$B(E2)$ value and configuration mixing in ^{32}Mg

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I demonstrate that the $B(E2)$ value in ^{32}Mg can be understood with a model in which both the ground and 2^+ first-excited states are predominantly of sd-shell character.

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I. INTRODUCTION

Several groups have equated a "larger than expected" $B(E2)$ value in ^{32}Mg with the conclusion that its ground state (gs) is dominated by an intruder configuration of two neutrons in the fp shell. It is true that the $B(E2)$ value is larger in ^{32}Mg than in ^{30}Mg, (see Table I) but it is significantly smaller than in ^{34}Mg—the last almost certainly consisting of the $(fp)^2$ configuration. It does appear that core excitation is larger in ^{32}Mg (and ^{30}Ne) than in other nearby nuclei. Except for one anomalously large value, [1] (which I ignore here) various experiments [2–5] agree on the $B(E2)$ measurements in ^{32}Mg. The analysis of the data divides the results into two camps—depending on the magnitude of the correction for feeding from above. Table I lists the $B(E2)$'s from gs to 2_1^+ in ^{30,32,34}Mg. It has been recently claimed [6] that ^{32}Mg(gs) is not dominated by the $(fp)^2$ configuration. Straightforward analysis [6] of data from the ^{30}Mg(t,p) reaction [7] has demonstrated that it is the excited 0^+ state that contains most of this intruder configuration.

Parameters in a shell-model calculation [8–12] can be adjusted to produce a ^{32}Mg (gs) that is predominantly $\nu(fp)^2(sd)^{-2}$, but that may not be necessary (or correct). It is possible that some of the shell-model calculations did not sufficiently renormalize the interaction when including different $\hbar\omega$'s. Inclusion of neutron excitations into the fp shell is important for understanding the properties of $N = 20$ neutron-rich nuclei. However, that need not imply that these excitations dominate the ground states. In some descriptions this $2\hbar\omega$ excitation arises from deformation, so that the $1/2^-$ Nilsson orbital is significantly lowered. Other descriptions ascribe this excitation to a pairing effect in spherical nuclei. Yamagami and Van Giai [13] performed Hartree-Fock-Bogoliubov (HFB) calculations for these nuclei with modified Skyrme interactions. They state that the $B(E2)$'s and 2_1^+ energies in ^{30,32}Mg are well described in calculations in which both nuclei are spherical. They find an fp-shell occupancy of $\sim$0.5 neutrons for ^{32}Mg(gs). Several calculations [14,15] found ^{34}Mg to be deformed, with β in the range 0.3 to 0.4; ^{30}Mg to be spherical, but β soft; and ^{32}Mg to be transitional with coexisting spherical and deformed shapes. With one choice of Skyrme force (SkM*), they were degenerate, but with all other forces [15] the deformed state was 2 to 4 MeV above the spherical gs. Reference [14] found for ^{32}Mg a minimum in the potential-energy surface at $\beta = 0$, and no other. A very recent paper [16] found ^{30}Mg to be very β soft, ^{34}Mg to be γ soft, and ^{32}Mg to have two minima—at 0 and 0.33.

II. THE MODEL

Here, we investigate whether we can understand the $B(E2)$ in a simple, consistent model. For ^{32}Mg, let

$$\text{gs} = a\,^{32}\text{Mg(gs},sd\text{shell}) + b\,^{30}\text{Mg(gs},sd\text{shell})\ \text{x}\ \nu(fp)_0^2,\ \text{and}$$

$$2_1^+ = A\,^{32}\text{Mg}(2_1^+,\,sd\text{ shell}) + B\,^{30}\text{Mg(gs},\,sd\text{ shell})\ \text{x}\ \nu(fp)_2^2.$$

Reference [6] found $a^2 = 0.74$–0.81. Later we consider adding a third term,

$$C\,^{30}\text{Mg}(2_1^+,\,sd\text{ shell})\ \text{x}\ \nu(fp)_0^2,$$

to the 2^+ state. We define $B(E2; i \to f) = M^2/(2J_i + 1)$, so that if $J_i = 0$, $B(E2) = M^2$. Then we have for ^{32}Mg

$$M(E2;\,^{32}\text{Mg}) = aAM(sd) + bBM(fp),$$

and the two terms are constructive. Now, we need to estimate $M(sd)$ and $M(fp)$. Because $M(sd)$ connects 2_1^+ and gs in the sd-shell ^{32}Mg, it must be a pure proton excitation, as the neutrons form a filled shell. Because $M(fp)$ connects $\nu(fp)_0^2$ to $\nu(fp)_2^2$, it is a pure neutron excitation. Now, look at ^{30}Mg. Is its $M(E2)$ larger or smaller than $M(sd)$? In the absence of cross-shell excitations, the proton part of $M(^{30}\text{Mg})$ should be similar to $M(sd)$ (the protons are similar in the two.), but $M(^{30}\text{Mg})$ can also contain some sd-shell neutron excitation. If the 2^+ and gs in ^{30}Mg also contain some $(fp)^2$ excitation, they will add to $M(^{30}\text{Mg})$. Because all the terms will add constructively, then we expect, quite rigorously, that $M(sd) \leqslant M(^{30}\text{Mg})$. For now we assume equality, but we return to this point later. For ^{34}Mg, both of the complicating terms will be smaller, so we expect $M(fp) \approx M(^{34}\text{Mg})$. This might be a topic for further study.

III. CALCULATIONS AND RESULTS

From Table I, the $B(E2)$ in ^{30}Mg is 295(26) e^2fm^4. The weighted average of the four large values in ^{32}Mg is 446(31) e^2fm^4. (We have ignored the much larger value of Chiste *et al.* [1] and have not averaged in the two values derived with large corrections for feeding from above.) We return to this point later. The weighted average in ^{34}Mg is 577(79) e^2fm^4. Thus, we have $M(^{30}\text{Mg}) = 17.2(5)$ and $M(^{34}\text{Mg}) = 24.0(16)$ $e$fm^2. The ratio is 1.40(12). We use these temporarily as $M(sd)$ and $M(fp)$, respectively, and investigate changing them later. Thus, we have

$$M(^{32}\text{Mg}) = aA\,[17.2(5)] + bB\,[24.0(16)].$$

TABLE I. $B(E2\text{:gs} \to 2^+_1)$ (in $e^2\text{fm}^4$) for ^{30,32,34}Mg.

Nucleus	$B(E2;\ \text{gs} \to 2^+{}_1)$	Reference
^{30}Mg	295(26)	[2]
^{32}Mg	454(78)	[3]
	440(55)	[2]
	330(70) [a]	[2]
	449(63)	[5]
	447(57)	[4]
	>328(48) [a]	[4]
^{34}Mg	<670	[2]
	631(126)	[5]
	541(102) [b]	[4]
	>438(83) [b]	[4]

[a]After correcting for feeding from above.

[b]No evidence of feeding from above, but if present, the lower limit applies.

We have no direct knowledge of the mixing in the 2^+ state and no way of deriving it from other data. We are attempting to determine if *some* mixed 2^+ wave function will reproduce the experimental $B(E2)$ when combined with the gs of Ref. [6]. First, we consider the dependence of $B(E2)$ on the 0^+ mixing. Many have claimed that the $B(E2)$ requires small a^2. We consider two determinations of A (with B following from $A^2 + B^2 = 1$): (i) the value of A that maximizes $B(E2)$ for a given value of a, and (ii) assuming the same mixing in the 0^+ and 2^+ states, i.e., $A = a$. [For assumption (i), we note that $A^2 < a^2$ throughout; i.e., maximizing $B(E2)$ requires more $(fp)^2$ mixing in the 2^+ than in the gs.] We return to this point in Sec. IV.

Results are displayed in Fig. 1, where we plot vs a^2, as a solid curve, the value of A^2 that maximizes $B(E2)$. Also plotted in Fig. 1 (long dashes) is the ratio $B(E2;^{32}\text{Mg})/B(E2;^{30}\text{Mg})$ that results from the given a,A combination. The value of the ratio that arises from the assumption of equal mixing in the gs and 2^+ states is also plotted (short dashes). The two results are not very different. This is the ratio of $B(E2;\,^{32}\text{Mg})$ to $B(E2;\,sd)$, where we have taken $B(E2;\,^{30}\text{Mg})$ for the latter. Remember, Ref. [6] found $a^2 \sim 0.75$. Near this region, this simple model predicts a $B(E2)$ ratio that is significantly larger than unity. In Fig. 2, we plot the predicted $B(E2)$ (solid) and the $\pm 1\ \sigma$ limit (dashed) curves arising from uncertainties in $M(sd)$ and $M(fp)$. Also shown there are both sets of experimental values for ^{32}Mg, as solid squares and open circles, with their uncertainties. We note that if the lower experimental $B(E2)$ value is correct, the $B(E2)$ requires $a^2 \geqslant 0.7$. We have agreement with the larger experimental value for a wide range of a^2 up to about 0.7. The simple model agrees well. Throughout the remainder of this article, the 0^+ mixing is held constant and the 2^+ mixing is investigated. Note that $a^2 = 0.75$ corresponds to $N_{fp} = 0.50$, consistent with the calculation of Ref. [13]. Remember that the present calculation, so far, does not contain any component in the 2^+ state in which the core is excited to 2^+, i.e., the term $C\ ^{30}\text{Mg}(2^+_1, sd\text{ shell})\ \text{x}\ \nu(fp)^2_0$. This term will serve to increase the calculated ^{32}Mg value further. Then $M(E2)$ is

$$M(^{32}\text{Mg}) = aAM(sd) + bBM(fp) + bCM(^{30}\text{Mg}).$$

For illustrative purposes we have computed the $B(E2)$, with this term added, for $a^2 = 0.75$ and $b^2 = B^2 = 0.25$, with $A^2 + B^2 + C^2 = 1$, for $C^2 = 0$–0.25 Results are displayed in Fig. 3. We note that any value of $C^2 \geqslant 0.025$ produces total agreement with the larger experimental value. The upper solid curve is not for fixed B, but for the A, B combination that maximizes $B(E2)$ for a given C^2, still with $a^2 = 0.75$ and $b^2 = 0.25$. These curves still have $M(sd) = M(^{30}\text{Mg})$. Even

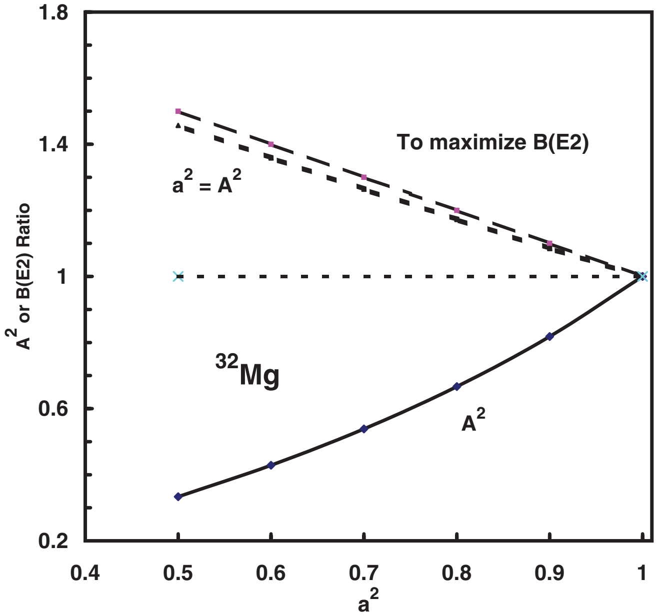

FIG. 1. (Color online) Solid curve is the value of A^2 that maximizes the ^{32}Mg $B(E2)$ for a given value of a^2. Long dashes are the ratio of that $B(E2)$ to the one in ^{30}Mg. Short dashes represent the ratio for equal 0^+ and 2^+ mixing $A^2 = a^2$.

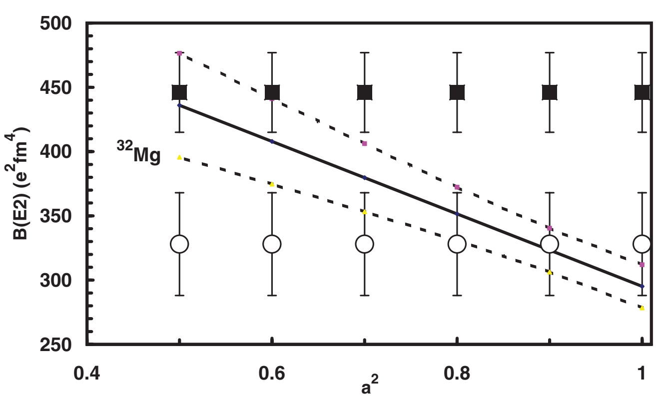

FIG. 2. (Color online) Calculated $B(E2)$ for ^{32}Mg (solid curve) with the $\pm\ 1\ \sigma$ limits (dashed) vs a^2, the amount of sd-shell component in the gs. Open circles are at the experimental value extracted with a large correction for feeding from above. Solid squares are at the experimental value if the analysis included little or no such correction.

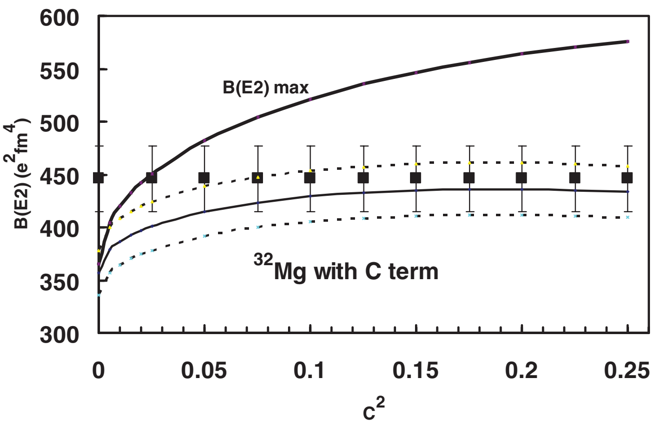

FIG. 3. (Color online) Squares are as in Fig. 2, but are now plotted vs C^2, using $a^2 = 0.75$ and $b^2 = B^2 = 0.25$, and with the C term (see text). The upper curve is for the A,B combination that maximizes $B(E2)$ for given C^2, still with $a^2 = 0.75$ and $b^2 = 0.25$.

with reasonably small values of C^2, a decrease in a^2 will produce a $B(E2)$ that is too large.

Concerning our assumption that $M(sd)$ can be approximated by $M(^{30}\mathrm{Mg})$, if it should turn out that $M(sd)$ is significantly smaller than $M(^{30}\mathrm{Mg})$, then the calculated $B(E2)$'s presented here will be smaller. But a smaller $M(sd)$ can be compensated by a slightly larger value of $M(fp)$ or a slightly larger value of C. We note from Fig. 3 that, with the C term present, it should be an easy matter to fit the larger of the two ^{32}Mg experimental values with a value for $M(sd)$ that is significantly smaller than that for $M(^{30}\mathrm{Mg})$. This point is clearly made in Fig. 4, where we plot, vs A^2, the value of $M(sd)$ needed to produce $B(E2;\,^{32}\mathrm{Mg}) = 446(31)\ e^2\mathrm{fm}^4$, for three different values of C^2. As expected, with a reasonably small C^2, the ^{32}Mg $B(E2)$ can be reproduced with an assumed $M(sd)$ that is quite a bit smaller than $M(^{30}\mathrm{Mg})$.

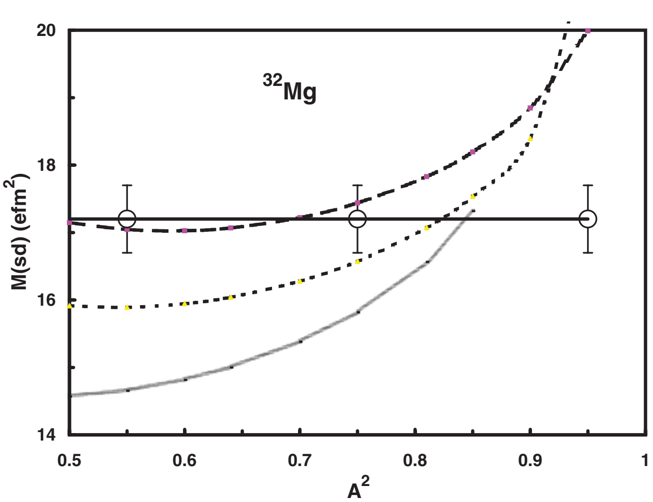

FIG. 4. (Color online) The value of $M(sd)$ (see text) necessary to fit the experimental $B(E2) = 446(31)\ e^2\mathrm{fm}^4$ for ^{32}Mg, plotted vs A^2 for three different small values of $C^2 = 0.025$ (long dashes), 0.05 (short dashes), and 0.10 (solid line). Open circles represent the experimental $M(^{30}\mathrm{Mg})$.

IV. THE MIXING

In a two-state model, if the mixed states are separated by an energy E, and the mixing amplitudes are a and b, then the matrix element responsible for the mixing is $V = abE$. Here, I use $a^2 = 0.81$ and $b^2 = 0.19$ [6], which were obtained assuming no core excitation in ^{30}Mg, because I intend to use the ^{30}Mg energies as representative of the sd shell 0^+–2^+ spacing. Then, with $E = 1.058$ MeV [7], we have $V = 0.415$ MeV. Thus, the unmixed 0^+ states are at 0.201 (sd shell) and 0.857 ($(fp)^2$) MeV. These energies are plotted in Fig. 5. For illustrative purposes we also show the 2^+ states. In Fig. 5, we place the sd shell 2^+ state 1.48 MeV above the sd shell 0^+ state, as in ^{30}Mg. And, we place the $(fp)^2$ 2^+ state 0.654 MeV above its 0^+ state, as in ^{34}Mg. This almost certainly is an oversimplification, and we do not intend to claim this determines the order of the two unmixed 2^+ states or their separation. But, it does show that the unmixed 2^+ states are closer to one another than the unmixed 0^+ states. Reference [2], while discussing ^{28}Ne, mentioned that the energy shift of the 2^+ states is larger than the 0^+ shift, because the 2^+ unmixed states are closer together than the unmixed 0^+ states. The same is true here. And, of course, the mixing amplitude will be larger for 2^+ than for 0^+. So, it is not surprising that, in the $E2$ analysis discussed above, the 0^+ gs was purer than the 2^+_1 state.

The mixing preserves summed energy, so these two 2^+energies, combined with $E(2^+_1) = 0.886$ MeV, give the second physical 2^+ state at 2.31 MeV. If these 2^+ basis energies are even remotely correct, then one of the two states at 2.321 and 2.551 MeV should be 2^+. The lower one has had several J^π suggestions, the latest being 4^+ [17]. The 2.55-MeV state

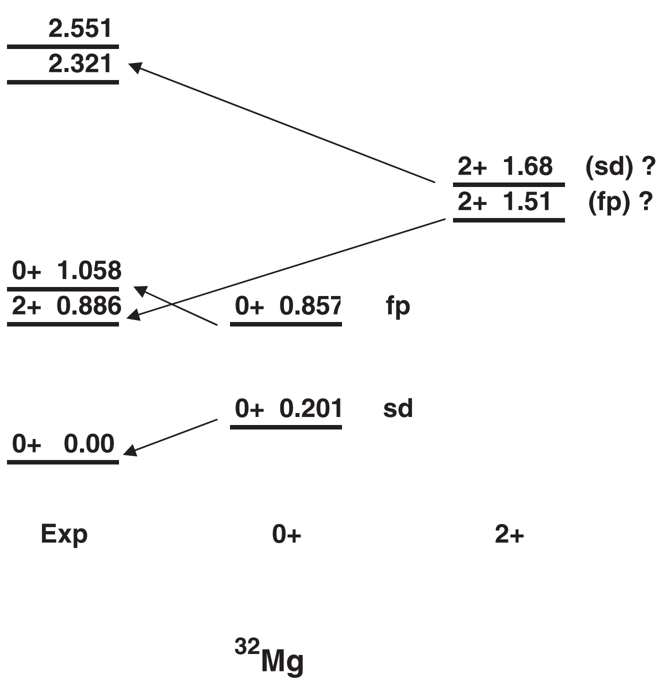

FIG. 5. Mixed and unmixed 0^+ and 2^+ states in ^{32}Mg. The left-hand column depicts the physical states, the middle column depicts the 0^+ basis states from the mixing in Ref. [6], and the right-hand column depicts one possibility for the 2^+ basis states.

is thought [18] to be (1^- or 2^+). It is unlikely that the 2^+ basis energies are lower than those in Fig. 5, because there are no other known states in ^{32}Mg below 2.3 MeV. If the basis 2^+ energies are significantly higher than those in Fig. 5, then the second 2^+ physical state would be above 2.6 MeV. The next known state that could be 2^+ is at 3.488 MeV [17].

With the 2^+ ordering shown in Fig. 5, the lower 2^+ physical state would have slightly more than 50% $(fp)^2$. But, small changes in the energy of either can easily change the order of the 2^+ basis states.

V. SUMMARY

The simpler model presented here (without the C term in the 2^+ state) has a slight preference for the lower $B(E2)$ value in ^{32}Mg, but calculated results are in rough agreement with either value. We conclude that we have no trouble understanding the $B(E2)$ in ^{32}Mg even with both 0^+ and 2^+ wave functions dominated by the sd-shell configuration. With the C term added we can reproduce the larger $B(E2)$, even with $M(sd)$ considerably smaller than $M(^{30}\mathrm{Mg})$. As noted in the Introduction, shell-model parameters can be adjusted to produce a ^{32}Mg(gs) that is predominantly of the configuration $\nu(fp)^2(sd)^{-2}$. Changes in these parameters should make it possible to obtain a gs that contains only about 25% of this configuration. A very recent paper [16], using HFB, concluded that for ^{32}Mg, "the interpretation of "deformed ground and spherical excited 0^+ states' based on the simple inversion picture of the spherical and deformed configurations does not hold."

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