Čerenkov radiation in chiral media[a)]

Nader Engheta
The Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, Pennsylvania 19104
Sassan Bassiri
Jet Propulsion Laboratory, Mail Stop 238-624, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California 91109

(Received 25 January 1990; accepted for publication 12 July 1990)

In this paper, the Čerenkov radiation in an unbounded homogeneous isotropic chiral medium is studied and analyzed classically. Starting from the Maxwell equations and the proposed constitutive relations for isotropic chiral media, we formulate the problem for the electric and magnetic fields emitted from a charged particle moving with a constant speed in a chiral medium, and find a formal solution for the electromagnetic field components and energy spectral density of radiation. Notable features, such as double cone of propagation, and important characteristics of the Čerenkov radiation in such media in terms of the relative velocity of the particle with respect to the two characteristic phase velocities in the medium are discussed.

I. INTRODUCTION

It is well known that accelerated charged particles radiate electromagnetic energy. It is also known that charged particles moving with constant velocities can, in certain circumstances, emit electromagnetic radiation. One such radiation is Čerenkov radiation which was discovered experimentally by Čerenkov[1] and was explained theoretically by Frank and Tamm[2] in 1937. This phenomenon was also studied by Heaviside[3] and Sommerfeld.[4] This radiation is observed when relativistic electrons are travelling in a dielectric medium with the velocity higher than the phase velocity of light in the medium. Since its discovery, Čerenkov radiation has been studied extensively and many theoretical and experimental problems in this radiative mechanism have been investigated. Among those, one should mention the problem of radiation from an electron moving with a constant velocity in a homogeneous dielectric medium treated by Frank and Tamm,[2] Čerenkov radiation in anisotropic and dispersive media by Pafomov,[5] Tanaka,[6] and Jenkins and White,[7,8] the problem of emission from a particle traversing a piecewise homogeneous dielectric medium by Fainberg and Khiznyak[9] and Garybyan,[10] and a similar radiative mechanism in a periodically inhomogeneous dielectric media by Ter-Mikaelyan[11] and Casey *et al.*[12]

In light of numerous applications of Čerenkov radiation in high-energy physics, nuclear and cosmic-ray physics, particle detectors, and nondestructive tests of properties of materials, it is of great theoretical and practical interest and importance to investigate the problem of Čerenkov radiation in homogeneous isotropic chiral materials. Chiral materials are characterized in classical electrodynamics by the following set of constitutive relations for the time harmonic case ($e^{-i\omega t}$)

$$\mathbf{D} = \epsilon \mathbf{E} + i\xi_c \mathbf{B}, \tag{1}$$

$$\mathbf{H} = i\xi_c \mathbf{E} + \mathbf{B}/\mu. \tag{2}$$

Here, ϵ, μ, and ξ_c represent, respectively, the permittivity, permeability, and chirality admittance of the chiral medium.[13] Such materials exhibit handed property and are categorized by an intrinsic right or left handedness due to the chirality or handedness of their constituent microstructures. It has been shown that light wave propagation in such media possesses two circularly polarized eigenmodes, a right- and a left-circularly polarized (RCP and LCP) wave with two unequal characteristic phase velocities v_+ and v_-, respectively[14,15]:

$$v_\pm = \omega/k_\pm = \left[\pm\mu\xi_c + \sqrt{\mu^2\xi_c^2 + \mu\epsilon} \right]^{-1}. \tag{3}$$

These unequal phase velocities (and wave numbers $k_\pm$) give rise to circular birefringence which results in optical activity and, for lossy chiral materials, circular dichroism. It is worth noting that, in isotropic chiral media, this birefringence is independent of the direction of wave propagation, whereas in anisotropic materials it does depend on the wave's direction of propagation. A variety of problems of electromagnetic and optical wave interactions with chiral materials have been investigated recently and the results have been reported in the literature. Among these, one notes dyadic Green's functions in chiral media,[14-16] waveguiding structures filled with chiral materials,[17-20] transition radiation caused by a chiral slab,[21] Doppler effects in chiral media,[22] wave propagation in periodic chiral structures,[23] point and distributed radiators embedded in chiral media,[24] and reflection and refraction at a chiral-nonchiral interface.[25-28]

In this paper, we investigate and analyze the Čerenkov

[a)] Parts of this work were presented by the authors at the 1988 International IEEE Antennas and Propagation Society/International Union of Radio Science (URSI) Symposium, Syracuse, NY, June 6–10, 1988, and at the Progress in Electromagnetics Research Symposium (PIERS), Boston, MA, July 24–26, 1989.

 0021-8979/90/214393-06$03.00

radiation emitted from a charged particle travelling with a constant velocity in an unbounded homogeneous isotropic chiral medium. A formal solution will be obtained for the electromagnetic field components, the energy spectral density of the radiation, and the state of polarization of radiated waves. These relations will demonstrate functional dependence of the above-mentioned quantities on material parameters.

II. FORMULATION OF THE PROBLEM

As mentioned earlier, the Čerenkov radiation would be emitted if a charged particle moved with a velocity greater than the light's speed in the medium. In chiral media, as given in Eq. (3), are present two unequal characteristic phase velocities for electromagnetic waves depending on their state of polarization. Therefore, it appears that in such media the Čerenkov radiation condition, where the particle's velocity must be greater than the medium's phase velocity, can be met for two different values of particle's velocity. This physical observation motivated us to examine thoroughly the problem of Čerenkov radiation in chiral media.

Consider a charged particle with charge q moving in the z direction with a constant speed v in an unbounded lossless homogeneous isotropic chiral material described by Eqs. (1) and (2). In a cylindrical coordinate system with z axis pointed in the direction of particle's motion, the current density $\mathbf{J}$ can be expressed as

$$\mathbf{J} = (qv/2\pi\rho)\delta(\rho)\delta(z - vt)\hat{\mathbf{z}}, \qquad (4)$$

where $\delta(\ )$ denotes the Dirac delta function, $\hat{\mathbf{z}}$ is a unit vector in the z direction, and the coordinate ρ is related to the coordinates x and y via $\rho^2 = x^2 + y^2$ (see Fig. 1). Equation (4), which expresses the current density $\mathbf{J}$ in the time domain, can be Fourier transformed into the frequency domain. Thus we have

$$\tilde{\mathbf{J}}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{J}(t)e^{i\omega t}\,dt = \frac{q}{4\pi^2\rho}\delta(\rho)e^{i(\omega/v)z}\,\hat{\mathbf{z}}. \qquad (5)$$

Considering the Fourier transform of $\mathbf{J}$ given in (5) and the constitutive relations of (1) and (2), Maxwell's equations can be written as

$$\nabla\times\tilde{\mathbf{E}} = i\omega\mu\tilde{\mathbf{H}} + \omega\mu\xi_c\tilde{\mathbf{E}},$$

$$\nabla\times\tilde{\mathbf{H}} = \omega\mu\xi_c\tilde{\mathbf{H}} - i\omega\epsilon\{1 + (\mu/\epsilon)\xi_c^2\}\tilde{\mathbf{E}} + \tilde{\mathbf{J}}. \qquad (7)$$

Here $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ represent the Fourier transforms of $\mathbf{E}(t)$ and $\mathbf{H}(t)$, respectively. From Eqs. (6) and (7) we find the following equation for the emitted electric field $\tilde{\mathbf{E}}$ in the frequency domain:

$$\nabla\times\nabla\times\tilde{\mathbf{E}} - \omega^2\mu\epsilon\tilde{\mathbf{E}} - 2\omega\mu\xi\nabla\times\tilde{\mathbf{E}} = i\omega\mu\tilde{\mathbf{J}}. \qquad (8)$$

Since this equation is linear, its solution can be expressed as the volume integral:

$$\tilde{\mathbf{E}}(\mathbf{r}) = i\omega\mu\int_{V'} \mathbf{\Gamma}(\mathbf{r},\mathbf{r}')\cdot\tilde{\mathbf{J}}(\mathbf{r}')\,dV', \qquad (9)$$

where $\mathbf{r}$ and $\mathbf{r}'$ are the observation and source points, respectively, V' is the source region over which the integral is carried out, and $\mathbf{\Gamma}(\mathbf{r},\mathbf{r}')$ is the dyadic Green's function in the unbounded chiral medium obtained in Ref. 14. Substituting (5) into (9), we get

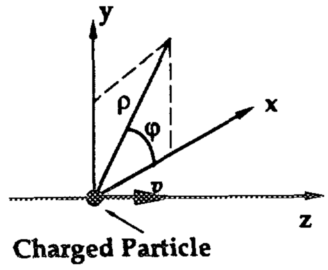

FIG. 1. Geometry of the problem. A charged particle is moving with a constant velocity v along the z axis in an unbounded homogeneous isotropic chiral medium.

$$\tilde{\mathbf{E}}(\rho,\varphi,z) = i\omega\mu\int_{-\infty}^{+\infty}\int_0^{2\pi}\int_0^{+\infty} \mathbf{\Gamma}(\rho,\varphi,z;\rho',\varphi',z')\cdot\hat{\mathbf{z}} \times\frac{q}{4\pi^2\rho'}\delta(\rho')e^{i(\omega/v)z'}\rho'\,d\rho'\,d\varphi'\,dz'. \qquad (10)$$

Using $\hat{\mathbf{\Gamma}}(\rho,\varphi,z;\rho',\varphi',\omega/v)$ as a shorthand for

$$\int_{-\infty}^{+\infty} \mathbf{\Gamma}(\rho,\varphi,z;\rho',\varphi',z')e^{i(\omega/v)z'}\,dz',$$

we can rewrite Eq. (10) as follows:

$$\tilde{\mathbf{E}}(\rho,\varphi,z) = i\omega\mu\int_0^{2\pi}\int_0^{+\infty} \hat{\mathbf{\Gamma}}(\rho,\varphi,z;\rho',\varphi',\omega/v)\cdot\hat{\mathbf{z}} \times\frac{q}{4\pi^2\rho'}\delta(\rho')\rho'\,d\rho'\,d\varphi'. \qquad (11)$$

Knowing that the problem is independent of the coordinate φ due to isotropy of the medium, the above integral can be further simplified to

$$\tilde{\mathbf{E}}(\rho,z) = i\omega\mu(q/2\pi)\hat{\mathbf{\Gamma}}(\rho,z;0,\omega/v)\cdot\hat{\mathbf{z}}. \qquad (12)$$

To obtain $\hat{\mathbf{\Gamma}}(\rho,\varphi,z;\rho',\varphi',\omega/v)$, we must multiply the three-dimensional dyadic Green's function[14] by $e^{i(\omega/v)z'}$ and integrate the product over z' from $-\infty$ to $+\infty$. This results in the following expression for $\hat{\mathbf{\Gamma}}(\rho,\varphi,z;\rho',\varphi',\omega/v)$;

$$\hat{\mathbf{\Gamma}}(\rho,\varphi,z;\rho',\varphi',\omega/v) = \alpha_+\mathbf{\Lambda}_+(k_+)\hat{G}_+(\rho,\varphi,z;\rho',\varphi',\omega/v) + \alpha_-\mathbf{\Lambda}_-(k_-)\hat{G}_-(\rho,\varphi,z;\rho',\varphi',\omega/v), \qquad (13)$$

where

$$\alpha_\pm = \pm\frac{k^2 - k_\pm^2}{k_-^2 - k_+^2}, \qquad (14)$$

$\mathbf{\Lambda}_\pm(k_\pm)$ is defined[14,24] as

$$\mathbf{\Lambda}_\pm(k_\pm) = [\mathbf{u} \pm (1/k_\pm)\mathbf{u}\times\nabla + (1/k_\pm^2)\nabla\nabla] \qquad (15)$$

and

$$\hat{G}_\pm(\rho,\varphi,z;\rho',\varphi',\omega/v) = (i/4)H_0^{(1)}(\gamma_\pm R)e^{i\beta z}, \qquad (16)$$

with

$$R = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\varphi - \varphi')},$$

$$\beta = \omega/v,$$

$$\gamma_\pm = \sqrt{k_\pm^2 - \beta^2}.$$

It is worth noting that when β equals zero, Eq. (13) yields the two-dimensional dyadic Green's function in an unbounded chiral medium already reported in the literature.[15,29]

Substituting (13) with $\rho' = 0$ into (12), we get

$$\widetilde{\mathbf{E}}(\rho,z) = -\omega\mu(q/8\pi)$$
$$\times[\alpha_+\mathbf{\Lambda}_+(k_+)\cdot\hat{\mathbf{z}}\, H_0^{(1)}(\gamma_+\rho)e^{i\beta z} + \alpha_-\mathbf{\Lambda}_-(k_-)\cdot\hat{\mathbf{z}}\, H_0^{(1)}(\gamma_-\rho)e^{i\beta z}]. \quad (17)$$

After a lengthy mathematical manipulation, the following explicit expression is obtained for $\widetilde{\mathbf{E}}(\rho,z)$:

$$\widetilde{\mathbf{E}}(\rho,z) = \frac{\omega\mu q}{8\pi(k_+ + k_-)}\Bigg[\left(\frac{\beta\gamma_+}{k_+}H_1^{(1)}(\gamma_+\rho) + \frac{\beta\gamma_-}{k_-}H_1^{(1)}(\gamma_-\rho)\right)e^{i\beta z - i\pi/2}\,\hat{\rho} + [\gamma_+ H_1^{(1)}(\gamma_+\rho) - \gamma_- H_1^{(1)}(\gamma_-\rho)]e^{i\beta z}\,\hat{\varphi} + \left(\frac{\gamma_+^2}{k_+}H_0^{(1)}(\gamma_+\rho) + \frac{\gamma_-^2}{k_-}H_0^{(1)}(\gamma_-\rho)\right)e^{i\beta z}\,\hat{\mathbf{z}}\Bigg], \quad (18)$$

where $\hat{\rho}$ and $\hat{\varphi}$ are the unit vectors along ρ and φ directions, respectively. The corresponding magnetic field $\widetilde{\mathbf{H}}$ can be obtained by substituting (19) into

$$\widetilde{\mathbf{H}} = i\xi_c\widetilde{\mathbf{E}} + (1/i\omega\mu)\nabla\times\widetilde{\mathbf{E}} \quad (19)$$

and arranging the terms. Thus we get

$$\widetilde{\mathbf{H}}(\rho,z) = \frac{q}{16\pi}\Bigg[\left(\frac{\beta\gamma_+}{k_+}H_1^{(1)}(\gamma_+\rho) - \frac{\beta\gamma_-}{k_-}H_1^{(1)}(\gamma_-\rho)\right)e^{i\beta z}\,\hat{\rho} + [\gamma_+ H_1^{(1)}(\gamma_+\rho) + \gamma_- H_1^{(1)}(\gamma_-\rho)]e^{i\beta z + i\pi/2}\,\hat{\varphi} + \left(\frac{\gamma_+^2}{k_+}H_0^{(1)}(\gamma_+\rho) - \frac{\gamma_-^2}{k_-}H_0^{(1)}(\gamma_-\rho)\right)e^{i\beta z + i\pi/2}\,\hat{\mathbf{z}}\Bigg]. \quad (20)$$

Equations (18) and (20) represent the explicit expressions of electric and magnetic fields in the frequency domain. It should be noted that for the case of nonchiral materials, i.e., where $\xi_c = 0$, Eqs. (18) and (20) reduce to

$$\widetilde{\mathbf{E}}(\rho,z) = \frac{q}{8\pi}\sqrt{\frac{\mu}{\epsilon}}\Bigg[\left(\frac{\beta\gamma}{k}H_1^{(1)}(\gamma\rho)\right)e^{i\beta z - i\pi/2}\,\hat{\rho} + \left(\frac{\gamma^2}{k}H_0^{(1)}(\gamma\rho)\right)e^{i\beta z}\,\hat{\mathbf{z}}\Bigg] \quad (21)$$

and

$$\widetilde{\mathbf{H}}(\rho,z) = \frac{q}{8\pi}[\gamma H_1^{(1)}(\gamma\rho)]e^{i\beta z + i\pi/2}\,\hat{\varphi}, \quad (22)$$

with γ and k being $\sqrt{\omega^2\mu\epsilon - \beta^2}$ and $\omega\sqrt{\mu\epsilon}$, respectively. These are electric and magnetic fields (in the Fourier domain) obtained in the Čerenkov radiation emitted from a moving charge in a simple homogeneous dielectric. By inverse Fourier transforming of Eqs. (18) and (20), one can obtain the time domain expressions for the electric and magnetic fields of the Čerenkov radiation in a homogeneous isotropic chiral material.

III. DISCUSSIONS

In the formal derivation of results for the electric and magnetic fields presented in the previous section, the particle was assumed to move at a constant velocity.[30] The relative value of this velocity with respect to the phase velocity of light in the medium plays an important role in determining the characteristics of the Čerenkov radiation. In a homogeneous isotropic nonchiral material, the phase velocity of light is a single value which may depend on the light's frequency. In that case, it is well known that if the particle's velocity were greater than the light's speed in the medium, the Čerenkov radiation would be emitted. In chiral media, however, as we pointed out in Eq. (3), there exist two unequal characteristic phase velocities depending on the state of polarization of light. Therefore, it is of great importance to find out how the relative value of particle's velocity with respect to the two characteristic phase velocities would determine the properties of the Čerenkov radiation in these media.

Three possible cases can be considered:

Case 1:

$$v_+ < v_- < v$$

In this case, the relativistic particle is moving with a velocity greater than both phase velocities.[31] Therefore, γ_+ and γ_- are real quantities and Eqs. (18) and (20) provide us with the detailed information on the field components. To gain more physical insight, it is useful to examine these solutions for observation points far from the particle's track. For $\gamma_+\rho \gg 1$ and $\gamma_-\rho \gg 1$, the large-argument asymptotic expression for the Hankel functions,

$$H_0^{(1)}(\gamma_\pm\rho) \approx \sqrt{(2/\pi\gamma_\pm\rho)}\; e^{i\gamma_\pm\rho - i\pi/4} \quad \text{for } \gamma_\pm\rho \gg 1, \quad (23)$$

$$H_1^{(1)}(\gamma_\pm\rho) \approx \sqrt{(2/\pi\gamma_\pm\rho)}\; e^{i\gamma_\pm\rho - i3\pi/4} \quad \text{for } \gamma_\pm\rho \gg 1, \quad (24)$$

can be used, and Eq. (18) can be modified and rearranged as follows:

$$\tilde{\mathbf{E}}(\rho,z) = \tilde{\mathbf{E}}_+ + \tilde{\mathbf{E}}_- = \frac{q\eta_c}{16\pi}\sqrt{\frac{2}{\pi\rho}}\Big\{\sqrt{\gamma_+}\Big[\Big(-\frac{\beta}{k_+}\hat{\rho} + \frac{\gamma_+}{k_+}\hat{\mathbf{z}}\Big) - i\hat{\varphi}\Big]e^{i(\beta z+\gamma_+\rho-\pi/4)} + \sqrt{\gamma_-}\Big[\Big(-\frac{\beta}{k_-}\hat{\rho}+\frac{\gamma_-}{k_-}\hat{\mathbf{z}}\Big)+i\hat{\varphi}\Big]e^{i(\beta z+\gamma_-\rho-\pi/4)}\Big\} \quad (25)$$

where $\eta_c = 1/\sqrt{\xi_c^2 + (\epsilon/\mu)}$. Similarly, the magnetic field is obtained for this asymptotic limit as

$$\tilde{\mathbf{H}}(\rho,z) = \tilde{\mathbf{H}}_+ + \tilde{\mathbf{H}}_- = \frac{q}{16\pi}\sqrt{\frac{2}{\pi\rho}}\Big\{i\sqrt{\gamma_+}\Big[\Big(-\frac{\beta}{k_+}\hat{\rho} + \frac{\gamma_+}{k_+}\hat{\mathbf{z}}\Big) - i\hat{\varphi}\Big]e^{i(\beta z+\gamma_+\rho-\pi/4)} - i\sqrt{\gamma_-}\Big[\Big(-\frac{\beta}{k_-}\hat{\rho}+\frac{\gamma_-}{k_-}\hat{\mathbf{z}}\Big)+i\hat{\varphi}\Big]e^{i(\beta z+\gamma_-\rho-\pi/4)}\Big\}. \quad (26)$$

Let us define two angles θ_+ and θ_- as

$$\cos\theta_\pm = \beta/k_\pm = v_\pm/v. \quad (27)$$

We can then rewrite $\tilde{\mathbf{E}}_+, \tilde{\mathbf{E}}_-, \tilde{\mathbf{H}}_+, \tilde{\mathbf{H}}_-$ in the following way:

$$\tilde{\mathbf{E}}_\pm(\rho,z) = \frac{q\eta_c}{16\pi}\sqrt{\frac{2}{\pi\rho}}\sqrt{\gamma_\pm}\,[(-\cos\theta_\pm\hat{\rho} + \sin\theta_\pm\hat{\mathbf{z}}) \mp i\hat{\varphi}] \times e^{ik_\pm(z\cos\theta_\pm + \rho\sin\theta_\pm) - i\pi/4} \quad (28)$$

and

$$\tilde{\mathbf{H}}_\pm(\rho,z) = (\pm i/\eta_c)\tilde{\mathbf{E}}_\pm(\rho,z) = \pm i\frac{q}{16\pi}\sqrt{\frac{2}{\pi\rho}}\sqrt{\gamma_\pm} \times [(-\cos\theta_\pm\hat{\rho} + \sin\theta_\pm\hat{\mathbf{z}}) \mp i\hat{\varphi}] \times e^{ik_\pm(z\cos\theta_\pm + \rho\sin\theta_\pm) - i\pi/4}. \quad (29)$$

We note that in this case, when the particle's velocity is greater than both intrinsic velocities, each frequency component of the radiated fields in the Čerenkov radiation consists of two cylindrical waves $\tilde{\mathbf{E}}_+$ and $\tilde{\mathbf{E}}_-$: one with the wave number k_+ and the other with the wave number k_-. These two cylindrical waves have two different wave fronts. The wave front of $\tilde{\mathbf{E}}_+$ is a single cone whose axis is the z axis and its flare angle is $(\pi/2) - \theta_+$, and the wave front of the other wave $\tilde{\mathbf{E}}_-$ is another single cone with the same axis as the first cone but with flare angle $(\pi/2) - \theta_-$. Thus the Čerenkov radiation in chiral media may have two cones of radiation. The direction of propagation of $\tilde{\mathbf{E}}_+$ makes angle θ_+ with the z axis whereas the direction of propagation of $\tilde{\mathbf{E}}_-$ makes angle θ_- with the z axis. Somewhat similar phenomenon is observed in four-parameter biisotropic media reported by Monzon.[32] The features of double cones of radiation is depicted in Fig. 2.

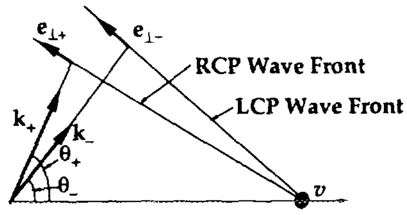

FIG. 2. A longitudinal cross section of the two cones of Čerenkov radiation in a chiral medium. The wave vectors $\mathbf{k}_+$ and $\mathbf{k}_-$ are normal to the RCP and LCP wave fronts, respectively. The angles θ_+ and θ_- are defined by $\cos\theta_\pm = v_\pm/v = \beta/k_\pm$. The unit vectors $\mathbf{e}_{\perp+}$ and $\mathbf{e}_{\perp-}$ are defined in the text.

Another important feature of the radiated field is their state of polarization. Let us introduce two unit vectors:

$$\hat{\mathbf{e}}_{\perp+} = (\cos\theta_+\hat{\rho} - \sin\theta_+\hat{\mathbf{z}}), \quad (30)$$

$$\hat{\mathbf{e}}_{\perp-} = (\cos\theta_-\hat{\rho} - \sin\theta_-\hat{\mathbf{z}}). \quad (31)$$

These unit vectors are perpendicular to $\hat{\varphi}$ and lie on the surfaces of the two cones representing wave fronts of $\tilde{\mathbf{E}}_+$ and $\tilde{\mathbf{E}}_-$, respectively. Using these unit vectors, we can express $\tilde{\mathbf{E}}_\pm$ as

$$\tilde{\mathbf{E}}_\pm(\rho,z) = -\frac{q\eta_c}{16\pi}\sqrt{\frac{2}{\pi\rho}}\sqrt{\gamma_\pm}\,(\hat{\mathbf{e}}_{\perp\pm} \pm i\hat{\varphi}) \times e^{ik_\pm(z\cos\theta_\pm + \rho\sin\theta_\pm) - i\pi/4}. \quad (32)$$

The above equation describes the state of polarization of the electric field of each conical wave. It shows that $\tilde{\mathbf{E}}_+$ is a right-circularly polarized (RCP) wave of wave number k_+ and that $\tilde{\mathbf{E}}_-$ is a left-circularly polarized (LCP) wave of wave number k_-.

The double cone of radiation is one of the notable features of Čerenkov radiation in a homogeneous isotropic chiral medium. This feature resembles that of the Čerenkov radiation emitted from a charged particle moving along the principal axis of a gyrotropic medium.[33,34] However, in the latter case, the radiation characteristics depend on the particle's direction of motion whereas in the former they are not.

The energy radiated by the charged particle through the surface of a cylinder of radius ρ and of length dl whose axis coincides with the z axis is

$$dW = 2\pi\rho\, dl\int_{-\infty}^{\infty}[\mathbf{E}(t)\times\mathbf{H}(t)]\cdot\hat{\rho}\,dt = 2\pi\rho\, dl\int_{-\infty}^{\infty}[E_\varphi(t)H_z(t) - E_z(t)H_\varphi(t)]dt. \quad (33)$$

Expressing the field components $E_\varphi(t), H_z(t), E_z(t)$, and $H_\varphi(t)$ in terms of their Fourier transforms and using the properties of the Dirac delta function the total energy radiated per unit path length yields[35]

$$\frac{dW}{dl} = 4\pi^2\rho\int_{-\infty}^{\infty} [\tilde{E}_\varphi(\omega)\tilde{H}_z(-\omega) - \tilde{E}_z(\omega)\tilde{H}_\varphi(-\omega)]d\omega. \quad (34)$$

From the above equation, we can obtain a general expression for the energy spectral density, i.e., the energy radiated per unit path length and per unit frequency interval (for positive frequency range) as

$$\frac{d^2W}{dl\,d\omega} = 8\pi^2\rho[\tilde{E}_\varphi(\omega)\tilde{H}_z(-\omega) - \tilde{E}_z(\omega)\tilde{H}_\varphi(-\omega)]. \quad (35)$$

Using the general expressions (18) and (20) for the electric and magnetic fields and the properties of the Wronskian of Hankel functions,[36] we can reduce Eq. (35) to the following equation:

$$\frac{d^2W}{dl\,d\omega} = \frac{\omega\mu q^2}{4\pi(k_+ + k_-)}\left(\frac{\gamma_+^2}{k_+} + \frac{\gamma_-^2}{k_-}\right). \quad (36)$$

It must be noted that for the isotropic nonchiral medium, ξ_c equals zero, and Eq. (36) [or Eq. (35) in general] reduces to the following well-known result:

$$\frac{d^2W}{dl\,d\omega} = \frac{q^2}{4\pi}\omega\mu\left(1 - \frac{\beta^2}{k^2}\right) \quad \text{for } \xi_c = 0, \quad (37)$$

where β and k are already defined.

Case 2:

$v_+ < v < v_-$

In this case, the particle's velocity is greater than only one of the characteristic phase velocities. Thus γ_+ is a real quantity while γ_- is an imaginary quantity denoted by $\gamma_- = i\alpha_- = i\sqrt{\beta^2 - k_-^2}$. As a result, the Hankel function associated with $\tilde{\mathbf{E}}_-$ should be replaced by the Modified Bessel function $K_n(\alpha_-\rho)$. For large arguments, $\gamma_+\rho \gg 1$ and $\alpha_-\rho \gg 1$, we follow the steps given in case 1. We then obtain

$$\tilde{\mathbf{E}}_+ = -i\eta_c\tilde{\mathbf{H}}_+ = \frac{q\eta_c}{16\pi}\sqrt{\frac{2}{\pi\rho}}\sqrt{\gamma_+}\left[\left(-\frac{\beta}{k_+}\hat{\rho} + \frac{\gamma_+}{k_+}\hat{\mathbf{z}}\right) - i\hat{\varphi}\right] \times e^{i\beta z + i\gamma_+\rho - i\pi/4}, \quad (38)$$

which is similar to that of case 1, and

$$\tilde{\mathbf{E}}_- = +i\eta_c\tilde{\mathbf{H}}_- = \frac{q\eta_c}{16\pi}\sqrt{\frac{2}{\pi\rho}}\sqrt{\alpha_-}\left[\left(-\frac{\beta}{k_-}\hat{\rho} + i\frac{\alpha_-}{k_-}\hat{\mathbf{z}}\right) + i\hat{\varphi}\right] \times e^{i\beta z - \alpha_-\rho}, \quad (39)$$

which, unlike $\tilde{\mathbf{E}}_-$ of case 1, decreases exponentially with ρ. From (38) and (39), we note that in this case, there exists only one cylindrical wave $\tilde{\mathbf{E}}_+$ with right-circular polarization, whose wave number is k_+. The wave front of this wave, like in case 1, is a single cone with its axis along the z axis. The other portion of the field, $\tilde{\mathbf{E}}_-$, is not an outgoing cylindrical wave, but it is a field attached to the charged particle and moving along the z axis with the same speed as the particle's velocity. The energy spectral density in this case, which is only due to $\tilde{\mathbf{E}}_+$, can be obtained following a similar method as given in case 1.

Case 3:

$v < v_+ < v_-$

In this case, both γ_+ and γ_- are imaginary quantities. Denoting γ_+ and γ_- by

$$\gamma_+ = i\alpha_+ = i\sqrt{\beta^2 - k_+^2},$$
$$\gamma_- = i\alpha_- = i\sqrt{\beta^2 - k_-^2},$$

we obtain the following expressions for the electric and magnetic fields in this case:

$$\tilde{\mathbf{E}}_+ = -i\eta_c\tilde{\mathbf{H}}_+ = \frac{q\eta_c}{16\pi}\sqrt{\frac{2}{\pi\rho}}\sqrt{\alpha_+}\left[\left(-\frac{\beta}{k_+}\hat{\rho} + i\frac{\alpha_+}{k_+}\hat{\mathbf{z}}\right) - i\hat{\varphi}\right] \times e^{i\beta z - \alpha_+\rho}, \quad (40)$$

$$\tilde{\mathbf{E}}_- = +i\eta_c\tilde{\mathbf{H}}_- = \frac{q\eta_c}{16\pi}\sqrt{\frac{2}{\pi\rho}}\sqrt{\alpha_-}\left[\left(-\frac{\beta}{k_-}\hat{\rho} + i\frac{\alpha_-}{k_-}\hat{\mathbf{z}}\right) + i\hat{\varphi}\right] \times e^{i\beta z - \alpha_-\rho}. \quad (41)$$

The above equations show that when the particle's velocity is less than both characteristic phase velocities in the chiral medium, no outgoing cylindrical wave exists. Like in the Čerenkov radiation in simple nonchiral media, the fields presented in Eqs. (40) and (41) decrease exponentially with ρ, and they are attached to the particle moving with velocity v. In this case, there is no energy emitted from the particle.

Summaries of parts of these results were presented by the authors in recent symposia.[37]

IV. CONCLUSIONS

We have analyzed classically the problem of Čerenkov radiation emitted from a fast moving charged particle with a constant velocity in an unbounded homogeneous isotropic lossless chiral medium. Starting from the Maxwell equations and the constitutive relations for chiral media, we formulated the problem in the frequency domain. We then obtained the formal solution for the electric and magnetic fields generated by this moving particle in the chiral medium. There are three distinct cases for the Čerenkov radiation depending on the relative value of the particle's velocity with respect to the light's phase velocities in the medium. In each case, the characteristics of the Čerenkov radiation have been discussed and the expressions of electric and magnetic fields along with their states of polarization have been given.

Čerenkov radiation has offered many applications in a variety of fields such as particle physics, high-energy physics, and cosmic-ray physics. Owing to the isotropic birefringence offered for this radiative mechanism in chiral media, such materials and their Čerenkov radiation characteristics can find applications in generation of microwave, millimeter wave, and far infrared with circular polarization.

ACKNOWLEDGMENT

This work was supported, in part, by the National Science Foundation under the Presidential Young Investigator Award No. ECS-8957434.

[1] P. A. Čerenkov, Phys. Rev. **52**, 378 (1937).

[2] I. Frank and I. Tamm, C. R. Acad. Sci. U.R.S.S. **14**, 109 (1937).

[3] See, e.g., P. J. Nahin, *Oliver Heaviside: Sage in Solitude* (IEEE, New York, 1988).

[4] A. Sommerfeld, *Optics* (Academic, New York, 1954).

[5] V. E. Pafomov, Zh. Eksp. Teor. Fiz. **32**, 610 (1957).

[6] K. Tanaka, Phys. Rev. **93**, 459 (1954).

[7] F. A. Jenkins and H. E. White, *Fundamentals of Physical Optics* (McGraw-Hill, New York, 1937).

[8] For more references in this subject, see J. V. Jelley, *Čerenkov Radiation and Its Applications* (Pergamon, New York, 1958).

[9] Ya. Fainberg and N. Khiznyak, Zh. Eksp. Teor. Fiz. **32**, 883 (1957) [Sov. Phys. JETP **5**, 720 (1957)].

[10] C. Garybyan, Zh. Eksp. Teor. Fiz. **35**, 1435 (1958) [Sov. Phys. JETP **8**, 1003 (1959)].

[11] M. Ter-Mikaelyan, Dokl. Akad. Nauk SSR **134**, 318 (1960) [Sov. Phys. Dokl. **5**, 1015 (1960)].

[12] K. F. Casey, C. Yeh, and Z. A. Kaprielian, Phys. Rev. **140**, B 768 (1965).

[13] D. L. Jaggard, A. R. Mickelson, and C. H. Papas, Appl. Phys. **18**, 211 (1979).

[14] S. Bassiri, N. Engheta, and C. H. Papas, Alta Freq. **LV-2**, 83 (1986).

[15] N. Engheta and S. Bassiri, IEEE Trans. Antenna Propagat. **AP-37**, 512 (1989).

[16] N. Engheta and M. W. Kowarz, J. Appl. Phys. **67**, 639 (1990).

[17] N. Engheta and P. Pelet, Opt. Lett. **14**, 593 (1989).

[18] P. Pelet and N. Engheta, IEEE Trans. Antenna Propagat. **AP-38**, 90 (1990).

[19] P. Pelet and N. Engheta, J. Appl. Phys. **67**, (1990).

[20] N. Engheta and P. Pelet, IEEE Trans. Microwave Theory Tech. (unpublished).

[21] N. Engheta and A. R. Mickelson, IEEE Trans. Antenna Propagat. **AP-30**, 1213 (1982).

[22] N. Engheta, M. W. Kowarz, and D. L. Jaggard, J. Appl. Phys. **66**, 2274 (1989).

[23] D. L. Jaggard, N. Engheta, M. W. Kowarz, P. Pelet, J. Liu, and Y. Kim, IEEE Trans. Antenna Propagat. **AP-37**, 1447 (1989).

[24] D. L. Jaggard, X. Sun, and N. Engheta, IEEE Trans. Antennas Propagat. **AP-36**, 1007 (1988).

[25] S. Bassiri, C. H. Papas, and N. Engheta, J. Opt. Soc. Am. A **5**, 1045 (1988).

[26] M. P. Silverman, J. Opt. Soc. Am. A **3**, 830 (1986).

[27] M. P. Silverman, J. Opt. Soc. Am. A **5**, 1852 (1988).

[28] A. Lakhtakia, V. V. Varadan, and V. K. Varadan, J. Opt. Soc. Am. A **6**, 23 (1989).

[29] A. Lakhtakia, V. V. Varadan, and V. K. Varadan, J. Opt. Soc. Am. A **5**, 175 (1988).

[30] This implies that the slowing down of particles due to ionization and the multiple Coulomb scattering are ignored.

[31] Without loss of generality, it is assumed here that ξ_c is a positive real quantity, and consequently $v_{+} < v$. For negative ξ_c the role of $+$ and $-$ subscripts will be interchanged. It is also assumed that the particle's velocity ν does not exceed the vacuum speed of light.

[32] J. C. Monzon, IEEE Trans. Antenna Propagat. **AP-38**, 227 (1990).

[33] A. A. Kolomenskii, Zh. Eksp. Teor. Fiz. **24**, 167 (1953).

[34] A. G. Sitenko and A. A. Kolomenskii, Zh. Eksp. Teor. Fiz. **30**, 511 (1956) [Sov. Phys. JETP **3**, 410 (1956)].

[35] It must be understood that for the case I the integration (38) is to be carried out only over those frequencies for which $v_{+} < v < \nu$.

[36] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1972).

[37] S. Bassiri, C. H. Papas, and N. Engheta, URSI Symp. Dig. **1**, 212 (1988). See also S. Bassiri, C. H. Papas, and N. Engheta, PIERS Dig. **1**, 52 (1989).