Abstract—In the model-based implementation methodology, the timed behavior of the software is typically modeled independently of the platform-specific timing semantics such as the delay due to scheduling or I/O handling. Although this approach helps to reduce the complexity of the model, it leads to timing gaps between the model and its implementation. This paper proposes a platform-specific timing verification framework that can be used to formally verify the timed behavior of an implementation that has been developed from a platform-independent model. We first describe a way to categorize the interactions among the software, a platform, and the environment in the form of implementation schemes. We then present an algorithm that systematically transforms a platform-independent model into a platform-specific model under a given implementation scheme. This transformation algorithm ensures that the timed behavior of the platform-specific model is close to that of the corresponding implementation. Our case study of an infusion pump system shows that the measured timing delay of the system is bounded by the formally verified bound of its platform-specific model.

I. INTRODUCTION

Model-based implementation is an effective approach to systematically develop real-time embedded software. In this approach, the timed behavior of the software is modeled using modeling languages (e.g., UPPAAL), and the model’s conformance to the timing requirements is formally verified using verification techniques (e.g., model checking). The code generation process then automatically generates source code from the verified model, so that the final implemented system (which executes the generated code on an embedded platform) also conforms to the timing requirements.

The timed behavior of the software is closely connected with the behavior of its platform; for instance, the response time of an input event from the environment depends on the task scheduling and I/O handling mechanisms of the platform. Hence, to ensure that the implemented system meets the timing requirements, the modeling and verification stage must consider the timing interactions between the software and the platform. This is non-trivial, as not only is the platform-specific timing information usually not available in this stage, but capturing all details of the platform would also make the model too complex to verify efficiently.

A common solution is to model the software and the platform separately, while considering their interactions when composing the generated code with the platform. However, this separation of concerns can lead to timing gaps between the platform-independent model and its implemented system. For example, depending on the specific platform support, the code may read environmental inputs through either a polling or an interrupt-based mechanism, each of which will add a different delay to the code’s outputs; as a result, even if the timing requirements between the code’s inputs and outputs are verified to satisfy in the platform-independent model, they may no longer hold in the implemented system.

A number of efforts have been made to bridge the above timing gaps, e.g., by adding platform aspects to the high-level models. Most notably, existing work on the implementability of timed automata (TA) incorporates the platform information by explicitly modeling the execution platform [3], [4], [18] or by modifying the TA semantics to reflect the implementation platform semantics (e.g., [13], [14], [19]). Real-time scheduling has also been combined with TA in [2], and a number of automata- and actor-oriented scheduling interfaces have also been developed [5], [8], [17]. However, these existing techniques consider rather restrictive platform semantics, such as assuming only a small subset of possible implementations [4] or ignoring various sources of timing delays [3], [18], [19].

In the prior work [9], we studied how the timing behavior of the implementation can be formally verified by measuring the platform-specific delays, and reflecting the measured delays to the original model for model checking. In [12], we presented a timing testing method that enables one to measure more refined platform-specific delay segments in a systematic way. However, we didn’t consider explicit platform models in those previous works. In order to reason about the timing behavior of the implementation more accurately, we need to build a more refined platform model that captures the complex interactions between platform-specific features.

In this paper, we present a platform-specific timing verification framework for verifying the timed behavior of an implemented system that is developed from a platform-independent model. In this framework, we model the timed behavior of a platform using an implementation scheme, which defines how the platform interacts with the platform-independent code and with the environment. Specifically, an implementation scheme specifies how a platform (i) executes the platform-independent code (e.g., periodic or aperiodic execution), (ii) reads sensor inputs from and writes actuator outputs to the environment (e.g., sampling or interrupt-based mechanism), and (iii) delivers the processed sensor inputs to and receives computed outputs from the platform-independent code (e.g., buffering or shared variable mechanism). From a platform-independent model (PIM) and an implementation scheme for the platform, we then systematically construct the corresponding platform-specific model (PSM) that captures the timed behavior of the corresponding implemented system. By verifying this PSM, we can now determine whether the implemented system satisfies the timing requirements.

In summary, we make the following contributions:

- a general category of implementation schemes for the platform based on Parnas’ four variable formalism [16] (Section III);
- a modular transformation algorithm for systematically transforming a PIM into a PSM that is extensible to a range of implementation schemes (Sections IV-V); and
- an infusion pump system case study, which demonstrates that the timed behavior of the PSM is sufficiently close to that of the implemented system (Section VI).

II. PROBLEM STATEMENT AND APPROACH OVERVIEW

A. Motivating Example: Infusion Pump System

An infusion pump is a safety-critical medical device that injects drugs into a patient’s body in a controlled manner for medical purposes, such as diabetes treatments or anesthesia. Its hardware platform is equipped with sensors (e.g., a bolus
request button) and actuators (e.g., a pump motor) to interact with its physical environment (e.g., a patient). For example, a patient presses a bolus request button to request an additional amount of drugs; a pump rotates a pump-motor that generates physical forces to move a loaded syringe so that drug can flow from the syringe to the patient. The software operating on the platform reads sensor inputs (e.g., a patient’s bolus requests) and writes actuator outputs (e.g., the rotation of a pump motor) to make infusion administration processes happen.

Consider the following timing requirement from [1]:

- (REQ1) “A patient requests a bolus, a bolus infusion should start within 500ms.”

Fig. 1 shows an UPPAAL [7] model that abstracts the timed behavior of the infusion pump software and its environment. This model is a parallel composition of two automata, \(M\) and \(ENV\). \(M\) models the software using a clock variable \(x\), input synchronizations (\(m\)-BolusReq and \(m\)-EmptySyringe), and output synchronizations (\(c\)-StartInfusion, \(c\)-StopInfusion and \(c\)-Alarm). \(ENV\) models the environment using a clock variable \((env\times)\) and the complementary synchronizations with that of \(M\).

One can easily verify that the model in Fig. 1 satisfies REQ1 (by describing REQ1 as a logic formula stating that the maximum delay between two successive synchronizations, \(m\)-BolusReq followed by \(c\)-StartInfusion, does not exceed 500 time units, and then verifying it using model checking).

![Diagram of platform-independent model](image)

**Fig. 1.** A platform-independent model of the infusion pump software

Using available code generators (e.g., [6] [15]), we can systematically generate the source code for the software from the verified model. The generated code repeatedly performs a series of interactions with a platform: (1) waiting for being invoked by a platform, (2) reading inputs from a platform, (3) computing outputs (which involves finding a transition to be taken using the inputs and the clocks’ values), and (4) writing the computed outputs to a platform.

Observe that the platform-specific interactions between the code and a platform are not specified in the above UPPAAL model (which is the reason we call this model a platform-independent model (PIM)). For instance, the model does not capture how the platform reads a bolus request from a patient and how the platform processes this input into a program input to be read by the code. Depending on which scheme the platform uses, the response time to the patient’s request can vary, which can make the timed behavior of the implementation deviate from that of the PIM. Hence, although REQ1 is satisfied in the PIM, this timing requirement may not hold in the implemented system due to the timing deviation.

### B. Problem Statement

To describe the problem more precisely, we first introduce some notations. \(\text{Code}(\text{PIM})\) denotes the \textit{platform-independent code} generated from a PIM. This code needs to be composed with platform-specific primitives (e.g., read/write API) to realize the platform-specific interactions. \(IS\) denotes an \textit{implementation scheme} that is used for such a composition. The implemented system is denoted by \(\text{Code}(\text{PIM})\)\(\mid\text{IS}\), which indicates that the resulting implementation is the platform-independent code executed with the support of an implementation scheme \(IS\). Finally, \(\mathcal{P}(\Delta_{mc})\) denotes a timing requirement that the delay between an input \(m\) and an output \(c\) must be within \(\Delta_{mc}\) time units (e.g., the delay between the bolus request and the start infusion must be within 500ms).

As explained in the infusion pump example, the timing information of a chosen \(IS\) is not explicitly modeled in the PIM and hence, the following claim may not always hold: \(\text{PIM} \models \mathcal{P}(\Delta_{mc}) \implies \text{Code}(\text{PIM})\)\(\mid\text{IS}\) \(\models \mathcal{P}(\Delta_{mc})\).

To better describe the timed behavior of the implementation, we need a platform-specific version of the PIM that captures the timed behavior of the IS, such that if this platform-specific model (PSM) is verified to meet a timing requirement, then its implementation also satisfies the requirement. In addition, as was discussed earlier, the IS may introduce an additional delay to the response time between an input \(m\) and an output \(c\), which can lead to violation of the timing requirement \(\mathcal{P}(\Delta_{mc})\), we would like to compute a new delay bound \(\Delta_{mc}'\) for which \(\mathcal{P}(\Delta_{mc}')\) holds in the implementation. In other words, we need to derive a platform-specific model (PSM) and a bound \(\Delta_{mc}'\) such that \(\text{PSM} \models \mathcal{P}(\Delta_{mc}') \implies \text{Code}(\text{PIM})\)\(\mid\text{IS}\) \(\models \mathcal{P}(\Delta_{mc}')\).

To this end, our goals are (1) to identify the necessary information to obtain such a PSM, (2) to develop a method for systematically transforming a PIM into a PSM based on the identified information, and (3) to compute the bound \(\Delta_{mc}'\).

### C. Approach overview

We assume that a platform consists of several building blocks to support the execution of \(\text{Code}(\text{PIM})\), including the \textit{Input-Device} that processes inputs generated from the environment, the \textit{Output-Device} that processes outputs generated from \(\text{Code}(\text{PIM})\), and the \textit{Code-Execution} that invokes \(\text{Code}(\text{PIM})\) to perform transitions based on the environmental inputs and the clocks’ values. Fig. 2-(a) shows the block diagram that illustrates these blocks in an implemented system.

![Diagram of implementation](image)

**Fig. 2.** The mapping between (a) the implemented system and (b) its platform-specific model

To model the platform-specific information, we propose a general category of implementation schemes that lists possible mechanisms to implement each interaction of the platform with the environment and with the \(\text{Code}(\text{PIM})\). A platform can select a particular combination of mechanisms from the category as its implementation scheme. Based on a chosen implementation scheme \(IS\), we can systematically transform a PIM into a PSM. The transformed PSM is the parallel composition of the UPPAAL models shown in Fig. 2-(b) (i.e., \(M_{IO} \parallel IF_{MI} \parallel IF_{OC} \parallel EXE_{IO} \parallel ENV_{MC}\)). Here, \(IF_{MI}\) and \(IF_{OC}\) model the platform-specific input and output processing mechanisms for interacting with the environment (modeled

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1Note that the specific timing parameter (500ms) is added to the original requirement to explain our work.

2More details about the model semantics can be found in the technical report [11].
as \( ENV_{MC} \). \( EXE_{IO} \) models the platform-specific invocation mechanism to schedule the platform-independent code so as to receive (delivered) the code’s inputs (outputs) from (to) a platform. Finally, \( M_{IO} \) models the timed behavior of the platform-independent code executing with the support of the implementation scheme \( IS \). Each of these models is matched to its respective building block shown in Fig. 2-(a).

Our transformation algorithm is modular and preserves the original structure of the \( PIM \). The algorithm ensures that the resulting \( PSM \) has a similar timed behavior3 to that of the \( Code(PIM) \)\(_{imp \ IS} \). Based on the obtained \( PSM \), we can verify whether the delay between an input \( m \) and an output \( c \) is bounded; if so, we derive the delay bound \( \Delta_{mc} \) from the total processing delays of the Input-Device, the Code-Execution, and the Output-Device. By verifying the \( PSM \) against the timing requirement \( \mathcal{P}(\Delta_{mc}) \), we can formally check whether the implementation meets this relaxed timing requirement.

III. Implementation Schemes

We define implementation schemes using the four-variables formalism proposed by Parnas [16]. As illustrated in Fig. 2-(a), the platform’s interactions occur at two system boundaries: (1) the \( mc \)-boundary, which is the boundary between the platform and the environment, and (2) the \( i/o \)-boundary, which is the boundary between the platform and the Code\((PIM)\).

At the \( mc \)-boundary, the Input-Device reads environmental inputs stored in the monitored variables \( m \), processes the inputs, and writes the processed inputs to the input variables \( i \). In addition, the Output-Device reads the code execution’s outputs from the output variables \( o \), processes these outputs, and delivers the processed outputs to the environment by writing to the controlled variables \( c \).

At the \( i/o \)-boundary, the Code\((PIM)\) is invoked by the Code-Execution to read the processed inputs stored in the \( i \)-variables. After performing the execution, the Code\((PIM)\) writes outputs to the \( o \)-variables. The information flows from the environment to the Code\((PIM)\) and vice versa are illustrated in Fig. 2-(a).

An implementation scheme defines a mechanism for implementing each interaction at the two system boundaries:

**Definition 1 (Implementation Scheme).** An implementation scheme is a pair \( \{MC, IO\} \), where

- \( MC \) specifies a reading (writing) mechanism and associated parameters for each variable \( v \in m \cup o \) (\( v' \in i \cup c \))
- \( IO \) specifies a reading (writing) mechanism and associated parameters for each variable \( v \in i \) (\( v' \in o \)), as well as an invocation mechanism for the Code\((PIM)\).

The category of reading, writing and invocation mechanisms, as well as the platform-specific parameters required by the mechanisms, is available in [11]. Example 1 gives a possible implementation scheme formed by selecting different mechanisms from this category. The timed behavior of the implementation with this scheme is illustrated by Fig. 3.

**Example 1 \( (IS_1) \).** The implementation scheme 1 is given by \( IS_1 = \{MC_1, IO_1\} \), where: (1) \( MC_1(v) = \langle pulse \text{ signal, interrupt, rising-edge) \rangle \) (\( delay_{\text{min}} = 1, delay_{\text{max}} = 3 \)) and \( MC_1(v') = \langle pulse \text{ signal) \rangle (delay_{\text{max}} = 1, delay_{\text{max}} = 3) \) for all \( v \in m \cup o \) and \( v' \in i \cup c \); and (2) \( IO_1(v) = \langle (Buffers, Read-all) \rangle \) (\( buffer-size = 5 \)) for all \( v \in i \cup o \) and \( IO_1(invoke) = \langle \text{(Periodic invocation)} \rangle \) (\( period = 100 \)).

We now explain the implementation schemes in detail.

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3We use the term “similar timed behavior” since it requires further assumptions to argue that the implementation shows exactly the “same timed behavior” with the \( PSM \) in a strict sense.
buffer. Due to space constraints, we do not discuss all possible combinations here.

Note that our platform characterization is motivated by Altené and Sifakis’s work [3], [4], [18] (which also studied how a platform can be abstracted and composed with the platform-independent model), but we aim at capturing more detailed timed behavior of different platforms. Specifically, the techniques in [3], [4], [18] consider a much smaller subset of possible implementations compared to our technique, and they also cannot capture scenarios that our framework can describe and formally verify. For example, they do not consider the buffer communication scheme, and they cannot capture the following scenario: Although a platform successfully detects an input from the environment, the platform-independent code may not be able to receive it due to a buffer overrun.

IV. MODULAR TRANSFORMATION FROM PIM TO PSM

Our transformation algorithm takes as inputs a PIM and an implementation scheme (IS), and it returns a PSM that has a similar timed behavior to that of the implementation Code(PIM)|∥o-IS. The algorithm is modular, as it preserves the structure of the PIM for any implementation scheme defined in Section III. This modularity makes the platform-specific timing verification possible for a range of implementation schemes that an arbitrary target platform may choose to execute the Code(PIM). Before describing the algorithm, we first discuss the timed behavior of the PSM that the algorithm constructs from the PIM and the IS.

Definition 2 (PIM). A PIM is defined as \( M \parallel ENV \), where \( M \) and \( ENV \) are the UPPAAL automata modeling the software and the environment, respectively. Further, \( M = (L, \mathcal{L}_0, C, A, E, I) \), where: \( L \) is a set of locations; \( \mathcal{L}_0 \) is an initial location; \( C \) is a set of clocks; \( A = A_m \cup A_c \), where \( A_m = \{ m_1, \ldots, m_k \} \) is a set of input synchronizations and \( A_c = \{ c_1, \ldots, c_j \} \) is a set of output synchronizations; \( E \) is a set of transitions; and \( I \) is a set of invariants.

Note that the input and output synchronizations in Definition 2 are expressed using \( m \) and \( c \) variables only, which implies that the interactions between \( M \) and \( ENV \) occur at the \( mc \)-boundary and that the interactions at the \( io \)-boundary are not captured by the PIM.

Definition 3 (PSM). A PSM is defined as the network of UPPAAL automata \( M_{IO} \parallel IF_{MI} \parallel IF_{MI1} \parallel \ldots \parallel IF_{MIk} \parallel IF_{OC1} \parallel \ldots \parallel IF_{OCj} \parallel EXE_{IO} \parallel ENV_{MC} \).

The definition of each automaton in Definition 3 is explained in Section II-C. Note that a subscript of each automaton specifies a system boundary where the interactions occur. For example, \( M_{IO} \) and \( EXE_{IO} \) model the interactions at the \( io \)-boundary; \( IF_{MI} \) and \( IF_{OC1} \) model the interactions at both the \( mc \)-boundary and \( io \)-boundary; and \( ENV_{MC} \) models the interactions at the \( mc \)-boundary. Such interactions are modeled using either channel synchronizations or variables in UPPAAL semantics.

Fig. 4 illustrates the different timed behaviors of the PIM and the PSM. As shown in the figure, in the PIM, \( M \) is directly synchronized with \( ENV \) at the \( mc \)-boundary: whenever an input is triggered \((m_k, i)\), \( M \) can immediately accept it \((m_k, ?)\); similarly, whenever \( M \) produces an output \((c_k, ?)\), the output is immediately visible to the environment \((c_k, ?)\).

However, in the PSM, \( M_{IO} \) (which will be constructed from \( M \)) is indirectly synchronized with the environment \( ENV_{MC} \) via a platform whose behavior is abstracted as the parallel composition of \( IF_{MI} \) (input interface), \( IF_{OC} \) (output interface), and \( EXE_{IO} \) (code execution) automata. Specifically, when an input is triggered \((m_k, i)\), it is first read \((m_k, ?)\), processed, and enqueued to a buffer by \( IF_{MI} \). The buffered input is then dequeued by \( EXE_{IO} \), which also performs the synchronization with \( M_{IO} \) \((i_k, ?)\). Subsequently, \( M_{IO} \) produces the corresponding output and dequeues the output to a buffer at the \( io \)-boundary. Finally, \( IF_{OC} \) dequeues the output, processes it, and makes the processed output visible to \( ENV_{MC} \) \((c_k, !)\).

We next explain the basic idea for constructing a PSM with the above timed behavior; a detailed construction algorithm and examples can be found in [11]. We focus on a transformation algorithm that is compatible with the implementation scheme \( IS_1 \) in Example 1; algorithms compatible for other schemes can be designed similarly.

(1) Construction of \( M_{IO} \) and \( ENV_{MC} \): \( M_{IO} \) remains syntactically the same as \( M \), except that its synchronizations are renamed from \( m \) to \( i \), and from \( c \) to \( o \). For example, the input synchronization \( m-BolusReq \) in Fig. 1-(1) is renamed to \( i-BolusReq \), and the output synchronization \( o-StartInfusion \) is renamed to \( o-StartInfusion \). In contrast, the environment model \( ENV_{MC} \) remains exactly the same under the current implementation scheme. The desynchronization between \( M_{IO} \) and \( ENV_{MC} \) described earlier results in some missing information in connecting the input and output data flows across the two system boundaries. We introduce the input and output interface automata that fill such information below.

(2) Construction of \( IF_{MI} \) and \( IF_{OC} \): \( IF_{MI} \) (\( IF_{OC} \)) is an input (output) interface automation that models the data flow from \( m \) to \( i \) (from \( o \) to \( c \)), which is performed by the Input-Device (Output-Device). This automation models (1) the platform-specific delays introduced when converting an environmental input to a program input (a program output to an environmental output), and (2) the communication mechanism used to deliver a program input to the Code(PIM) (a program output to the platform). We explain the construction of \( IF_{MI} \) below; \( IF_{OC} \) can be constructed in the same manner.

Fig. 5-(1) shows the automaton \( IF_{MI-BolusReq} \) constructed from \( M \) in Fig. 1 and \( IS_1 \). At the location \( Idle \), the Input-Device is ready to read an input \( m_j \) from \( ENV_{MC} \). At the location \( Processing \), the Input-Device is currently processing the input \( m_j \) that has been read from \( ENV_{MC} \). Once the
input \( m_j \) is read (when a transition from Idle to Processing
is taken), a processed input is ready within delay\textsubscript{min} and delay\textsubscript{max} time units, as specified by \( IS_j \). A processed input is
delivered to the Code(PIM) through a finite buffer whose
size is defined in the parameter buffer-size. Therefore, there
are two cases when the processed input needs to be inserted
into the buffer: (1) the buffer has an empty slot, and (2) the
buffer is full. These two cases are modeled as the two
respective transitions from Processing to Idle. Fig. 5-(2)
shows the automaton \( IF\textsubscript{OC}\text{\_fromInfusion} \), which models the output
data flow from the Code(PIM) to the platform. (More details
can be found in [11].)

(3) Construction of \( EXE_{IO} \): Observe that the buffer com-
munication scheme prevents \( M_{IO} \) from directly synchronizing
with \( IF\text{\_MC} \) and \( IF\text{\_OC} \); for instance, the automaton \( IF\text{\_MC}\text{\_BolusReq} \) in
Fig. 5-(1) is synchronized with \( ENV\text{\_MC} \) through the \( m\text{-BolusReq} \) channel, but there is no synchronization with
\( M_{IO} \) over the \( i\text{-BolusReq} \) channel. In other words, once an
input (output) is written to a buffer by the Input-Device
(Code(PIM)), the Code(PIM) (Output-Device) does not need to
read the input (output) immediately. These situations are
illustrated in Fig. 4 as the two timing gaps: (1) the time
passage between \( eng(i_k) \) and \( deq(i_k) \), and (2) the time pas-
 sage between \( eng(o_k) \) and \( deq(o_k) \). The timing for these
buffer read/write operations is closely tied to the invocation
mechanism of Code(PIM), since these operations can only
occur while Code(PIM) is being invoked by a platform. Our
algorithm constructs an automaton \( EXE_{IO} \) that models the
resulting timed behavior under the periodic invocation and the
synchronizations with \( M_{IO} \).

Fig. 6 shows the \( EXE_{IO} \) automaton, which is constructed
as follows. We first create six locations that correspond to
the execution stages of the Code(PIM): Waiting indicates
that the Code(PIM) is waiting for an invocation; Active
indicates that the Code(PIM) has been invoked and is ready
for execution; the remaining four locations indicate that the
respective computations -read input, compute transitions, write
output- are being performed. Then, necessary input and output
transitions are added to the constructed locations in a way
the following semantics are realized: For each transition associated
with either an input or an output synchronization in \( M_{IO} \), we
add a complementary transition in \( EXE_{IO} \) and associate it
with the conjunction of three guard conditions: (1) \( M_{IO} \) is in
a location that can read the input (or write the output), and (2)
the original guard condition of the transition in \( M_{IO} \), and (3)
the input is in the buffer (or the output buffer is not full).

V. PROPERTY OF THE TRANSFORMED PSM

Given \( PIM \models P(\Delta_{mc}) \), our goal is to find a relaxed timing
constraint \( \Delta_{mc} \) from the original constraint \( \Delta_{mc} \) such that
\( PSM \models P(\Delta_{mc}) \) and \( \Delta_{mc} \geq \Delta_{mc} \).

We first highlight the platform-specific delays that any pair
\((i)\) of input and output experiences due to the platform-specific
parts of the PSM (i.e., \( ENV\text{\_MC}, IF\text{\_MC}, IF\text{\_OC}, EXE_{IO} \)):

1) M-C Delay \( \Delta_{mc} \): specifies the maximum time passage
from the instant the environment triggers an input \((m_j, t_{m_j})\)
until the instant the environment observes an output \((o_j, t_{o_j})\) at
the \( mc \)-boundary, i.e., \( \Delta_{mc} = t_{o_j} - t_{m_j} \); this delay is illustrated in
Fig. 4 as the time passage between the two synchronizations
\((mc_k, t_{mc_k}) \) and \( \Delta_{mc} \).

2) Input-Delay \( \Delta_{mi} \): specifies the maximum time passage
from the instant the environment triggers an input \((m_j, t_{m_j})\)
at the \( mc \)-boundary until the instant the Code(PIM) reads the
input \((i_j, t_{i_j})\) at the \( io \)-boundary, i.e., \( \Delta_{mi} = t_{i_j} - t_{m_j} \); this
delay is illustrated in Fig. 4 as the guarded section of \( IF\text{\_MC} \).

3) Output-Delay \( \Delta_{mo} \): specifies the maximum time passage
from the instant Code(PIM) produces an output \((o_j, t_{o_j})\) at
the \( io \)-boundary until the instant the environment observes the output \((o_j, t_{o_j})\) at the \( mc \)-boundary, i.e., \( \Delta_{mo} = t_{o_j} - t_{o_j} \); this
delay is illustrated in Fig. 4 as the guarded section of \( IF\text{\_OC} \).

The relaxed constraint \( \Delta_{mc} \) is computed in terms of the
Input-Delay and the Output-Delay described above.

Remark 1. In general, we cannot determine the bound of \( \Delta_{mc} \),
because some combinations of platform-specific parameters
make \( \Delta_{mc} \) unbounded. However, we can derive the timing
constraints on implementation schemes that make \( \Delta_{mc} \) bounded.
If these constraints are satisfied, then we can find such a bound.

We now show the four constraints that make \( \Delta_{mc} \) bounded:

- (Constraint 1) Detection of all input signals: Given an input
  pattern generated from \( ENV\text{\_MC} \), \( IF\text{\_MC} \) can detect
  all input signals, and (2) the maximum input processing delay
  of \( IF\text{\_MC} \) is shorter than the minimum inter-arrival time.

- (Constraint 2) No overflow of the input buffer: The invoca-
tion interval of \( EXE_{IO} \) should be small enough w.r.t. the
  input processing speed of \( IF\text{\_MC} \) so that each input can be read
  by \( EXE_{IO} \) before the input buffer overflows.

- (Constraint 3) No overflow of the output buffer: Given an output
  pattern generated by \( M_{IO} \), \( IF\text{\_OC} \) has sufficient
  processing speed to process all outputs before the output
  buffer overflows, and make them visible to \( ENV\text{\_MC} \), and (2)
  \( ENV\text{\_MC} \) can read the produced outputs by \( IF\text{\_OC} \) fast enough.

- (Constraint 4) No internal transition occurrences: Since
  \( ENV\text{\_MC} \) generates an input, \( M_{IO} \) does not take internal
  transitions until the input is processed by \( IF\text{\_MC} \) and read by
  \( M_{IO} \).

Lemma 1. If the system constraints are verified in PSM, then
(1) the Input-Delay is bounded by the function of all maximum
platform-specific parameters that are used for \( ENV\text{\_MC}, IF\text{\_MC},
EXE_{IO} \), (2) the Output-Delay is bounded by the function of
all maximum platform-specific parameters that are used for
\( ENV\text{\_MC}, IF\text{\_OC}, EXE_{IO} \).

Proof: Refer to [11].
be the maximum internal delay of the PIM for processing the input and output pair \((i, o)\). The following lemma holds.

**Lemma 2.** If the system constraints are verified in PSM, then a possible value of \(\Delta_{mc}'\) for which \(PSM \models \mathcal{P}(\Delta_{mc}')\) is given by \(\Delta_{mc}' = \Delta_{mc} + \Delta_{io} + \Delta_{internal}\).

**Proof:** The rest of delays that contribute to \(\Delta_{mc}'\) is the internal delay of PIM for processing the input \(i\) and the output \(o\). Therefore, the maximum possible M-C delay is bounded by the summation of these three types of delays.

**Theorem 1.** Suppose PSM verifies the system constraints and a platform is correctly described using the implementation scheme IS. Then, \(PSM \models \mathcal{P}(\Delta_{mc})\) implies \(\text{Code}(PIM) || \text{imp} IS \models \mathcal{P}(\Delta_{mc}')\).

The assumption “if a platform is correctly described using the implementation scheme” can be validated by testing.

**VI. CASE STUDY**

We present a case study of infusion pump system (c.f. Section II-A) to demonstrate the utility of the proposed framework.

**Setting:** We created the UPPAAL model \(PIM\) of the software, which is verified to meet REQ1 (infusion always starts within \(\Delta_{mc} = 500\)ms from the instant a patient requests a bolus), i.e., \(PIM \models \mathcal{P}(\Delta_{mc})\). We used the TIMES tool [6] to automatically generate the Code\((PIM)\) from the PIM. The Code\((PIM)\) was then integrated with an infusion pump platform\(^4\) that follows the implementation scheme IS\(_1\), except that the polling scheme is used to read the bolus request input. (A summary of relevant platform-specific parameters is available in [11].) Finally, we constructed the PSM from the PIM and IS\(_1\) using the algorithm in Section IV.

In this setting, we performed 60 times of the bolus request scenarios with the implementation and measured the timing delays using an oscilloscope. Some of these parameters (e.g., input processing delay, WCET) were obtained from testing, and the rest (e.g., polling interval, invocation period) was set manually. Throughout the testing, the M-C delay, the Input-Delay and the Output-Delay are measured together; their average, maximum, and minimum delays are summarized in the Measured Delay-(IMP) rows in Table I.

**Verification of PSM against \(\mathcal{P}(\Delta_{mc})\):** REQ1 is not satisfied by the PSM (i.e., \(PSM \not\models \mathcal{P}(\Delta_{mc})\)). In other words, when the PIM is composed with the implementation scheme IS\(_1\), there are scenarios where the infusion starts after more than 500ms since a bolus is requested. This prolonged delay is caused by the additional delays originated from the platform-specific parts in the composition of PIM and IS\(_1\). Out of 60 test scenarios, we observed 53 scenarios in which the timing requirement REQ1 is violated. Hence, we can conclude that the implementation introduces a case where the actual delay is greater than the delay (\(\Delta_{mc}\)) that has been verified in the PIM, i.e., \(\text{Code}(PIM) || \text{imp} IS \not\models \mathcal{P}(\Delta_{mc}')\).

| Verification of PSM against \(\mathcal{P}(\Delta_{mc})\): We verified that PSM does satisfy the four conditions for bounded delay (c.f. Section V). Hence, \(\Delta_{mc}'\) can be determined from the platform-specific parameters, Input-Delay (490ms) and Output-Delay (440ms), using Lemmas 1 and 2 as follows: \(\Delta_{mc}' = 490 + 440 + 500 = 1430\)ms. Using verification, we verified that the PSM satisfies the relaxed timing requirement \(\mathcal{P}(\Delta_{mc}')\). Thus, assuming that the platform-specific parameters obtained through testing are correct, we can conclude \(\text{Code}(PIM) || \text{imp} IS \models \mathcal{P}(\Delta_{mc}')\). Note that this verified result is also consistent with the testing result, i.e., all measured delays are bounded by the verified bound (1430ms).

**VII. CONCLUSION**

We have presented a framework for formally verifying the timed behaviors of implementations that are developed from a platform-independent model (PIM). We presented a general category of implementation schemes that a platform can choose from to execute the platform-independent code, as well as a modular algorithm for systematically generating a platform-specific model (PSM) from a PIM and an implementation scheme. In order to explain the framework, we chose one particular implementation scheme from the general category to show how the corresponding PSM is constructed according to the transformation algorithm, and presented a case study to show that the timed behavior of the generated PSM is similar to that of the implementation. Our framework can be used to add formal timing assurance to implementations and to formally verify subtle timing errors through a model checking technique, which cannot be achieved using testing [12]. Even though we only presented the verification framework using a particular combination of the implementation scheme, we believe that the transformation can be applied for other implementation schemes in a similar way, and leave the research about the applicability of this framework for general cases as a future work.

**REFERENCES**


\(^4\)This platform has been used for the Generic Patient-Controlled-Analgesia (GPCA) infusion pump reference implementation [10]