

Revenue Management with Strategic Customers: Last-Minute Selling and Opaque Selling

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First version: March 2008; This version: August 2009.¹

Abstract

Companies in a variety of industries (e.g., airlines, hotels, theaters) often use last-minute sales to dispose of unsold capacity. Although this may generate incremental revenues in a short term, the long-term consequences of such a strategy are not immediately obvious: more discounted last-minute tickets may lead to more consumers anticipating the discount and delaying the purchase rather than buying at the regular (higher) prices, and hence potentially reducing revenues for the company. To mitigate such behavior, many service providers have turned to opaque intermediaries such as hotwire.com that hide many descriptive attributes of the service (e.g., departure times for airline tickets) so that the buyer cannot fully predict the ultimate service provider. Using a stylized economic model, this paper attempts to explain and compare the benefits of last-minute sales directly to consumers vs. through an opaque intermediary.

We utilize the notion of rational expectations to model consumer purchasing decisions: consumers make early purchase decisions based on expectations regarding future availability, and these expectations are correct in equilibrium. We show that direct last-minute sales are preferred over selling through an opaque intermediary when consumer valuations for travel are high and/or there is little service differentiation between competing service providers; otherwise, opaque selling dominates. Moreover, contrary to the usual belief that such sales are purely mechanisms for disposal of unused capacity, we show that opaque selling becomes more and more preferred over direct last-minute selling as the probability of having high demand increases. When firms randomize between opaque selling and last-minute selling strategies, they are increasingly likely to choose opaque selling strategy as the probability of high demand increases. When firms with unequal capacities use opaque selling strategy, consumers know more clearly where the opaque ticket is from and the efficacy of opaque selling decreases.

Keywords: distribution channels, competition, revenue management, strategic consumer behavior, rational expectations.

¹This research was supported by The Mack Center for Technological Innovation at the Wharton School. The authors would like to acknowledge the faculty of the OPIM and Marketing departments at the Wharton School, seminar participants at University of British Columbia and New York University, the participants of the Mini-Conference on Customer-Oriented Operations Models at Washington University in St. Louis, the participants of the Marketing Science Conference 2008 and the participants of the Revenue Management and Pricing Conference 2008 for their helpful comments. We also thank two anonymous reviewers and the Associate Editor for their extremely valuable suggestions for improving the paper.

1 Introduction

Firms in the travel industry (e.g., airlines, hotels and car rentals) face the problem of uncertain demand for their services. While these firms typically begin by selling regularly priced products through direct channels as well as through a variety of intermediaries, they often later utilize last-minute sales discounts to sell off their leftover capacity (in the airline industry, it is often termed “distressed inventory”). Since service capacity in these industries is hard to adjust in the short term and the marginal cost of providing service is negligible, such dynamic discounts are pervasive. One common practice that firms adopt is to sell last-minute tickets at low prices through their own web sites. For instance, in the case of US Airways, consumers can visit <http://www.usairways.com/awa/faresale/eSaver.aspx> and find out the current week’s discounted fares. Unfortunately, last-minute sales directly to consumers are dangerous in that they condition potential consumers to expect that there might be a deal available at the last moment at a much lower price. As a result, some consumers may prefer to wait for last-minute sales rather than purchasing earlier at a higher price. Many industry analysts, observers and executives have questioned the last-minute sales approach altogether since it “... starts a cycle of price degradation that will eventually lead to ... destroying the airlines” (Sviokla 2004).

Another mechanism for selling distressed inventory appeared more recently under the name of *opaque* selling. Before purchasing the opaque product from an intermediary, a consumer does not know which company will ultimately provide the service, when exactly will the service commence and how long will it take (for air tickets) etc. In the travel industry, firms like hotwire.com, cheaptickets.com, priceline.com², onetravel.com and many others have popularized this selling mechanism in which the intermediaries hide some attributes of the product from the consumers and reveal them only after the purchase has been made.

This is the opposite of *transparent* sales discussed above, whereby all attributes are observable up front. One frequently cited reason for the existence of the opaque channel is that “... it enables airlines to generate incremental revenue by selling distressed inventory cheaply without disrupting existing distribution channels or retail pricing structures” (Smith et al. 2007).

²In addition to opacity, the “Name-Your-Own-Price” (NYOP[®]) is an interesting parallel concept that priceline.com uses (i.e., consumers can haggle for ticket prices). In this paper we focus on studying opacity and therefore abstract away from the bargaining/haggling process; see Terwiesch et al. (2005) for further discussion of online haggling.

Using opaque sales can have two opposing effects: by hiding key attributes of the product the firms may persuade some consumers to buy regularly priced service (and, to some extent, mitigate the “cycle of price degradation”), but there is also a chance that consumers indifferent among multiple service providers may be diverted to the opaque channel if “the price is right.” Given that 60% of consumers shopping online buy the lowest fare available (PhoCusWright 2004), and that opaque selling of products has been widely prevalent in recent years among travel companies (Lambert 2006, Harrison 2006), it is important to understand how channel choice affects profitability, prices and consumer segmentation.

The goal of this paper is to understand the dynamics of the last-minute discounts and to shed some light on the relative merits of last-minute sales directly by the firm vs. through an opaque intermediary. To analyze these issues we propose a stylized economic model in which two firms sell horizontally differentiated products to consumers on a Hotelling line in two time periods. The firms have fixed capacities but can adjust prices from period to period. The firms use transparent sales in the first period, and may use either transparent last-minute sales or opaque last-minute sales in the second period, if there is left-over inventory. Our model yields the following major findings:

1. When firms sell through opaque channels, consumers (who prefer a ticket from one firm over the other) form expectations about the availability of the tickets from either firm and factor this into their purchase decisions. When firms have symmetric capacities, we demonstrate that, in equilibrium the consumers expect that the opaque product comes from either of the two firms with equal probability and the two firms supply equal quantities of their products to the opaque channel, i.e., the firms achieve “perfect masking” of the product identity.

2. When there is no uncertainty in demand, selling through the opaque channel weakly increases the firms’ profits compared to selling only through the direct channel. This is because using the opaque channel as a “clean up” mechanism by charging a low opaque price but masking the product attributes leads to ex ante expected surplus of zero for all consumers, who, therefore, purchase the product. This does not distort the regular pricing structure (i.e., in the absence of the opaque channel), while consumers who would not have purchased otherwise, if any, now purchase the opaque product.

3. When demand is uncertain, both opaque last-minute sales and direct last-minute sales can lead to higher profits under different conditions. Under uncertain demand, consumers trade off

the possibility of buying later at a lower price with the risk of not buying at all (if demand turns out to be high and inventory runs out). In opaque selling, the identity of the product is masked, which makes the ex ante expected surplus from an opaque product zero. Hence, consumers do not benefit from waiting and the firms use this to charge high prices in the first period, even when consumer valuations for tickets are low. In direct last-minute sales, the firms price in the first period to extract the surplus from consumers who have a high preference for purchasing, while low-preference consumers choose to wait and buy at low prices through the last-minute sales if any products are left over. In this strategy, the firms make the bulk of their profits through first-period prices. However, if consumer valuations are low, these first-period prices are low, and as valuations increase, these prices (and, correspondingly, firm profits) increase. Hence, opaque sales are preferred over transparent last-minute sales when consumer valuations are low, and vice versa.

4. We demonstrate that, as the probability of having high demand realization increases, opaque selling becomes more and more preferred over direct last-minute selling. This finding is contrary to the traditional understanding of opaque selling as a mechanism to clear leftover inventories when demand is low. The intuition behind this result is, again, that masking the product identity leads to ex ante expected surplus of zero from an opaque product, so that consumers do not have a benefit from waiting and they prefer to purchase in the first period. As the probability of high demand (and, therefore, the possibility of not getting a product) increases, the competition (or “clamor”) among consumers for products in the first period increases, which enables the firms to charge higher first-period prices and increase profits.

5. We analyze a situation in which firms randomize between selling through opaque or through their own channels in the second period. We find that as probability of high demand increases, firms are more likely to adopt opaque selling strategies (i.e. they sell through opaque intermediaries with higher probability).

6. When firms are asymmetric with one firm having larger capacity than the other, they will not be able to achieve perfect masking of the opaque product because consumers will rationally expect that the probability that it is from the larger firm is higher. Hence, because of the imperfect masking in the opaque channel, the pricing power of the opaque intermediary and the two firms will diminish. We find that the firm with a larger capacity is at a greater disadvantage. In spite of this finding, using opaque sales still helps asymmetric firms to increase their profits.

To summarize, our study is the first to shed light on the comparative advantages of direct selling versus opaque selling as last-minute sales strategies. We explicitly model the effect of consumers' strategic behavior regarding product availability on firms' sales strategies to dispose of inventories, and we characterize conditions under which firms prefer one strategy over the other. The rest of the paper is organized as follows. In Section 2, we review related literature from economics, operations and revenue management. In Section 3, we describe our model, in Section 4, we analyze the case of deterministic demand and in Section 5, we analyze the case of stochastic demand. In Section 6, we extend our basic model to the case of asymmetric firms and in Section 7, we consider the consequences of relaxing several other assumptions. We conclude in Section 8 with a discussion.

2 Related Literature

The work on inter-temporal sales started with the seminal Coase conjecture (Coase 1972) which postulates that, given an infinite number of selling opportunities over time, a monopolist will eventually decrease a product's price to its marginal cost, and all consumers will anticipate this decrease and delay their purchases. Numerous subsequent papers modeled scenarios in which the Coase conjecture may not hold (Stokey 1979, Besanko and Winston 1990, DeGraba 1995). Specifically, DeGraba (1995) suggested that under uncertain demand, if product availability is limited, consumers might not have an incentive to wait for a lower price because of the threat of unavailability. We build on this trade-off between price and availability in the context of opaque sales when consumers form rational expectations about future availability.

Our paper contributes to the small but growing literature in Operations Management that models strategic consumer behavior. Recent papers in this stream that explicitly incorporate product availability considerations and demand uncertainty into last-minute sales models include Cachon and Swinney (2008), Lai et al. (2008), Su (2007), Su and Zhang (2008) and Yin et al. (2009). For instance, Cachon and Swinney (2008) demonstrate that the value of quick response strategies is higher in the presence of consumers who strategically wait for sales. However, none of aforementioned papers consider competition and opacity in product attributes. For a rich compilation of recent work in operations management which incorporates strategic consumer behavior the reader is referred to Netessine and Tang (2009).

There is a rich literature studying revenue management practices in the travel industry (Talluri and van Ryzin 2004) which is the primary, but not the only, adopter of opaque selling. This literature usually assumes that availability of products or competition do not affect consumer demand (see Liu and van Ryzin (2008) and Netessine and Shumsky (2005) for exceptions). Koenigsberg et al. (2008) consider the impact of airline capacity and the number of customer segments (differing in price sensitivity) on the pattern of sales by airlines. However, strategic waiting by consumers for low prices, an extremely important issue that airlines face every day, has not been considered in the revenue management literature. This paper is an attempt to unravel the impact of strategic consumers on airline firms' selling strategies.³

To our knowledge, only a few very recent papers explicitly consider opaque selling mechanisms. Jiang (2007) considers opaque selling for two flights owned by the same monopolist firm but scheduled at different times throughout the day. Fay and Xie (2008) study "probabilistic selling" under which a monopolist creates a probabilistic good by clubbing several distinct goods together. Fay (2008) considers a model of competition with deterministic demand and shows that the opaque channel increases the degree of price rivalry and reduces industry profits unless firms have very loyal customers. Shapiro and Shi (2008) model competing firms selling opaque products through a passive intermediary which posts prices dictated by firms while hiding the identity of the products. There are two segments of consumers with different strengths of brand preference. They find that opaque sales intensifies competition for customers with low brand preferences, and enables firms to commit to high prices for customers with high brand preference so that the total profit can increase.

All of these papers utilize single-period models in which opaque sales occur simultaneously with transparent sales. Our model is very different in many ways. First, we explicitly recognize that opaque sales is a last-minute sales mechanism and we study it using the resulting dynamic model. The dynamic aspect is an important practical consideration for opaque selling (Elkind 1999, Harrison 2006): in the travel industry, opaque products are sold only a few days before service delivery⁴ while transparent products are sold at regular prices up to a year in advance. Second, we

³Xie and Shugan (2001) consider the strategy of advance selling of tickets to strategic consumers. Here, consumers' valuations for the product are uncertain in advance and are realized after the purchase decision has been made. In our model, both consumers and firms are certain about consumers' valuations for the product; the uncertainty is on the realization of demand and, correspondingly, on future product availability. Su (2009) considers a two-period model where firms face inertial and rational customers. Inertial customers have a tendency to refrain from purchasing. Levin et al. (2008) consider dynamic pricing of limited capacity sold over multiple periods to strategic customers.

⁴To be precise, opaque products are sometimes available in advance but they are priced at the same level as

also allow for demand uncertainty which is only resolved very late in the selling horizon. By virtue of the above two aspects, consumers face a trade-off between buying at a higher price early versus at a lower price (under the threat of stockout) later, which is the key trade-off of our model. Third, we compare opaque selling with another commonly observed last-minute sales mechanism, namely, direct last-minute sales, and characterize the conditions under which one is better than the other for firms involved.

Versioning (Varian 2000) and “damaged goods” (Deneckere and McAfee 1996) are related price discrimination strategies in which a high-quality product is sold in its original form and also in an inferior version with some of its features disabled. Opaque selling is related to these strategies because (i) the same product is sold both through a transparent channel and through an opaque channel with some attributes hidden and (ii) typically, airline reservations made through opaque channels cannot be modified or exchanged without an extra fee. However, the key difference for opaque selling comes by virtue of the fact that it requires the availability of an alternative competing product. Furthermore, all consumers who purchase the inferior product in the form of versioning or damaged goods receive lower utility from consuming it (relative to consuming the original product), while a fraction of consumers who purchase the opaque product end up receiving the same utility (and higher net surplus) relative to what they would have received by purchasing a transparent product. Finally, rational expectations regarding product availability (due to limited capacity and demand uncertainty) never come into consideration in either the versioning or the damaged goods literature.

3 The Model

Two competing firms, A and B , each hold a quantity $K/2$ of inventory (later, we consider firms with asymmetric capacities). The inventories can be service capacities, e.g., seats on flights operated by the firms on a particular date, or rooms for a particular day in similar hotels run by the firms. The products are perishable in the sense that they have to be sold before a certain time and have no value if they remain unsold. For travel services, this is a reasonable assumption (e.g., products

transparent products which makes them unattractive to any consumer. This can be observed through a simple experiment on hotwire.com which starts selling deeply discounted opaque tickets only a few days before the service date.

have no value after the flights take off and hotel rooms have no value if they remain unfilled by the day under consideration).

Consumers have heterogeneous preferences between firms. The reason might be loyalty to the firm, preference for a brand, or simply an established relationship with the company (e.g., through rewards programs). We capture this consideration by invoking a horizontal differentiation model similar to Fay (2008) and other papers on opaque selling.⁵ We assume that the two competing firms A and B are located at each end of a Hotelling line of length 1 and a continuum of consumers is spread on the horizontal line over the interval $[0, 1]$ with uniform density. A population of J consumers is spread uniformly over the entire line. We consider cases of deterministic low demand ($J < K$), deterministic high demand ($J > K$), and random demand ($J = H > K$ with probability α , and $J = L < K$ with probability $1 - \alpha$).

Each consumer has a valuation V for the product and purchases at most one unit. The brand preference of every consumer is completely characterized by his location $x \in [0, 1]$ on the line which influences the utility a consumer derives when he purchases a product from a firm. The parameter t denotes the strength of brand preference in the market. A consumer at x incurs a disutility tx when buying a product from firm A and a disutility $t(1 - x)$ when buying a product from firm B . Thus, the customers have varying preferences towards the competing firms. If firms A and B charge prices p_A and p_B respectively, then a consumer located at x receives net utility $V - tx - p_A$ when purchasing a product from firm A , and receives net utility $V - t(1 - x) - p_B$ when purchasing a product from firm B . We assume that $V/t \geq 1/2$ so that every consumer would receive non-negative utility from the product from at least one of the firms if it were offered for free by both firms.

We will encounter the ratio V/t frequently in the analysis to follow. This ratio can be interpreted as a “brand preference adjusted valuation” for a product and it reflects the degree of competition between the firms. If V is large, the valuation for a product in the market is high and the market will be competitive, and vice versa. Further, if t is small, the consumers do not care about the firm that they buy from and competition will be high, and vice versa. Overall, as V/t increases, the market becomes more competitive.

⁵We assume that there is no vertical differentiation between products of the two firms, i.e., one product is not inherently superior to the other for all customers.

We divide the selling horizon into two periods so that each firm has two pricing opportunities. We assume no discounting between the two periods. The firms choose one of the following strategies for selling products:

1. The firms can sell the products through their own channels in both periods and offer different prices in each period of sale.
2. The firms can sell in the “transparent” channel in the first period and sell opaque products through an intermediary (such as hotwire.com) in the second period. The intermediary, denoted by I , makes its own pricing decision p_I . It keeps a fraction $1 - \delta$ of the revenue it makes. The remaining fraction of the revenue δ from each product goes to the airline whose product the intermediary sells (see Elkind (1999), Priceline (2006) and Phillips (2005) for a description of such arrangements in the travel industry).

Since the firms sell products over two periods, possibly at different prices, the consumers strategically time their purchases based on their valuations, expectations of future inventory availability, and the firms’ pricing strategies. In other words, a consumer might decide to purchase products in the second period rather than in the first period. A consumer purchasing from the opaque intermediary does not know from which firm the product is coming, but develops expectations about the service provider based on expectation of capacity/inventory supplied to the intermediary by the two competitors. All consumers have the same beliefs. To differentiate from the actual equilibrium outcomes, we represent beliefs (or expectations) by using the superscript e .

We assume consumers are forward-looking and have rational expectations, i.e., the availability of tickets from each firm in every period matches consumers’ expectations on the availability.⁶ Amaldoss and Jain (2008) have a similar model with two time periods where consumers in the first time period form expectations about the future popularity of a product and make their purchase decisions accordingly. Diamond and Fudenberg (1989), Stokey (1981) and several others solve similar multi-period games using the rational expectations concept.

⁶The rational expectations concept assumes strong rationality on the part of the agent (Muth 1961), i.e., an agent can correctly expect the future equilibrium path in a multi-period game and acts on these expectations, so that this equilibrium indeed arises. There is a debate in the literature around this strong rationality assumption. Experimentally, Amaldoss and Jain (2005), Sunder (1995) and others show that markets can converge to predictions of rational expectations equilibria, even though Garner (1982), Smith et al. (1988) and others show that expectations at the individual level may not always correspond to theoretically predicted individual-level rational expectations. Liu and van Ryzin (2007) show that adaptive learning mechanisms can also lead to an equilibrium identical to the rational expectations equilibrium.

We first consider the case of deterministic demand, with both low demand (demand is lower than capacity) and high demand (demand is higher than capacity) and then consider the case of uncertain demand. In all cases, the firms and consumers know the values of all parameters. The above is a general description of the model. We provide specific details for each case as appropriate.

4 Deterministic Demand

In this section, we explore the two strategies of the firms when demand is deterministic: (i) the firms can sell through their own channels and have the option of offering different prices in the two periods of sale, and (ii) the firms can sell opaque products in the second period, after sales in the first period have concluded. We consider two possible scenarios for each strategy: (i) low-demand scenario ($J < K$), and (ii) high-demand scenario ($J > K$). The deterministic demand model helps us gain insights into the players' decisions when demand is lower/higher than capacity and it serves as a logical building block for the more complex model with demand uncertainty (Section 5).

4.1 Selling through firms' direct channels

When firms sell only through their direct channels and demand is deterministic, we find that each firm charges the same price in the two periods.⁷ Intuitively, if the firms were to try and charge a higher price in the first period and a lower price in the second period, the consumers, being strategic and having full information about demand, would wait to buy products until the prices were lowered. (In case of the uncertain demand, we will see that this result changes.) We provide the detailed analysis in the appendix in Section A1.1 and only provide the main insights here.

The prices charged and market covered depend on whether demand is low or high and on the value of the quantity V/t . In the case of low demand ($J < K$), when V/t is small ($1/2 \leq V/t < 1$), the firms act as local monopolies and each firm prices at $V/2$ to cover the length $V/(2t)$ on its side of the Hotelling line, which is less than half. As V/t increases, the market covered increases, until the firms start to compete — specifically, when $1 \leq V/t < 3/2$, each firm prices at $V - t/2$, covers half the market and sells $J/2 < K/2$ tickets; and when $V/t \geq 3/2$, each firm prices at t , covers half

⁷Prices would not be identical across periods if consumers discounted their second period utility. However, the *discount-adjusted* prices would be identical across periods. Introducing discounting makes the analysis more tedious, while all our insights continue to hold.

the market and sells $J/2 < K/2$ tickets.

In the case of high demand ($J > K$), the same pattern of coverage which increases with V/t holds. However, in this case, the firms never get into competition and always act as local monopolies because each firm can sell a maximum of $K/2$ tickets. Hence, if V/t is large enough (specifically, $V/t \geq K/J$), the firms charge a price of $V - K/(2J)t$ and cover exactly $K/(2J) < 1/2$ on their side of the market.

4.2 Opaque selling

As we described in the introduction, firms often sell products/services through opaque intermediaries very close to the terminal time, i.e., after consumers have bought in the transparent channel but the firms still have some inventory of products left over. In this case, we assume that the firms declare that they might sell through an opaque channel late in the selling horizon (e.g., airlines selling opaque tickets through Hotwire declare this by having their names listed on the web site hotwire.com). Firms will engage in opaque selling only if there are products that are left unsold through their own direct channels. More formally, the game proceeds in the following manner.

1. Every consumer is endowed with expectations, given by γ_A^e and γ_B^e , about the probabilities that the product they will obtain from the opaque seller will be from firm A or firm B, respectively. In the first period, firms A and B set prices p_A and p_B in the direct-to-consumers channel (transparent channel).
2. Given prices p_A and p_B and his expectations about availability in the opaque channel from both firms, every consumer makes a purchase decision in the transparent channel.
3. After the transparent channel sales are over, the leftover products are made available to the opaque intermediary I by both firms. The opaque intermediary sets a price p_I for the opaque product. Consumers who did not purchase in the transparent channel now make their purchase decisions in the opaque channel. A consumer may not obtain an opaque product if the number of leftover products is less than the number of consumers who are willing to purchase at price p_I . We denote the probability that the consumer can obtain an opaque product by β so that each consumer desiring a product is equally likely to obtain it.
4. The opaque intermediary keeps a fraction $1 - \delta$ of the revenues from the opaque channel. The remaining fraction δ is distributed between firms A and B in proportion to the products sold for

each firm.⁸

Consequently, for the consumer at x_A who is indifferent between buying from firm A and buying in the opaque channel, the following condition holds in equilibrium:

$$V - p_A - tx_A = \beta(V - p_I - \gamma_A^e tx_A - \gamma_B^e t(1 - x_A)).$$

We now characterize the equilibria under low-demand and high-demand cases. Without loss of generality, we focus on $\delta = 1$; any $\delta \in [0, 1]$ yields the same insights. We provide the detailed analysis in the appendix in Section A1.2 and provide the main insights here.

First, consider the case of low demand ($J < K$). To solve for the equilibrium, we use backward induction. We start with the opaque intermediary's problem in the second period — the intermediary has access to consumers on the line segment $[x_A, x_B]$. (x_A and x_B are arbitrary and the intermediary can choose to cover this market fully or partially). This determines the actual number of leftover tickets from each firm and, therefore, the realized probability of obtaining tickets from firm A in the opaque channel through the expectation function as $\gamma_A = \frac{K/2 - x_A J}{K/2 - x_A J + K/2 - (1 - x_B) J}$. We solve for the equilibrium through the following procedure.

The opaque intermediary sets the price p_I it will charge to sell the leftover tickets from the two firms. Since this is the case of low demand, there is no shortage of tickets, so that $\beta = 1$. Based on the above, the intermediary sets the price p_I as function of location of the indifferent customers x_A , x_B , and first-period prices p_A^1 and p_B^1 . In the first period, both firms set their prices, based on the prices set by the opaque intermediary under all second-period possibilities.

As we demonstrate in the Section A1 in the appendix, in equilibrium, the rational expectations of the fraction of opaque products from each firm are $\gamma_A^e = \gamma_B^e = 1/2$. This implies that, if the firms have equal capacities, then it is rational for consumers to expect that in the opaque channel half of the products come from one firm and the other half come from the other.

This result further implies that, in equilibrium, at price p_I the ex ante utility of each consumer from purchasing in the opaque channel is $V - p_I - t/2$, so that the intermediary charges a price of $p_I = V - t/2$ to make every consumer's ex ante surplus equal to zero. The benefit from the opaque channel lies in the fact that by masking the identity of the product, the ex ante surplus from

⁸This revenue sharing contract with opaque intermediaries is consistent with observations and industry practice (see Elkind 1999, Priceline 2006, Phillips 2005).

purchasing an opaque ticket is zero for all consumers in $[x_A, x_B]$ and hence they purchase tickets. Recall that in the transparent channel, the surplus from purchasing a ticket is zero only for the customers at x_A and x_B who are indifferent between purchasing from firms A and B, respectively. For every other customer in $[x_A, x_B]$ the surplus is negative and they do not purchase tickets in the transparent channel.

The price charged by the opaque intermediary, $p_I = V - t/2$, is always lower than the price charged in the transparent channel in the first period, which is equal to $p_A = p_B = V/2$ for $1/2 \leq V/t \leq 1$. Hence, the opaque channel serves as a “clean up” mechanism to increase profits by selling leftover tickets at a low price to consumers who would otherwise not have purchased at all, and masking the identity of the tickets enables this. When $V/t \geq 1$, there are no opaque sales because each firm covers half the market directly with the transparent sales. Hence, firms use opaque selling for the range $(1/2 \leq V/t < 1)$.

In the case of high demand ($J > K$), the same insights hold in equilibrium, $\gamma_A = \gamma_B = 1/2$, $p_I = V - t/2$ and the ex ante expected surplus of consumers in the opaque channel is zero because of masking the product identity. However, in this case, one difference is that as V/t increases and firms increase their market coverage, they stock out when they cover a length of $K/(2J)$, which occurs for $V/t \geq K/J$. Hence, firms use opaque selling for a smaller range $(1/2 \leq V/t < K/J)$. The second difference is that when firms do use opaque sales, all the consumers in $[x_A, x_B]$ are willing to purchase opaque tickets, but only a fraction $\beta = (K - (V/t)J)/(J - (V/t)J) < 1$ actually obtain opaque tickets.

4.3 Comparison of strategies under deterministic demand

In both high- and low-demand scenarios, the opaque channel acts as a “clean up” mechanism to dispose of unsold products by selling them to consumers who would otherwise not have purchased at all. Hence, if opaque selling is used (when the market is not fully covered by the transparent channels), it will strictly improve firm profits.

Figure 1 depicts the optimal strategies for the firms given different values of consumer valuations (the ratio V/t) and inventory availability relative to demand (the ratio K/J). Under both high and low demand, firms sell products through the opaque channel only if V/t is small enough because in this case the firms do not cover the full market in the transparent channels and use opacity as a

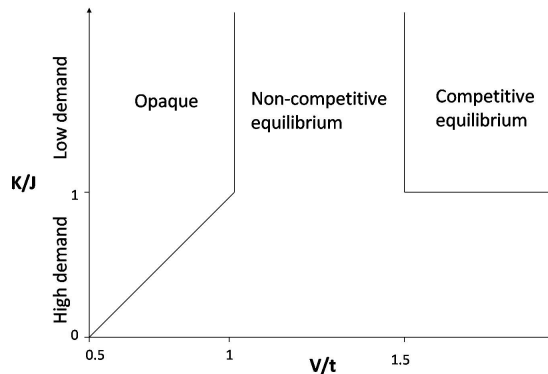


Figure 1: Strategy space for different valuations and capacity/demand scenarios.

mechanism to dispose of unsold products. As the ratio V/t increases above a threshold, the firms have the option of using an opaque channel, but price in the transparent channels to cover the market anyway, and do not need to resort to selling cheaper opaque products. Figure 1 also shows that if demand is high, opaque sales will be seen less frequently (for a smaller range of V/t), than if demand is low. This is consistent with the notion that the opaque channel is used to dispose of distressed inventory (Harrison 2006).

5 Uncertain Demand

Uncertainty in demand volume is a pervasive feature in many industries. In the travel industry, for example, firms usually can estimate the demand distribution for a given airline route or hotel using historical records but the precision of such estimates is quite limited (see Talluri and van Ryzin 2004). As the departure date approaches, the firms can improve the forecast and therefore project with a higher degree of confidence whether the demand for the route is higher or lower than the available capacity. Building on the analysis in previous sections, this section extends our model to incorporate demand uncertainty.

Due to the presence of demand uncertainty, consumers cannot always credibly adopt a strategy of waiting in the early stages of the game because market demand could be high and products could be unavailable later. However, a consumer can form expectations about future availability and buy early if the expected utility from doing so is higher than the expected utility from waiting. These dynamics capture the practical consideration that not all consumers wait for last-minute discounts

and allow us to derive several insights beyond the model with deterministic demand.

The specifications of the model remain the same, except that the level of demand is now variable. We assume that, with probability α the total number of consumers in the market is $H(> K)$ and, with probability $1 - \alpha$ the total number of consumers in the market is $L(< K)$. As before, each firm has capacity $K/2$. The parameters α, L, H and K are common knowledge. The selling horizon is divided into two periods. In the first period, the firms and the consumers know the distribution of demand, but do not know the state of nature (whether demand is H or L). At the end of the first period, but before the second period begins, the realization of demand is observed by the firms and the consumers.⁹

We assume that, in any selling period, if the number of consumers who are willing to buy a product is higher than the capacity available, products are allocated randomly to the consumers. In other words, if a certain number of consumers desire products at the announced price but the number of products available is lower than the number of products demanded (which can be the case if demand is high), it is possible that consumers with a lower expected (but positive) surplus obtain products at the expense of consumers with a higher expected surplus. In the following sections, we analyze the two strategies of selling through the firms' direct channels ("Last-Minute Sales Strategy" or LMSS) and opaque selling ("Opaque Sales Strategy" or OpSS).

5.1 Selling through firms' direct channels

The following is the order of events in the game when firms adopt a LMSS.

1. In the first period, firm A prices its products at p_A^1 and firm B prices its products at p_B^1 and both firms declare that there might be last-minute sales.
2. All consumers form expectations about the number of consumers purchasing in each period (and therefore the corresponding future availability), and strategically make or postpone their purchase.
2. At the end of period 1 and before period 2 begins, demand uncertainty is fully resolved. The level of demand is determined as H or L , and is observed by both the firms and the consumers. The firms then set their prices (firm A sets price p_A^{2L} if demand is low and p_A^{2H} if demand is high, and similarly for firm B).
3. The consumers who postponed their purchase in the first period decide to purchase or not in

⁹In practice, some residual uncertainty in demand would remain.

the second period at the announced prices.

The equilibrium solution LMSS strategy is provided in the following proposition.

Proposition 5.1 *When the firms sell products through their own channels, the following equilibrium always exists: In the first period both firms set prices to cover $x_A = 1 - x_B = K/(2H)$ of the market. If demand is high, no products are sold in the second period since the firms stock out in the first period. If demand is low, consumers located between $x_A = K/(2H)$ and $x_B = 1 - K/(2H)$ buy in the second period. The first-period and second-period prices are as follow:*

$\frac{V}{t}$	First-period prices ($p_A^1 = p_B^1$)	Second-period prices when demand is low ($p_A^{2L} = p_B^{2L}$)
$\frac{1}{2} \leq \frac{V}{t} < 1 - \frac{K}{2H}$	$(\frac{1+\alpha}{2})(V - \frac{K}{2H}t)$	$\frac{1}{2}(V - \frac{K}{2H}t)$
$1 - \frac{K}{2H} \leq \frac{V}{t} < \frac{3}{2} - \frac{K}{H}$	$\alpha(V - \frac{K}{2H}t) + (1 - \alpha)(V - \frac{t}{2})$	$V - \frac{t}{2}$
$\frac{V}{t} \geq \frac{3}{2} - \frac{K}{H}$	$\alpha(V - \frac{K}{2H}t) + (1 - \alpha)t(1 - \frac{K}{H})$	$t(1 - \frac{K}{H})$

In the equilibrium, all consumers who attempt to buy a product in the first period obtain a product, but pay the high price p_A^1 or p_B^1 as in the proposition above. Note that the first-period prices are such that the customer who is indifferent between purchasing in the first period from A and waiting for the second period to purchase from A has a positive (and, of course, equal) surplus from purchasing the ticket in either period. If demand is high, firm A sells to $H(K/2H) = K/2$ consumers in the first period and thus exhausts its capacity so there are no products sold in the second period through last-minute sales. If demand is low, firm A sells to $K/(2H) \cdot L (< K/2)$ consumers in the first period, and will have some products left over to sell in the second period. Moreover, there are more of these leftover products than the number of unserved consumers in the market in the second period. Therefore, the consumers who waited for the “last-minute” products obtain them at lower prices only if demand is lower than capacity. The situation is symmetric for firm B.

To summarize, in the first period all consumers with “high brand preference” (located in the interval $[0, K/(2H)]$) purchase at a high price from firm A. If there are any leftover products, the consumers with “low brand preference” (located in the interval $[K/(2H), 1/2]$) purchase from firm A during the last-minute sales at lower prices. If there are no leftover products, there are no sales in the second period. In effect, the firms are separating out consumers who are ready to pay a

higher price under the threat of stockout and making most of their profits from the higher prices charged to the high-preference consumers in the first period.

5.2 Opaque selling

The following is the order of events in the game when the firms adopt an opaque sales strategy.

1. Every consumer is endowed with expectations about the probabilities that the product they will obtain from the opaque seller in the second period will be from firm A or firm B for both high and low demand realizations. In the first period, firm A prices its products at p_A^1 and firm B prices its products at p_B^1 and both firms declare intention of sales through an opaque channel.
2. Consumers strategically purchase or postpone purchasing based on expectations about availability in the second period (β) and about the probability with which they will obtain the opaque product from each firm $(\gamma_A^{H,e}, \gamma_B^{H,e}, \gamma_A^{L,e}, \gamma_B^{L,e})$.
3. At the end of period 1 and before period 2 begins, demand uncertainty is resolved, the level of demand is determined as H or L and is observed by the firm and the consumers. The leftover products, if any, are made available to the opaque intermediary I , who then sets a price p_I^H if the demand realization is H or a price p_I^L if the demand realization is L .
4. Consumers who have not purchased in the transparent channel now make their purchasing decision in the opaque channel. The intermediary commits to a credible opaque strategy and sells products from both firms at price p_I . For every product sold, it keeps a fraction $1 - \delta$ of the revenue accrued from the opaque channel and distributes the remaining fraction δ to firm A or B whose product it sold.

Let the locations of consumers indifferent between buying in the first period and buying opaque tickets in the second period be x_A and x_B . Then we have the following two cases based on the realization of demand.

1. If the level of demand is low, then leftover products for firm A must be $l_A^L = \max\{K/2 - x_A L, 0\}$ and leftover products for firm B must be $l_B^L = \max\{K/2 - (1 - x_B)L, 0\}$. In equilibrium, then the customers will obtain a product from firm A with probability $\gamma_A^L = l_A^L / (l_A^L + l_B^L)$ and from B with probability $\gamma_B^L = l_B^L / (l_A^L + l_B^L)$. (If there are no left over products the probabilities are set to zero).
2. If the level of demand is high, then the leftover for firm A is $l_A^H = \max\{K/2 - x_A H, 0\}$ and the leftover for firm B is $l_B^H = \max\{K/2 - (1 - x_B)H, 0\}$. In equilibrium, if the opaque channel exists,

the customers will obtain a product from firm A with probability $\gamma_A^H = l_A^H / (l_A^H + l_B^H)$ and from firm B with probability $\gamma_B^H = l_B^H / (l_B^H + l_A^H)$, if defined.

Based on the demand realization and availability, opaque intermediary fixes opaque ticket prices. Based on the second period decision of the opaque intermediary, the firms optimally fix the first-period prices. The equilibrium of the above game is characterized in Proposition 5.2. (To keep results simple, we present the case with $\delta = 1$. The analysis for any $\delta \in [0, 1]$ yields the same insights).

Proposition 5.2 *When the firms sell products through the opaque intermediary, the following rational expectations equilibrium exists:*

$\frac{V}{t}$	First-period prices ($p_A = p_B$)	Opaque prices (p_I^L, p_I^H)
$\frac{1}{2} \leq \frac{V}{t} < \frac{K}{H}$	$\frac{V}{2}$	$V - \frac{t}{2}, V - \frac{t}{2}$
$\frac{K}{H} \leq \frac{V}{t} < \frac{K}{H} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}$	$V - \frac{K}{2H}t$	$V - \frac{t}{2}, -$
$\frac{K}{H} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L} \leq \frac{V}{t} < 1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}$	$\frac{V}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{4L}t$	$V - \frac{t}{2}, -$
$1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L} \leq \frac{V}{t} < \frac{3}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}$	$V - \frac{t}{2}$	$-, -$
$\frac{V}{t} \geq \frac{3}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}$	$\left(1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}\right) t$	$-, -$

Note that in equilibrium, consumers have the same probability of obtaining a ticket from either firm in the opaque market (if it exists), irrespective of whether the demand realization is high or low. This also implies that the consumer who is indifferent between purchasing a transparent ticket in the first period and an opaque ticket in the second period has the ex ante expected surplus of zero.

There is, however, a critical difference from the deterministic demand case. Under deterministic demand consumers know the state of demand, and the opaque channel is primarily a clearance mechanism used when the entire market could not be covered by the firms through transparent prices. In contrast, when demand is uncertain, there is an additional factor — since the consumers do not know the state of demand in the first period, they face the possibility of the firms stocking out if demand is high. In other words, if a consumer has non-negative utility in the first period at the price offered by a firm, then he will purchase the product, inferring that he might not obtain it at all later if the demand turns out to be high. This consideration allows the firms to charge higher prices in the first period as compared to the prices in the deterministic case. Consequently,

if demand is low, only a few products will be sold in the first period. However, in this eventuality, the firms can use the opaque channel to “clean up” the leftover products if any. Selling to a smaller population at higher prices in the first period helps the firms to increase the expected profit across two periods.

The above argument naturally leads to the interesting insight that, as the probability of high demand increases, the firm will rely more and more on the opaque channel (the formal proof is straightforward and is therefore omitted). The reason is that if there is a greater chance that demand is high, the “competition for products” among consumers in the first period will be higher, which means that the firms will be able to raise the first-period prices. If demand turns out to be high, the firms will exhaust their capacities. On the other hand, even if demand turns out to be low, there will still be some consumers left in the market because of high first-period prices. Consequently, there will be some leftover products, and the firms will sell them through the opaque channel.

5.3 Opaque Sales vs. Last-Minute Sales

We saw in the previous two sections that both LMSS and OpSS can increase firms’ profits. In this section, we can use our analytical results to answer the following question: When should firms employ LMSS versus OpSS? To illustrate our insights better, we compare the profits of the firms for these two strategies for a representative set of parameter values ($\alpha = 1/2, K = 1, L = 1/2, H = 3/2, t = 1$) in Figure 2. We use numerical and graphical illustration for expositional simplicity. Qualitatively, the results do not change for other values of the parameters. From Figure 2, we obtain the insight that if V is low, the profits from OpSS are higher than the profits from LMSS which is consistent with the intuition stated above. As V increases, the profits from OpSS flatten out, while the profits from LMSS keep increasing. Above a certain threshold for V , LMSS profits become higher than OpSS profits.

To see why the above result holds, note that under LMSS the bulk of a firm’s profits comes from products sold in the first period to the consumers that are closer to the firm on the Hotelling line. If the valuation for products in the market is high (i.e., V is high), the first-period prices are high. However, if the valuation for products is low, the first-period prices are low, the second-period prices are even lower, and hence profits from LMSS are low. In OpSS, on the other hand, the

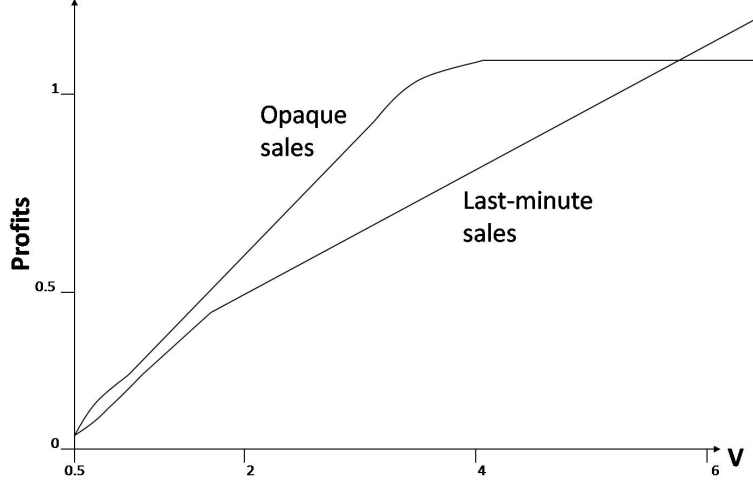


Figure 2: Profits accrued by a firm (π_A, π_B) under uncertain demand, when the firm employs the last-minute sales strategy and the opaque sales strategy. For the figure, we use $\alpha = 1/2, K = 1, L = 1/2, H = 3/2, t = 1, \delta = 1$.

first-period prices are higher than in LMSS for low V because each firm is choosing to cover only a small portion of the market in the transparent channel by charging a price that makes the surplus of the marginal consumer equal to zero and covers the rest using the opaque channel. Moreover, note that the second-period prices in the opaque channel (if opaque sales are present) are equal to or higher than the second-period prices for LMSS (except when $V/t < 1 - K/(2H)$). As in the deterministic demand case, by masking the identity of the product the opaque intermediary can sustain relatively higher second-period prices. Hence, for low V/t , OpSS yields higher profits.

As V increases, the revenue from LMSS increases faster, because the firms are able to separate out the consumers with a high preference for a particular firm and charge these consumers higher prices even if demand is low. In OpSS, on the other hand, prices are such that the firms cover a large portion of the market at lower prices if demand is low. In fact, if V is high enough, the firms are in a competitive equilibrium in the first period itself under OpSS when demand is low, so that prices are very low. (In Figure 2, this is the region where the OpSS profits level off.) Hence, when V is high, LMSS yields higher profits because it allows the firms to “milk” the high-preference consumers in the first period, even if it has to charge lower prices in the second period when demand turns out to be low.

What happens as the probability of high demand realization increases? As we discussed earlier

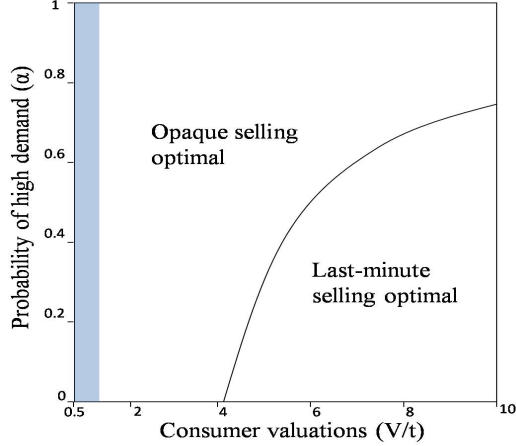


Figure 3: Strategies the firms should adopt for different consumer valuations (V/t) and different probabilities of high demand (α). The shaded area denotes the region where the opaque selling market exists for deterministic low demand (i.e., when $V/t \leq 1$). For the figure, we use $K = 1, L = 1/2, H = 3/2, t = 1, \delta = 1$. Qualitatively, the results do not change for other values of the parameters.

for OpSS, as the probability of high demand realization increases, consumers are under a higher threat of stockout in the first period. Thus, many more consumers prefer to buy in the first period and therefore the firms increase prices. In other words, not only is there a higher chance that demand is high, the prices are also high. If demand turns out to be low, the first-period sales suffer, but the leftover capacity is cleared through the opaque channel. Over the two periods, expected profits increase.

In LMSS, however, the firms charge a first-period price that increases at a slower rate with an increase in the probability of high demand. Further, consumers with low firm preferences buy only if demand is low, which now happens with lower probability. Hence, even though expected profits increase (because there is a higher chance of high demand) the increase is slower than in OpSS. Figure 3 summarizes the comparison between the opaque strategy and the last-minute direct sales strategy for various probabilities of high demand ($\alpha \in [0, 1]$ on the y -axis) and consumer valuations (V on the x -axis).

Results for subgame perfect Nash equilibrium:¹⁰ In the rational expectations equilibrium that we have derived above, consumers are assumed to rationally predict the full equilibrium path of

¹⁰We thank an anonymous reviewer and the Associate Editor for suggesting this line of analysis.

the dynamic game, i.e., in the beginning of the first period, they develop correct *point* expectations regarding ticket availability (in equilibrium) from each firm in the second period. In contrast, in a subgame perfect Nash equilibrium, expectations on ticket availability from each firm are developed at the end of the first period as a *function* of the prices charged by the firms in the first period.¹¹ The formulation using the subgame perfect Nash Equilibrium concept is algebraically intractable for our model, so we conduct a numerical study to analyze this equilibrium. We find that the results are qualitatively the same, i.e., as the probability of high demand (α) increases, opaque selling is preferred for larger values of consumer valuation of the product (V/t). Details on the numerical analysis are available in Section A2.4 in the appendix.

5.4 Mixed-Strategy Equilibria¹²

So far, we modeled a problem with two firms that announce their strategies in the first period (OpSS or LMSS), implicitly assuming that both firms select the same strategy. This is a natural assumption since if one firm does not select the opaque selling strategy, the other firm cannot implement it alone. One way to circumvent this difficulty and allow firms to pick different selling strategies is to consider mixed strategies — both firms randomize between the strategies under consideration and the relevant strategy is chosen in the second period, with some probability.

Suppose firm A plays OpSS with probability $q_A \in [0, 1]$ and LMSS with probability $1 - q_A$, and firm B plays OpSS with probability $q_B \in [0, 1]$ and LMSS with probability $1 - q_B$. Let π_A^1 denote firm A's profit in the first period, $\pi_A^{O,2}$ denote firm A's expected profit in the second period when opaque tickets are sold in the second period (so that total expected profit is $\pi_A^O = \pi_A^1 + \pi_A^{O,2}$) and $\pi_A^{T,2}$ denote firm A's expected profit in the second period when transparent tickets are sold in the second period (so that total expected profit is $\pi_A^T = \pi_A^1 + \pi_A^{T,2}$). If firm A plays OpSS, with probability q_B opaque tickets will be sold in the second period (if firm B also plays OpSS) and with probability $1 - q_B$ transparent tickets will be sold in the second period (if firm B plays LMSS), and firm A's expected profit will be equal to $q_B \pi_A^O + (1 - q_B) \pi_A^T$. If firm A plays LMSS, only transparent tickets can be sold in the second period and its expected profit will be π_A^T . Hence, for firm A, we

¹¹Both rational expectations equilibrium and subgame perfect Nash equilibrium have been used as solution concepts for dynamic games in the extant literature. For our model, invoking the rational expectations assumption affords analytical tractability, while we also confirm numerically that our results remain qualitatively the same in a subgame perfect Nash equilibrium.

¹²We thank the Associate Editor for suggesting this line of analysis.

obtain the condition for mixing between OpSS and LMSS as:

Expected Profit from OpSS = Expected Profit from LMSS

$$\begin{aligned}
q_B \pi_A^O + (1 - q_B) \pi_A^T &= \pi_A^T \\
\Rightarrow \pi_A^O &= \pi_A^T & \Rightarrow \pi_A^1 + \pi_A^{O,2} &= \pi_A^1 + \pi_A^{T,2} \\
&& \Rightarrow \pi_A^{O,2} &= \pi_A^{T,2}
\end{aligned} \tag{1}$$

Similarly, for firm B, we obtain $\pi_B^{O,2} = \pi_B^{T,2}$. The above result provides the interesting insight that each firm will adopt a mixed strategy when the profits from opaque sales and transparent sales are equal in the second period (condition (1)). We obtain this condition due to the fact that one firm alone cannot implement the opaque selling strategy. We provide the rest of the analysis in Section A2.2 in the appendix and provide two salient insights here.

First, we obtain the mixing probabilities for both firms as $q_A = q_B = \sqrt{\alpha(H - K)/((1 - \alpha)H + \alpha K)}$. This expression implies that, as the probability of high demand (given by α) increases, the firms choose the opaque-selling strategy with higher probability. This is in line with the insight from Figure 3, which shows that firms prefer opaque selling as α increases. Furthermore, as capacity becomes more constrained (H increases or, alternatively, K decreases) the firms choose the opaque-selling strategy with higher probability. Note, however, that the mixing probabilities do not depend on L since $L < K$ and the market is always unconstrained in the low-demand state. Second, when the firms mix between opaque and transparent sales, the prices charged for transparent tickets in the second period are equal under high and low demand (and both are equal to $\delta(V - t/2)$). This is derived indirectly from condition (1) which states that, in both high- and low-demand cases, the second-period profits from transparent sales and opaque sales are equal. Under opaque selling, in turn, prices are equal to $V - t/2$ in both the high- and low-demand cases (because, for symmetric firms, tickets come from either firm with equal probability). This leads to the result that the prices charged for transparent tickets in the second period are equal in high- and low-demand cases.

6 Asymmetric Firms With Unequal Capacities¹³

So far, we assumed the two firms to be symmetric in all respects which leads to “perfect masking” in the opaque channel, i.e., all consumers obtain a ticket from either firm with equal probability. If firms are asymmetric, with one firm having larger capacity than the other, then consumers will expect the probability that an opaque ticket is from the larger firm to be higher. In this section, we extend the basic model to investigate the implications of the inability to achieve perfect masking in the opaque channel. Broadly, we find that using opaque sales still helps firms to increase their profits. However, the pricing power of the opaque intermediary and of the two firms indeed diminishes, and the firm with a larger capacity is at a greater disadvantage.

6.1 Opaque Selling with Deterministic Demand

Consider first the case of deterministic low demand, i.e., the total capacity of the firms is more than the total demand in the market. Here we generalize the model in Section 4.2 by assuming that the capacities of firms A and B are given by K_A and K_B , respectively, and $K_B \geq K_A$. Our solution approach is also the same as in Section 4.2. We provide the details in Section A3.1 of the appendix and discuss some salient insights here.

Since firm A has smaller capacity, consumers rationally expect that the probability that an opaque ticket is from firm A is less than half, i.e., $\gamma_A^e < 1/2$.¹⁴ This implies that, in the opaque channel, the surplus for a consumer located at x , given by $V - p_I - \gamma_A^e tx - (1 - \gamma_A^e)t(1 - x)$, increases with x . In other words, the leftmost consumer on the Hotelling line who purchases an opaque ticket will have the lowest surplus and all other consumers will have surplus greater than the surplus of this consumer. The opaque intermediary prices such that the consumer at x_A , who is indifferent between purchasing a transparent ticket from firm A and an opaque ticket, has zero surplus. (This gives us the equalities $V - p_A - tx_A = V - p_I - \gamma_A^e tx_A - (1 - \gamma_A^e)t(1 - x_A) = 0$, which also implies that firm A prices its transparent ticket to make the surplus of its marginal consumer equal to zero.)

The above arguments also imply that all other consumers closer to firm B will have positive

¹³We thank an anonymous reviewer for suggesting this analysis.

¹⁴Note that we also check that the case when the firm with the larger capacity has fewer tickets in the opaque market (i.e., the probability of obtaining an opaque ticket from the larger-capacity firm is $< 1/2$) is off the equilibrium path.

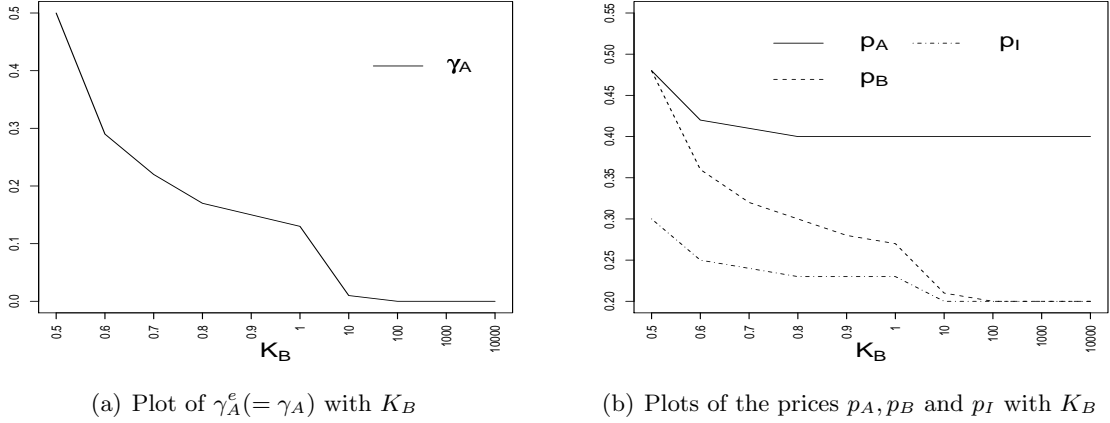


Figure 4: Trends in equilibrium expectations of product availability and prices in the opaque channel as the capacity of firm B increases. Note the abrupt change in scale on the x -axis. For the plots above, the values of the other parameters are chosen as $V = 0.8, t = 1, J = 1, \delta = 1$ and $K_A = 0.5$. Other choices of the parameters yield qualitatively the same plots.

surplus in the opaque channel. Now, consider the consumer at x_B who is indifferent between purchasing a transparent ticket from firm B and an opaque ticket. This consumer prefers a ticket from firm B and there is a higher chance that the opaque ticket is from firm B. Hence, this consumer has a positive surplus in the opaque channel, i.e., $V - p_I - \gamma_A^e t x_B - (1 - \gamma_A^e) t (1 - x_B) > 0$ since $x_B > x_A$. Hence, firm B will charge a price p_B such that $V - p_B - t(1 - x_B) = V - p_I - \gamma_A^e t x_B - (1 - \gamma_A^e) t (1 - x_B) > 0$. This implies that, unlike firm A, firm B cannot price in the first period to extract full surplus from its marginal consumer and will therefore charge a lower price for its transparent ticket than firm A.

The above discussion also clarifies why “imperfect masking” in the asymmetric-capacities case reduces the profits in opaque selling as compared to the symmetric-capacities case. The reason is that under imperfect masking the larger-capacity firm cannot extract full surplus from its indifferent consumer. In contrast, recall that with perfect masking in the symmetric-capacities case, both firms were able to extract full surplus from their indifferent consumers. Hence, in opaque selling with asymmetric capacities, the larger firm has to leave some “surplus on the table” for its indifferent consumer, while when the firms have symmetric capacities, neither firm leaves any surplus on the table for its indifferent consumer.

Figure 4(a) shows how the probability of obtaining a ticket from firm A in the opaque channel varies with the capacity of firm B. (Other choices of the parameters yield qualitatively the same

insights. Notably, if V/t is higher, the prices are higher but follow the same patterns.). When firms have equal capacity, the probability is equal to half, and as firm B's capacity increases, the probability goes down. At the extreme, as firm B's capacity becomes much larger than firm A's capacity, this probability tends to zero. Figure 4(b) shows the corresponding prices. When the capacities are equal, firms A and B charge equal prices. As firm B's capacity increases and consumers become more certain about the identity of the product in the opaque channel, all prices decline. Firm B's price is always lower than that of firm A, and as its capacity becomes much larger than the capacity of firm A, its price approaches the opaque channel price. Hence, in equilibrium, the firm with the larger capacity is forced to price lower than the firm with the lower capacity. Furthermore, even though the larger firm has a greater total sales volume, its profit is lower than that of the firm with the lower capacity due to lower prices.

Other specifications, such as when the total capacity in the market is held fixed but the percentage of capacity held by one firm is increased, yield similar insights. The case of asymmetric firms when demand is deterministic and high (i.e., the total demand in the market is more than the combined capacity of the two firms) also yields similar insights. Due to space considerations, we do not discuss these cases and proceed directly to the more interesting scenario when demand is uncertain.

6.2 Opaque Selling with Uncertain Demand

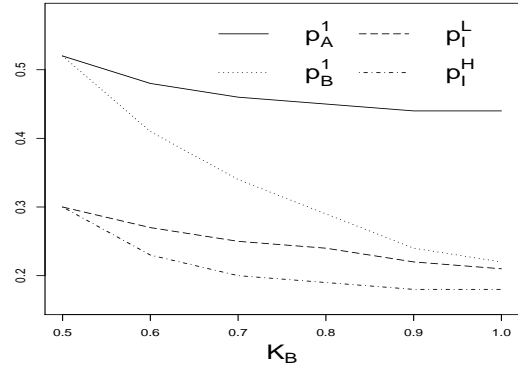
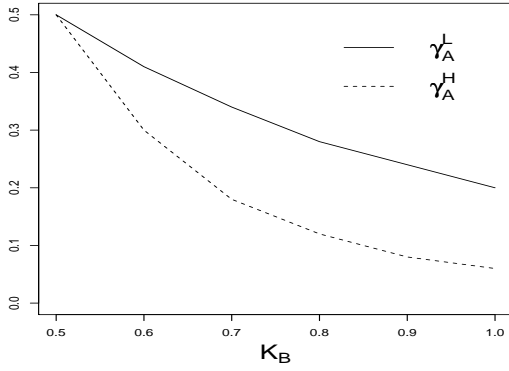
We now generalize the model in Section 5.2 by assuming that the capacities of firms A and B are given by K_A and K_B , respectively, and $K_B \geq K_A$. The details of the solution are in Section A3.2 in the appendix.

As in the deterministic-demand case, the masking of the opaque ticket's identity is imperfect in the uncertain-demand case, i.e., consumers know that there is a higher probability that the opaque ticket is from the larger-capacity firm (i.e., firm B), which hurts the profits in the opaque channel (as compared to profits with perfect masking in the symmetric-capacities case). Overall, this imperfect masking leads to lower prices charged by the intermediary in both the high- and low-demand opaque channels and, in turn, for both firms in the first period, with firm B charging a price lower than firm A. Furthermore, unlike in the symmetric case, consumers can have different expectations of product availability in the high- and low-demand opaque channels, and this can

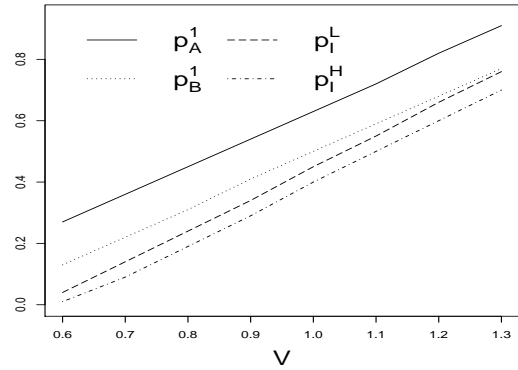
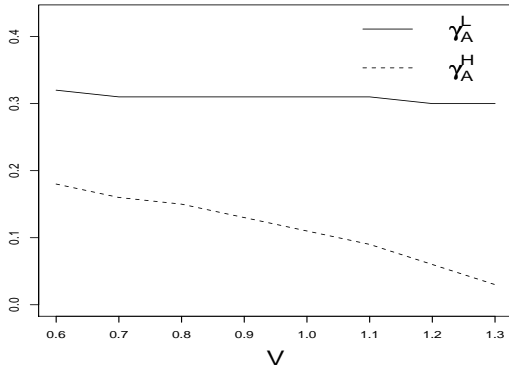
lead to different prices of the opaque product in the two cases.

We now develop some salient insights with the help of an illustrative example. Using other values for these parameters yields qualitatively similar insights. Note from Figure 5(a) that as the capacity of firm B increases, the probability that an opaque ticket is from firm A decreases for both the high-demand and low-demand opaque channels. Moreover, this probability is always lower in the high-demand opaque channel. This is because when demand is high both firms sell more tickets (compared to the case when demand is low) in the first period and, therefore, the proportion of tickets in the opaque channel is larger (compared to the case when demand is low) for the firm with higher capacity. In other words, the probability that an opaque ticket (if available) is from firm B (the larger-capacity firm) is higher when demand is high than when demand is low, the masking of the opaque product is less perfect, and the consumers know more clearly where the opaque ticket is from. This implies that, ironically, the opaque intermediary will charge a lower price for the opaque product in the high-demand opaque channel than in the low-demand opaque channel. From Figure 5(b) we see that this is indeed the case. We also see that the first-period prices for both firms are higher than the second-period opaque channel prices, with firm B charging a lower price than firm A. (Note that, in Figure 5, the relationships between the probabilities of availability discussed above hold in plots (c) and (e), and the relationships among prices hold in plots (d) and (f).)

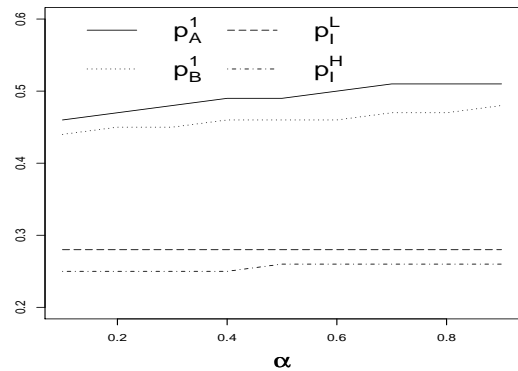
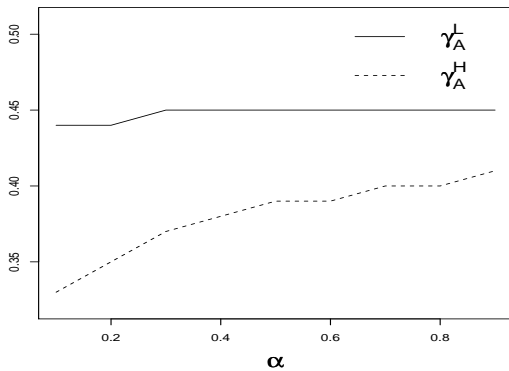
As the consumers' valuation for tickets (V) increases, keeping other parameters constant, all prices increase (as shown in Figure 5(d)) and firms A and B cover a larger part of the Hotelling line through transparent sales. Hence, both firms sell a larger number of tickets in the first period, the overflow of tickets into the second period decreases for both firms and the proportion of tickets in the opaque channel increases for the larger firm with increasing V . Hence, as V increases, the probability that an opaque ticket is from firm A decreases, and this decrease is sharper in the case of high demand. This is shown in Figure 5(c). Finally, as the probability of high demand (α) increases, customers are under a larger threat of stockout in the second period. Hence, both firms charge higher transparent channel prices in the first period (Figure 5(f)) and cover a smaller market. As firms sell a smaller number of tickets in the first period, the overflow of tickets into the second period increases for both firms and the proportion of tickets in the opaque channel decreases for the larger firm with increasing α for both high- and low-demand scenarios. Hence, as α increases, masking of the opaque product improves (Figure 5(e)), which leads to increasing



(a) Plots of the probabilities γ_A^H and γ_A^L with K_B ; $V = 0.8, \alpha = 0.5$ (b) Plots of the prices p_A^1, p_B^1, p_I^H and p_I^L with K_B ; $V = 0.8, \alpha = 0.5$



(c) Plots of the probabilities γ_A^H and γ_A^L with V ; $K_B = 0.75, \alpha = 0.5$ (d) Plots of the prices p_A^1, p_B^1, p_I^H and p_I^L with V ; $K_B = 0.75, \alpha = 0.5$



(e) Plots of the probabilities γ_A^H and γ_A^L with α ; $K_B = 0.75, V = 0.8$ (f) Plots of the prices p_A^1, p_B^1, p_I^H and p_I^L with α ; $K_B = 0.75, V = 0.8$

Figure 5: Trends in equilibrium expectations of product availability in the opaque channel and prices as the capacity of firm B increases. For the plots above, we use $t = 1, L = 0.75, H = 1.25, \delta = 1$ and $K_A = 0.5$. Values of the other parameters are specified next to the corresponding plots. Other choices of the parameter values yield qualitatively the same plots.

opaque prices (Figure 5(f)).

To summarize the results above, opaque selling always increases the profits of both firms in the asymmetric-capacities scenario, as compared to not using opaque selling at all. However, imperfect masking of the opaque product reduces the efficacy of the opaque channel, as compared to perfect masking in the symmetric-capacities scenario. Furthermore, this imperfect masking hurts the prices and profit of the larger firm more than it hurts the prices and profit of the smaller firm, to the extent that the larger firm makes overall lower profit across the two periods than the smaller firm.

6.3 Comparison with LMSS

The above analysis leads to a natural question: Will asymmetric firms prefer the OpSS strategy or the LMSS strategy? To answer this question, we solve the LMSS game with asymmetric firms and obtain the following insights.¹⁵

The main difference of the asymmetric-capacities case from the symmetric-capacities case is that, due to the fact that firm B has larger capacity, it charges lower prices than firm A and sells more than firm A. Due to its higher sales, it makes a larger profit than firm A (though this is lesser than the symmetric-case profit due to reduced prices). However, even with asymmetric capacities, the nature of the LMSS equilibrium remains similar to that in the symmetric case (described in Proposition 5.1). Specifically, both firms price in the first period such that all consumers with “high brand preference” purchase at high prices from their preferred firms in the first period (due to the threat of stock out later) and, if demand is high, both firms stock out in the first period itself. If demand turns out to be low, there is leftover capacity for both firms and they compete and price low in the second period. The bulk of the profits for each firm come from the first period.

From a numerical comparison with the OpSS strategy, we find that asymmetric firms prefer the LMSS strategy over a larger region of the (V/t) - α space. (Specifically, the equal-profit contours in the (V/t) - α space in Figure 3 shift more and more outward (in the top-left direction) as K_B/K_A increases.) The reason is that in LMSS the nature of the equilibrium remains the same, as described above. In OpSS, however, asymmetric firms cannot achieve perfect masking in the opaque channel in the second period. Since the opaque-channel profits hinge upon how well product identity

¹⁵We characterize the equilibrium in Appendix A3.3 but do not provide the details of the solution due to space constraints and because the derivation is almost exactly as in Appendix A2.3.

can be masked, imperfect masking significantly reduces the efficacy of the opaque channel. As a consequence, there is reduced preference for opaque selling with asymmetric firms.

7 Other Modeling Considerations

Opaque sales and last-minute sales are encountered in a variety of practical situations, many of which are not fully reflected in our stylized model. Thus, apart from analyzing the core model above, we point out some modeling variations that might form a good starting point for various future research considerations.

Damaged Opaque Goods: In our basic model, we assume that consumers derive the same utility from flying with a firm regardless of whether they buy in the direct channel or the opaque channel. However, firms sometimes force opaque sellers to sell “damaged” goods. For instance, firms might disallow re-booking or charge high cancellation fees for opaque goods. One way to make this consideration consistent with our model is to impose that “damaging” the goods affects valuations that consumers obtain on opaque purchases. In our model, this can be admitted by discounting the valuation in the opaque channel to $\delta_o V$, where $\delta_o \in (0, 1)$ is a factor that accounts for the possible disutility from the reduced flexibility in the opaque channel. As a consequence of reduced valuation, the intermediary charges a reduced price $p_I = \delta_o V - t/2$ in equilibrium. We note that, in the symmetric model, this change affects the prices only on the opaque selling channel, whereas the LMSS prices remain unaffected. As a result, the revenues for the intermediary are reduced in the opaque channel, and more consumers purchase directly from the firms in the first period. Hence, firms are more likely to sell in their own channels. Thus, if opaque goods are damaged, LMSS strategy may be preferred to OpSS over a larger range of problem parameters.

Concentrated versus monopolistic markets: In this paper, we analyzed competing firms selling through an opaque intermediary. In some situations, however, the same service providers sell opaque products as well. E.g., Norwegian Cruise Line offers both specific staterooms on their cruise ships as well as opaque staterooms which guarantee certain minimal amenities but not a specific location on the ship. Similarly, one airline firm might be selling opaque tickets with different departure timings on the same route (e.g., morning vs. evening flights). We considered a variation of our model in which both transparent products are managed by the same firm which

maximizes its total profit. We find that the monopoly firm is able to derive higher profit from the LMSS strategy (because second-period prices for transparent tickets are set in a monopolistic rather than a competitive scenario), while our other results remain qualitatively unchanged. Thus, without competition, LMSS strategy may be preferred to OpSS over a larger range of problem parameters.

Heterogeneous values for the core product: In the basic model, we assumed that consumers are homogeneous in their preference for the core product, i.e., valuation V does not vary by consumer. In practice, some companies (e.g., airlines) derive significant profits by discriminating between “business” and “leisure” travelers who typically have drastic different preferences for travel.

We believe that these considerations will not impact the main insights from the model because consumers with high utility for product consumption are likely to purchase the product at full price, and would not participate in either opaque or last-minute sales channels. Thus, our model focuses exclusively on price-conscious consumers with relatively low value for the product itself. It is, however, possible to incorporate into our model heterogeneous consumers that differ in their core value for the product. For instance, we could introduce a second Hotelling line with a much larger core value \bar{V} representing consumers with high valuation for the product. (We observe that this approach introduces more complexity to the model.) Since these consumers have high willingness to pay, the firms will allocate capacity to satisfy these consumers first, and then sell to consumers with lower V . Future research can carefully explore how this might affect the tradeoff between OpSS and LMSS.

Multiple hidden product attributes and vertical product differentiation: In our model, we assume that products are characterized by a single attribute: the company that sells it. In practice, however, products may differ along multiple dimensions. Hotel rooms purchased on hotwire.com differ in size, location and amenities. Airline tickets differ in the number of stops, departure times and trip lengths. All these different attributes can be hidden from (or revealed to) consumers in the opaque selling channel. Some opaque intermediaries allow consumers to select the level of opacity. E.g., priceline.com lets its consumers specify whether a “red eye” flight is acceptable and it also allows to set the upper bound on the number of stops. The issue of selecting the optimal level of opacity and the right attributes to hide provides potential for future research but is outside the scope of this study. We believe adding multiple dimensions of opacity might shift consumer

preferences towards the direct last-minute sales channel. In our model, consumers are certain that the firms are selling products of identical valuations in both the channels. However, if there is additional uncertainty about exact features of the product purchased from the opaque channel, then the consumers will be more likely to purchase directly from the firms.

Channel design and intermediary selection decisions: Our paper models the decision of firms to choose between LMSS (selling directly) and OpSS (an opaque intermediary) to sell limited inventories to strategic consumers. However, channel design and choosing between several available intermediaries remain as important decisions for a large number of firms. In a sequence of papers, Rangan (1986) and Rangan et al. (1987) discuss a prescriptive model for designing channels (with wholesalers and retailers), and choosing the right intermediary for the channel designed. Consumer demand is modeled exogenously in those papers. How strategic consumer behavior affects airline channel design (e.g., selling online through expedia.com vs. selling through travel agents) is a pertinent question that we leave for future research.

8 Discussion and Conclusions

Due to uncertain demand and (short-term) inflexible capacity, firms in travel industries often end up with one of the two extremes — a shortfall of capacity due to high demand, or leftover unused (and expensive) capacity due to low demand. To deal with the mismatch between demand and supply, firms have implemented a variety of strategies, and two of the most prominent strategies are direct last-minute sales at reduced prices and sales through an opaque intermediary. However, consumers are becoming more and more strategic — they have learned to anticipate this last-minute distress selling and might decide to postpone their purchase in expectation of future lower prices. The risk the consumers face while making this decision is of not being able to obtain a product if demand turns out to be high.

In this paper, we model this strategic interaction between competing firms and consumers, and shed light on the following question: when should firms offer last-minute sales directly to consumers versus through an opaque intermediary? We find that the answer depends on at least three factors: (1) the valuations that consumers have for the service, (2) the strength of brand preference that consumers have for competing firms (alternatively, the extent of service differentiation between

competing firms), and (3) the probability that demand in the market exceeds capacity. If consumer valuation for product is high and/or the strength of brand preference of the consumers in the market is low, firms prefer direct last-minute sales over opaque sales. Furthermore, as the probability of high demand increases, firms start to prefer opaque sales over direct last-minute sales. At the extreme, if market demand is deterministic, direct last-minute sales are never offered while opaque sales can be offered if consumer valuations for travel are very low. These findings immediately translate into empirically testable hypotheses.

We find that the dynamics underlying these two selling strategies are very different. By using direct last-minute sales, each firm prices in the first period so that only consumers with high preference for the firm buy the product. Thus, each firm derives the bulk of its profits primarily by charging high prices to these consumers, while second-period prices are very low (however, these cheap products are available only if demand turns out to be lower than capacity). Quite differently, in the opaque selling strategy, if the consumer valuations are very low, the firms set first-period prices to extract maximum profits from consumers and then clear any remaining products through the opaque channel. When valuations are high, the firms price in the first period to ensure that the number of consumers who want to buy products exceeds supply if demand is high (which, at this point of time, is unknown to everybody), introducing clamor for the limited number of products, and leveraging the risk of product shortage to charge higher first-period prices. To summarize, the direct last-minute sales strategy can be construed as extracting profits from high-preference consumers, while the opaque sales strategy can be thought of as creating a frenzy for products to raise prices. Clearly, both strategies in our paper are far from simple “inventory clearance mechanisms” — they are indeed strategic responses by the firms to consumers making strategic purchasing decisions.

We omitted several considerations from the model in order to obtain sharper insights, and these considerations pose several interesting questions for future research. First and foremost, how opaque should the opaque product be? Should consumers be able to specify at least some time intervals for the departure or not? Addressing this important question is beyond the scope of the current study. Second, we simplified the decision for the firms by allowing for two sales opportunities: one “regular” and one “sales.” In practice, for example, airlines offer many fares, and prices tend to increase until the very last moment when last-minute sales are announced.

In our model, all consumers fully know their brand preference adjusted valuations. Those customers with higher valuations naturally buy earlier. However, customers may not always know their valuation for flying. For instance, some customers, despite having high valuation for flying for an occasion, may realize their need late in the selling season. This remains a challenging problem, and is explored in a stream of research by Akan et al. (2008), Courty and Li (2000) and others, where factors such as product opacity and competition are not considered. It is fruitful to combine the research issues to explore the impact of learning behavior on opaque selling. Due to the complexities involved with dynamic learning models, we leave the solution of this problem for future research.

Incorporating the above considerations into modern decision support systems remains a challenge. Finally, although numerous studies have modeled airline revenue management decisions, there have been very few attempts to verify these findings empirically (see Koenigsberg et al. (2008) for an exception). Empirical studies tend to be limited by data availability — although airlines periodically share data with regulatory authorities, these data are not precise enough to distill specific pricing strategies employed by an airline. All these directions are promising areas of future research in years to come.

References

- Akan, M., B. Ata and J. Dana. 2008. Revenue Management by Sequential Screening. *Carnegie Mellon University*, working paper.
- Amaldoss, W. and S. Jain. 2005. Conspicuous Consumption and Sophisticated Thinking. *Management Science*, 51(10), 1449-1466.
- Amaldoss, W. and S. Jain. 2008. Trading Up: A Strategic Analysis of Reference Group Effects. *Marketing Science*, 27(5), 932-942.
- Besanko, D. and W. L. Winston. 1990. Optimal Price Skimming by a Monopolist Facing Rational Consumers. *Management Science*, 36(5), 555-567.
- Cachon, G. and R. Swinney. 2009. Purchasing, Pricing, and Quick Response in the Presence of Strategic Consumers. *Management Science*, 55(3), 497-511.
- Coase, R. H. 1972. Durability and Monopoly. *Journal of Law and Economics*, 15(1), 143-149.
- Courty, P. and H. Li. 2000. Sequential Screening. *Review of Economic Studies*, 67(4), 697-717.
- DeGraba, P. 1995. Buying Frenzies and Seller-Induced Excess Demand. *Rand Journal of Economics*, 26(2), 331-342.

- Deneckere, R. J. and R. P. McAfee. 1996. Damaged Goods. *Journal of Economics & Management Strategy*, 5(2), 149-174.
- Diamond, P. and D. Fudenberg. 1989. Rational Expectations Business Cycles in Search Equilibrium. *The Journal of Political Economy*, 97(3), 606-619.
- Elkind, P. 1999. The hype is big, really big, at Priceline. *Fortune*, September 6, 1999.
- Fay, S. 2008. Selling an Opaque Product through an Intermediary: A Case of Disguising One's Product. *Journal of Retailing*, 84(1), 59-75.
- Fay, S. and J. Xie. 2008. Probabilistic Goods: A Creative Way of Selling Products and Services. *Marketing Science*, 27(4), 674-690.
- Garner, C. 1982. Experimental Evidence on the Rationality of Intuitive Forecasters. *Research in Experimental Economics*, (2) 113-128.
- Gibbons, R. 1992. *Game Theory for Applied Economists*. Princeton University Press.
- Harrison, J. 2006. Traveling 'blind'. *The Ottawa Citizen*, September 26, 2006.
- Jiang, Y. 2007. Price Discrimination with Opaque Products. *Journal of Revenue and Pricing Management*, 6(2), 118-134.
- Koenigsberg, O., E. Muller and N. J. Vilcassim. 2008. EasyJet Pricing Strategy: Should Low-Fare Airlines Offer Last-Minute Deals? *Quantitative Marketing and Economics*, 6(3), 279-297.
- Lambert, M. 2006. Last-minute gamble: take a chance, but be ready for anything. *Miami Herald*, November 26, 2006.
- Lai, G., L. G. Debo, K. Sycara. 2009. Buy Now and Match Later: The Impact of Posterior Price Matching on Profit with Strategic Consumers. *Manufacturing and Service Operations Management*, forthcoming.
- Levin, Y., J. McGill and M. Nediak. 2009. Dynamic Pricing in the Presence of Strategic Consumers and Oligopolistic Competition. *Management Science*, 55(1), 32-46.
- Liu, Q. and G. van Ryzin. 2007. Strategic Capacity Rationing When Customers Learn. *Columbia University*, working paper.
- Liu, Q. and G. van Ryzin. 2008. Strategic Capacity Rationing to Induce Early Purchases. *Management Science*, 54(6), 1115-1131.
- Muth, J. F. 1961. Rational Expectations and the Theory of Price Movements. *Econometrica*, 29(3), 315-335.
- Netessine, S. and R. Shumsky. 2005. Revenue Management Games: Horizontal and Vertical Competition. *Management Science*, 51(5), 813-831.
- Netessine, S. and C. Tang. 2009. Operations Management Models with Consumer-Driven Demand. Forthcoming in Springer International Series. Ed.: Hillier, Frederick S.
- Phillips, R. L. 2005. *Pricing and Revenue Optimization*. Stanford Business Books, Stanford, CA.

- PhoCusWright. 2004. The *PhoCusWright* consumer travel trends survey, sixth edition.
- Rangan., V. K. 1986. The Channel Design Decision: A Model and an Application. *Marketing Science*, 6(2), 156-174.
- Rangan, V. K., A. A. Zoltners and R. J. Becker. 1986. The Channel Intermediary Selection Decision: A Model and an Application. *Management Science*, 32(9), 1114-1122.
- Shapiro, D.A. and X. Shi. 2008. Market Segmentation: The Role of Opaque Travel Agencies. *Journal of Economics and Management Strategy*, 17(4), 803-837.
- Smith, B. C., R. Darrow, J. Elieson, D. Gunther, B. V. Rao and F. Zouaoui. 2007. Travelocity becomes a travel retailer. *Interfaces*, 27(1), 68-81.
- Smith, V., G. Suchanek, A. Williams. 1988. Bubbles, Crashes and Endogenous Expectations in Experimental Spot Asset Markets. *Econometrica*, 56, 1119-1151.
- Stokey, N. 1979. Intertemporal Price Discrimination. *Quarterly Journal of Economics*, 93(3), 355-371.
- Stokey, N. 1981. Rational Expectations and Durable Goods Pricing. *The Bell Journal of Economics*, 12(1), 112-128.
- Su, X. 2007. Inter-Temporal Pricing with Strategic Consumer Behavior. *Management Science*, 53(5), 726-741.
- Su, X. 2009. A Model of Consumer Inertia with Applications to Dynamic Pricing. *Production and Operations Management*, forthcoming.
- Su, X. and F. Zhang. 2008. Strategic Consumer Behavior, Commitment, and Supply Chain Performance. *Management Science*, 54(10), 1759-1773.
- Sunder, S. 1995. Experimental assets markets: A survey. J. Kagel, A. Roth, eds. *The Handbook of Experimental Economics*. Princeton University Press, Princeton, NJ, 445-500.
- Sviokla, J. 2004. Value Poaching. *Conference Board Review*, June 2004.
- Talluri, K. and G. van Ryzin. 2004. *The Theory and Practice of Revenue Management*. Kluwer Academic Press, Boston, MA.
- Terwiesch, C., S. Savin and I. Hann. 2005. Online Hagglng at a Name-Your-Own-Price Retailer: Theory and Application. *Management Science*, 51(3), 339-351.
- Varian, H. 2000. Versioning Information Goods. In "Internet Publishing and Beyond: The Economics of Digital Information and Intellectual Property," 190-202, MIT Press, Cambridge, MA.
- Xie, J. and S. M. Shugan. 2001. Electronic Products, Smart Cards and Online Prepayments: When and How to Advance Sell. *Marketing Science*, 20(3), 219-243.
- Yin, R., Y. Aviv, A. Pazgal and C.S. Tang. 2009. Optimal Markdown Pricing: Implications of Inventory Display Formats in the Presence of Strategic Customers. *Management Science*, 55(8), 1391-1408.

Technical Appendix

A1 Deterministic Demand

We consider two strategies: (i) The firms can sell through their own channels and have the option of offering different prices in each period of sale. (ii) The firms can sell opaque products in the second period, after sales in the first period have concluded. We consider two possible scenarios for each strategy: low-demand scenario ($J < K$), and high-demand scenario ($J > K$).

A1.1 Selling Through the Firms' Direct Channels

The demand is deterministic and equals J . The firms and all consumers know J . Assume that firm i (where $i \in \{A, B\}$) charges p_i^1 in the first period and p_i^2 in the second period. Each consumer buys a product, if available, from the firm that provides him with the highest net utility (conditional on it being positive), either in the first period or the second period. This is formalized in the following Lemma.

Lemma A1.1 *When customers are rational, the equilibrium prices are such that $p_i^1 = p_i^2 = p_i, i \in \{A, B\}$.*

Proof: We begin the proof by observing that the two periods are identical in the information all the players (firms and consumers) have and there is no stochastic component in demand or utilities. Further, there is no discounting.

Consider the case of firm A . First, consider the case when demand is low ($J < K$). Suppose, in equilibrium, the consumer located at x_A is indifferent between buying from firm A in the first period at price p_A^1 and in the second period at price p_A^2 .

Could a consumer at x_A be indifferent between purchasing from firm A in period 1 at price p_A^1 and firm B in period 2 at price p_B^2 ? We can show such consumers who are indifferent between purchasing from one firm in the first period and the other firm in the second period do not exist.

For this consumer at x_A , the following indifference condition holds when he is making his purchase or postpone decision in the first period ($p_A^{2,e}$ is his first-period expectation of the second-period price):

$$V - p_A^1 - tx_A = V - p_A^{2,e} - tx_A$$

which implies that $p_A^1 = p_A^{2,e}$ in equilibrium, regardless of the location of the indifferent customer. Further, in equilibrium, the expectations will be correct, i.e., $p_A^{2,e} = p_A^2$. Therefore, firm A will offer the same price in both periods.

When demand is high ($J > K$), there are two kinds of consumers based on their locations — those who will obtain firm A 's product (located in the region $[0, K/(2J)]$), and those who will not obtain firm A 's product (located to the right of $K/(2J)$). When demand is high, firm A maximizes revenues by selling to the consumers located to the left of $K/(2J)$. The second kind of consumers therefore would be unable to buy products in the high-demand scenario. For the first set of consumers, the indifference condition above again holds and we have $p_A^1 = p_A^2$. Further, firm A sets these prices so that the consumer at $K/(2J)$ is indifferent between purchasing and not purchasing a ticket, i.e., $V - p_A^1 - tK/(2J) = V - p_A^2 - tK/(2J) = 0$, which yields $p_A^1 = p_A^2 = V - K/(2J)$.

Similar arguments apply for prices offered by firm B . ■

A1.1.1 Low Demand

Firms A and B set revenue maximizing prices p_A and p_B and accrue profits $\pi_A = p_A x_A J$ and $\pi_B = p_B(1 - x_B)J$, where x_A and x_B represent the locations of the farthest consumers who bought products from firms A and B , respectively, on the Hotelling line. The solution to the game is formalized in Proposition A1.1.

Proposition A1.1 *When demand is deterministic, there is ample capacity ($J < K$) and firms sell only through their own channels, the optimal prices, market coverage and profits in the equilibrium are as follow:*

$\frac{V}{t}$	Prices ($p_A = p_B$)	Market Coverage ($x_A = 1 - x_B$)	Profits ($\pi_A = \pi_B$)
$\frac{1}{2} \leq \frac{V}{t} < 1$	$\frac{V}{2}$	$\frac{V}{2t}$	$\frac{V^2}{4t} J$
$1 \leq \frac{V}{t} < \frac{3}{2}$	$V - \frac{t}{2}$	$\frac{1}{2}$	$(V - \frac{t}{2}) \frac{J}{2}$
$\frac{V}{t} \geq \frac{3}{2}$	t	$\frac{1}{2}$	$\frac{t}{2} J$

Proof: We prove the proposition for low demand. Note that the total capacity of the two firms (K) is more than the total demand (J). Let $V/t \geq 1/2$ as described in the paper.

First, consider the case in which the firms are acting as local monopolies. We consider the decision of firm A in detail, and the analysis will be identical for firm B . If firm A chooses the price p_A , the right-most consumer to buy from the firm will be at x_A such that $V - tx_A - p_A = 0$, i.e., the utility of the consumer at x_A is zero. The price charged by the firm to all consumers will then be $p_A = V - tx_A$, and the demand will be $x_A J$. Thus, the profit for the firm will be $\pi_A = p_A x_A J = (V - tx_A)x_A J$. This profit is maximized at $x_A = V/(2t)$, and the maximized profit is given by $\pi_A = JV^2/(4t)$. However, to ensure that the firms are local monopolies, we need to ensure that at the optimum $x_A < 1/2$, which yields $V/t < 1$.

When $V/t \geq 1$, the above equilibrium does not hold, since the firms are not local monopolies (the optimal coverage for each firm will be $> 1/2$). We propose that for $1 \leq V/t < 3/2$ both firms charge prices $p_A = p_B = V - t/2$ in equilibrium, cover half the market and make profits $\pi_A = \pi_B = (V - t/2)J/2$.

Suppose firm A raises its price and charges $p_A^+ = V - t/2 + \epsilon t$ where $\epsilon > 0$, while firm B still charges $p_B = V - t/2$. Then, firm A covers $x_A = 1/2 - \epsilon$ and makes a profit $(1/2 - \epsilon)(V - t/2 + \epsilon t)J$. However, under the condition $V/t \geq 1$, this profit is lower than the equilibrium profit, so that the firm does not have an incentive to raise its price above the equilibrium price. Now, consider the case in which the firm lowers its price and charges $p_A^- = V - t/2 - \epsilon t$. The point \tilde{x} at which the indifferent consumer is located is then found by solving the condition $V - p_A^- - t\tilde{x} = V - p_B - t(1 - \tilde{x})$, which yields $\tilde{x} = (1 + \epsilon)/2$, and the profit for firm A is given by $\frac{1}{2}(1 + \epsilon)(V - t/2 - \epsilon t)J$. However, under the condition $V/t < 3/2$, this profit is always lower than the equilibrium profit, so that the firm does not have an incentive to lower its price below the equilibrium price. Hence, the equilibrium proposed above holds for the range $1 \leq V/t < 3/2$.

Now consider the case in which the two firms are in direct competition. Firm A charges a price p_A and firm B charges a price p_B . Assume that the indifferent consumer is located at \tilde{x} . Since this consumer is indifferent to buying from A or B , the following condition holds for him: $V - p_A - t\tilde{x} = V - p_B - t(1 - \tilde{x})$, which gives $\tilde{x} = 1/2 + (p_B - p_A)/(2t)$. The profits for firms A and B are given, respectively, by $\pi_A = p_A \tilde{x} J$ and $\pi_B = p_B (1 - \tilde{x}) J$. Maximizing the profits jointly, we obtain $p_A = p_B = t$, $\tilde{x} = 1/2$ and $\pi_A = \pi_B = Jt/2$. Under our assumption that the outside utility of a consumer is zero, we need to ensure that $V - p_A - t\tilde{x} = V - p_B - t(1 - \tilde{x}) \geq 0$, which gives the condition $V/t \geq 3/2$. This completes the proof. ■

A1.1.2 High Demand ($J > K$)

Since demand is larger than capacity available in this scenario, full market coverage cannot occur. To maximize revenues, each firm will then cover the $K/2$ consumers located closest to it. The location of the farthest consumer covered by firm A (when valuation is high enough) is, therefore, $x_A = K/(2J) < 1/2$. (Similarly, $x_B = 1 - K/(2J) > 1/2$.) The following proposition lays out the solution to the game.

Proposition A1.2 *When demand is deterministic, capacity is a constraint ($J > K$) and firms sell only through their own channels, the optimal prices, market coverage and profits in the equilibrium are as follow:*

$\frac{V}{t}$	Prices ($p_A = p_B$)	Market Coverage ($x_A = 1 - x_B$)	Profits ($\pi_A = \pi_B$)
$\frac{1}{2} \leq \frac{V}{t} < \frac{K}{J}$	$\frac{V}{2}$	$\frac{V}{2t}$	$\frac{V^2}{4t} J$
$\frac{K}{J} \leq \frac{V}{t}$	$V - \frac{K}{2J}t$	$\frac{K}{2J}$	$(V - \frac{K}{2J}t)\frac{K}{2}$

Proof: Note that $V/t \geq 1/2$. In this proposition, we analyze the high demand case. The total capacity of the two firms (K) is less than the total demand (J) and firms will act as local monopolies. Again, we consider firm A and the analysis is identical for firm B . If firm A chooses the price p_A , the right-most consumer to buy from the firm will be at x_A such that $V - tx_A - p_A = 0$. The price charged by the firm to all consumers will then be $p_A = V - tx_A$, and the demand will be $x_A J$.

Thus, the profit for the firm will be $\pi_A = p_A x_A J = (V - tx_A)x_A J$. This profit is maximized at $x_A = V/(2t)$, and the maximized profit is given by $\pi_A = JV^2/(4t)$. However, to ensure that the firms do not stock out, we need to ensure that at the optimum $x_A \leq K/(2J)$, which gives $V/t \leq K/J$.

For $V/t > K/J$ each firm will charge the price $p_A = p_B = V - tK/(2J)$, cover $x_A = 1 - x_B = K/(2J)$ and make profits $\pi_A = \pi_B = (V - Kt/(2J))\frac{K}{2}$. Note that the firm cannot lower its price below this level, since it does not have the capacity to serve the expanded market. It can be easily shown, using an ϵ -deviation argument as in the proof of proposition A1.1, that the firm does not have an incentive to lower its price below this level. This specifies the equilibrium for all values of $V/t \geq 1/2$ and completes the proof. ■

A1.2 Opaque Selling

The firms sell transparent tickets in the first period and opaque tickets through an intermediary in the second period. The stages of the game and the backward induction solution approach have been described in the main paper in Section 4.2. Here, we provide a formal analysis of opaque selling in low and high deterministic demand.

A1.2.1 Low Demand

Proposition A1.3 *When demand is deterministic, there is ample capacity ($J < K$), and firms can utilize the opaque channel, the equilibrium prices charged in the first period by the firms, the price charged in the opaque channel by the intermediary, and the opaque market coverage in the equilibrium (for $\delta = 1$) are as follow:*

$\frac{V}{t}$	First-period prices ($p_A = p_B$)	Opaque prices (p_I)	Opaque coverage ($x_B - x_A$)
$\frac{1}{2} \leq \frac{V}{t} < 1$	$\frac{V}{2}$	$V - \frac{t}{2}$	$1 - \frac{V}{t}$
$1 \leq \frac{V}{t} < \frac{3}{2}$	$V - \frac{t}{2}$	-	-
$\frac{3}{2} \leq \frac{V}{t}$	t	-	-

Proof: We first prove that the rational expectations equilibrium does not exist for $\gamma_A^e \in [0, 1] \setminus \{\frac{1}{2}\}$, and only $\gamma_A^e = \gamma_B^e = 1/2$ are supported in equilibrium. We establish the rational expectations equilibrium by first analyzing the second period, and then the first period.

We consider the case in which the firms have ample capacity, i.e., $J < K$. Let us consider the *second* period. Without loss of generality, let x_A, x_B be the location of the consumers closest to the firm who did not buy in the first period. Hence interval $[x_A, x_B]$ denotes the location of all the consumers remaining in the second period. Consider any consumer located at $x \in [x_A, x_B]$. x_A and x_B are the left-most and right-most points on the line available to the intermediary to sell opaque products.

Since we are in the second period, demand realization has occurred, and opaque seller has announced price p_I . The consumer has expectations over availability. Upon buying the opaque product at price p_I , the surplus a consumer located at x expects to attain is

$$V - p_I - \gamma_A^e t x - \gamma_B^e t (1 - x)$$

which, using $\gamma_B^e = 1 - \gamma_A^e$, can be written as

$$V - p_I - t(1 - \gamma_A^e) + (1 - 2\gamma_A^e)tx.$$

$0 < \gamma_A^e \leq 1/2$: It suffices to consider $0 < \gamma_A^e \leq 1/2$ because, if $\gamma_A^e = 0$, the market is not opaque since the consumers believe that all the products in the opaque channel are coming from firm B . The analysis for $1/2 \leq \gamma_A^e < 1$ is identical to the analysis for $0 < \gamma_B^e \leq 1/2$ (which is the same as the analysis below by symmetry).

For a given p_I , the surplus for a consumer purchasing in the opaque market is increasing in his location x , as long as $\gamma_A^e < 1/2$. In other words, the minimum surplus is obtained by the consumer located at x_A . We consider two cases, namely, when the intermediary wants to cover the full market from x_A to x_B and when the intermediary considers covering this interval partially.

The intermediary may not necessarily cover the full market $[x_A, x_B]$ available to him. Suppose that the intermediary only aims to cover the market $[x', x_B]$, where $x' > x_A$. Note that the surplus of a consumer is increasing in his location x . Therefore the opaque intermediary will price such that $p_I = V - (1 - \gamma_A^e)t + (1 - 2\gamma_A^e)tx'$, $x_A < x' < x_B$. Then the consumers in the interval $[x_A, x']$ (which is defined to be null if $x' < x_A$) do not buy because they have negative utility. The consumers in the interval $[x', x_B]$ buy in the opaque channel because they have non-negative utility. The profit of the intermediary is then $\pi_I = (V - (1 - \gamma_A^e)t + (1 - 2\gamma_A^e)tx')(x_B - x')J$. To maximize this profit the intermediary sells to the market $[x'^*, x_B]$ where $x'^* = \frac{(1 - \gamma_A^e)t - V + (1 - 2\gamma_A^e)tx_B}{2(1 - 2\gamma_A^e)t}$. This implies that $p_I = \frac{1}{2}(V - (1 - \gamma_A^e)t + (1 - 2\gamma_A^e)tx_B)$.

Now consider the analysis for firm A selling in the transparent channel. The person located at x_A has negative utility in the opaque channel. Thus, to this consumer, firm A selling in the transparent channel can charge $p_A = V - x_A t$ and make him indifferent between buying and not buying. This gives firm A a profit of $\pi_A = (V - tx_A)x_A J + \delta\gamma_A^e(V - (1 - \gamma_A^e)t + (1 - 2\gamma_A^e)tx_B)(x_B - x')$.

Now consider firm B . The consumer at x_B has to be indifferent between purchasing in the first period and in the second period. The consumer at x_B solves $V - p_B - t(1 - x_B) = V - p_I - \gamma_A^e tx_B - (1 - \gamma_A^e)t(1 - x_B)$, which, using the value of p_I from above, gives $p_B = \frac{1}{2}(V + t(-1 + x_B - \gamma_A^e + 2x_B\gamma_A^e))$. The profit for firm B is $\pi_B = p_B(1 - x_B)J + \delta(1 - \gamma_A^e)p_I(x_B - x')J$. Maximizing π_A and π_B w.r.t. x_A and x_B simultaneously, we obtain $x_A = \frac{V}{2t}$ and $x_B = \frac{V(1 + (-1 + \gamma_A^e)\delta) + t(-2 + \delta + (\gamma_A^e)^2\delta - \gamma_A^e(3 + 2\delta))}{t(-2 + \delta + 2(\gamma_A^e)^2\delta - \gamma_A^e(4 + 3\delta))}$.

Using these values of x_A and x_B , we obtain the expectation function $\gamma_A = \frac{\frac{K}{2} - x_A J}{\frac{K}{2} - x_A J + \frac{K}{2} - (1 - x_B) J}$.

In the rational expectations equilibrium, the beliefs have to be consistent with the outcome. It must be that $\gamma_A = \gamma_A^e$. Imposing this condition we solve for $\gamma_A = \gamma_A^e = \gamma_A^e(V, t, J, K, \delta)$. The value of $\gamma_A(V, t, J, K, \delta)$ is algebraically complicated and we do not present it here. However, we check that imposing the condition $0 < \gamma_A^e < 1/2$ implies $V/t < 1/2$, which is a contradiction. (Recall that we require $V/t \geq 1/2$ as a “sanity condition” to ensure that if the firms sell products for free, then everybody in the market will have positive evaluation to obtain the product from at least one of the firms. In other words the condition ensures some market coverage at zero prices.) Thus, when the intermediary sets prices such that the intermediary’s market coverage is partial, then the rational expectations equilibrium does not exist.

Let us now analyze the case when the intermediary prices to cover the entire market $[x_A, x_B]$. If the intermediary wants to cover the full opaque market, he will price so as to make the surplus of the consumer at x_A equal to zero, i.e., $p_I = V - t(1 - \gamma_A^e) + x_A(1 - 2\gamma_A^e)t$.

Since the consumer x_A is indifferent between the opaque and first period market, firm A sets its price p_A by solving the following equation:

$$V - p_A - tx_A = V - p_I - \gamma_A^e tx_A - (1 - \gamma_A^e)t(1 - x_A).$$

To extract maximum revenues in the opaque market, the intermediary sets p_I such that the right hand side of the above equation is zero. Therefore the value that x_A receives is zero.

Therefore in the first period, if firm A covers the interval $[0, x_A]$, the price is $p_A = V - tx_A$. In the first period firm A then maximizes its profit $\pi_A = (V - tx_A)x_A J + \delta\gamma_A^e p_I(x_B - x_A)J$, where p_I is as above.

Firm B solves

$$V - p_B - t(1 - x_B) = V - p_I - \gamma_A^e tx_B - (1 - \gamma_A^e)t(1 - x_B).$$

Restricting to $\gamma_A^e \in (0, \frac{1}{2}]$ and using p_I above, we obtain $p_B = V + t(-1 + x_A - 2x_A\gamma_A^e + 2x_B\gamma_A^e)$.

The profit for firm B is given by $\pi_B = p_B(1 - x_B)J + \delta(1 - \gamma_A^e)p_I(x_B - x_A)J$.

Maximizing π_A and π_B simultaneously for the firms w.r.t. x_A and x_B gives:

$$x_A = \frac{V(4 - (1 + 2\gamma_A^e)\delta + (1 - 3\gamma_A^e + 2(\gamma_A^e)^2)\delta^2) + t\delta(1 - \delta + 2(\gamma_A^e)^3\delta + 4\gamma_A^e(1 + \delta) - (\gamma_A^e)^2(8 + 5\delta))}{t(8 + (1 + 4\gamma_A^e - 12(\gamma_A^e)^2)\delta + (1 - 2\gamma_A^e)^2(-1 + \gamma_A^e)\delta^2)}$$

$$x_B = \frac{t(2 - 2\delta + 2(\gamma_A^e)^4\delta^2 - 5(\gamma_A^e)^3\delta(2 + \delta) + (\gamma_A^e)^2\delta(1 + 4\delta) + \gamma_A^e(4 + 5\delta - \delta^2)) + V(3(-1 + \delta) + (\gamma_A^e)^2(4 - 3\delta)\delta + 2(\gamma_A^e)^3\delta^2 + \gamma_A^e(2 - 6\delta + \delta^2))}{t\gamma_A^e(8 + (1 + 4\gamma_A^e - 12(\gamma_A^e)^2)\delta + (1 - 2\gamma_A^e)^2(-1 + \gamma_A^e)\delta^2)}$$

Using the above values, we obtain $\gamma_A = \frac{\frac{K}{2} - x_A J}{(\frac{K}{2} - x_A J) + (\frac{K}{2} - (1 - x_B)J)}$. In the rational expectations equilibrium, $\gamma_A = \gamma_A^e$. Upon solving this, we obtain $\gamma_A = \gamma_A^e = 1/2$ as the only real-valued solution. Hence, the equilibrium does not exist for $0 \leq \gamma_A^e < 1/2$ when the intermediary wants to cover the full market between $[x_A, x_B]$. Only $\gamma_A^e = \gamma_B^e = 1/2$ can be supported in the equilibrium.

$1/2 < \gamma_A^e \leq 1$: When $1/2 < \gamma_A^e \leq 1$, for a given p_I , the surplus decreases with x . In other words, the minimum surplus is obtained by the consumer located at x_B . The analysis proceeds as above, except the subscripts A and B are suitably interchanged. Using identical arguments, we show that there is no equilibrium such that $1/2 < \gamma_A^e \leq 1$. Further, the analysis for $\gamma_A^e > 1/2$ is the same as the analysis for $\gamma_B^e < 1/2$.

In summary, all consumers develop rational expectations $\gamma_A^e = \gamma_B^e = 1/2$, which are realized in equilibrium.

In other words, the rational expectations equilibrium does not exist for $\gamma_A \in [0, 1] \setminus \{\frac{1}{2}\}$ and only $\gamma_A^e = \gamma_B^e = 1/2$ are supported in the rational expectations equilibrium.

Since $\gamma_A^e = \gamma_A = 1/2$ for every consumer in the market and the probability of getting an opaque product $\beta = 1$ (since there is ample capacity), the ex-ante expected surplus for each consumer buying from the opaque channel is simply $V - p_I - t/2$ and is independent of the location of the consumer. Therefore intermediary prices at $p_I = V - t/2$ and attains the revenue $\pi_I = (1 - \delta)(V - t/2)(x_B - x_A)J$ by selling to the entire remaining market. Note that the revenue-maximizing action in the opaque channel is independent of the fraction of revenues $(1 - \delta)$ held by the intermediary.

We now analyze the optimal choices of the firms in their own transparent channels before the opaque sales. The consumers located between x_A and x_B prefer to buy from the opaque channel. As before, for firm A the consumer located at x_A must be indifferent between buying from the firm now or in the opaque channel later. In a low-demand state, the leftover products are sufficient to

cover all the remaining demand. Hence, we have:

$$V - p_A - tx_A = V - p_I - \gamma_A^e tx_A - \gamma_B^e t(1 - x_A).$$

Since $\gamma_A^e = 1/2$ and $p_I = V - t/2$, the right-hand side of the equation above is zero. Hence, the price charged by firm A is $p_A = V - tx_A$. Firm A , which covers the market till x_A and charges price $p_A = V - tx_A$, makes a profit of

$$\pi_A = (V - tx_A)x_A J + \delta \gamma_A^e \left(V - \frac{t}{2} \right) (x_B - x_A) J.$$

The value of x_A that maximizes the profit for firm A is $x_A = \frac{V}{2t} - \frac{\delta}{4} \left(\frac{V}{t} - \frac{1}{2} \right)$, which is decreasing in δ . As the ability to earn more revenues from the opaque channel increases, the firm chooses to cover less through its own channels. This does not imply that the firm is generating smaller revenue through its own channel. The corresponding optimal price $p_A = V - tx_A = \frac{V}{2} + \frac{\delta}{4} \left(\frac{V}{t} - \frac{1}{2} \right) t$ is increasing in δ . Because of the presence of the opaque channel, firms sell fewer products in their own channels at *higher* prices. Proceeding with a similar analysis for firm B , we obtain $x_B = 1 - \left(\frac{V}{2t} - \frac{\delta}{4} \left(\frac{V}{t} - \frac{1}{2} \right) \right)$. Thus, the coverage by the intermediary is $x_B - x_A = 1 - \frac{V}{t} + \frac{\delta}{2} \left(\frac{V}{t} - \frac{1}{2} \right)$. For the above expression, we need to ensure that $x_A \leq \frac{1}{2} \Rightarrow \frac{V}{t} \leq \frac{4-\delta}{2(2-\delta)}$.

For $V/t \geq 3/2$, the competitive equilibrium holds, with both firms covering exactly half the market at prices $p_A = p_B = t$ and obtaining profits $\pi_A = \pi_B = tJ/2$ while $\pi_I = 0$. For $\frac{4-\delta}{2(2-\delta)} \leq \frac{V}{t} \leq \frac{3}{2}$ we construct the non-competitive equilibrium as both firms charging $p_A = p_B = \left(V/t - \frac{1}{2} \right) t$, covering exactly half the market, and therefore making profits $\pi_A = \pi_B = \left(\frac{V}{t} - \frac{1}{2} \right) \frac{t}{2} J$ with $\pi_I = 0$ (see table below). Thus, for $\frac{V}{t} \geq \frac{4-\delta}{2(2-\delta)}$ it turns out that nothing is allocated to the opaque channel in equilibrium.

In the above analysis, we see that firms charge a higher price in the first period as compared to the direct selling case. Note, however, that consumers are strategic. They recognize that the firms could rely upon the opaque channel and increase prices in the transparent channels. Further, consumers know that they can prevent the firms from implementing the opaque channel if they delay their purchases (in the extreme, delay purchases until right before the selling horizon ends). Effectively, through this strategic behavior, the consumers will make the firms charge first-period

transparent prices no higher than the price in the direct channel equilibrium (characterized in Proposition A1.1). In this equilibrium, however, the firms can still use the opaque channel when $1/2 \leq V/t < 1$ and the market is not covered with transparent prices. In the equilibrium, the firms will charge a price $V - t/2$ in the opaque channel to all remaining consumers, who will all buy products. The proposition provides the expressions for the equilibrium. ■

A1.2.2 High Demand

Proposition A1.4 *When demand is deterministic, capacity is a constraint ($J > K$) and firms can utilize the opaque channel, the equilibrium prices charged in the first period by the firms, the price charged in the opaque channel by the intermediary, and the opaque market coverage in the equilibrium (for $\delta = 1$) are as follow:*

$\frac{V}{t}$	First-period prices ($p_A = p_B$)	Opaque prices (p_I)	Opaque coverage ($x_B - x_A$)
$\frac{1}{2} \leq \frac{V}{t} < \frac{K}{J}$	$\frac{V}{2}$	$V - \frac{t}{2}$	$K - \frac{V}{t}J$
$\frac{K}{J} \leq \frac{V}{t}$	$V - \frac{K}{2J}t$	-	-

Proof: This proof proceeds in a manner very similar to the proof of Proposition A1.3. We first prove that the rational expectations equilibrium does not exist for $\gamma_A^e \in [0, 1] \setminus \{\frac{1}{2}\}$, and only $\gamma_A^e = \gamma_B^e = 1/2$ are supported in equilibrium.

In the case in which the firms have limited capacity, i.e., $J > K$, we need to impose the conditions $x_A \leq K/(2J)$ and $1 - x_B \leq K/(2J)$ while optimizing the profits for firms A and B respectively. Let us consider the second period when opaque products are being offered. Without loss of generality, let x_A, x_B be the locations of the consumers who were indifferent between purchasing and not purchasing from the firms A and B in the first period, respectively. Hence $[x_A, x_B]$ denotes the interval of all the consumers remaining in the second period. Now, consider any consumer located at $x \in [x_A, x_B]$.

Since we are in the second period, demand realization has occurred, and opaque seller has announced price p_I . The consumer located at x has beliefs over availability. Upon buying the opaque product at price p_I , the surplus he expects to attain is $V - p_I - \gamma_A^e tx - \gamma_B^e t(1 - x)$ which, using $\gamma_B^e = 1 - \gamma_A^e$, can be written as $V - p_I - t(1 - \gamma_A^e) + (1 - 2\gamma_A^e)tx$.

If capacity is not binding, then the analysis is no different from the previous analysis of the low demand case. (i.e., in equilibrium, the expectations $\gamma_A = \gamma_B = 1/2$).

Suppose capacity is limited in the second period. In other words, the residual capacity in the second period is less than the unfulfilled demand in the second period, i.e. $(K - x_A J - (1 - x_B)J) < (x_B - x_A)J$. Let us assume that the opaque intermediary covers interval $[x', x'']$ where $x' > x_A$ and $x'' < x_B$.

Consider $0 < \gamma_A^e < 1/2$. p_I is obtained by solving $V - p_I - t(1 - \gamma_A^e) + (1 - 2\gamma_A^e)tx' = 0$. Since the indifferent consumer is located at $x_A < x'$, we have that the net valuation of the consumer purchasing from firm A is zero. (If the consumer has positive utility, then a consumer to the right of x_A would also buy with positive utility.) The optimal revenue of firm A is achieved by maximizing $\pi_A = (V - tx_A)x_A J + \delta\gamma_A^e p_I(x'' - x')J$ w.r.t. x_A . This implies $x_A^* = \min\{K/(2H), V/(2t)\}$.

Since capacity is binding, all the seats with the opaque intermediary are sold. Hence, $x'' < x_B$ is determined by $(K - x_A J - (1 - x_B)J) = (x'' - x')J$. Then firm B maximizes $\pi_B = (V - t(1 - x_B))(1 - x_B)J + \delta(1 - \gamma_A^e)p_I(x'' - x')J$ w.r.t. x_B . This implies $x_B = 1 - \min\{K/(2H), V/(2t)\}$. Hence $x_A = 1 - x_B$, and the rational expectations regarding the probabilities of availability of the left over products from each firm will be symmetric. Hence only $\gamma_A = \gamma_B = 1/2$ is sustained in equilibrium in the high-demand environment.

Now, consider a consumer at $x \in [x_A, x_B]$ and note that $K < J$. The probability this consumer obtains a product is $\beta = \frac{(K - x_A J - (1 - x_B)J)}{(x_B - x_A)J}$. If this consumer obtains a product from the opaque seller at price p_I , the surplus he attains is $V - p_I - \gamma_A^e tx - \gamma_B^e t(1 - x)$.

Using the result above, $\gamma_A^e = 1/2$. In this case, the expected surplus in the equilibrium is simply $V - p_I - t/2$ and is independent of the location of the consumer. The opaque intermediary prices the products at $p_I = V - t/2$ and the total revenue accrued in the channel is $\pi_I = (V - t/2)(K/J - x_A - (1 - x_B))J$. Note that the intermediary can sell to *any* consumer located between x_A and x_B even though he is unable to cover the full market between these two points due to constrained capacity.

Now consider firm A in the transparent channel. The consumer at x_A (the right-most consumer that buys from A) solves

$$V - p_A - tx_A = \beta(V - p_I - \gamma_A^e tx_A - \gamma_B^e t(1 - x_A)).$$

Since $\gamma_A^e = 1/2$ and $p_I^e = p_I = V - t/2$, the right-hand side of the equation above is zero.

The rest of the analysis proceeds exactly as above in the proof for Proposition A1.3, except that the firm stocks out if the optimal value of x_A is greater than $K/(2J)$. After imposing $x_A \leq \frac{K}{2J}$ in the solution above due to capacity constraints, we obtain $\frac{V}{2t} - \frac{\delta}{4} \left(\frac{V}{t} - \frac{1}{2} \right) < \frac{K}{2J} \Rightarrow \frac{V}{t} \leq \frac{4\frac{K}{J} - \delta}{2(2-\delta)}$. Upon imposing the condition $\frac{V}{t} \geq \frac{1}{2}$, we obtain a lower bound on $\frac{K}{J}$, i.e., $\frac{K}{J} \geq \frac{1}{2}$. Thus, for $\frac{1}{2} \leq \frac{V}{t} \leq \frac{4\frac{K}{J} - \delta}{2(2-\delta)}$ (ensuring the firm does not stock out in the transparent channel) we obtain

$$\begin{aligned}\pi_A &= \pi_B = \left(\frac{V}{2} + \frac{\delta}{4} \left(\frac{V}{t} - \frac{1}{2} \right) t \right) \left(\frac{V}{2t} - \frac{\delta}{4} \left(\frac{V}{t} - \frac{1}{2} \right) \right) J + \frac{\delta}{2} \left(\frac{V}{t} - \frac{1}{2} \right) \left(K - \left(\frac{V}{t} - \frac{\delta}{2} \left(\frac{V}{t} - \frac{1}{2} \right) \right) J \right) t, \\ \pi_I &= (1 - \delta) \left(\frac{V}{t} - \frac{1}{2} \right) \left(K - \left(\frac{V}{t} - \frac{\delta}{2} \left(\frac{V}{t} - \frac{1}{2} \right) \right) J \right) t.\end{aligned}$$

For the case in which $\frac{V}{t} \geq \frac{4\frac{K}{J} - \delta}{2(2-\delta)}$, we construct the non-competitive equilibrium as follows: both firms charge a price $p_A = p_B = \left(\frac{V}{t} - \frac{K}{2J} \right) t$ and cover $x_A = 1 - x_B = \frac{K}{2J}$. The profits are given by $\pi_A = \pi_B = \left(\frac{V}{t} - \frac{K}{2J} \right) \frac{K}{2} t$ and $\pi_I = 0$.

As in the case of low demand, we see that firms charge a higher price in the first period as compared to the direct selling case. On the lines of the arguments in the proof of the low demand case, the consumers will make the firms charge first-period transparent prices no higher than the price in the direct channel equilibrium (characterized in Proposition A1.2). Hence, when $1/2 \leq V/t < K/J$, the firms charge a price $V/2$ and cover $V/(2t)$, which is less than $1/2$. The remaining $K/2 - VJ/t$ portion of the market $\left((V/(2t), K/(2J)] \text{ and } 1 - K/(2J), 1 - V/(2t) \right)$ is covered in the opaque channel by charging a price $V - t/2$. When $V/t \geq K/J$, the full market is covered in the transparent channel in the first period at price $V - K/(2J)t$ (i.e., the firms sell all $K/2$ products), and there are no products left to be allocated to the opaque channel. The proposition provides the expressions for the equilibrium. ■

A2 Uncertain Demand

This section provides the proofs for the propositions in Section 5.

A2.1 Proof of Proposition 5.2

We characterize the equilibrium for the case $\delta = 1$; the intuition remains similar for all $\delta \in [0, 1]$, because changing δ only changes the profits transferred from the opaque intermediary to the firms.

The analysis below is for $\gamma_A^{H,e} = \gamma_A^{L,e} = 1/2$, which are rational expectations in equilibrium. To see why this is the case, assume that $\gamma_i^e = 1/2$. Consider the case when demand is low and consumers purchase in the opaque channel. Suppose that the intermediary has the market $[x_A, x_B]$ available to it (and enough capacity to fulfil this demand) and offers a price p_I^L . For any customer at $x \in [x_A, x_B]$, the ex ante surplus from purchasing an opaque ticket is $V - p_I^L - \gamma_A^{L,e}tx - \gamma_B^{L,e}t(1-x) = V - p_I^L - t/2$. This is independent of position x , or, stated differently, all consumers have the same ex ante utility from purchasing an opaque ticket. The intermediary will then price at $p_I^L = V - t/2$, since at this price all consumers will purchase.

Now, consider the case when demand is high and consumers purchase in the opaque channel. Suppose that the intermediary has the market $[x_A, x_B]$ available to it (but not enough capacity to meet all this demand, so that a consumer who wants to purchase an opaque ticket will only get it with probability β) and offers a price p_I^H . For any customer at $x \in [x_A, x_B]$, the ex ante surplus from an opaque ticket is $\beta(V - p_I^L - \gamma_A^{L,e}tx - \gamma_B^{L,e}t(1-x)) = \beta(V - p_I^L - t/2)$. Once again, the intermediary will then price at $p_I^H = V - t/2$, since at this price all consumers will want to purchase (while only a fraction β of them will actually get tickets).

Now, using the results above, we solve for firm prices p_A and p_B in the first period for the cases in Proposition 5.2. Finally, in each of the cases below we will confirm that the realized probabilities of availability, γ_i , are all equal to $1/2$, thus ensuring consistency of beliefs.

- $1/2 \leq V/t < K/H$

The firms will not stock out even when demand is high. As in the deterministic demand case, consumers will not let the firms leverage the opaque channel to increase first-period prices, so that

$$\begin{aligned}\pi_A &= p_A x_A (\alpha H + (1 - \alpha)L) + \delta \gamma_A \left(\alpha p_I^H \left(\frac{K}{2} - x_A H + \frac{K}{2} - (1 - x_B)H \right) + (1 - \alpha) p_I^L (x_B - x_A)L \right), \\ \pi_B &= p_B (1 - x_B) (\alpha H + (1 - \alpha)L) + \delta \gamma_B \left(\alpha p_I^H \left(\frac{K}{2} - x_A H + \frac{K}{2} - (1 - x_B)H \right) + (1 - \alpha) p_I^L (x_B - x_A)L \right) \\ \pi_I &= (1 - \delta) \left(\alpha p_I^H \left(\frac{K}{2} - x_A H + \frac{K}{2} - (1 - x_B)H \right) + (1 - \alpha) p_I^L (x_B - x_A)L \right).\end{aligned}$$

The profit π_I is shared by firms A and B in proportion to the products sold by each. The no-stockout situation implies $x_A \leq K/(2H)$ and $x_B \geq 1 - K/(2H)$. Firm A sets $p_A = V - x_A t$ and firm B sets $p_B = V - (1 - x_B)t$. Optimizing the above two expressions w.r.t. x_A and x_B , we obtain $x_A = 1 - x_B = V/(2t)$ and $p_A = p_B = V/2$. After imposing $x_A \leq K/(2H)$ we obtain $V/t \leq K/H$, which we have already assumed. Moreover, $1/2 \leq V/t \Rightarrow 1/2 \leq K/H \Rightarrow K/H \geq 1/2$, which is required, but is a mild assumption.^{A1}

Using these values of x_A and x_B , we obtain $\gamma_A^L = \frac{\frac{K}{2} - x_A L}{\frac{K}{2} - x_A L + \frac{K}{2} - (1 - x_B)L} = \frac{1}{2}$ and $\gamma_A^H = \frac{\frac{K}{2} - x_A H}{\frac{K}{2} - x_A H + \frac{K}{2} - (1 - x_B)H} = \frac{1}{2}$. This confirms that $\gamma_i^e = 1/2$ are indeed equilibrium expectations.

If demand is high, the capacity sold in the opaque channel is the leftover from the transparent channel which is $K - VH/(2t)$, and if demand is low, the amount sold in the opaque channel is $L - VL/(2t)$. The prices in the opaque channel are $p_I^H = p_I^L = V - t/2$ and the profits are $\pi_I^H = (V/t - 1/2)(K - VH/(t))t$ and $\pi_I^L = (V/t - 1/2)(1 - V/(t))Lt$. In equilibrium, half of the above profits will be transferred to each firm. Hence, the profits for firms A and B are:

$$\pi_A = \pi_B = \frac{V^2}{4t}(\alpha H + (1 - \alpha)L) + \frac{1}{2} \left(\frac{V}{t} - \frac{1}{2} \right) \left(\alpha \left(K - \frac{V}{t}H \right) + (1 - \alpha) \left(1 - \frac{V}{t} \right) L \right) t.$$

In the cases that follow, we construct the equilibria and it can be shown using an ϵ -deviation argument that these are indeed equilibria. Further, we do not explicitly show that $\gamma_i^e = \gamma_i^e$ in all these cases; the reader can, however, check that this always holds.

- $\frac{K}{H} \leq \frac{V}{t} < \frac{K}{H} + \left(\frac{\alpha}{1 - \alpha} \right) \frac{K}{2L}$

Here, we construct the equilibrium as follows: both firms charge $p_A = p_B = V - Kt/(2H)$ and cover exactly $x_A = 1 - x_B = K/(2H)$ in both high and low demand. In high demand there is no leftover for the opaque channel, while in low demand the total leftover is $(x_B - x_A)L = (1 - K/H)L$. In high demand the opaque channel profit is zero (since nothing is left over to be allocated to it). In low demand the opaque channel price is $p_I^L = V - t/2$ and the profit is $\pi_I = (V/t - 1/2)(1 - K/H)Lt$. The profit for firms A and B is therefore

$$\pi_A = \pi_B = \left(V - \frac{K}{2H}t \right) \left(\alpha \frac{K}{2} + (1 - \alpha) \frac{K}{2H}L \right) + \frac{1}{2}(1 - \alpha) \left(\frac{V}{t} - \frac{1}{2} \right) \left(1 - \frac{K}{H} \right) Lt.$$

^{A1}This condition is required because we assume that $V/t > 1/2$ to ensure that if tickets are free, consumers located farthest from both firms (at $1/2$) have positive utility for them.

- $\frac{K}{H} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L} \leq \frac{V}{t} < 1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}$

Here, we construct the equilibrium as follows: both firms charge $p_A = p_B = \frac{V}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{4L}t$ and cover exactly $\frac{K}{2H}$ when demand is high and $\frac{V}{2t} - \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{4L}$ when demand is low. (For $\frac{V}{t} < 1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}$ this is $\leq \frac{1}{2}$.) In the high-demand state there is no leftover capacity for the opaque channel, while in the low-demand state the total uncovered market is $\left(1 - \frac{V}{t} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}\right)$. Thus, in the high-demand state the profit from opaque channel is zero. In the low-demand state the opaque channel price is $p_I^L = V - t/2$ and the profit is $\pi_I = \left(\frac{V}{t} - \frac{1}{2}\right) \left(1 - \frac{V}{t} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}\right) Lt$. The profit for firms A and B is therefore

$$\begin{aligned} \pi_A = \pi_B = & \left(\frac{V}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{4L}t\right) \left(\alpha \frac{K}{2} + (1-\alpha) \left(\frac{V}{2t} - \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{4L}\right) L\right) \\ & + \frac{1}{2}(1-\alpha) \left(\frac{V}{t} - \frac{1}{2}\right) \left(1 - \frac{V}{t} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L}\right) Lt. \end{aligned}$$

- $1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{2L} \leq \frac{V}{t} < \frac{3}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}$

We construct the equilibrium as follows: both firms charge $p_A = p_B = V - t/2$ and cover exactly $K/(2H)$ in the high-demand state and $1/2$ in the low-demand state. In both high- and low-demand cases, there is no leftover for the opaque channel and the opaque channel profit is zero. The profit for firms A and B is therefore

$$\pi_A = \pi_B = \left(\frac{V}{t} - \frac{1}{2}\right) \left(\alpha \frac{K}{2} + (1-\alpha) \frac{L}{2}\right) t.$$

- $\frac{V}{t} \geq \frac{3}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}$

We begin by assuming that the firms cover $K/(2H)$ each in the high-demand state but are in the “competitive equilibrium” in the low-demand state. Thus, for prices p_A and p_B , $x_A = 1/2 + (p_B - p_A)/(2t)$ in the low-demand state. In either demand state, nothing is left over for the opaque channel. Thus, the firms’ profits are $\pi_A = p_A \left(\alpha \frac{K}{2} + (1-\alpha) \left(\frac{1}{2} + \frac{p_B - p_A}{2t}\right) L\right)$ and $\pi_B = p_B \left(\alpha \frac{K}{2} + (1-\alpha) \left(\frac{1}{2} + \frac{p_A - p_B}{2t}\right) L\right)$. Optimizing the above expressions simultaneously w.r.t. p_A and p_B , we obtain $p_A = p_B = \left(1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}\right) t$, $x_A = x_B = \frac{1}{2}$ and the optimal profits are $\pi_A = \pi_B = \left(1 + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}\right) \left(\alpha \frac{K}{2} + (1-\alpha) \frac{L}{2}\right) t$. For the equilibrium to exist, the consumer located at $1/2$ should have a non-negative utility. Mathematically, $V - p_A - \frac{t}{2} \geq 0 \Rightarrow \frac{V}{t} \geq \frac{3}{2} + \left(\frac{\alpha}{1-\alpha}\right) \frac{K}{L}$,

which we have already assumed.

The above analysis characterizes the equilibria for the full range of $V/t \geq 1/2$.

A2.2 Analysis of Mixed-Strategy Equilibrium

In this appendix, we provide a sketch of the derivation of the equilibrium quantities when firms follow mixed strategies.

Consider the second period when demand is low and the second-period market is represented by the interval $[x_A, x_B]$. First, take the case of transparent sales. Suppose firm A covers the market from x_A to x' and firm B covers the market from x' to x_B . Then, the profit of firm A is $p_A^{2,L}(x' - x_A)L$ and firm B is $p_B^{2,L}(x_B - x')L$. Next, take the case of opaque sales. If the opaque price is p_I^L and a ticket comes from firm A with probability γ_A , firm A's profit is $\delta\gamma_A p_I^L(x_B - x_A)L$ and firm B's profit is $\delta(1 - \gamma_A)p_I^L(x_B - x_A)L$. From this point on, we use $\delta = 1$ to conform with the rest of the paper (δ is the share of the revenues that the firms obtain from the opaque intermediary). Using the result in (1), for firm A we obtain

$$\gamma_A p_I^L(x_B - x_A)L = p_A^{2,L}(x' - x_A)L, \quad (\text{A1})$$

and, for firm B, we obtain

$$(1 - \gamma_A)p_I^L(x_B - x_A)L = p_B^{2,L}(x_B - x')L, \quad (\text{A2})$$

Adding (A1) and (A2) and using symmetry to argue $p_A^{2,L} = p_B^{2,L} = p^{2,L}$ we obtain

$$\begin{aligned} p_I^L(x_B - x_A)L &= p^{2,L}(x_B - x_A)L, \\ \Rightarrow p^{2,L} &= p_I^L \end{aligned} \quad (\text{A3})$$

Similarly, for the high demand case, if a market exists, firm A's profit from transparent sales is $p_A^{2,H}(K/2 - x_A H)$, firm A's profit from opaque sales is $\gamma_A p_I^H(K - x_A H - (1 - x_B)H)$, firm B's profit from transparent sales is $p_B^{2,H}(K/2 - (1 - x_B)H)$ and firm B's profit from opaque sales is $(1 - \gamma_A)p_I^H(K - x_A H - (1 - x_B)H)$. As before, equating the profit from different strategies for each

firm and adding, we obtain

$$\begin{aligned}
p_I^H(K - x_A H - (1 - x_B)H) &= p^{2,H}(K - x_A H - (1 - x_B)H), \\
\Rightarrow p^{2,H} &= p_I^H
\end{aligned} \tag{A4}$$

In the case of opaque selling in the second period with symmetric firms, in the cases of low and high demand (if a market exists), consumers expect that tickets come from either firm with probability equal to $1/2$ and hence the price in the opaque channel is $p_I^L = p_I^H = V - t/2$. (We do not show the details here due to space considerations; the proof proceeds exactly as in Appendix A1.2.) Hence, this implies that the prices in the second period in transparent sales are $p^{2,L} = V - t/2$ and $p^{2,H} = V - t/2$. (For an arbitrary value of δ , $p^{2,L} = p^{2,H} = \delta(V - t/2)$. Note that this is derived from condition (1) — the second-period profits from opaque and transparent sales are equal in both high and low demand and, due to the nature of the opaque market, prices are equal in the high- and low-demand opaque markets. This leads to the result that the prices charged for transparent tickets in the second period are equal under high and low demand.) At these prices, consumers in $[x_A, 1/2]$ purchase from firm A and consumers in $[1/2, x_B]$ purchase from firm B if sufficient tickets are available. Hence, for the case $x_A \leq K/(2H)$ the total profit for firm A across two periods is given by^{A2}

$$\pi_A = p_A^1 x_A (\alpha H + (1 - \alpha)L) + \alpha \left(V - \frac{t}{2} \right) \left(\frac{K}{2} - x_A H \right) + (1 - \alpha) \left(V - \frac{t}{2} \right) \left(\frac{1}{2} - x_A \right) L.$$

Now, consider the customer at x_A who is indifferent between purchasing a ticket in the first period from firm A and, in the second period, purchasing either an opaque ticket (with probability $q_A q_B$ if both firms choose opaque strategies) or a transparent ticket (with probability $(1 - q_A q_B)(1 - \alpha)$ if both firms use transparent sales and demand is low, and with probability $(1 - q_A q_B)\alpha(K/2 - x_A H)/((1/2 - x_A)H)$ if both firms use transparent sales and demand is high, assuming allocation

^{A2}Note that we can also obtain the profit expression by computing second-period profits from opaque sales. Both approaches give the same profit expression, and this is in compliance with (1).

is random when tickets are limited). For this customer, we have the indifference condition

$$V - p_A^1 - tx_A = q_A q_B \left(\alpha \left(V - p_I^L - \frac{t}{2} \right) + (1 - \alpha) \left(V - p_I^H - \frac{t}{2} \right) \right) \\ + (1 - q_A q_B) \left(\alpha \left(\frac{\frac{K}{2} - x_A H}{\left(\frac{1}{2} - x_A\right) H} \right) (V - p_A^{2,H} - tx_A) + (1 - \alpha) (V - p_A^{2,L} - tx_A) \right)$$

Using the values of the second-period prices ($p_I^L, p_I^H, p_A^{2,L}$ and $p_B^{2,H}$) and the above indifference condition, we obtain the profit expression for firm A in terms of the first-period prices and the mixing probabilities. Similarly, we obtain the profit expression for firm B. We then optimize the profits simultaneously to obtain the profit-maximizing values of the first-period prices p_A^1 and p_B^1 and the mixing probabilities q_A and q_B as

$$q_A = q_B = \sqrt{\frac{\alpha(H - K)}{(1 - \alpha)H + \alpha K}} \quad \text{and} \quad p_A^1 = p_B^1 = V - \frac{t}{2} \left(1 - \alpha \left(\frac{H - K}{H} \right) \right).$$

A2.3 Proof of Proposition 5.1

Consider any consumer at some position $x \leq 1/2$ who has the following beliefs about the location of the indifferent consumers: The consumer x believes that the consumer indifferent between buying in the first period from firm A and buying in the second period from firm A is located at x_A^e . Further, x believes the consumer indifferent between buying and not buying from A in the second period is located at $y_A^{H,e} \geq x_A^e$ when demand is high and at $y_A^{L,e} \geq x_A^e$ when demand is low.

We begin the analysis for firm A in the second period, with the indifferent customer located at x_A^e .

Second Period: In the second period, if demand is high, the number of products available is $(K/2 - x_A^e H)^+$ and the number of products demanded is $(x_A^e H - K/2)^+ + (y_A^{H,e} - x_A^e)H$. Hence, the probability of obtaining a product is

$$\min \left\{ 1, \frac{\min \left\{ (K/2 - x_A^e H)^+, (x_A^e H - K/2)^+ + (y_A^{H,e} - x_A^e)H \right\}}{(x_A^e H - K/2)^+ + (y_A^{H,e} - x_A^e)H} \right\}.$$

If demand is low, the number of products available is $(K/2 - x_A^e L)^+$ and the number of products demanded is $(x_A^e L - K/2)^+ + (y_A^{L,e} - x_A^e)L$ so that the probability of obtaining a product is

$$\min \left\{ 1, \left(\frac{K}{2} - x_A^e L \right)^+ / \left((x_A^e L - \frac{K}{2})^+ + (y_A^{L,e} - x_A^e) L \right) \right\}.$$

The expected surplus for the consumer at x for the second period is, therefore,

$$\begin{aligned} &= \alpha \min \left\{ 1, \frac{\min \left\{ \left(\frac{K}{2} - x_A^e H \right)^+, (x_A^e H - \frac{K}{2})^+ + (y_A^{H,e} - x_A^e) H \right\}}{(x_A^e H - \frac{K}{2})^+ + (y_A^{H,e} - x_A^e) H} \right\} (V - xt - p_A^{2H}) \\ &+ (1 - \alpha) \min \left\{ 1, \frac{\left(\frac{K}{2} - x_A^e L \right)^+}{(x_A^e L - \frac{K}{2})^+ + (y_A^{L,e} - x_A^e) L} \right\} (V - xt - p_A^{2L}). \end{aligned}$$

In the first period, if demand is high, the probability that a consumer will obtain a product is $(\min\{\frac{K}{2}, x_A^e H\} / (x_A^e H))$ and, if demand is low, the probability that this consumer will obtain a product is 1. Hence, the expected surplus for the consumer at x from buying in the first period is $(\alpha (\min\{K/2, x_A^e H\} / (x_A^e H)) + (1 - \alpha)) (V - xt - p_A^1)$.

Let x_A be the actual location of the indifferent consumer. Therefore, in the equilibrium, we can write

$$\begin{aligned} &\left(\alpha \left(\frac{\min\{\frac{K}{2}, x_A^e H\}}{x_A^e H} \right) + (1 - \alpha) \right) (V - x_A^e t - p_A^1) \\ &= \alpha \min \left\{ 1, \frac{\min \left\{ \left(\frac{K}{2} - x_A^e H \right)^+, (x_A^e H - \frac{K}{2})^+ + (y_A^{H,e} - x_A^e) H \right\}}{(x_A^e H - \frac{K}{2})^+ + (y_A^{H,e} - x_A^e) H} \right\} (V - x_A^e t - p_A^{2H}) \\ &+ (1 - \alpha) \min \left\{ 1, \frac{\left(\frac{K}{2} - x_A^e L \right)^+}{(x_A^e L - \frac{K}{2})^+ + (y_A^{L,e} - x_A^e) L} \right\} (V - x_A^e t - p_A^{2L}). \end{aligned} \quad (\text{A5})$$

Note that, trivially, we can ensure $y_A^{L,e} \geq x_A^e$ and $y_A^{H,e} \geq x_A^e$. The condition above looks quite imposing to solve, but we can simplify it considerably by dividing it into two cases: (1) when $x_A^e \leq K/(2H)$ and (2) when $x_A^e > K/(2H)$.

Assuming $x_A = x_A^e < K/(2H)$, the above simplifies to

$$V - x_A t - p_A^1 = \alpha (V - x_A t - p_A^{2H}) + (1 - \alpha) (V - x_A t - p_A^{2L}).$$

Assuming $x_A = x_A^e > K/(2H)$ (and $y_A^{L,e} L < K/2$, i.e., no stockout in the low-demand state), the

above simplifies to

$$\left(\alpha \frac{K}{x_A H} + (1 - \alpha) \right) (V - x_A t - p_A^1) = (1 - \alpha)(V - x_A t - p_A^{2L}).$$

In the second period, the indifferent customer x_A could be located anywhere. We analyze all possible realizations separately in the following subsections. We consider firm A but the analysis for firm B is identical.

- $x_A = x_A^e < K/(2H)$

Suppose all consumers correctly believe that $x_A = x_A^e < K/(2H)$. In the first period, denote the price charged by firm A by p_A^1 . In the second period, the firms know the state of demand to be high or low. Denote the prices charged by firm A in high- and low-demand states by p_A^{2H} and p_A^{2L} , respectively. Let the indifferent consumer be located at x_A . For this consumer, in the equilibrium and when expectations are consistent, we have (this consumer can always obtain a product if he wishes to buy)

$$\begin{aligned} V - p_A^1 - t x_A &= \alpha(V - p_A^{2H} - t x_A) + (1 - \alpha)(V - p_A^{2L} - t x_A) \\ \Rightarrow p_A^1 &= \alpha p_A^{2H} + (1 - \alpha) p_A^{2L}. \end{aligned}$$

Now consider a consumer at $x_A + \delta$ where $\delta > 0$ and such that $x_A + \delta < K/(2H)$. This consumer has the same belief x_A^e , which is consistent. Moreover, for this consumer the net utility from buying a product in the first period is $U_1 = V - p_A^1 - t(x_A + \delta)$, and the net expected utility from waiting to buy in the second period is $U_2 = \alpha(V - p_A^{2H} - t(x_A + \delta)) + (1 - \alpha)(V - p_A^{2L} - t(x_A + \delta))$. Using the fact that $p_A^1 = \alpha p_A^{2H} + (1 - \alpha) p_A^{2L}$, these utilities are equal. Hence, the consumer at $x_A + \delta$ is also indifferent between buying in the first period or waiting to buy in the second period. This argument can be extended to any consumer in the range $[0, K/(2H))$, which means that the belief x_A^e is incorrect in the equilibrium. Hence, an equilibrium with $x_A = x_A^e < K/(2H)$ does not exist.

- $x_A = x_A^e > K/(2H)$

The goal of this section is to show that this equilibrium does not exist for all values of the parameters V, t, K, L, H and α .

Consider the second period. Suppose demand is high. Then the firm stocks out in the first period, because the number of products is less than the demand in the first period $x^A H > HK/(2H) = K/2$. Now, suppose demand is low. We consider firm A and limit ourselves to the case when, even in low demand, it is a local monopoly and covers the line till $y_A^L \leq 1/2$. The firm charges a price $p_A^{2L} = V - y_A^L t$ where $y_A^L \leq 1/2$ and sells $(y_A^H - x_A)L$ products to make a profit of $\pi_A^{2L} = (V - y_A^L t)(y_A^L - x_A)H$, which is maximized at $y_A^L = (V + x_A t)/(2t)$, with $p_A^{2L} = (V - x_A t)/2$ and $\pi_A^{2L} = (V - x_A t)^2 L/(4t)$.

Next, consider the equation for the indifferent consumer. Under the assumption $x_A^e \geq K/(2H)$ and $y_A^{L,e} L < K/2$, i.e., no stock out in the low-demand state, the indifference condition is

$$p_A^1 = \frac{\alpha K/(x_A^e H) + (1 - \alpha)}{\alpha K/(x_A^e H) + 2(1 - \alpha)}(V - x_A t).$$

Writing the expression for the total expected profit of firm A as $\pi_A = p_A^1(\alpha K/2 + (1 - \alpha)x_A L) + (1 - \alpha)\pi_A^{2L}$ and differentiating w.r.t. to x_A , we obtain

$$x_A = \frac{\alpha K(VL(1 - \alpha) - \alpha K t - x_A^e H(1 - \alpha)t}{L(1 - \alpha)t(2H(1 - \alpha)x_A^e + 3K\alpha)}.$$

In equilibrium, we have $x_A^e = x_A$, which yields

$$x_A = x_A^e = \frac{\sqrt{\alpha K(\alpha K(H^2 + 9L^2 - 2HL) + 8(\frac{V}{t})HL^2(1 - \alpha) - \alpha K(H + 3L))}}{4HL(1 - \alpha)}.$$

Note that we need the following conditions to hold: $V/t \geq 1/2$, $x_A \geq K/(2H)$ and $y_A^L \leq 1/2 \Rightarrow V/t \leq 1 - x_A$. This equilibrium does not always exist. For instance, when $L = 1/2$, $K = 1$, $H = 3/2$, $\alpha = 1/2$, $t = 1$ and $V = 2/3$, the equilibrium does not hold.

- $x_A = x_A^e = K/(2H)$

Suppose all consumers correctly hold the belief that $x^A = x_A^e = K/(2H)$. Consider the indifferent consumer at $x_A = K/(2H)$. In the first period, irrespective of demand being high or low, this consumer can obtain a product at price p_A^1 . In the second period, if demand is high, no products are being sold. If demand is low, the consumer can obtain a product at price p_A^{2L} . For this consumer,

we can therefore write

$$\begin{aligned} V - tx_A - p_A^1 &= (1 - \alpha)(V - tx_A - p_A^{2L}) \\ p_A^1 &= \alpha(V - Kt/(2H)) + (1 - \alpha)p_A^{2L}. \end{aligned}$$

Next, consider a consumer to the left of this indifferent consumer, at $x_A - \epsilon$, $\epsilon > 0$, who holds the belief $x_A = x_A^e = K/(2H)$. In the first period, this consumer can obtain a product irrespective of high or low demand at price p_A^1 , which gives him net utility $U_1 = V - t(x_A - \epsilon) - p_A^1$. If the consumer waits for the second period, he can obtain a product only in the case of low demand at price p_A^{2L} . His net expected utility from waiting is $U_2 = (1 - \alpha)(V - t(x_A - \epsilon) - p_A^{2L})$. Then, $U_1 - U_2 = \alpha\epsilon t > 0$, which means that this consumer prefers to buy in the first period rather than wait.

Next, consider a consumer to the right of the indifferent consumer, at $x_A + \epsilon$, $\epsilon > 0$ who holds the belief $x_A = x_A^e = K/(2H)$. In the first period, this consumer can obtain a product if demand is high. (In the case of high demand, $K/2$ products are being bought by $K/(2H) \cdot H = K/2$ consumers and if this consumer wants to buy a product, $K/2$ products will be bought by $K/2 + \epsilon'$ consumers, and he will obtain a product with probability $\lim_{\epsilon' \rightarrow 0} \frac{K/2}{K/2 + \epsilon'} = 1$.) The consumer can also obtain a product in the low-demand state. In other words, he can obtain a product in the first period irrespective of high or low demand at price p_A^1 , which gives him net utility $U_1 = V - t(x_A + \epsilon) - p_A^1$. If he waits for the second period, he can obtain a product only in the case of low demand at price p_A^{2L} . His net expected utility from waiting is $U_2 = (1 - \alpha)(V - t(x_A + \epsilon) - p_A^{2L})$. Then, $U_1 - U_2 = -\alpha\epsilon t < 0$, which means that this consumer prefers to wait and buy in the second period.

Hence, a consumer to the left of the indifferent consumer prefers to buy in the first period, and a consumer to the right of the indifferent consumer prefers to wait for the second period, which is consistent with equilibrium beliefs. Hence, this equilibrium always exists. It now remains to characterize the equilibrium.

We first limit ourselves to the case in which, even in the low-demand state, each firm is a local monopoly. If demand is low, the firm charges a price $p_A^{2L} = V - y_A^L t$ where $y_A^L < 1/2$, and sells $(y_A^H - \frac{K}{2H})L$ products to make a profit of $\pi_A^{2L} = (V - y_A^L t)(y_A^L - \frac{K}{2H})L$. This profit is maximized at $y_A^L = \frac{V + Kt/(2H)}{2t}$, with $p_A^{2L} = \frac{V - Kt/(2H)}{2}$ and gives $\pi_A^{2L} = \frac{(V - Kt/(2H))^2}{4t}L$. Using $p_A^1 =$

$\alpha(V - Kt/(2H)t) + (1 - \alpha)p_A^{2L}$, we obtain $p_A^1 = ((1 + \alpha)/2)(V - (K/(2H))t)$. The firm's first-period profit is given by $\pi_A^1 = p_A^1(\alpha H + (1 - \alpha)L)K/(2H)$, and total profit is given by $\pi_A = \pi_A^1 + (1 - \alpha)\pi_A^{2L}$. However, we need to impose $y_A^{2L} \leq 1/2$, which gives the restriction $V/t < 1 - K/(2H)$.

For $1 - K/(2H) \leq V/t < 3/2 - K/H$, $y_A^{2L} = 1/2$, $p_A^{2L} = V - t/2$, $p_A^1 = \alpha(V - Kt/(2H)) + (1 - \alpha)(V - t/2)$, $\pi_A^{2L} = (1 - \alpha)(V - t/2)(1 - K/H)L/2$ and $\pi_A^1 = p_A^1(\alpha H + (1 - \alpha)L)K/(2H)$. This is the non-competitive equilibrium with each firm covering exactly half the line.

For $V/t \geq 3/2 - K/H$, $y_A^{2L} = 1/2$, $p_A^{2L} = t(1 - K/H)$, $p_A^1 = \alpha(V - Kt/(2H)) + (1 - \alpha)t(1 - K/H)$, $\pi_A^{2L} = (1 - \alpha)t(1 - K/H)^2 L/2$ and $\pi_A^1 = p_A^1(\alpha H + (1 - \alpha)L)K/(2H)$. This is the competitive equilibrium with each firm covering half the line. This completely characterizes the equilibrium for all values of V/t .

A2.4 Subgame Perfect Nash Equilibrium

In this section, we provide a sketch of the numerical analysis using the subgame perfect Nash equilibrium concept.

Numerical Analysis

The following is the order of events in the game when the firms adopt an opaque sales strategy.

1. In the first period, firm A prices its product at p_A^1 and firm B prices its product at p_B^1 and both firms declare intention of sales through an opaque channel.
2. Consumers strategically purchase or postpone purchasing. In making this decision, every consumer makes the calculation that if the consumer who is indifferent between purchasing from firm A and waiting is at x_A and the consumer who is indifferent between purchasing from firm B and waiting is at x_B then, if demand is low, the availability in the opaque market from firm A will be $l_A^L = \max\{K/2 - x_A L, 0\}$ and from firm B will be $l_B^L = \max\{K/2 - (1 - x_B)L, 0\}$, respectively, and, if demand is high, the availability in the opaque market from firm A will be $l_A^H = \max\{K/2 - x_A H, 0\}$ and from firm B will be $l_B^H = \max\{K/2 - (1 - x_B)H, 0\}$, respectively, where x_A and x_B are functions of p_A and p_B .
3. At the end of period 1 and before period 2 begins, demand uncertainty is resolved, the level of demand is determined as H or L and is observed by the firm and the consumers. All consumers who did not purchase in the first period have observed the first-period prices and now know the

realization of demand, so they have expectations of product availability as described above. I.e., the probabilities of availability from firm A in the opaque market will simply be the ratios of leftovers, i.e., $\gamma_A^L = l_A^L / (l_A^L + l_B^L)$ (and $\gamma_B^L = 1 - \gamma_A^L$) in the low demand state and $\gamma_A^H = l_A^H / (l_A^H + l_B^H)$ (and $\gamma_B^H = 1 - \gamma_A^H$) in the high demand state. The leftover products, if any, are made available to the opaque intermediary I , who then sets a price p_I^H if the demand realization is H or a price p_I^L if the demand realization is L .

4. Consumers who have not purchased in the transparent channel now make their purchasing decision in the opaque channel.

We solve this game by backward induction. Below, we discuss the case in which there is a stockout in the second period in high demand. (The analyses of other cases, e.g., the case in which there is no stockout in the second period in high demand, are similar.) First, consider the opaque intermediary's problem given the market $[x_A, x_B]$. Suppose $\gamma_A^L = \frac{K/2 - x_A L}{K/2 - x_A L + K/2 - (1 - x_B)L} \leq 1/2$ and the opaque intermediary wants full coverage. Then, at price p_I^L , the surplus for a consumer at x is $V - p_I^L - \gamma_A^L t x - (1 - \gamma_A^L)t(1 - x)$, which is increasing in x . Hence, the intermediary will price such that the consumer at x_A has zero surplus, i.e., $p_I^L = V - \gamma_A^L t x_A - (1 - \gamma_A^L)t(1 - x_A)$. (Note that here we are using the *function* γ_A^L as defined above, while in the rational expectations equilibrium we were using the *point* expectation $\gamma_A^{L,e}$.) The intermediary's profit will be $\pi_I^L = p_I^L(x_B - x_A)L$.

In the first period, the firms A and B price so that their indifferent customers are located at x_A and x_B . Therefore, firm A prices such that $V - p_A^1 - t x_A = 0$ which gives $x_A = \frac{V - p_A^1}{t}$, and firm B prices such that $V - t(1 - x_B) - p_B^1 = \alpha(0) + (1 - \alpha)(\gamma_A^L(V - p_I^L - t x_B) + (1 - \gamma_A^L)(V - p_I^L - t(1 - x_B)))$ which gives

$$x_B = \frac{\left[\sqrt{4L\alpha(Kt(p_B^1 + t - V) + L((p_A^1)^2(\alpha - 1) + p_A^1(p_B^1 + t\alpha - 2V\alpha + V) - p_B^1(t + V) - (t - V)(t + V\alpha))) + (Kt - L(-p_A^1 + p_B^1 + t\alpha + t))^2} - Kt - Lp_A^1 + Lp_B^1 + Lt\alpha + Lt \right]}{2Lt\alpha}$$

Using the above expressions, we can write the profit function for the firms as

$$\begin{aligned}\pi_A &= p_A^1(\alpha K/2 + (1 - \alpha)x_A L) + \delta(\alpha(0) + (1 - \alpha)\gamma_A^L p_I^L(x_B - x_A)L) \\ \text{and } \pi_B &= p_B^1(\alpha K/2 + (1 - \alpha)(1 - x_B)L) + \delta(\alpha(0) + (1 - \alpha)(1 - \gamma_A^L)p_I^L(x_B - x_A)L),\end{aligned}$$

where γ_A^L is expressed in terms of x_A and x_B , and x_A and x_B are, in turn, expressed in terms of p_A^1 and p_B^1 . To find the equilibrium, we take the derivative of π_A wrt p_A^1 and the derivative of π_B wrt p_B^1 . We are able to obtain closed-form expressions for the derivatives. However, we are unable to algebraically solve the equations $\frac{\partial \pi_A}{\partial p_A^1} = 0$ and $\frac{\partial \pi_B}{\partial p_B^1} = 0$ simultaneously. Therefore, we resort to a numerical solution. To this end, we numerically characterize the price-response curve of firm A to firm B's price, and vice versa, for given values of the exogenous parameters. The point of intersection of these curves gives the first-period equilibrium prices, and all other quantities can be computed from these prices.^{A3} In particular, to generate the analog of Figure 3, we use the values $L = 1/2, K = 1, H = 3/2, t = 1, \delta = 1$ and vary the values of α between 0.05 and 0.95 in steps of 0.05 and V between 0.5 and 10 in steps of 0.5. (We also use other values of the parameters L, K and H and obtain qualitatively the same results.)

In the last-minute selling strategy, we effect a similar change in the game as in the opaque selling strategy. Specifically, we do not assume that consumers develop any point expectations of future availabilities at the beginning of the game. At the end of period 1, consumers still in the market know the prices charged by both firms in period 1 and can derive the availabilities from each firm as a function of these prices. We find that the indifferent customer is located at $K/(2H)$ for firm A and $1 - K/(2H)$ for firm B, and we obtain the same solution as in Section A2.3.

Upon comparing the equilibrium profits of the opaque selling strategy and the last-minute selling strategy computed as described above, we obtain qualitatively the same insights as in the main paper (where the rational expectations equilibrium concept is used). Specifically, we find that as the probability of high demand (α) increases, opaque selling is preferred for larger values of consumer valuation of the product (V/t). To show this with an example, we generate Figure A1, the analog of Figure 3. It is clear from comparing Figure 3 and Figure A1 that the choice of

^{A3}Note that, in this model with symmetric firms, we do not find any asymmetric equilibria for firm strategies from our numerical analysis. While this still leaves open the possibility that asymmetric equilibria might exist, we focus on the symmetric equilibrium alone.

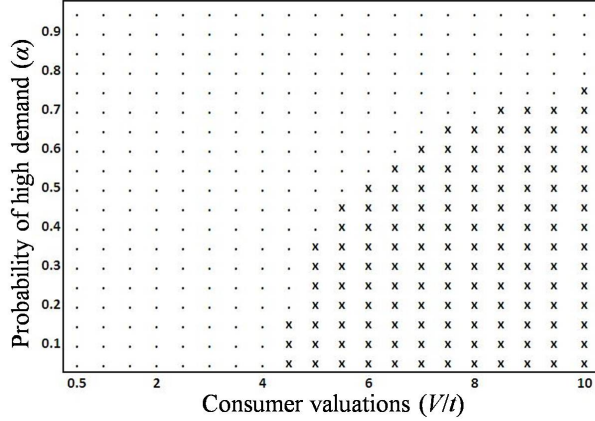


Figure A1: The analog of Figure 3 using a numerical solution of the subgame perfect Nash equilibrium. Opaque selling is optimal in the region with dots (\cdot) and last-minute selling is optimal in the region with crosses (\times). We use the values $L = 1/2, K = 1, H = 3/2, t = 1, \delta = 1$ for this figure. Qualitatively, the results of the subgame perfect Nash equilibrium and the rational expectations equilibrium are the same.

equilibrium concept does not change the insights from our model.

A3 Asymmetric Firms with Unequal Capacity

In this section, we provide a sketch of the solutions of the games when the firms are asymmetric with unequal capacity.

A3.1 Opaque Selling with Deterministic Demand

Consider the case when demand is low, i.e., the total capacity of the firms exceeds the maximum demand in the market. The solution is by backward induction.

In the second period, the opaque intermediary has access to customers in the range $[x_A, x_B]$. All customers have the belief that an opaque ticket comes from firm A with probability γ_A^e and from firm B with probability $\gamma_B^e = 1 - \gamma_A^e$. Since firm A is the smaller firm, consumers believe that $\gamma_A^e < 1/2$. The intermediary has to charge a price p_I and can choose to cover all or part of the market. A consumer in the opaque market at $x \in [x_A, x_B]$ has the ex ante expected surplus $V - p_I - \gamma_A^e tx - (1 - \gamma_A^e)t(1 - x)$. This can be written as $V - p_I - (1 - \gamma_A^e)t + (1 - 2\gamma_A^e)tx$, which implies that the surplus increases with x when $\gamma_A^e < 1/2$. Hence, in this market, the leftmost customer who purchases an opaque ticket obtains the minimum surplus and all other consumers

obtain a higher surplus.

Suppose the intermediary covers the full market, i.e., it prices such that the consumer at x_A has zero surplus. To achieve this, p_I is set by imposing $V - p_I - \gamma_A^e t x_A - (1 - \gamma_A^e)t(1 - x_A) = 0$, which gives $p_I = V - \gamma_A^e t x_A - (1 - \gamma_A^e)t(1 - x_A)$. The profit of the opaque intermediary is given by $\pi_I = (1 - \delta)p_I(x_B - x_A)J$.

Now, in the first period, firm A prices such that the consumer at x_A is indifferent between purchasing a transparent ticket from firm A and an opaque ticket. Since this consumer's ex ante expected surplus in the opaque channel is zero (as explained above), firm A sets its price by imposing $V - p_A - t x_A = 0$, which gives $p_A = V - t x_A$ and its profit is given by $\pi_A = p_A x_A J + \delta \gamma_A^e \pi_I$. Firm B prices such that the consumer at x_B is indifferent between purchasing a transparent ticket from firm B and an opaque ticket. This is ensured by imposing $V - p_B - t(1 - x_B) = V - p_I - \gamma_A^e t x_B - (1 - \gamma_A^e)t(1 - x_B) = 0$. Plugging in p_I from above, we obtain $p_B = V - t(1 - x_A - 2\gamma_A^e(x_B - x_A))$ and its profit $\pi_B = p_B(1 - x_B)J + \delta(1 - \gamma_A^e)\pi_I$.

The two firms simultaneously set prices p_A and p_B to maximize their profits, which determine the points $x_A(\gamma_A^e)$ and $x_B(\gamma_A^e)$ as functions of γ_A^e . The realized value of $\gamma_A(\gamma_A^e)$ is given by $\gamma_A(\gamma_A^e) = \frac{K_A - x_A(\gamma_A^e)J}{K_A - x_A(\gamma_A^e)J + K_B - (1 - x_B(\gamma_A^e))J}$. We now impose the rational expectations condition $\gamma_A(\gamma_A^e) \equiv \gamma_A^e$ to solve for the value of $\gamma_A^e (= \gamma_A)$, which completes the specifications of all the quantities.

In the case when the intermediary does not cover the full market, it prices such that a consumer at $x' \in (x_A, x_B]$ has zero surplus from an opaque ticket by imposing $V - p_I - \gamma_A^e t x' - (1 - \gamma_A^e)t(1 - x') = 0$, which gives $p_I = V - \gamma_A^e t x' - (1 - \gamma_A^e)t(1 - x')$. The profit of the intermediary is then $\pi_I = (1 - \delta)p_I(x_B - x')J$ and it maximizes this profit by choosing $x' = \frac{t(1 - \gamma_A^e) + t x_B(1 - 2\gamma_A^e) - V}{2t(1 - 2\gamma_A^e)}$. The rest of the solution proceeds in the same manner as above. Note that only one of the two equilibria, i.e., either the equilibrium with full coverage in the opaque market or the equilibrium with partial coverage in the opaque market, will exist for a given set of values of the model parameters.

The above calculations are analytically intractable, so we solve the game numerically. We confirm in our numerical experiments that $\gamma_A^e (= \gamma_A)$ is indeed less than 1/2 when firm A has lower capacity. We also rule out the case when $\gamma_A^e (= \gamma_A) > 1/2$. In other words, the second-period possibility that the larger-capacity firm has fewer tickets in the opaque market is off the equilibrium path. We present the results for a representative set of values in Section 6.1. Other values of the parameters yield qualitatively the same results.

A3.2 Opaque Selling with Uncertain Demand

There are two cases in the second period with opaque sales: either the demand is high or it is low. The intermediary has access to the market from $[x_A, x_B]$ and can choose to cover this market fully or partially. Consider the case when demand is low. All consumers in the market believe that the probability that an opaque ticket comes from A is $\gamma_A^{L,e} < 1/2$. In the case of full coverage, it prices such that the consumer at x_A has zero surplus. This is ensured by imposing $V - p_I^L - \gamma_A^{L,e}tx_A - (1 - \gamma_A^{L,e})t(1 - x_A) = 0$, which gives $p_I^L = V - \gamma_A^{L,e}tx_A - (1 - \gamma_A^{L,e})t(1 - x_A)$. The profit of the opaque intermediary in the low-demand case is given by $\pi_I^L = (1 - \delta)p_I^L(x_B - x_A)L$. In the case of high demand and full coverage, the intermediary covers the market $[x'_H, x_B]$ such that $(x_B - x'_H)H = K_A - x_AH + K_B - (1 - x_B)H$ (where $x'_H > x_A$ because demand is high and the number of tickets available are not enough to cover the full market $[x_A, x_B]$) and the price is therefore $p_I^H = V - \gamma_A^{H,e}tx'_H - (1 - \gamma_A^{H,e})t(1 - x'_H)$. (This ensures that, under the given beliefs on availabilities that consumers have, the price is the highest while remaining capacity is exhausted. Note that full coverage here does not mean coverage from $[x_A, x_B]$; it means that the intermediary prices to sell all tickets available.) Note that $\gamma_A^{H,e} < 1/2$ and the intermediary's profit is given by $\pi_I^H = (1 - \delta)p_I^H(K_A - x_AH + K_B - (1 - x_B)H)$.

In the first period, firm A prices to make the surplus of the consumer at x_A equal to zero, i.e., $p_A^1 = V - tx_A$. Firm B prices such that the consumer at x_B is indifferent between purchasing a transparent ticket from it or waiting for the opaque channel. The surplus from purchasing from firm B is $V - p_B^1 - t(1 - x_B)$. If the consumer waits for the opaque sales, then with probability $1 - \alpha$ the demand will be low and surplus will be $\gamma_A^{L,e}(V - p_I^L - tx_B) + (1 - \gamma_A^{L,e})(V - p_I^L - t(1 - x_B))$ and with probability α the demand will be high and surplus will be $\beta \left(\gamma_A^{H,e}(V - p_I^H - tx_B) + (1 - \gamma_A^{H,e})(V - p_I^H - t(1 - x_B)) \right)$, where $\beta = \frac{K_A - x_AH + K_B - (1 - x_B)H}{(x_B - x'_H)H}$ indicates the probability that the consumer gets a ticket if he wants one. (Note that the manner in which we have set x'_H , $\beta = 1$. This is different from the symmetric case where $\beta < 1$ in high demand in the opaque channel.) Hence, p_B^1 is set by solving

$$\begin{aligned} V - p_B^1 - t(1 - x_B) = & \alpha\beta \left(\gamma_A^{H,e}(V - p_I^H - tx_B) + (1 - \gamma_A^{H,e})(V - p_I^H - t(1 - x_B)) \right) \\ & + (1 - \alpha) \left(\gamma_A^{L,e}(V - p_I^L - tx_B) + (1 - \gamma_A^{L,e})(V - p_I^L - t(1 - x_B)) \right) \end{aligned}$$

From the above, the profit expressions for firm A and firm B are, respectively,^{A4}

$$\begin{aligned}\pi_A &= p_A^1 x_A (\alpha H + (1 - \alpha)L) + \delta(\alpha p_I^H \gamma_A^{H,e} (K_A - x_A H + K_B - (1 - x_B)H) + (1 - \alpha) p_I^L \gamma_A^{L,e} (x_B - x_A)L) \\ \pi_B &= p_B^1 (1 - x_B) (\alpha H + (1 - \alpha)L) + \delta(\alpha p_I^H \gamma_B^{H,e} (K_A - x_A H + K_B - (1 - x_B)H) + (1 - \alpha) p_I^L \gamma_B^{L,e} (x_B - x_A)L).\end{aligned}$$

The firms simultaneously solve for the prices to maximize the above profits, which gives us $x_A(\gamma_A^{L,e}, \gamma_A^{H,e})$ and $x_B(\gamma_A^{L,e}, \gamma_A^{H,e})$. The realized values γ_A^L and γ_A^H are given by

$$\begin{aligned}\gamma_A^L(\gamma_A^{L,e}, \gamma_A^{H,e}) &= \frac{K_A - x_A(\gamma_A^{L,e}, \gamma_A^{H,e})L}{K_A - x_A(\gamma_A^{L,e}, \gamma_A^{H,e})L + K_B - (1 - x_B(\gamma_A^{L,e}, \gamma_A^{H,e}))L}, \\ \gamma_A^H(\gamma_A^{L,e}, \gamma_A^{H,e}) &= \frac{K_A - x_A(\gamma_A^{L,e}, \gamma_A^{H,e})H}{K_A - x_A(\gamma_A^{L,e}, \gamma_A^{H,e})H + K_B - (1 - x_B(\gamma_A^{L,e}, \gamma_A^{H,e}))H}\end{aligned}$$

We now impose the rational expectations conditions $\gamma_A^L(\gamma_A^{L,e}, \gamma_A^{H,e}) \equiv \gamma_A^{L,e}$ and $\gamma_A^H(\gamma_A^{L,e}, \gamma_A^{H,e}) \equiv \gamma_A^{H,e}$ to solve for the values of $\gamma_A^{L,e}(= \gamma_A^L)$ and $\gamma_A^{H,e}(= \gamma_A^H)$, which completes the specifications of all the quantities.

In the case when the intermediary does not cover the full market in low demand, it prices such that the consumer at $x'_L \in (x_A, x_B]$ has zero surplus from an opaque ticket which gives the price $p_I^L = V - \gamma_A^{L,e} t x'_L - (1 - \gamma_A^{L,e}) t (1 - x'_L)$ and the profit $\pi_I^L = p_I^L (x_B - x'_L)$. In the case of high demand, $p_I^H = V - \gamma_A^{H,e} t x'_H - (1 - \gamma_A^{H,e}) t (1 - x'_H)$ and the profit $\pi_I^H = p_I^H (x_B - x'_H)$. The intermediary chooses prices to maximize these profit expressions. The rest of the solutions proceed in the same way.

Hence, there are four cases: (i) the opaque intermediary covers the market fully in both high and low demand, (ii) the opaque intermediary covers the market partially in both high and low demand, (iii) the opaque intermediary covers the market fully in low demand but partially in high demand and (iv) the opaque intermediary covers the market fully in high demand but partially in low demand. We have discussed the first case in full detail and the other cases are solved similarly. For a given set of parameter values, only one equilibrium exists.

The above calculations are analytically intractable, so we solve the game numerically. We confirm in our numerical experiments that $\gamma_A^{L,e}(= \gamma_A^L)$ and $\gamma_A^{H,e}(= \gamma_A^H)$ are indeed less than 1/2

^{A4}Here, we are assuming no stockout in the first-period transparent sales for either firm to focus on the dynamics in the opaque market. If either firm stocks out, there is no opaque market in the second period.

when firm A has lower capacity. We also rule out the cases when either is greater than $1/2$. In other words, the second-period possibility that the larger-capacity firm has fewer tickets in the opaque market is off the equilibrium path. We present the results for a representative set of values in Section 6.2. Other values of the parameters yield qualitatively the same results.

A3.3 Equilibrium for LMSS with Uncertain Demand

When $K_B \geq K_A$ and the firms sell products through their own channels in both periods, the equilibrium characterized below always exists. In the first period both firms set prices to cover $x_A = K_A/H$ and $1 - x_B = K_B/H$ of the market. If demand is high, no products are sold in the second period since the firms stock out in the first period. If demand is low, consumers located between $x_A = K_A/H$ and $x_B = 1 - K_B/H$ buy in the second period. Let the second-period coverage for firm A and firm B be denoted by $y_A^L - x_A$ and $x_B - y_B^L$, respectively, and the second-period prices in low demand be denoted by p_A^{2L} and p_B^{2L} , respectively. For different ranges of V/t , these quantities are as follow:

$\frac{V}{t}$	$y_A^L,$ y_B^L	$p_A^{2L},$ p_B^{2L}
$\frac{1}{2} \leq \frac{V}{t} \leq 1 - \left(\frac{K_A+K_B}{2H}\right)$	$\frac{V}{2t} + \frac{K_A}{2H},$ $1 - \frac{V}{2t} - \frac{K_B}{2H}$	$\frac{V}{2} - \frac{K_A}{2H}t,$ $\frac{V}{2} - \frac{K_B}{2H}t$
$1 - \left(\frac{K_A+K_B}{2H}\right) < \frac{V}{t} \leq \frac{3}{2} - \left(\frac{5K_A+3K_B}{4H}\right)$	$\frac{1}{2} - \frac{K_B-K_A}{4H},$ $\frac{1}{2} - \frac{K_B-K_A}{4H}$	$V - \frac{t}{2} + \left(\frac{K_B-K_A}{4H}\right)t$ $V - \frac{t}{2} - \left(\frac{K_B-K_A}{4H}\right)t$
$\frac{V}{t} > \frac{3}{2} - \left(\frac{5K_A+3K_B}{4H}\right)$	$\frac{1}{2} - \frac{K_B-K_A}{4H},$ $\frac{1}{2} - \frac{K_B-K_A}{4H}$	$\left(1 - \frac{3K_A+K_B}{2H}\right)t,$ $\left(1 - \frac{K_A+K_B}{H}\right)t$

In each of the above cases, the first-period prices in terms of p_A^{2L} and p_B^{2L} are as follow:

$$p_A^1 = \alpha \left(V - \frac{K_A}{H}t \right) + (1 - \alpha)p_A^{2L}$$

and $p_B^1 = \alpha \left(V - \frac{K_B}{H}t \right) + (1 - \alpha)p_B^{2L}.$

Note that the nature of the equilibrium characterized above is the same as in Proposition 5.1 and if $K_A = K_B = K/2$ then the values above also assume exactly the values in Proposition 5.1.