

Planning Math Language in the United States, 1650 to 1945

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Within mathematics education in the United States, educators and scholars have worked to identify ways of using language that students of mathematics must perform. I describe how mathematics educators from 1650–1945 have argued whether or how language is important for learning and doing mathematics. Framing these arguments as a form of language policy and planning, I apply intertextual research methods (Johnson, 2015) and the framework of enregisterment (Agha, 2007) to present explicit and implicit policy and planning for math language as intertextually linked models of linguistic behavior. I also summarize the gradual development of math language alongside wider shifts in the structure and philosophy of education in the United States. While early attention to language and mathematics learning did not produce expectations for student language use, student-regulating models of math language eventually solidified through the context of progressive education scholarship in the early 20th century.

In a second grade Philadelphia, Pennsylvania classroom in 2015, during a mathematics lesson on the symbols $>$ and $<$, students watched a cartoon that included two characters, Dizzy and Bitz, each with piles of objects called *Umies* they were throwing at each other (see Figure 1). The characters argue about their piles but are soon interrupted by an anthropomorphic $>$ symbol:

Dizzy: Bitz, you've got way more Umies than me.

$>$: You mean, Bitz's Umi pile is greater than yours.

Bitz and Dizzy: Huh?!

The teacher soon paused the video and followed up with an additional explanation of this specific terminology: "So he's saying, you can talk about something being greater, or less. We talked about last week that greater means more, less means smaller."

As I observed this lesson with my own second grade teaching experience in mind, I instantly recognized the task that the teacher had undertaken. Even elementary mathematics curricula sometimes demand uses of new language, and teachers act deliberately and carefully to ensure students grasp new ways of using language without confusion. While jotting impressions of the video into my notebook, I was reminded of all the attention that I and my fellow teachers put into explaining phrases or constructions that we felt the students would need extra support to understand, such as "How many more pretzels does Javier have than Amanda?" or "two out of three equal parts." The curriculum seemed to contain precise ways of talking about math that were essential to mastering new content, yet also worryingly unfamiliar to students.



Figure 1. A still image from an Umigo video, showing the three characters.¹

However, when I sat in this classroom as a visiting researcher, I was taken aback by the sight of a cartoon mathematical symbol correcting the language of a cartoon monster to guide him toward academic norms. The language at issue was not a new term for an unfamiliar concept (such as might be said of *magma* in a second grade science unit), but instead a new phrase (*X greater than Y*) for a concept that kids already were imagined to know (*quantity*) and already had ways of describing (*X is more than Y*, *A has more than B*, etc.). This case of *mathematical prescriptivism* (Chrisomalis, 2015) highlights the intersection of language policy and academic curricula: directives for teachers to establish linguistic norms of a particular content area. Such norms are continuous with ways that language is regimented in any part of our life, yet combined with the exclusionary power of schools, they are also always potentially implicated in the marginalization of students and the production of inequality. Although the singing $>$ had only a fleeting presence in the classroom, the short video invited me to explore how potentially exclusionary linguistic norms have figured into elementary mathematics education in the United States.

The specific history of the $>$ symbol reveals little on its own. Thomas Harriot invented the symbol in his work on algebra, eventually published in 1631 after his death as *Artis Analyticae Praxis* (Cajori, 1993/1928). However, this fact in itself does not explain the importance given to vocabulary with which students are taught to refer to $>$ or why a sunglasses-wearing $>$ sang to Philadelphia school children. The symbol $>$ only became a cartoon character because it had already entered the repertoire that second grade students are held responsible for being able to use when they learn and perform mathematics—a process that unfolded over generations of mathematics teachers and learners in the United States. After all, second grade teachers in the United States do not read Harriot's *Artis Analyticae Praxis* as part of their training. They read professional journals, go to classes in college, attend professional development sessions and workshops, and

¹ From "UMIGO - Greater Than Less Than (COMPARISON)," by UMIGO, 2013 (<https://www.youtube.com/watch?v=Zzf1-bGNLW4>).

use other materials disseminated by local school administration (e.g., published resources purchased by a school district, a recommendation from a single school's leadership) or obtained through less formal networks (e.g., Teachers Pay Teachers, Pinterest, independent specialty teacher stores). These texts and events include representations of what belongs in a repertoire of learning mathematics—a crucial part of how mathematics education in the United States has changed over time.

In this paper, I describe the development of professional conversation about this repertoire from 1650–1945. I use the term *math language* to denote the target of efforts within mathematics education to identify and govern ways of using language that students of mathematics must perform. Such efforts by mathematics educators, both explicit and implicit, have been undertaken in pursuit of what has been called *mathematical language*, *standard mathematical terminology*, *the language of arithmetic*, *the language of algebra*, and so on. Despite differences in terminology for the target of these efforts, these concerns appear linked by a common goal and similar motivations. Such efforts continue in the contemporary era in the form of scholarship that seeks to objectively determine ways of using language that are essential to performing mathematics (e.g., Halliday, 1974; Pimm, 1987), which has subsequently supported scholarship that highlights complexities of mathematics education in multilingual contexts or simultaneously with learning an additional language (e.g., Barwell, Barton, & Setati, 2007; Moschkovich, 2007; Moschkovich & Setati, 2013). A parallel line of work clarifies the roles that communication and problem solving plays in learning and doing mathematics (Ball, 1993; Leinwand, 2014; Wagganer, 2015). The work I present here is not yet in direct conversation with either body of work in that I do not directly argue whether or how language is important for learning and doing mathematics. Instead, by examining math language from the perspectives of language policy and planning (LPP), I present selections of professional discourse from 1650–1945 to show how mathematics educators themselves have engaged in these arguments throughout a specific period. I apply intertextual research methods (Johnson, 2015) and the framework of enregisterment (Agha, 2007) in order to present the explicit and implicit policy and planning for math language as intertextually linked models of linguistic behavior. I summarize major threads of the typification of math language alongside wider shifts in the structure and philosophy of education in the United States.

Conceptual Framework: LPP for Mathematics Education

The practices and forms that have comprised math language in the eyes of mathematics educators are dwarfed by the range of practices normally called languages (such as Spanish, Tagalog, and Mandarin). However, the boundaries of math language—a model of the appropriate ways for students to speak about mathematics—are closely guarded in much the same that the boundaries of languages are maintained, especially in cases where there is a diversity of language practices sufficient to make policies referring to static and uniform languages ambiguous or difficult to implement (García, 2009; Irvine & Gal, 2000; Makoni & Mashiri, 2007; Makoni & Pennycook, 2007; Valdés, 2016). In the context of education, the boundaries drawn around math language are implicated in broader processes by which schooling functions to continually assess language use by students against norms of academic or otherwise appropriate behavior

(e.g., Chrisomalis, 2015; Duff, 2010; Flores, Kleyn, & Menken, 2015; Flores & Rosa, 2015; Heath, 1983; Mertz, 1998; Wortham, 2005). In other words, math language is one of many models of language involved in the ways that teachers, curriculum directors, textbook writers, and others plan for education. The teacher in the opening vignette (and the writers of the video she showed) employed a model of language, specifically math language, that held students accountable for knowing and using *greater*, not *more*, when discussing differences of quantity.

LPP frameworks offer possibilities for investigation of math language as language policy because mathematics educators' understanding of math language are plans to "influence linguistic behavior" of students learning mathematics in the United States (Cooper, 1989, p. 35). Professional discourse about math language can be seen as language policy when we consider the many forms taken by policy. Shohamy (2006) argues that "[language policy] should not be limited to the examination of declared and official statements" and describes the numerous institutional *mechanisms* through which language policies are carried out "by all groups in society, top-down and bottom-up, whenever they use language as a means of turning ideology into practice and of creating de facto policies" (p. 54). Tracking the development of math language continues the expansion of LPP scholarship in the ways that Shohamy discusses because math language is mainly a matter of *de facto* policy, and because ideologies of math language are put into practice through a variety of mechanisms: textbooks, tests of mathematics, teacher training materials, classroom instruction, and so on. Shohamy's call for the expansion of language policy scholarship still focuses on policies about languages, but many of the essential issues that Shohamy and others highlight apply to math language as well. As the field of LPP increasingly considers activities beyond those of actors traditionally understood as policymakers, to also include actors in classrooms and other educational spaces (Hornberger & Johnson, 2007; McCarty, 2011; Ricento & Hornberger, 1996), it becomes more able to include descriptions of objects such as math language.

A key articulation of this broader focus is seen in Schiffman's (1996) argument that language policy is grounded in *linguistic culture*. Schiffman argues we cannot understand policy without understanding its situation within the linguistic culture in which it is pursued, defined partly as "the set of behaviors, assumptions, cultural forms, prejudices, folk belief systems, attitudes, stereotypes, ways of thinking about language, and religio-historical circumstances associated with a particular language" (p. 5). Schiffman's understanding of linguistic culture includes the need to treat language as more than a code, and he argues that it can only be thoroughly documented through ethnographic observation. This vision of LPP widens our gaze to see more aspects of language, including models of specialized disciplinary language practices like math language, as subject to policymaking processes. It also indicates how an examination of language in education must account for the social and political context of schooling. In this way, teachers' day-to-day activity with respect to language cannot be separated from other ways in which language is regulated and reflexively organized (Agha, 2007; Bauman & Briggs, 2000; Lucy, 1993; Rymes, 2014; Silverstein, 1993). Teachers' physical presence in the classroom rather than the halls of a policymaking body does not make them incapable of re-interpreting or even re-making policy, nor does it remove them from understandings of language that pertain to contexts in addition to school

(e.g., García & Menken, 2010; Hélot, 2010; Lo Bianco, 2010). At the same time, with an expanded understanding of language policy, we can recognize that there are many simultaneous policies at issue in the classroom.

On this issue, there is continuity between LPP and classroom discourse analysis which shows how learning, socialization, and identity develop through everyday interaction (Cazden, 2001; Gallas, 1995; Martin-Jones, 2015; Palmer, 2008; Rymes, 2009; Wortham, 2005). When these processes occur through descriptions of discourse produced by students (e.g., Au, 1980; Heath, 1983; Michaels, 1981; Philips, 2009; Rymes, 2009), LPP frameworks are well poised to ask how student discourse is understood in the classroom along with the histories and politics that inform these understandings, even when these do not just refer to languages. Descriptions of math language are a crucial part of language-in-education planning in the sense that they involve “selecting the language media for education, the languages taught, and the varieties used in education,” and they are also part of language education policy in the sense that “most education decisions are, in effect, language education policy” (García & Menken, 2010, pp. 252–254).

In addition to being a product of policy within mathematics education, math language can also be seen as a *register*, insofar as the use of particular discursive forms is able to contextualize an occasion as one in which mathematics is being done or position the user as mathematically competent (Agha, 2007). A register is only definable and bounded “to a degree set by sociohistorical processes of *enregisterment*, processes whereby its forms and values become differentiable from the rest of the language (i.e., recognizable as distinct, linked to typifiable social personae or practices) for a given population of speakers” (Agha, 2007, p. 168). Mathematics instruction in schools (events with teachers and students as participants) comprises a large fraction of this process of *enregisterment*, but professional activities of teacher preparation, professional discourse on the best forms of math pedagogy, conversations in the teachers’ lounge, and the activity of textbook writers are also sites of the process. Math language is like all other registers to the extent that it requires activities of use and typification for its existence. Agha (2007) stresses that metapragmatic stereotypes (or regularities of typification) are necessarily “expressible in publicly perceivable signs” (p. 154). If such stereotypes were not publicly perceivable through artifacts of interaction, we would never get a chance to learn to recognize and produce samples of registers. The framework of *enregisterment* highlights how communicative forms are linked to socially meaningful roles or behaviors. Complementing LPP, *enregisterment* as a framework highlights similar issues about the distributed, multi-site process of typifying linguistic and other communicative forms, reinforcing the need for ethnography or other methods that reveal ways that language practices are brought into being as recognizable through their description.

Data Collection

This paper presents the earliest efforts to establish math language as well as the historical context of how these efforts arose. To repeat, I define *math language* as the target of efforts within mathematics education to identify and govern ways of using language that students of mathematics must perform. Language policy frameworks help characterize the importance of math language as a component of

mathematics education. The framework of enregisterment helps establish ways of collecting data that illuminate the implicit and extended policymaking process of producing and disseminating models of math language. Documenting a register requires attention to typification and metapragmatic description, which can be both explicit or implicit.

One difficulty of this project is that over the period surveyed, schooling took radically different forms, not only with regards to pedagogy or curriculum but also in terms of who was imagined to require (or deserve) schooling. From 1650 to the present, there is not consensus about which people should be included as students: discrimination, segregation, and restricted access to schooling in the United States are fundamental aspects of its history. Yet while different in form, schooling is intertextually continuous across the history of discourse on education insofar as it is referenced in linked descriptions by teachers and policymakers. In other words, while a math lesson in 1780 was very different from a math lesson in 1830 or 1850, there is a continuity of reference forged by the fact that educators and commentators in the intervening years would use descriptions of past schooling practices to frame descriptions or proposals about current or proposed practices. An intertextual approach fits the need to review a broad range of material produced by many authors over time.

Johnson (2015) proposes that attention to intertextuality in policy allows an analyst to establish “the multiple and potentially conflicting meanings, voices, and styles in a text” (p. 166). Intertextuality as a methodological framework draws on the work of Bakhtin (1981, 1986) and emphasizes the fundamental unfinishedness and multiplicity of meaning of any single utterance or text. Because all discourse relies on other discourse for its interpretation, our understanding of one text must always involve understanding their necessary connections to other texts. As Johnson (2015) summarizes:

Language policies are linked to past policy documents, such as earlier policies and earlier versions of the same policy (vertical intertextuality) and current policies (horizontal intertextuality), and they may be connected to a variety of past and present discourses (interdiscursivity). (p. 168)

Because I am examining samples of professional discourse and textbooks rather than policy documents per se, the professional conversation is spread over a larger distribution than a change in law might be. Thus, it seems even more important to adopt an intertextual approach.

The time period surveyed is large (295 years), and not all work within it constitutes math language efforts per se. The data collection strategies used for different periods of this date range varied according to the types of documents that had been published during those periods (see Table 1). For example, secondary sources on the history of mathematics education highlighted widely published textbooks and major works in pedagogy. Some sources from the early 20th century were collected through reviews of academic journals on education, which largely did not arise until that time. The sources I present offer a representative portrait of mathematics education discourse in the period described. While not sufficient to describe the history of mathematics education in detail, they do capture what I understand to be the development of math language in the time period surveyed.

Table 1
Summary of Changes in Mathematics Education, Attention to Language, and Sources Used for Analysis

Time period	Mathematics education	Math language	Key sources
c. 1650–1800	Formal schooling was extremely limited and sporadic, and mathematics was not a priority for those uninvolved in commerce.	Scripted discourse about mathematics rules are the largest extent of attention to language.	Arithmetic textbooks
1800–1890	Ciphering declines as a method while new inductive and analytic pedagogies begin to take hold. A larger portion of the population gains access to school. Mathematics gains ground as a fundamental subject, becoming taught earlier and to girls as well as boys.	While ciphering specified nothing about math language, writings on the inductive and analytic method show greater attention to language. However, this attention is directed at the language of teachers and textbooks (how best to communicate to a child) not the language of students (how to show that one has learned mathematics).	Arithmetic textbooks, teachers' guides, other writings on mathematics pedagogy
1890–1945	Progressive reforms reshape policies toward children and schooling. Progressive education continues emphasis on a student's mind and understanding, but this focus is interpreted through opposing perspectives.	Projects to scientifically determine the language necessary to learn mathematics emerge for the first time, intertwined with cultural and racial politics of the period.	Journal articles, teachers' guides, other writings on mathematics pedagogy, early child psychology

The Colonial Era: Talk About Mathematics (c. 1650–1800)

Among the growing European settlements of eastern North America in the 17th century, formal schooling of young people was rare and did often not include arithmetic (Dauben & Parshall, 2014). To keep in mind, the total population of settlers and enslaved Africans in 1700 eastern North America has been estimated about 250,000 (United States Census Bureau, 2004, p. 1168). Due to this relatively small population, the forms and aims of settler schooling were greatly influenced by local circumstances, especially the differing religious and national groups that settled in different areas. Generally, those who learned arithmetic usually did so as part of apprenticeship into trade and commerce and not as a component of grammar school. In the colonial era,

the appropriate pupil for [arithmetic] was the twelve-to-fourteen-year old boy, judged to be mature enough to absorb the arcane techniques of computation as well as sufficiently competent in writing to create a permanent copybook. (Cohen, 2003, p. 44)

Arithmetical fluency served to “help sellers, buyers, and investors calculate quantities and prices” and deal with “the computational complexity of weights, measures, and monetary systems” (Cohen, 2003, p. 49). Thus for the most part, anyone who did not play a direct role in commerce was not likely to be educated in mathematics. For example, the boys in school tracks leading to college and careers in the law and the clergy “barely studied arithmetic at all” (Cohen, 2003, p. 52). Boys to be trained as navigators, surveyors, and mapmakers, all professions that required algebra, geometry, and trigonometry, were some exceptions to the focus on commerce (Dauben & Parshall, 2014). Clements and Ellerton (2015) point to the contractually obliged education of urban apprentices in reading, writing, and arithmetic as a common form of mathematics education in the 18th century and point to only a diffuse infrastructure outside urban centers:

Indeed, it is likely that there were hundreds of itinerant teachers moving from village to village, offering tuition in reading, writing and occasionally cyphering [an arithmetic teaching method]. Some of these itinerants did not read, write, or cypher well, themselves.... Textbooks were not readily available. Even if textbooks were available, the complex language that authors used to describe rules and cases rendered them incomprehensible for most unassisted readers—and that statement applied as much to inexperienced teachers, who had studied very little arithmetic in the past, as it did to school students. (p. 7)

In this period, no systems of certification or accreditation existed resembling those of today, but even if the professional community of mathematics educators seems extremely limited, it is possible to explore precursors to a professional conversation about math language in this period within mathematics textbooks. Though Clements and Ellerton (2015) note that textbooks in this period, called *arithmetics*, were rare, those that were circulated are recoverable examples of math education discourse and comprised the major infrastructure of the profession. Unlike the textbooks of today, arithmetics did not serve foremost as workbooks for the student but as resources for the teacher, with direct explanations of the

content, “little more than a collection of rules” (Cohen, 2003, p. 47) or at times as scripted models of teacher-student conversations. Therefore, when textbooks did not comment on a separate practice of math language, they still modeled the use of terminology and other elements of a repertoire of discussing mathematics.

Like other arithmetics of the period, Dilworth’s *Schoolmaster’s Assistant*, first published in London around 1740 but reprinted dozens of times up until the early 19th century, contained hundreds of catechism-like recitations to serve as components of lessons, such as the following:

Q. What is Simple Subtraction?

A. Simple or Single Subtraction is the finding a difference between any two numbers, whose signification is the same as the difference between 6 yards and 4 yards is 2 yards.

Q. How are numbers to be placed in Subtraction?

A. With units under units, tens under tens, &c. as in addition.

Q. What rule have you for the operation of subtraction in general?

A. When the lower number is greater than the upper, take the lower number from the number which you borrow, and to that difference add the upper number, carrying one to the next lower place. (Dilworth, 1825, p. 21)

Dilworth’s text was one of the most widely circulated textbooks of the 18th and 19th centuries, and the question and answer format was widely used (Clements & Ellerton, 2015; Dauben & Parshall, 2014). Such recitations seem to have been the greatest extent of professional attention to specific language to be used by students, yet in representing teacher-student conversations they still function as a policy that designates language practices appropriate to students being recognized as learning and doing mathematics. However, the policy embedded in the textbook recitations does not imply that students must adopt linguistic practices that differ from their present capacities. In other words, to the extent that there is a register of math language implied by Dilworth’s text, it is much more scripted and much less elaborated than registers that include a vast repertoire that a user is expected to employ flexibly but appropriately, such as in legal practice. Policies on language and mathematics in this period did not yet invoke forms of specialized language to learn or perform mathematics. Policies of math language had not yet been developed, but mathematics educators in the next century soon expanded their attention beyond pedagogical recitations and debated ever more detailed models of teacher-student discourse.

Emerging Intersections Between Language and Mathematics (1800–1890)

In the wake of the new independent United States government’s founding, both the reach of schooling and its inclusion of mathematics increased, until by 1800 “a solid grounding in elementary mathematics became increasingly central to North American educational objectives” (Dauben & Parshall, 2014, p. 179). There were still differences compared to today’s math instruction however—typically arithmetic instruction did not begin until the age of 10 (Cohen, 2003). Textbooks proliferated “not merely because there was a growing commercial market for

them but because there was a widespread dissatisfaction with the way the subject was taught, as well as a growing sense of the importance of the subject," a sense linked to notions of what participation in the emerging market economy required (Cohen, 2003, p. 56). Cohen reviews several of the new textbooks circulating early in the 19th century and argues that in their content, the texts did not "escape the traditional pathway" (p. 56). Among the textbooks that Cohen reviews is a notable example of attention to language: Goodrich's (1818) *The Child's Arithmetic*, which makes the unusual proposal of instructing children with physical objects long before abstract numbers. It also explicitly critiques earlier arithmetics (such as those of Dilworth or Pike) for the style of language they employed:

Our school arithmetics are written in a rigidly technical style, which, however clear and philosophical to mature and cultivated minds, is utterly incomprehensible to children. The consequence is, they are discouraged at the very threshold. They indeed learn the rules and definitions by rote, but they still do not know their meaning: they therefore despair of helping themselves with the book; they run to the teacher in every new case; they follow his directions mechanically, and thus go through a volume, without comprehending the principles of a single page. (Goodrich, 1818, p. iii)

These complaints about technical language are early documented examples of an explicit discussion approximating math language. Because Goodrich's description of language centers on language used to instruct students, rather than language students themselves use, it is still quite distinct from the policy of the singing > described in the opening vignette. In that sense, it is not yet math language, but just as it is intertextually linked to the recitations of previous arithmetics, it presages later models of language. Goodrich proposes these improvements:

The principal points of difference between this and other school arithmetics, are (1) The definitions are given in plain and simple language, and are then illustrated to the senses of the child and (2) The rules are also given in intelligible language, and are then clearly and minutely exemplified. (Goodrich, 1818, p. iii, punctuation modified for modern readers)

It is difficult for us to grasp the distinction Goodrich makes, since comparing a 200-year-old text to a 250-year-old text does not replicate what Goodrich feels in comparing his own writing to that of 50 years prior. However, it is easy enough to place one of Goodrich's rules against one of Pike's, which Goodrich claims was incomprehensible to children. Here are the two writers providing rules for addition:

Add the right hand column as in Case 1 ["Add the bottom figure to the one above." (p. 10)]. Set down what is over ten, or twenty, or thirty, or forty, &c. And carry to the next column, one for ten, two for twenty, three for thirty, four for forty, &c. Then add the next column in the same way, adding what is brought from the other column; proceed in this manner through the sum, and under the last column set down the whole amount of that column. (Goodrich, 1818, p. 12)

Having placed units under units, tens under tens, &c. draw a line underneath, and begin with the units; after adding up every figure in that column, consider how many tens are contained in their sum, and, placing

the excess under the units, carry so many as you have tens, to the next column, of tens: Proceed in the same manner through every column, or row, and set down the whole amount of the last row. (Pike, 1808, p. 16)

While there are some stylistic differences, Goodrich's innovations do not seem to have broken new ground in conversations about language in mathematics education. Crucially, distinguishing his work from that of later writers, he does not explore questions of how students themselves ought to use language as they learn and use mathematics.

Until the late 19th century, the predominant method of teaching and engaging with rule-based textbooks remained *ciphering*. The ciphering (or, *cyphering*) approach consisted of developing students' ability to perform calculations on paper according to prescribed algorithms. Born of pedagogical philosophy as well as material circumstances, ciphering was the dominant method of teaching mathematics from the early 19th century until around 1870 (Ellerton & Clements, 2012). Where teachers lacked printed textbooks providing guidance or carefully crafted explanations of mathematical concepts, ciphering books were capable on their own of providing instructional models, "with strings of problems worked out in detail and by hand but with no additional explanation" (Dauben & Parshall, 2014, p. 177). Students were charged with working out the same problems, after which they "checked them with the teacher against the solutions in the teacher's cipher book, and copied those solutions into their own evolving books" (Dauben & Parshall, 2014, p. 177). Thus, mathematics education through ciphering involved comparing two text artifacts: one produced by the student and one produced, or at least furnished, by the teacher. Despite this emphasis on exact duplication, there is little evidence of specific policies about language in mathematics education as undertaken through ciphering.

By 1860, ciphering as a teaching approach had dramatically declined, for reasons reflective of wider social and educational changes. Ellerton and Clements (2012) identify three factors responsible for the decline: (1) the advent of written examinations whose results were reported publicly meant that carefully prepared cipher books were no longer "likely to be rewarded"; (2) the advent of age-graded classrooms and the new whole-class teaching methods accompanying them; (3) changing needs of United States society that meant an "emphasis on business-related arithmetic was seen to be losing relevance" (pp. 145–146). Broadly speaking, the decline of ciphering was accompanied by a reorganization of pedagogy and teacher training in other subjects besides math.

While the prevalence of ciphering declined, new approaches to mathematics teaching developed. Textbooks adopting the rule method, exemplified by Goodrich's and earlier arithmetics, were gradually replaced with texts adopting the *inductive* and *analytic* methods, which while "antithetical" to the rule method were "complementary to each other and thus... sometimes combined in a text" (Michalowicz & Howard, 2003, p. 80). The differences between these two methods do not figure into the present discussion. In terms of the development of a professional conversation on math language, both methods marked a shift toward pedagogical theories becoming more explicit and more interested in student capacities, believing "that children could *understand*, not just *do* arithmetic, if taught properly" (Michalowicz & Howard, 2003, p. 105).

The *inductive* method of Warren Colburn came to prominence in the 1820s. Like the algorithms prescribed in *The Schoolmaster's Assistant* and other similar texts, Colburn's inductive method emphasizes the accurate performance of calculation as both the end goal of an education in mathematics and the primary means of mathematics instruction. However, unlike the rule-based methods of the past, Colburn sought to develop student's understanding of calculation by slowly adding challenge and complexity to mathematical work, beginning at a basic but thorough understanding of numeration (p. 80). Colburn's influential work makes limited but revealing reference to language. For example, he argued for avoiding all symbolic notation—including numerals, which Colburn called "a new language, which the pupil has to learn" (Colburn, 1829, p. viii). Colburn's method proposed that students perform mathematical tasks without labeling any one in particular as *addition*, *subtraction*, *division*, or *multiplication*. Like Goodrich, Colburn's discussion of language is limited to a claim that there are ways of using language that are more and less comprehensible to children. Colburn's model of language and math still does not yet propose that there are ways a student should and should not use language when learning and using mathematics. However, though he does not place demands on students' language use, he is suspicious of approaches that use figures and abstract numbers because he believed that they violate a more natural way of reasoning and communicating about math that young children already use. This implication that there was a truer or more authentic way of talking about math would later be understood as something that must be imparted upon children (as by a singing cartoon > symbol), not as something children perhaps already possessed.

Edward Brooks's work on the *analytic* method also played a role in the developing mathematics pedagogies of the 19th century. As an associate of the common school movement, Brooks rejected the widespread ciphering approach and its emphasis on the memorization and application of rules. Similar to Colburn, Brooks emphasized number facts as the basis of arithmetic but also explored questions about the meaning numbers have for the students who use them. Brooks was a trained mathematician as well as an educator who served (at different times) as both a mathematics professor at the University of Pennsylvania and the superintendent of Philadelphia's public school system (Cooper-Twamley & Null, 2009). Also similar to Colburn, Brooks' limited references to language illustrate the emerging connections between communication, mathematics, and the capacities of students.

Brooks (1876) devotes a full section of his work *The Philosophy of Arithmetic*, considered to be "the first arithmetic methods book for elementary level teachers" (Cooper-Twamley & Null, 2009, p. 193), to what he calls "the language of arithmetic," comprised of a very basic catalog: the names for numbers. As he explains in the opening of the section: "Beginning at the Unit, we obtain, by a process of synthesis, arithmetical objects which we call Numbers. These objects we distinguish by names, and thus obtain the language of arithmetic" (Brooks, 1876, p. 92). What follows is an astonishingly dry description of the principles that Brooks finds essential for understanding the importance of numbers, and their names, in arithmetic:

[A] single thing is called one; one and one more are two; two and one more are three; and in the same manner we obtain four, five, six, seven, eight,

and nine, and then adding one more and collecting them into a group, we have ten. Now, regarding the collection ten as a single thing, and proceeding according to the principle stated, we have one and ten, two and ten, three and ten, etc., up to ten and ten, which we call two tens. Continuing in the same manner, we have two tens and one, two tens and two, etc., up to three tens, and so on until we obtain ten of these groups of tens. These ten groups of tens we now bind together by a thread of thought, forming a new group which we call a hundred. (Brooks, 1876, pp. 92–93)

Though he is not concerned with any other element of language other than the words we use for numbers, he echoes Colburn's insistence that labeling tasks or questions with such terms as addition represents a needless barrier to student understanding. In *addition*, Brooks foreshadows the concern of later writers on the potential for language to obscure mathematical concepts. Immediately following the quote above, he turns the troublesome issue of numbers that do not transparently fit the language of arithmetic as Brooks sees it:

This is the actual method by which numbers were originally named; but unfortunately, perhaps, for the learner and for science, some of these names have been so much modified and abbreviated by the changes incident to use, that, with several of the smaller numbers at least, the principle has been so far disguised as not to be generally perceived. If, however, the ordinary language of arithmetic be carefully examined, it will be seen that the principle has been preserved, even if disguised so as not always to be immediately apparent. (Brooks, 1876, p. 93)

He goes on to discuss the historical linguistic origins of such troublemakers as the word *eleven* (rather than *one-teen*, or *ten and one*). He even takes the time to explain why it is best that we use compositional names for numbers rather than give a unique word to every number like we do for one through ten:

This would, of course, require a vocabulary of names as extensive as the series of natural numbers,—a vocabulary which, even for the ordinary purposes of life, could be learned only by years of labor. By the method of groups, the vocabulary is so simple that it can be acquired and employed with the greatest ease. (Brooks, 1876, p. 97)

Brooks' great attention to what he calls the language of arithmetic bears similarities to Colburn and Goodrich in that he does not prescribe how students should use language to do mathematics. However, he still invokes one, a way of using language that is uniquely mathematical in ways that are relevant for mathematics pedagogy. In typifying names for numbers as the language of arithmetic, rather than as continuous with everyday life and reasoning as in Colburn's thinking, Brooks' work contributes to the establishment of math language.

Overall, in 19th century approaches to mathematics education, there is little evidence of mathematics educators working to establish any one version of math language in which students will be instructed. To clarify, teachers and students at this time certainly talked about mathematical problems and mathematics lessons, but there appears no sense that they were engaging in a special variety of English when they did so. While both teachers and students would have employed models

of appropriate conduct for teacher-student discourse, there does not appear to be recoverable professional discourse from the time advocating particular norms of language for learning and demonstrating competence in mathematics. Several changes in mathematics education culminated around the turn of the century. Donoghue (2003) names 1890 as the beginning of the math education profession in United States, pointing to new formal organizations for math teachers as well as rapidly expanded systems of teacher preparation accompanied by rapid expansion of school attendance (high school enrollment doubled between 1890 and 1900). By 1910, 25 higher education institutions included a program to prepare high school math teachers, whereas in 1890 no institution had such a program (Donoghue, 2003, p. 165). These shifts were connected to efforts to teach mathematics earlier in a child's education:

[B]y the end of the nineteenth century, beginning arithmetic had become a subject taught to six-year-olds, both boys and girls, and algebra and geometry were routine high school subjects, taken by both sexes. (Cohen, 2003, p. 46)

This would align with Lagemann's (2000) dating of the beginning of teaching and learning becoming a subject of university research in 1890. However, Ellerton and Clements (2012) argue that Donoghue's date of 1890 is too late and point to the foundation laid by "reflection on the role of the normal schools,² and on curriculum, pedagogical and assessment issues in relation to school mathematics" from 1840–1890 and to the work of Edward Brooks in particular (Ellerton & Clements, 2012, pp. 136–137).

Math Language in Progressive Education (1890–1945)

In the close of the 19th century, the rise of a corporate industrial economy dramatically shifted the division of labor and wealth in the United States and concentrated new population growth in cities, from both immigration and migration within the country. Reformers aligned with progressivism believed this new social and economic landscape warranted sweeping changes in policy, even as there were diverse approaches to the specifics of those policies (Labaree, 2010). Mandatory schooling, citizenship training, and anti-child labor laws were the major permanent changes brought to the lives of children at the height of the progressive era, from 1880–1920 (Fass, 2014). Progressive reform efforts in education included not only new pedagogical philosophies but also more centralized forms of school governance and teacher preparation (Labaree, 2010; Schneider, 2014; Tyack & Hansot, 1982). Educational practices became more regulated, and pedagogical texts became more widely disseminated. Within mathematics education specifically, the progressive era solidified trends toward philosophies that considered children as mathematical thinkers and assumed language used in instruction had an impact on a learner's mind. Progressive writers were the first to impose a model of math language that took for granted that learning and doing mathematics required particular ways of using language that children would mainly learn in school.

² Normal schools were sites of teacher training that were part of the common school movement in some states in the early to mid-19th century, pioneered by Horace Mann and others, which would later become widely implemented.

Early progressive writers framed their vision of new forms of math teaching by referring to methods that had previously been common. Their work was in many ways continuous with the development of the analytic and inductive methods associated Brooks and Colburn in the 19th century. David Eugene Smith, an active mathematics education scholar early in the progressive era, criticized the rule-based recitations once commonly adopted by textbooks but not yet eliminated from educational practice: "The glib recitation of rules for long division, greatest common divisor, etc., which one hears in some schools—what is all this but a pretence of knowledge?" (Smith, 1906, p. 31). Smith makes a pedagogical point but also a language planning one. The language needed to demonstrate knowledge of math was no longer a call and response style description of rules, but something new. Hartung (1939) describes progressive mathematics in this way: "Instead of stressing specific habits, drill, and similar isolated psychological aspects, their approach was from the standpoint of the whole rather than the part" (p. 265). B. Buckingham (1938) similarly emphasizes the importance of "inner meaning" and "experience" that "ensure ability to solve problems" (p. 30). The pedagogical progressive vision of mathematics education articulated by B. Buckingham and Hartung is echoed by other writers of the period who highlighted ways in which language could obscure or hinder conceptual understanding (e.g., Shaver, 1911; Walker, 1925; Williams, 1910).

Edward Thorndike is most well-known for his contributions to the founding of educational psychology and arguments for use of intelligence testing in education, but he also helped develop an influential model of math language. He presented the contributions of his new educational psychological approach to mathematics education by distinguishing it from earlier approaches to teaching arithmetic specifically highlighting the students' understanding as a central concern, in keeping with his fellow progressives (Thorndike, 1922, p. 74). He identifies four specific flaws of previous approaches to teaching arithmetic, one of which is that "it does not take account of the very large amount of teaching of language which is done and should be done as a part of the teaching of arithmetic" (Thorndike, 1922, p. 2). In his argument for an improved mathematics education that centers on student understanding, he highlights specific words of which students must have knowledge in order to be successful:

The understanding of such words as *both, all, in all, together, less, difference, sum, whole, part, equal, buy, sell, have left, measure, is contained in*, and the like, is necessary in arithmetic as truly as is the understanding of numbers themselves. It must be provided for by the school; for pre-school and extra-school training does not furnish it, or furnishes it too late. It can be provided for much better in connection with the teaching of arithmetic than in connection with the teaching of English.

It has not been provided for. An examination of the first fifty pages of eight recent textbooks for beginners in arithmetic reveals very slight attention to this matter at the best and no attention at all in some cases. Three of the books do not even use the word *sum*, and one uses it only once in the fifty pages. In all the four hundred pages the word *difference* occurs only twenty times. When the words are used, no great ingenuity or care appears in the means of making sure that their meanings are understood. (Thorndike, 1922, p. 8)

Like Goodrich in 1818, Thorndike shares in the tradition of criticizing other math texts for their use of language. Although Thorndike disagreed with other progressive writers on mathematics education about how student understanding could best be facilitated, there was broad agreement about the potential for language or specific words to confound students (Cooper-Twamley & Null, 2009). Indeed, despite their other disagreements, Thorndike and Brooks share nearly identical concerns about potential confusions around the names for numbers (e.g., *eleven* vs. *onety-one*; a child writing *61* for *sixteen* because *six* is said first). However, distinct from other writers, Thorndike expands the repertoire of terms he identifies as crucial to mathematics and problematic for student understanding.

Thorndike's more significant break is to describe students as entering school lacking the language necessary to learn mathematics without impediment. His mention of "pre-school" and "extra-school" training point to students' homes and communities. It is not clear from Thorndike's text if he believes school children lack some specialized mathematical knowledge of words such as *both* or *all* or if he is making the much more extraordinary claim that they do not know these words at all. These claims were made within a political and scholarly response to increased immigration and the expansion of schooling to greater numbers of students (Marten & Gallagher, 2014). No different from any other language policy, Thorndike's contributions to math language responded to his political and intellectual context. Although progressives such as Dewey attacked some aspects of scientific racism, progressive reforms nevertheless adopted some eugenicist premises and were not a complete break from ideologies justifying racial inequality (Fallace, 2015). Thorndike himself, like many white intellectuals of the time, was a proponent of eugenics. In this context, circulating models of students, the people whose linguistic behavior he sought to change, represented people in great need of assimilation into a new social and economic order and potentially hindered by inherited or racial differences in ability.

Other scholars soon replicated Thorndike's attempts to empirically establish language necessary for mathematics. Motivated by the conclusion that "a too difficult vocabulary would be a serious handicap in the learning process" (p. 76) in algebra, G. Buckingham (1937) pursued a correlational study of how vocabulary knowledge was associated with performance in algebra. Building on Thorndike's (1921) *The Teacher's Word Book*, a laboriously crafted corpus³ of commonly used words intended as an aid to teachers, G. Buckingham (1937) sought specialized algebra words by identifying those words contained in an algebra textbook but not contained in Thorndike's list. He isolated "13 non-technical words [e.g., *sil*], 10 technical mathematical terms [e.g., *cube*], and 16 technical Algebraic terms [e.g., *monomial*]" (p. 77). Based on his assessments, he looked askance, but also with scholarly interest, at student-provided definitions such as "a circle is a round line" and "a parallelogram is a cone shaped angle with four square corners" (p. 79). On the basis of his investigation, he proposed that "vocabulary peculiar to Mathematics

³ Thorndike's corpus included "an alphabetical list of the 10,000 words which are found to occur most widely in a count of about 625,000 words from literature for children; about 3,000,000 words from the Bible and English classics; about 3,000,000 words from elementary-school text books; about 50,000 words from books about cooking, sewing, farming, the trades, and the like; about 90,000 words from the daily newspapers; and about 500,000 words from correspondence" (p. iii). He later published a 20,000-word version!

in general needs attention” referring to what may have been the widest array of math terminology important for education yet identified at the time (p. 79).

Progressive era writers tried to understand students’ mathematical thinking in order to support it as best they could, and new and more detailed models of math language were part of new approaches to math education. By the end of the progressive era, scholarly consensus had emerged that particular ways of using language constituted a necessary tool for students learning and doing mathematics. While mathematics education scholars pursued scientific studies of language used in mathematics, math language was at times defined largely in opposition to whatever it was that students were imagined to already know.

Implications

The history of policy on math language in the United States reveals the enormous work done to isolate what uses of language should be learned by mathematics learners. Policymaking on math language was articulated through pedagogical principles for teaching mathematics and then explicit investigation of language, all of which responded to social and political changes shaping education as a whole. Mathematics classrooms today depend on the lasting products of this policymaking and the structures of schooling that allow their distribution. Math language owes its function as a register to the generations of educators who defined and described it. The record of mathematics education presented here suggests that the development of math language as a norm of student-produced language followed decades of pedagogical research and theory on language and mathematics education that did not place specific demands on student language. This suggests a counterexample to contemporary understandings of academic language that express instructional needs by invoking elusive desired future language practices.

Historical investigation of language policy embedded in educational materials can aid efforts to understand how naturalized representations of language are used to marginalize students. Subsequent investigation is required to follow how progressive-era efforts to empirically investigate math language continued and grew more complex throughout the 20th century and up until today. For example, contemporary discourses of the “word gap” (Avineri et al., 2015) share many assumptions with Thorndike’s determination that children’s communities and homes did not offer opportunities to learn the set of words needed to do arithmetic. In both cases, scholars argue that home and community language practices are deficient in comparison to academic language. In order to understand how linguistic norms marginalize students in schools, we must continue to acknowledge and investigate the ways that educational research itself can support or resist this marginalization.

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