unrelated to content, but I contend that they too shaped the students' experiences of doing math because they shaped the tone of the classroom. When the tone was positive, the potential for students to develop self-efficacy – to believe that they were capable of solving problems and making sense of mathematical situations – was different than when the tone was negative, when interactions negated or overwrote students' agency or ability to make sense of the math. To capture this idea

of tone, I reorganized the patterns in interactions to develop the social culture of the classroom into 3 distinct groupings: *positive*, characterized by energy and enthusiasm; *neutral or business-like*, oriented towards compliance to facilitate information delivery; or *negative*, primarily corrective or frustrated. Table 13 presents the constellations of moves that made up these classroom tones. Kyra's classroom stood out as a site in which there were few if any interactions related to developing social culture; together the teacher and students seemed to be in tacit agreement about what they needed to do, casting a tone best described as neutral. Becca's instruction similarly fell into this neutral or business-like category, while Faith's instruction took on a negative tone given her emphasis on correcting behaviors and her irritated or frustrated voice. Marie and Kevin both established norms that resulted in a positive tone, and Noah's instruction was a mix of these negative and positive norms. Later in this chapter I use these descriptions to suggest how social culture shapes math learning opportunities and signals messages about why doing math is worthwhile and who can do math.

Table 13

Fall Year 1 Snapshot of Instruction: Developing the Social Culture and Tone (***Bold** indicates routinely. Did not include interactions that happened rarely/once).

	Negative	Neutral			Positive		
	Behavior (Corrective or negative)	Acknowledged or Praises Behavior or Effort	Encouraged (General/ Coaching)	Empathized or Commiserated	Acknowledged Contribution to the Math Content	Encouraged (math-specific)	Acknowledged Math Ability (specific)
Faith	x (-)	x		x			
Marie	x	x			x	x	
Becca	x	x		x			
Kevin	x	x	x	x	x	x	x
Kyra				x			
Noah	x	x	x		x		

Responding to Students and Participation Structures

As I identified and analyzed normative practices related to the two primary roles of the teacher – to guide the development of mathematical ideas and to develop the social culture of the classroom – a third category of practices emerged that was relevant to both roles but also distinct: how the teacher responded to students and structured opportunities for participation. Responding happened throughout the lessons, as the teacher responded both to students’ answers, understandings and questions about math, and also more generally to students’ actions and behaviors. Responding moves overlapped with moves to guide the development of mathematical ideas when teachers evaluated student answers for correctness, clarified themes or content in

strategies, and posed questions to probe student responses for further explanation or reasoning. Similarly, when teachers addressed student misbehavior or praised student effort, those responding moves co-occurred with interactions to develop the social culture of the class. Furthermore, when they called on students – either those who volunteered or through a predetermined manner, like using popsicle sticks labeled with student names to make sure all students were accountable and had an opportunity to speak – they were setting up structures to guide participation. How, when, and to what teachers responded to students and the ways teachers structured participation emerged as a third category of normative practices that shaped mathematical learning opportunities and signaled messages about what it meant to know and do math.

To understand this grouping of normative practices, I considered multiple features of responses and participation. To what did a teacher routinely respond: student actions and comments related to behavior or to content? In the course of a single class, when did most of the opportunities for student participation in a whole-class discussion happen: during the launch, notetaking, or guided practice, or after work time, in a summary or review, or throughout? Were there opportunities for students to participate during notetaking –for example, providing definitions from prior knowledge? To what extent did discourse and a whole-class discussion happen after work time, and were students asked to share, explain, or critique others' strategies or answers? Throughout the lesson, for what reasons did the teacher call on students? To solicit ideas? To evaluate answers? To share strategies? To connect or comment on others' strategies? Finally, in all examples of practice, the teachers in this study routinely revoiced or repeated what students said, but even that action had varying purposes: to repeat and recap, to clarify, and to advance or articulate ideas for further consideration by the class as a whole. Table 14 provides an overview of each teachers' practices for responding to students and structuring participation.

Table 14

Normative Practices Responding to Students and Structuring Participation

Teacher	Routinely, responses to what students said or did were		When did whole-class discourse take place?	Teacher called on students to				Purpose of revoicing moves: <i>Repeat</i> <i>Clarify</i> <i>Advance for further discussion</i>
	<i>related to Behavior or Management</i>	<i>related to Content</i>		<i>Solicit ideas, background knowledge</i>	<i>Share answers</i>	<i>Share strategies</i>	<i>Connect or comment on others' strategies</i>	
Faith	Yes Negative/ Correcting	Occasionally Brief responses	Notetaking Guided Practice <i>before</i> Work	Occasionally	Occasionally	--	--	Repeat
Marie	Occasional redirecting	Yes Both how to and explanation	Both <i>before</i> and <i>after</i> Work Time	Routinely	Routinely	Routinely	Routinely	Clarify Advance
Becca	Yes Correcting or neutral	Yes Brief steps or "how to"	Notetaking Guided Practice <i>before</i> Work Time	--	Yes (Brief or choral responses)	Occasionally	--	Repeat Clarify
Kevin	Yes Positive Occasional redirecting	Yes Both how to and explanation	Both <i>before</i> and <i>after</i> Work Time (Discussion)	Routinely	Routinely	Routinely	Routinely	Repeat Clarify Advance
Kyra	--	Occasionally Mostly brief steps or "how to"	Notetaking Guided Practice <i>before</i> Work Time	Routinely in first lesson Occasionally	Routinely (Mostly brief/ choral responses)	Rarely	--	Repeat
Noah	Yes Correcting Also positive	Yes Mostly "how to" or "what you did"	Interspersed with Work Time (task scaffolded)	Occasionally	Routinely	Occasionally	--	Repeat Clarify Advance

Again, patterns in practices were apparent, and perhaps not surprisingly they held across groupings of other normative practices. Notably, in Faith, Becca, and Noah's classrooms, routinely the teachers responded to negative behaviors, correcting or redirecting. Another similarity between Becca and Faith's instruction was the occurrence of opportunities for participation in whole-class discussion before work time; rarely was time allocated for students to share strategies after work time. In this way, their instruction mirrored Kyra's. Noah's instruction stood out in that when he was not addressing behavior issues, he attempted to have students share not only answers but strategies or ideas. The classroom management issues, however, stymied the majority of these efforts.

Earlier in the chapter I provide examples of how Marie and Kevin responded to students with more energetic and enthusiastic tones; only occasionally did they redirect behavior and often they praised effort and welcomed contributions. With respect to how they responded to students and structured participation, these teachers' instruction shared common patterns: they allocated time for whole-class discussion both during the launch of the lesson or activity and during the summary or discussion after students had time to work. They routinely invited students to share not only answers but also strategies. They both asked questions to connect different strategies, although both tended to make those connections themselves if students did not. The focus for this category of normative practices was on opportunities students had to participate based on the teachers' actions; this was not an assessment of how the students responded or whether they provided reasoning in a mathematically efficient way, but rather of how the teachers encouraged or limited participation. In fact, when asked to explain their answers, students in all classes tended to begin with explaining *how* they arrive at answers (the steps they took, the formula they applied, and so on) rather than *why* their strategy made sense mathematically. Regardless of the nature of student responses, the participation structures matter because they created an opportunity, a

potential, for explanation, reasoning, and sense-making, especially when they took place after students have had time to wrestle with problems.

Additionally, what happened after teachers revoiced what students said conformed to the other patterns observed in their instructional interactions. In classrooms in which notetaking preceded any work, in which students were called on to share only answers, in which brief or choral responses to questions were sufficient, when the teacher revoiced what students said, s/he repeated utterances verbatim, in effect acting as a megaphone. And, in the classrooms in which teachers called on students to share strategies and explain answers, the teachers' revoicing moves served a slightly different purpose: clarifying or articulating ideas for the class to take up in an ensuing discussion. The practice of revoicing can take on different functions, with consequences for how student voices are invited into the discussion: in an echoing way or a participatory way.

From these different patterns of responses, a continuum of teacher responses and participation structures emerged. Figure 10 illustrates this continuum and shows where each teacher's practice fell. On one end of the continuum was teacher-oriented or teacher-directed participation: when the teacher responded either to the whole class or to individual students. In the middle of the continuum were those interactions in which teachers initiated the discussion – asking for an answer, for example – and students responded and then the teachers followed up with a recap, additional comments, or extending questions. Finally, at the other end of the continuum were response and participation structures initiated or guided by the students, with the teacher stepping in and out of the discussion. Progressively the continuum moved from class talk during which teachers made most of the statements – often in declarative or imperative form – to shared dialogues in which students had opportunities to participate or even lead. In the fall of their first year of teaching, Faith, Becca, and Kyra's instruction fell on the teacher-guided side, while Marie, Kevin, and to some extent Noah's instructional practices moved towards more

behavior, encourage students, and acknowledge student effort, work, and contributions. When, how, and to what teachers respond foster different kinds of participation opportunities, structures and storylines of whose voices are heard and when. By studying recorded episodes of instruction, I developed snapshots of the normative practices or “regularities” in novice middle school mathematics teachers’ instruction in their first semester of teaching along these three dimensions. Using a light beam as a metaphor for instruction, this initial analysis was akin to a process of splitting that light beam into three parts to be developed or understood separately. Now, in this section, I pull these parts together, as in a Technicolor process, to approximate a composite of instruction once more and to assess how these strands of normative practices, when interwoven, shaped the mathematical learning opportunities accessible to students and signaled messages about what it meant to know and do math, which I suggest is integral to cultivating productive dispositions towards mathematics.

The first question guiding this analysis was “How does the teacher’s instruction shape the kinds of opportunities to learn mathematics?” or, stated another way, “Given the normative practices of the classroom, what mathematical learning opportunities were available and accessible to students?” After grouping teachers by the ways in which the normative practices in their instruction shared features, two broad conceptions of the nature of school mathematics, and ensuing learning opportunities, surfaced: math as rule-bound, with the primary classroom activities or learning opportunities consisting of notetaking, guided examples, and practice problems; and math as a sense-making endeavor, in which students’ work and ideas were integrated into the class’s work and engagement with tasks and concepts as part of a process of co-constructing mathematical meanings with the teacher. In this conception of mathematics knowledge as co-constructed, I considered how compositions of normative practices had the potential to foster learners’ capacity a) with mathematical practices like representation and

justification, for example, and b) with argumentation, by gauging the extent to which the class moved towards assessing the validity of solutions and strategies based on the underlying mathematics (and not based solely on evaluation by the teacher or the textbook). Students sharing strategies served as a necessary first step in the process of developing capacity for mathematical argumentation.

The second guiding question for this analysis was “What does the instruction signal about what it means to know and do math?” or, phrased another way, “What signals or messages about knowing and doing math were conveyed through the normative practices?” For this analysis, I used the lens of a *productive disposition*, defined as the “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (Kilpatrick et al., 2001, p. 5), to understand effects of different compositions of normative practices. I considered both explicit messages and those that may have been unstated or unintended but were conveyed through the class norms. For clarity it is helpful to note the two distinct parts of the description of a productive disposition. The first part, the “inclination to see mathematics as sensible, useful, and worthwhile,” addresses the question “Why know and do math?” The second part, “a belief in diligence and one’s own efficacy,” speaks to how students perceive their own capabilities and capacities to know mathematics and solve problems, even if part of that “solving” process involves some struggle or effort. Messages related to dimensions of a productive disposition are interwoven within the math learning opportunities; for example, classes in which discussions include sharing and critiquing of student strategies by their peers cultivate a different sense of efficacy than those classrooms in which students practice a method demonstrated by the teacher. However, the teacher may also explicitly address student efficacy through remarks about what she or he believes the students are able to do through their own initiative.

Earlier, in Chapter 2, I distinguish between different *practice* constructs. I reprise those distinctions here to clarify the relationship between normative practices and mathematical practices and messages. Norms are distinct from mathematical practices. Teachers calling on many students to answer questions is a kind of normative practice, an explicit way of structuring participation to enforce an expectation that all students are accountable for being prepared to talk about their work in class. When that normative practice of calling on students facilitates sharing multiple strategies or representations in a public way, the norm shapes the mathematical learning opportunity because there is the potential to emphasize connections between different strategies and concepts, or to assess and critique reasoning based on the validity of the mathematical work. Similarly, when correct answers are accepted without question or explanation, and the teacher is the only one to identify and correct errors – again, both possible examples of classroom norms – the opportunity for students to practice reasoning and communication as mathematical skills is limited. Emphasizing notetaking and vocabulary before an activity is a normative practice, just as connecting to conventions after students have had time to work with problems is; they are both examples of how teachers may provide relevant information. These norms may point to consistent mathematical messages: for example, that precision and accuracy are important. They also provide different learning opportunities and emphases: for example, notetaking before work on an activity may suggest that the teacher’s providing of information takes precedence over students’ opportunities to pursue alternate methods or persevere through problems drawing on their own existing knowledge and strategies. An unstated assumption may be that students are not able or capable of doing the work without explicit guidance from the teacher. Guided problems may funnel work on a task towards specific procedures or algorithms. Or, when precision and conventions are emphasized with and through student work, there may be an opportunity to cultivate a different kind of mathematical sense-making, one in which students suggest and assess

possible solutions for validity and accuracy based on the mathematical evidence. Finally, norms to develop the social culture of the classroom may complicate this relationship: a positive tone may suggest at least an environment in which students are perceived as capable, whereas a negative or neutral tone may further cap the ways in which students engage with content. In this study, one finding suggested there was a constructive role that non-content normative practices played to further support mathematical sense-making, and, conversely, that when those social culture norms took on a more negative or neutral tone they capped or limited the potential for the math work that students did.

For this discussion of the analysis of Fall Year 1 instruction, I drew on the findings presented in the previous sections and represented in Tables 11 and 13, and Figure 10 (Normative Practices for Guiding the Development of Mathematical Ideas, Developing the Social Culture of the Classroom, and Responding to Students and Structuring Participation, respectively). I grouped the teachers by shared norms along the different dimensions, and then considered distinctions or dissimilarities with the groupings. For example, Faith, Becca, and Kyra's instruction shared guiding norms, such as emphasizing notetaking before student work time, but Faith's instruction stood out because of the negative tone cast both by the numerous ways and time spent addressing disruptive behaviors or perceived lack of students' preparedness. I then considered what the implications of these groupings, based on overlaps in normative practices across three dimensions, were for both the math learning opportunities and the messages conveyed about who knows and does math. Two groupings emerged: in the first (Becca, Kyra, and Faith), normative practices effectively *capped* learning opportunities and features of productive dispositions, and in the second (Marie, Kevin, and to some extent Noah) normative practices offered *promising* opportunities.

Normative Practices Setting a Ceiling or Capping Learning Opportunities and Messages

As stated earlier, the normative practices in Faith, Becca, and Kyra's instruction were similar in the ways in which they guided the development of mathematical ideas and structured responses and participation. Taken together, these normative practices suggested a construction of school mathematics as rule-bound, with learning opportunities funneled towards those kinds of activities that conveyed rules (for example, note-taking) and then emphasized repeated practice of the application of those rules. In these classrooms the teacher, in conjunction with the text, served as the authority assessing and evaluating solutions produced by procedures which had been demonstrated by the teacher. The routine pattern of revoicing as repeating what was said resulted in the teacher acting as a kind of speaker or megaphone for correct responses. The teachers' implicit approval of a response was the primary thread connecting correct answers, rather than the underlying mathematics. While connections may be emphasized even in a class in which the focus is on rules and procedures, in these classes, that did not happen. For example, in both of Becca's observed classes the videos she showed provided visual representations and real-world examples of the Pythagorean Theorem. The ensuing discussion and practice problems, however, did not advance connections between the formulas and the representations. The emphasis in both Becca and Kyra's lessons was on delivering information prior to work time so that students could then practice applying those formulas (using the Pythagorean Theorem to calculate the length of a missing side in a triangle, or calculating surface area of prisms and cylinders). The teachers showed and repeated how to apply the formulas, and then the students did the same, repeatedly. Faith's lesson differed in that the task, the mathematical activity, had the potential to be more open because there was not a set procedure for determining coordinates of a square park. Through her normative practices, however, Faith steered the lesson in this direction; she showed and told students what to do.

There were differences between these teachers' instruction, too, however, that evinced the relative importance of the third category of normative practices: developing the social culture. In Becca and Kyra's classes, students had time to work on mathematical problems without direct guidance from the teacher. This independent practice time, in fact, was an integral component of the routine of guided practice: demonstrate how to apply a procedure first before providing students with independent practice problems to develop mastery. The neutral tone in their classrooms reinforced this routine – or at least did not disrupt or contradict the implicit routine. Kyra's instruction barely addressed social culture in overt ways; already, a tacit agreement appeared between teacher and students about what was expected in math class: she guided and students followed. Becca redirected certain off-task behaviors, but her attempts to acknowledge contributions even briefly, with "Great" and "Good", set an overall neutral tone about the business of doing math work in math class. In Faith's classroom, ultimately the behavior issues she routinely addressed limited students' time to work on math independently, and frustration in her voice cast a negative tone over much of the class period. Students did very little math work themselves.

These differences in tone, even in the non-content interactions, shaped messages about efficacy differently. A consistent message in Becca's and Kyra's classes was that mathematical prowess developed through practice, and skills the students practiced would prepare them for math in high school. The teachers repeatedly reminded students about how to record their work and answers and how to organize their materials; in a sense, math work and class served as the setting to practice more general study skills. Faith also mentioned preparing for high school as a rationale for working on math, but in the ways she addressed students' behaviors and study skills she sent mixed messages about the students' capabilities. The negative tone in the class social culture diminished the sense of efficacy of students. For example she interrupted the warm up

work time to chastise students who did not have their papers, reminding them they were expected to keep their work together. Then, the very next thing she said was “You need a lab sheet? OK, I’m going to give it to you this time but you cannot lose it. If you think you are going to lose it you give it to me. No, don’t blame others, you are responsible for yourself” (Fall Year 1, Day 2). Later, during the same lesson, she connected a general lack of preparedness – not having pencils – to mistakes in math, but not in a way that suggested mistakes could be sites for learning. She said, in a loud voice,

K, just a general reminder: in this class you have to have pencils. If you do not have pencils and you ask me to borrow one I’m going to write you guys a detention because you guys need to come to class prepared. OK? I let you guys borrow pencils and I never get them back. This is math. You cannot do math in pen. Because I know, I make mistakes still too, but you guys make mistakes and you are not able to erase mistakes with pen. OK? That is why you have to have pencils. If you have some hardship that you cannot bring me pencils or you cannot bring your own pencils, you need to talk to me about it. But I expect every single one of you guys to be bringing pencils. You guys are in the 8th grade. Next year you guys are going to be going to the high school and you’re gonna have to use pencils in math.

The neutral tone in the social culture in Becca and Kyra’s classroom set an almost business-like environment: the teachers did not voice doubts about the students’ capabilities to do math, but they emphasized practice as the key, and through the other norms established a cap on the ways students could engage with the content. In Faith’s classroom, the negative tone cast doubt on what the students could do, period.

Promising Possibilities for Learning Opportunities and Developing a Productive Disposition

In contrast to the capped learning opportunities and messages, the normative practices shaping Marie and Kevin’s instruction (and Noah’s to some extent, but with mixed results) suggested a conception of mathematical meaning as co-constructed by teacher and students. This conception of school mathematics fostered learning opportunities with greater potential for developing capacity with practices like representation, explanation, justification, generalization. The norms for how the teachers responded to students and how they guided the development of

mathematical ideas complemented each other. Revoicing moves were not limited to repeating verbatim what students said, but instead advanced ideas for the whole class to consider. In this way the students' mathematical work was incorporated in to the progression of the lesson. At a minimum this developed capacity with practices of communication and representation. As students' presented different approaches, both Kevin and Marie would identify those approaches by the students' names ("This is Fretinisha's strategy", "what Marley did"). Both Kevin and Noah alluded to making connections between strategies and even generalizing from these presentations of work; Kevin described this as developing a "kind of arsenal of tools to be able to figure out these [problems]" (Fall Year 1, Day 1).

Noah attempted to do this by asking what patterns or relationships students noticed, for example with rectangles of fixed areas, but the mathematical thread typically was lost amidst behavioral or management interruptions. Finally, norms that supported a conception of mathematics as co-constructed also provided an opportunity to develop students' reasoning and argumentation. While an eventual mathematical knowledge goal of argumentation is for students to evaluate and assess the accuracy of a solution based on the mathematics (and not the external approval of the teacher or textbook), before that can happen, students need opportunities explaining and extending multiple solutions or approaches. In these teachers' classrooms, normative practices supported making students' work public, with accompanying social culture norms to encourage and acknowledge mathematical contributions without immediately evaluating. The responding norms complemented the guiding norms in that when the teachers articulated strategies to the whole class, in effect they made common information from more sources than just the teacher or the text. Co-constructing meaning entailed building concepts on and with the progression of student ideas.

In the observed lessons some of the teachers' interactions explicitly addressed dimensions of a productive disposition: why do math, and who can do math? Marie made repeated references to using maps in the real-world when launching activities that used coordinate grids. Kevin's rationales focused more on grades; he repeatedly reminded students that their efforts were part of their participation grade and, when redirecting students who had been distracted, he emphasized the role of working on math as part of an overarching responsibility of school:

You guys need to work on this. I'm disappointed you don't care much about your grade right now. Take a breath, grow up, and be eighth graders. It's a hard truth, but it's important that you know that. (Fall Year 1, Day 2)

Through the normative practices that facilitated student participation and acknowledged students' contributions, both Marie and Kevin's instruction had the potential to develop a positive sense of efficacy, supporting the view that the students were capable of doing the math work and adding to the classes' understanding. In his responses to students and moves to develop the social culture by establishing rapport, Kevin explicitly emphasized different ways students might view themselves as mathematicians, too:

So which way is correct? They both are correct. In math so often there's only one way to do something, but this way, in geometry – that's why I love geometry, because you all have choice. If you're visual people, if you're multipliers, whatever. ... I want you to turn to your neighbor and ask them this question: what is your strategy for finding area of irregular figures? Are you a counter? Are you a multiplier? Some people are counters? Some people are multipliers? Some people are visual. You take it like a puzzle. ... I personally am a multiplier. I like making rectangles around it and divide it in half. ... [After taking a poll by asking students to raise their hands to indicate their preference] OK, so we've got a good mix of people in here. Are any of those wrong? NO! (Fall Year 1, Day 2)

Kevin also emphasized that students could be knowledgeable authorities in the class, when he appointed students who had finished work on the task as resources for other students:

Kevin: I have duplicated myself. Somehow I've duplicated myself into 4 people total.

Student: You've quadrupled yourself.

Kevin: Yes, good point. I've quadrupled myself. Ronan, Marshall, and Elijah, if you have a question raise your hand and they'll help you too, 'cause they're doing really well. And getting antsy. (Fall Year 1, Day 1)

Both Marie and Kevin treated mistakes as part of learning. Kevin emphasized effort, reminding students more than once, "You don't have to be masters at it yet but you do have to give it a shot" (Fall Year 1, Day 1). Finally, in Noah's case, while his guiding moves did not yield the same results as Marie and Kevin's, the ways he scaffolded tasks still put the responsibility for trying and doing the mathematical work on the students. He did not show students every step, which was also important to supporting student autonomy and fostering a sense of efficacy.

When learning opportunities included the potential for students to co-construct mathematical meaning, the norms to develop the social culture seemed to play a different role in the class learning environment. With students' work and contributions made public, teachers made remarks about students' ability that furthered the development of rapport with students. Kevin explicitly affirmed students' effort: "Butler is showing a lot of effort today. He should be proud of that. You should" (Fall Year 1, Day 2). Noah, too, used instances of students' mathematical talk to provide positive reinforcement to students. One student in particular, Brian, had received repeated warnings and consequences for disruptive behavior, but Noah also persisted in structuring opportunities for the student to participate, to suggest a strategy, to check that strategy, until finally he had to say, "Brian, you participate great but I want to get other voices in the room" (Fall Year 1, Day 1). Even as Brian's behavior had warranted addressing and correcting, Noah also used participation opportunities to acknowledge and praise efforts in the math work. The negative behavior interactions did not set the same negative tone as in Faith's class; attempts to encourage participation and to value contributions seemed to counter the tone, mitigating the potential negativity from addressing the multiple behavior infractions. Finally, while Marie and Kevin both shared an upbeat tone in their demeanors, Marie did not exude praise

in quite the same way; however, she consistently invited students to talk and student contributions grounded and drove the class's engagement with the tasks. Marie and Kevin's enacted instruction shared several features: Both provided repeated openings and opportunities for students to talk about their own work and their peers' strategies. Their instruction differed, however, in the ways the teachers acknowledged behaviors or norms they wanted to promote.

Summary

In this chapter I have presented findings from a close examination of the normative practices that comprised novice middle school mathematics teachers' instruction in the fall semester of their first year of teaching. In attending to normative practices I focused on those patterns in instruction that reflected the regularities in the classroom about how the community worked together on mathematical tasks. I have suggested that different compositions of norms around guiding the development of mathematical ideas, developing the social culture, and structuring participation and responding to students shaped markedly different learning opportunities for students and signaled substantively distinct messages about who can know and do mathematics – who was capable, and in what ways. The portraits of instruction rendered both promising practices – such as those captured in Kevin and Marie's instruction – and more limiting or capped practices – such as those evinced in Faith, Becca, and Kyra's classrooms. At times, normative practices grouped in one specific strand outweighed, impeded, or reinforced the other practices. For example, in Noah's instruction, promising inclinations towards developing capacity with mathematical practices like generalization were impeded at almost every turn and ultimately stalled by disruptive behavioral issues that required addressing because they affected the well-being and safety of the learning environment for the students. Similarly, in Faith's class, addressing management issues happened repeatedly in a negative or frustrated way, such that the mathematical work that students could do on their own – and, perhaps more importantly, that the

teacher saw as possible for her students to do at all – was constrained to a “show and tell” set up, with the teacher, Faith, doing all the showing and telling. The students’ role was confined to copying what they were shown or told. In contrast, in Kevin’s classroom, norms fostering a positive social culture possibly had an amplifying or magnifying effect on how tasks provided opportunities for students to co-construct meaning and develop a positive view of their own capacity to solve challenging problems. I suggest that these varied compositions of normative practices suggest different messages both about the purpose of doing mathematics (in what ways was it worthwhile, relevant, and/or useful to learners) and about who was capable of doing specific kinds of mathematics work. In the next chapter, I present findings from an analysis of the teachers’ practice over time and suggest both story lines of change and explanations for those story lines.

CHAPTER 5: GROWING PRACTICES – STUDYING INSTRUCTION OVER TIME

In the previous chapter I presented findings from a close examination of teachers' instructions that illuminated not only the different kinds of normative practices that comprised instruction, but also how those norms, woven together, shaped mathematical learning opportunities and signaled messages about who could do and know math, and for what purposes. Mathematical learning opportunities were connected to two conceptions of mathematics: school mathematics as rule-bound, and school mathematics with meanings co-constructed by teacher and students together through work on mathematical tasks. These conceptions of school mathematics are not necessarily separate; they may coexist throughout a yearlong course. In the classrooms of the novice teachers featured in this analysis, however, these conceptions of mathematics were distinct, fostered by qualitatively different (and at times diametrically opposite) normative practices related to guiding the development of mathematical ideas, developing the social culture, and responding to students and structuring participation opportunities. In addition to shaping learning opportunities, the normative practices signaled messages about doing and knowing mathematics, which I framed as molding the potential to develop productive dispositions towards learning and doing mathematics. The normative practices related both to the ways in which students might have seen mathematics as useful, worthwhile, and relevant, and to the extent to which they might have viewed their own efficacy with respect to doing mathematical work.

After looking at the composites of the different groups of normative practices I suggested two descriptions of instructional practices: practices which were “capped” in their potential to undergird work and discussion around rich mathematical tasks (as demonstrated in Faith, Becca, and Kyra’s instruction) and “promising” practices, through which there were openings for the class community to engage in mathematical work with more of an emphasis on sense-making, reasoning, explanation, and the beginning of argumentation (sharing and examining possible

solutions and strategies). These promising practices were evident in Marie and Kevin's instruction. Noah's instruction blurred lines between these groupings in that while there was evidence of possible "promising" openings, management and behavior conditions and interruptions typically disrupted those openings, lapsing back to either incomplete or capped practices.

Now, in this chapter, I pivot to consider another question this dissertation addresses: what changes and what stays the same in novice teachers' instruction over time? Drawing on a similar close analysis of recorded observations of instruction from the spring of the first and second years of teaching, I identify storylines of change in the novice teachers' instruction. Specifically with respect to the teachers featured in this study, this analysis was a response to the "What happens next?" question. Did Kevin and Marie, both of whose instruction shared promising opportunities for student-oriented discourse, maintain those practices? Did they continue to solicit multiple strategies and explanations and make connections between those? And, over time, did students take on those roles and further develop their capacity for mathematical reasoning and argumentation? As the first year progressed, did management and behavior issues become less of an issue for Faith and Noah? Was Noah able to leverage his positive reinforcement to support student talk about content? Did Becca and Kyra move from presenting vocabulary and formulas and then assigning practice problems to making room for more student talk and eliciting students' prior knowledge? As I present change storylines (including those suggesting a lack of change, or a solidification or convergence of practice around specific patterns), I also consider more generally the ways that the push and pull of these individual threads on the "loom" of instruction may vary. For example, in what ways can changing participation structures lead to changes in other normative practices? In what ways can norms for guiding the development of mathematical ideas converge with norms for developing social culture in ways that value student talk and

contributions? And what are the implications of these changes in normative practices on the mathematical learning opportunities, including the ways in which a productive disposition may be cultivated (or capped).

Findings related to these ideas of changing practice are presented in this chapter. First, drawing on an analysis of observations from the spring of the teachers' first and second years of teaching, I describe changing or recurring patterns of practice. For example, Marie's efforts to elicit student talk and explanations and to make connections to students' lives or interests continued to support guiding norms that undergirded messages that math was connected to students and that students' work and ideas were imperative to the work of the class, to constructing meaning. In contrast, while a major structural change affected Faith's teaching context in her second year (she was reassigned to a different team to teach sixth grade, in an effort to reduce some of the management issues she encountered), change in practice was most evident in the norms related to developing the social culture and structuring participation. The shift to a more positive tone in the social culture, however, in which students presented their solutions for the teacher to evaluate, was not matched by similar shifts in the norms guiding the development of mathematical ideas and as such, learning opportunities remained capped. Becca and Kyra both maintained practices of presenting information and then providing time for students to work on practice problems; while capping the learning opportunities, however, these practices were consistent with the teachers' explicitly expressed goals of preparing students to be efficacious and successful in future math classes, in high school. Throughout this analysis of change, I considered both questions of which practices changed and to what extent, and whether changes in practices co-occurred with shifts in learning opportunities and messages about mathematics relevant to two dimensions of a productive disposition (the purpose for doing mathematical work in school and the development of a sense of efficacy).

After presenting these storylines of change, I shift the analytical focus to understand these narratives of change. First, I compare and contrast these findings with another measure of instruction, the Instructional Quality Assessment (IQA; Boston & Wolf, 2006), both to consider alternate assessments of instruction and also to distinguish the ways in which an analysis of normative practices may contribute to understanding the enactment of mathematics instruction. Next, drawing on an analysis of interview data with teachers, their administrators, and other mentors and colleagues during the first two years of teaching, I present both contextual and individual explanatory factors to understand the differences in instructional practices (and how they shaped and signaled mathematics). Included in the contextual factors were professional development opportunities and resources at the school and district levels, and also school structures that were perceived as constraining. I then suggest points of coherence and dissonance between novice teachers' orienting views towards their work (beliefs about what teaching mathematics entails and perceptions about their own successes and challenges) and the practices observed. In this way I work towards the dual purposes of this dissertation: to understand how normative practices shape and signal mathematics, and to frame those practices within the teachers' development of practice over time, for which teaching becomes a site for practice, to hone future practice.

Studying Practices over Time: Changing or Holding Patterns

To render the story lines of change for these novice teachers, I applied the same analytical lens to recorded observations at two additional time points in the novice teachers' careers: during the spring semester of the first year and the second year. Throughout this progression of time I attended not only to occurrences and patterns of normative practices for guiding the development of mathematical ideas, developing the social culture, and structuring participation and responding to students, but also to recurring patterns, to determine which

practices seemed to take hold over time. In this way this analysis attended to Lampert's (2010) conceptions of teaching practices as both the "things that people do, constantly and habitually" (p. 25) and as rehearsals for future performances, or, for these beginning teachers, future instruction.

While I examine changes in the infrastructure or contexts in which the teachers worked later in this chapter, a few contextual changes are worth noting before the descriptions and analysis of change. First, two teachers, Faith and Noah, shared similar experiences in that, in response to substantial management disruptions in their classes and across their grade-level team, their administrators took steps to modify their teaching contexts and assignments. By the spring of year 1, for example, Faith's schedule was changed from the original block schedule, in which she met her classes for up to two-hour lesson blocks every other day, to meeting classes daily for 70- to 80-minute periods. Noah's structural change was more drastic: due to ongoing behavior disruptions across all four subjects (language arts, mathematics, science, and social studies), the administrators at his school combined all the classes on the grade-level team so that one teacher would teach the subject area to all students on the team, while the other teachers and an administrator provided support. This resulted in Noah teaching one math class a day to as many as 50 students. This adjustment caused a lot of tension, but, since he wanted to continue participating in the induction study, Noah proposed that, for his observed lessons, he teach a smaller group of ten students whose assessment results indicated they were on the cusp of being considered "proficient" (as opposed to "improving"). For their second year of teaching, both Faith and Noah were reassigned to teach 6th grade, the first middle school grade, because their administrators believed a) they would have an easier time developing a positive social culture and well-managed class with the younger students, and b) they would then be able to devote more attention to improving their content instruction. The second major change, while at the district level, permeated half of the teachers' classes in the second year. In the 2010-2011 school year

the district implemented a pre-Algebra curriculum, *College Preparatory Mathematics* (CPM), to complement the existing CMP2 curriculum. The teachers affected by this change in their second year of teaching were Becca, Kevin, and Kyra. Because Faith and Marie were part of an earlier cohort in the induction study, they taught the CMP2 curriculum exclusively in their first two years, as did Noah, since he was reassigned to sixth grade.

The changes in the district-required curriculum and in teaching assignment were examples of the complexity of observing instruction and teacher practice in real time, over time; variables, like grade level or curriculum, changed due to circumstances and realities external to both the teachers themselves and the research study. I suggest, however, that examining normative practices still provides a window into the development of instruction and the ways practice shapes and signals mathematics because in attending to the practices as conveyed through interactions, the focus is on the teacher and her/his role mediating interactions with and between students and content. While the students changed every year, as did the curriculum for some teachers, there were consistent patterns in *how* the teachers engaged with students as a class around content. For this analysis, the normative practices that emerged from the teachers' interactions with students and content served as the evidence of change (or lack of change) in the teachers' developing practice. In the next section, I focus on the teachers' contexts and their orienting views or perspectives to understand the intentions underlying their actions.

In this section, I present findings from this examination of changes in normative practices in instruction over time. First, in an extension of the normative practices tables first introduced in Chapter 4, I describe the changes in normative practices based on the analysis of instruction from spring observations of the teachers' classes in the first and second years of teaching. Corresponding to Table 11, Table 15 depicts changes in normative practices to guide the development of mathematical ideas. Similarly, Table 16 and Figure 11 correspond to Table 13

and Figure, depicting changes in normative practices to develop the social culture and in the continuum of responding to students, respectively. I then outline three story lines of change (or lack of change): continued “capped” practices (as reflected in Becca, Kyra, and Faith’s instruction) despite changes in some categories of normative practices; continued “promising” practices (as reflected in Marie’s instruction); and “capped changes” to promising practices (as reflected in Kevin and Noah’s instruction). In framing these story lines of change I considered both patterns in normative practices and also how those practices shaped mathematical learning opportunities and presented messages relevant to developing a productive disposition towards doing math (specifically, signaling the purpose for doing math in school and supporting a sense of efficacy). I use examples from the teachers’ instruction to illustrate the different narratives of change.

Table 15

Snapshot of Change in Normative Practices – Guiding the Development of Mathematical Ideas

(√ indicates consistent presence in the teacher's instruction; ~ indicates attempts or incomplete/unrealized efforts; + indicates increased or extensive occurrence)

	Routinely repeated non-math procedural directions	Routinely provided vocabulary, conventions, procedures, formulas as lesson/launch, before work time	Mathematical emphasis on knowing definitions, applying formulas	For most questions, brief or choral responses sufficient (including listing of procedural steps)	Errors or mistakes were identified by teacher, who then demonstrated correction	Interspersed language and conventions with student work, discussion of strategies	Mathematical emphasis on making sense of problem or connections	Questions had potential for higher-level extensions, representations, explanations	Errors or mistakes part of class discussion, with teacher soliciting corrections or alternate methods
Faith FY1	√	√	√	√					
SY1/2	√	√	√	√	√				~
Marie FY1						√	√	√	√
SY1/2						√+	√	√	√
Becca FY1	√	√	√	√	√				
SY1/2		√+	√+	√	√				
Kevin FY1	√					√	√	√	√
SY1/2	√		√	√		√			
Kyra FY1		√	√	√	~				
SY1/2		√	√	√	~				
Noah FY1	√			√			~	~	~
SY1/2	√			√	√	~	~	~	

Table 16.

Snapshot of Change in Normative Practices – Developing the Social Culture

(***Bold** indicates routinely. Did not include interactions that happened rarely/once).

	Negative	Neutral			Positive		
	Behavior (Corrective or negative)	Acknowledged or Praises Behavior or Effort	Encouraged (General/ Coaching)	Empathized or Commiserated	Acknowledged Contribution to the Math Content	Encouraged (math-specific)	Acknowledged Math Ability (specific)
Faith	x (-)	x		x			
<i>SY1/2</i>	x (-)	x	x				
Marie	x	x			x	x	
<i>SY1/2</i>		x	x	x	x	x	x
Becca	x	x		x			
<i>SY1/2</i>		x	x	x			
Kevin	x	x	x	x	x	x	x
<i>SY1/2</i>	x	x	x	x	x	x	
Kyra				x			
<i>SY1/2</i>		x	x				
Noah	x	x	x		x		
<i>SY1/2</i>	x	x	x		x		

Becca's guiding math norms were also amplified, but in a very different way; while she called on more students to show work, she took on the role of narrating every step, further amplifying her role as providing information *including* verbalizing students' own work. Some of the changes regarding the norms to develop the social culture of the class were expected: by the time observations were recorded in the spring, what transpired with respect to addressing behavior tended to be more consistently positive or redirecting, as the teachers had established expectations and consequences with the class over a longer period of time. In the fall of the first year, for many these efforts were still fledgling or developing, and as such addressing behavior more frequently was corrective. Finally, Figure 11 depicts the continuum of responding to students and structuring participation practices. Notably Becca and Kyra both extended their revoicing patterns to clarifying ideas for students, but these clarification patterns were different than Kevin's and Marie's in that Becca and Kyra tended to narrate students' approaches for them, rather than using student words to develop a progression of ideas.

From these patterns of normative practices over time, I developed story lines of change based on an analysis of the groupings and nuances within groupings, especially when taking implications for math learning opportunities (math as rule-bound or math as co-constructed) and messages related to productive dispositions into account. As in the analysis of Fall Year 1 instruction, math learning opportunities and messages about math both were shaped by combinations of normative practices, but, in this analysis of instruction over time, I also considered whether the changes in normative practices amplified or attenuated opportunities or messages, and in what ways. Amplification of patterns in normative practices suggested a stability or recurrence of normative practices that continued to shape either promising learning opportunities or capped ones. For example, Marie's instruction illustrated this idea of amplification in a promising way. Even when the lessons themselves seemed to involve a fair

amount of demonstration (for example, introducing reflection or rotation symmetry), the amplified way that Marie introducing ideas by interweaving them with students' contributions or interests created avenues for mathematical meaning to be co-constructed. In contrast, in Faith, Becca, and Kyra's instruction, shifts in practices to develop the social culture, to acknowledge effort more, did not sufficiently counter the amplification of guiding norms in which the teachers asserted their role of telling students what to do to practice procedures, and this amplification had consequences for the message about how students developed efficacy (through extensive practice of what the teacher demonstrated). Finally, changes in practices in Kevin and Noah's instruction sometimes attenuated or countered promising practices: I called these "capped changes". Kevin, for example, maintained several of the positive social culture and participation practices from Fall Year 1: he acknowledged and valued student effort and contributions and structured multiple opportunities for student talk. But, when the curriculum tasks were less exploratory (especially when the curriculum changed in Year 2), he shifted from emphasizing making sense of problems and strategies to funneling students towards a single strategy or "rule" for his students' "arsenal" of math tools. Similarly Noah persisted in scaffolding tasks and not showing students how to solve problems step-by-step, thus distinguishing his practice from Faith, Becca, and Kyra's, but he also asked questions for which brief or choral responses were sufficient, thus attenuating the extent to which students engaged in reasoning or argumentation practices.

In the following sections I provide illustrative examples of amplification or attenuation in the changes in normative practices. I then consider effects of these story lines of change on the math learning opportunities and messages related to purposes for doing math and building students' sense of efficacy.

Repeated “Capped” Practices: Capped Learning Opportunities & Messages

Becca, Kyra, and Faith’s story lines of change in normative practices ultimately suggested continued or amplified “capped” practices, learning opportunities and messages. There were some shifts in social culture norms and participation norms. Becca, for example, acknowledged students’ efforts more consistently and encouraged their efforts. By the Spring of Year 1, she set up participation structures in which students were called on at random to show their work (typically an application of a formula) on the board; this was in contrast to the Fall Year 1, when she typically demonstrated problems and steps. In attempts to establish rapport, she made supporting comments when students were struggling, affirming for example, “K, we’ll help you! Come on, you can phone a friend, it’s okay” (Spring Year 1, Day 2). These changes, however, did not shift the math learning opportunities; they did not alter the direction of the norms guiding the development of mathematical ideas, which had tended towards a conception of math as a rule-bound subject for which the teachers explained or demonstrated the rules and then students practiced. Becca amplified her ways of providing information, so that not only was she demonstrating or modeling sample problems, but also, even when students showed their work, she narrated each step of the problem. The following excerpt illustrates this amplified pattern. The problem asked students to evaluate the expression $-x - 6$, if $x = -8$. Becca explained,

So, Alicia is going to put her negative out front, and then she’s going to put parentheses, and then she’s going to put negative 8, subtract 6, K? So she put little plus signs through the 2 negatives because they’re next door to each other, right? So then she has 8 minus 6. Which one’s bigger: the 8 or the 6? So it’s going to be positive, right? (Spring Year 1, Day 1)

Kyra’s normative practices guiding the development of mathematical ideas also emphasized the idea of rules governing math work. For example, during a lesson on reflection symmetry of points and figure on a coordinate grid, she introduced and reiterated the following rules before and during student work time:

Kyra: OK, so if it's y-axis, and we just want to find the reflection, then we negate the what? Negate the x, make the x a negative. Does the y change? No. So let's think about it, if we're reflecting over the y-axis, and we have to make the x negative and the y remains the same, what do you think would happen when we reflect over the x-axis?

Student: The y turns negative.

Kyra: The y turns negative and what remains the same? The x. If you study and complete the study guide, you will be successful on the [upcoming assessment]. (Spring Year 1, Day 1)

The mention of the tests was one of many versions of a consistent message about the purpose of doing math work: to pass tests successfully and be prepared for high school. The messages about mathematics in these classrooms were fairly static. By the Spring of Year 1, Kyra mentioned the importance of being able to explain reasoning:

But again, it's very important that you not only know how to recognize translation symmetry but that you are able to verbalize what translation symmetry is and be able to defend your answer as far as if somebody says, "Well, why do you feel like this is translation symmetry?" We should be able to communicate our ideas effectively. (Spring Year 1 Day 2)

However, the practice of explaining was never woven into the work of the class. Instead, the message (often stated explicitly) was that through extensive practice students could develop fluency or efficacy or capacity with solving math problems so that they would be prepared for high school. When Kyra introduced a topic, for example, she noted, "It's nothing very strenuous, but it's all to help us get prepared for the [test]" (Spring Year 1, Day 1). This reiterated the conception of math as rule-bound; students had opportunities to practice and develop fluency with procedures but they did not apply that knowledge to novel problems. Furthermore, despite saying explanations and reasoning were important, Kyra focused students on writing answers using mathematical conventions, stating that this was preparation for expectations in high school:

OK, either answer is going to be correct, but I want you to know how to write it both ways, cause when you start doing as freshmen, sophomores, when you start getting into geometry, you're going to probably see the rules more so than you will see the explanations, okay? I want you to be familiar with those ... OK, so I want us to get used to writing it in the parentheses, okay, because when you get to high school I don't want

your high school teacher to think that we didn't learn it this way. And that all you can do is describe. (Spring Year 1, Day 2)

In concert with this focus on learning math to be prepared for high school, both Kyra and Becca emphasized ways of organizing and recording as means for students to develop study skills, and conversely that those study skills, rather than reasoning or representations or connections, would support their math proficiency. Throughout work on reflections and translations, Kyra emphasized, "The key to this is really going to be organization. K? To make sure that you really organize your coordinates so that you can keep them on track" (Spring Year 1, Day 2).

Over time, despite major changes in structures (in the duration of class in Spring year 1, and in grade assignment in Spring Year 2), Faith's instruction resembled Becca and Kyra's. With the structural changes there were some shifts in the tone of the social culture in her class, from distinctly negative and corrective to more neutral. As that shift happened, she made efforts to ask students more questions, though they tended to require brief responses. Her revoicing patterns shifted from only repeating or recapping to revoicing to clarify steps of a problem. Ultimately, however, the math learning opportunities were consistent with a conception of math as rule-bound; she still focused on telling students what to do and showing procedures. There may have been more opportunities for students to share answers, but Faith still controlled the evaluation of those answers for correctness, again narrating the steps. She marked errors and either demonstrated or guided students through the descriptions. She positioned herself as the guide between the students and the content, through statements like, "I'm gonna describe it for you," "This is what you guys need to do," and "I would let you write up here but we are running short on time, so if you will tell me what I'm going to write for A, that is what I will write" (Spring Year 1, Day 2).

Amplifying “Promising” Practices: Making Math Work Public and Connecting to Students’

Interest

Throughout the observed lessons in her first two years teaching, Marie continued to enact several of the normative practices that had supported a conception of mathematical meanings as co-constructed, had allowed for opportunities for students to explain reasoning and make their mathematical ideas public, and had fostered promising dimensions of a productive disposition.

In the Fall of Year 1, Marie had typically interspersed introducing mathematical language and conventions with students’ ideas and work. In this way the class co-constructed meaning, in contrast to lessons in which conventions were presented first for students to take in. Marie continued this in lessons in both the spring of year 1 and year 2, often drawing on humorous reference points or student interests to further anchor the math topic. For example, during a lesson on rotational symmetry (Spring Year 1, Day 2), Marie and the class informally discussed the center of rotation with humor; she reminded them of something they did at the beginning of the year that distracted her.

Marie: Think about how, when I give your rulers - remember, at the beginning of the year? Shea? When I gave your rulers and you guys would always take your pencil and put it on that, that ruler and spin it like a helicopter. That would always really annoy me?! When you spun it, where did you usually put that pencil in order to make it rotate evenly?

Different students call or shout out. The class consensus is in the middle or in the center.

Marie: In the exact middle. What would happen if you put it on the other side?

Again, different students call out, “It would be lopsided.”

Marie: It would be lopsided, right? It wouldn’t rotate evenly. So the center of rotation has to be in the exact middle - why?

Class: So it can be even/equal.

Marie: Good, so it can rotate evenly around the middle. So that’s something to remember.

Marie eventually introduced the terminology she wanted the students to know, but not until after the class as a whole had co-constructed images like this through connections to students' prior knowledge or experiences. Similarly in year 2, Marie introduced a symmetry project, this time with a Selena Gomez music video. She asked students to identify all the different examples of symmetry they saw in the video, explicitly connecting school math content to something that felt relevant to the students' lives.

Another promising practice that grew over time was the continued effort not only to give students the opportunity to share their math work publicly, but also to use those contributions to develop students' capacity to assess the validity or correctness of a solution. This happened throughout different activity structures. For example, just as the other teachers had, by the spring semester Marie was emphasizing testing; it was a reality on the horizon for her class, so warm up tasks were sample multiple choice or extended response review problems. Her method for going over the answers, however, was different than that of any of the other teachers. Without giving any indication of the correct answer, she would ask students to raise their hands to show which answer they picked, stating, "Raise your hand high, it doesn't matter if you guys are wrong, just looking to see where you guys are at." In this way she informally assessed who had figured out the correct answer, but did not call out or make an example of those students who had made a mistake. Mistakes were treated as sites for learning, as the class would review each answer choice and eliminate incorrect answers. Then, as some answer choices were eliminated, Marie would ask, "Who can take us through the thought process?" What was notable was that the students would then take over the explanation, and the evaluation of the correctness. They would hold the floor for 30 seconds or more, making their strategy public. This extended air time for student talk offered a contrast to the classes in which brief or choral responses made up the majority of student talk. For example, students worked on a probability multiple choice problem: "If the

letters of the word MATHEMATICS were cut apart and placed in a bag, which of the following would be LEAST LIKELY to occur if I only drew one letter from the bag?” After students had eliminated some choices, Brianna explained,

All right, well, I picked D because I looked at the little Mathematics word and then I seen how many, like, how many letters there were of each. And like, then I looked at that thing, all the answer choices, right, and then I was like, well, H only has 1 and so that’s least likely to occur. (Spring Year 1, Day 1)

Marie asked if any students had questions; the class as a whole accepted Brianna’s extended, if informal, reasoning, as did Marie. She did not restate or re-explain Brianna’s strategy; the implicit expectation was that students would listen to each other’s explanations and hold each other accountable. This accountability not only to what the teacher said but what peers said was evident during other activity structures. In a lesson on reflection symmetry (Spring Year 1, Day 1), Marie asked students to find lines of symmetry for different images. During the discussion, students clarified errors or misunderstandings for the whole class; for example, one student challenged a proposed line symmetry, noting that it did not work because in the image not all the pieces would be reflected. Through these opportunities to make their mathematical thinking public, students not only co-constructed meaning, but they also developed their sense of argumentation around critiquing and assessing the validity of proposed strategies and solutions.

These opportunities to co-construct meaning and make mathematical ideas public also suggested different attributes of a productive disposition that could be fostered through Marie’s normative practices. First, through her frequent efforts to connect the math topics to students’ background knowledge or interests, Marie’s practice suggested a different answer to the question, “Why do math?” The answer was not focused explicitly on developing skills for high school; rather, Marie emphasized that math was relevant and connected to student lives. At times, as she wove students’ background knowledge or experiences with topics, it seemed like she noticed connections big and small. For example, in the same lesson when she drew on “pencil wheelies”

to illustrate the center of rotation, Marie noticed that a student's belt buckle was a hubcap. At the end of the lesson she called the class's attention to this:

Um, this is Rachel's belt. Rachel's belt, no it doesn't spin, but it's a hubcap. K? Can you look and see how many sections you would divide this into? How many sections can you divide it into to get the exact same picture? ... Is this a little hard? What about if you look at it this way? How many sections are there? There are 3. If you look at this way you can see there are 3 parts that are going to give you the same image (turned belt buckle three times). These are all around you, so next time you go to your car you can look at that. (Spring Year 1, Day 2)

Marie's practices also reinforced a slightly different message about efficacy. While there were lessons in which Marie asked the students to practice particular problems or strategy (for example, identifying the angle of rotation, or finding the center of rotation, for a given set of images), her efforts to keep student work and explanations public set an implicit message that students were capable of contributing to the math work of the class by persevering with problems. Marie made explicit statements to support students' sense of efficacy, too. For example, when posing the warm up problem to students, she noted, "We have two problems up here that you should be able to struggle your way through. I understand if it's a little challenging for you but, um, you have enough background knowledge that you should be able to get these" (Spring Year 1, Day 1). Notably, she did not demonstrate how to solve problems before students had time to try them.

Marie's development over time provides an illustration of how when certain normative practices recurred or were amplified, promising learning opportunities and messages about math abounded. In the last story line of change, I present the cases when, despite some evidence of normative practices that were "promising", over time change was "capped", resulting in an attenuation of the "promise."

“Capped” Changes: Attenuated Learning Opportunities & Messages

In this story line of change, “capped” changes refers to narratives in which, despite evidence of inclinations towards practices that would support promising openings or learning opportunities, shifts in the norms resulted in either attenuated or unrealized opportunities to learn and mixed messages. Both Kevin and Noah’s instruction over time showed evidence of these “capped” changes. In Kevin’s case, while the promising social practices he enacted to cultivate talk and engagement recurred, there were shifts in the normative practices to guide the development of mathematical ideas, and those led to somewhat narrowed learning opportunities and mixed messages to develop dimensions of a productive disposition towards doing math. In Noah’s situation, while he continued to attempt to support student engagement and struggle with problems, he did not always see his plans and practices to completion, resulting in unrealized opportunities. What distinguished these teachers’ practice from the “capped” practices, however, was that there were openings or opportunities for students to take some part in co-constructing meaning.

In his observed lessons in the spring of year 1 and year 2, Kevin maintained the promising normative practices he had enacted in the fall of year 1. He continued to acknowledge and appreciate student contributions and ideas, and regularly set aside time for a class discussion or summary. He regularly elicited student contributions with enthusiasm and emphasized expectations that students were accountable to attending not only to what he said, but to what their peers put forward in class discussions. For example, before a lesson on reflections over a line, he posed an open warm up task, asking students to draw a shape and then reflect it across a vertical line of reflection. During the warm up review, he encouraged and praised students’ presenting their work and emphasized expectations for how students should listen to each other.

Kevin: Who feels awesome about what they drew, they are so proud, so proud? Let’s do Miguel. Will you go up to the board and draw it for me - or my computer? And as an

active audience it's not just sit time. As an active audience raise your hand if you feel you might see an error and we can help Miguel out. Keep in mind what he's doing. Doesn't have to be perfect, doesn't have to be perfect.

Student call out: No, it's wrong.

Kevin: See, all the respect just went out the window. What happened? I am so happy you are doing what you're doing, Miguel, I'm going to show you in a second. (Miguel was labelling vertices.) Great. Go ahead, and stop there. Take a seat. I love it, I love it. I love how you're participating.

Student call out: Give him a round of applause.

Kevin nods, then pauses: I'll wait on you all. I'll wait on you all. OK. Thumbs up if you agree with Miguel, thumbs down if you disagree. No, we're not going to be splitting hairs here. Good, good. I agree with him, I think it's awesome. Now, it's not going to be exactly perfect because first of all it's hard to write on that, isn't it? I have to do it all day long. But secondly, do you know what I love about Miguel's drawing?

Student call out: He labeled his –

Kevin: He labeled it! But some people, they might put this is B and this is A (left to right – a correct reflection would show right to left). So you gotta think about that. If you took this shape and you mushed it on top of itself – there's actually a special way that we note a reflected image. (Spring Year 1, Day 1)

Then, building on the progression of ideas from how Miguel labelled his reflected image, Kevin introduced the convention of how to label the corresponding vertices of a reflected image. Kevin made repeated attempts to establish rapport with his students, to recognize their effort and contribution, and to address in an explicit way social norms such as how the class would listen and respond to students presenting their work. In Year 2 he continued to introduce vocabulary and conventions after the class had spent some time working with a topic; for example, he introduced the term “reciprocal” after a student had noticed and suggested the more colloquial word “opposite.”

However, Kevin's normative practices to guide the development of mathematical ideas underwent some changes, especially when the curriculum changed in Year 2. Even in the Spring of Year 1, Kevin had shifted towards emphasizing rules. Occasionally he provided information

before work time, and then, as students worked, repeatedly funneled student attention towards general rules they could apply. This could be viewed as an approximation of the mathematical practice of generalization; what made Kevin’s approach distinct, however, was that he presented these rules as tricks or “short cuts”, and sometimes seemed to separate the rule from the problem context in ways that de-emphasized sense-making. When launching or assigning a task, Kevin also made modifications to the task in ways that shifted the emphasis from sense-making to finding short cuts. For example, an open-ended task, “Write commands that will draw the image of the original flag under a reflection in the x-axis” was changed to a brief and more specific, “Reflect the flag and write down the coordinates” (Spring Year 1, Day 1). Before students began working on the task, Kevin reviewed the “rule” for which coordinates change: “When you reflect over the y-axis, the sign of the x-value changes” and “when you reflect over the x-axis, the sign of the y-coordinate value changes.” He emphatically reminded students, “Once you remember this pattern, you don’t even need to plot the point!” After students had worked and shared answers, Kevin reiterated the rule, connecting it to the work the students had just finished:

Kevin: Do you understand what we did there? Stop, stop, take a breath. I reflected over my x-axis. If I folded it right down the middle, this would be on top of this, but what value changed?

Students called out: The y, the y.

Kevin: The x did not change, still stayed 2 and 2. But what changed?

Students called out: The y.

Kevin: The y. So reflect over the y-axis, what changes? (Louder) Reflect over the y-axis, what changes? X. Reflect over the x-axis, what changes?

Students called out: The y.

Kevin: If you understand that, you will get a perfect score on your homework tonight. That's it. It's that easy.

While Kevin's normative practices for responding to students maintained some opportunities for co-constructing meaning, the changes in norms to guide the development of mathematical ideas attenuated those learning opportunities and resulted in some mixed messages about knowing and doing math. For example, similar to Kyra, Kevin framed the purpose for doing math in terms of pending tests:

Everybody, please watch because you will be assessed on this and this is part of your quiz, it's part of your test, it's part of everything. Please listen. I want you to reflect this image over the y-axis for me. Over the y-axis. Do your best, do your best. (Spring Year 1, Day 1)

Kevin continued to provide affirmations of his students' math ability, which could shape their sense of efficacy, but, again like Kyra, he related that efficacy to practicing applying the tools or shortcuts he demonstrated:

So if you remember that if we reflect over the y-axis that the x-values change and that if we reflect over the x-axis the y-values change, this homework will be super easy for you. I hope you guys are proud of yourselves today because this is actually pretty tough stuff. Yep. And if you don't think it's tough that's even better. That's even better.

In praising students for the math work they did, he conveyed a message that math was a hard subject, but that they could be successful. This message became a little convoluted, however, by more dismissive remarks he made, perhaps to establish rapport with students by joking about himself. For example, he commented, "I've got a dot on the board. I'm going to call it a point. 'Cause I'm a math teacher, right, we use our own little nerdy terminology" and later repeated that dismissive joke again when introducing how to label points of a reflected image: "Call that A prime, B prime, C prime, D prime, and E prime. Makes you sound smart when you say it. Makes you sound a little nerdy and math-like, right?" In sum, while Kevin maintained some promising practices that created a learning environment in which students could have experiences with constructing meaning, with representations, with some forms of argumentation, other changes in norms attenuated or diminished the ways math was shaped and signaled.

In Noah's case, his continued efforts to scaffold rather than demonstrate and guide students through applying procedures marked his instructional practices as different from the "capped" practices; he did not show students what to do. The resulting learning opportunities and messages were mixed, however, often with unrealized potential. In his school and class context, disruptions in management quickly derailed promising practices by limiting follow through. For example, in his observed small-group lessons in the Spring Year 1, he tried to connect the math work to student interests by having them create data sets based on surveys they conducted around taste tests. He utilized a scaffolded approach to move the class through the problems in a coordinated way. He continued to reinforce behavior expectations with praise when possible, but there were still enough disruptions that most of his questions and the answers required became brief, terse, and the potential for connections or explanations was left unrealized. In the Spring of Year 2, when he was teaching sixth grades, he made efforts to have students share strategies, but while he would pay attention and make connections, he did not always make sure that these math ideas were public to the entire class. And, when behaviors became disruptive, Noah resorted to assigning individual work time on work sheets to manage the class. The challenging context seemed to cap or place a ceiling on the promise of his norms to support student talk, engagement, and perseverance with worthwhile tasks.

Looking Across Story Lines of Change

In this section I describe three story lines of change both in enacted normative practices and the ways they shaped math learning opportunities and signaled messages about knowing and doing mathematics. Looking across the different story lines, some common threads emerged. First, changes in normative practices did not always guarantee changes in learning opportunities. For example, Becca increased the opportunities for students to participate by presenting their solutions publicly, writing on the Smart Board, but since she directed and narrated all the steps

and explanations, the learning opportunities were capped at applying rules and procedures. Changes to structuring participation opportunities were important but insufficient to raise the potential of capped practices when other norms remained unchanged. Second, repetition of normative practices could have an amplifying or attenuating effect on the learning opportunities. Even when teaching seemingly closed tasks (like finding an angle of reflection), Marie infused each activity structure –from the launch to review of terminology to work time and discussion – with connections to students’ lives and interests, and this amplified a distinct message of math as relevant and connected to students. Meanwhile, Kevin’s focus on finding rules that could serve as shortcuts attenuated some of the ways his response structures could support students’ mathematical sense-making. Finally, teachers conveyed different messages about the purpose for doing math and how students could develop fluency or a sense of efficacy, sometimes explicitly, sometimes in unstated ways, through the norms they enacted.

In the next section I consider the validity of these assertions about change by comparing and contrasting them with another measure of the quality of instruction and then by situating these practices within the teachers’ contexts and orienting views.

Understanding Change: Comparing Analyses of Instruction over Time

Thus far I have presented different storylines of normative practices and how they shaped learning opportunities and signaled messages about doing and knowing mathematics. After studying the classroom normative practices fostered by novice middle school mathematics teachers in their first two years of teaching, I suggested a) different groupings and features of normative practices that composed mathematics instruction, b) ways those strands of practices, when interwoven, both shaped mathematical learning opportunities and signaled messages about the purposes for learning mathematics and how learners could develop efficacy, and c) that over time, practices changed or, more often, recurred, but with disparate significance on the math

learning opportunities and messages. In this section, I reflect upon possible explanations of the practices and changes in practices. I consider how these portraits of instruction cohered or aligned with another analysis of instruction, the IQA, which was designed to assess dimensions of the mathematical learning experiences and opportunities in the classroom. Then, in the next section, drawing on an analysis of interviews with the teachers, their administrators, and their mentors and colleagues in the first and second years of teaching, I present findings of the contextual and structural factors that framed the novice teachers' experiences and, on an individual level, the perceptions of challenges, both mathematical and non-mathematical, that affected the teachers' priorities and, as such, influenced how and about what they interacted both with students and with content.

Comparing Analyses of Instruction (and Narratives of Change)

A guiding assumption of the analytical framework for this study is that instruction is composed of interactions (Cohen & Ball, 1999) that take place across different problem spaces (Lampert, 2001) between the teacher, the students, and the content. Within the different relational spaces, normative practices push and pull in ways that shape the mathematical learning opportunities. For example, the tasks a class may do are written in the curriculum, but during instruction, during the enactment of specific tasks, norms for guiding the development of the mathematical ideas frame whether the task is worked on in a procedural or conceptual way, the cognitive demand of the task is maintained, and solutions or strategies are discussed in a rich, connected way (see research on the Mathematical Task Framework; e.g., Stein et al., 1996; Stein & Henningsen, 1997; Stein & Lane, 1996). Norms for developing the social culture and for structuring participation shape opportunities for discourse to be mathematically rich, for example, when strategies are shared, discussed, and connected to representations or other approaches, or to be more rote, such as when brief answers suffice and funnel ideas towards a single approach or

formula (see, for example, Wood, 1996). By affecting tasks and talk in the classroom, normative practices mediate the nature of the mathematical experiences in which students engage. In this section, I present findings from an analysis of the change trajectories of these novice teachers based on another measure of instruction, the Instructional Quality Assessment (IQA; Boston & Wolf, 2006), which was used in the induction study (the source of the data for this study). For this analysis, I examined the extent to which the story lines of change based on studying normative practices aligned to the trajectories that emerged from the IQA ratings, which addressed comparable dimensions of the mathematical learning experiences in the classrooms.

As described earlier in Chapters 2 and 3, the IQA instrument was used in the induction study to assess two dimensions of instructional quality: academic rigor and accountable talk. These dimensions provided a window into the quality of the mathematical learning experience by assessing the enactment of mathematical tasks (the extent to which the tasks were and remained cognitively demanding) and the classroom talk (the extent to which students were held accountable both to the learning community and to the knowledge and rigorous thinking of the discipline). Both dimensions impact the mathematical learning opportunities, and connections between normative practices and each dimension are discernible. Normative practices, such as asking for explanations or presenting vocabulary or formulas before work time to funnel students' attention towards a specific method or algorithm, may support or inhibit whether the cognitive demand of a task is maintained, which is a focus of the Academic Rigor rubrics. Practices such as acknowledging contributions and structuring opportunities for extended review to clarify or articulate key ideas shape the talk in the classroom community and speak to the Accountable Talk rubrics.

For the induction study, each recorded observation was rated with respect to both dimensions, with the following guidelines: a) the task that took up the majority of class time was

the mathematical task assessed using the rubrics, and b) each observation was treated as a distinct data point. Given that the observations took place in real-time, in natural settings, adhering to these guidelines yielded portraits of instruction that at times were incomplete. For example, if the activity taking up the majority of class time was notetaking, then scores on the Accountable Talk rubrics were not applicable since those rubrics focused specifically on the talk from discussions that occurred after work time. As noted throughout this analysis, in these novice teachers' classrooms, opportunities for student talk were not confined to discussions after work time, and in fact, in several of the teachers' classrooms, most opportunities for students to talk happened during other activity structures, like the warm up or the lesson launch. Or, if the class ran out of time, and discussion of a problem was continued on the following day, that discussion was not reflected in the Accountable Talk scores for the given task. Given these constraints on assessing the Accountable Talk in the novice teachers' classes, for this analysis, I focused on the ratings of the three rubrics of Academic Rigor (AR): Potential of the Task (AR1), Implementation of the Task (AR2), and Student Discussion Following Task (AR3). The AR1 rubric assessed the potential of the mathematical task *as written* to engage students in cognitively demanding thinking. The AR2 and AR3 rubrics spoke to the implementation or enactment of the task, to whether the lesson and ensuing discussion provided opportunities for students to engage with the high-level demands of the task. For the analysis in the induction study, two coders (typically graduate students in education trained to use the IQA rubrics) scored each observation; the ratings for these six teachers may be found in Appendix B. In the following paragraphs I describe each of the Academic Rigor rubrics, present findings of patterns in the IQA ratings, and compare and contrast these findings with the findings from the analysis of normative practices. Table 17 presents the patterns or stories of change with respect to the IQA ratings for each of the AR rubrics.

Table 17.

Changes in ratings of Academic Rigor Rubrics using the IQA

Teacher	Academic Rigor 1 Task Potential	Academic Rigor 2 Task Implementation	Academic Rigor 3 Whole-Class Discussion
<i>Faith</i>	Most observed tasks had potential for connections (CMP2)	Cognitive demand of tasks either declined or maintained (to procedural without connections)	Brief, one-word answers or non-mathematical responses, or presentations of work only
<i>Marie</i>	Most tasks had potential for connections (CMP2)	When tasks had potential for connections, most of the time cognitive demand maintained	Presentations of work but limited discussion noted following task
<i>Becca</i>	Most tasks emphasized procedures without connections explicit	Cognitive demand of tasks maintained (procedural without connections)	Presentations of work typically limited to showing one strategy or representation
<i>Kevin</i>	Most tasks had potential for connections (CMP2)	When tasks had potential for connections, most of the time cognitive demand maintained	Evidence of explanations with presentations of work (but often incomplete or preliminary)
<i>Kyra</i>	Most tasks had potential for connections (CMP2)	Cognitive demand of tasks either declined or maintained (to procedural without connections)	Presentations of work typically limited to showing one strategy or representation
<i>Noah</i>	Most tasks had potential for connections (CMP2)	Sometimes cognitive demand maintained, sometimes declined	Presentations of work but limited discussion noted; for scaffolded tasks, brief answers accepted

Academic Rigor 1 (AR1): Potential of the Task

The first dimension of the Academic Rigor rubric rates the potential of the task itself to provide opportunities to engage students in complex thinking. It assesses the level of thinking required to produce a complete and thorough response to the task, to satisfy the stated demands of the task. For AR1 ratings coders evaluated, when available, the written instructions of the task as they appeared in curricular materials. As noted earlier, there was a critical distinction between ratings of 1 or 2 and ratings of 3 or 4. Tasks rated as a 1 or 2 were limited in their cognitive demand, in that the tasks focused student work and engagement on either memorizing or reproducing facts, rules, or formulae, or on using a procedure specifically called for, with little ambiguity about what needed to be done and how to do it. The tasks did not require students to make connections to concepts or underlying meanings of procedures being used. In contrast, tasks rated as a 3 or 4 had high levels of cognitive demand because they had the potential to engage students in complex thinking, in making connections between mathematical concepts, procedures, relationships, and/or representations, and, in the case of level 4 tasks, explicitly prompted for evidence of students' reasoning.

Examining the AR1 ratings provided an indicator of which teachers started off with rich, cognitively demanding tasks. Faith, Marie, Kevin, Kyra, and Noah regularly started with tasks scored at a level 3 or 4. Given the context in which these teachers worked – ongoing efforts at the district-level to support an inquiry-oriented curriculum, CMP2, with a specific launch-explore-summary sequence for most investigations – this finding was not surprising, especially in Year 1 of the study, when CMP2 was the primary mathematics curriculum in middle school. (For further reference, the topics or investigations from each observed lesson are recorded on a table in Appendix A.) In Year 2, with the introduction of the pre-Algebra CPM curriculum, the materials changed, but even these tasks had potential to engage students in making connections to concepts. The CMP2 curriculum did include ancillary “additional practice” material, which Kyra and Becca

assigned; these materials were more skill-based or practice-oriented. Becca also supplemented her lessons with materials from other textbooks, often following a presentation of a specific rule or formula (for example, a video demonstrating how to apply the Pythagorean Theorem to find the length of a missing side of a right triangle). In this way her tasks channeled students towards a specific strategy, and as such rated a Level 2 (low-cognitive demand) for task potential.

Certain tasks supported developing multiple approaches or establishing connections between representations better than others. For example, the first task observed in Kevin's classroom, to find the area of irregular polygons, offered more openings for students to try different strategies than a lesson on finding coordinates of a park; the higher cognitive demand of the latter was due to the explicit emphasis on explaining reasoning, including assessing the validity of proposed coordinate pairs. In these ways, the mathematical richness of the task, the potential to engage in conceptually challenging work, mattered to which normative practices *could be* cultivated. However, even with the curriculum and pacing calendar determined by the district, implementation varied. The CMP2 teacher guides may have suggested questions for teachers to pose, but what the teachers did varied. During Fall Year 1 observations, Marie and Faith were observed teaching the same investigation, "Planning Parks", in very different ways, further highlighting the need to attend to task enactment or implementation. Minimally, the AR1 rubric ratings shared a common feature with norms for guiding the development of mathematical ideas, in that they provided an indicator of the potential mathematical emphasis in the lesson for tasks with high (or low) levels of cognitive demand.

Academic Rigor 2 (AR2): Implementation of the Task

The second dimension of the Academic Rigor rubric addresses whether the opportunities for students to engage in complex thinking were maintained and achieved as the task was enacted or implemented during the lesson. It assesses the level at which the teacher guides the students to engage with the task within the lesson. For AR2 ratings coders evaluated the highest the level of

thinking demonstrated by the majority of the students both during work time and discussion. A task may have had high potential, however, students may not have had the opportunity to engage in the rigorous thinking afforded by the task; this would be noted as a decline in the demand of the task. As with AR1, there was a critical distinction between ratings of 1 or 2 and ratings of 3 or 4. Tasks rated as a 1 or 2 were limited in their cognitive demand, in that the implementation of the tasks did not facilitate or maintain any high-level demands of the task. For example, feedback or modeling or examples were so directive that students were not pressed for complex thinking, or students were not given enough time to deeply engage with the task. In contrast, tasks rated as a 3 or 4 were implemented in such a way that students had opportunities to engage with the high-level demands of the task; for example, teachers provided time for students to grapple with demanding aspects of the task, or students had opportunities to serve in a capacity as a mathematical authority, presenting, defending, or critiquing their own strategies and those of others.

Examining the AR2 ratings and changes in the ratings over time provided another view into the ways the novice teachers guided the mathematical work in the class. While all the teachers were observed implementing CMP2 investigations – with their specific launch, explore, and summarize sequence - at some point in the first two years, that investigative sequence was observed most clearly in Kevin and Marie’s instruction. Faith and Noah both alluded to that sequence but typically did not implement it, though Noah’s scaffolded approach approximated it. Based on the AR2 ratings for their lessons, Marie and Kevin tended to maintain the cognitive demand of the task at some high-level (a 3 or a 4). In contrast, even if the task had the potential to engage students in complex thinking, during implementation the task became more procedural in the other teachers’ classrooms, for a variety of reasons. In some cases, as in Becca and Kyra’s classes, the teachers emphasized a set procedure for solving the task, or students were not pressed or held accountable for explanations. The focus was on the correctness of the answer rather than

the meaning or process. In Faith's class, students were not always given enough time to complete the task to the extent expected. In Noah's classes, management problems interrupted the work. And, as explored later in the analysis of the teachers' perspectives, many of the teachers felt the students did not have the requisite knowledge to engage with the task at a high level.

The patterns in the AR2 ratings held across the three observed time points and cohered with the stories of change in practice. Marie, Kevin, and Noah were the three teachers who implemented tasks regularly in ways that maintained opportunities for students to engage in complex thinking, and as the normative practices analysis showed, these were the teachers whose practice displayed promising openings. The lens afforded by the analysis of normative practices highlighted the ways in which these teachers maintained the demands of the task, for example, in the ways they structured participation, used student responses to clarify concepts or articulate reasoning, and, especially in Kevin's class, solicited and acknowledged student contributions in a way that encouraged more participation. For Faith, Becca, and Kyra, the findings from the IQA ratings confirmed the stories of capped practices and learning opportunities in that they tended to teach with a procedural emphasis; the IQA ratings, however, did not capture the small, if capped, shifts in encouraging more participation, for example. The analysis of normative practices confirmed and informed analysis of the third AR rubric about student discussion.

Academic Rigor 3 (AR3): Student Discussion Following Task

The third dimension of the Academic Rigor rubric addresses the extent to which students show their work and explain their thinking about the important mathematical content. It focuses on students' contributions to the whole-group discussion following the task. Points of overlap between groupings of normative practices and the AR3 rubric are evident: social culture norms to acknowledge or encourage student contributions, for example, would support aspects of student talk differently than acknowledgements of effort alone. As with the AR1 and AR2 rubrics, there is a critical distinction between ratings of 1 or 2 and ratings of 3 or 4. A discussion would be

scored a 1 if students provide brief or one-word answers, while a 2 would indicate that there is some presentation of students' work but no discussion, or perhaps only one strategy or representation for solving the task is shown. Discussions would be rated a 3 or 4 when students discuss more than one strategy or representation for solving the task and provide explanations of how, and in the case of level 4, why that strategy is applicable.

Unsurprisingly, since this rubric focuses on discussion following work time, little new information was revealed in the ratings of lessons for those teachers who typically did not guide extended discussions after work time. Among the teachers who demonstrated promising practices – Kevin and Marie – there were two interesting findings. First, Kevin's lesson discussions were rated a 3 in both the Fall Year 1 and the Spring Year 2, but while the IQA scores suggested a consistency, the normative practices analysis provided insight into what remained consistent: the norms for developing social culture and for structuring participation. As discussed earlier, over time the norms for guiding the development of mathematical ideas in Kevin's classroom more often funneled students towards a particular strategy, especially once the curriculum changed. Second, the scores rating Marie's discussions were lower – levels 2 or 1 – than the implementation scores suggested, in contrast to the analysis or normative practices. This may be attributed to the emphasis of the AR3 rubric on discussion following the task; Marie elicited student talk throughout the warm up and launch, but these interactions (and the related norms they promoted) were not captured in the AR3 rating.

Looking Across the IQA Ratings and the Analysis of Normative Practices

The analysis of the IQA ratings of Academic Rigor for the observed lessons cohered for the most part with the findings from the analysis of normative practices. First, the findings from the analysis of the potential and implementation of the task confirmed findings from research on mathematical tasks: when the teachers started with cognitively demanding tasks, there was the possibility for engagement in a connected way. The tasks Becca taught typically involved

practicing a specific skill or procedure and as such, the AR scores for implementation and discussion were capped. Second, those teachers whose normative practices fostered “promising” learning opportunities – Marie and Kevin, and to some extent Noah – were more likely to have implementation scores of 3, suggesting that the cognitive demand of tasks were maintained and students had some opportunity to provide reasoning, to make connections, and even to engage in productive struggle to some extent.

The normative practice lens, however, did offer some important clarifications and exceptions to the narratives of change underscored in the IQA ratings. First, even when implementation scores were 3’s, what the teachers *did* to maintain cognitive demand was different in each classroom. In Kevin’s classroom, non-content specific normative practices, like his moves to acknowledge contributions and structure participation, supported discussion and presentations of work. In Marie’s classrooms, the normative practices to guide the development of mathematical ideas were evident more than specific social culture norms, supporting cognitive demand in a different way. In Noah’s classroom, social culture norms alternately supported and undermined mathematical discussions, depending on the day and the disruptions. And finally, as noted, the AR rubrics focused on discussion after student work time, but in doing so the ratings rendered most discussions at level 2, but, as evident from the normative practice analysis, the talk in Marie’s classroom both after the work time and during the launch was substantially different than the talk in Becca, Kyra, or Faith’s classrooms. Finally, while the IQA rubric ratings clearly aligned with representations of math as rule-bound or co-constructed, they did not speak to the different interactions that supported developing a robust sense of efficacy. Ratings of 3s or 4s would indicate more opportunities for students to develop a sense of efficacy because of the implied descriptions and explanations of work, but, as the normative practice analysis demonstrated, even in classrooms with ratings of 2, there were openings to support developing

dimensions of a productive disposition. To understand how teacher practice might support teacher learning, it is helpful to identify openings or preliminary practices upon which to build.

The findings from the IQA analysis also raised questions I explore in the next section about understanding the teachers' access to resources, contexts, and orienting views as explanatory. The AR1 ratings illustrated the ways in which curriculum mattered; the district-wide focus on implementing CMP2 meant that the teachers were at least starting with cognitively demanding tasks and lesson structures that, at least in the curriculum materials, supported opportunities for student talk. Beginning with cognitively demanding tasks did not always translate to implementation in which the complex thinking was maintained or supported. To understand why this happened – whether the teachers enacted normative practices to support participation, or engagement, or sense-making, or focus on a single strategy – it is helpful to consider individual factors, including teacher perceptions of whether their students had the requisite knowledge to work on the tasks as framed in the curriculum materials. In the next section I present access to resources, contexts, and orienting views as possible explanatory factors for both the observed normative practices and the narratives of change.

Understanding Change: Considering Teachers' Resources, Contexts, & Orienting Views

To understand the patterns of practice and the change trajectories over the novice teachers' first two years of teaching, I turn to the third research question framing this study: What are the personal and contextual factors that influence novice teachers' instruction and changes in instruction? Drawing on data from individual interviews with the teachers, their administrators, and their mentors and colleagues, I considered factors, both contextual and individual, that contributed to and may help explain the storylines of change in the teachers' developing practice over the course of their first two years of teaching. Contextual factors included both opportunities and access to structures and resources and also constraining or limiting structures or external realities (as perceived and presented by the teachers and/or their administrators and support

networks). Individual factors focused primarily on the teachers' orienting views or mindsets towards their work and contexts. In the analysis of individual factors I studied the ways the teachers' perceptions of what teaching math entailed and their own challenges and successes reinforced the story lines of practice shaping and signaling mathematics, as "narrated" through normative practices. I analyzed interviews with teachers at three time points: the fall of year 1 and in the spring of years 1 and 2. Interviews were not timed with observations and typically took place after the observations, at the end of the semester. The purpose of this interview analysis was to understand, from the teachers' perspectives, the orienting views that undergirded the normative practices cultivated in their classrooms and the story lines of change.

Opportunities for and Constraints on Teacher Learning of Practice

As described in Chapter 3 (Research Design and Methodology), in the district in which these teachers taught, there were multiple initiatives in place both to support mathematics instruction, with attention to the role of task and task in students' mathematical learning, and new teachers' induction. Table 18 provides an overview of the resources and supports to which the novice teachers had access during their first two years of teaching. Curriculum and content support included participation in monthly CMP2 curriculum cohort meetings, support from a district resource teacher, and in-school math support through the math lead teacher or department chair. Induction, or *learning teaching*, support included participation in the state internship program with an assigned mentor and mentoring committee, the assignment of a mentor outside or in lieu of the internship, and support by colleagues or teams (in the middle school structure the teams often consisted of 4 teachers, each assigned a different subject matter, working with the same group of students).

Table 18.

Resources and Supports to Which Novice Teachers Had Access over the First Two Years

(x indicates teacher described as a resource or supporting experience; + indicates when support described as positive; - indicates when intended support posed challenges)

Teacher	During Year 1						Summer Preceding and During Year 2					
	Curriculum and Content Support			Learning Teaching Support			Curriculum and Content Support			Learning Teaching Support		
	CMP2 8 th Gr. Cohort	District Resource Teacher	In-School Math Support	Team Support (+/-)	State Internship (mentor)	Other Assigned Mentor	CMP2 Weeklong Summer PD	District Resource Teacher	In-School Math Support	Team Support (+/-)	State Internship (mentor)	Other Assigned Mentor
<i>Faith</i>	x	x	x	-	x		x	x		-		
<i>Marie</i>	x	x	x	+	x		x	x	x	+		
<i>Becca</i>			x	+		x			x	+	x	x
<i>Kevin</i>		x	x	+		x		x	x	+	x	x
<i>Kyra</i>		x		+		x	x		x	+	x	x
<i>Noah</i>			x	-		x	x		x	+	x	x

Supporting Mathematics Instruction. Within the district, teachers, including all the teachers featured in this study, had access to the many professional development resources related to implementing the CMP2 curriculum. A major corporate foundation grant provided support for district resource teachers to lead monthly curriculum cohort sessions in which teachers at the

same grade level met to examine the upcoming CMP2 book and unit. Marie described the cohort sessions:

And those have really helped me be successful in, in teaching CMP because I'm able to actually, cause it's, they say work out all the problems and yeah, you can work them out but what the cohorts do is help us think like kids. And so you're not just working out the problem, you're anticipating student problems. You're figuring out what homework would be good, working out the homework problems, you're getting really in depth with it, stuff that if you just sat down and did it on your own, you wouldn't do. (Spring Year 1)

District resource teachers also were assigned to several schools to work with administrators and math lead teachers to support teachers implementing the CMP2 curriculum through observations, model lessons and co-teaching. Finally, funding was available to send at least two teachers from each school to the CMP2 summer training institute at Michigan State University. Table 18 shows the curriculum and content support teachers described accessing. The professional development support around implementing the CMP2 curriculum checked many of the criteria identified for professional development more likely to be effective in changing teachers' classroom practices (Desimone et al., 2002): teachers participated collectively, with school and/or grade level colleagues, and engaged in active learning opportunities (for example, through the co-teaching models with the resource teachers), and through its specific focus on the curriculum the learning opportunities were coherent and relevant to the teachers' work.

In addition to the curriculum resources for teachers, at the district-level there was a sustained (over a year) math professional development effort to support principals' (instructional leaders') understanding of the role of rigorous tasks and discourse in mathematics classroom. These principal training sessions marked an effort to move school leaders' understanding beyond naming a "form" of an instructional practice (Spillane, 2000) to addressing the underlying function of practices like encouraging student talk, to support mathematics learning goals. In interviews, the principals explicated talk and struggle as important in mathematics classrooms.

For example, Noah's principal articulated themes of questioning and even productive struggle when asked what he looked for as evidence of high-quality mathematics instruction:

I would look at the level of questioning, you know, to see if high-level questions were being asked to students. I would look at the expectation level of the teachers, what type of, you know, did they have high expectations for their students and what they could be able to do, and what they could perform. I would look and see if they were allowing students to, to struggle or grapple with concepts, see if they could figure out things or whether or not teachers were just giving them answers. You know I would hope that they were letting students work through to find and see the bigger picture and try to understand the math concepts. (Noah's Principal Interview, Fall Year 1)

Similarly, Kyra's principal described a shift in focus to the role of questions and talk to support alternate strategies rather than funneling students towards a specific algorithm:

We've been listening to questioning also. So what is the questioning like? Are the teachers just asking questions that could be answered with a yes or no? Are the teachers asking questions in the form where they just get an answer? We're trying to steer our teachers away, and we're looking to see if the teachers are asking questions that make the kids think and not just give an answer but, are they open-ended questions? Are we steering kids towards one solution, or is it open enough where the kids, is your question open enough where the kids can have multiple solutions? (Kyra's Principal, Fall Year 1)

This awareness of the role of student talk in mathematics framed the shifts in thinking about mathematical practices at the class, school, and district levels to improve mathematics instruction. As the findings of the analysis of instruction demonstrated, however, access to these instructional learning opportunities and awareness of the role of task and talk in shaping and signaling math did not guarantee *how* these dimensions of instruction emerged in the teachers' practice. Both principals and teachers alluded to challenges implementing the CMP2 curriculum, from a perceived lack of alignment between the curriculum and the state standards and assessments at the time to issues of student readiness and management of classroom talk. Even when instructional leaders talked about promoting discourse, and teachers attended professional development to learn more about implementing a curriculum that emphasized student talk as a means to learning, actually enacting these kinds of practices in local contexts was not straightforward (Herbel-Eisenmann et al., 2006), especially when the teaching and learning

practices seemed to conflict with perceptions and prior experiences with math. Becca's principal explained this dilemma she observed new teachers like Becca encountering:

I've actually had situations where the teacher knew math, the teacher knew high-level math, but there was no connection between what the teacher was doing and what the students were receiving. So there's got to be a connectedness. It's student engaged. It's student, it's teacher with-it-ness and awareness and making that connection so that it is real for the students. So using real examples, doing some actual application of the math, not just drill and kill. If they just did Connected Math, I'd be on the right path, because it, the advantage with the Connected Math, if it's done correctly, is that it provides them the information to make the application of the skills. ... The problem is for teachers that have not gone through that type of learning themselves; they keep looking for the math. And I had a teacher this year that's no longer working, and he was an unusual case, because as a student himself, if you can imagine a twenty-three year old that has three degrees and math is one of them, and he is very intelligent, very bright, and his question to me was when was he gonna get to the math. And I said, "You couldn't have started without being in the math." And he said, "This math book doesn't look like my math book." He could not, he could not wrap his head around that at all. ... And for our teachers, their thing is it's, they have in their head, many of them, this notion that there's a ready, set, go button, and when the kids are ready, then we can do this other. But if they haven't got to the ready yet, we shouldn't be doing this other. And they don't understand that's a tool to get to where they're trying to go. (Becca's Principal, Fall Year 1)

The principal continued to explain how a perception of student readiness for a curriculum like CMP2 affected instructional decisions: "For Becca, she's really gonna have to be one that we push to get to understand that all students are capable of doing the math." The following year, when the pre-algebra text (CPM) was added to the 8th grade curriculum, Becca's principal identified its appeal, based on her observations of the teachers in her school:

I think the teachers understood the teacher's role, and they could pick up. They do some exploratory type things, but it is not as "turn loose from the teacher" as it is with C, CMP. And I think the teachers just felt a little more in control. And for new teachers to give up control when they've never had control is a lot more difficult. They got into the lessons a lot more, and it, you could see in them that they were excited about what they were doing. (Becca's Principal, Spring Year 2)

Supporting New Teachers in Specific School Contexts. Table 18 also shows the different learning supports to which the new teachers had access. During their first two years of teaching each teacher participated in the state internship program; for Faith and Marie this took place during their first year, while the other teachers, all alternatively certified, were interns in

their second year (once they had completed a teacher education program). When they were not enrolled in the internship program, the teachers were assigned mentors through other avenues. The mentors' role was not always content-specific. In Kevin's case, however, his principal arranged his schedule so that he could observe a master teacher teaching math every day: "He has the benefit of sitting back and watching her daily on the same team, every day. And that would just be an ideal situation for any new teacher" (Kevin's Principal, Fall Year 1). Marie, Kyra, and Becca also spoke of seeking out colleagues on their grade-level teams for support in both years.

Faith and Noah encountered challenges within their specific school contexts. Both the teachers and their principals identified challenging situations related to their "team" assignments. In Noah's case, he began teaching after the school year had started; in fact, he was the 5th teacher assigned to his group of students (the third teacher of record, but the 5th when taking long-term substitutes into account). He described the challenges this posed from a management perspective alone:

And I think for me, like the biggest mistake I made was coming in as the third math teacher, being told you're a month behind, and on the first day feeling pressure to play catch-up. I never felt comfortable taking the time to establish the management, so it became this huge snowball where I'm trying to rush them to get caught up, and there's no, I'm told at first not to worry about the management pieces, and then by November and December, you know, everything's falling apart. (Spring Year 1)

Faith's team challenge was different but no less extreme. Her principal outlined the challenges – namely, that on her team the other teachers had opted for a two-hour block schedule, which for Faith meant she would see classes for two-hour blocks every other day. Her principal was candid in her description of the struggles Faith encountered and the attempts to support her:

She's struggling with management. Her team; they are in two hour blocks which is tough I think for seasoned teachers, but for a newbie, you know... She's got it in her mind I think that she's supposed to only teach one lesson inside of that two hour block; therefore her pacing is very slow. The kids are not engaged, probably bored and so we're working-right now what we're doing is we're meeting with her team and her mentor went in there and spent the whole day with her. I sent the math resource teacher in there one day, not this week but the past week, and she was in there and they're working on some of that, the management pieces. ... We're working with her team to change the schedule, but

three of them on that team are okay with two hour blocks and she is not. (Faith's Principal, Fall Year 1)

Both Faith and Noah's situations served as examples of the non-content issues within the school contexts that impacted the classroom learning environment.

In another point of commonality between Faith and Noah, both of their principals attempted to mitigate the challenges in Year 1 by assigning them mentors, both school-based and external, and in Year 2 by reassigning them to different grades. The principals suggested that both teachers would be able to develop their content teaching prowess once management issues were tended, and both principals speculated that the teachers would find openings to form better relationships with younger students. Noah's principal described,

We [the administrative team] just kind of felt with, he gets kind of intimidated with the bigger guys a little bit. So I think for him he kind of wanted to try sixth grade, and we thought that maybe his personality cause he, you know, sixth graders need much more nurturing than eighth graders. I think Noah has a nice, nurturing side to him. We felt like that might play well with sixth grade kids. So we wanted to give him an opportunity with that" (Noah's Principal, Spring Year 1)

Similarly, Faith's principal explained,

Faith needs to go to sixth grade. Because then the discipline stuff is not so much in your face on the first day of school. You've got some, you know, those kids are still new, they're sort of scared, so you've got some time to kind of build some confidence in yourself and build your classroom they want. They have no preconceived notions about what middle school is like. (Spring Year 1)

From the analysis of normative practices, this change in grade assignment seemed to change some normative practices – specifically, for both teachers the tone of the classroom shifted from negative and corrective to more redirection of behavior, and then more opportunities for teachers to encourage and acknowledge effort. However, these changes did not affect or mitigate guiding practices in a way that changed math learning opportunities substantially. This lack of change coheres with findings from recent research on whether grade-assignments are an effective lever of change to improve student achievement (Blazar, 2015).

Individual Factors: Teacher Perceptions of Challenges

In the previous section I outlined several contextual factors that would appear to support teaching math in a way that featured developing student capacity with mathematical practices. There were sustained efforts to support curriculum implementation, addressing both teachers and their instructional leaders, both in and out of the classroom. Furthermore, there were structural factors in the schools that could support or impeded instruction; and, over time, there were attempts by instructional leaders to mitigate or shift these impediments (for example, grade reassignment in year 2). Despite all these contextual factors, the overarching story of change was not one of improved opportunities to learn, but rather maintained or capped practices and opportunities. In this section I present findings from the analysis of individual factors. The teachers came to their contexts with beliefs that informed and were informed by how they perceived their circumstances – the expectations in their schools, the capabilities of their students, and the fit of their curricula with their students’ needs and interests, for example. Thompson (1992) described these conceptions or orienting views as “what a teacher considers to be desirable goals of the mathematics program, his or her own role in teaching, the students’ role, appropriate classroom activities, desirable instructional approaches and emphases, legitimate mathematical procedures, and acceptable outcomes of instruction.” Munter (2014) reported how beliefs and perceptions, or visions, related to teaching practices as described by the teachers: teachers who described traditional visions valued and emphasized clear, explicit instruction (providing and modeling steps to carry out a procedure, for example). However, Wilhelm (2014) reported nuances to these findings about the relationship between teacher beliefs or visions and practice, noting that teachers who espoused productive views of how to support struggling students were more likely to maintain tasks with high cognitive demand. In this section I draw on the interview data to understand to what extent or in what ways teacher perspectives reinforced narratives of the features of normative practices and change.

For this analysis, I focused specifically on what teachers identified as challenges to teaching, both math-related and non-math. I framed these as proxies or approximations for what the teachers' prioritized addressing through their instructional decisions. While some of the challenges identified were not surprising – concern about the curriculum, for example, and how to address learners' different needs given the perception that in math, knowledge builds over time, so students who had fallen behind suffered from an accumulating deficit of knowledge and skills – there were interesting commonalities among the teachers who had established promising practices (Marie, Kevin, and Noah to some extent). Namely, the teachers connected effort or struggle and talk to approaches to address challenges – and the ways they attended to these themes had implications for both the math learning opportunities and the messages signaled about developing capacity or efficacy to do math.

Perceived Challenges Specific to Teaching CMP2. All the teachers identified teaching the CMP2 curriculum as a challenge, though they differed in the degrees to which they perceived it as a possible impediment to teaching and student learning. The teachers described student readiness (or lack of) as a major concern; they worried that their students lacked the requisite knowledge for investigating math tasks and learning from a curriculum like CMP2, and they worried that CMP2 was too hard to implement in a differentiated way to meet the needs of all learners – that it did not provide students with opportunities to practice or master concepts after investigating topics. Kevin described, “It’s all inquiry-based, it’s all real life, and that, that is great. But I would say that it’s so much that way, that students don’t get repetition. They don’t get the practice they need on doing the same thing over and over and over again” (Spring Year 1). Kyra questioned whether the instructional approach for CMP2 fit with the specific needs of her students:

The problem with the Connected Math, it's anything that is learned comes from my instruction because the books do not really offer any examples or anything like that. So, absenteeism, we have high absenteeism. If you're absent, yes, you can get notes from a

friend, but that really isn't the same as being in house to hear and really go through the problems. (Spring Year 2)

Noah connected this challenge to the pressures beginning teachers encountered:

What I found, what I finally realized was that I'm trying to teach something inquiry-based to students who really don't have the fundamentals of math. So I don't think the, I don't think the curriculum is a match for the students I had. And so I think that was the biggest challenge because I, for whatever reason, I felt like I had to rely on the book and this comes from kind of hearsay where I heard that one of the people before me, one of the reasons they got rid of the teacher was because he wasn't following the Connected Math curriculum. So I felt obligated to follow it, like exactly as it states, but I mean, I think it was challenging, I think it's just challenging to try and use that. And especially for a first year teacher when, let's face it, you're really kind of living by day-to-day, and Connected Math is something that expects you to plan your next six weeks. (Noah, Spring Year 1)

The teachers all described ways they changed their enactment of the CMP2 curriculum to address the needs they perceived from their students. For example, Kyra identified scaffolding as her approach to differentiating instruction:

[In our school] You have anywhere from lower level learners to advanced learners all in the same class, and how do you make it interesting for all parties? And so, some difficulty with the Connected Math is that it might not, it breaks down the questioning, but it doesn't really break down how it's taught. So we might have to use more scaffolding techniques that are kind of created, created by myself to kind of give those lower level learning, learners up to speed but still, while maintaining higher level thinking, order thinking for those that are advanced learners so that they don't become bored with the material. Because one thing with CMP 2 is that it's very repetitive and they'll use the same themes throughout the book, and for some students that really get math, that becomes boring very quickly. (Spring Year 1)

As he grew his pedagogical repertoire over years 1 and 2, Kevin looked ahead to what his students would do in high school to inform his decisions about what and how to teach:

If I know as a student that they're not really going to use that anymore, then I would just cover it a little bit, make sure they know enough for the test but, you're not going to use this later, guys. So I truly think about when are they going to use this, not only in real life, but I also think about in high school, are they going to use this? And I actually have some teachers, just friends of mine, who teach high school. Sometimes I'll call 'em up, and I'll say, "Hey, I'm supposed to be teaching this. Do they need this in the next couple of years?" And so like, "No, we don't teach that at all," then I'll still teach it, but I'll reallocate my time more toward other things that I know they will need. (Spring Year 1)

Only Marie offered a more positive view of CMP2, noting, “I learned it different than the book is teaching it so if they don’t get how the book is teaching it, I can talk about how I learned it. Then they’ll pick up those other ones” (Spring Year 1).

While the principals and mentored spoke in aligned ways about teaching with a curriculum like CMP2, emphasizing at least in form student work with ambitious tasks, productive struggle and the role of talk, they acknowledged the challenges. Kyra’s principal expanded on this idea of student readiness and connected it to decisions teachers would make on a day-by-day basis:

Sometimes you don’t always see the disc-, the student discourse [in Kyra’s class]. Sometimes it’s still too teacher-directed, but Kyra’s also good at gauging the behavior of the kids and what’s gone on in the neighborhood and what type of day it is. So I do know on some of those days that that rigor might not be as consistent as it was, but it’s not because of her planning or her teaching. It might be today is just a better day that we come in the room and we don’t have any discourse at all because any discourse is going to lead to some, some issues. And she does an outstanding job of being able to gauge students. ... I think sometimes it [the level of rigor] becomes inconsistent but it’s not because of her skills. It’s just the situation of the day. (Kyra’s Principal, Spring Year 1)

There was a common thread in the interviews that teachers had to make decisions based on their students’ long-term learning needs and daily needs, and that the way to do this was to adjust the curriculum enactment, but this was not perceived as *changing* the curriculum.

Underlying Promising Practices: Themes of Effort, Opportunities to Struggle, and an Emphasis on Talk. As described in the previous section, student readiness for an investigation-themed curriculum like CMP2 was described as a challenge (or even an impediment) by all the teachers; only Marie stood out as a teacher not too troubled by this challenge. In a sense, this preoccupation with student readiness evoked a deficit view in that the teachers ascribed the challenge to student characteristics: they did not hold the requisite knowledge to learn with a curriculum like this and therefore the teachers needed to modify how they enacted the curriculum. The typical modification was to default to teaching with a conception of math as a rule-bound discipline. The implication was that if the students could learn all the rules, then maybe they

could work on CMP2-like tasks. Through the analysis of what the teachers described as challenges, however, two compelling threads emerged from the perspectives of those teachers who cultivated normative practices that offered some promising openings for students to co-construct meaning in math: encouraging effort and even productive struggle in math, and managing talk in productive ways to support math learning.

Kevin, Marie, and Noah each spoke of effort and struggle as both challenges to teaching and learning and also priorities they held for developing their students' capacity to do math. Kevin's explained his "Grading 1 and 1" strategy, which he used to address the dilemma of wanting to recognize student effort, balanced with the disciplinary need to emphasize precision and accuracy:

Math is one of those subjects that's totally different than any of the other ones because there's definitely right and definitely wrong answers. Whereas other ones like social studies, it's more subjective. And math is very, you know, two plus two equals four, not five, because oh, well, you know, something like that. ... But with certain students, that's not their strong suit and they feel like they can justify a wrong answer with their words or something like that. That's difficult for me to be able to deal with the students. Like sometimes they'll write a totally wrong answer but then they'll write this great explanation for it, and say, well, doesn't that count for anything? And I'm like yeah I, I guess but the answer is wrong. I mean so how can I give you credit for that? So that's, that's me dealing with that. I need to figure out some system of yes, they put forth some effort. ... I call it grading one and one. Grading one and one and that means that you get one point for attempting it and giving it a shot, and you get one point for getting it right. So if you miss everything but you give it a shot, you'll at least get a fifty percent. So I do that a lot so my kids' grades don't get totally slammed if they don't understand something. (Fall Year 1)

Kevin's preoccupation with acknowledging his students effort was consistent with his norms of acknowledging effort and contribution regularly in class. And, his focus on grades and what he could do to ensure both his students' learning and their success, defined as progression through eighth grade on to high school, was consistent with the ways his normative practices fostered a sense of learning math to do future math, as part of a school progression.

In contrast, Marie and Noah both described perspectives more open to the idea of letting students struggle, and even that this struggle was part of learning. Marie described as a challenge her students' "learned helplessness":

Personally I'm struggling with getting the students to work hands-on because they have the worst case of learned helplessness ever! Even when I give them their work they're like, "Oh man, I don't know how to do this. I don't know what the answer is. Will you do it for me? Can we go over it? Are we gonna go over this? They're doing what they can *not* to work. I'll deduct participation points. I'll do whatever I can to get them involved, but if they don't want to work they don't want to work. At the end we'll go over the answers to make sure that everybody's right on track. Then they'll be like, "Oh, answers! Got it! Good to go!" That's what I'm struggling with right now. I'm working through it, like using their classmates as resources and working in pairs. (Fall Year 1)

Noah similarly described wanting to give students space and time to work on their own, even with others, so that they could develop a sense of their own efficacy when doing math:

I want to see more evidence of students just being able to do it on their own, being able to think. I mean a good example is the other day we had a graph and they had to make a table out of the graph and I had kids, like the graph had scale on it, they basically said the scale was a table, and I pretty much looked at them and I said, "When I give you fifteen minutes to write down the numbers that were in front of you" and some of them thought – well, "No, you're going to have to think a little bit" and they wanted me, they didn't know how to do it, they wanted me to basically do it for them and they would get frustrated. Some of them would – because I'm like, "I'm not going to show you how to do any of that for the first eighty minutes or so" – and they don't, they get very discouraged and kind of shut down and don't like those kinds of challenges and just kind of give up and so working on teaching them how to go through that struggle I guess. (Spring Year 2)

In her second year, Marie suggested that if struggling with ideas was part of learning, then addressing that challenge meant she as the teacher needed to keep things interesting and introduce novel ideas:

I always try to have something new and a little extension. Like if we're, I'm very specific with it, like it's not like work on reflection symmetry. It'll be like learn how to calculate the angles of reflection or the lines of reflection symmetry. Or just stuff like that, so that they always feel like they're learning something new. (Spring Year 2)

For these teachers, an emphasis on effort or time to struggle made addressing and managing talk all the more important. Interestingly, both Marie and Kevin ascribed a management purpose to talk, though they approached talk differently. Both suggested that

students could easily be distracted, especially if they did not understand the lesson. For Marie, a strategy to counter this inclination to talk, especially about non-mathematics topics, was to emphasize connections to students during the launch, from the beginning of the lesson, so she did not lose students' interest at the outset:

If they don't understand the introduction, they're just gonna tune you out and expect that they're never gonna get it. They give up way too easily and then that's when they get frustrated and say, "Well I'm not gonna get it anyway, hey my neighbor's wearing a cool shirt" so they start talking about their neighbor's shirt. (Fall Year 1)

She emphasized "students engaging in math and relating it to their own lives" as a priority in instruction. Meanwhile, Kevin was more blunt in his rationale for encouraging talk – and while he did relate student talk to learning math, he was also pragmatic. He presumed middle school students would talk about something, and he figured his role as teacher was to structure outlets for that talk:

I think they need to be talking to one another because I mean, I think they need, their task needs to be not only between me and them and themselves, but they gotta be talking to each other about it. And if you don't have 'em talking, on another note, if they're not talking about that, they're gonna talk about what they want to talk about. So might as well channel that talking somewhere. So I do a lot of Think-Pair-Share's where, you know, their task would be, okay, think about it, write it down, you've got thirty seconds to share with the people around you, asking questions, bring it back. (Spring Year 1)

Both Marie and Kevin's stances towards the role of talk cohered with the normative practices they cultivated, even if the resultant learning opportunities were somewhat different. In Marie's case she saw talk as a means for students "to think for themselves and not rely on the teacher" (Spring Year 1). Kevin connected talk to a kind of formative assessment for both him and the students, so that both teacher and students articulated ideas, questions, and confusions.

Making Room for Listening. Finally, given all this attention to talk, it seems appropriate to mark when the teachers, if at all, mentioned listening or noticing. This was evident most clearly throughout Marie's interviews, raising the question of whether this stance towards listening was also an important stance underlying her promising practices. For Marie, talk was

closely related to listening to students, both to make connections and to understand their feelings and dispositions towards math. In the fall of her first year, Marie attributed her establishing rapport with her students to the ways she could listen and relate to their interests outside of math class:

I've found that it's relatively easy to connect to the students because I still listen to the same music as they do. I still watch the same shows. I'm pretty much almost in their generation because I'm only ten years apart from them. (Fall Year 1)

She reprised this theme in the spring of her second year, but was more specific about her goals of listening to students to connect their interests with the content on which they were working.

Furthermore, she actively sought out student feedback:

I also gave my students a survey – what they liked about my room. I called it my report card. I said, “I've been giving you guys report cards all year long, now you guys get to give me one.” And what this is, is it's basically a, just a recap of how they think the year went, and what they think I could have done better, what they thought, all that kind of stuff. And, I'm pulling it up now, but a lot of people showed a huge improvement whether, from when they liked math before to how they like math now. Only one person out of a hundred and twenty-five said that I did not make math interesting. Let's see, the question, how much did you like math before you had me as a teacher, three people had a 7, it was a scale of 1 to 7, 7 being the highest. Three people had a 7, twenty-nine had a 1. And after, there was one person that had a 1 and fifty-two had a 7. So there's a remarkable shift, you can kind of see this here. ... Here, this is if I made math interesting, everybody said that I care about them. They feel like I care about them as a person, that I treat all the students with respect and that I enjoy teaching, a hundred percent. So it's kind of cool, because it's, I like the one that, do I care about them because it's, a lot of people care about them for their test score, and they don't usually make personal connections with teachers, and so it shows that they have. (Spring Year 2)

What stood out in Marie's description of her interactions with students was the way she emphasized connections between students' interests and content, noting both as important and related to each other – distinct, but related. Through Marie's teaching and reflection on her teaching, the ways in which normative practices both to support content and to develop social culture are interwoven become apparent.

Summary

In Chapter 4, I presented findings from a close scrutiny of instructional episodes, using normative practices to look at math instruction, and specifically the math learning opportunities supported or constrained by particular combinations of normative practices. In this chapter, I took a longer view, presenting findings from an analysis of what changed in novice teachers' instruction over time and identifying implications for learning opportunities and messages signaled about mathematics, especially those relevant to developing (or capping) a productive disposition.

In the next chapter, I discuss these findings and consider the following questions:

- What ideas and questions about instruction did a close examination of normative practices raise? When did the normative practices the teachers established provide openings to expand or extend to richer mathematical tasks and discussion?
 - Are there ways that seemingly non-mathematical interactions (the social norms) shaped the mathematical learning experiences, and how?
 - Are there ways that the mathematical teaching actions supported teachers' social (or nonacademic) priorities such as motivating students' interest or cultivating study skills to support the high school transition?
- How might an analysis of normative practices inform other analyses of instruction?
- How might this analysis of normative practices help further conceptualize the relationship between teaching practices and the cultivation of students' productive dispositions? For example, would they have the opportunity to see math as relevant to their own lives, and for what purpose? To make sense of the world? Or to support capacity to do future math in school?

- How might promising openings in novice teachers' practice be used to inform and improve future practice?

CHAPTER 6: DISCUSSION

In this dissertation I have presented findings from a close analysis of novice middle school mathematics teachers' instruction over time to illuminate the normative practices that shaped mathematical learning opportunities and signaled messages about what it meant to know and do mathematics, and to frame those practices within the beginning teachers' learning and development of their pedagogical repertoire. In this chapter I reflect on how the findings contribute to ongoing conversations in mathematics education and teacher education. While I pull together threads that surfaced through the extensive analyses of the teacher cases, to understand *these* teachers and their practices, I also adjust the analytical lens and zoom out to consider how these findings inform understandings about mathematics teaching and learning to teach mathematics. I consider the ways in which this work may suggest further directions for research in order a) to understand how normative practices matter to mathematical learning opportunities, including those which relate to the development of students' productive dispositions towards doing mathematics, and b) to identify promising practices and openings that may lead to generative learning opportunities for novice teachers. Throughout the discussion, key themes of the role of talk and the interplay of effort, (productive) struggle, and supporting a sense of efficacy surface. In addition, I consider the extent to which non-mathematical moves and interactions can shape the mathematical learning opportunities accessible to students, and conversely whether math moves and interactions might serve and support teachers' priorities that are social or management-related.

I organize this discussion in the following sections. First I address contributions of this study's findings to existing mathematics education research on the role of normative practices in mathematics instruction. The attention to normative practices suggests a way to operationalize dimensions of developing productive dispositions as another goal of mathematics instruction. I then consider how the norm of making students' mathematical thinking public – as represented

specifically in the cases of Marie and Kevin – suggest a kind of high-leverage practice. I also consider how normative practices may align with and support novice teachers’ priorities, which are set as much by the teachers’ contexts, including mandates of the curriculum and the pacing calendar, as they are by the teachers’ perceptions and orienting views of the needs of their students and the class as a whole. Next, I revisit the conceptual framework introduced in Chapter 2 and suggest ways in which norms may be understood as levers to improve teaching practice in small and discrete but also concrete ways, given that each strand of normative practices, while distinct, is intricately interwoven with other strands to make up instruction. Finally, I consider implications, questions raised, and directions for further research, and acknowledge limitations of this study.

Exploring Ways Normative Practices Shape & Signal Mathematics

Teaching involves orchestrating the content, the representation, and the people in relation to one another. ... Supporting learning comes from knowing the students, the situation, and the content and then making decisions that support interaction that productively engages students in moving their ideas forward. (Franke et al., 2007, p. 228)

Analyzing normative practices provides a window to understanding how enacted instruction unfolds. Normative practices are interwoven with other dimensions of instruction, such as task and discourse. Identifying and studying norms informs our understanding of what may need to happen at the level of classroom interactions – interactions between teacher and content, teacher and students, and teacher and students around content – to maintain the rigor or cognitive demand of the task and to create a space in which students have opportunities to engage in mathematical practices like reasoning and to practice argumentation by sharing and critiquing solutions. Furthermore, normative practices include math moves as well as those that are relational (Battey, 2013) but nevertheless important to students’ learning opportunities because of the implicit (or explicit) messages about the purpose for doing math and about who is capable of doing math.

In the cases of these novice teachers, attending to the normative practices undergirding the classroom learning environment shed light on promising instructional practices. Consistently Marie demonstrated how a teacher could support student engagement in math tasks by making connections between curricular content and students' lives and interests. Furthermore, by frequently making student math work public, Marie wove that work into the progression of the lesson, so that as a class students co-constructed meaning as she provided relevant information, terms, or context. This instruction provided a contrast to the more traditional model of a guided introduction lesson followed by time to practice. The implicit assumption about learners embedded in of the traditional model was not that students could not develop a sense of efficacy, but that such a sense of competence or capability was contingent on teachers guiding the students through the math activity. Whereas through her actions Marie attempted to harness students' thinking and prior knowledge, even if emergent or incomplete, to construct new meanings – with the implication that they were capable of participating in this work *with* her. With the practices in place that allowed student thinking to be public, even if incorrect or incomplete, Marie both attuned her instruction based on her students' thinking and released some pedagogical control. These practices stood out in contrast to the instruction of Faith, Becca, and Kyra, for example, whose more restrictive practices kept student thinking invisible and rendered the students as recipients for knowledge transmission rather than co-constructing participants.

While structuring opportunities for participation was an important actions the teachers could undertake to increase students' engagement, analysis of the different cases indicated that opportunities to participate alone were not sufficient to change math learning opportunities from rule-bound conception to one more practice-oriented and co-constructed. For example, as they grew their teaching expertise, Becca, Kyra, and Faith all took steps to have students demonstrate solutions and strategies. However, when these demonstrations were limited to presentations without explanations, or with the teachers narrating the reasoning, the opportunity for students to

practice communication or justification, or to develop argumentation skills, was capped. In contrast, Kevin's instruction illustrated the interplay between norms to develop the social culture and math learning opportunities. Through his consistent attempts to establish rapport, to acknowledge students' ideas and contributions with the same positive reinforcement he used to promote behavior and work expectations, Kevin created multiple entry points for students to participate in and shape the class discussion and progression of ideas.

Kevin's case, however, also highlighted the important role of curricular materials to support teachers' practice and the limiting or constraining effect of teacher beliefs. As a first-year teacher Kevin adhered to the curriculum, which provided extended guides to teachers for questioning to support student investigations and articulation of ideas. In his second year, teaching a different curriculum more aligned with his expectations for math work, Kevin was more inclined to teach in a manner consistent with his orienting views towards what mattered to math instruction (more opportunities for practice after teacher-guided demonstrations). Along with other factors, the change in materials played a mediating role in the norms Kevin cultivated. However, even though his guiding math norms became more rule-oriented, the norms he cultivated to develop the social culture and respond to students somewhat attenuated the math shift so that there were still opportunities for students to talk; and that was consistent with Kevin's view of how to manage a classroom of middle school students. Kevin's routines for encouraging participation served as a foundation for different mathematical learning opportunities through student talk, but they were not generative in the same way as Marie's were. He did not seem convinced that students could do the hard work in math; while he cheered them on with positive affirmations, like a coach, he did not set a routine expectation for explanation or justification. His view of how students developed a sense of efficacy aligned more with that of Becca and Kyra.

The teacher cases pointed to the dynamic nature and relationship between the different categories of normative practices and their potential effects on math learning opportunities and messages. From a methodological perspective, through this analysis I suggest a way to conceptualize groupings of normative practices that make up the regularity of classroom life and a way to operationalize those constructs for study. By using the two key roles of the teacher to organize the categories of normative practices – to guide the development of mathematical ideas and to develop the social culture – this analysis offered a way to answer questions about the extent to which teachers’ non-math moves or practices shaped math, and the way math teaching moves or practices could affect teachers’ non-math priorities. For example, while several of the teachers identified managing student talk as a behavior challenge, in Marie and Kevin’s classrooms the practices to encourage, elicit, and invite student talk quite possibly diminished talk as a distraction. As Kevin noted, he assumed middle school students would be chatty and so he tried to provide opportunities to direct that. This disposition towards talk was different from Faith’s; she assumed she had to manage off-topic talk before she could allow students to talk about content. Challenging that perspective could be useful, especially for new teachers who encounter multiple dilemmas and novel situations, especially in the first year: shifts in how students engage with content could result in promising shifts for non-math-specific priorities, including managing student distractions and motivating student interests.

Studying and Assessing Mathematical Learning Opportunities

To understand the nature of the mathematical learning opportunities and the messages about knowing and doing math embodied in the novice teachers’ instruction, I analyzed normative practices, those “regularities in classroom social interactions that constitute the grammar of classroom life” (Franke, Kazemi & Battey, 2007, p.238). In both the conceptual framework and throughout the analysis and presentation of findings, I framed normative practices as a thread interwoven with other dimensions of instruction, the task and the discourse. Of course,

there are many ways to study practice, including the Instructional Quality Assessment (IQA; Boston & Wolf, 2006), which also was structured to provide insight on the mathematical learning experiences cultivated through instructional moves. Here I consider how this study of normative practices complements the IQA and adds to that portrait of instruction.

The IQA focuses on both the academic rigor of the mathematical activity and the manner in which the students are held accountable through their discourse to both the discipline and the learning community. In many ways the IQA ratings for the Academic Rigor rubrics matched the portraits of instruction conveyed through the analysis of normative practices. The lessons scored as implementing tasks with low cognitive demand matched those lessons I assessed as “capped” in practices and in learning opportunities. With the IQA ratings, however, that low cognitive demand rating of a level 2 was applied to many of the teachers’ classes, but, as the normative practice analysis indicates, there were major differences within these classrooms. Faith’s classes in the Fall of Year 1 were markedly different than either Kyra or Becca’s because the negative tone that arose from the social culture had different implications for developing students’ sense of efficacy, of being able and capable of doing math. Furthermore, in the spring of Year 1, Marie’s lesson on rotational symmetry was rated a 2, but the norms established in her class created markedly different learning opportunities than those accessible in Becca or Kyra’s instruction. In these instances, the normative practices analysis offered a window into the subtle ways teachers’ interactions shaped the learning opportunities by providing more details about *how* instructional moves maintained or caused a decline in cognitive demand.

The differences between the portraits of instruction conveyed through the IQA analysis and the normative practices analysis could be attributed to methodological decisions, both in this study and in the induction study for which the data were collected. For example, in focusing on class discussions following work time, the IQA did not assess the talk that happened during other activity structures, such as the warm up or the launch. Because of this focus on one specific part

of the lesson, the IQA would not capture the connections Marie made between students and content during the launch, or the ways she encouraged students to explain their thinking during the warm up reviews. This distinction may be especially important when studying novice teachers' instruction; for many different reasons novice teachers may not get to student reasoning in a summary discussion, but if they foster opportunities for student participation and talk at other times in the lesson, those opportunities may serve as openings upon which the new teachers can build their repertoire of practice.

The analysis of normative practices also drew attention to how social or non-content-related interactions nonetheless affected task implementation and the related ratings, especially for tasks with high levels of cognitive demand. For example, Kevin shifted towards funneling students towards a specific rule to apply, but even with these guiding norms, he continued to encourage student talk and acknowledge and incorporate student contributions into the progression of the lesson. His emphasis on social norms shaped a learning environment in which students could make connections between their ideas and those of their peers'. To some extent it seemed as if the ratings Kevin received for task implementation may have been higher not because of how he guided the math, but because of how he structured talk and participation. This suggests that access to opportunities to participate may serve as a foundation for tasks to be implemented in a way that supports the cognitive demand.

Insights into Developing a Sense of Efficacy

The analysis of normative practices also provided insight into how classroom interactions may cultivate particular messages relevant to developing productive dispositions towards learning and knowing math. Norms may govern the grammar of classroom life – or how the class works – but they also speak to questions including “What is the purpose for doing math?” and “Who is capable of doing math?” These dimensions of a productive disposition may be signaled explicitly or through normative practices in an unstated but regularly-occurring way. As such, it may be

helpful to use the decomposition of normative practices proposed from this analysis to consider whether specific normative practices may have unintended consequences for the development of a productive disposition towards mathematics. For example, in the classrooms in which math was conveyed as rule-bound, developing students' sense of efficacy was addressed primarily through an emphasis on practice. In other classrooms, norms that encouraged making student work public to be taken up by the class as part of the progression of ideas suggested a different sense of efficacy, one in which students' prior knowledge and current effort were valued as part of the class sense-making process. Even seemingly minor shifts in norms may lead to different messages about efficacy; for example, when teachers provide relevant information as needed in response to student questions during work time, the message about efficacy is different than when the teachers provide demonstrations or examples before student work time. In the former setting, there is a sense that students are capable and have the requisite knowledge to attempt a task, even if it may be new or novel; whereas in the latter setting, the unintended message may be that the only way for students to develop fluency and efficacy with respect to doing mathematics is for them to do what the teacher demonstrates.

Another interesting finding related to cultivating productive dispositions was that the teachers whose practices created opportunities for students and teacher to co-construct meaning were also the ones who emphasized the importance of some struggle in math learning in addition to opportunities to practice: Marie, Kevin and Noah. Furthermore, when teachers identified student readiness as the key challenge (versus more content-student oriented challenges like motivating students or connecting math to student interest) – as Faith, Kyra, and Becca did – then the extent to which instruction can support students' sense of efficacy may be stilted. By focusing on student readiness, the teachers set as their goals preparing students to be proficient applying rules and operations. They may have opted to avoid more open tasks because they did not perceive the students as capable without explicit directions or instruction. In sum, the teachers'

orienting views may have set a ceiling on the kind of work they believed students could do, which in turn signaled a deficit view of students' efficacy or capability. Wilhelm's (2014) analysis of the relationship between teachers' vision and the enactment of cognitively demanding tasks found a connection between teachers' productive views of students and enacting cognitively demanding tasks, especially for struggling students. Further study of how the normative practices teachers enact may yield differential opportunities for productive struggle, and how those opportunities map to both teacher and student perceptions of efficacy, is warranted.

Understanding How Novice Teachers Develop Their Repertoire of Practice

Rather than teaching being a summation of individual acts, it becomes multifaceted in that relationships among people and content are constantly negotiated. As this negotiation takes place, teachers must learn and adjust to the changing nature of individual and collective learning. (Franke et al., 2007, p. 228)

The conceptual framework, introduced in Chapter 2, acknowledges the complexity inherent in a multi-layered conception of teaching. Normative practices are rooted within relational spaces, and as novice teachers adjust their normative practices based on their assessments of the context, they build their repertoire of practice. The findings from this study speak to different facets of new teacher learning: trajectories of change (growth or decline) or stasis in teachers' practice, and what mattered to that change; the role of teachers' orienting views towards talk as potentially fertile ground for generative or high-leverage practices around making students' mathematical thinking public and creating opportunities for experiences with argumentation; and the extent to which teachers' inclinations to provide students with time to engage in productive struggle yielded opportunities to support students' sense of efficacy.

Conceptualizing three distinct strands of normative practices making up mathematics instruction allowed for multiple change or growth trajectories for novice teachers. It was possible, for example, for a teacher to change practices developing the social culture of the classroom – indeed, this would make sense as a teacher grew more familiar with the context and expectations

of the school, the administration, the student body, and external stakeholders over time – while not significantly changing guiding norms. The findings from this analysis demonstrated, however, that not all changes in normative practices are equivalent in their effect on learning opportunities and messages about mathematics. For example, while Becca increased the number of students she asked to participate, and encouraged them more, the opportunities to learn math in her class remained essentially unchanged. Additionally, infrastructure changes in the teachers' work settings to support one strand of normative practices – for example, Faith and Noah's grade reassignment in year 2 to give them the opportunity to develop positive social cultures in classes with younger students – did not guarantee changed teaching or normative practices. They may have created opportunities for teachers to encourage more participation, but Faith's more teacher-centered guiding norms capped the influence on learning opportunities. In other words, norms had differential impacts on learning opportunities; just because students talked did not guarantee that the talk was mathematically rigorous.

Making room for students to talk about mathematics is hard, and can happen in varying degrees, with limited or incremental effects. The findings from the analysis of teacher practices and their orienting views suggest that supporting talk is easier perhaps when that practice coheres with the teacher's mindset about student engagement, as in the cases of Kevin and Marie. Letting students talk requires sharing some space, air time, and authority, which can be difficult for novice teachers, especially as they are developing their own sense of efficacy in a professional capacity. Encouraging discussions becomes a management feat as much as a math teaching action; the findings from the interview analysis suggested that this was more likely when the emphasis on talk cohered with teachers' orienting views towards engaging students, even if those views were not rooted in a particular content-specific emphasis like developing the practice of justification or critiquing reasoning. Furthermore, considering how novice teachers may support mathematical talk and the issues and challenges they encounter sheds light on the way teachers'

social goals may support development of content, and conversely, how content may support social goals. It is not just that non-mathematical normative practices can and do shape learning opportunities, but also that math learning practices may serve non-math purposes. For example, Marie identified managing student off-task talk as a challenge, and it was the behavior she addressed most often, though in brief or nonverbal ways, with a look, or a move towards a student, or by saying a student's name. However, she spent more time channeling student talk in productive ways because she created so many openings for students to talk, and explicitly acknowledge contributions by interweaving them with the progression of content in the class.

Finally, teacher mindsets or orienting views seemed to outweigh infrastructure or contextual emphases with respect to giving students room to show effort and time to struggle. The teachers' mindsets about their students' readiness to do the math work of the class most frequently aligned with the findings of whether normative practices "capped" learning opportunities or offered "promising" openings. This suggests that before teachers can enact learning experiences or implement tasks in which students may need to play around with strategies, struggle, and persevere, the teachers themselves need to see this productive struggle as an important dimension of math learning. The challenge, the problem of practice, is for teachers to reconcile this idea of struggle with very real pressures to differentiate instruction, to meet the many different needs of diverse learners.

Informing & Refining the Conceptual Framework

In Chapter 2 I introduced the theoretical and conceptual frameworks that formed the foundation for this analysis of normative practices. Key themes from the different perspectives included a) conceptualizing instruction as composed of interactions between teacher, students, and content (Cohen & Ball, 1999), all of which exact pushes and pulls on the other elements, so that an interaction between teacher and content, for example, affects students, and b) recognizing teaching as relational work, with practice as a composite of the many relations and interactions

that take place across different “problem spaces” (Lampert, 2001). Here I address what this analysis of novice teachers’ normative practices offered to the framework.

Categorizing normative practices in math instruction initially aligned with instruction as interaction framework. It was straightforward to assign some normative practices to specific vectors and relational spaces. For example, how the teacher launched the task intuitively seemed to fit on the Teacher – Content vector. Or, how the teacher addressed behavior or encouraged students were examples of normative practices along the Teacher – Students vector. Quickly, however, the extended analysis suggested nuances or shifts that may be important to account for in a math learning environment that supports developing capacity with reasoning or with argumentation. For example, Marie’s instruction illustrated vividly how, when steps were taken to connect content to students during a task or topic launch, those interactions created potential both for eliciting student engagement and contributions by accessing prior knowledge and for promoting math as relevant and connected to student lives. The work she did to engage with the content, with her particular students in mind, fell along the Teacher – Content and Teacher – Students|Content vectors in ways that fostered potentially high-leverage practices of connecting math to student knowledge and interest and making a public space for student mathematical thinking to be aired and vetted not only by the teacher but by peers. Similarly, when Kevin continued to emphasize student talk as a management or engagement strategy, the normative practices fell along the Teacher – Students vector because they were part of his moves to develop social culture. These social moves created a foundation for some opportunities for students to practice reasoning, but the impact of this “lever” was constrained by the guiding moves funneling students towards specific “rules”. Meanwhile, in other classrooms, like Becca’s or Faith’s, there was not a similar emphasis on talk as part of rapport or social norms, and as such the possibility of cultivating even preliminary forms of argumentation was further diminished by the absence of these moves.

The findings from this study suggested ways to further break down relational spaces for analysis of the interactions and how they shape and signal mathematics. In turn, this decomposition of the normative practices corresponding to and comprising the different relation spaces can inform our understanding of the ways changes in interactions may have differential influences on the mathematical learning opportunities; changes in normative practices may function or proceed differently or at a different rate in the ways they shape and signal mathematics. (For example, in Becca's class, increasing student participation by calling on students to show answers, and offering more encouragement was not sufficient to attenuate the overarching guiding norm of the teacher providing all the relevant information, including the narration of steps of a problem. In this example, those changes in normative practices were not leveraged to cultivate reasoning or to co-construct math meaning.) It makes sense intuitively that some changes in practice may matter more than others; by focusing on normative practices or classroom norms, both content- and non-content-oriented, this analysis suggests a way to understand how different norms might be leveraged to sustain learning opportunities that encourage reasoning. The analytical approach guided by the framework and implemented in this analysis informs our understanding of when there may be openings for interactions that support connected ways of learning mathematics, ones in which students co-construct meaning (in keeping with push to develop mathematical habits of mind or mathematical practices). Notably, these interactions are not confined to the teacher-content vector or relational space; practices that merge into other spaces, are also pivotal. In other words, non-content-specific interactions matter, too, and the analysis of novice teachers' practice shed light on the different ways these practices could amplify or attenuate promising norms to guide the development of mathematical ideas.

Limitations

I acknowledge the limitations of this study of normative practices. First, the identification of normative practices, and of changes in practice, were based on enacted instruction as it was

observed in two class periods at 3 distinct time points in the teachers' first and second years of teaching. The process of establishing a pattern as a norm or regularity of the classroom was reasoned, with attention both to those interactions that could be named and counted and those interactions that were presumed typical because they happened without prompting and in that sense captured the just-in-time response to unplanned events (for example, a response to an error). However, in only looking at the normative practices emerging in one class, while I was able to make assertions supported about evidence of the features and prevalence of those practices, I did not have insight into the prevalence of those practices across the teachers' classes. Future studies could provide additional insight into the development of the norms by looking at practice at more frequent intervals, or by examining the practices across multiple classes, to determine a sense of the regularity or consistency in practice on a daily basis. Such an analysis might contribute further to understanding the role of norms to develop the social culture; it is reasonable to assume teachers have different interactions with different groups of students, thus raising the question of what works, for whom, and under what conditions.

A second limitation of the study was that while an instruction-as-interaction perspective framed the analysis, teacher and student perceptions timed with the observed lessons were not available. I used the teacher interviews to consider the extent to which the practices observed aligned with the teachers' stated priorities, but I did not have data on the teachers' perspectives intent, and reflections with respect to the observed lessons. A kind of member check (Maxwell, 2005) could have provided insight into whether normative practices were intentional or habitual but unintended. Given that classroom instruction is multi-faceted, the study had to be bounded in some ways, and for this analysis the study focused on the teacher. Clearly, however, the teacher interacted with students, and it would also be helpful to know more about the students' perspectives. What were their expectations? How would they describe their sense of efficacy, or their disposition towards doing mathematics? An analysis of normative practices including the

student perspective could speak to the effects of the “local” contexts on the novice teachers’ developing repertoire of practice: how did this specific class, this group of students, influence the patterns in practices, and what norms or expectations did they perceive and enact?

In this analysis three strands or categories of normative practices emerged: guiding the development of mathematical ideas, developing the social culture, and responding to students and structuring participation. These practices were observed in a context selected specifically because of the distinct structures in place at the time of the study to support ambitious mathematics instruction and teacher induction. In addition to these efforts, the district also was piloting programs to address social culture issues – for example, a conflict resolution program that established specific approaches and strategies for cultivating productive social norms. There is evidence that the social norms established by such programs may support mathematical teaching practices in vital ways because they create a learning environment in which accountable talk and productive struggle are more easily developed (Ottmar et al., 2015). Data about these programs and how the teachers perceived and implemented them would provide additional insight into how non-content interactions might be leveraged in ways to support rich(er) mathematical tasks and talk. Also, the findings of this study suggested that the infrastructure supports played less of a role in changing teachers’ practice than expected (or hoped for). A comparison of the normative practices cultivated in classrooms in a district in which these dimensions of support were different or not present could provide more insight into the extent to which these infrastructure dimensions (curriculum, professional development, mentoring) mattered to the normative practices captured within daily interactions in the classroom.

Implications

Here I consider questions for research on learning to teach and implications for practice. With respect to supporting novice teachers learning to teach math in the field, in their classrooms, the findings from this analysis yielded questions about the role of curriculum and other

infrastructure elements that affected the teachers' normative practices. For example, Kevin's norms to guide the development of mathematical ideas changed as the curriculum changed. He perceived this curriculum change in a positive way because it aligned with his orienting views towards math as a discipline and what he believed to be integral to math teaching and learning: opportunities to practice skills. However, when he started teaching, he relied on his curricular materials, and in those lessons observed in the Fall Year 1, his social norms worked in concert with the guiding math norms to create multiple opportunities and openings for students to co-construct meaning. This raises questions about the role of curriculum in supporting the development of normative practices that offer promising learning opportunities. When tasks are less open-ended, can certain social norms or participation structures compensate and maintain or increase the cognitive demand by creating opportunities for students to share and critique each other's reasoning and co-construct meaning? And, with respect to teachers' contexts, for novice teachers, which infrastructure levers support the development of promising normative practices and learning opportunities, and perhaps mitigate or change teachers' orienting views? Marie spoke affirmatively of the grade level curriculum cohorts, and many of the teachers attended the curriculum weeklong summer training. While this was beyond the scope of this study, it would be useful to know more about these teacher learning opportunities, whether they endorsed or supported particular practices, the extent to which teachers tried to enact those practices, and the challenges teachers encountered to enacting those norms.

The current focus on mathematical practices, as embodied in the Standards for Mathematical Practice (CCSSO, 2010), represents the continued focus on the interplay of learning experiences – what students are doing as they learn mathematics – with the content itself to make up the learning opportunities. Of course, this has implications for practice, at the classroom level, because student learning of mathematics “depends fundamentally on what happens inside the classroom as teachers and learners interact over the curriculum” (Ball &

Forzani, 2011, p. 17). This analysis of normative practices may speak to practitioners through a connection to the Standards for Mathematical Practice. For example, norms guiding the development of mathematical ideas may support the development of Standard for Mathematical Practice 1: Make sense of problems and persevere in solving them. When teachers provided relevant information in concert with student work, rather than as a demonstration preceding student work, they set up conditions in the learning environment for students to make sense and persevere. Similarly the norms to respond to students and structure participation formed the foundation for the talk that took place in the classroom, a necessary precursor for students to develop Standard for Mathematical Practice 3: Construct viable arguments and critique the reasoning of others. The lens of analyzing normative practices offers a means to assess how the regularities of the classroom, both mathematics-related and social, may be necessary precursors to instruction that develops these mathematical practices.

Supporting students learning in ways consistent with the portrait of doing math that is conveyed through the Standards for Mathematical Practice is challenging work. Connecting to efforts in teacher education to identify and cultivate high-leverage practices that underlie effective teaching (Ball & Forzani, 2010), through, for example, rehearsals of practices such as eliciting and responding to student performance (Lampert et al., 2013), the normative practice analysis suggests other teacher practices to develop explicitly in both in-service and preservice teacher education settings. Identifying the norms that support making students' mathematical thinking public beyond brief or choral responses may be a necessary first step to creating learning opportunities to develop students' capacity with reasoning and argumentation. Teacher learners (both preservice and novice teachers) may need many opportunities to rehearse this practice to interrupt traditionally-held views of the roles of teachers and students. Allowing class time and a public space for students' mathematical thinking – whether correct, incorrect, incomplete, or in progress – has the potential to be a high-leverage practice; it may be generative in that through

this practice teachers may further their own understanding both of content and of the different ways their students may make sense of problems. Making room for students' mathematical thinking to be public, however, is a skill that requires continued practice or rehearsal (Ball & Forzani, 2011); it is not intuitive.

Conclusion

This study bridged questions and problems of practice in mathematics instruction and teacher learning. In addition to the specific findings presented in detail throughout this dissertation, more broadly the analysis raises questions for further reflection and study. For example, the findings suggest how the social interactions that beginning teachers identify as key or important (such as addressing behavior, encouraging students or establishing rapport) might constrain or impede more ambitious learning opportunities, and how even seemingly small social interactions might support more student engagement and autonomy with respect to math learning. Teachers may share as a priority the desire to support students becoming more independent workers and learners, but the ways they go about this has implications for the sense of efficacy students may develop. Are students only capable after the teacher guides them through what to do? Or, are they encouraged to draw on their own knowledge, ideas, and capacities – and are they allowed some autonomy in tackling problems? Are they given room to struggle, and what conditions and normative practices support productive struggle?

Finally, just as the social interactions may support interactions with content, the study of normative practices raises the question of whether content practices can support teachers' non-content goals. If students are talking about math and seeing it as connected or relevant to their lives, will there be fewer off-topic disruptions – and will students be prepared for future math and school experiences differently than when the content is merely a backdrop or context for developing more general study skills? With teachers, especially novice teachers juggling multiple responsibilities and competing demands, this study and its findings hopefully provide a lens into

the ways math learning opportunities and messages can serve social goals, and, in concert, norms to develop the social culture can help shape a space for productive engagement with mathematics problems.

APPENDICES

List of Appendices & Content

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Appendix A: Tasks/Topics for each Observation, by Teacher

WU stands for Warm Up Activity

MT stands for Main Task/Topic

CPM stands for *College Preparatory Mathematics*

CMP2 stands for Connected Math Project 2

Inv. stands for Investigation

Teacher	Fall Year 1		Spring Year 1		Spring Year 2	
	Day 1	Day 2	Day 1	Day 2	Day 1	Day 2
<i>Faith</i>	WU: Graph Coordinate Pairs MT: Properties of squares, right triangles, rectangles, and parallelograms	WU: Properties of shapes MT: CMP2, <i>Looking for Pythagoras</i> , Inv. 1.2 Planning Parks	WU: Naming symbols used in math MT: Preparing to write symbolic expressions	WU: More with math symbols MT: CMP2, <i>Say it with Symbols</i> , Inv. 1.1 Tiling Pools	WU: Change decimals to percents MT: CMP2, <i>Bits and Pieces III: Computing with Decimals and Percents</i> , Prob. 4.1 Determining Tax	WU: Word problem calculating total cost of a CD with sales tax MT: CMP2, <i>Bits and Pieces III</i> , Prob 4.2 Computing Tips
<i>Marie</i>	WU: Naming points on a coordinate grid MT: CMP2, <i>Looking for Pythagoras</i> , Inv. 1.1 Driving around Euclid	WU: Drawing different polygons MT: CMP2, <i>Looking for Pythagoras</i> , Inv. 1.2 Planning Parks	WU: Multiple choice test preparation MT: CMP2, <i>Kaleidoscopes, Hubcaps & Mirrors</i> , Reflectional Symmetry	WU: Multiple choice test preparation MT: CMP2, <i>Kaleidoscopes, Hubcaps & Mirrors</i> , Rotational Symmetry	WU: Multiple choice test preparation MT: CMP2, <i>Kaleidoscopes, Hubcaps & Mirrors</i> Prob. 2.3 Describing Translations	WU: Multiple choice test preparation MT: Continued work on Student Labsheet related to Problem 2.3 Translations
<i>Becca</i>	WU: Quick review, 3 short questions MT: Solving problems using the Pythagorean Theorem	WU: Quick review, 3 short questions MT: Solving problems using the Pythagorean Theorem	WU: 3 problems related to functions MT: <i>Glencoe</i> Simplifying algebraic expressions by combining like terms	WU: 3 problems: functions, tables, expressions MT: Evaluate expressions to complete function tables	WU: -- MT: <i>CPM Algebra Connections: Setting up and solving equations based on proportional relationships</i>	WU: 6 Review problems (mixed) <i>CPM Algebra Connections: Writing and solving equations using proportions</i>

	Fall Year 1		Spring Year 1		Spring Year 2	
Teacher	Day 1	Day 2	Day 1	Day 2	Day 1	Day 2
<i>Kevin</i>	WU: Finding areas of regular polygons on grids MT: CMP2, <i>Looking for Pythagoras</i> , Inv. 1.3 Areas of Irregular Polygons	WU: Finding areas of regular polygons on grids MT: CMP2, <i>Looking for Pythagoras</i> , Inv. 1.3 Areas of Irregular Polygons	WU: Reflect a shape across a vertical line of reflection MT: CMP2, <i>Kaleidoscopes, Hubcaps & Mirrors</i> , Prob 5.1 Coordinate Rules for Reflections	WU: -- MT: Review of reflection and translation symmetry	WU: Graph 2 lines on the same plane MT: <i>CPM Algebra Connections</i> : Comparing slopes of perpendicular lines	WU: Given 2 points, write an equation of the line MT: <i>CPM Algebra Connections</i> : Find the equation of a line when you know its graph (with graphing calculators)
<i>Kyra</i>	WU: Solve $3x-5=1$ for x MT: Identify basic geometric elements like rays, line segments, lines	WU: Multiple choice question on equation of a line and slope MT: Finding surface areas	WU: Convince someone that a figure has reflection symmetry MT: (Find rule for) Reflecting points on a coordinate grid over the x-axis, y-axis, line $x = y$	WU: Convince someone that a figure has translation symmetry MT: (Find rule for) Translating figures on a coordinate plane	WU: “Brainstarter” Expense/income word problem MT: CMP2 (“Predicting Profits”) Rewrite 2-variable equation by replacing 1 variable with an equivalent expression	WU: “Brainstarter” MT: CMP2, <i>Say it with Symbols</i> , Inv. 2.2, 2.3 (Modeling situations with symbolic statements, writing expressions)
<i>Noah</i>	WU: Writing algebraic expressions MT: CMP2, <i>Thinking with Mathematical Models</i> , Inv. 3.1 Rectangles with Fixed Areas	MT: Slope/Equation of a line problem CMP2, <i>Thinking with Mathematical Models</i> , Inv. 3.2 Bridging the Distance (Inverse Variation Patterns)	MT: CMP2, <i>Samples and Populations</i> , Modified from Investigations 1 and 2 Taste Testing Orange Drink Survey	MT: CMP2, <i>Samples and Populations</i> , Modified from Investigations 1 and 2 Taste Testing Barbecue Chips Survey	WU: Find area and perimeter of figures MT: CMP2 <i>Prime Time Patterns</i> and Sequences Problem “for use before Inv 2 Whole Number Patterns and Relationships”	WU: Find area and perimeter of irregular figure MT: Ratio and Proportions Review (write ratios as fractions in simplest form, setting up proportions)

Appendix B: IQA Scores for each Observation, by Teacher

Academic Rigor Rubric							Accountable Talk Rubric									
AR1 Potential of Task							AT1 Participation									
AR2 Implementation of Task							AT2 Teacher Linking									
AR3 Discussion of Task (Post-work time)							AT3 Student Linking									
							AT4 Teacher Ask									
C1 is coder 1; C2 is coder2							AT5 Student Provide									

Fall Year 1, Day 1

Teacher	AR1		AR2		AR3		AT1		AT2		AT3		AT4		AT5	
	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2
<i>Faith</i>	1	3	1	2	1	1	2	0	2	2	1	1	1	1	1	1
<i>Marie</i>	4	4	2	3	1	2	1	1	2	2	1	1	1	1	1	1
<i>Becca</i>	2	2	2	2	1	0	1	0	1	0	0	0	0	0	0	0
<i>Kevin</i>	3	3	3	3	3	3	4	4	2	2	1	1	4	4	3	4
<i>Kyra</i>	4	4	3	3	2	3	2	3	2	2	2	2	1	2	2	3
<i>Noah</i>	3	3	2	3	1	1	3	4	2	3	1	1	2	3	1	2

Fall Year 1, Day 2

Teacher	AR1		AR2		AR3		AT1		AT2		AT3		AT4		AT5	
	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2
<i>Faith</i>	3	4	2	2	1	1	1	1	2	2	1	1	1	1	1	1
<i>Marie</i>	4	4	3	3	2	2	2	2	2	2	1	1	2	2	2	2
<i>Becca</i>	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1
<i>Kevin</i>	3	2	3	2	3	2	3	4	2	2	1	1	3	2	2	2
<i>Kyra</i>	2	2	2	2	2	2	1	1	2	2	1	1	2	2	1	2
<i>Noah</i>	2	3	2	2	1	2	2	4	2	3	1	1	2	2	1	2

Spring Year 1, Day 1

Teacher	AR1		AR2		AR3		AT1		AT2		AT3		AT4		AT5	
	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2
<i>Faith</i>	1	1	1	1	2	2	1	2	2	2	1	1	2	2	1	1
<i>Marie</i>	3	3	3	2	2	2	2	2	2	3	1	2	1	2	1	2
<i>Becca</i>	2	2	2	2	2	2	1	1	2	2	1	1	1	1	1	1
<i>Kevin</i>	2	2	2	2	2	2	2	3	2	2	1	1	1	2	1	2
<i>Kyra</i>	2	3	2	2	1	2	3	3	2	2	1	1	1	1	1	1
<i>Noah</i>	3	2	2	2	1	2	4	3	2	1	1	1	1	1	1	1

Spring Year 1, Day 2

Teacher	AR1		AR2		AR3		AT1		AT2		AT3		AT4		AT5	
	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2
<i>Faith</i>	2	3	2	3	2	2	1	2	2	2	1	1	1	2	1	2
<i>Marie</i>	3	2	2	1	1	1	2	2	2	2	1	1	1	1	1	1
<i>Becca</i>	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0
<i>Kevin</i>	2	2	2	2	2	1	2	0	2	1	1	1	1	1	1	1
<i>Kyra</i>	2	2	2	2	1	2	1	2	2	2	1	1	1	1	1	1
<i>Noah</i>	2	2	2	2	2	3	3	4	2	2	1	1	1	2	1	2

Spring Year 2, Day 1

Teacher	AR1		AR2		AR3		AT1		AT2		AT3		AT4		AT5	
	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2
<i>Faith</i>	3	2	2	2	2	2	2	2	2	2	1	1	2	2	1	1
<i>Marie</i>	3	3	3	3	1	2	1	3	2	2	1	1	2	2	1	2
<i>Becca</i>	2	4	2	3	2	2	4	3	2	2	1	1	2	2	2	2
<i>Kevin</i>	3	3	3	3	3	2	2	2	2	2	1	1	2	3	2	2
<i>Kyra</i>	4	4	2	2	1	1	1	2	2	2	1	1	1	1	1	1
<i>Noah</i>	3	3	2	3	1	2	4	4	2	2	1	1	2	2	2	2

Spring Year 2, Day 2

Teacher	AR1		AR2		AR3		AT1		AT2		AT3		AT4		AT5	
	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2
<i>Faith</i>	2	2	2	2	0	1	0	1	0	2	0	1	0	1	0	1
<i>Marie</i>	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0
<i>Becca</i>	2	2	2	2	2	1	2	1	2	2	1	1	1	1	1	1
<i>Kevin</i>	2	3	2	1	2	1	2	2	2	2	1	1	2	2	2	1
<i>Kyra</i>	4	4	2	2	1	1	2	1	2	2	1	1	1	1	1	1
<i>Noah</i>	2	2	2	2	0	1	0	1	0	2	0	1	0	1	0	1

Appendix C: Observation Analysis Field Note Coding Guide

Observation Analysis Process

- Complete an observation summary sheet (drawing on data from Video Summary Sheets and observation artifacts) to identify activity structures within the lesson.
- Create an InqScribe document (to note time points in video as necessary)
- Take field notes by time using Observation Coding Worksheets
- At the end of an observation, enter normative practices data, by category, onto Observation Event Listing/Assertions Table by activity structure to identify patterns, connected to counts/prevalence

When taking field notes:

- If a move or section is illustrative of a norm, transcribe with time codes
- A move may have multiple codes (fit in more than one category)
- Focus on what is said in the “public” sphere – in other words, what the whole class hears or is intended to hear.

Coding for actions that fall into two roles of the teacher: Guiding the Mathematics Work (Hiebert et al: “Providing direction for mathematical activities”) and Cultivating the Social Culture of the Classroom (Hiebert et al: Guiding Development of Classroom Culture)

(Abbreviations)

- **HL/LL** = Higher level/ Lower level
- **M/NonM** = Math/Non-Math
- **+ / -** = Positive/Negative

Guiding the Mathematics Work

Procedural: Two kinds of actions:

- For process or pedagogical interactions, ranging from assigning the work (“you will do 3 problems”) to assigning how the work will be completed (“you will work in groups”). In other words, the management aspect of instruction. These kinds of interactions are **NonM**.
- Also, *if* a procedure or algorithm is presented as part of instruction (for example, going through the steps of factoring a binomial expression) note it here but demarcate as **M** for math. Or, if teacher provides a hint in a funneling sense, or reviews an answer, note that here (because action may be proceduralizing the math).

Marking

LL = Repeating what a student says, or IRE evaluating (“that’s correct”)

HL = Making a connection (either to what another student said or to a concept) – e.g. “That’s an interesting point. Can you explain how that helps us?”

Higher Level (extension, representation)

Suggesting a representation, or extending an idea, or asking for justification or reasoning

Error

How are errors addressed? What kinds of errors, if any, are not addressed?

Language/Conventions

Does the interaction emphasize mathematical language or a convention? And is this appropriate/accurate?

Developing/Cultivating the Social Culture

Rapport: Coaching/Contribution: These are not specific to content. They may be an exhortation like “Let’s have a good day”, a general compliment like “Good job”, a general appreciation like “Thank you.” These are + **positive**. May want to note when these are **M** – related to the math work.

Rapport: MathAbility: References to students’ ability specific to the math work (“Good job using a figure” or “You should be able to do this.” Mostly + **positive** though there could be instances when this is – **negative** (for example, “There’s no reason you can’t factor this equation in the next 2 minutes” may sound aggressive – the tone matters).

Behavior: Addressing + **positive** or – **negative** behaviors (talking, calling out, etc.) Management/discipline related.

Norms: Expectations for Work: This may include the purpose of the work. May overlap with Procedural (See above)

Norms: Doing Math: Sociomathematical. How we do math in the classroom; What I expect to see; What you should include; Etc. Note if HL (related to argumentation, reasoning/why) or LL (when a “how I did it” answer suffices).

Norms: Participation: How and which Ss participate (calling on, drawing sticks, asking for volunteers)

Is participation rooted in engagement with math ideas/what students are offering? Or is participation structured guided by different priorities?

Open Codes (to note points of interest)

Real World Connection: describe this to consider appropriateness (is it a meaningful connection), relevance (to students), and breadth/variety (for example, are all real-world applications/connections related to money and finance?)

Connection to Students

Curriculum: For example, may include direct reference (remember what we did/see page ##/ use your book to help you...), or oblique/implied “they”

Appendix D: Template – Observation Coding Worksheet

Observation Live Coding Worksheet v2: _____

Page ____ of ____

Time Codes	Activity Structure	Guiding the Mathematics Work						Cultivating the Social Culture					Particip.	Open Codes (Connections to RW, to S, or to Curr.)
		Procedural Non M	Procedural M	Marking (LL/HL)	Higher Level (Ext., Rep.)	Error	Language	Rapport: Coaching/Contrib.	Rapport: Ability	Behavior + or -	Norms: Exp. for Work	Norms: Doing Math		

Appendix E: Example – Observation Event Listing/Assertions Table

Excerpt from Marie, Fall Year 1, Day 1

Activity Structure (duration)	Guiding the (Development of) Content		Notes/ Extended Examples	Developing Social Culture of Classroom		Notes/ Extended Examples	Other Notes
	Non-Math (pedagog.)	Math		Non-Math	Doing Math		
Warm up WT (12 m)	5		N-M Procedural – after about 5 min WT, selecting Ss to write coordinates on white board	3 (1+, 2 -) behavior 4 exp for work		N-M reminders about using time, sitting so others can see	T selected Ss for participation
Warm up D (5 m)	5	3 marking, overlapping with error 1 lang: y-int “Does anyone have a different answer?” x2	Initial question: “Does anyone have anything different? To address error: Who agrees with D? Anyone have a different answer? Also language: picking out the y-intercept	2 (+ hands)	1 (+ ability)	“Nice job, Devon”	
Lesson Intro (10 m)	9	4	Marking: Repeat/describe in different way” Emphasize directions to note different places with similar streets	2 (+)	2	Richard can you repeat?, J can you describe in a different way?	RW example: Chicago, Washington DC (in text)
Launch Task (10 m)	2	4 builds on S examples	Distance as crow flies from block	2 (-) 2 don’t work ahead, follow directions	1+ you gave good example	Uses S example to emphasize different kinds of routes (as crow flies/helicopter, on the road, then the emergency route)	
Work Time							
Discussion (4 m)	7	4 builds on what Ss say	How did we determine when to stop?			Calls on 4 Ss, builds on what they say, uses class terms (helicopter)	

Appendix F: Interview Analysis Coding Guide

Purpose for coding teacher interviews

- To gather information for describing novice teacher’s orienting views towards content, students, and teaching
- To ascertain if there is change in views over time, and to what teacher attributed that change
- To identify potential relationships between teacher orienting views and practice (normative, mathematical) as manifested through patterns of instruction (i.e., to understand what motivates/contributes to the normative practices, mathematical and not, that developed)

CODE	Definition
Teacher.Background	Description of background – experiences as a student, for example, and also college experience and teacher training
Teacher.Challenges.Math	Description of challenges to teaching math
Teacher.Challenges.NotMath	Description of teaching challenges other than math-related ones
Teacher.Resources	Description of resources and sources of support (may include lack of)
Teacher.Successes	Description of perceived successes
Teacher.Beliefs.Math	Beliefs about the subject (“cumulative”, building blocks)
Teacher.Perceptions.Curriculum	Perceptions of curriculum materials
Teacher.Beliefs.Teaching	Beliefs about what teaching entails/requires (general)
Teacher.Beliefs.MathInstruction	Beliefs about how math should be taught, how math is learned (Role of practice, activities, etc.)
Teacher.Perceptions.Students	Perceptions of students (interests, ability, etc.)
Teacher.Perceptions.Context	Perceptions of the school context, including norms/expectations
T.Perception.TeacherKnowledge	Perception of teacher’s own knowledge (“I know what to do” and also “I don’t know...”)

Clarify! How is a belief distinct from a perception? A belief is rooted “within” the teacher, though influenced of course by experiences, whereas a perception inherently is informed by the immediate circumstances, how the teacher “processes” or makes meaning of those circumstances (perceptions rooted in the experiences connected to the act of teaching, whereas the beliefs existed in some form prior to the work of teaching).

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