

The Effect of Risk on the CEO Market*

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Abstract

This paper presents a market equilibrium model of CEO assignment, pay, and incentives under risk aversion and moral hazard. Each of the three outcomes can be summarized by a single closed-form equation. In the presence of moral hazard, assignment is distorted from positive assortative matching on firm size as firms with higher risk or disutility choose less talented CEOs. Such firms also pay higher salaries in the cross-section, but economy-wide increases in risk or the disutility of being a CEO do not affect pay. The strength of incentives depends only on the disutility of effort and is independent of risk and risk aversion. If the CEO can affect firm risk, incentives rise and are increasing in risk and risk aversion. We calibrate the losses from various forms of poor corporate governance, such as failures in monitoring and inefficiencies in CEO assignment.

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This paper presents a market equilibrium model of CEO assignment, pay and incentives. Risk-averse managers of different talents are hired in a competitive market by heterogeneous firms, which vary in their size, risk, and level of effort required. The level of pay drives the assignment of talent to firms. The strength of incentives induces the efficient effort level, and is determined by an optimal contracting approach.

Our main contribution is to incorporate risk into a CEO market equilibrium in a tractable manner. A large empirical literature has shown that risk is a first-order determinant of compensation contracts (Demsetz and Lehn (1985), Garen (1994), Core and Guay (1999), Oyer and Schaefer (2004), Peters (2009), Peters and Wagner (2009)) but most theories assume risk-neutrality. This assumption is often necessitated the fact that adding risk aversion is typically a non-trivial extension that leads to highly complex contracts. Often, the contract cannot be solved in closed form, which makes it difficult to understand the economic intuition and see which features of the environment are driving which features of the contract. While Holmstrom and Milgrom (1987) develop frameworks that allow closed-form contracts under risk aversion, both consider only a single agent rather than a market containing multiple principals and multiple agents. Thus, most papers are forced to assume risk neutrality, for tractability rather than empirical realism.

We attempt a first step towards bridging the gap between theory and empirics by adding risk aversion into a model that features both a talent assignment problem and moral hazard. In an extension, we also allow risk to be influenced by the CEO rather than being an exogenous parameter. Despite the rich setup, the equilibrium can be summarized by three simple, closed-form equations, one for each of assignment, pay, and incentives. We achieve this by using the tractable incentive contracts developed by Edmans and Gabaix (2010). The model's tractability allow its economic forces to be transparent, yields clear empirical predictions for which factors do and do not matter for the three outcomes, and allows analysis of welfare consequences. Combining these three questions within a unifying framework generates a number of new implications unattainable from piecing together the results of individual models of each issue in isolation.

Despite the potential complexity caused by combining a talent assignment model with an agency problem under risk aversion, the equilibrium can be summarized by three simple, closed-form equations, one for each of assignment, pay, and incentives. The model's tractability allow its economic forces to be transparent, yields clear empirical predictions for which factors do and do not matter for the three outcomes, and allows analysis of welfare consequences. Combining these three questions within a unifying framework generates a number of new implications unattainable from piecing together the results of individual models of each issue in isolation.

First, we start with talent assignment. As in standard assignment theories, we model talent as affecting the maximum firm value that can be achieved in the absence of an agency problem. Without moral hazard, more talented CEOs work at larger firms to allow their talent to have the greatest impact. We show that this allocation is distorted in the presence of an agency problem.

A talented manager is a mixed blessing for two reasons. First, if utility is multiplicative in cash and effort, exerting effort is more costly to a talented and thus wealthy manager (e.g., a day of leisure is particularly valuable to a rich CEO as he can enjoy his wealth in leisure time). Thus, the firm must pay a rich CEO a greater premium for disutility of effort. Second, a manager who is already wealthy is less motivated by incentive pay and more willing to sacrifice it for leisure. The firm must therefore provide him with stronger incentives, which requires paying him a premium for risk. With multiplicative preferences, stock holdings must rise in proportion to his wage; combined with CRRA utility, this means the risk premium is also proportional to the wage. Thus, firms involving greater risk or disutility must pay particularly high premiums to hire talented managers, and so may prefer to appoint a “poor-and-hungry” CEO rather than a “rich-and-contented” alternative.¹ Some talented managers are hired by small firms, where their talent affects fewer assets, if such firms involve lower risk or disutility. While it is well-known that moral hazard under risk aversion leads to inefficient risk-sharing, we show in a market equilibrium that it also distorts real production.

We obtain closed-form solutions for the losses in total surplus due to inefficient risk-sharing and talent misallocation. The former depends on the average level of risk in the economy; the latter depends on the cross-sectional variance of risk but not its mean. If risk is high but constant across firms, it has no effect on a CEO’s choice of employer and so assignment is not distorted. The losses from misallocating managers are also increasing in the dispersion of managerial ability, as is intuitive. More surprisingly, they are decreasing in the dispersion of firm size and the size elasticity of talent. When size is more dispersed, or talent has a particularly strong impact on large firms, size becomes more important than risk in determining the equilibrium matching. Thus, assignment becomes closer to the efficient positive assortative matching on size. The sum of both inefficiencies is a measure of the losses from boards’ failure to control moral hazard through monitoring. If they instead must solve moral hazard through contracts, such contracts create distortions even if they are set optimally. Thus, direct monitoring and incentives are not perfect substitutes as governance mechanisms.

Second, the level of pay is increasing in firm size as in a pure assignment model. The addition of an agency problem means that pay also depends on a firm’s disutility and risk, as the CEO demands a premium for bearing them. Thus, firms with high risk or disutility not only hire less talented CEOs, but also pay their CEOs more relative to their skill level. Cross-sectionally, riskier firms pay more as found by Garen (1994); greater disutility has the same effect. Gayle and Miller (2009) show that firms that are more complex to manage or have greater agency problems (and thus stronger required incentives) pay their executives more.

However, what matters is not the absolute level of these parameters, but their magnitudes

¹Note that CEOs in our model have the same utility function. Thus, it is not that a “poor-and-hungry” CEO has a different cost of effort or risk aversion coefficient. CEOs differ only in their talent and thus reservation wage. Owing to multiplicative preferences, differences in the reservation wage translate into differences in the tendency to shirk, even though the utility function is not CEO-specific.

compared to other firms in the economy. Thus, aggregate changes in risk or the disutility of being a CEO (e.g., due to regulation or increasingly activist shareholders) do not affect pay: while working for one's current firm becomes less attractive, so do the outside options. This conclusion differs from the partial equilibrium model of Hermalin (2005), who argues that the recent strengthening in corporate governance increases the level of effort the CEO must exert and the risk of dismissal, and thus may explain the rise in pay over time. We show that in a market equilibrium, such economy-wide changes have no effect. Indeed, Peters and Wagner (2009) find that the effect on pay of dismissal risk is eight times as high along the cross-section as over the time series. The dependence of pay on outside options also highlights the importance of controlling for aggregate conditions (or at least time trends) in empirical analyses of pay.

Third, the strength of incentives is measured by the percentage change in CEO pay for a percentage firm return. It depends only on the disutility of effort and is independent of both risk and risk aversion.

The above core model is presented in Section 1. In Section 2, we extend the model to allow the CEO's actions to affect the variance as well as mean of firm returns. Specifically, actions that improve the average return also increase risk, such as the undertaking of a risky, positive-NPV project. While diversified shareholders do not care about idiosyncratic risk, a risk-averse CEO has private incentives to inefficiently forgo such a project. Therefore, if the CEO is more risk averse than a log agent, the contract becomes more convex to give the CEO a benefit from risk to offset his risk aversion. This result contrasts the argument that powerful incentives induce the CEO to take excessive risk, and thus if the CEO has control over risk, incentives should be weaker. Moreover, incentives are now *increasing* in risk and risk aversion, contrary to traditional models, which assume exogenous risk and predict a negative relationship. When the CEO is more risk-averse or the firm is riskier, it is necessary to give him even more convexity (and thus stronger incentives) to induce him to undertake a value-creating risky project. Indeed, Demsetz and Lehn (1985), Core and Guay (1999), Oyer and Schaefer (2004), and Coles, Daniel, and Naveen (2006) find a positive relationship between incentives and risk. For the same reason, incentives are increasing in the marginal increase in risk caused by value-enhancing actions. If the CEO mainly affects firm value by consuming perks, these actions have little effect on risk and so incentives are weaker, but if the CEO creates value by choosing risky projects, incentives are stronger. The link between incentives and the effect of value-enhancing actions on risk has both cross-sectional and time-series implications. Along the cross-section, "new economy" firms require the pursuit of risky growth opportunities (e.g., investing in R&D has little payoff if it fails). Indeed, Ittner, Lambert, and Larcker (2003) and Murphy (2003) find stronger incentives in new economy firms. Over time, as industries mature and competition intensifies due to globalization, "sure-fire" projects, which generate value with little risk become scarce, and enhancing firm value increasingly requires risk-taking. This may account for the rise in incentives, and in particular options, over time (e.g., Jensen and Murphy 2004.)

Our final theoretical extension allows for an elastic supply of CEO talent. We introduce a second labor market involving non-CEO jobs (e.g., entrepreneurship or consulting), which we call the non-corporate sector. This market provides both a secondary source from which corporate firms can hire, and an outside option for CEOs. An aggregate increase in the disutility of being a CEO (while holding constant the disutility of working in the non-corporate sector) now augments CEO pay, as firms must compensate CEOs to deter them from leaving to the non-corporate sector. Since the additional disutility is particularly costly for talented CEOs, corporate firms hire less skilled managers, reducing the value created by the corporate sector. The magnitude of the rise in pay, downgrade in talent, and value loss are all increasing in the size of the non-corporate sector, as this represents the extent of CEOs' outside options.

The two-sector model can also be used to analyze the effect of trends in a specific industry. For example, one "sector" could represent the financial industry, and the second all alternative jobs for such CEOs. An increase in regulation of the financial industry (e.g., in response to the recent crisis) may cause talented CEOs to leave. Since the outside options for financial CEOs are extensive (hedge funds and private equity houses in addition to executive positions at non-financial corporations), the value loss to the financial industry may be substantial.

Finally, our model's closed form solutions allow a calibration of the inefficiencies from various forms of poor corporate governance. Aggregating over the 500 largest firms in ExecuComp, if boards fail to monitor CEOs directly and instead must solve agency problems by contracting, we estimate losses from inefficient risk-sharing at \$2.0 billion per year, and misallocation at \$7.7 billion; the latter is an upper bound. The total inefficiency of \$9.7 billion is approximately twice the aggregate CEO salary. However, it is not substantial since firms are contracting and selecting CEOs optimally (given they cannot directly observe effort) and so the outcome is second-best efficient. A social planner also unable to observe effort would not be able to improve on the allocation. By contrast, if board failures instead lead to CEOs being randomly assigned to firms while retaining optimal contracting, the losses are approximately \$16 billion per year as a *lower* bound. Our model thus allows analysis of the losses from various manifestations of poor corporate governance. Naturally, all of these losses would be significantly higher when considering all top executives rather than just CEOs. Recent critics of governance focus on inefficiencies in contracting (see, e.g., Bebchuk and Fried 2004). We show that, even if boards set optimal contracts which are fully effective at preventing the CEO from taking pet projects, there can still be significant value loss resulting from misallocation of talent. Our paper thus highlights the importance of boards not only contracting efficiently, but also making correct hiring decisions – a role that has received relatively less attention. The absence of corporate scandals, excessive pay or perk consumption does not automatically mean that boards are performing effectively.

In addition to the results above, our paper makes two methodological contributions. One is solving an assignment problem whereby firms differ in disutility and risk, as well as size. In existing assignment models (e.g., Sattinger 1993; Gabaix and Landier 2008 ("GL"); Terviö

2008), both firms and workers differ in a single dimension (size and talent, respectively) and thus can be unambiguously ranked.² This allows for a simple solution to the assignment problem – positive assortative matching between the ranks. Assignment models are typically complex to solve if one or both sides vary across multiple dimensions, because this makes ranking difficult. We show that risk and disutility can be combined with size into a single dimension, which we call “effective” size, which we can use to unambiguously rank firms and thus achieve a tractable solution to a multidimensional allocation problem.

A second methodological contribution is achieving a closed-form solution to a model in which the agent affects the volatility, as well as mean of firm returns. Antecedents include Sung (1995) and Ou-Yang (2003), who use the Holmstrom and Milgrom (1987) framework that requires exponential utility, a financial cost of effort, continuous time, and Gaussian noise, and Dittmann and Yu (2010) who assume separable preferences and Gaussian noise. We allow for general noise distributions and non-separable utility.

This paper is related to a number of models of executive compensation. Himmelberg and Hubbard (2000) is an early attempt to jointly model pay and incentives, but the level of pay is not an equilibrium and the absence of closed-form solutions renders drawing implications difficult. Gabaix and Landier (2008), Terviö (2008), and Eisfeldt and Kuhnen (2010) present competitive assignment models of the managerial labor market, absent an agency problem. Edmans, Gabaix, and Landier (2009), Acharya and Volpin (2010), Axelson and Bond (2010), Dicks (2010), Falato, Li, and Milbourn (2010) add moral hazard but assume risk-neutrality and thus cannot investigate the effect of risk, risk aversion or risk-taking.³ Adding risk aversion is typically a non-trivial extension that leads to very complex contracts. Holmstrom and Milgrom (1987) derive simple contracts under the aforementioned assumptions of exponential utility, a financial cost of effort, continuous time, and Gaussian noise. However, as shown by Edmans, Gabaix and Landier (2009), a multiplicative non-financial cost of effort is necessary to generate realistic income effects and empirically consistent scalings of incentives with firm size. We thus use the modeling setup of Edmans and Gabaix (2010, “EG”) which yields closed-form contracts without restrictions on the utility function or cost of effort, while retaining the clarity of discrete time. As a result, the equilibrium can be summarized by three closed-form equations. Plehn-Dujowich and Subrahmaniam (2010) and Tsuyuhara (2010) consider a market equilibrium with risk aversion, where both firms and workers are ex ante homogeneous. Like us, Baranchuk, Macdonald, and Yang (2010) and Bandiera, Guiso, Prat, and Sadun (2010) allow

²In Eisfeldt and Kuhnen (2009), workers (not firms) differ on multiple characteristics; the model specifies that productivity is a weighted average of these characteristics, thus effectively representing a single dimension. Antràs, Garicano, and Rossi-Hansberg (2006) consider the allocation of workers to tasks, where both differ along a single dimension (skill and complexity, respectively). Kihlstrom and Laffont (1979) study the allocation of agents to jobs (either worker or entrepreneur) according to a single dimension, risk aversion. Galichon and Salanie (2009) do consider matching where both parties vary according to multiple dimensions, but require utility to be transferable across the matching parties and are unable to obtain closed-form solutions.

³de Bettignies and Chemla (2008) study the effect of competition for the CEO among principals on the power of incentives and the form of the contract. They consider a single manager.

for heterogeneous managers in a market equilibrium with risk aversion, but assume linear contracts in contrast to our optimal contracting approach. Acharya, Gabaar, and Volpin (2010) extend the standard assignment model to incorporate heterogeneity in corporate governance and show the allocation depends on governance, as well as size.

1. The Model

1.1 Incentive pay in partial equilibrium

We commence with a one-period model featuring a single firm and a single CEO (also referred to as the manager). This section is similar to EG; the main results come in Section 1.2 where we extend the model to a market equilibrium with multiple firms and CEOs. The Appendix provides proofs not given in the main text. The firm’s end-of-period stock price is given by:

$$P_1 = se^{a-\bar{a}+\eta}/E[e^\eta], \tag{1}$$

where s represents baseline firm size and $a \in [\underline{a}, \bar{a}]$ is the CEO’s action (“effort”). The action a refers to any decision that improves the stock price but is costly to the manager, such as exerting effort, forgoing private benefits, or choosing not to consume perks. Since there is a limit to the number of productive activities the agent can undertake to benefit the principal, we specify the firm’s end-of-period fundamental value as:

$$V_1 = se^{\min(a,\bar{a})+\eta-\bar{a}}/E[e^\eta]. \tag{2}$$

\bar{a} is the maximum productive effort level. For example, \bar{a} reflects zero stealing in a cash flow diversion model, taking all positive-NPV projects (while rejecting negative-NPV ones) in a project selection model, or a limit to the number of hours the CEO can work while remaining productive in an effort model. Actions $a > \bar{a}$ do not benefit the principal but improve the stock price, such as manipulation. We allow for the maximum feasible action $\bar{\bar{a}}$ to exceed the maximum productive action \bar{a} purely for technical reasons – when \bar{a} is an interior action, the incentive compatibility (IC) constraint to implement \bar{a} becomes an equality, which substantially simplifies the proofs. Shareholders maximize expected fundamental value net of CEO pay. We prove in the Appendix that, if firm size s is sufficiently high, maximum productive effort \bar{a} is optimal for the firm because the benefits of effort (which are multiplicative in s) outweigh the costs (disutility and inefficient risk-sharing, which are multiplicative in the CEO’s wage).⁴ We assume that V_1 is non-contractible and so CEO pay can only be made contingent on P_1 .

The variable η is mean-zero noise with standard deviation σ and bounded interval support.

⁴Implementing $a > \bar{a}$ is inferior as it does not improve firm value but imposes greater disutility and risk on the CEO.

The normalization by $E[e^\eta]$ in (1) and (2) ensures that expected firm value does not depend on the noise distribution. The CEO privately observes η before choosing a . EG show that this assumption leads to closed-form contracts in discrete time, as well as consistency with the optimal contract in continuous-time, where noise and actions are simultaneous.⁵ Note that the CEO remains exposed to risk, since he does not observe η until after signing the contract – as we will see, risk affects virtually all of our results.

On the equilibrium path where $a = \bar{a}$ is exerted, the initial stock price is $P_0 = e^{-\delta} E[P_1]$, where δ is the continuously compounded discount rate. Thus, the firm’s log stock return is:

$$r = \ln \frac{P_1}{P_0} = a + \eta + \mu, \quad (3)$$

with $\mu = \delta - \bar{a} - \ln E[e^\eta]$.

The CEO has no pre-existing wealth, and his utility is given by:

$$\begin{aligned} U(c, a) &= \frac{(ce^{-g(a)})^{1-\Gamma}}{1-\Gamma} \text{ for } \Gamma \neq 1 \\ &= \ln c - g(a) \text{ for } \Gamma = 1. \end{aligned} \quad (4)$$

c is the CEO’s monetary compensation. $g(a)$ captures the disutility of effort and is increasing and convex; in Section 1.2 we allow the cost function $g(\cdot)$ to depend on the firm that the CEO is working for, i.e., it is a firm rather than CEO characteristic.⁶ $\Gamma \geq 0$ denotes relative risk aversion. The CEO’s reservation utility is \underline{u} , which is exogenous in this section.

As in Edmans, Gabaix, and Landier (2009), effort has a multiplicative effect on both CEO utility (equation (4)) and firm value (equation (2)). When effort has a percentage effect on firm value, the dollar benefits of working are higher for larger firms. Most CEO actions can be “rolled out” across the entire firm and thus have a greater effect in a larger company. Multiplicative preferences consider private benefits as a normal good, i.e., the utility they provide is increasing in consumption. This is consistent with the treatment of most goods and services in consumer theory; they are also commonly used in macroeconomics (e.g., Cooley and Prescott 1995). This specification is also plausible under the literal interpretation of effort as forgoing leisure: a day of vacation is more valuable to a richer CEO as he has wealth to enjoy during it. Thus, the CEO’s expenditure on leisure and private benefits rises in proportion to his wealth – just as

⁵This timing assumption is also featured in models in which the agent sees total output before deciding how much to divert (e.g. Lacker and Weinberg 1989; DeMarzo and Fishman 2007; Biais, Mariotti, Plantin, and Rochet 2007), or observes the “state of nature” before choosing effort (e.g., Harris and Raviv 1979; Sappington 1983; Baker 1992; and Prendergast 2002). As in most of these papers, to focus on a single source of imperfection (unobservability of effort) we abstract from commitment problems and assume that the CEO cannot quit after η is realized. Quits can be prevented by raising the fixed component of pay (see Appendix F of EG.)

⁶More formally, the utility function is $\frac{(ce^{-G})^{1-\Gamma}}{1-\Gamma}$, where G is the disutility that working in the firm imposes on the CEO. Exerting effort a in firm n entails disutility $G = g_n(a)$, where g_n is a function denoting the cost of effort from working in firm n .

with CRRA preferences, an investor's allocation to risky assets rises in proportion to his wealth. Indeed, it is multiplicative preferences that generate the CRRA utility function (4).⁷ Thus, just as CRRA is typically favored over CARA in asset pricing and macroeconomics because it leads to realistic income effects, the same considerations motivate the use of a multiplicative rather than financial cost of effort here. In addition, Edmans, Gabaix, and Landier (2009) show that multiplicative preferences and production functions are necessary to deliver empirically consistent predictions for the scaling of various incentive measures with firm size.

We take an optimal contracting approach that does not restrict the contract to specific functional forms.⁸ The optimal contract is a general function $c(r)$ that implements $a = \bar{a}$, satisfies the participation constraint $E[U] \geq \underline{u}$, and has the minimum cost $w = E[c]$ to the firm. From Theorem 1 of EG, the optimal contract is as follows:

Proposition 1 (CEO Pay in Partial Equilibrium) *The optimal contract pays the CEO an amount c defined by⁹:*

$$\ln c = \Lambda r + K, \tag{5}$$

where $\Lambda = g'(\bar{a})$ and K is a constant that makes the participation constraint bind ($E \left[(ce^{-g(\bar{a})})^{1-\Gamma} / (1-\Gamma) \right] = \underline{u}$).

Proof The full proof is in EG; a heuristic proof is in The Appendix. ■

The contract in Proposition 1 has a simple form. It is attainable in closed form, and its slope depends only on the cost of effort Λ , but not on risk σ nor risk aversion Γ – these only affect the scalar K . The sensitivity Λ represents the percentage change in pay c for a given return r . The contract can thus be implemented by giving the CEO Λw of stock and $(1-\Lambda)w$ of cash.¹⁰ When considering the contract in terms of the effect of firm returns on dollar pay, Λ reflects the convexity of the contract. Thus, changes in Λ affect both the sensitivity of the contract (in percent terms) and its convexity (in dollar terms).

We parameterize $\eta = \sigma\varepsilon$, where ε has unit variance, and define

$$\bar{\Gamma}(\sigma^2) = 2 \left(\ln E[e^{\sigma\varepsilon}] - \frac{1}{1-\Gamma} \ln E[e^{(1-\Gamma)\sigma\varepsilon}] \right).$$

⁷Consider the general utility function $U(c, a) = \frac{e^{(1-\Gamma)(v(c)-g(a))}}{1-\Gamma}$. Our utility function (4) is a special case of this with $v(c) = \ln c$ (multiplicative preferences), which leads to CRRA. By contrast, Holmstrom and Milgrom (1987) assume that the cost of effort is financial, i.e., $v(c) = c$ and so the utility function becomes $e^{(1-\Gamma)(c-g(a))} / (1-\Gamma)$, which is CARA.

⁸Even though this is a hidden information model (the CEO learns η before choosing a), the optimal contract does not involve messages, as proven in EG. Intuitively, the firm wishes to implement \bar{a} for all η . Thus, on the equilibrium path, there is a one-to-one correspondence between r and η , which makes messages redundant.

⁹Our contract is written for r in the domain observed in the equilibrium path, i.e., for the domain observed conditional on $a = \bar{a}$. For values (not observed in equilibrium) outside of that domain, the contract pays a very low consumption ($c \rightarrow 0$) to the agent.

¹⁰Since r is a continuously compounded return, the contract must be rebalanced continuously so that the percentage of stock remains constant at Λ .

If ε is a standard Gaussian, then

$$\bar{\Gamma}(\sigma^2) = \Gamma\sigma^2$$

while for any distribution with finite expectations, we have

$$\bar{\Gamma}(\sigma^2) \sim \Gamma\sigma^2$$

for $\sigma \rightarrow 0$. $\bar{\Gamma}(\Lambda^2\sigma^2)/2$ is the risk premium required by a CEO receiving the contract in Proposition 1, in the sense that $\bar{\Gamma}(\Lambda^2\sigma^2)/2 = \ln E[c] - \ln U^{-1}(E[U(c)])$, where $U(c) = \frac{c^{(1-\Gamma)}}{1-\Gamma}$. This interpretation motivates our notation $\bar{\Gamma}$.

1.2 Incentive pay in market equilibrium

The simplicity of the contract in Proposition 1 allows it to be embedded into a market equilibrium where the expected wage w is endogenously determined. We use the equilibrium model of GL, which we summarize here. There is a continuum of firms of different size and managers with different talent. Firm $n \in [0, N]$ has size $S(n)$ and CEO $m \in [0, N]$ has talent $T(m)$. Low n denotes a larger firm and low m a more talented CEO: $S'(n) < 0$, $T'(m) < 0$. The CEO's talent increases firm value according to:

$$s = S + CTS^\gamma, \tag{6}$$

where γ parameterizes the size elasticity of the impact of talent and C the productivity of talent, which we later allow to be heterogeneous across firms. Since talented CEOs are more valuable in larger firms, the n th most talented manager is matched with the n th largest firm to allow their talent to have greatest impact. The variable s considered in Section 1.1 thus refers to firm size gross of talent and S refers to net size; going forward, unless otherwise stated, the term “size” will refer to S .

GL assume a Pareto firm size distribution $S(n) = An^{-\alpha}$, and the following asymptotic value for the spacings of the talent distribution: $T'(n) = -Bn^{\beta-1}$. As in GL, we consider the limit as $n/N \rightarrow 0$, i.e., the upper tail of the pay distribution. The equilibrium expected pay is:

$$w(n) = D(n_*) S(n_*)^{\beta/\alpha} S(n)^{\gamma-\beta/\alpha}, \tag{7}$$

where $S(n)$ is the size of firm n , n_* is the index of a reference firm (e.g., the median firm in the economy), $S(n_*)$ is the size of that reference firm, and $D(n_*) = -Cn_*T'(n_*) / (\alpha\gamma - \beta)$ is a constant. CEOs at large firms earn more as they are the most talented.

GL do not feature an agency problem and only specify the expected level of pay. We now incorporate the incentive model of Section 1.1 to determine the sensitivity of pay. We index the maximum effort level by \bar{a}_n to allow for heterogeneity in the level of effort required. Firms may also differ in their cost of effort, $g_n(a_n)$ (e.g., a firm in a regulated industry or headquartered

in an unattractive location is unpleasant to work for regardless of the effort a exerted by the CEO). The marginal cost of effort at the implemented effort level becomes $\Lambda_n = g'_n(\bar{a}_n)$. Risk may also vary and is indexed σ_n . We need not make any assumptions on how these parameters vary with n : since the contract implements $a = \bar{a}_n$, from (2), gross firm value remains at s as in the GL market equilibrium.

The expected utility of firm n 's CEO is given by:

$$U_n = \frac{(w_n e^{-\chi_n})^{1-\Gamma}}{1-\Gamma},$$

where:

$$\chi_n = g_n(\bar{a}_n) + \frac{\bar{\Gamma}(\Lambda_n^2 \sigma_n^2)}{2} \quad (8)$$

denotes the “equivalent variation” (“EV”) associated with firm n , i.e., the loss suffered by the manager from disutility (the $g_n(\bar{a}_n)$ term) and risk (the $\bar{\Gamma}(\Lambda_n^2 \sigma_n^2)/2$ term). The latter arises because the CEO has a fraction Λ_n of his pay invested in the firm, and firm returns have volatility σ_n . After adjusting for the EV, CEO n 's “effective” wage is:

$$v_n = w_n e^{-\chi_n}. \quad (9)$$

Define $\bar{\chi}$ as the average of the firms' EVs:

$$e^{-\bar{\chi}} = E[e^{-\chi_n/(\alpha\gamma)}]^{\alpha\gamma}. \quad (10)$$

CEO assignment, pay, and incentives in market equilibrium are given below:

Theorem 1 (CEO Pay in Market Equilibrium) Rank managers by their talent T_n and firms by their “effective size” defined by:

$$\widehat{S}_n = S_n e^{-\chi_n/\gamma}. \quad (11)$$

In equilibrium, the manager of rank n runs a firm whose effective size is ranked n , and receives an expected pay:

$$w_n = D(n_*) S(n_*)^{\beta/\alpha} S_n^{\gamma-\beta/\alpha} \exp\left(\frac{\beta}{\alpha\gamma}(\chi_n - \bar{\chi})\right), \quad (12)$$

where χ_n and $\bar{\chi}$ are defined by (8) and (10), $S(n_*)$ is the size of the reference firm, and $D(n_*)$ is a constant independent of firm size. The actual pay c_n is given by:

$$\ln c_n = \Lambda_n r_n + \ln w_n - \ln E[e^{\Lambda_n r_n}]. \quad (13)$$

Proof (Sketch). Assume that in market equilibrium, a CEO of talent $T(m)$ receives an effective wage (adjusted for effort and risk) of $v(m)$. If firm n wishes to hire manager m , it must pay

him a effective wage $v(m)$ and thus a dollar wage $v(m)e^{x_n}$. It solves:

$$\max_m E \left[(S(n) + CS(n)^\gamma T(m)) \frac{e^\eta}{E[e^\eta]} - v(m)e^{x_n} \right]$$

i.e.

$$\max_m Ce^{-x_n} S(n)^\gamma T(m) - v(m). \quad (14)$$

Firm n behaves like a firm with “effective” size $(e^{-x_n})^{1/\gamma} S(n)$. The Appendix proves that it will pay the effective wage $v_n = D(n_*) (e^{-\bar{x}} S(n_*))^{\beta/\alpha} (e^{-x_n/\gamma} S)^{\gamma-\beta/\alpha}$. Taking into account the EV, the dollar wage is $w_n = v_n e^{x_n}$, which yields (12); (13) flows directly from Proposition 1. ■

Theorem 1 shows that CEO assignment, pay, and incentives in competitive market equilibrium can be summarized by three simple closed-form equations, (11)-(13). This tractability allows for clear comparative statics. Starting with managerial assignment, in standard models, firms and CEOs each vary along a single dimension (size and talent, respectively). This allows for a relatively simple solution to the assignment problem – positive assortative matching, where the CEO with the highest attribute is matched to the firm with the highest attribute. Assignment models are typically difficult to solve where there is firm heterogeneity along multiple dimensions, since it is unclear how to rank the firms and determine which is the “best” firm to be matched with the most talented CEO. The above proof sketch shows that risk σ_n and the marginal cost of effort Λ_n can be combined with size S_n into a single dimension, “effective” size $S_n e^{-x_n/\gamma}$, which can be unambiguously ranked and determines the equilibrium matching.

In assignment models without moral hazard, more talented managers are assigned to larger firms; this is efficient because talent has a greater impact in a bigger firm. We show that adding an agency problem distorts this efficient allocation. A firm with a higher cost of effort must pay a greater salary to compensate. Given multiplicative preferences, exerting effort is particularly costly for talented, highly-paid CEOs. For example, a day of vacation yields high utility to a rich CEO as he has income to spend during it. Therefore, the compensation for disutility is proportional to the CEO’s wage. The required compensation for risk is also proportional to the CEO’s wage. The incentive contract (13) pins down the fraction Λ_n of the CEO’s salary that must be paid in stock. CEOs that are already wealthy are less motivated by incentives, and thus must be given a greater dollar amount of stock to induce effort. Therefore, an increase in firm risk has a greater dollar effect on the variability of their pay, and requires the firm to pay them a higher dollar risk premium.¹¹ With multiplicative preferences, the CEO’s dollar stock holding is a percentage of his wage. Combined with CRRA, this means that the required risk premium is also a percentage of the wage. In sum, both disutility and risk force a firm to increase the

¹¹Bandiera, Guiso, Prat, and Sadun (2010) find that managers with steeper contracts are paid more, and Conyon, Core, and Guay (2010) shows that the higher salary of U.S. CEOs compared to their U.K. counterparts can be explained by the fact that the former hold greater equity incentives, and thus require compensation for risk.

salary of any manager that it hires by a given *proportional* amount, e^{χ_n} . Since this additional compensation is proportional to the CEO’s salary, it is higher for more talented managers and so skilled managers become relatively more expensive. Therefore, the firm chooses to hire a lower ability manager. Acharya, Gabarro, and Volpin (2010) find that firms with weaker governance employ high-talent managers. This is consistent with the model as tightly-governed firms impose constraints on the CEO and so are less attractive to work for. The prediction is also shared by the model of Hermalin and Weisbach (1998), in which skilled CEOs bargain for weaker governance as it imposes disutility on them. Similarly, Palia (2000) shows that firms in regulated industries hire lower-quality CEOs.

In sum, managerial talent is a double-edged sword. While a talented manager has the potential to improve firm value to a greater degree, he is also more expensive to incentivize: since he already commands a high salary, he is willing to forgo incentive pay to enjoy leisure. Indeed, Malmendier and Tate (2009) find that winning awards (which may lead to an upward revision of the market’s perception of the CEO’s talent) leads to CEOs pursuing outside opportunities, such as writing books and assuming board seats. Falato, Li, and Milbourn (2010) document that more talented CEOs, who are more difficult to incentivize through pay, are instead disciplined through greater turnover-performance sensitivity. The incentive problem is particularly severe if the firm involves high effort or risk. Thus, start-ups in particular may prefer to hire a “poor-and-hungry” CEO rather than a “rich-and-contented” alternative.

Turning to expected pay, (12) shows that the wage depends not only on firm size S_n as in GL, but also on how the firm’s cost of effort and risk (χ_n) compare to other firms in the economy ($\bar{\chi}$). Thus, CEOs are not paid only for their talent, but also to compensate for bearing risk and disutility. Nguyen and Nielsen (2010) use sudden deaths to identify managerial ability and show that, while there is generally a positive correlation between talent and salary, a significant number of low-ability managers are well paid. Holding $\bar{\chi}$ constant, an increase in χ_n augments the wage as a recompense for risk and disutility. Therefore, in the cross-section, firms with high EVs pay more. Indeed, Garen (1994) finds empirically that CEOs of riskier firms command higher pay. Note that there are two effects of risk on pay – on the one hand, it causes a firm to hire a less talented and thus cheaper manager (from (11)); on the other hand, it must pay the manager more. Equation (12) takes both effects into account. If a firm’s cost of effort and risk rose from $\bar{\chi}$ to χ_n and the firm did not change its manager, its pay would rise by $\exp(\chi_n - \bar{\chi})$. However, since the firm will trade down to a less talented manager, it rises by only $\exp\left(\frac{\beta}{\alpha\gamma}(\chi_n - \bar{\chi})\right)$.

Note that it is only the *relative* EV, $(\chi_n - \bar{\chi})$, that matters. Thus, disutility and risk only matter in the cross-section but not in the aggregate. If there was an economy-wide increase in risk or the disutility of being a CEO (e.g., due to regulation or activist shareholders), which increases the EV of all firms by the same absolute amount ξ , both χ_n and $\bar{\chi}$ increase by ξ ; $(\chi_n - \bar{\chi})$ and thus wages are unaffected – even though working for one’s present firm becomes

less attractive, so do the outside options.¹² Regarding the own-firm prediction, Peters and Wagner (2009) find that a one percentage point increase in the risk of firing augments pay by 4%-8% along the cross-section, but only 0.2%-1.3% over the time series. Peters (2009) shows that risk can explain the higher moments of the cross-sectional pay distribution. Regarding the cross-firm prediction, Acharya, Gabaar, and Volpin (2010) find that a firm pays higher salaries if its competitors are worse governed (and thus more attractive to work for). More generally, the dependence of pay on the aggregate variable $\bar{\chi}$ highlights the importance of controlling for economy-wide variables such as average risk (or at least time trends) in empirical analyses of the determinants of pay.

The effect of changes in $(\chi_n - \bar{\chi})$ on expected pay is scaled by $\beta/\alpha\gamma$. A higher α raises the dispersion of firm sizes, and a higher γ augments the size elasticity of talent.¹³ Both factors increase the importance of size for CEO assignment and pay, and mean that variations in χ_n are relatively unimportant – as can be seen in (11), the effect of χ_n on “effective” size is decreasing in γ . Hence α and γ appear in the denominator of (12). By contrast, a higher β raises the dispersion of CEO talent. When talent is more variable, firms are more willing to pay the required compensation to attract a talented CEO (rather than “trading down” to the next best CEO), and so $(\chi_n - \bar{\chi})$ has a higher effect on the wage.

Moving to the strength of incentives, (13) shows that it depends only on Λ_n , the cost of effort, and is independent of risk and risk aversion. Hence, Theorem 1 shows which parameters do and do not matter for the different components of the contract. The cost of effort affects the strength of incentives and the level of pay in the cross-section but not in the aggregate. Risk and risk aversion also augment the level of pay in the cross-section, but not in the aggregate. However, they have no effect on the strength of incentives. The familiar trade-off between incentives and risk, predicted by standard contracting models, may not apply to CEOs. Since CEOs impact the entire firm, if the firm is sufficiently large, the benefits of effort are sufficiently strong that the firm implements maximum effort regardless of risk or risk aversion.

Theorem 1 can be extended to allow for firm heterogeneity not only in total disutility \bar{a}_n and risk σ_n , but also the impact of CEO talent. This extension is given in the following Remark.

Remark 1 (*Heterogeneous Talent Impact*) *Let the effect of talent on firm value (6) be given by:*

$$s_n = S_n + C_n T S_n^\gamma, \tag{15}$$

where C_n parameterizes the productivity of talent in firm n . In equilibrium, the manager of

¹²This prediction assumes that a CEO’s only outside option is to become a CEO of another firm. If CEOs can find a job outside of the CEO market (as considered in Section 2.2), the more general prediction is that the cross-sectional elasticity of the wage to disutility and risk is higher than the market-wide elasticity.

¹³In mathematical terms, it is S^γ that matters for assignment, given equation (6). In turn, $S^\gamma(n) = An^{-\alpha\gamma}$: S^γ has a Pareto distribution with exponent $1/(\alpha\gamma)$. The higher $\alpha\gamma$ is, the more dispersed the distribution of elasticity-adjusted sizes S^γ .

rank n runs a firm whose “effective size”

$$\widehat{S}_n = S_n C_n^{1-\gamma} e^{-\chi_n/\gamma}$$

is ranked n .

A firm with high C_n particularly benefits from a talented manager and thus has a higher effective size. For example, firms with high growth opportunities or in an unregulated industry have significant scope for a talented manager to add value.¹⁴ Note also that talent impact C_n is a quite different concept from disutility $g_n(\bar{a}_n)$, and so the differential effects of these variables on the talent of the CEO hired are mutually consistent. $g_n(\bar{a}_n)$ reflects the total disutility the CEO must suffer when working for the firm (e.g., from regulation, being headquartered in an unfavorable location, or having to exert effort or forgo vacation days). These are inconveniences that are not mitigated by talent; in fact, they are particularly severe for talented managers owing to multiplicative preferences. By contrast, C_n reflects the impact that a talented CEO has on firm value if he exerts maximum effort: recall that gross firm value s only becomes (15) if $a_n = \bar{a}_n$, so it is C_n not \bar{a}_n that parameterizes the maximum potential value.¹⁵ Thus, C_n reflects the potential for the manager to add value through exploiting growth opportunities, innovating or changing strategy. We previously noted that Palia (2000) finds that regulated firms hire low-talented managers, which is consistent with the disutility caused by regulation. Remark 1 provides an additional reason for this result: regulated firms have lower talent-sensitivity C_n , since regulation limits the actions a talented manager can undertake to increase firm value. In sum, Remark 1 predicts that talented managers will be hired by firms with high growth potential, low risk, and low disutility.

One might also think that talent might affect the CEO’s productivity of effort, in addition to its effect on maximum firm value as measured by T . Unfortunately, it is very difficult to solve tractably an assignment model in which both sides differ along multiple dimensions; while we are able to go beyond prior literature by allowing for firm heterogeneity across multiple dimensions, CEOs can only differ along a single dimension (the parameter T) and so we cannot introduce a separate manager-specific parameter for the productivity of effort. However, note that T already captures the productivity of effort to a degree: since $V_{1n} = (S_n + C_n T S_n^\gamma) e^{\min(a, \bar{a}) + \eta - \bar{a}} / E[e^\eta]$, the marginal effect of increasing a on firm value is increasing in T . Hence, our single source of manager heterogeneity does incorporate the realistic notion that effort is more productive if the manager is talented.¹⁶

¹⁴Since growing firms are also likely to be risky, and risk *reduces* the talent of the CEO hired from Theorem 1, empirical testing of this prediction will have to control for risk.

¹⁵From equation (2), we have $E[V_1] = s e^{\min(a, \bar{a}) - \bar{a}}$ and so \bar{a} denotes the range of actions the CEO can take to destroy firm value (compared to the maximum effort benchmark) rather than create value. For example, \bar{a}_n is high in firms with free cash flow problems or weak governance.

¹⁶Even though effort has a higher dollar productivity for a talented manager, its percentage productivity (i.e.,

We conclude this section by highlighting the features in the model that generate our results. First, the positive *qualitative* relationship between the CEO’s wage and the required compensation for disutility and risk, and thus distortions in allocation, can be generated by other utility functions and do not require multiplicative preferences. (See the Appendix for a proof.) Multiplicative preferences are only necessary to deliver the *quantitative* result that, as the CEO’s wage rises, his dollar stock holdings must increase in direct proportion. Thus, if Λ_n is constant across firms, the fraction of pay that is in stock is independent across firms of different size, as found empirically by Gibbons and Murphy (1992) and Murphy (1999). This empirical consistency is not a new result – Edmans, Gabaix, and Landier (2009) already showed that multiplicative preferences are necessary to generate the size-independence of the stock fraction (albeit in a risk-neutral model) – instead, it provides the justification for using multiplicative preferences here. In turn, the direct proportionality (that results from multiplicative preferences and CRRA) leads to substantial tractability, as it means that many key variables scale with CEO pay. In particular, the required compensation for risk and disutility is proportional to the wage, so the effective wage is proportional to the actual wage. This is critical for the derivation of the “effective” size variable \widehat{S}_n that allows a tractable solution to a multidimensional allocation problem (see the proof sketch of Theorem 1).

Second, in standard models, the optimal effort level for the firm is a trade-off between the costs (disutility plus the risk imposed by incentives) and benefits of effort, and is typically very difficult to solve (see, e.g., Grossman and Hart 1983). Since CEOs can affect the entire firm value, the benefits of effort outweigh the costs and so the firm always implements maximum effort. This removes the need to analyze small trade-offs and leads to a simple optimal contract.

Third, the CEO observes the noise before taking his action. As shown in the heuristic proof and in EG, this leads to simple contracts such as (5). The intuition is that, since the CEO has observed η when taking his action, the IC condition ((33) in The Appendix) must hold state-by-state (i.e., for every possible realization of η). This tightly constrains the set of contracts available to the principal. If η was realized after a , the IC condition would only need to hold on average. Many contracts would satisfy the IC condition, and the problem becomes complex as the principal must solve for the cheapest contract out of this continuum.

1.3 Efficiency analysis

1.3.1 Losses from moral hazard under optimal contracting

In a pure assignment model, the efficient allocation involves positive assortative matching between talent and size. With an effort decision, the first-best allocation (that would occur if effort was observable) now involves assigning CEOs to firms based on their size and disutility.

the effect of effort on firm returns) is independent of talent, and so the incentive contract (13) is independent of T . Intuitively, since the market already knows that the manager is talented, the firm’s stock price is already high; thus, to increase the stock return, he has to work just as hard as an untalented manager.

It may be efficient for a talented CEO not to work for a large firm if it involves high disutility, because working is particularly painful for wealthy CEOs. If all CEOs are risk-neutral, then “effective size” becomes $S_n e^{-g_n(\bar{a}_n)/\gamma}$ and is based on size and disutility alone, and so the market equilibrium allocation is first-best efficient, just as in the risk-neutral model of Edmans et al. (2009). Thus, the addition of an effort decision without risk aversion does not lead to distortions: analogously, in a standard effort model, the first-best can be achieved if the agent is risk-neutral and does not face limited liability.

However, when risk aversion is added, the market allocation now depends on risk aversion, as well as size and disutility, and is second-best. Large firms that would benefit highly from a talented CEO nevertheless choose to hire a lower-ability CEO if they are risky. Thus, risk aversion leads to two sources of inefficiency. The first is inefficient risk-sharing between firms and CEOs, which also exists in a single-firm moral hazard model and does not affect production. The second, which is specific to a market equilibrium, is distortions in talent assignment that affect real productive activity.

We now derive closed-form expressions for both sources of inefficiency to analyze the cost of the moral hazard problem, even when it is fully solved by contracts. If corporate governance were perfect, boards would monitor the manager’s actions directly, achieving first-best. Given imperfect monitoring, moral hazard must be addressed with incentive pay. Even if such contracts are set optimally, the above inefficiencies remain. Direct monitoring and incentives are sometimes seen as substitute governance mechanisms; however, the former is more efficient as it does not lead to distortions.

Since inefficiency stems solely from risk, not disutility, for simplicity we set $g_n(\bar{a}_n) = 0 \forall n$. The EV thus becomes $\chi'_n = \bar{\Gamma}(\Lambda_n^2 \sigma_n^2)/2$; its mean $\bar{\chi}'$ is defined analogously using (10). Let

$$W = \int w(n) dn \tag{16}$$

denote the total salary received by CEOs, and normalize the wage of the least talented manager, $w(N)$, to 0. Since a wage of w is worth an “effective” wage of $w e^{-\chi'_n}$, the total loss due to inefficient risk-sharing is:

$$L_{RA} = \int [w(n) - w(n) e^{-\chi'_n}] dn.$$

If $\hat{T}(n)$ denotes the talent of the CEO assigned to firm n under the second-best allocation, the loss due to inefficient talent assignment is given by:

$$L_{Alloc} = \int CS(n)^\gamma T(n) dn - \int CS(n)^\gamma \hat{T}(n) dn.$$

The losses are given in Proposition 2.

Proposition 2 (*Losses from Moral Hazard under Optimal Contracting*) *The loss due*

to inefficient risk-sharing is:

$$L_{RA} = E \left[1 - e^{-\chi'_n} \right] W \quad (17)$$

and the loss due to inefficient talent assignment is:

$$L_{Alloc} = \frac{1}{\beta} E \left[e^{\frac{\beta}{\alpha\gamma}(\chi'_n - \bar{\chi}')} - 1 \right] W. \quad (18)$$

For small distortions, these expressions become:

$$L_{RA} \sim E[\chi'_n] W \quad (19)$$

$$L_{Alloc} \sim \frac{1 + \beta}{2\alpha^2\gamma^2} \text{var}(\chi'_n) W. \quad (20)$$

Proof See the Appendix. ■

Both sources of inefficiency are proportional to W , the total wage bill. This is intuitive: the economic importance of a distortion to a factor of production is proportional to its marginal product; for a worker this is measured by his wage. From (19), the approximate loss due to inefficient risk-sharing depends on the mean of χ'_n , since this affects the amount of risk the average CEO has to bear. By contrast, from (20), the approximate loss due to misallocation is proportional to the variance of χ'_n . If $\chi'_n = \bar{\chi}'$, the rankings of effective size $S_n e^{-\chi'_n/\gamma}$ coincide exactly with the rankings of size S and there is no distortion. It is *relative* differences in χ'_n that cause the rankings to differ and the assignment to be affected.

Holding W constant, α , β , and γ have the same effects on allocational efficiency L_{Alloc} as they do for the dispersion of wages in (12). The intuition is similar: when α and γ are high, distortions due to differences in χ'_n have a small effect. The ranking of effective size is similar to the ranking of unadjusted size and so assignment is little affected. By contrast, a higher β means that talent is more dispersed, and so the losses from misallocation of talent are greater.

Section 2.3 calibrates the magnitude of these losses. Note that we can already draw some conclusions from the analytical expressions in Proposition 2: the inefficiencies will be moderate as they are proportional to the total wage bill W rather than firm size. This is because firms are contracting efficiently and making optimal hiring decisions, given the need to pay a premium for risk and disutility. Indeed, the allocation is second-best efficient: given the existence of a moral hazard problem (the unobservability of effort), a social planner who has the same information as firms (and is thus also unable to observe effort) could not improve on the outcome. A formal proof is in The Appendix.

1.3.2 Losses from random assignment

For comparison with the above moderate losses, we now conduct the following thought experiment. Assume that poor corporate governance instead manifests in CEOs being randomly allocated, rather than a second-best optimal assignment, i.e., in addition to being unable to

monitor, boards make errors in hiring. Each firm in the top N by size hires a CEO at random from the top MN CEOs by talent, where $M \geq 1$ is a parameter we discuss below. The efficiency loss is

$$L_{Rand} = \int CS(n)^\gamma T(n) - \int CS(n)^\gamma \bar{T} dn,$$

where $\bar{T} = \frac{1}{MN} \int_0^{MN} T(n) dn$ denotes the mean talent.

Proposition 3 (Losses from Random Assignment) *If $\alpha\gamma > 1$, the losses from a random allocation of CEOs are infinite, $L_{Rand} = +\infty$. If $\alpha\gamma < 1$,*

$$L_{Rand} = \left[\frac{1 - \alpha\gamma + \beta}{(1 + \beta)(1 - \alpha\gamma)} M^\beta - 1 \right] \frac{W}{\beta}. \quad (21)$$

For $M = 1$, this specializes to:

$$L_{Rand} = \frac{\alpha\gamma}{(1 - \alpha\gamma)(1 + \beta)} W. \quad (22)$$

Proof See the Appendix. ■

Equation (20) showed that losses due to misallocation of talent resulting from moral hazard are decreasing in α and γ (holding W constant). By contrast, the losses due to random assignment are increasing in α and γ . In Proposition 2, assignment is second-best optimal. Thus, when effective firm size is more dispersed (α and γ are higher), variation in χ_n has a relatively small effect on the rankings of effective firm size and we remain close to positive assortative matching. The losses from second-best matching are thus lower. In Proposition 3, assignment is random. Thus, when effective firm size is more dispersed, the losses from random matching are higher. Since talent has a multiplicative effect on scaled firm size S^γ (equation (6)), the cost of random assignment of talent is a function of scaled firm size. When $\alpha\gamma > 1$, this mean firm size $E[S^\gamma]$ is infinite and so losses are infinite.

There are two natural choices for M . One is $M = 1$, i.e., the top N firms randomly choose from the top N CEOs, in which case the losses are given by (22). However, this is not an equal comparison with the losses from second-best assignment given in Proposition 2. With $M = 1$, all firms are guaranteed a CEO in the top N . By contrast, in the allocation of Proposition 2, a firm of size rank N hires a CEO of talent rank $N e^{\chi_n - \bar{\chi}}$. Therefore, the worst manager that can be hired has rank MN , where:

$$M = \sup e^{\chi_n - \bar{\chi}}. \quad (23)$$

Thus, a second natural choice for M in Proposition 3 is given by (23), in which case the worst manager that can be hired also has rank $N \sup e^{\chi_n - \bar{\chi}}$, just as in Proposition 2. With this choice of M , the losses under random and second-best allocation can be directly compared. Since

$1 < \frac{1-\alpha\gamma+\beta}{(1+\beta)(1-\alpha\gamma)}$, we have:

$$\begin{aligned} L_{Alloc} &= E \left[e^{\frac{\beta}{\alpha\gamma}(x'_n - \bar{x}')} - 1 \right] \frac{W}{\beta} \\ &\leq (M^\beta - 1) \frac{W}{\beta} = \left[\frac{1 - \alpha\gamma + \beta}{(1 + \beta)(1 - \alpha\gamma)} M^\beta - 1 \right] \frac{W}{\beta} = L_{Rand}. \end{aligned}$$

Thus, $L_{Alloc} \leq L_{Rand}$ as is intuitive: second-best matching is superior to random matching. The difference is increasing in α and γ , as these variables raise the dispersion of effective firm size and thus importance of second-best matching.

Recent criticism of corporate governance has centered around inefficiencies in pay-setting (e.g., Bebchuk and Fried 2004). The results of Section 1.3 show that, even if boards are able to set optimal contracts that are fully effective at preventing the CEO from shirking and consuming perks, there may be significant value losses. Section 1.3.1 shows that, if boards are unable to monitor and must use contracts to solve agency problems, there can be non-trivial losses, even if their hiring decisions are second-best optimal. Section 1.3.2 shows that the losses are even greater if inability to monitor is compounded by errors in hiring.

2. Extensions

2.1 Providing risk-taking incentives

2.1.1 General theorem

In the core model, the CEO can improve the mean return r without changing risk, which is exogenous at σ . In reality, increasing firm value may require taking on risky, positive-NPV projects. Indeed, many commentators argue that a major goal of incentive compensation is to induce managers to take actions that improve firm value even if they augment risk (see, e.g., Core, Guay and Larcker 2003). In this section, we endogenize risk so that it depends on the mean return chosen by the CEO. In a standard model with effort and risk-taking where noise follows the action (e.g., Dittmann and Yu 2010), the above choice can be modeled by allowing the CEO to choose a single action a , which affects both the mean and volatility of the return. Since the action affects the distribution of the noise, the noise must follow the action. However, the framework we use to achieve tractability requires no noise to follow the CEO's final action, so that the IC constraints hold state-by-state. We therefore operationalize the CEO's risk choice by extending the model to two periods. The first-period action a_1 affects both the mean of the first-period signal r_1 and the volatility of the second-period signal r_2 . (In this subsection, subscripts index time periods rather than the rank of a firm or CEO.) The second-period action a_2 affects the mean of r_2 only, since there is no noise to follow this action.

As with earlier, we solve for the cheapest contract that implements \bar{a} in each period. We

conjecture that $a_1 = \bar{a}$ remains optimal in the two-period model if the firm is sufficiently large and so the benefits of effort outweigh the costs. Given the high complexity of proving the optimality of maximum effort in a multi-period context (see Edmans, Gabaix, Sadzik, and Sannikov 2010), we do not attempt a formal proof here.

The full timing is as follows:

1. Noise η_1 is privately observed by the CEO.
2. The CEO chooses a_1 .
3. The signal $r_1 = a_1 + \eta_1$ is publicly observed.
4. Noise η_2 is privately observed by the CEO.
5. The CEO chooses a_2 .
6. The signal $r_2 = a_2 + \sigma(a_1)\eta_2 + \mu(a_1)$ is publicly observed.

While r_2 depends directly on a_1 , the support of η_2 does not depend on a_1 . We have $\sigma'(a_1) \geq 0$, so that actions to improve firm value also entail augmenting risk (e.g., taking on risky, positive-NPV projects). To ensure that $E[e^{r_2} | a_1]$ is independent of a_1 (so that a_1 affects the volatility of firm value proportionally to e^{a_1}), we assume $E[\eta_2] = 0$ and take:

$$\mu(a_1) = -\ln E[e^{\sigma(a_1)\eta_2}]. \quad (24)$$

To our knowledge, the contracting problem where the agent affects the volatility as well as the mean has only been solved in specific cases. Sung (1995) and Ou-Yang (2003) study the Holmstrom and Milgrom (1987) case of exponential utility, a financial cost of effort, continuous time, and Gaussian noise, and Dittmann and Yu (2010) consider separable preferences and Gaussian noise in a one-period model.¹⁷ Therefore, before specializing to the utility function (4) used in this paper, we first derive the result for the more general utility function:

$$U(c, a_1, a_2) = u[(v(c) - g_1(a_1) - g_2(a_2))], \quad (25)$$

where $u(x) = e^{(1-\Gamma)x}/(1-\Gamma)$ for $\Gamma \neq 1$ and $u(x) = x$ for $\Gamma = 1$. The only assumption we make on v is that it is increasing and weakly concave. The utility function (4) corresponds to $v(c) = \ln c$ and a single action.

Theorem 2 (*Optimal Contract, Endogenous Risk*) *The optimal contract pays the CEO an amount c defined by:*

$$c(r_1, r_2) = v^{-1}(\Lambda_1 r_1 + \Lambda_2 r_2 + K) \quad (26)$$

¹⁷Lambert (1986) considers a model in which the agent takes separate effort and volatility decisions, in a model where output is restricted to three possible levels.

with

$$\Lambda_1 = \begin{cases} g_1'(\bar{a}) - \Lambda_2 \mu'(\bar{a}) - \frac{1}{1-\Gamma} \frac{d}{d\bar{a}_1} \ln E \left[e^{(1-\Gamma)\Lambda_2(\sigma(a_1)\eta_2)} \right]_{a_1=\bar{a}} & \text{if } \Gamma \neq 1 \\ g_1'(\bar{a}) - \Lambda_2 \mu'(\bar{a}) & \text{if } \Gamma = 1, \end{cases} \quad (27)$$

$$\Lambda_2 = g_2'(\bar{a}), \quad (28)$$

and K is a constant that makes the CEO's participation constraint bind.

For the particular case where η_2 is Gaussian, or the limit of small noises, then $\mu'(\bar{a}) = -\sigma(\bar{a})\sigma'(\bar{a})$, and so

$$\Lambda_1 = g_1'(\bar{a}) + [\Lambda_2 + (\Gamma - 1)(\Lambda_2)^2] \sigma(\bar{a})\sigma'(\bar{a}). \quad (29)$$

Proof See the Appendix. ■

2.1.2 Application to CRRA preferences

The utility function (4) corresponds to $v(c) = \ln c$. Applying Theorem 2 to this case yields the following result.

Proposition 4 (CEO pay in Partial Equilibrium, Endogenous Risk) *The optimal contract pays the CEO an amount c defined by:*

$$\ln c = \Lambda_1 r_1 + \Lambda_2 r_2 + K,$$

where Λ_1 and Λ_2 are given by (27) and (28). On the equilibrium path this can be rewritten:

$$\ln c = k + \eta,$$

where k is a constant that makes the CEO's participation constraint bind, and

$$\eta = \Lambda_1 \eta_1 + \Lambda_2 \sigma(\bar{a}) \eta_2 \quad (30)$$

is the total noise to which the contract exposes the agent.

The market equilibrium allocation and wage are given by equations (11) and (12) in Theorem 1, with the EV now defined by

$$\chi_n = g_{1n}(\bar{a}_n) + g_{2n}(\bar{a}_n) + \frac{\bar{\Gamma}(\eta_n)}{2},$$

where n indexes firm n 's risk and cost of effort, $\eta_n = \Lambda_{n1}\eta_{n1} + \Lambda_{n2}\sigma(\bar{a}_n)\eta_{n2}$, and we define (with a slight abuse of notation):

$$\bar{\Gamma}(\eta_n) = 2 \left(\ln E[e^{\eta_n}] - \frac{1}{1-\Gamma} \ln E[e^{(1-\Gamma)\eta_n}] \right).$$

Proposition 1 shows that, under exogenous risk, $\Lambda_1 = g'_1(\bar{a})$. We compare this with our slope under endogenous risk with small or Gaussian noises, equation (29). The core case is $\Gamma \geq 1$. Λ_1 is higher when the CEO affects firm risk, since the contract must now induce not only effort but also risk-taking. A risk-averse CEO may forgo risky, positive-NPV projects. To induce him to accept such a project, it is necessary to give him a more convex payout so that he benefits from risk. Since the strength of incentives Λ_1 also represents the convexity of dollar pay to firm value, this increased convexity is achieved by raising Λ_1 . Indeed, Gormley, Matsa, and Milbourn (2010) use a natural experiment to identify a positive causal impact of convexity on risk-taking.

The strength of incentives Λ_1 is increasing in four parameters. First, it is increasing in risk aversion Γ : the more risk-averse the CEO, the greater the convexity needed to overcome his risk aversion. For similar reasons, it is increasing in $\sigma(\bar{a})$ (the level of firm risk) and Λ_2 (the CEO's exposure to the risk induced by a_1 .) The positive relationship between incentives and risk $\sigma(\bar{a})$ contrasts the negative association predicted by standard models, which assume exogenous risk and posit a trade-off between incentives and risk-sharing, but is consistent with the empirical findings of Demsetz and Lehn (1985), Core and Guay (1999), Oyer and Schaefer (2004) and Coles, Daniel and Naveen (2006). Finally, Λ_1 rises in the marginal increase in risk caused by implementing all positive-NPV projects $\sigma'(\bar{a})$. The intuition is similar: the greater the risk imposed by a positive-NPV project, the greater the convexity the CEO must be given to induce him to take it. If the main way in which the CEO affects firm value is by not diverting cash flows, there is no link between risk and return and so $\sigma'(\bar{a}) = 0$ and $\Lambda_1 = g'_1(\bar{a})$. By contrast, if the key CEO action is the choice of risky projects, $\sigma'(\bar{a}) > 0$ and Λ_1 increases. $\sigma'(\bar{a})$ is likely to be high in new economy firms since they have little tangible capital and so enhancing firm value involves greater risk – investing in R&D has a zero payoff if it fails, whereas investing in an old economy plant generates liquidation value upon failure. Indeed, incentives are stronger in new economy firms (Ittner, Lambert, and Larcker 2003; Murphy 2003) and have risen over time (Jensen and Murphy 2004). Guay (1999) finds that incentives are more convex in firms with valuable risk-increasing projects.

An interesting benchmark case is that of a risk-neutral CEO. Plugging $\Gamma = 0$ into (29) gives

$$\Lambda_1 = g'_1(\bar{a}) + [\Lambda_2 - \Lambda_2^2] \sigma(\bar{a}) \sigma'(\bar{a}),$$

which is *lower* than $g'_1(\bar{a})$ if and only if $\Lambda_2 > 1$. A risk-neutral CEO only cares about the expected value of his compensation. If $\Lambda_2 > 1$, then his compensation is a convex function of

the firm’s market value¹⁸, and thus he has incentives to take excessive risk (i.e., choose an a_1 above the maximum productive level \bar{a}). A lower Λ_1 offsets this tendency and induces the CEO to reduce a_1 to the optimal level. In sum, our results contrast the argument (often made by critics of executive pay) that powerful incentives induce the CEO to take excessive risk, and thus if the CEO is able to affect risk as well as the average return, incentives should be weaker. For the core case of $\Gamma \geq 1$, incentives are unambiguously stronger; only if Γ is sufficiently low and Λ_2 is sufficiently high will incentives be shallower.

We note two additional points. First, even if $g_1(a) = 0 \forall a$ (i.e., the risk-increasing action is costless to the CEO), Λ_1 is non-zero – incentives are necessary not because the efficient action requires the CEO to exert effort, but because it exposes him to risk. This is consistent with the idea mentioned at the start of this section, that incentives are used to induce risk-taking, rather than solely to induce effort. Second, since a_1 affects $r_2 (= a_2 + \sigma(a_1)\eta_2 + \mu(a_1))$, it may seem that Λ_2 could be used to control the CEO’s choice of a_1 . However, Λ_2 is unchanged at $g'_2(\bar{a})$. This is because the time-2 IC condition must hold state-by-state (i.e., for every possible realization of η_2). In turn, this forces the slope of the contract (i.e., benefits from effort) to equal the marginal cost of effort, $g'_2(\bar{a})$. This is a similar intuition to the contract’s tractability, described at the end of Section 1.2 – since the IC conditions must hold state-by-state, the principal has little freedom in designing the contract.

2.2 Outside options

The core model considered a single labor market (CEOs) in fixed supply. In reality, CEOs may be able to find jobs outside the CEO market, and firms may hire managers currently employed in other sectors. We thus extend the model to allow for an elastic supply of talent. To do so in a tractable way, we assume the existence of an integrated market between the “corporate sector” and the “non-corporate sector.” The former represents the CEO labor market, and the latter represents alternative jobs, such as entrepreneurship or consulting. We assume that firms in both sectors initially have identical characteristics, and that the fraction of firms in the corporate and non-corporate sectors are respectively $1 - \pi$ and π . The probability that a firm is in the corporate sector is drawn independently from the distribution of firm sizes.

Theorem 1 showed that, if the disutility of being a CEO at any firm rises from $g_n(\bar{a}_n)$ to $g_n(\bar{a}_n) + \xi$, the level of pay is unchanged: while working for one’s own firm becomes less attractive, the outside option of being a CEO at another firm also becomes undesirable. We revisit this prediction in the case where the CEO has an additional outside option, the non-corporate sector, in which disutility is unchanged.

Proposition 5 (Outside Options) *Suppose that the disutility of working in the corporate sector rises from $g_n(\bar{a}_n)$ to $g_n(\bar{a}_n) + \xi$ for a small ξ . Then:*

¹⁸The dollar pay received by the CEO as a result of second-period performance is $e^{\Lambda_2 r_2}$. Substituting $r_2 = \ln(P_2/P_1)$ gives $(P_2/P_1)^{\Lambda_2}$, which is convex in P_2 if and only if $\Lambda_2 > 1$.

- (i) Log pay in the corporate sector increases by $\frac{\beta\pi}{\alpha\gamma}\xi$.
- (ii) The talent rank of a manager hired by firm n in the corporate sector rises from n to $n\left(1 + \frac{\pi}{\alpha\gamma}\xi\right)$, i.e., the sector hires less talented workers.
- (iii) The total loss of value creation (aggregate firm value gross of wages) by the corporate sector is $W\frac{\pi}{\alpha\gamma}\xi$, where W is the initial amount paid to CEOs in the corporate sector.

Proof See the Appendix. ■

Part (i) of Proposition 5 shows that the log wage increases by $\frac{\beta}{\alpha\gamma}\pi\xi$. The intuition behind the effect of α , β , and γ is the same as for their effect on the pay equation (12), discussed earlier: when α and γ are large, ξ has a small effect on the distribution of scaled size S^γ ; when β is high, firms are more willing to pay to retain talent. Part (ii) shows that a corporate firm hires a less talented manager. The intuition is similar to the distortion to CEO assignment caused by moral hazard, discussed in Theorem 1. Since corporate firms must pay a premium for the increased disutility of being a CEO, and the premium is multiplicative in the wage and thus greater for more talented workers, corporate firms hire less skilled agents. The intuition behind the effect of α and γ is the same as in equation (12); the dispersion of talent β has no effect since part (ii) refers to the talent rank of a manager. Part (iii) shows that the total loss in value created by the corporate sector is increasing in the aggregate pay of corporate CEOs W , for the same reason as in Proposition 2.

All three outcomes are increasing in π , the size of the non-corporate sector, as this represents the outside option. When outside options are larger, a higher wage premium is required to keep a CEO within the corporate sector (part (i)). (When the corporate sector is the entire economy ($\pi = 0$), pay does not change, since CEOs have no outside option; this is the result from Theorem 1.) This greater premium in turn leads to greater distortions in CEO assignment (part (ii)) and consequently more value loss (part (iii)).

Finally, the model can be extended to consider other top management positions than the CEO. For example, assume that there are integrated markets for all senior management positions (e.g., CEO, COO, CFO). If the disutility of being a CEO increases more than for other executive positions (due to regulation or shareholder activism), then the pay for CEOs will rise faster as a recompense. Similarly, if the CEO's potential impact on firm value has increased faster than other managers (e.g., due to opportunities resulting from globalization), then the optimal level of incentives becomes stronger, which necessitates paying the CEO a premium for risk. Bebchuk, Cremers, and Peyer (2010) argue that the CEO's high "pay slice" (proportion of total executive pay earned by the CEO) may reflect agency problems, but an alternative explanation is that it represents compensation for disutility and risk.¹⁹

¹⁹See Dasgupta and Ding (2010) for another explanation for the increasing gap between the pay of CEOs and other executives, based on the rise of search intermediaries for CEOs.

2.3 A calibration

We now undertake an approximate calibration of the efficiency losses in Section 1.3. We start with the losses from second-best assignment under moral hazard, given in Proposition 2. As in GL, we take $\alpha = \gamma = 1$ and $\beta = 2/3$ and consider the top 500 firms in Execucomp by aggregate value. For 2005, aggregate flow compensation (“tdc1” in Execucomp, winsorizing at the 5th and 95th percentiles) is $W = \$5$ billion.

We start with the estimation of L_{Alloc} , given by equation (18). The key challenge is to estimate $\chi'_n - \bar{\chi}'$. This depends on the marginal cost of effort, which is inherently unobservable and we are unaware of any previous studies that estimate it. However, an advantage of our unifying framework is that we can infer $\chi'_n - \bar{\chi}'$ by using the wage equation (12). Taking logs of this equation and rearranging yields:

$$y_n = \ln w_n - (\gamma - \beta/\alpha) \ln S_n,$$

where $y_n = k + \frac{\beta}{\alpha\gamma} (\chi_n - \bar{\chi})$ and k is a constant. We have:

$$E [e^{-y_n/\beta}] = e^{-k/\beta} E [e^{-(\chi_n - \bar{\chi})/\alpha\gamma}] = e^{-k/\beta}$$

by definition of $\bar{\chi}$, so $e^{-k} = E [e^{-y_n/\beta}]^\beta$. Using $E [e^{\frac{\beta}{\alpha\gamma}(\chi_n - \bar{\chi})}] = E [e^{y_n - k}]$, we obtain:

$$E [e^{\frac{\beta}{\alpha\gamma}(\chi_n - \bar{\chi})}] = E [e^{y_n}] E [e^{-y_n/\beta}]^\beta. \quad (31)$$

We use (31) as an estimate of $E [e^{\frac{\beta}{\alpha\gamma}(\chi'_n - \bar{\chi}')}]$ in equation (18). This yields an estimate of L_{Alloc} as \$7.7 billion. We note two potential issues with our approach. The first is that equation (18) contains $\chi'_n - \bar{\chi}'$, but (31) contains $\chi_n - \bar{\chi}$. Recall that $\chi_n = g_n(\bar{a}_n) + \frac{\bar{\Gamma}(\Lambda_n^2 \sigma_n^2)}{2}$ and $\chi'_n = \frac{\bar{\Gamma}(\Lambda_n^2 \sigma_n^2)}{2}$, so we are implicitly assuming that $g_n(\bar{a}_n)$ is the same across firms. Empirically, cross-sectional variation in χ_n from the average $\bar{\chi}$ may stem from variation in $g_n(\bar{a}_n)$, but the above approach attributes it entirely to differences in χ'_n . Since L_{Alloc} is increasing in the variance of χ'_n , this has the potential to overstate L_{Alloc} . One goal of the calibration is to highlight that the losses from random assignment in Proposition 3 are significantly greater than those from second-best assignment in Proposition 2. Thus, by providing an upper bound on L_{Alloc} , this approach works against us by underestimating the differences in losses.

The second caveat is that y_n may be mis-measured in practice: although firm size and the CEO’s wage are observable, it may be that the CEO’s actual wage differs from his market wage (e.g., if he is given deferred compensation). Again, measurement errors will overstate L_{Alloc} ; moreover, we can estimate the likely magnitude of the resulting bias. Let y_n^* denote the true value and y_n the observed value, and assume the classic errors-in-variables structure

$y_n = y_n^* + u_n$ where y_n^* and u_n are independent. This yields the decomposition:

$$E[e^{y_n}] E[e^{-y_n/\beta}]^\beta = E[e^{y_n^*}] E[e^{-y_n^*/\beta}]^\beta E[e^{u_n}] E[e^{-u_n/\beta}]^\beta.$$

Using the notation $\Phi(y) = E[e^{y_n}] E[e^{-y_n/\beta}]^\beta$, the decomposition can be rewritten:

$$\Phi(y) = \Phi(y^*) \Phi(u).$$

The measured $\Phi(y)$ overstates $\Phi(y^*)$ by a factor $\Phi(u)$, which exceeds 1 by Jensen's inequality. To estimate the magnitude of the bias, suppose that $u \sim N(0, \sigma_u^2)$. Then,

$$\Phi(u) = e^{\sigma_u^2/2} \left(e^{\sigma_u^2/2\beta^2} \right)^\beta = e^{(1+1/\beta)\sigma_u^2/2} = e^{\frac{5}{4}\sigma_u^2}.$$

If the measurement error is moderate (e.g., $\sigma_u = 0.2$), the bias is $e^{\frac{5}{4}0.2^2} = 1.05$ (i.e., only 5%). Indeed, replacing w by a three-year average has little effect on the results.

We now turn to L_{RA} , given by (17). This requires an estimate of χ'_n alone, rather than $\chi'_n - \bar{\chi}'$, and so we cannot use the above method. We thus infer the marginal cost of effort from observed contracts, under the assumption that firms are contracting efficiently. In our one-period model, where incentives stem only from newly-granted stock and options, Λ_n equals the percentage change in pay for a percentage point return. In reality, the bulk of a CEO's incentives stems from previously granted stock and options (see, e.g., Hall and Liebman 1998; Core, Guay, and Verrecchia 2003). Hence, Edmans, Gabaix, and Landier's (2009) measure of incentives is the dollar change in *wealth* for a one percentage point return, scaled by annual pay, which they call B^I .²⁰ While B^I measures the sensitivity of CEO wealth to the current period return, the CEO bears risk from changes in the stock price during his entire tenure as CEO. To convert B^I into an estimate of Λ , we assume the CEO works for L years and consumes only at the end (we suppress the dependence on n for brevity). Then, the contract becomes:

$$\ln c = \Lambda \sum_{t=1}^L r_t + K.$$

Thus, $\text{var}(\ln c) = \Lambda^2 \sigma^2 L$, and $\chi' = \frac{\Gamma(\Lambda^2 \sigma^2 L)}{2}$. We also have

$$B^I = \frac{d\text{Wealth}/dr}{w} = \Lambda \frac{\text{Wealth}}{\text{Wage}} = \Lambda R,$$

where $R = \text{Wealth}/\text{Wage}$, and since $\Lambda = (d\text{Wealth}/\text{Wealth})/dr$ in a multi-period model.

To estimate the CEO's wealth, we start by taking data from Dittmann and Maug (2007),

²⁰This dataset is available at <http://finance.wharton.upenn.edu/~aedmans/data.html>. Its construction is described in Appendix B of Edmans, Gabaix, and Landier (2009).

who estimate the CEO's non-firm wealth that results from past salary and bonus awards, and sales of stock and options previously granted by the firm.²¹ We then add the CEO's current wealth invested in the firm, from stock and options, to give a total wealth measure. Unfortunately, it is not possible to obtain data on the wealth of U.S. CEOs that does not stem from past or current executive compensation (e.g., real estate ownership or holdings of other securities), but this is a reasonable benchmark. CEOs in 2005 have been in their current position for a median of 5.0 years. This is an estimate of $L/2$ (since most CEOs will continue in office after 2005) and so it corresponds to $L = 10$. Taking the benchmark case of log utility ($\Gamma = 1$) and using the exact expression (17) yields $L_{RA} = 0.40W = \$2.0$ billion.²² In sum, the preliminary calibration suggests total efficiency losses from imperfect monitoring (while retaining optimal contracting) of approximately \$9 billion, twice the aggregate CEO salary.

We now turn to the losses from random allocation, given in Proposition 3. We focus on the case of $M = 1$ since this does not require estimation of the χ_n 's. Since this means that each firm is guaranteed to end up with a top-500 CEO, our results will represent a lower bound. To estimate $\frac{1}{1-\alpha\gamma}$ (in the denominator of (22)), we use the identity²³:

$$\frac{1}{1-\alpha\gamma} = E \left[\left(\frac{S}{S_*} \right)^\gamma \mid S \geq S_* \right]. \quad (32)$$

Indeed, for an arbitrary size cutoff S_* in the support of S , $P(S \geq xS_* \mid S \geq S_*) = x^{-1/\alpha}$, so

$$E \left[\left(\frac{S}{S_*} \right)^\gamma \mid S \geq S_* \right] = \int_1^\infty x^\gamma x^{-1/\alpha-1} \frac{1}{\alpha} dx = \frac{1}{\alpha} \left[\frac{-x^{-1/\alpha+\gamma}}{\frac{1}{\alpha} - \gamma} \right]_1^\infty = \frac{1}{1-\alpha\gamma}.$$

Taking $\gamma = 1$, the term $E \left[\left(\frac{S}{S_*} \right)^\gamma \mid S \geq S_* \right]$ can be estimated as the mean firm size above a cutoff S_* , divided by S_* . We define S_* as the size of the median firm in our top 500 (the 250th largest firm), which yields 5.3. From (22) we have $L_{Rand} \sim 5.2W / (5/3) = 3.1W \sim \16 billion. This is markedly greater than the combined losses from second-best assignment in Proposition

²¹This dataset is available at <http://people.few.eur.nl/dittmann/data.htm>. We thank Ingolf Dittmann for generously making this data available.

²²Note that this method is not suitable for estimation of L_{Alloc} : since χ'_n enters with a positive exponent, L_{Alloc} would be extremely sensitive to outliers in χ'_n resulting from outliers in volatility, wages, or stock and option holdings. The estimation of L_{RA} is much less affected since χ'_n enters with a negative exponent. In addition, as with any calibration, we assume real-world data is optimal and thus can be used to estimate Λ_n . If, however, governance failures manifest in suboptimal contracting, our measure of Λ_n is inaccurate.

²³Our calibration uses Zipf's law and constant returns to scale ($\alpha = \gamma = 1$). L_{Rand} in (22) is thus on the cusp of being divergent and so we empirically implement $\frac{1}{1-\alpha\gamma}$ with (32). (Note that the divergence is weak and logarithmic: the empirical counterpart of $E \left[\left(\frac{S}{S_*} \right)^\gamma \mid S \geq S_* \right]$ is proportional to $\ln N$ when drawing from a sample of N firms, as in Gabaix (2010): indeed our estimate of 5.3 is close to $\ln 500$.) However, we directly use the values $\alpha = \gamma = 1$ to calibrate $\alpha\gamma$ in the numerator of (22). Using the empirical average $E \left[\left(\frac{S}{S_*} \right)^\gamma \mid S \geq S_* \right]$ to estimate $\alpha\gamma$ (by backing it out from equation 32) would be a very poor estimator as it has a formally infinite variance.

2. Moreover, since our estimate of L_{Rand} is a lower bound (it assumes $M = 1$) and the estimate of L_{Alloc} is an upper bound, the true difference is likely to be significantly greater. Naturally, the losses from both imperfect monitoring and random allocation will be significantly higher if we consider all top executives, rather than just the CEO.

Our analysis is related to some recent papers that estimate efficiency effects in a market equilibrium, albeit of government intervention rather than governance failures. Dittmann, Maug, and Zhang (2010) calibrate the impact of various forms of restrictions on CEO compensation, such as taxes and limits on both ex ante salary and ex post realized pay, in an assignment model with loss aversion and moral hazard; Llense (2009) calibrates the effect of a pay cap in the absence of an agency problem (in which case there is no distinction between ex ante and ex post pay). Thanassoulis (2010) theoretically studies the effect of restrictions on bankers' pay, such as taxes and limits on bonuses.

3. Conclusion

This paper studies how CEO assignment, pay, and incentives depend on talent, talent impact, firm size, risk, and disutility in market equilibrium. The model's closed-form solutions allow the determinants of these three outcomes to be transparent, and clear empirical predictions. In talent assignment models without an effort conflict, the most talented managers are assigned to the largest firms. We show that this efficient allocation is distorted in the presence of moral hazard – a firm that is riskier or involves greater disutility hires a less talented CEO. The loss in efficiency is decreasing in the dispersion of firm size and size elasticity of talent, and increasing in the dispersion of managerial ability. If poor corporate governance instead manifests in a random assignment of CEOs to firms, the losses are significantly higher, and affected by the above parameters in the opposite direction.

Cross-sectional changes in risk and disutility increase the level of pay. Thus, risky firms not only hire less talented CEOs, but also pay their CEOs highly (relative to their skill level) as a recompense. However, aggregate changes in these variables have no impact as they affect the current firm and outside options equally. The strength of incentives is increasing in the disutility of effort, but independent of risk and risk aversion if the CEO only affects mean returns. If value-enhancing actions by the CEO also increase firm risk, the contract slope generally rises and exhibits a positive relationship with both risk and risk aversion.

While a number of the model's predictions regarding pay and incentives are consistent with existing empirical findings, some predictions regarding talent assignment are yet to be tested (given the difficulties of measuring talent)²⁴ and are potentially fruitful topics for future empirical research. In terms of future theoretical directions, it would be interesting to extend the

²⁴However, see Palia (2000), Acharya et al. (2010), Chang, Dasgupta, and Hilary (2010), Falato, Li, and Milbourn (2010), and Nguyen and Nielsen (2010) for promising approaches to measuring talent.

analysis to a dynamic model where CEOs can be fired or voluntarily move between jobs. Axelson and Bond (2009) consider a dynamic market equilibrium under risk-neutrality, and Tsuyuhara (2010) assumes homogeneous agents and firms. Whether tractability can be preserved under the combination of dynamics, risk aversion, and skill differences is an open question.

Appendix: Proofs

Proof of Proposition 1

This is a special case of Theorem 1 of EG. EG have the utility function $u(v(c) - g(a))$; our utility function (4) is a particular case of this with $u(x) = \frac{e^{(1-\Gamma)x}}{1-\Gamma}$ and $v(c) = \ln c$. We refer the reader to EG for the full proof, which does not use first-order conditions and rules out contracts that are stochastic or depend on messages. Here we give a heuristic proof that conveys the intuition, so that the intuition is self-contained within this paper. Given $r = a + \eta + \mu$, the agent's expected utility is given by:

$$E[U] = E \left[\frac{(c(a + \eta + \mu) e^{-g(a)})^{1-\Gamma}}{1-\Gamma} \right].$$

Since η is known when the agent takes his action, we can remove the expectations operator. The IC condition is thus:

$$\bar{a} \in \arg \max_{a \in [\underline{a}, \bar{a}]} c(a + \eta + \mu) e^{-g(a)}. \quad (33)$$

Taking the first order condition yields:

$$c'(\bar{a} + \eta + \mu) e^{-g(\bar{a})} - g'(\bar{a}) e^{-g(\bar{a})} c(\bar{a} + \eta + \mu) = 0,$$

i.e.,

$$\frac{c'(r)}{c(r)} = g'(\bar{a}) = \Lambda.$$

Since this must hold state-by-state (i.e., for every possible η and r found on the equilibrium path), this integrates to:

$$\ln c = \Lambda r + K. \blacksquare$$

Proof that maximum effort is optimal if s is sufficiently large

Consider a CEO with a reservation utility $\underline{u} = u(\ln v_n)$, where v_n is the “effective” dollar wage (defined later in equation (9)). Call $[\underline{\eta}, \bar{\eta}]$ the support of η , $f(\eta)$ its density, and $\bar{F}(x) = P(\eta > x)$ the complementary cumulative distribution function. Define

$$Q_n \equiv \frac{1}{e^{\bar{a}_n + \eta_n}} \left(g'_n(\bar{a}_n) + g''_n(\bar{a}_n) \sup_{\eta} \frac{\bar{F}_n(\eta)}{f_n(\eta)} \right) e^{g'_n(\bar{a}_n) + (\bar{\eta}_n - \eta_n) g'_n(\bar{a}_n)}.$$

From condition (28) in EG (applied to $b(a, \eta) = e^{a+\eta}$ and $v(c) = \ln c$, and $e^{u^{-1}(\underline{u})} = v_n$), the firm wishes to implement \bar{a} for all η if:

$$s_n > Q_n v_n.$$

This condition requires firm value s_n to be sufficiently large compared to the CEO's effective

wage v_n . ■

Proof of Equation (8)

The CEO's wage is:

$$\ln c = \Lambda r + K = \Lambda(a + \eta) + K = \eta' + K',$$

with $\eta' = \Lambda\eta$, $K' = \Lambda a + K$. Thus his expected wage is:

$$w = E[c] = E[e^{\eta'+K'}] = E[e^{\eta'}] e^{K'} = e^{\ln E[e^{\eta'}]+K'}.$$

His expected utility is:

$$\begin{aligned} U_n &= \frac{1}{1-\Gamma} E \left[(ce^{-g(a)})^{1-\Gamma} \right] = \frac{1}{1-\Gamma} E \left[e^{(\ln c - g(a))(1-\Gamma)} \right] \\ &= \frac{1}{1-\Gamma} E \left[e^{(\eta'+K'-g(a))(1-\Gamma)} \right] = \frac{1}{1-\Gamma} E \left[e^{(\eta')(1-\Gamma)} \right] e^{(K'-g(a))(1-\Gamma)} \\ &= \frac{1}{1-\Gamma} e^{(1-\Gamma) \left[\frac{1}{1-\Gamma} \ln E[e^{\eta'(1-\Gamma)}] + K' - g(a) \right]}. \end{aligned}$$

Hence, with:

$$\bar{\Gamma}(\Lambda^2\sigma^2)/2 = \ln E[e^{\eta'}] - \frac{1}{1-\Gamma} \ln E[e^{(1-\Gamma)\eta'}],$$

we have:

$$\begin{aligned} U_n &= \frac{1}{1-\Gamma} e^{(1-\Gamma) \left[\ln E[e^{\eta'}] - \bar{\Gamma}(\Lambda^2\sigma^2)/2 + K' - g(a) \right]} \\ &= \frac{1}{1-\Gamma} e^{(1-\Gamma) \left[-(g(a) + \bar{\Gamma}(\Lambda^2\sigma^2)/2) + \ln E[e^{\eta'}] + K' \right]} \\ &= \frac{1}{1-\Gamma} e^{(1-\Gamma)[- \chi + \ln w]} \\ &= \frac{(we^{-\chi})^{1-\Gamma}}{1-\Gamma}. \end{aligned}$$

The CEO receives the same utility as if he had to exert no effort, and received a fixed wage $we^{-\chi}$. ■

Proof of Theorem 1

This proof consists of four steps.

Step 1: Effective sizes. This is derived in the proof in the main paper.

Step 2: Distribution of effective sizes. We use the notations

$$\alpha' = \alpha\gamma, \quad \kappa_n = \chi_n/\alpha', \quad \bar{\kappa} = \bar{\chi}/\alpha'.$$

We use the interpretation of n as a quantile to simplify the algebra. Since $S(n) = An^{-\alpha}$, the

distribution of sizes follows $P(S \geq x) = (x/A)^{-1/\alpha}$. Averaging over all χ_n yields the following distribution function for effective sizes $\widehat{S}_n = S_n e^{-\chi_n/\gamma}$:

$$\begin{aligned}\widehat{F}(x) &= P\left(\widehat{S}_n \geq x\right) = P\left(S_n e^{-\chi_n/\gamma} \geq x\right) = P\left(S_n \geq x e^{\chi_n/\gamma}\right) \\ &= E\left[\left(x e^{\chi_n/\gamma}/A\right)^{-1/\alpha}\right] = \left(\frac{x}{A}\right)^{-1/\alpha} E\left[e^{-\chi_n/(\alpha\gamma)}\right] \\ &= \left(\frac{x}{A}\right)^{-1/\alpha} e^{-\bar{\kappa}}.\end{aligned}$$

We will use the r to denote the rank in effective size, and n for the rank in actual size. The effective size $\widehat{S}(r)$ of the firm of rank r satisfies $\widehat{F}\left(\widehat{S}(r)\right) = r$, i.e., the effective size of the firm of quantile rank r is:

$$\widehat{S}(r) = A e^{-\alpha\bar{\kappa}} r^{-\alpha}.$$

Step 3. Assignment in effective sizes. A firm with effective rank r optimizes over the talent rank q of the manager it wishes to hire:

$$\max_q C \widehat{S}^\gamma(r) T(q) - v(q),$$

which yields $C \widehat{S}^\gamma(r) T'(q) - v'(q) = 0$. In the competitive equilibrium, there is matching between talent and effective size, $q = r$. Hence:

$$C \widehat{S}^\gamma(r) T'(r) = v'(r). \quad (34)$$

Let \underline{v}_N denote the effective reservation wage of the least talented CEO ($n = N$). We obtain the classic assignment equation (Sattinger 1993; Terviö 2008):

$$v(r) = - \int_r^N C \widehat{S}(u)^\gamma T'(u) du + \underline{v}_N.$$

Using the functional forms $\widehat{S}(u) = A e^{-\alpha\bar{\kappa}} u^{-\alpha}$ and $T'(u) = -B u^{\beta-1}$, we obtain:

$$\begin{aligned}v(r) &= A^\gamma e^{-\alpha'\bar{\kappa}} BC \int_r^N u^{-\alpha\gamma+\beta-1} du + \underline{v}_N = A^\gamma e^{-\alpha'\bar{\kappa}} BC \left[\frac{u^{-\alpha'+\beta}}{-\alpha'+\beta} \right]_r^N + \underline{v}_N \\ &= \frac{A^\gamma BC}{\alpha' - \beta} e^{-\alpha'\bar{\kappa}} \left(r^{-(\alpha'-\beta)} - N^{-(\alpha'-\beta)} \right) + \underline{v}_N.\end{aligned}$$

In the limit $(r/N) \rightarrow 0$, the term $r^{-(\alpha'-\beta)}$ dominates the other two, and we have:

$$v(r) = \frac{A^\gamma BC}{\alpha' - \beta} e^{-\alpha'\bar{\kappa}} r^{-(\alpha'-\beta)}.$$

Step 4. Wages. The rank of a firm with effective size $S_n e^{-\chi_n/\gamma}$ is:

$$r = \widehat{F} \left(S_n e^{-\chi_n/\gamma} \right) = \left(\frac{S_n e^{-\chi_n/\gamma}}{A} \right)^{-1/\alpha} e^{-\bar{\kappa}} = \left(\frac{S_n}{A} \right)^{-1/\alpha} e^{\kappa_n - \bar{\kappa}}.$$

In other words, a firm with size rank n hires a manager with size talent rank $r = n e^{\kappa_n - \bar{\kappa}}$ (at least in the upper tail, i.e., in the domain of the power law specification). It pays an effective wage of $v(r)$, and thus a monetary wage of:

$$\begin{aligned} w_n &= v(r) e^{\chi_n} \\ &= \frac{A^\gamma BC}{\alpha' - \beta} e^{-\alpha' \bar{\kappa}} \left(\left(\frac{S_n}{A} \right)^{-1/\alpha} e^{\kappa_n - \bar{\kappa}} \right)^{-(\alpha' - \beta)} e^{\alpha' \kappa_n} \\ &= \frac{A^{\beta/\alpha} BC}{\alpha' - \beta} S_n^{\gamma - \beta/\alpha} e^{\beta(\kappa_n - \bar{\kappa})}. \end{aligned}$$

Finally, substituting $S_{n_*} = A n_*^{-\alpha}$, we obtain:

$$\begin{aligned} w_n &= \frac{(S_{n_*} n_*^\alpha)^{\beta/\alpha} BC}{\alpha' - \beta} S_n^{\gamma - \beta/\alpha} e^{\beta(\kappa_n - \bar{\kappa})} \\ &= D(n_*) S_{n_*}^{\beta/\alpha} S_n^{\gamma - \beta/\alpha} e^{\beta(\kappa_n - \bar{\kappa})}, \end{aligned}$$

with $D(n_*) = \frac{n_*^\beta BC}{\alpha' - \beta}$. ■

A sufficient condition for the risk premium to increase with the wage

Consider a general utility function $U = \frac{e^{(1-\Gamma)(v(c)-g(a))}}{1-\Gamma}$ where v is concave; the core model corresponds to $v(c) = \ln c$. From EG, the optimal contract is $c = v^{-1}(\Lambda r + K)$. From the proof of equation (8) in The Appendix, we have:

$$E[U] = \frac{e^{(1-\Gamma)(\Lambda a + K + H)}}{1-\Gamma},$$

where $H = -g(a) + \ln E[e^{\Lambda \eta}] - \bar{\Gamma}(\Lambda^2 \sigma^2)/2 < 0$. Hence, v (effective wage) $= \Lambda a + K + H$, and so the risk premium is given by:

$$w - f(\Lambda a + K + H),$$

where $f = v^{-1}$.

Suppose now that w increases a small amount Δ and $\Lambda r + K$ increases δ such that $w + \Delta = E[f(\Lambda r + K + \delta)]$. The risk premium becomes $w + \Delta - f(\Lambda a + K + \delta + H)$. For the risk premium to be increasing in w , we require:

$$w + \Delta - f(\Lambda a + K + \delta + H) > w - f(\Lambda a + K + H),$$

i.e.:

$$\Delta > f(\Lambda a + K + \delta + H) - f(\Lambda a + K + H) = \delta f'(\Lambda a + K + H)$$

since δ is small. We have:

$$\Delta = E[f(\Lambda r + K + \delta)] - E[f(\Lambda r + K)] = \delta E[f'(\Lambda r + K)]$$

and so we require:

$$E[f'(\Lambda r + K)] > f'(\Lambda a + K + H).$$

Since v is concave, f is convex and so $f'' > 0$. Since also $H < 0$, it is sufficient to show that $E[f'(\Lambda r + K)] \geq f'(\Lambda a + K)$. Thus, it is sufficient for f' to be weakly convex (i.e., $f''' \geq 0$) for the risk premium to be increasing in the wage; multiplicative preferences are not necessary. ■

Proof that the market equilibrium is constrained-efficient

We prove that if a social planner faces the same informational constraints as the agents in the model (in particular, she cannot observe CEO effort), she cannot find a Pareto-dominant allocation. Using the same argument as in the “Proof that maximum effort is optimal if s is sufficiently large,” for a given CEO-firm pair, the social planner wishes the CEO to exert maximum effort (because the benefits of effort are sufficiently large) and seeks the cheapest contract that implements this effort level. We have shown that this is the contract in Proposition 1. Given this, the market assignment is Pareto optimal, as is well-known (e.g., Gretsky, Ostroy and Zame (1999)). For completeness, we provide a proof sketch.

Consider two firms a and b , who are matched in a decentralized equilibrium with two CEOs, a and b . We normalize $\gamma = 1$ and define $A_i = e^{x_i}$. Thus, if firm i hires CEO j , it must pay an effective wage v_j and a dollar wage $A_i v_j$. Since firm a appoints CEO a rather than CEO b :

$$S_a T_a - v_a A_a \geq S_a T_b - v_b A_a, \tag{35}$$

and likewise because firm b appoints CEO b rather than CEO a :

$$S_b T_b - v_b A_b \geq S_b T_a - v_a A_b. \tag{36}$$

We study whether the social planner can achieve a Pareto improvement, i.e., increase total production $\sum T_i S_j$ net of wages, subject to each CEO j receiving a utility at least v_j , the utility given by the market outcome. If the planner pairs firm a with CEO b , while paying CEO a at least $v_a A_b$ (and doing the symmetrical arrangement for firm b and CEO a), the surplus he achieves is $S_a T_b + S_b T_a - v_a A_b - v_b A_a$. By adding (35) and (36), this is weakly less than the initial surplus, $S_a T_a + S_b T_b - v_a A_a - v_b A_b$. This completes the proof. ■

Proof of Proposition 2

We start with L_{RA} . We have:

$$\begin{aligned} L_{RA} &= \int \left[w(n) - w(n) e^{-\chi'_n} \right] dn \\ &= W - \int w(n) e^{-\chi'_n} dn \\ &= W - \left(\int w(n) dn \right) E \left[e^{-\chi'_n} \right], \end{aligned}$$

since χ'_n and $w(n)$ are independent. This yields $L_{RA} = WE [1 - e^{-\chi'_n}]$.

Turning to L_{Alloc} , as we have shown in the full proof of Theorem 1, a firm with size rank n hires a manager with talent rank $ne^{\kappa_n - \bar{\kappa}}$. Thus, $\hat{T}(n) = T(ne^{\kappa_n - \bar{\kappa}})$ and the value loss is:

$$L_{Alloc} = \int CS(n)^\gamma (T(n) - T(ne^{\kappa_n - \bar{\kappa}})) dn.$$

Since $T(n) = T_{\max} - \frac{B}{\beta} n^\beta$, we have:

$$L_{Alloc} = \int CS(n)^\gamma \frac{B}{\beta} n^\beta (e^{\beta(\kappa_n - \bar{\kappa})} - 1) dn.$$

As the κ_n are drawn independently of $S(n)$, we have:

$$L_{Alloc} = \left(\int CS(n)^\gamma B n^\beta dn \right) \frac{1}{\beta} E [e^{\beta(\kappa_n - \bar{\kappa})} - 1].$$

Next, we observe that $Bn^\beta = -T'(n)n$, and so:

$$\begin{aligned} \int_0^N CS(n)^\gamma B n^\beta dn &= - \int_0^N CS(n)^\gamma T'(n) n dn \\ &= - \int_0^N w'(n) n dn \text{ by (34)} \\ &= [- (w(n) - w(N)) n]_0^N + \int_0^N (w(n) - w(N)) dn = W \\ \int_0^N CS(n)^\gamma B n^\beta dn &= \int_0^N (w(n) - w(N)) dn = W. \end{aligned} \tag{37}$$

This yields:

$$L_{Alloc} = \frac{1}{\beta} E [e^{\beta(\kappa_n - \bar{\kappa})} - 1] W.$$

With small distortions, we can take Taylor expansions. As only $\kappa_n - \bar{\kappa}$ matters, it is sufficient to consider the case $E[\kappa_n] = 0$. The definition of $\bar{\kappa}$ gives:

$$e^{-\bar{\kappa}} = E [e^{-\kappa_n}] = E \left[1 - \kappa_n + \frac{\kappa_n^2}{2} + o(\kappa_n^2) \right] = 1 + \frac{\text{var}(\kappa_n)}{2} + o(\text{var}(\kappa_n)),$$

$$\bar{\kappa} = \frac{-\text{var}(\kappa_n)}{2} + o(\text{var}(\kappa_n)).$$

We next have:

$$\begin{aligned} E \left[e^{\beta(\kappa_n - \bar{\kappa})} - 1 \right] &= E \left[\beta(\kappa_n - \bar{\kappa}) + \frac{\beta^2}{2} (\kappa_n^2 - 2\kappa_n\bar{\kappa} + \bar{\kappa}^2) \right] \\ &= -\beta\bar{\kappa} + \frac{\beta^2}{2} E[\kappa_n^2] + o(\text{var}(\kappa_n)) \\ &= \frac{\text{var}(\kappa_n)}{2} (\beta + \beta^2) + o(\text{var}(\kappa_n)), \end{aligned}$$

and hence:

$$L_{Alloc} \sim (1 + \beta) \frac{\text{var}(\kappa_n)}{2} W = \frac{(1 + \beta) \text{var}(\chi'_n)}{2\alpha^2\gamma^2} W.$$

Proof of Proposition 3

Using again $\alpha' = \alpha\gamma$, we observe that (37) yields:

$$\begin{aligned} W &= \int_0^N CS(n)^\gamma Bn^\beta dn = \int_0^N A^\gamma BCn^{-\alpha\gamma+\beta} dn \\ &= A^\gamma BC \frac{N^{1-\alpha'+\beta}}{1-\alpha'+\beta}. \end{aligned}$$

Given $T(n) = T_{\max} - \frac{B}{\beta}n^\beta$, we have

$$\bar{T} = \frac{1}{MN} \int_0^{MN} T(n) dn = T_{\max} - \frac{B}{\beta(1+\beta)} (MN)^\beta.$$

Thus:

$$\begin{aligned} L_{Rand} &= \int_0^N CS(n)^\gamma T(n) - \int_0^N CS(n)^\gamma \bar{T} dn \\ &= \int_0^N CS(n)^\gamma \left(\frac{B}{\beta(1+\beta)} M^\beta N^\beta - \frac{B}{\beta} n^\beta \right) dn \\ &= \int_0^N \frac{A^\gamma BC}{\beta} n^{-\alpha\gamma} \left(\frac{1}{1+\beta} M^\beta N^\beta - n^\beta \right) dn \\ &= \frac{A^\gamma BC}{\beta} \left[\frac{n^{1-\alpha'}}{1-\alpha'} \frac{M^\beta N^\beta}{1+\beta} - \frac{n^{1-\alpha'+\beta}}{1-\alpha'+\beta} \right]_0^N \\ &= \frac{A^\gamma BC}{\beta} \frac{N^{1-\alpha'+\beta}}{(1-\alpha'+\beta)} \left[\frac{1-\alpha'+\beta}{(1-\alpha')(1+\beta)} M^\beta - 1 \right] \\ &= \frac{W}{\beta} \left[\frac{1-\alpha'+\beta}{(1+\beta)(1-\alpha')} M^\beta - 1 \right]. \blacksquare \end{aligned}$$

Proof of Proposition 5

(i) In the corporate sector, χ_n increases by ξ , while it remains constant in the non-corporate sector. $\bar{\chi}$ thus changes to:

$$\begin{aligned}\bar{\chi}' &= -\alpha\gamma \ln E \left[e^{-\tilde{\chi}'_n/(\alpha\gamma)} \right] \\ &= -\alpha\gamma \ln E \left[e^{-\tilde{\chi}_n/(\alpha\gamma)} \left((1-\pi) e^{-\xi/(\alpha\gamma)} + \pi \right) \right] \\ &= -\alpha\gamma \ln E \left[e^{-\tilde{\chi}'_n/(\alpha\gamma)} \right] - \alpha\gamma \ln \left((1-\pi) e^{-\xi/(\alpha\gamma)} + \pi \right) \\ &= \bar{\chi} - \alpha\gamma \ln \left((1-\pi) e^{-\xi/(\alpha\gamma)} + \pi \right).\end{aligned}$$

Therefore, in the limit of small ξ , we have:

$$\bar{\chi}' = \bar{\chi} + (1-\pi)\xi + O(\xi^2).$$

From equation (12), the wage in the corporate sector changes by:

$$\begin{aligned}\Delta \ln w_n &= \frac{\beta}{\alpha\gamma} (\Delta\chi_n - \Delta\bar{\chi}) \\ &= \frac{\beta}{\alpha\gamma} [\xi - (1-\pi)\xi] + O(\xi^2) \\ &= \frac{\beta}{\alpha\gamma} \pi\xi + O(\xi^2).\end{aligned}$$

(ii) Given the Pareto firm size distribution $S(n) = An^{-\alpha}$, the number of firms with a size greater than S is $KS^{-1/\alpha}$ for a constant $K = NA^{1/\alpha}$.²⁵ We normalize the initial χ to 0. For a non-corporate firm, the effective size equals its actual size. Given the increase in disutility, a corporate firm with effective size S has real size $Se^{\xi/\gamma}$. Thus, the probability that a firm has an effective size greater than S is:

$$(1-\pi)K(Se^{\xi/\gamma})^{-1/\alpha} + \pi KS^{-1/\alpha}.$$

Thus, the talent corresponding to a firm with effective size S is:

$$n' = n + \Delta n = KS^{-1/\alpha} \left(1 - \frac{(1-\pi)\xi}{\alpha\gamma} \right) + O(\xi^2).$$

Hence, a corporate firm of size S and thus effective size $Se^{-\xi/\gamma}$ hires a manager of talent (for

²⁵The proof is thus. $S = An^{-\alpha}$ implies $n = (S/A)^{-1/\alpha}$ and thus $n/N = (S/A)^{-1/\alpha}/N$. The left-hand side n/N is the number of firms larger than S , and the right-hand side can be rewritten $KS^{-1/\alpha}$.

small ξ):

$$\begin{aligned}
n' &= K (S e^{-\xi/\gamma})^{-1/\alpha} \left(1 - \frac{(1-\pi)\xi}{\alpha\gamma} \right) \\
&= K S^{-1/\alpha} \left(1 + \frac{\pi}{\alpha\gamma} \xi \right) \\
&= n \left(1 + \frac{\pi}{\alpha\gamma} \xi \right).
\end{aligned}$$

(iii) The value created by the corporate sector is given by $X = \int CS^\gamma(n) T(n) dn$, for n in the corporate sector. The loss of value creation in the corporate sector is:

$$\begin{aligned}
-\Delta X &= \int CS^\gamma(n) T(n) dn - \int CS^\gamma(n) T(n + \Delta n) dn \\
&= - \int CS^\gamma(n) T'(n) n \frac{\pi\xi}{\alpha\gamma} dn + O(\xi^2) \\
&= \frac{\pi\xi}{\alpha\gamma} W + O(\xi^2),
\end{aligned}$$

since (37) showed that $-\int_0^N CS(n)^\gamma T'(n) n dn = W$. ■

Proof of Theorem 2

We define $V(r_1, r_2) = v(c(r_1, r_2))$. At $t = 2$, the IC condition is:

$$\bar{a} \in \arg \max_{a_2} u(V(r_1, a_2 + \sigma(a_1)\eta_2 + \mu(a_1)) - g_1(a_1) - g_2(a_2)).$$

Note that there is no expectations operator here, since all noise has been realized when the CEO chooses a_2 : this highlights the role of our timing assumption in achieving tractability. We can thus remove $u(\cdot)$ to yield:

$$\bar{a} \in \arg \max_{a_2} V(r_1, a_2 + \sigma(a_1)\eta_2) + \mu(a_1) - g_1(a_1) - g_2(a_2).$$

The first order condition is:

$$\frac{\partial}{\partial r_2} V(r_1, r_2) - g_2'(\bar{a}) = 0,$$

which integrates to:

$$V(r_1, r_2) = K(r_1) + g_2'(\bar{a}) r_2, \tag{38}$$

for some function $K(r_1)$ to be determined.

We now consider the $t = 1$ IC constraint:

$$\bar{a} \in \arg \max_{a_2} E_1 [u(K(a_1 + \eta_1) + \Lambda_2(\bar{a} + \sigma(a_1)\eta_2 + \mu(a_1)) - g_1(a_1) - g_2(\bar{a}))], \tag{39}$$

where E_1 is the expectation conditional on η_1 . From $u(x) = e^{(1-\Gamma)x}/(1-\Gamma)$, we have the following certainty equivalent formula for any constant x and random variable \tilde{y} :

$$E[u(x + \tilde{y})] = u\left(x + \frac{1}{1-\Gamma} \ln E[e^{(1-\Gamma)\tilde{y}}]\right).$$

Applying this to (39) yields:

$$\bar{a} \in \arg \max_{a_1} u\left(K(a_1 + \eta_1) + \Lambda_2 \bar{a} + \frac{1}{1-\Gamma} \ln E[e^{(1-\Gamma)\Lambda_2(\sigma(a_1)\eta_2 + \mu(a_1))}]\right) - g_1(a_1) - g_2(\bar{a})$$

for any η_1 . As above, we can remove the u function to yield:

$$\bar{a} \in \arg \max_{a_1} K(a_1 + \eta_1) + \Lambda_2 \bar{a} + \frac{1}{1-\Gamma} \ln E[e^{(1-\Gamma)\Lambda_2(\sigma(a_1)\eta_2 + \mu(a_1))}] - g_1(a_1) - g_2(\bar{a}).$$

Hence we must have:

$$K'(r_1) + \frac{d}{da_1} \frac{1}{1-\Gamma} \ln E[e^{(1-\Gamma)\Lambda_2(\sigma(a_1)\eta_2 + \mu(a_1))}] - g_1'(a_1) \Big|_{a_1=\bar{a}} = 0,$$

which implies:

$$K(r_1) = K_0 + \Lambda_1 r_1$$

for some constant K_0 , and:

$$\begin{aligned} \Lambda_1 &= g_1'(\bar{a}) - \frac{1}{1-\Gamma} \frac{d}{da_1} \ln E[e^{(1-\Gamma)\Lambda_2(\sigma(a_1)\eta_2 + \mu(a_1))}] \Big|_{a_1=\bar{a}} \\ &= g_1'(\bar{a}) - \Lambda_2 \mu'(a_1) - \frac{1}{1-\Gamma} \frac{d}{da_1} \ln E[e^{(1-\Gamma)\Lambda_2(\sigma(a_1)\eta_2)}] \Big|_{a_1=\bar{a}}. \end{aligned}$$

Combining this with (38) yields:

$$V(r_1, r_2) = K(r_1) + \Lambda_2 r_2 = K_0 + \Lambda_1 r_1 + \Lambda_2 r_2,$$

which generates the contract in Theorem 2.

Note that the above proof considers contracts that are message-free, deterministic, and differentiable. The techniques in EG formally prove that the optimal contract satisfies all of these criteria.

In the limit of small noises, we have:

$$\ln E[e^{(1-\Gamma)\Lambda_2(\sigma(a_1)\eta_2)}] \sim \frac{1}{2} (1-\Gamma)^2 (\Lambda_2)^2 \sigma(a_1)^2$$

and

$$\frac{1}{1-\Gamma} \frac{d}{da_1} \ln E[e^{(1-\Gamma)\Lambda_2(\sigma(a_1)\eta_2)}] \Big|_{a_1=a_1^*} \sim (1-\Gamma) (\Lambda_2)^2 \sigma(a_1) \sigma(a_1)'$$

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