

ESSAYS IN MACROECONOMIC AND INTERNATIONAL FINANCE

Zachary R. Stangebye

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2015

Supervisor of Dissertation

Jesús Fernández-Villaverde, Professor of Economics

Graduate Group Chairperson

George Mailath, Professor of Economics

Dissertation Committee

Guillermo Ordoñez, Assistant Professor of Economics

Ufuk Akcigit, Assistant Professor of Economics

Satyajit Chatterjee, Senior Economist at the Federal Reserve Bank of Philadelphia

ABSTRACT

ESSAYS IN MACROECONOMIC AND INTERNATIONAL FINANCE

Zachary R. Stangebye

Jesús Fernández-Villaverde

In the first two chapters of this dissertation, I explore the potential for sovereign debt markets to experience a new type of dynamic lender coordination problem in sovereign debt markets that I call a dynamic panic. During a dynamic panic, expectations of future negative investor sentiments reduce the willingness of the sovereign to repay in the future and thus translate to negative investor sentiments today. I find conditions under which such sentiment dynamics can be active and document their presence in standard models. When the debt is of longer maturity I show that such panics resemble the recent Eurozone crisis, and so I explore policy implications in this environment. I find that interest rate ceilings are an ineffective policy tool but that liquidity provision by the ECB could be welfare-improving. Motivated by this result, in the second chapter I perform a structural estimation exercise to determine investors' ex ante forecast of such panics and the concomitant welfare consequences of liquidity provision. Using Bayesian methods and Spanish CDS spreads, I find that investors' forecast of such a crisis ex-ante was once every 7.37 years, which is in close accordance with the realized frequency of 7.5 years. I also find that liquidity provision by the ECB was likely welfare-improving. In the last chapter, my co-author and I document a downward trend in the leverage ratios of innovative firms. We argue that this trend could be the result of either a reduction in the cost of external equity finance or from a shifting the in the risk-frontier associated with innovation. To disentangle these alternative stories, we develop an equilibrium model of optimal finance choice. The estimated model suggests that a reduction in the cost of external equity finance is the more dominant driver of the observed trend: From the first time period to the next, the per-unit cost of equity falls by 31.6% while the risk frontier increases by only 4.7%.

Contents

1	Dynamic Panics in the Eaton-Gersovitz Model	1
1.1	Introduction	1
1.2	Literature Review	6
1.3	Simple Model	9
1.3.1	Environment	9
1.3.2	Characterizing Confidence-Waves Equilibria	11
1.4	Full Theoretical Model	15
1.4.1	Augmented Environment	15
1.4.2	Theoretical Results	18
1.5	Conclusion	28
2	Dynamic Panics: Application to the Eurozone	30
2.1	Introduction	30
2.2	Literature Review	32
2.3	Model Description	33
2.3.1	Specification	36
2.3.2	Parameter Instability	40
2.3.3	Model Solution	41
2.3.4	Estimation Procedure	47
2.3.5	Results	50

2.3.6	Policy Experiment: Liquidity Provision	53
2.4	Conclusion	57
3	Capital Structure and Innovation Riskiness	58
3.1	Introduction	58
3.2	Relation to Literature	65
3.3	Model	68
3.3.1	Financing Game	69
3.3.2	Equilibrium in the Financial Game	70
3.3.3	Firm's Optimal Capital Structure Choice	77
3.4	Empirics	83
3.4.1	R&D Trends	85
3.4.2	Innovation Risk Trends	86
3.5	Quantitative Results and Theory Tests	87
3.5.1	Policy Functions and Payoffs	88
3.5.2	Quantitative Analysis	89
3.5.3	Estimation Strategy	93
3.6	Conclusion	96
	Appendices	98
4	List of Tables	99
4.A	Tables for Chapter 2	99
4.B	Tables for Chapter 3	100
5	List of Illustrations	103
5.A	Figures for Chapter 1	103
5.B	Figures for Chapter 2	107

5.C	Figures for Chapter 3	109
6	Appendix for Chapter 1	115
6.A	Theoretical Proofs	115
6.A.1	Proof of Theorem 1.3.1	115
6.A.2	Proof of Proposition 1.4.2	123
6.A.3	Proof of Proposition 1.4.6	123
7	Appendix for Chapter 2	125
7.A	Computation and Estimation	125
7.A.1	Model Solution	125
7.A.2	Proof of Theorem 2.3.1	127
7.A.3	Fixed Parameters	133
7.A.4	Kalman Filter: Observables	134
7.A.5	Kalman Filter: Model Components	135
7.A.6	Posterior Distribution	136
7.B	Robustness and Model Fit	136
7.B.1	Posterior Predictive Distributions	136
7.B.2	Alternative Model Specifications	139
7.B.3	Alternative Example: Italy	140
8	Appendix for Chapter 3	142
8.A	Empirics	142
8.A.1	Summary Statistics	142
8.A.2	Definition of Data Items	142
8.A.3	Composition of Firms in the Compustat	145
8.A.4	Additional Capital Structure Trends	145
8.A.5	Additional Innovation Trends	151

8.B	Solution Method	152
8.C	Theoretical Proofs	153

Chapter 1

Dynamic Panics in the Eaton-Gersovitz

Model

1.1 Introduction

Since the seminal contribution of Eaton and Gersovitz (1981), there has been a folk intuition in the sovereign debt and default literature that market sentiments can have real effects. The intuition is as follows: When investors expect the sovereign to default they demand a high spread, which makes repayment costly and raises the default frequency; on the other hand, when lenders do not expect a default they demand a low spread, which reduces the burden of repayment and with it the default frequency.*

However, the recent quantitative literature in this field lacks these sentiment dynamics. For instance, in the seminal models of Aguiar and Gopinath (2006) and Arellano (2008), which have become the workhorse models for a vast quantity of applied work, default episodes always result from an unfortunate sequence of fundamental shocks. Furthermore, the underlying fundamental Markov-Perfect equilibrium in these models tends

*This intuition was formalized in some form in the models of Calvo (1988) and Cole and Kehoe (1996).

to be unique[†], which has motivated the application of method of moments in quantitative analysis.

Although this determinacy result is appealing from a tractability and econometric perspective, the lack of sentiment dynamics could be problematic since many empirical crises seem to resemble some sort of a market panic. Two noteworthy examples are Mexico's 1994 Tequila Crisis or the recent crisis in the Periphery Eurozone countries. This paper reconciles this apparent inconsistency by showing that such confidence-driven crises can still arise in this canonical sovereign debt framework.

In particular, I outline a new type of dynamic lender coordination problem that I call a *dynamic panic*. During a dynamic panic, lenders today anticipate that, for no fundamental reason, lenders tomorrow will demand a very high spread on debt issued by the sovereign. In the face of these high spreads, the sovereign's value of repaying existing debt obligations tomorrow will fall, which will induce him to default more often as a consequence. As a result, lenders today panic and demand higher spreads. Therefore a dynamic panic arises when expectations of lender panics tomorrow cause such panics today.

A simple illustration is provided in Figure 5.A.1. The red and green circles denote non-fundamental confidence, which exhibits some persistence governed by the thickness of the transition lines. Suppose that when the confidence light is green tomorrow, lenders offer the sovereign a high price, q , on his debt. This expands his consumption possibilities set under repayment, which reduces his propensity to default, p_D . The opposite is true when the light is red.

If the confidence light is green today it is likely to remain green tomorrow, which implies that the sovereign is more likely to repay debt issued today. The opposite is true when

[†]The term 'Markov-Perfect Equilibrium' has taken different meanings in the literature. Some, such as Arellano (2008), have taken it to mean an equilibrium dependent only on contemporaneous fundamentals, while others such as Conesa and Kehoe (2012) allow the equilibrium to depend on additional contemporaneous states that encode relevant information, such as a sunspot. In this paper, I will distinguish between these types by calling the former a *fundamental* Markov-Perfect Equilibrium.

the light is red in that lenders expect him to default more often. Lenders, internalizing this in the price, offer the sovereign a high q when the light is green and a low q when it is red.

Therefore, it is possible to have two stable, non-fundamental regimes: A high-confidence regime with high debt prices and repayment frequencies; and a low-confidence regime with low debt prices and repayment frequencies. A dynamic panic is a transition from the high confidence regime to the low one.

All that is needed to induce such crises is some persistent, non-fundamental object on which lenders can coordinate such as market sentiment or confidence. A persistent notion of confidence gives lenders *across time*[‡] a way to communicate with each other and thus to coordinate on malignant spread dynamics.[§] These dynamics are sustained in turn by optimal sovereign default behavior.

In this paper, I provide conditions under which non-fundamental confidence fluctuations have real effects and characterize their basic properties. I also explore how these crises differ with maturity structure of the debt. Unlike a rollover crisis in the tradition of Cole and Kehoe (1996), dynamic panics can exist even when the debt is of very long maturity and consequently there is almost nothing to roll over. When the debt is of longer maturity, instead of inducing outright higher default frequencies, a lender panic can instead generate a persistent period of costly and excessive borrowing. This occurs for two complementary reasons: First, the average cost to the sovereign of issuing new debt in the face of a panic is lower when the debt is of longer maturity because only a small fraction of the debt stock needs to be rolled over at these high spreads. Second, the spreads induced by such a panic will necessarily be smaller than those induced by a fundamental shock, since panics of this kind will increase future but not contemporaneous default frequencies

[‡]Static investor coordination problems have been studied in some depth in such models as Diamond and Dybvig (1983), Obstfeld (1996), and Cole and Kehoe (1996) but little attention has been paid to the possibility of coordination failures over time. A recent exception is Lorenzoni and Werning (2013), who highlight the potential for dynamic lender coordination failures when the debt is of longer maturity.

[§]There is empirical evidence to suggest that confidence or beliefs exhibit some persistence even when fundamentals are accounted for. See, for example, Barsky and Sims (2009) or Lubik and Schorfheide (2004).

default frequencies. Thus, the sovereign is much more willing to borrow into these spreads rather than default or reduce consumption, and this excessive borrowing raises the default frequency and justifies the lender panic.

In a sense, such long-term dynamic panics are the opposite of investor ‘runs’ in the tradition of Diamond and Dybvig (1983) or Rodrik and Velasco (1999). Rather than fearing that future investors will pull their funds out of the country, investors today are afraid of precisely the reverse: They are afraid that lenders tomorrow will lend to the sovereign far too liberally, which will raise the future default frequency and destroy the future price of their debt, about which long-term bondholders care. In fear of this, lenders demand high spreads, which forces the sovereign to borrow more today and, in a Markovian framework, fulfills those negative expectations.

This unusual feature of borrowing into high spreads, which comes quite naturally out of my model, is a hallmark feature of the Eurozone crisis and the one that has attracted the most attention from the recent literature (see Lorenzoni and Werning (2013), Corsetti and Dedola (2013), or Conesa and Kehoe (2012)). In light of this similarity, I explore the policy implications of dynamic panics. I find that in this environment an interest rate ceiling, which has been proposed as a potential policy, would be ineffective.

To see why an interest rate ceiling is ineffective during a dynamic panic, it is helpful to understand why it would normally work. The justification for such a policy is grounded in Calvo’s (1988) framework in which there are two ways a sovereign can generate the same of revenue: Issue a small amount of debt at low spreads, which are low because the probability of default is low, or a large amount of debt at high spreads, which are high because the probability of default is high. Through the lens of this model, distressed Eurozone countries were ‘stuck’ in the latter situation and therefore a simple, credible cap on the market rate would be enough to rule out this sub-optimal equilibrium. However, during a dynamic panic, the sovereign is always borrowing on the left side of this ‘Laffer curve’, even during

a crisis, since it can freely choose its debt level and would never place itself on the Pareto-dominated right side.[¶] Thus, an interest rate ceiling in this environment is isomorphic to a revenue cap on debt issuance and will only reduce government consumption and increase the likelihood of default.

Even though rate ceilings are ineffective, I do find that liquidity provision, which is a policy that the ECB undertook in its OMT program, was effective. In providing liquidity, the central bank credibly pledges to purchase sovereign debt at potentially sub-market rates. The model suggests that such a policy is effective at removing sentiment fluctuations, but that its welfare consequences are ambiguous, since some limited-commitment based default will remain after implementation. This trade-off can be understood in the context of the debate between the core and the periphery: The periphery wants the central bank to provide liquidity to protect them from malignant market sentiments, while the core fears that with such a backstop the periphery will rack up unsustainable debt levels and bring about another, more fundamental crisis. Both channels are active in the model and so the welfare consequences of such provision will depend on the underlying parameterization.

While in this paper I consider the special case of sovereign debt markets and in particular the case of the periphery Eurozone, the dynamic lender coordination problem I document is far more general. These coordination failures could plague markets for municipal or even corporate bonds and thus understanding them could provide new insights into the debt dynamics in these markets.

In summary, this paper makes the following points: First, it outlines a new dynamic lender coordination problem, which is the susceptibility of the standard sovereign debt environment to dynamic panics; second, it provides conditions for the real effects of non-fundamental dynamics in this environment; third, it generates borrowing into high spreads endogenously in the context of an already canonical model; fourth, it suggests that interest

[¶]For a graphical illustration of this, refer to Figure 5.A.6.

rate ceilings would be an ineffective policy in combating such crises and highlights the trade-off faced in the provision of liquidity

The rest of the paper is organized as follows. In Section 1.2, I review the relevant literature. In Section 1.3, I describe a simple model with only extrinsic uncertainty and completely characterize the set of sunspot equilibria, highlighting several necessary conditions for sunspot activity. In Section 1.4, I embellish the simple model by adding intrinsic uncertainty, borrowing choice, and potentially longer maturities. I then explore several necessary features of dynamic panics in this environment and their relationship to the Eurozone crisis. Lastly, I explore the impact of several plausible policies designed to counter such panics. Section 1.5 concludes.

1.2 Literature Review

This paper contributes to several different strands of the literature. First, it contributes to the quantitative literature outlining the dynamics of sovereign debt and default episodes. This literature takes the seminal framework of Eaton and Gersovitz (1981) and applies it quantitatively to primarily Latin American economies to match business cycle statistics and the empirical regularities of developing nations. Noteworthy papers in this vein include Aguiar and Gopinath (2006) and Arellano (2008). There is a nice summary of this tradition in Aguiar and Amador (2014).

This literature has also developed a branch that explicitly considers debt of longer maturities, of which prominent examples include Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012), and Arellano and Ramanarayanan (2012). The lesson from this branch is that, apart from expected default, the movements in the expected future price of the debt can have a significant impact on spreads today. This effect has been called ‘dilution risk’, and Chatterjee and Eyigungor (2012) show that it accounts for a substantial fraction of long-term spreads. Dilution risk will feature prominently in my exploration of dynamic

panics at longer maturities.

The framework developed by this literature has also been the benchmark for a string of recent applied work studying default episodes. This is in large part because of the tractability of the assumption of Markov-Perfection. Some prominent examples include Mendoza and Yue (2012), Gornemann (2014), Salomao (2014), and Na et al. (2014), who study respectively the impact of default on international private lines of credit, long-term growth, credit default swaps, and optimal devaluation policy.

A common thread in this tradition is the lack of multiplicity or self-fulfilling dynamics. In this class of models, default is always driven by an unfortunate sequence of fundamental shocks and often the underlying equilibrium is unique. This uniqueness result was recently formalized by Auclert and Rognlie (2014). A recent exception is Passadore and Xandri (2014), who find sufficient conditions under which multiplicity of equilibria can exist for the case of short-term debt and use these conditions to bound the impact of sunspot activity. This paper will have a similar goal, but will focus primarily on the recursive dynamics of sunspot activity itself rather than its bounds. It also explores the interaction of sunspot activity with the unique features of long-term debt.

This paper also contributes to the recent literature on the Eurozone. As of yet, the academic literature has had little time to keep pace with developments that took place in the Eurozone over the past 6 years or so. However, several noteworthy pieces have emerged that have tried to deal seriously with the peculiar circumstances surrounding the sovereign debt crisis in the Eurozone. These papers have been both empirical and structural. On the empirical side, recent work has taken aim at demonstrating the confidence-driven nature of this crises by documenting an unusually weak correlation between economic fundamentals and CDS spreads. Some prominent examples include De Grauwe and Ji (2013) and Aizenman et al. (2013). This work will rely in some sense on these empirical findings in its placement of malignant market sentiments at the heart of the story in its

theory of the crisis.

The only other authors that have highlighted the role of dynamic coordination problems in the recent Eurozone crisis are Lorenzoni and Werning (2013). These authors also argue for a Laffer-curve type multiplicity in the spirit of Calvo (1988), but with the explicit inclusion of long-term debt. In their environment, as in mine, a dynamic lender coordination failure can place the economy on a malignant trajectory of high spreads and debt ratios. They term such a crisis a ‘slow-moving’ crisis. The key difference between their paper and mine is that I place more focus on the dynamic coordination failure and give it the form of a persistent sunspot. This allows me to explore the possibility of such crises in the standard Eaton-Gersovitz framework in which the government can commit to a level of debt as well as to a level of revenue. This small difference has important policy implications. For instance, in their environment, an interest rate ceiling would be an effective tool at alleviating slow-moving crises, while in mine such a policy will tend to induce more default.

On the theoretical frontier, this paper also contributes the literature on sunspots, since that is the tool I choose to model non-fundamental confidence. This literature started with the works of Azariadis (1981) and Cass and Shell (1983). Shell (2008) provides a nice summary of the prerequisite conditions for the existence of sunspot activity. Gottardi and Kajii (1999) demonstrate that multiplicity of equilibria is not necessary for the presence of sunspot activity and Hoelle (2014) discusses in depth the relationship between sunspot activity and multiplicity of equilibria when markets are incomplete. My work contributes to this literature by providing a complete set of conditions for sunspot dynamics in a simple sovereign debt environment and also characterizing features of sunspot equilibria in more complex environments.

1.3 Simple Model

1.3.1 Environment

In this section I construct a simple sovereign debt environment in the tradition of Eaton and Gersovitz (1981) and Arellano (2008) but with no intrinsic uncertainty and then characterize completely how the model reacts to non-fundamental confidence. In the subsequent section I will relax many of the restrictive assumptions and explore quantitatively how a richer, more standard model reacts to confidence shocks.

In particular, consider an infinitely-lived sovereign borrower that receives a constant endowment, y , in each period. This sovereign has an increasing flow utility, $u(\cdot)$, over consumption in each period and discounts the future at a rate $\beta < 1$. The only uncertainty in this model is extrinsic confidence, ξ , which can take one of two values, $\{\xi_L, \xi_H\}$. I assume that ξ follows a symmetric Markov process with transition probability η .^{||}

He has a constant stock of debt, b , and he can either choose to roll over that debt at an exogenously given price, q , which may depend on the level of confidence. If he does not roll over this debt, then he defaults on it. He makes this default decision as soon as the extrinsic uncertainty is realized i.e. before he goes to the auction to roll over his debt. When he defaults, he is excluded from credit markets forever and pays a constant additive cost $\phi(y)$ in every subsequent period. I will restrict attention to equilibria that are Markov-Perfect in confidence, and so we can write his Bellman equation, conditional on repayment, as follows:

$$V(\xi) = u(y - b + q(\xi)b) + \beta E_{\xi|\xi}[\max\{V(\tilde{\xi}), X\}] \quad (1.1)$$

^{||}The assumption of symmetry can be easily relaxed at the cost of a lengthier exposition.

where X , which is the value of default, can be computed as follows:

$$X = u(y - \phi(y)) + \beta X$$

Notice that because there is no re-entry the value of X is independent of any particular equilibrium. Hence, it is not an equilibrium object. The sovereign will borrow from a unit mass of risk-neutral, deep-pocketed lenders with an outside option with return R . These lenders price default risk according to a no-arbitrage condition, so in equilibrium the price must be given by:

$$q(\xi) = \frac{1}{R} E_{\tilde{\xi}|\xi} [\mathbf{1}\{V(\tilde{\xi}) \geq X\}] \quad (1.2)$$

We are now ready to define an equilibrium in our simple environment. In particular, a **Markov-Perfect Equilibrium** will be a pair of functions $\{V(\xi), q(\xi)\}$ such that

1. Given $q(\xi)$, the value function $V(\xi)$ solves Recursion 1.1
2. Given $V(\xi)$, the pricing function $q(\xi)$ solves Recursion 1.2

I will call a Markov-Perfect Equilibrium a **Fundamental Markov-Perfect Equilibrium** if in it confidence has no real effects. If confidence does have real effects, I will call the Markov-Perfect Equilibrium a **Confidence-Waves Equilibrium**. In a Confidence-Waves Equilibrium, a shift from ξ_H to ξ_L will be a **Dynamic Panic**.

1.3.2 Characterizing Confidence-Waves Equilibria

In this simple environment, it is possible to make analytic statements regarding the entire set of Confidence-Waves Equilibria. In particular, the following theorem can be established:

Theorem 1.3.1 *The set of parameters over which a Confidence-Waves Equilibrium exists is completely characterized by the following two conditions:*

1. $\frac{R-1+\eta}{R}b \leq \phi(y)$
2. $\frac{\beta\eta}{1-\beta(1-\eta)} \left[u\left(y - \frac{R-1+\eta}{R}b\right) - u(y - \phi(y)) \right] < u(y - \phi(y)) - u\left(y - \frac{R-\eta}{R}b\right)$

Proof See 6.A

Theorem 1.3.1 provides a set of both necessary and sufficient conditions for the potential for sentiment dynamics in this simple model of sovereign default.** The first condition will ensure that repayment is optimal for some value of $\xi = \xi_H$ without loss of generality. The second condition ensures that, in addition, default is optimal in ξ_L . Since there is no other source of uncertainty, this is the only way in which confidence can have real effects in a Markov-Perfect and thus these conditions completely characterize the conditions necessary for sunspot activity.

From these conditions, we can derive several useful corollaries that highlight key model elements required for confidence to have a real impact. The first is that confidence must be persistent.

Corollary 1.3.2 *In any Confidence-Waves Equilibrium, $\eta < 1/2$.*

**Note that we cannot say that such conditions are absolutely necessary for sunspot activity since, as I will show, non-sunspot equilibria will also exist for these same parameterizations.

Proof See 6.A

Why must confidence be persistent? When default is driven by market sentiment, it must be that high spreads themselves cause the default and low spreads themselves cause repayment. However, spreads reflect anticipated default *in the future*, not contemporaneous default.

Suppose that we were trying to establish an equilibrium in which default only occurred in the face of low confidence. If confidence was transient, then high confidence today would imply that quite likely default would occur tomorrow. But this would drive down the price of debt today relative to the low confidence state. Since the price discrepancy can be the sole driver of default discrepancies, this would induce default in the high-confidence state instead of the low confidence state, which is a contradiction. Thus, if confidence has real effects, it must be persistent.

So confidence must be persistent. But there is another important characteristic about the sovereign debt environment that is not immediately obvious. It is summarized in the following corollary:

Corollary 1.3.3 *Let $\Theta(u)$ be the set of parameters for which a Confidence-Waves Equilibrium exists given a utility function, u . If \hat{u} is more concave than u , then $\Theta(u) \subseteq \Theta(\hat{u})$.*

Proof See 6.A

In words, any parameterization for which a Confidence-Waves Equilibrium exists will continue to do so as you increase the degree of risk-aversion of the sovereign. Further, the set of parameters over which confidence fluctuations can occur expands with the degree of risk-aversion. Why is this? It is because a more concave utility functions will punish a repaying sovereign in utility terms more severely when debt service costs are expensive and thus when consumption is low. This will occur when confidence is low. This punishment

must be severe enough such that default is optimal even though there is some probability of re-entering the high-confidence, no-default regime.

Relationship to Rollover Crises and Multiplicity

An easy misinterpretation of Theorem 1.3.1 is that I am simply finding conditions for a ‘rollover crisis’ as in Cole and Kehoe (1996). This is not at all what I am describing here. During a rollover crisis, an individual investor fears that other investors will not show up to the auction to roll over the sovereign’s short-term debt. If the sovereign defaults in response, then the investors’ fears are justified and they do not show up.

This is not what is going on with my dynamic panics. First, this is because the timing is different: The default decision occurs *prior* to the sovereign’s debt auction. Thus, lenders cannot experience such a contemporaneous coordination failure in my set-up. Second, lenders in my set-up are not panicking about the behavior of other lenders today; rather, they are panicking about the behavior of lenders *tomorrow*. Because the sentiment shock is persistent, when it is shocked today, lenders anticipate that lenders tomorrow will offer a low price, which will induce default tomorrow. This fear induces them to offer a low price today, which in turn induces default today. It is for this reason that I call my confidence crises dynamic panics, since the intertemporal dimension is crucial.

This is formalized in the following corollary. During a rollover crisis, the equilibrium price of debt, q , equals zero i.e. the sovereign cannot raise any revenue at the auction. However, during a dynamic panic the price of debt falls, but it is not zero.

Corollary 1.3.4 *During a dynamic panic, debt can be auctioned at $q(\xi_L) > 0$.*

Thus, unlike a rollover crisis, it is *possible* to raise revenues at auctions; it is simply sub-

optimal to do so.

The last necessary condition I will discuss is the relationship of Confidence-Waves Equilibria to multiplicity. In particular, we can claim the following:

Corollary 1.3.5 *If a Confidence-Waves Equilibrium exist, then the full-default and full-repayment equilibrium both exist as well.*

Proof See 6.A

This finding accords with the recent findings of Passadore and Xandri (2014), who find that multiplicity is a necessary pre-condition to sunspot activity. In other words, there must be some room for strategic complementarities of the sort that would induce multiple equilibria in order for a Confidence-Waves Equilibrium to exist.

This is not to say that Confidence-Waves Equilibria are simply randomizing over existing multiplicity. While this is one way that they can be generated, it is also possible that the sunspot is randomizing over pricing schedules *that are not themselves equilibria*. In 6.A I provide an analytic example of such a case when there is some intrinsic uncertainty as well.

This result is similar in kind to that of Gottardi and Kajii (1999), who argue that sunspots need not randomize over existing multiplicity, but only over ‘potential multiplicity’. They define to potential multiplicity to be the existence of multiple equilibria for a reallocation of endowments. In my set-up, such ‘potential multiplicity’ arises if there exist pricing schedules and default strategies that are close to satisfying the equilibrium conditions. Confidence fluctuations can randomize over these schedules even though they themselves are not equilibria.

1.4 Full Theoretical Model

In the last section I highlighted several features of the sovereign default environment that are necessary to generate confidence-driven fluctuations. In particular, I argued that the persistence of the confidence process is necessary and that the steepness of the flow utility function can expand the parameterizations that are subject to sentiment shocks. As it turns out, these two features will be crucial for finding these equilibria computationally using the tools developed by the literature.

I now embellish the simple model until it subsumes several of the standard sovereign default models in the literature. I then explore quantitatively the potential for Confidence-Waves Equilibria in these already standard models and characterize several necessary features of dynamic panics.

This full theoretical model will be similar to Chatterjee and Eyigungor (2012). In particular, there will be intrinsic as well as extrinsic uncertainty. There will also be an endogenous borrowing choice and debt of longer maturities. In this more sophisticated environment, I will no longer be able to characterize fully the set of Confidence-Waves Equilibria, but I will be able to find them computationally and derive several illuminative necessary conditions.

1.4.1 Augmented Environment

The full theoretical model will allow for three stochastic processes: A fundamental endowment shock, $y \in \mathcal{Y}$, a continuous, fundamental consumption preference shock, $\tilde{m} \in [\underline{m}, \bar{m}]$, and a non-fundamental confidence shock, $\xi \in \Xi$. Both the endowment and the confidence shocks are assumed to be persistent and \mathcal{Y} and Ξ are assumed to be discrete sets. In particular, I continue to assume that confidence is binary i.e. $\xi \in \{\xi_L, \xi_H\}$. The preference shock,

\tilde{m} , is assumed to be iid over time and will therefore not impact the price in equilibrium. Its continuous nature, however, will smooth over the discrete nature of sovereign's decisions in expectation, which helps both with existence and computation. Its range is assumed to be fairly small.

The sovereign borrower chooses a level of consumption, c , how much to borrow from abroad, $b' \in \mathcal{B}$, and whether or not to default. He receives a flow utility, $u(\cdot)$ from consumption, which I assume is increasing and strictly concave. Debt is long-term as in Chatterjee and Eyigungor (2012) i.e. debt pays a coupon κ for every period in which it does not mature and matures stochastically with a probability λ .

I will focus on Markov-Perfect Equilibria and so I can write the sovereign's problem recursively. Taking as given the demand schedule for its debt from foreign investors, $q(y, \xi, b')$, the government solves the following Bellman, which is conditional on repayment this period:

$$V(y, \xi, m, b) = \max_{c \geq 0, b' \in \mathcal{B}} u(c) + \beta \mathcal{V}(y, \xi, b') \quad (1.3)$$

$$\text{s.t. } c - m \leq y - [\lambda + (1 - \lambda)\kappa]b + q(y, \xi, b')[b' - (1 - \lambda)b]$$

I assume that if the sovereign faces an empty budget set, he must default. The continuation value allows for default is ex-post optimal, and is thus given by

$$\mathcal{V}(y, \xi, b') = E_{(\tilde{y}, \tilde{\xi}, \tilde{m})|(y, \xi)} [\max\{V(\tilde{y}, \tilde{\xi}, \tilde{m}, b'), X(\tilde{y})\}]$$

I will denote the sovereign's borrowing function to be $a(y, \xi, m, b)$.

Unlike the simple model, I assume that when the country defaults, it is excluded from credit markets only *temporarily*. It will continue so suffer some additive output loss, $\phi(y)$, in each period of this exclusion. It re-enters stochastically at a rate π_{RE} and it does so with a high level of confidence ξ_H , after which confidence follows its typical Markov-process.

This simple assumption will allow the default value to be independent of ξ while retaining its non-fundamental nature, which will be important both computationally.^{††} Also upon re-entry, it has no debt obligations.^{‡‡}

Under these assumptions, we can express the value of default as

$$X(y) = u(y + \underline{m} - \phi(y)) + \beta E_{\tilde{y}, \tilde{m}|y}[(1 - \pi_{RE})X(\tilde{y} - [\underline{m} - \tilde{m}]) + \pi_{RE}V(\tilde{y}, \xi_H, \tilde{m}, 0)] \quad (1.4)$$

Notice that the value of default is *no longer independent* of the equilibrium, since the value of default will depend on the equilibrium pricing function through its continuation value. Thus, it is an equilibrium object. Notice further that, as in Chatterjee and Eyigungor (2012), during the first period of default the sovereign faces the worst \tilde{m} shock but experiences it as its normal stochastic process thereafter.

The final augmentation that needs to be made is to the foreign lenders' no-arbitrage condition. These lenders now care about the persistent part of the endowment today, y , and level of borrowing today, b' , since both of these determine provide information regarding the default frequency tomorrow. Since the debt is long-term, they also care about the expected future price of their debt and thus about the degree to which the sovereign borrows tomorrow. The pricing recursion thus becomes

$$q(y, \xi, b') = \frac{1}{R} E_{(\tilde{y}, \tilde{\xi}, \tilde{m})|(y, \xi)} \left[1\{V(\tilde{y}, \tilde{\xi}, \tilde{m}, b') \geq X(\tilde{y})\} \times \right. \\ \left. [\lambda + (1 - \lambda)(\kappa + q(\tilde{y}, \tilde{\xi}, a(\tilde{y}, \tilde{\xi}, \tilde{m}, b')))] \right] \quad (1.5)$$

^{††}There will also be interesting theoretical implications of this assumption, but they are largely tangential. For instance, since the value of default is an equilibrium object, it cannot be the case the Confidence-Waves Equilibria are randomizing over two distinct, non-sunspot equilibria, since these equilibria would have to share a common value of default.

^{‡‡}This last assumption can be relaxed to allow for haircuts provided we restrict attention to the set of equilibria for which demand curves for debt are always downward sloping.

Equilibrium Definition

A **Markov-Perfect Equilibrium** is a set of functions $V(y, \xi, m, b)$, $a(y, \xi, m, b)$, $X(y)$, and $q(y, \xi, b')$ such that

1. $V(y, \xi, m, b)$ satisfies Recursion 1.3 when given $X(y)$ and $q(y, \xi, b')$ and implies the borrowing policy function $a(y, \xi, m, b)$
2. $X(y)$ satisfies Recursion 1.4 when given $V(y, \xi, m, b)$
3. $q(y, \xi, b')$ solves Recursion 1.5 given $V(y, \xi, m, b)$, $X(y)$, and $a(y, \xi, m, b)$

A **Confidence-Waves Equilibrium**, **Fundamental Markov-Perfect Equilibrium**, and **Dynamic Panic** are defined relative to the Markov-Perfect Equilibrium as in the previous section.

1.4.2 Theoretical Results

In what follows I characterize the theoretical properties of Confidence-Waves Equilibria. In particular, I will show that dynamic panics at longer maturities exhibit several features peculiar to the Eurozone crisis, including excessive persistence and borrowing into high spreads. Because of this, I explore several policy implications of long-term dynamic panics and discuss briefly their applicability to the Eurozone crisis.

The first result tells us that confidence shocks are indeed an equilibrium phenomenon: Both lenders and the sovereign must actively respond to the shock in order for them to have any real effects:

Proposition 1.4.1 *Let $d(y, \xi_L, m, b)$ denote the default policy of the sovereign. In any CW Equilibrium, **both** of the following must hold*

1. $\exists(y, m, b)$ such that either $d(y, \xi_L, m, b) \neq d(y, \xi_H, m, b)$ or $a(y, \xi_L, m, b) \neq a(y, \xi_H, m, b)$

2. $\exists(y, b')$ such that $q(y, \xi_L, b') \neq q(y, \xi_H, b')$

Proof This can be shown by contradiction. Suppose that we had a Confidence-Waves Equilibrium with a pricing schedule that never depended on ξ . Once given the pricing schedule, the sovereign's Bellman equation becomes a contraction on V and X . Since the only channel through which ξ can affect the sovereign's payoff is through the price, the resulting, unique fixed point will not depend on fluctuations in ξ . Thus, ξ would have no effect in equilibrium, which contradicts the fact that the equilibrium is a Confidence-Waves Equilibrium.

The same argument holds if we suppose that sovereign behavior never depended on ξ , since the lenders' pricing recursion is also a contraction on q . Thus, any Confidence-Waves Equilibrium must feature an active change of behavior on both sides of the market in response to sentiment shocks. ■

Having established the equilibrium nature of these sentiment shocks, I now begin to characterize them by defining a new term:

Definition A Confidence-Waves Equilibrium is **Default-Relevant** if the realization of ξ matters for the default decision of the sovereign at some point in the fundamental state space.

In other words, if the equilibrium is default-relevant then there is a fundamental state for which the sovereign defaults when confidence is low and repays when confidence is high. It is possible to have an equilibrium that is non-default-relevant even though \tilde{m} is continuously distributed provided that the range of \tilde{m} is fairly small. I will provide an example momentarily.

With this definition in hand, we can distinguish how maturity will influence the underlying impact of confidence.

Proposition 1.4.2 *If the debt is short-term i.e. $\lambda = 1, \kappa = 0$, then any Confidence-Waves Equilibrium must be default-relevant.*

Proof See 6.A.

Proposition 1.4.2 tells us that if the sunspot has any real effects, it must at some point make the difference between the sovereign defaulting and repaying. This proposition disappears when we extend the maturity of the debt. It will still be the case that default-relevance is sufficient to have an active sunspot, but it will no longer be necessary, since the sunspot can affect the future price of the debt if it affects borrowing behavior.

All of the results in the simple model outlined before were driven by default-relevance, which accords with the proposition. Investors today feared that investors tomorrow would panic and that this panic would induce more frequent default, which gave investors today a reason to panic. This proposition tells us that, for debt of longer maturities, this is not the only way that dynamic panics can unfold.

So how can we have a non-default-relevant dynamic panic? Figure 5.A.2 demonstrates. With longer term debt lenders care not only about whether the sovereign defaults tomorrow, but about the future price of the debt. Thus, if lenders in period t anticipate lenders in $t + 1$ to panic, it need not be the case that default probabilities actually rise in $t + 1$, since lenders in t already care about the price of debt in $t + 1$. But the sovereign must respond somehow to this shock, even though he does not change his default behavior. Thus, he must change his borrowing behavior, and in particular he must borrow in the face of a panic to justify the panic occurring in the first place. He is willing to do this because the marginal cost of borrowing into high spreads is lower at longer maturities, since less debt is actually issued at these low prices.

In a sense, a non-default-relevant dynamic panic is the opposite of an investor ‘run’ as in Diamond and Dybvig (1983) or Rodrik and Velasco (1999). Investors in such a panic do not fear that investors tomorrow will withdraw lending from the sovereign. They are afraid of quite the reverse i.e. that lenders tomorrow will lend excessively to the sovereign, which will drive down the expected future price of the debt. Since long-term bondholders care about this future price, they too will panic and demand higher spreads.

Although the causal chain of a long-term dynamic panic is driven by borrowing, in equilibrium we will simply have two different regimes: One in which spreads, default probabilities, and borrowing are all low and another in which all of these objects are high.^{§§} This can be seen in the numerical example provided in Figures 5.A.3 and 5.A.4.^{¶¶} This example shows how a non-default-relevant dynamic panic can affect borrowing and pricing behavior. In particular, we get two distinct pricing and borrowing regimes: One in which low borrowing occurs at low spreads and one in which high borrowing occurs at high spreads.

Further, we can see from a simulation path that a regime change is associated with higher levels of borrowing.^{***} Figure 5.A.4 compares the spreads and consumption paths of two sample economies which face the same endowment shocks. The first economy does not respond to non-fundamental activity and the second economy is in a confidence-waves equilibrium, experiencing a dynamic panic. One can see that when the confidence falls, it immediately and substantially increases spreads; by a factor of about two in this example. However, consumption changes very little; in other words, the sovereign undergoes little to no fiscal consolidation in response to this shock. Rather, he borrows more from abroad

^{§§}The only time this will not be true is in the period of the initial panic itself, since here default probabilities will not increase, though spreads and borrowing will.

^{¶¶}To calibrate these examples, I simply use the parameterization of Chatterjee and Eyigungor (2012) and add a large but non-binding subsistence level of consumption to induce greater absolute risk-aversion in the flow utility function, which is assumed to be CRRA.

^{***}Notice that both spreads and debt ratios are contemporaneous. High confidence-driven spreads induce excessive borrowing in the next period while high fundamental-driven spreads induce deleveraging in the following period.

to fill the budgetary gap. This is seen in Figure 5.A.5, which plots the response of the debt level to the same sequence of shocks as Figure 5.A.5.

I will address the optimality of excessive sovereign borrowing in a moment, but before I do I will define one more term to describe the sort of panic outlined in this quantitative model.

Definition A dynamic panic is **monotone** if $q(y, \xi_H, b') \geq q(y, \xi_L, b')$ for every $(y, b') \in \mathcal{Y} \times \mathcal{B}$.

There are numerous necessary features of monotone, non-default-relevant dynamics panics that formalize the intuition just outlined. I will now outline each of them in turn.

Proposition 1.4.3 *If, in a non-default-relevant Confidence-Waves Equilibrium, the dynamic panic is monotone, then there is a subset of fundamental states, $S \subset \mathcal{Y} \times \mathcal{M} \times \mathcal{B}$ such that if $(y, m, b) \in S$, we must have that*

$$a(y, \xi_H, m, b) < a(y, \xi_L, m, b)$$

In these states, future default frequencies must rise.

Proof First, note that under the restriction of Markov-Perfection, the default policy of the sovereign is increasing in b ,^{†††} We know that, in equilibrium, the price schedule q must reflect default risk in *all* future periods, not just the subsequent period. Further, since the equilibrium is non-default-relevant, we know that a confidence-driven price discrepancy in period t does not reflect an difference in the default probabilities in period $t + 1$.

^{†††}See Chatterjee and Eyigungor (2012) for a proof of this claim. Their results generalize to my environment.

Thus, the confidence-driven price discrepancy must reflect increased default risk in periods $t + 2, t + 3, \dots$. Since confidence never drives default, it must be the case that the distribution over fundamental states induced by ξ_L implies a higher default frequency than that induced by ξ_H . However, since the distribution of future endowment realizations is identical across the two regimes, it must be the case that debt levels are higher in *some* states of the world under ξ_L than ξ_H . Since confidence is persistent, this must have been generated by strictly higher borrowing in ξ_L than ξ_H for some fundamental states in the future, in particular, those states which are most likely. ■

This behavior of borrowing into high spreads is a *necessary* condition of non-default-relevant long-term dynamic panics. The sovereign has three options in the face of an adverse shock: Default, delever, or borrow into the spreads. This third option, in which little to no domestic fiscal adjustment takes place, *is a requirement* to generate non-default-relevant dynamic panics, since this is the only response that justifies the lender panic. This is one of the key features of such panics that I will utilize in my structural estimation.

But why would such excessive borrowing behavior be the optimal response of the sovereign? The reason is twofold. First, at longer maturities, the amount of debt that needs to be rolled over at panic spreads will be lower than it will be for shorter maturities; thus, such panic borrowing is on average less costly.

Second, the typical default-risk channel that generates overly volatile consumption is absent without default-relevance. To understand why, consider what usually happens in response to an adverse fundamental shock in period t in the style of Arellano (2008). Such a shock has two consequences: First, it reduces income in period t , generating a borrowing motive; and second, it increases default risk in period $t + 1$, which in equilibrium raises interest rates and generates a saving or delevering motive. In quantitative applications, this latter effect dominates and saving occurs in bad times, generating consumption volatility

that is greater than output volatility.

However, during a monotone, non-default-relevant confidence shock, default risk remains unchanged in period $t + 1$; instead, it increases in periods $t + 2, t + 3, \dots$ as a result of higher future borrowing. Thus, the magnitude of the equilibrium rate effect is necessarily smaller for a confidence shock of this kind, and so too is the saving motive. In equilibrium, it is necessarily dominated and higher borrowing occurs.

In practice, this result is far stronger than the fairly weak claim of this proposition: Such borrowing into high spreads tends to happen in nearly every state of the world that is realized on the equilibrium path. In fact, in the face of a panic the sovereign tends to not change his consumption or default behavior at all and instead borrows additionally to fill the same primary deficit.

This result is quite striking upon reflection. It tells us that during a dynamic panic, the sovereign will *willfully increase his debt position and default probability*. This stands in stark contrast to the behavior sovereigns in such models as Cole and Kehoe (1996), in which case the sovereign either delevers or defaults in response to the negative shock. The intuition behind this result is that the debt here is of longer maturity and therefore the average cost of new debt issuance, which is what is directly affected by the price, is not nearly as high as it is for short-term debt, the stock of which the sovereign must roll over every period. It is therefore much more willing to increase its debt position in the hopes of recovery.

The last useful property of dynamic panics is that they are in fact rationally anticipated. Because of this, they will be priced into the spreads. I can exploit this in the estimation.

Corollary 1.4.4 (Rationally Priced) *Consider a Confidence-Waves Equilibrium in which the sovereign always borrows additionally in the face of a monotone, non-default-relevant dynamic panic. Then, fixing the value and policy functions of the sovereign, $q(\cdot, \xi_L, \cdot)$ is*

decreasing in η and $q(\cdot, \xi_H, \cdot)$ is increasing in η .

This last result will be invoked heavily during the empirical exercise, in which I will assume the presence of such a panic and use spread data to identify η .

Policy Implications

I now explore two new policy implications in this environment. The first is the efficacy of a rate ceiling. Many authors, including Corsetti and Dedola (2013) and Lorenzoni and Werning (2013) have argued that an interest rate ceiling could have been an effective tool in combating malignant market sentiments. The reason is the following: A graph of revenue versus debt at any debt-auction ought to be parabolic since low levels of debt will have high prices and thus raise revenue but high levels of debt will have lower prices due to increased default probabilities and thus actually lower revenue. Some examples of such Laffer-curves can be seen in Figure 5.A.6.

These authors follow Calvo (1988) in asserting that the confidence crisis experienced by the Eurozone was a result of the sovereign winding up on the right-hand side of this Laffer-curve i.e. raising the same amount of revenue but with higher debt and worse prices. In the presence of such a crisis, an interest rate ceiling can be an effective tool since it forces the investors to coordinate on the good equilibrium on the left-hand side of the Laffer curve.

This policy implication is lost when the crisis at hand is a dynamic panic and not a Calvo-style crisis, as is made clear by the following proposition.

Proposition 1.4.5 *During a dynamic panic, a binding interest rate ceiling is equivalent to a revenue cap on debt issuance. Thus a binding, temporary interest rate ceiling will increase the probability of default.*

Proof First, note that whether the economy is in a crisis or not the sovereign optimally borrows on the left-hand side of the ‘Laffer curve’, which plots revenue against debt issuance. This is because the sovereign can commit to not only to revenue raised at debt auctions, but also to the amount of debt issued. Thus, given to issuance options yielding the same revenue, the sovereign will always choose the one with less debt, since the value function is decreasing the level of debt.

During a dynamic panic, the entire Laffer curve shifts but the sovereign continues to remain on the left-hand side of it. Therefore, if we implement a binding interest-rate ceiling, it will necessarily lower the quantity of debt that can be issued. This is because the demand curve for debt is downsloping^{†††}, so a price floor (rate ceiling) translates directly to a ceiling on debt issuance. A ceiling on debt issuance also places a ceiling on the revenue that can be raised, since the sovereign is located on an upsloping portion of the Laffer curve. Since the ceiling is temporary, tomorrow the sovereign can expect to resume with the equilibrium dynamics.

Denote the value of the sovereign who faces the original equilibrium demand functions with an interest rate ceiling as $\hat{V}(y, \xi, m, b')$. Note that since $\hat{V}(y, \xi, m, b')$ is the objective function of the same maximization as $V(y, \xi, m, b')$ but with an additional constraint, specifically one on revenue, we will necessarily have $\hat{V}(y, \xi, m, b') \leq V(y, \xi, m, b')$. Therefore, the probability of default has risen. ■

There is an intuitive graphical exposition of Proposition 1.4.5 in Figure 5.A.6. The black line represents the debt cap imposed by the rate ceiling. Consider the level of revenue raised by the horizontal dashed line. The typical Calvo-style multiplicity dictates that the sovereign is on the far-right intersection with the blue curve, and thus a rate ceiling such as this forces investors to coordinate back on the good equilibrium on the left side of the

^{†††}See Chatterjee and Eyigungor (2012) for a proof of this.

curve.

However, during a dynamic panic, we are not on the right side of the blue curve; we are in fact on the left side of a new red curve that implies less revenue raised for any given level of debt. The debt cap imposed by the black line then simply implies a revenue cap. Given this revenue cap, consumption must drop in the case of repayment and so repayment becomes less attractive and default frequencies rise.

It is important to note that an interest rate ceiling is different than the arguably successful measures that the ECB took to avert the crisis such as the Outright Monetary Transactions (OMT) bond-buying program. As noted by Corsetti and Dedola (2013), these programs were not rate ceilings but guarantees that the ECB would purchase government debt at sub-market interest rates. I follow De Grauwe (2011) in calling such a policy *liquidity provision*. The next proposition demonstrates that in fact such a policy may be effective.

Proposition 1.4.6 *Liquidity provision can eliminate the impact of confidence fluctuations without the need to actually purchase any assets. The resulting economy will still suffer from a weakly positive probability of default driven by the problem of limited commitment.*

Proof See 6.A.

Proposition 1.4.6 tells us that the ECB can in fact judiciously apply provision of liquidity to eliminate the impact of malignant market sentiments, as they effectively did with the OMT Program. Such a policy does not actually require a purchasing of the debt, so long as the implied demand schedule is consistent with an equilibrium not subject to confidence fluctuations.

The proposition also tells us that under the resulting economy will continue to suffer from a weakly positive probability of default. This is actually a likely description of the

OMT program, which did not provide unconditional liquidity, but required that certain sustainability measures be met. Wolf (2014) highlights that this aspect of the program drew criticism from its opponents, since it would presumably not be there to provide liquidity precisely when member countries needed it most: During a crisis. My modelling choice for liquidity provision allows for precisely such a crisis to occur while simultaneously eliminating the direct impact of sentiment fluctuations.

It is clear from this proposition then that the central bank has the capacity to eliminate confidence fluctuations. What is less clear from this proposition alone are the welfare implications of such a policy. It is not clear that the ECB would want to remove confidence fluctuations in the first place, since it is not clear that the equilibrium with confidence fluctuations is better than the one without. This trade-off can be phrased more colloquially in terms of the position of the core countries relative to the periphery with regards to provision of liquidity. Periphery countries argue that such provision is necessary to protect them from malignant market sentiments, while core countries argue that if such provision were in place periphery countries would build up unsustainable debt-to-GDP ratios and ultimately threaten the stability of the currency union. This trade-off is exactly highlighted by Proposition 1.4.6. Thus, to determine whether the provision of liquidity was effective or not, we will need to take a stand on the model's parameters, especially with regards to the stochastic structure of the confidence shocks. In the next chapter, I estimate a model on Spanish data to do just that.

1.5 Conclusion

In this paper, I characterized a new type of dynamic lender coordination problem, which I call dynamic panics. I demonstrated their relevance in a standard quantitative sovereign debt model as well as characterized their basic properties. I also showed that such panics can affect both long-term and short-term debt, but if they affect short-term debt it must at

some point act through the default channel. However, with long-term debt we can have crises driven solely by borrowing behavior, which I argue occurred in the Eurozone periphery.

I further demonstrated that in this environment interest rate ceilings are ineffective but that liquidity provision can eliminate the impact of market sentiments. The intuition for the first was that the form of multiplicity is not of a Laffer-curve type, since the sovereign has the ability to commit to a level of borrowing today; and the intuition for the second is that the central bank, acting as a lender of last resort, has the ability to costlessly coordinate the economy on an equilibrium free of sentiment dynamics, though it remains unclear if such coordination is welfare-improving.

This paper lays the groundwork for much potential future research. For instance, I have only just begun to outline the theoretical properties of these confidence-waves and have only been able to prove their existence for short-term debt, though computational examples with long-term debt can be found. An existence theorem for the case of long-term debt would likely be quite enlightening.

Further, the dynamic panics may have significantly broader implications than simply in sovereign debt markets. There is no reason why we would not expect such panics in markets for, say, municipal debt or commercial paper. A more in-depth exploration of the potential for dynamic panics in these markets would also prove illuminating.

Chapter 2

Dynamic Panics: Application to the Eurozone

2.1 Introduction

Motivated by the applicability of dynamic panics to the recent crisis in Peripheral Europe and the ambiguous nature of the key policy implications, in this paper I explore quantitatively the empirical and policy implications of non-default-relevant dynamic panics in a structural estimation exercise. In particular I estimate a quantitative business cycle model specially designed to isolate empirically non-default-relevant dynamic panics. While this model will be more complex and substantially different in nature than the theoretical model presented in the last chapter, non-default-relevant dynamic panics in both models will affect observables through the same mechanism, and so I can meaningfully interpret parameter estimates from this latter model.

I address the principal difficulties associated with the introduction of default into this class of models by implementing a new solution method outlined by Foerster et al. (2013). Further, I develop a new algorithm to expedite the estimation that I call the *Capital-Motion Algorithm* and which applies to a wider class of models than the one I consider.

I take this model to the Spanish time-series data with two key questions: First, what was the ex-ante likelihood of a transition into a dynamic panic? And second, was liquidity provision by the ECB welfare-improving? My structural estimation on Spanish data suggests that these crises may in fact occur frequently. The probability of switching confidence regimes in any given quarter is estimated to be around 3.39%, which is roughly once every 7.37 years. This figure is quite robust and is computed from spread data before the crisis. It does not rely whatsoever on the relative frequency of these events in the data. Given that the crisis took place roughly 7.5 years after the inception of the monetary union,* this figure is in close accord with realized events and tells us that if anything, investors anticipated such crises more often than they occurred.

This result also helps to solve the puzzle of low sovereign debt spreads throughout the early 2000's as well. Lane (2012) articulates this puzzle as follows: "(T)he low spreads on sovereign debt...indicated that markets did not expect substantial default risk and certainly not a fiscal crisis of the scale that could engulf the euro system as a whole." On the contrary, I argue such low spreads could be compatible with a rational long-term dynamic panic. This is because during normal times investors did not fear default, but the possibility that the economy would enter a regime in which default is more likely. Since such a regime-shift was unlikely and during one debt would still have substantial value, the implied spread in non-crisis times would have been very small. However, a simple drop in investor confidence was sufficient to tip the whole economy into a high-spread panic of the magnitude we observed.

To answer the second question, the estimation suggests that so long as liquidity provision does not induce limited-commitment-based default more than once every 9.7 years that it will indeed improve welfare. This is substantially less than historical default trends

*'Inception' is a vague term here, since the launch of the Euro took several years following its initial circulation in 1997. The external validity measure of 7.5 years assumes the initial date is the full launch, which occurred in 2002Q1, and that the crisis occurred in 2009Q3.

outlined by Reinhart and Rogoff (2010), and so the provision of liquidity was likely a welfare-improving policy.

In summary, this paper makes the following points: First, it develops a new type of Markov-switching dynamic equilibrium model and implements a new solution method to isolate empirical dynamic panics; second, it develops several widely applicable techniques that can be used to reduce the computational burden of approximating Markov-switching dynamic equilibrium model solutions; and third, it uses empirical estimates of the frequency of dynamic panics to argue that the provision of liquidity during the recent crisis was likely welfare improving.

2.2 Literature Review

This paper contributes to two primary literatures: First, it furthers the recent literature on the Eurozone crisis. As of yet, the academic literature has had little time to keep pace with developments that took place in the Eurozone over the past 6 years or so. However, several noteworthy pieces have emerged that have tried to deal seriously with the peculiar circumstances surrounding the sovereign debt crisis in the Eurozone. These papers have been both empirical and structural. On the empirical side, recent work has taken aim at demonstrating the confidence-driven nature of this crises by documenting an unusually weak correlation between economic fundamentals and CDS spreads. Some prominent examples include De Grauwe and Ji (2013) and Aizenman et al. (2013). This work will rely in some sense on these empirical findings in its placement of malignant market sentiments at the heart of the story in its theory of the crisis.

On the structural side, much emphasis has been placed on the unusual phenomenon of borrowing into high spreads and its concomitant drastic effect on debt-to-GDP ratios. Conesa and Kehoe (2012) have termed this phenomenon ‘gambling for redemption,’ and have argued that being mired in a deep recession is a necessary condition for such behavior.

Broner et al. (2014) and Corsetti and Dedola (2013) have also built models featuring borrowing into high spreads. The former emphasizes the crowding out effect of sovereign debt issuance when there is domestic preference for debt and the latter argues that the access to liquidity that the central bank provides is more important for preventing such crises than the printing press.

The other relevant strand of literature that this work advances is that of Markov-switching dynamic equilibrium models. This literature, which started with Hamilton (1989), has made the case that parameter instability in the form of regime-switching is often key to understanding macroeconomic time-series. One recent work, Foerster et al. (2013), solves these Markov-switching models from the full initial model by perturbing the underlying parameters of the model governed by regime-switching. I apply their framework in my solution of the model that I estimate. But further, I provide a set of tools to broaden the applicability of their method and demonstrate how it could be applied to models of sovereign default, provided that default behavior is taken as exogenous.

2.3 Model Description

In the previous chapter, I outlined the theoretical properties of Confidence-Waves Equilibria and showed that confidence shocks manifest themselves as dynamic panics. Now, I seek to understand the recent Eurozone crisis quantitatively under the assumption that member countries experienced such a panic. After the theoretical analysis in the previous chapter I am left primarily with two lingering questions with regard to the Eurozone: What was the anticipated frequency of a long-term dynamic panic? And, given this frequency, was the provision of liquidity through programs such as OMT actually welfare-improving? To answer these questions, I will construct a stylized business cycle model that incorporates monotone, non-default-relevant dynamic panics in a way that can be taken directly to Eurozone data.

My approach is motivated by a careful consideration of the impact of long-term dynamic panics on the key observable variables, debt levels and spreads. In the theoretical section, I first established that in the face of a non-default-relevant dynamic panic, the sovereign *must* maintain roughly the same primary surplus and borrows additionally to fill the gap; in other words, he follows a roughly constant domestic fiscal rule and borrows additionally from abroad in the face of a lender panic to do so. Next, I demonstrated that as a consequence of this additional borrowing the benefits from default increase, which increases the future default frequency.

Rather than solve the fully endogenous model with confidence, when I go to the data I will take these two mechanisms as primitives and estimate the parameters governing them. In particular, I will assume that default occurs at some exogenously given frequency that varies across confidence regimes and that the fiscal rule governing domestic consumption and taxation is constant across these regimes. Doing so will allow me to nest dynamic panics in a much richer and more flexible business cycle model that will be much more conducive to empirical inference. Further, there will tend to be a one-to-one welfare map[†] between the fully endogenous model and the empirical one, since in both certainty equivalent consumption will be decreasing in expected default frequencies. This will allow for plausible policy experimentation and welfare analysis.

It can be shown at great difficulty that all of the theoretical results outlined in the previous section continue to go through in a modification of this empirical environment in which the government and households make decisions separately, provided the households value government spending separably and the government maximizes only this portion of household utility.

[†]One potential map between these two models could be the mapping average frequency of exogenous default in the empirical specification to the degree of lender risk-aversion in the fully endogenous model. In Aguiar et al. (2015), it is shown that with long-term debt, risk-aversion acts as a disciplinary device that reduces default frequencies without changing sovereign tastes or technology. This discipline tends to be welfare-improving quantitatively.

My identification of the stochastic structure of the confidence regimes is motivated by Corollary 1.4.4, which tells us that the probabilities of transitions are priced into the spreads. Thus, I can use spread data to estimate the probabilities of regime switching. Since I am interested in only a few novel parameters of an otherwise standard model, I will employ full-information techniques to derive my estimates. Several authors interested in full-information estimation have taken the approach of estimating models with sovereign default by specifying an exogenous rule for fiscal policy and default, instead of actually allowing for endogenous default choice, as I have done in the theoretical section. Some noteworthy examples include Bi and Traum (2012) and Bocola (2014). With an exogenous fiscal rule, the equilibrium is determinate and the likelihood function is well-defined.

However, just because the equilibrium is determinate does not mean that the model exhibits steady-state dynamics. These authors address this problem by applying the particle filter method of Fernandez-Villaverde and Rubio-Ramirez (2007), which uses simulated ‘particles’ to approximate the likelihood of a given parameterization from the full non-linear specification. Rather than taking this approach, I modify the model such that it can be linearized and adopt the method of Foerster et al. (2013).

With the perturbation method described in Foerster et al. (2013), I can capture all of the essential dynamics of confidence-waves equilibria while taking confidence and default as exogenous regime shifts. This approach will result in determinacy and substantially faster computation time. This simple model will produce spreads that explicitly price not only the probability of a sovereign default, but also of a dynamic panic.

In employing this technique, I make a handful of technical contributions to speed up the computation as well. In particular, they show in their paper that the primary bottleneck to solving a standard dynamic equilibrium model with parameter instability involves solving a quadratic system. I demonstrate that, for a large class of models, many of the unknowns in this quadratic system can be determined before explicitly solving the system. Specifically,

I argue that the response of investment to changes in the capital stock is the only object that needs to be solved for in the quadratic system. Once this rule is known, the rest of the equilibrium decision rules can be derived from a simple linear system. This exponentially reduces computational time while retaining the useful property that one still derives all possible first-order approximations.

2.3.1 Specification

In this section I present a standard business cycle model such that it retains the impact of non-default-relevant dynamic panics while simultaneously being suited to estimation methods. First, I will unpack the endowment fluctuations assumed in the theoretical model and have a unit mass of standard, neoclassical growth households with endogenous labor supply and preferences as in Greenwood et al. (1988). They have a constant degree of relative risk aversion, σ , and a Frisch elasticity of labor supply, χ . These households save in capital and can only trade in domestic markets.

There is a unit mass of competitive final goods firms with a Cobb-Douglas technology that experience an aggregate productivity shock, z_t . There is also a competitive investment goods sector that produces subject to convex adjustment costs $\Phi\left(\frac{i_t}{k_{t-1}}\right)$.

The government's budget constraint remains the same, but it will follow a simple fiscal rule instead of maximizing household utility.[‡] In particular, government expenditures will follow an exogenous AR(1) process in its log:

$$\log(g_t) = (1 - \rho_g) \log(g^*) + \rho_g \log(g_{t-1}) + \sigma_g \varepsilon_t \quad (2.1)$$

To generate non-trivial borrowing behavior, I also specify the tax policy rule. What the

[‡]It is not hard to show that all of the theoretical results in the previous section go through in this environment if households value government spending separably and the government is quasi-benevolent in the sense that it only maximizes that separable portion of household utility via endogenous default and borrowing decisions.

government does not raise in domestic taxes it borrows from abroad in defaultable debt. In the event of default, it is assumed that the government sets $\tau_t = g_t$ i.e. it must use current taxes to finance all expenditures. When it is not in default, it sets taxes in response to its current debt level and the exogenous interest rate it faces from the outside investors. In particular, I assume that the government chooses a lump-sum tax policy of the following form:

$$\tau_t - \tau^* = \gamma_b(b_{t-1} - \frac{\hat{\gamma}_b}{\gamma_b}b^*) + \gamma_R(R_t - R^*) + \gamma_g(g_t - g^*) \quad (2.2)$$

τ^* , b^* , R^* , and g^* are target levels that will, in equilibrium, reflect the steady state values. The term $\hat{\gamma}_b$ simply adjusts the fiscal rule for the maturity of the debt and possibility of default. It is given by

$$\hat{\gamma}_b = \gamma_b - \lambda - (1 - \lambda)\kappa + \lambda q_{ss}$$

where q_{ss} is the steady state price of foreign government debt. This rule is similar to those in Schmitt-Grohé and Uribe (2007) and Leeper (1991), but with the addition of explicit consideration of the foreign interest rate that will induce a downward sloping demand curve for foreign assets in response to exogenous fluctuations in the interest rate.

The parameter γ_R governs the response of the government to exogenous changes in the interest rate that it faces. When $\gamma_R > 0$, then the government raises taxes and thus lowers its debt issuance in response to interest rate shocks, generating a downsloping demand curve for foreign assets as a function of the exogenous interest rate.

γ_g determines the extent to which the government responds to shocks in spending with taxes and γ_b governs how aggressively the government responds its debt: If γ_b is high, then high debt levels are quickly adjusted; if γ_b is low, then large debt levels will linger

longer.[§] Taken together, the magnitude of $(\gamma_b, \gamma_R, \gamma_g)$ can be interpreted as a measure of fiscal discipline, since they determine the extent to which adverse shocks are funded by painful domestic taxes relative to defaultable foreign debt.

The price of debt, q_t , continues to reflect the probability of default, but to generate more plausible debt price dynamics I allow for haircuts after the stochastic re-entry after default. All of the theoretical results will continue to go through in this environment provided we restrict our attention to the plausible case in which the demand for debt is downward-sloping. In particular, I assume that in each period of default a fraction $1 - \hat{\delta}$ of the face value of the bond is destroyed. This implies a pricing recursion as follows:

$$q_t = \frac{1}{R_t} E \left[[1 - d_{t+1}] [\lambda + (1 - \lambda)(\kappa + q_{t+1})] + d_{t+1} \hat{\delta} q_{t+1} \right]$$

Allowing for I allow for the outside option, R_t to fluctuate over time. Notice that this recursion is valid when the sovereign is in default as well.

Lastly, I assume that non-default-relevant dynamic panics enter the model exogenously as a regime shift. However, I invoke their properties from the theoretical model. In particular, we know the following from Proposition 1.4.3:

1. During a non-default-relevant dynamic panic, the government maintains a roughly constant primary deficit and borrows more to fill it
2. **During a long-term dynamic panic, the future probability of default rises.**

To generate exogenously a long-term dynamic panic, I take this last characteristic to be fundamental i.e. I assume that default occurs stochastically but that it has a greater likelihood during such a panic.[¶]

[§]The theoretical model will imply that $\gamma_b < 1$, since borrowing increases in response to higher debt levels. This is proven in Chatterjee and Eyigungor (2012).

[¶]In 7.B, I present several empirical measures of the robustness of this specification and show that the data strongly favor it over several alternative models.

This simple assumption will generate a long-term dynamic panic along the model observables. First, it will induce higher spreads since investors demand compensation for the higher possibility of default, and thus both parties will change their behavior during a crisis. Second, it will dictate increased borrowing on the part of the government, since it must fill the same primary deficit with lower-priced debt. These trends can be seen in the simulation averages of the modified model in Figure 5.B.1.^{||}

Not only will a long-term dynamic panic will be associated with higher spreads, higher external borrowing, and greater default probabilities, but we will also see a slump in investment and a concomitant contraction in output coming from the private sector. This happens for two complementary reasons: First, expected productivity falls during such a panic, since default productivity costs are more likely in the future; second, consumption may actually be higher during a default, since foreign debt obligations are repudiated. Provided the household is patient enough to internalize these expected changes, both of these effects create a strong disincentive to save that is reflected in low investment. This can be seen in Figure 5.B.2.

Lastly, for the purposes of the estimation, I assume that foreign interest rates and labor productivity follow AR(1) processes as well and that productivity drops during a default.

$$\begin{aligned}\log(z_t) &= (1 - \rho_z) \log(z^*(s_t)) + \rho_z \log(z_{t-1}) + \sigma_z \varepsilon_{z,t} \\ \log(R_t) &= (1 - \rho_R) \log(R^*) + \rho_R \log(R_{t-1}) + \sigma_R \varepsilon_{R,t}\end{aligned}$$

where s_t denotes the current regime, of which default is a possibility.

Note that a lender shock will increase the spread today even though the risk-free rate is differenced out. This is because the lender shocks are persistent i.e. $\rho \in (0, 1)$. While higher rates today are differenced out of the spread, higher anticipated future rates are not.

^{||}The trajectories in Figure 5.B.1 are computed using the estimated parameters I later obtain, but the pattern looks the same for any standard parameter values.

Rather, they bring down the price of debt tomorrow and thus drive down the price of debt today.

2.3.2 Parameter Instability

I consider an equilibrium in which four key parameters are subject to switching: (z^*, rr, d, p_D) , where rr is the recovery rate on bonds in default. I denote these parameters by the vector $\theta(s_t)$, where $s_t \in \{1, 2, 3\}$ i.e. there are three distinct regimes. The parameters take the following values in the three different regimes:

$$\begin{pmatrix} z^*(s_t) \\ rr(s_t) \\ d(s_t) \\ p_D(s_t) \end{pmatrix} \in \left\{ \begin{pmatrix} \mu \\ \delta_u \\ 0 \\ p_H \end{pmatrix}, \begin{pmatrix} \mu \\ \delta_u \\ 0 \\ p_L \end{pmatrix}, \begin{pmatrix} \mu_D \\ \hat{\delta} \\ 1 \\ 1 - \pi_{RE} \end{pmatrix} \right\} \quad (2.3)$$

where $\mu_d < \mu$. $\hat{\delta}$ governs the recovery rate of bonds in default; δ_u is never observed on the equilibrium path and so can be judiciously chosen to ensure a well-defined steady state. The change from $s_t = 1$ to $s_t = 2$ will behave as the completely endogenous confidence-switching regimes described in the general theoretical model. Estimating the transition between these regimes and the implications for policy of this switch is the primary goal of this exercise, since a default has yet to occur in the Spanish data. In particular, the transition matrix $P = (p_{s',s})_{s',s=1,2,3}$ will be given by:

$$P = \begin{bmatrix} 1 - \eta - p_H & \eta & p_H \\ \eta & 1 - \eta - p_L & p_L \\ \pi_{RE} & 0 & 1 - \pi_{RE} \end{bmatrix} \quad (2.4)$$

It is assumed that $p_L > p_H \geq 0$ i.e. default is more likely in the low-confidence regime. I will also assume that $\eta < .5$ i.e. the regimes are persistent. Notice that I allow for stochastic

re-entry with probability ξ in the event of a default and that it is assumed, as is the case in the general model, that the sovereign re-enters credit markets with high confidence.

2.3.3 Model Solution

The equilibrium conditions can be written in the following form:

$$E_t[f(y_{t+1}, y_t, x_t, x_{t-1}, \chi \varepsilon_{t+1}, \varepsilon_t, \theta_{t+1}, \theta_t)] = \mathbf{0}_{n_x+n_y} \quad (2.5)$$

where $y_t = (c_t, i_t, Q_t)$ are the primary control variables, $x_t = (k_t, b_t, R_t, z_t, g_t)$ are the endogenous state variables, and $\theta_t = (p_D(s_t), z^*(s_t), d(s_t))$ are those parameters that are subject to regime switching, and s_t denotes the current regime. $\bar{\chi}$ is the perturbation parameter. It is convenient to write the equilibrium conditions in this way because I can then apply the

method of Foerster et al. (2013). Note that the function f looks as follows:

$$\begin{aligned}
 & \left[\begin{aligned}
 \text{(1)} & \left[c_t - \kappa_1 (z_t^{1-\alpha} k_{t-1}^\alpha)^{\frac{1+\chi}{1+\alpha\chi}} \right]^{-\sigma} - \beta \left[c_{t+1} - \kappa_1 ([z^*(s_{t+1})^{1-\rho_z} z_t^{\rho_z} e^{\sigma_z \bar{\chi} \varepsilon_{z,t+1}}]^{1-\alpha} k_t^\alpha)^{\frac{1+\chi}{1+\alpha\chi}} \right]^{-\sigma} \times \\
 & \left(\kappa_2 \left[[z^*(s_{t+1})^{1-\rho_z} z_t^{\rho_z} e^{\sigma_z \bar{\chi} \varepsilon_{z,t+1}}]^{1-\alpha} k_t^{\frac{\alpha-1}{1+\chi}} \right]^{\frac{1+\chi}{1+\alpha\chi}} + (1-\delta) \left(1 + \Phi \left(\frac{i_{t+1}}{k_t} \right) + \Phi' \left(\frac{i_{t+1}}{k_t} \right) \frac{i_{t+1}}{k_t} \right) \right) \\
 \text{(2)} & c_t + \left[1 + \Phi \left(\frac{i_t}{k_{t-1}} \right) \right] i_t + g_t - [1 - d(s_t)] [\lambda + (1-\lambda)(\kappa + q_t)] b_{t-1} + q_t b_t - \kappa_0 [z_t^{1-\alpha} k_{t-1}^\alpha]^{\frac{1+\chi}{1+\alpha\chi}} \\
 \text{(3)} & - q_t b_t + d_t q_t r r(s_t) b_{t-1} + \\
 & [1 - d(s_t)] [(\lambda + (1-\lambda)(\kappa + q_t) - \gamma_b) b_{t-1} + \gamma_R R_t + (\gamma_g - 1) g_t + (\tau^* + \hat{\gamma}_b b^* - \gamma_R R^* - \gamma_g g^*)] \\
 \text{(4)} & k_t - (1-\delta) k_{t-1} - i_t \\
 \text{(5)} & \log(R_t) - (1-\rho_r) \log(R^*) - \rho_R \log(R_{t-1}) - \sigma_R \varepsilon_{R,t} \\
 \text{(6)} & \log(g_t) - (1-\rho_g) \log(g^*) - \rho_g \log(g_{t-1}) - \sigma_g \varepsilon_{g,t} \\
 \text{(7)} & \log(z_t) - (1-\rho_z) \log(z^*(s_t)) - \rho_z \log(z_{t-1}) - \sigma_z \varepsilon_{z,t} \\
 \text{(8)} & q_t - \frac{1}{R_t} \left[[1 - d(s_{t+1})] [\lambda + (1-\lambda)(\kappa + q_{t+1})] + d(s_{t+1}) r r(s_{t+1}) q_{t+1} \right]
 \end{aligned} \right]
 \end{aligned} \tag{2.6}$$

I seek a solution to this model of the following form:

$$y_t = g(x_{t-1}, \varepsilon_t, \bar{\chi}, s_t), y_{t+1} = g(x_t, \bar{\chi} \varepsilon_t, \chi, s_{t+1}), x_t = h(x_{t-1}, \varepsilon_t, \bar{\chi}, s_t) \tag{2.7}$$

An exact solution to this model is computationally burdensome and, given the model's design, unnecessary. Instead, I will find a linear approximation to the model around a non-stochastic steady state. I will search for a set of matrices $\{g_{ss}(s_t), h_{ss}(s_t)\}_{s_t=1, n_s}$, where

$g_{ss}(s_t)$ has dimension $n_y \times (n_x + n_\varepsilon + 1)$ and $h_{ss}(s_t)$ has dimension $n_x \times (n_x + n_\varepsilon + 1)$. When in a regime s_t , $g_{ss}(s_t)$ will map deviations in $(x_{t-1}, \varepsilon_t, \bar{\chi})$ from their non-stochastic steady state into deviations of y_t from its non-stochastic steady state. Thus, if \hat{z}_t is the steady-state deviation of an equilibrium object, z_t , then $\hat{y}_t = g_{ss}(s_t)[\hat{x}'_{t-1}, \varepsilon_t, \bar{\chi}]'$ and $\hat{x}_t = h_{ss}(s_t)[\hat{x}'_{t-1}, \varepsilon_t, \bar{\chi}]'$ when in a regime s_t .^{**}

In order to perturb this model, I must have a well-defined notion of a steady state that is independent of the Markov-switching regimes. To do so, I follow Foerster et al. (2013) and perturb the parameters $(d(s_t), z^*(s_t))$ as follows:

$$\begin{pmatrix} z^*(\chi, s_t) \\ rr(\chi, s_t) \\ d(\chi, s_t) \end{pmatrix} = \begin{pmatrix} \bar{z} \\ \bar{r}\bar{r} \\ \bar{d} \end{pmatrix} + \bar{\chi} \begin{pmatrix} \hat{z}(s_t) \\ \hat{r}\hat{r}(s_t) \\ \hat{d}(s_t) \end{pmatrix} \quad (2.8)$$

where $\hat{z}(s_t) = z^*(s_t) - \bar{z}$, $\hat{r}\hat{r}(s_t) = rr(s_t) - \bar{r}\bar{r}$, and $\hat{d}(s_t) = d(s_t) - \bar{d}$ and $(\bar{z}, \bar{r}\bar{r}, \bar{d})$ are taken to be the ergodic mean of $(z^*(s_t), rr(s_t), d(s_t))$. I construct the steady state of the dynamic system in terms of the $(\bar{z}, \bar{r}\bar{r}, \bar{d})$, and thus the steady state is independent of the current regime. I calibrate $\hat{\delta}$ and choose $\delta_u(\hat{\delta})$ to ensure that $\bar{r}\bar{r} = 1$, which guarantees that the following expression holds:

$$f(\bar{y}, \bar{y}, \bar{x}, \bar{x}, 0, 0, \bar{\theta}, \bar{\theta}) = \mathbf{0}_{(n_x+n_y) \times 1}$$

i.e. the equilibrium conditions equate to zero at the non-stochastic steady state.

In order to solve this system, I must take a series of derivatives of Equation 2.6 with respect to all endogenous objects and the Markov-switching parameters and evaluate them at the steady state. Foerster et al. (2013) demonstrate that a first-order approximation to the solutions g and f can then be obtained in two steps. The first step entails solving the following quadratic system for $\{D_{1,n_x}g_{ss}(s_t), D_{1,n_x}h_{ss}(s_t)\}_{s_t=1}^{n_s}$, which are the first n_x

^{**}The non-stochastic steady state of ε_t and $\bar{\chi}$ are 0. $\bar{\chi}$ is 1 in the perturbation solution.

columns of the approximated policy rules and laws of motion, respectively, for each state. The relevant quadratic system is given below:

$$A(s_t) \begin{bmatrix} \mathbf{I}_{n_x} \\ D_{1,n_x} g_{ss}(1) \\ \dots \\ D_{1,n_x} g_{ss}(n_s) \end{bmatrix} D_{1,n_x} h_{ss}(s_t) = B(s_t) \begin{bmatrix} \mathbf{I}_{n_x} \\ D_{1,n_x} g_{ss}(s_t) \end{bmatrix} \quad (2.9)$$

for all s_t . Where $A(s_t)$ is an $(n_x + n_y) \times (n_x + n_s n_y)$ matrix and $B(s_t)$ is an $(n_x + n_y) \times (n_x + n_y)$ matrix. Both are functions of the derivatives of Equation 2.6 and their full specification can be found in Foerster et al. (2013).

Once a solution to Equation 2.9 has been found, the remaining elements of the matrices h_{ss} and g_{ss} can be found by solving a simple linear system that is provided in the computational appendix. If there are multiple mean-square stable approximations, I denote the one that provides the *higher* likelihood to be the true one.^{††}

I validate the accuracy of the first-order approximation by checking the unconditional Euler Equation errors, as suggested by Foerster et al. (2013). I find that at the posterior mean, $\log_{10}(EE \text{ Error}) = -3.6078$.^{‡‡} To put this figure in perspective, note that when this object is -3 (-4), there is a \$1 error for every \$1,000 (\$10,000) of consumption determined by the Euler Equation.

Reducing the Dimensionality

Equation 2.9 is the matrix representation of a quadratic system with $n_s n_x (n_x + n_y)$ equations and the same number of unknowns. Foerster et al. (2013) suggest the use of Grobner bases to solve for all possible solutions to this system. While this method is satisfyingly

^{††}Note that a multiplicity of first-order approximations *does not imply* a multiplicity of equilibria. The equilibrium of the model is demonstrably determinant.

^{‡‡}This error is much smaller than comparable models described by Foerster et al. (2013) that also have some form of adjustment costs.

exhaustive, its full implementation can be burdensome, as computational time is for most algorithms is exponential or even doubly exponential in the number of potential solutions. In their expository examples, the number of unknowns i.e. $n_s n_x (n_x + n_y)$ is never more than 8.

The solution I seek has a substantially larger dimensionality. In particular, $n_s n_x (n_x + n_y) = 3 \times 5 \times 8 = 120$. Given an exponential rate, even one iteration could take hundreds of thousands of years to compute, which is clearly impractical. However, there is much that we know about how the equilibrium operates that can be imposed on the solution before we even begin that allow me to reduce the dimensionality of the system. In particular, I develop a new method called the **Capital-Motion Algorithm** that reduces the quadratic system to 3 equations in 3 unknowns, which can be solved in hundredths of a second. The Capital-Motion Algorithm can be implemented in a wide class of models beyond the one at hand, and thus render this solution method more applicable in general.

The **Capital-Motion Algorithm** proceeds in 4 steps:

1. Fix the coefficients governing exogenous laws of motion.
2. Use the resource constraint to express consumption in terms of investment.
3. Solve a smaller quadratic system to determine the derivative $i_k(s_t)$ in each state i.e. how investment responds to a shock to capital.
4. For each solution $i_k(s_t)$, solve a linear system to determine all other model derivatives.

We can demonstrate a very useful property of the Capital-Motion Algorithm as it pertains to our case:

Theorem 2.3.1 *The Capital-Motion Algorithm reduces the dimensionality of the quadratic system governing the model solution from 120 equations/unknowns to 3 equations/unknowns and still delivers all solutions to the original system.*

Proof See 7.A.

Theorem 2.3.1 is tailored to the model at hand for simplicity, but it can be generalized to a wider class of models: Essentially any model for which the crux of the intertemporal dynamics is the Euler equation. It relies on the fact that the key unknowns in the quadratic system are the coefficients governing the future capital choice in each state with respect to shocks to current capital, of which there are n_s i.e. one for each state. The decision of the agent to consume or to save today will reflect his propensity to consume or save tomorrow *in response to the same shock*; hence, when the rule is linear the relevant system is quadratic. Given the information imposed on the system so far, knowledge of saving behavior tomorrow in response to a capital shock in each of the different states is sufficient for determining saving behavior today in response to the same shock, and thus we can solve for these objects.

Once the saving response to a capital shock has been determined, the other equilibrium objects can be solved for linearly, since none of the others require solving an intertemporal problem, which is the source of the problem's quadratic nature.^{§§} In other words, a shock to any other object in the model will only be affected by future objects insofar as it has adjusted investment *today*.

Theorem 7.A.4 reduces the dimensionality of the problem drastically, to the point where there are only n_s coefficients that must be solved for, which is 3 in this model. Upon

^{§§}A general form of Proposition 7.A.4 would entail solving for the number of endogenous equilibrium objects that jointly affect the intertemporal decision. For instance, if we were to introduce cyclical fiscal policy into the model, then we would need to solve a quadratic system of $2n_s$ equations and unknowns, since the dynamics of investment and debt be interdependent.

reaching this point, I can easily solve the model for all possible solutions in fractions of a second and proceed with the rest of the solution as described in the previous section.

2.3.4 Estimation Procedure

Data

I use three quarterly time series data from Spain from 2001 until 2012: GNP, 5-year CDS spreads, and the public current account as a fraction of GNP. I take gross external government debt and GNP from the ECB Statistical Data Warehouse and I take spread data on 5-year debt from the MARKIT database. The first two objects require detrending, for which I use a Hodrick-Prescott filter (Hodrick and Prescott (1997)) to remain agnostic. The spread data requires no de-trending, though I reduce the frequency from daily to quarterly by means of an average. The filtered data can be found in Figure 7.A.1 in 7.A.^{¶¶}

I use Spanish data because De Grauwe (2011) and others have noted that Spain's crisis seems most confidence-driven i.e. spreads tend to co-move the least with its fundamentals relative to other countries in the Eurozone periphery, although they all exhibit these trends. Since it is this shift in confidence that I seek to estimate, I find Spanish data to be the most appropriate tool available.

Identification

The shocks are all identified because they have orthogonal impacts on the observable variables: An interest rate shock today will impact positively debt and spreads but leave contemporaneous output untouched; a government policy shock will have a negative impact on debt but leave spreads and output untouched; and a productivity shock will increase contemporaneous output without impacting either the debt level or the spreads.

There are two potential sources of identification of the probability of regime switching:

^{¶¶}In this figure, I mark the start date of the crisis as 36 quarters, which corresponds to 2009Q4.

First, there is the length of time that the economy spends in one period versus another; and second, there is the fact that the probability of a regime switch is priced into the spreads. Given the relatively short span of the data, I follow the second approach for identification.

To identify this probability from spread data, I make a couple of identifying assumptions. First, I assume that the probability of regime switching is symmetric. This condition, although perhaps not necessary, is the condition under which I can ensure existence in the theoretical model. The results change only negligibly if I assume that confidence regime shifts are asymmetric for reasons I will discuss below. Second, I assume that there is no probability of default in normal times i.e. $p_H = 0$. Thus, any positive spread that we see in normal times reflects the possibility of a regime shift into a panic state in which a default is possible. The divergence in the spreads in the panic state plus the symmetry assumption will allow me to jointly pin down the probability of default and the probability of a confidence shift. Note that the probability of a default in a panic is not uniquely determined by the spread, however. Private sector expectations, through investment and consumption, help provide additional identification regarding this probability.

Last, I denote the start of the crisis i.e. the regime shift as occurring in 2009Q4, which roughly dates the start of the sovereign debt crisis by most accounts. Since there is little dispute regarding the start date of the crisis, I prefer this route to estimating the start date, an approach taken by, for instance, Bianchi (2013).

Fixed Parameters

I fix many of the model's key parameters and estimate only 5: η , p_L , γ_b , γ_g , and γ_r . The parameterization is given in Table 7.A.1, which can be found in 7.A. A few require discussion.

First, I choose z_L based on Mendoza and Yue (2012) i.e. 5% less than normal times. I choose the mean risk-free rate to be 0.01. To match the volatility, I choose to set the

volatility of this shock to the estimated volatility of a T-bill to an emerging market from Neumeyer and Perri (2005). As for the defaultable debt, I assume a 4% coupon, which is in line with the average for Spanish long-term debt, and choose λ to match the average maturity of Spanish debt, which was 6.5 years at the time of the crisis.

The specification of the government spending and TFP shocks is taken from Arias et al. (2007) who calibrate these parameters in a simple RBC model. Though they calibrate to the US, several authors have noted that the Spanish economy before the crisis was not terribly different from the US in its cyclical properties (see Puch and Licandro (1997)).

I calibrate π_{RE} such that the average default lasts 2 years, which is roughly the length of Grecian exclusion from international credit markets following its default in 2012. The results do not substantially change even when this parameter varies widely. I also calibrate the haircut following default to match that of Greece on average. In particular, note that the face value of a bond in default is given by

$$\begin{aligned} \tilde{b}_t &= \sum_{\tau=1}^{\infty} \pi_{RE} (1 - \pi_{RE})^{\tau-1} \left(\frac{\hat{\delta}}{R^*} \right)^{\tau} b_t \\ \rightarrow \frac{\tilde{b}_t}{b_t} &= \sum_{\tau=1}^{\infty} \pi_{RE} (1 - \pi_{RE})^{\tau-1} \left(\frac{\hat{\delta}}{R^*} \right)^{\tau} \end{aligned} \quad (2.10)$$

I equate this expression to the average recovery rate of the face value of Greek debt, which was 29.5%, to determine the value of $\hat{\delta}$.

I calibrate impatience,^{***} adjustment costs, the Frisch elasticity of labor supply, labor disutility, capital share of income, intertemporal elasticity of substitution, and capital depreciation to standard values in the RBC literature.

I take the fraction of average foreign debt to GNP as well as government spending to GNP direction from Spanish data.

^{***}This parameter is on the high end of standard discount rates. This simply helps to ensure that household cares enough about the future to disinvest when a default is likely. A low depreciation rate serves this same purpose.

Estimation Procedure

The remaining 5 parameters, which are those of most interest, are estimated using a Bayesian approach. I specify the priors in Table 4.A.1. I take a fairly agnostic stance with regard to the prior distributions, assuming that they all fall in the range $[0, 1]^{\dagger\dagger}$ and that the government does not respond to adverse shocks 1 to 1 with taxes i.e. it smooths such shocks with debt. Changing these priors change the results only negligibly.

Their distribution is attained via a Random-Walk Metropolis Algorithm as outlined in Schorfheide and An (2007). To derive the likelihood of a particular parameterization, I solve the model to a first-order approximation using the algorithm described previously. I then place this approximation into state-space form and evaluate the likelihood with the Kalman filter, taking the series on output, the public current account, and spreads to be my observables.^{‡‡‡}

After specifying the prior, I follow the RWM outlined in Schorfheide and An (2007) to obtain a modal estimate and simulate draws from the posterior distribution. Although the entire distribution is of interest for constructing credible sets and understanding how the data operate, I am most interested in the point estimates given by the mode, since they will provide my estimates of probability of regime switches.

2.3.5 Results

In this section I present the results of the estimation. The mean and the median are similar in all regards, but for the purposes of inference I will take the posterior median to be my primary estimate. The posterior distributions can be found in Figure 7.A.5 in 7.A.

First, notice that the estimate of the probability of regime-switching is actually not that

^{†††}The domain restriction imposed by the priors helps deliver comparable estimates across different models.

^{‡‡‡}I initialize the Kalman filter mean at the non-stochastic steady state and the variance according to the non-crisis regime.

unlikely: The data suggest a median estimate of 3.39%, which corresponds to roughly 7.37 years. Recall that this figure is identified from spread data prior to the crisis and spread levels during the crisis. It does not use the relative frequency of the regimes. If we take 2002 to be the initial date at which investors fully anticipated monetary union for the foreseeable future, this figure seems quite reasonable: Investors expected such a crisis to occur on average every 7.37 years and it took about 7.5 years for one to occur. The closeness of this non-targeted moment speaks strongly to the validity of the model.

The mean estimate of η is actually slightly higher than the median estimate: 4.72%. If we took this to be our estimate, it would imply that investors anticipated a crisis once every 5.3 years. This suggests that, if anything, investors anticipated such crises to occur more frequently than they actually did, not less frequently. Even though the credible set reaches all the way to 13.94%, Figure 7.A.5 shows that most of the mass is concentrated around the mean and median and that there is a long, thin tail on the right-hand side.

This high-frequency estimate helps us to reconcile the ‘low spreads puzzle’ of the Eurozone in the mid-2000s. Lane (2012) articulates this puzzle as follows: “(T)he low spreads on sovereign debt...indicated that markets did not expect substantial default risk and certainly not a fiscal crisis of the scale that could engulf the euro system as a whole.” My empirical analysis suggests quite the contrary: The Eurozone crisis may have been rationally anticipated and in fact priced. To see why, consider the position of an investor in the early 2000s. Such an investor did not fear an outright default, but rather a shift into a low-confidence regime next period. In such a regime, spreads, borrowing, and default probabilities will be significantly higher, but debt will still hold substantial value. Thus, it is the anticipation of this *regime shift* and not of a default that drove the spreads in this early period.

And these spreads were quite low for two complementary reasons. First, confidence processes are by necessity persistent, and so such a shift was an unlikely event. Second,

even when confidence drops, debt still has value. Thus, the return on debt falls, but the value of the debt is not completely obliterated. These two forces can generate a very small but significant spread that is completely consistent with a panic of massive proportions unfolding quickly afterward.

I provide the model-implied shocks at the median estimate in Figure 7.A.3 in 7.A. Two interesting results can be gleaned from this figure. First, notice that the main driver of activity in this model is the government spending shock. This is because government spending in the model, as in the data, accounts for a large fraction of output: 40% for Spain. Also, in the baseline model government spending shocks directly translate to higher debt levels while productivity shocks do not since there is no cyclical policy. Since the model is matching the public current account, which moves cyclically, it will naturally place more weight on these shocks to explain this variation.

Second, the model suggests that the crisis was accompanied by a period of excessively low risk-free rates. It has generally been thought that the low risk-free rates during this period were associated with a monetary policy response. While this response did in fact lower borrowing costs, the model suggests that it may have *raised* spreads. This is because a negative shock to the risk-free rate will reduce the investors' outside option today and tomorrow, but it will reduce it by more today than it will tomorrow because of the mean-reverting nature of these shocks. Thus, the price of debt today will rise by more today than it is expected to rise tomorrow, which raises the dilution spread.

Last, let us turn to the estimated probability of default. Notice first that it is quite high in the model with confidence waves, with default expected to occur with probability 22.04% in any given quarter. This is in accordance with Proposition ???. However, this default probability is not terribly well-identified, as can be seen by the broad span of the credible set. This is simply because the transition probabilities are pinned down by the mean of the spreads in each regime, and there are only 9 quarters in the data used to pin down the default

transition. The credible set for the confidence-wave estimate is smaller because there are 36 quarters over which to average, implying greater accuracy in the face of unrelated lender shocks.

Notice, however, that the large credible set on p_L does not translate to η , even though they are jointly identified.^{§§§} This is because information travels from p_L to η through the price, and there are several forces in the model that dampen the impact of p_L on the price of debt. First, the assumption of long-term debt implies that the spread is partially dictated by the expectation of default and partially dictated by the expected future price of debt. Thus, changes in the default frequency may not impact the spread as much for long-term debt.^{¶¶¶} Second, the inclusion of re-entry and haircuts mitigates the impact of default on investors' return, since investors know that a default will not destroy all of their wealth, but only part of it.

In 7.B I consider a slew of robustness exercises and tests of model fit. First, I examine the fit of the model along several relevant dimensions with posterior predictive distributions; next, I use posterior odds ratios to gauge the adequacy of my model against other candidate models, including a model without regime shifts and one driven by sudden loss of bailout expectations. By these metrics, I find that the data strongly favor my model.

2.3.6 Policy Experiment: Liquidity Provision

Given a set of estimated parameters, we can now begin to think seriously about the model's implications for policy. The key policy question tends to revolve around the provision of liquidity by the European Central Bank: Should the ECB act as the lender of last resort in sovereign debt markets for the constituent countries of the Eurozone? We know that it did with the Outright Monetary Transactions bond-buying program, but was this optimal? The

^{§§§}This is the reason why the specification of the exit probability from a crisis regime has little to no impact on ex-ante forecasts.

^{¶¶¶}This is the typical reason given for why yield curves invert during crises.

theoretical model of dynamic panics is ambiguous on this point, and so we need the data to provide the answer.

So-called fundamentalists, such as Issing (2011) have argued no, citing both the problem of unsustainable fiscal policy in the periphery and cross-subsidization from fiscally responsible countries to fiscally irresponsible ones. Others, such as De Grauwe (2011), take a multiple-equilibrium view, arguing that panic in financial markets caused self-fulfilling increases in the likelihood of default, since such panic raises the cost of debt repayment.

We can analyze the provision of liquidity in the context of this model with a simple exercise. Suppose that if the ECB begins providing liquidity, two things happen: First, confidence-waves are eliminated, since the ECB is not affected by market sentiments;¹⁷ and second, the country defaults more often because of greater limited commitment issues, since such provision encourages fiscal irresponsibility since member countries can rely on the ECB to purchase the debt and fill revenue gaps if times turn bad.¹⁸

We can approximate the value function of the household in each regime, compute its certainty-equivalent consumption,¹⁹ and then ask ourselves whether provision of liquidity could improve on this in each regime. We certainly do not know by exactly how much the limited commitment problem would increase the default probability, but we can ask ourselves *how bad* it would have to be for liquidity provision to become sub-optimal. The estimated model suggests that liquidity provision can improve welfare so long as the limited-commitment-induced likelihood of default does not exceed 2.59% in any given quarter.

¹⁷It is not difficult to demonstrate that an equilibrium always exists in the absence of confidence fluctuations. Chatterjee and Eyigungor (2012) provide a proof of this when the state space is discrete. An equilibrium of this type would come into action if confidence-waves were eliminated.

¹⁸Since there is no default in the baseline model when confidence is high, it is assumed here that the limited commitment problem would be worse in non-crisis times, though it may be better than non-crisis times.

¹⁹Given the GHH preferences, I take certainty-equivalent consumption to be the constant stream of consumption the household must receive if it provides $l = 0$ forever to be indifferent with its situation in the recursive equilibrium.

Procedure

In computing the welfare statistics, I need to ensure that my approximation of the value function is accurate for the task at hand. This is not an easy task, given that the regime-specific ‘steady states’²⁰ may be far away from the ergodic distribution of model objects.

I overcome this problem via simulation. In particular, I approximate the unconditional value of being in one regime or another as follows:

1. Simulate the model for a long time e.g. $N_{inner} = 10,000,000$ quarters
2. Compute the average household value conditional on being in a given regime for each regime.
3. Invert this value to obtain the certainty equivalent consumption implied by the simulation.
4. Repeat Steps 1-3 many times e.g. $N_{outer} = 1000$
5. Take as an estimate of the certainty-equivalent consumption in each regime its average over the N_{outer} simulations.

Given the stationarity of the model, this estimate will converge to the true certainty-equivalent consumption as $N_{inner} \rightarrow \infty$. The use of an outer loop N_{outer} helps to speed up the process by providing independent estimates instead of relying on ergodicity. In practice, this generates estimates for CEC which vary from each other by less than $1e - 4$, which I take to be the desired tolerance when comparing the efficacy of policy.

To determine the threshold default frequency under liquidity provision, I repeat the above procedure several times in the context of an interval bisection, since welfare will be decreasing in the likelihood of default.²¹

²⁰These should not be confused with the model’s non-stochastic steady state, which is regime-independent.

²¹Though I have no guarantee that welfare is decreasing in the default probability, in every numerical

Welfare Results

Figure 5.B.3 shows the difference in certainty-equivalent consumption between the model with liquidity provision and the baseline model for different degrees of the limited commitment problem. The threshold default frequency will equate the certainty-equivalent consumption under the liquidity policy to that in the baseline model. The graph shows this to be 2.59%, which translates to once every 9.65 years.

This figure is the same for crisis times and non-crisis times, so there is no time-inconsistency. The reason for this is because of the endogenous labor supply. When the economy enters a crisis, consumption and investment fall as taxes rise to service mounting debt burdens; however, labor supply also falls. The net effect of these two on welfare is essentially zero, though they have large implications for other model objects.

The default frequency in the absence of confidence-waves is lower than the frequency of confidence-waves in the baseline model. This does not speak to any particular welfare implications of the model, but simply reflects the fact that with confidence-waves it takes at least two periods to default instead of one. To match in welfare terms this default structure, defaults must occur less frequently if they are not preceded by a confidence-waves crisis.

So can we expect liquidity provision to be welfare-improving? The model puts no rigor on the frequency of such default as it pertains to the Eurozone, but we can compare it to the historical experience of other countries. For instance, Reinhart and Rogoff (2010) show that the average external default rates for Brazil and Greece are once every 20 and 31.3 years, respectively over the past two centuries. Since the welfare threshold for our estimated Spanish data is once every 9.65 years, the provision of liquidity by the central bank in the Eurozone is quite likely to be welfare-improving for its member countries.

It is important to note that the exercise here does not explicitly incorporate the welfare application it is. This is because the costs of default together with the higher ex-ante spreads tend to drain welfare much more than expected debt repudiation raises it.

of all member of a monetary union, but only those in distress. There is a fear, express by Issing (2011) and others, regarding the cross-subsidization of member countries implied by liquidity provision facilities. However, as history has shown and as the model predicts, it is sufficient for the ECB to declare credibly that it is willing to purchase the debt of distressed member countries. They need not actually do it.²²

2.4 Conclusion

In this paper, I performed a structural estimation on a standard business cycle model with time-varying default probabilities and used Spanish data to estimate the ex-ante frequency of such non-default-relevant dynamic panics as well as fiscal parameters. I outlined a new algorithm to speed up the computation of this class of models that can be generalized to increase their applicability. The estimation of this model told us that the median estimate of the probability of a confidence-wave crisis, as determined by spreads, is 3.39%, which is validated externally by the actual frequency of such crises relative to the inception of the Euro and helps to resolve the low-spreads puzzle. Further, I demonstrated that the provision of liquidity by the ECB is welfare-improving provided it does not induce limited-commitment-based default more than once every 9.65 years on average.

The structural model I've developed here that incorporates exogenous default into a standard business cycle model can be expanded and used for forecasting. For instance, the existence of a non-stochastic steady state in a model with default can allow for a well-specified Taylor rule and would allow for a tractable exploration of the joint dynamics of monetary and default policy in more developed economies.

²²Some distressed countries, such as Greece and Ireland did receive rescue packages from the ECB's Emergency Lending Facility. However, it is not clear that at the time these amounted to liquidity provision, since the sovereign did not purchase the debt in secondary markets but rather lent directly to the country *when that country faced exclusion from capital markets*.

Chapter 3

Capital Structure and Innovation

Riskiness

3.1 Introduction

Since the 1970's, US firms that engaged in innovative behavior displayed a marked decrease in their choice of leverage. This is seen clearly in Figure 5.C.1 and is quite robust to both the measure of leverage and the definition of 'innovative'. The change was fairly gradual and observed at both margins, the extensive and the intensive i.e. new cohorts of firms entered with smaller levels of leverage and existing cohorts of firms found it optimal to delever over time*.

In Figure 5.C.1, innovative firms are distinguished from their counterparts by whether or not they engage in R&D in a given year, but we claim that the key difference between these two types of firms is the level of risk in the projects that they undertake. There are both theoretical and empirical reasons to believe that innovative projects are riskier, such as a longer time horizon, a greater asymmetry of information, or simply the fact that R&D investments are in general intangible (see Aghion et al. (2005) or Hall (2002)).

*8.A explores these trends in some detail and documents the robustness of these facts.

We will demonstrate in this paper that when risk is a distinguishing characteristic of innovation projects, we can expect this divergence in capital structures over time. Equity finance is preferred to debt finance for risky projects because of the dividend structures of the two assets i.e. equity has a *convex* payoff structure and debt has a *concave* payoff structure. This notion is not new in the finance literature (See Dewatripont and Tirole (1994) or Boyd and Smith (1999)), but to the authors' knowledge, the literature has yet to explore the implications of this dichotomy for specific, economically relevant project types e.g. innovation.

Given that risk is the relevant characteristic of innovation, there are two potential explanations for the leverage phenomenon we observe: One on the supply-side and one on the demand-side.

On the supply-side there were numerous tax changes and deregulations that could have increased the supply of equity. In 1980, the corporate tax rate fell by 12%, from 46% to 34%. In addition, in 1982 an R&D tax credit of 20% was introduced, which for innovative firms is isomorphic to a corporate tax reduction. Lastly, the effective marginal dividend tax rate fell substantially during this time, from 43% to 17% (See McGrattan and Prescott (2003) for details). Since the interest on debt payments is tax-deductible, a reduction in the corporate tax rate would simultaneously make debt more expensive for the firm and equity cheaper since the supply of funds would increase. The decrease in the dividend tax would further increase the supply of equity and drive down the price of issuances. Since equity is used to fund riskier projects, such tax changes would allow for more risky projects to be undertaken using primarily equity finance.

Further, throughout this period there were manifold deregulations that effectively relaxed constraints on equity financing. For instance, in 1982 the SEC adopted a 'safe harbor' rule that protected firms from manipulation charges when they repurchased their stock, and in 1983 the SEC adopted a 'shelf offering rule' that allowed firms up to a two year window

between the disclosure of the firm's financial condition and the issuance of common stock[†].

Certainly there were financial innovations that reduced the price of issuing debt as well, which would seem to have the opposite effect on leverage. However, Philippon (2012) argues that aggregate efficiency in the financial sector has actually *decreased* over the past century or so. His argument is not that technological innovations in the financial sector have not taken place, but that these innovations have not kept pace with the increasing difficulty of monitoring new types of projects. We take Phillipon's stance here as well in arguing that stationarized monitoring costs likely did not change substantially over time, but that perhaps the *composition* of those costs did. Phillipon's time-series analysis of financial sector inefficiencies does not account for these compositional changes.

The last effect that could have an impact from the supply side is the rise in institutional investors. Institutional investors are large entities, such as insurance companies or pension funds, that actively participate in the market for private equity. Aghion et al. (2013) document that in 1970, these institutions owned 10% of publicly traded equity and by 2006 they owned 60% of publicly traded equity. They argue that this rise is significant for innovative projects because institutional investors can more easily monitor manager activities because there is less difficulty in gathering a consensus vote from shareholders when the number of *individual* shareholders is smaller. We will interpret this rise in institutional investors in the same way that they do: As some sort of reduction in the costs of equity monitoring.

It is important to note that the all of these supply-side mechanisms would not simply suggest that a reduced cost of equity implies lower leverage; this is trivially true. Rather, the significant nuance is that these changes would reduce leverage differentially *across project risk*. Risky, innovative firms would delever more quickly than their safer, more capital-intensive counterparts. The particular reasons why will be discussed shortly.

In addition to these supply-side changes in the availability of equity financing, there are

[†]See Itenberg (2013) for a more complete list of deregulations and innovations.

possible demand-side changes that could cause the leverage trends as well. The leverage trends could simply be due to an exogenously (or endogenously) shifting ‘research frontier.’ It could be the case that the research frontier, which R&D was used to advance, became more complex over time. Thus, the risk associated with an R&D project grew over time. Lanjouw and Schankerman (2004) explore the increasing trends in the R&D to patent ratio, claiming that perhaps there are diminishing returns in the ‘knowledge production function,’ and that more human capital is thus required to produce the same quantity of patents. Jones (2009) argues that inventors face an increasingly difficult burden over time as knowledge accumulates, and thus it is more difficult and requires more specialized knowledge to advance the research frontier. In the context of finance, this would imply greater idiosyncratic project risk resulting from greater asymmetry of information.

We verify this empirically in Section 3.4 using both patent citation volatility and sales growth volatility as proxies for risk, and show that innovative firms have *increased* the degree of project risk undertaken throughout this time period. If this is the case, and these changes are indeed exogenous as the above authors have claimed, then innovation projects are simply becoming riskier and as such will exhibit a greater preference for equity finance over time.

In this paper we employ a new structural model to parse apart quantitatively the contributions of these two forces, the supply-side and the demand-side, to the leverage trends observed in the data. In addition, our model will present financial microfoundations for a large literature regarding technology adoption in economies (see Greenwood and Jovanovic (1990) or Greenwood and Smith (1997)). Many of these works focus on the project-level correlation of risk and return, which we will have as well, but don’t consider the endogenous optimal capital structure of the firm. They show that a greater capacity to undertake ‘high risk/return’ projects via financial development can have a significant impact on economic growth. We will show this to be true as well, but will also provide an explicit

characterization of the *firm's* problem of choosing an optimal capital structure.

Under either of the two theories mentioned, our model certainly suggests that the development of private equity markets and economic development are intimately related, especially in a framework in which innovation drives the endogenous growth of the economy, such as Romer (1986) or Aghion and Howitt (1990). However, understanding which of the theories above is the most relevant or at least most quantitatively important can have significant policy implications. For instance, if supply-side changes can cause the innovative sector to grow by orders of magnitude, then simple adjustments of the targeted tax rates or other deregulations aimed at increasing efficiency of this market can be used to attain a higher rate of growth[‡]. On the other hand, if the leverage trends are simply due to an shifting research frontier, then there is not much to be done from a policy perspective to assist this process save perhaps innovation tax credits and subsidies.

The intuition of our model is as follows: Risk-neutral debt-holders become risk-averse toward the project in which they invest because they absorb the downside of the risk when the firm defaults, but don't see the upside of the risk when the firm does well since their returns are capped; similarly, risk-neutral equity-holders become risk-seeking toward the project because the debt-holders absorb all bad realizations with a default, placing a floor on bad outcome realizations while realizing the full potential of good realizations. Given these forces, when the firm chooses its optimal capital structure subject to the zero-profit condition of its investors, riskier projects will imply lower leverage ratios.

It is important to note that in our model of mixed capital structure, risk enters in two ways: Through the risk-aversion of the debt-holders and the risk-seeking of the equity holders. Many authors have examined models of capital structure in which only one of these two channels is active. For instance, Covas and Haan (2012) consider an RBC model

[‡]Empirical work has found that the efficiency and 'intensity' of private equity markets has lasting implications for economic growth, productivity growth, capital accumulation, and per capita income (See Levine and Zervos (1998) and Rousseau and Wachtel (2000)). Our work provides some structural foundations for these results.

in the style of Bernanke et al. (1999), which suggests that leverage ratios are decreasing in risk; however, equity in that model is simply a costly ‘residual,’ and so the fact that leverage is decreasing in risk is due only to the risk-aversion of the debt-holders. There is no role for active risk-seeking on the part of the equity-holders, which would amplify this effect. In our model we allow for both effects to be at work.

In our model, a reduction in the costs of equity finance, via either deregulation, tax changes, or monitoring cost reductions, will disproportionately affect risky firms, since they optimally choose to use more equity finance. Safe, non-innovative firms, will exhibit a greater preference for debt finance and will be affected little, if at all, by a reduction in these costs. This will happen at the extensive margin as well: Some entirely debt financed-firms will find it optimal to mix their capital structure following a reduction in the costs of equity finance.

The framework of our model will be in the costly-state-verification spirit of Townsend (1979). Unlike Townsend, we will not solve an optimal contracting problem, but instead will take the dividend structure of the securities as fundamentals and find the equilibrium of a dynamic signaling game, much like Hvide and Leite (2008). There will be an equilibrium of this game for every possible capital structure, each with an implied payoff to the firm, and the firm will choose the capital structure that maximizes its expected payoff.

We will demonstrate that, in the absence of tax considerations and asymmetric information, this economy will boil down to a Modigliani-Miller (1958) world in which the optimal capital structure of the firm is indeterminate. We will also show that when financial frictions matter, this framework will induce some firms to choose only debt finance while others will prefer a mixed capital structure based on the investment requirements and associated risk.

The model’s quantitative solution lends insight into the potential causes of the leverage divergence. In the model, both explanations will decrease leverage ratios and interest

rates while increasing project risk at the *intensive* margin; however, the demand-side effect drives down the *extensive* margin i.e. how many firms find it optimal to innovate in the first place. A higher project risk raises the expected costs of intervention under a debt financing scheme. Since a non-trivial fraction of innovative firms choose to be entirely debt-financed[§], those firms will find it optimal to not to innovate and instead to choose the safe project. Increases in the mass of mixed-capital structure firms will not be large enough to offset this fall. With supply-side increases this effect is reversed: Since equity financing effectively becomes cheaper, more firms choose to undertake these risky, equity-financed projects.

Since, in the data, the trends we see are increased innovation at both the intensive and the extensive margin, our analysis suggests that if indeed the risk frontier is changing, which in the context of most endogenous growth models is certainly occurring, then this effect must be complemented by supply-side changes in the financial market as well in order to match the patterns in the data. Our calibrated results indeed demonstrate this to be true. We find that equity issuance costs must have fallen by 31.2% while idiosyncratic project risk only increased by 4.7%.

The rest of the paper is organized as follows. Section 3.2 places our work in the context of the literature; Section 3.3 presents and characterizes the theoretical model; Section 3.4 documents empirically the relationship between capital structure and risk, especially as it pertains to innovation, and also demonstrates that the risky activity of innovative firms has actually increased as well; Section 3.5 uses the model to investigate the plausibility of the theories put forth and discusses the quantitative implications of the model; and Section 3.6 concludes.

[§]We will discuss why this is when characterizing the model, but the intuition is simple: Townsend (1979) demonstrated that debt contracts minimize monitoring costs. Thus, equity finance is sub-optimal. For low-risk projects, firms will gain little from the increased willingness to pay of risk-seeking equity holders and will find it optimal to instead will choose to minimize monitoring costs with a debt-type contract.

3.2 Relation to Literature

Our work is primarily motivated by strong, currently undocumented trends in the data. There have been several papers that have explored the global rise in corporate saving, such as Armenter and Hnatkowska (2012) and Karabarbounis and Neiman (2012), but to our knowledge our work is the first to consider the *difference* in these trends across firm types e.g. innovative versus non-innovative firms. Our approach in understanding these trends is in the tradition of at least three distinct literatures.

Our model is grounded in the finance literature originating with Townsend (1979), Gale and Hellwig (1985), and Williamson (1986) among others that emphasizes the role of asymmetric information as the key financial friction in otherwise smoothly operating competitive markets. The asymmetry of information can be dissipated, but at a cost. We do not, however, adopt an optimal contracting approach with an incentive constraint, as is commonly done in this literature. Instead, we consider a signaling game. This approach was first taken by Leland and Pyle (1977) and followed by others such as Grinblatt and Hwang (1989) and Bajaj et al. (1998) in their exploration of equity finance, though we will consider a vastly different game from theirs.

We will explore in some depth the significance of mixed capital structures, which has been done before in this finance literature. For instance, Aghion and Bolton (1992) and Dewatripont and Tirole (1994) develop models of incomplete contracts in which the firm's choice of leverage can be used as an additional mechanism to implement the optimal contract. Boyd and Smith (1999) also argue that a mixed capital structure can implement an optimal contract when the firm has access to multiple technologies, the returns to only some of which are unobservable. Our model will abstract from the contracting problem considered by these authors and instead focus on an equilibrium outcome, since we are not interested in justifying the *coexistence* of debt and equity, as these authors were, but rather

exploring their *implications* for risky projects in response to several proposed exogenous changes. For our purposes, we are willing to take their joint coexistence, and indeed their joint optimality, as given.

Our model will differ from the models of both Dewatripont and Tirole (1994) and Boyd and Smith (1998) not only in methodology, but also in implication. For instance, in the Dewatripont-Tirole paradigm, increased project risk will imply *higher* choice of leverage, since leverage is a disciplinary tool used to incentivize managers and this incentive problem becomes more pronounced as the project risk increases. In our model, increased project risk will imply lower leverage ratios, since more equity issuance is a less costly method of financing for these risky firms. In the Boyd-Smith model, equity financing is used primarily to finance projects whose returns are low and publicly observable, and is thus used more as high-returns from unobservable debt projects are driven down from increased project demand. Thus this model suggests that equity financing is more frequently employed for projects with *less* asymmetric information. On the contrary, in our model, equity finance will be more frequently employed for projects with greater risk, which translates in our model to greater adverse selection problems.

We follow most closely a literature that seeks to remedy the incentive problems associated with Townsend's basic model. The problem is that ex post intervention costs may be sufficiently high that the creditor may wish to accept a haircut instead of intervening, even though ex ante incentive constraints are satisfied. Boyd and Smith (1994) lift the assumption of the contractibility of intervention and allow for randomized interventions and find that contracts in this scenario are more plausible. Gale and Hellwig (1989) do this as well, but also seek randomized intervention that is stable in a strategical sense, noting that in some frameworks there may be a trade-off between stability and efficiency.

The model we propose is most closely related to Hvide and Leite (2008), who set up a dynamic finance game with both debt and equity securities and find assumptions under

which a mixed capital structure is stable and optimal. Our finance game is essentially the same as theirs, but the equilibria that we find and analyze are different. They analyze the Perfect Bayesian Equilibrium of the game and are most interested in its stability and relationship to the literature; on the other hand, we weaken our equilibrium notion to a Weak Perfect Bayesian Equilibrium and in exchange get more empirically relevant shareholder behavior in the case of absolute priority violations[¶] as well as a demonstrable preference for risk on the part of the share-holders.

Besides the finance literature, our work is related to a large body of work on endogenous growth. This literature, which started with Romer (1986) and Aghion and Howitt (1990), seeks to understand the precise mechanisms by which economies grow and the factors that influence their growth rates. It emphasizes the role of innovation as the driver of growth. Our work, at least empirically, is quite concerned with the dynamics of innovation as well, and indeed many of our empirical results are derived from the NBER patent database. However, this literature is concerned mostly with the positive externalities and strategic dynamics associated with innovation itself and places little emphasis on the relevance of its finance. Some key exceptions are King and Levine (1993b) and Itenberg (2013).

Lastly, our work is related to a large literature on the joint dynamics of economic and financial development. This literature documents empirically strong correlations between measures of financial development and economic growth and posits theoretical models to explain this phenomenon. Classic examples of this literature are Greenwood and Jovanovic (1990), King and Levine (1993a), Levine and Zervos (1998), and Rajan and Zingales (1998). This literature often employs optimal contracting techniques in their treatment of financial markets.

There has been work in this literature similar to ours in that it tries to explain the distinct

[¶]An Absolute Priority (AP) Violation occurs when a firm defaults on its debt obligations, which should suggest that the creditors seize ownership of the firm and yet the shareholders still retain some ownership or control rights.

properties of equity and debt that lead to financial development. Boyd and Smith (1998) develop a model with two different technologies, one of which is optimally financed with debt and another optimally financed with equity, and show that there is a threshold level of income below which debt is the only method of finance employed and above which an equity market grows. Blackburn et al. (2005) show that a debt-equity mixture can be optimal if the optimal contracting problem faces enforcement constraints along multiple dimensions. To our knowledge, however, our work is the first to explore in a macrofinancial context the intimate relationship between risk and equity finance and its implications for optimal project choice and growth.

3.3 Model

In this section we present a theoretical model that will allow us to explore in depth the financial market changes proposed in the introduction.

We consider a unit mass of overlapping, heterogeneous firms, each of which has access to two distinct projects, a safe one and a risky one. Each incur the same investment cost, I ^{||}. The firm must decide which project to pursue and can only pursue one. Firms are heterogeneous along the level of risk and the absolute return of the safe project. For firm $i, j \in [0, 1] \times [0, 1]$, the risky project has a mean given by \bar{x} and a variance σ_i^2 , while the safe project yields a certain return of $\gamma_j \bar{x}$, where $\gamma_j < 1 \forall j \in [0, 1]$. Suppose that the outcome of the risky project is continuously distributed and bounded in $[x_L, x_H]$, with positive density on the entire domain.

^{||}This assumption can be relaxed. We can assume that the firms enter with some net worth, n , and that the project costs some $K > n$. We then simply take $I = K - n$ i.e. the portion of the project that requires external finance.

3.3.1 Financing Game

Firm ij has only the technology, and requires external finance to cover the cost of investment, I . Upon choosing the project, the firm must make use of either or both of two financing technologies, debt and equity, to undertake its project. Investors of both types know the type of the firm, ij , but not the realization of the risky project, x , which only the firm observes. We assume a risk-free interest rate of 0 and that investors are competitive and thus pushed to zero net returns.

We assume for now that safe projects are financed with debt. Since there is no risk, there is no meaningful distinction between debt and equity finance, but we will show later that the optimal capital structure in the limit as we take the variance of a project to zero involves no outside equity. To model the financing of the risky project, we consider the CSV debt-equity game of Hvide and Leite (2008), henceforth H&L.

In this game, a capital structure consists of (D, β) , where D is the promised repayment to the debtors and where the shareholders are entitled to a fraction β of the firm's residual earnings. Because of the asymmetry of information, investors must pay a cost to learn of the project realization. For debt-holders, this cost is c_D and for equity-holders this cost is c_E . The monitoring cost is assumed to be paid by the firm**. From now on, the action of monitoring will be referred to as an 'intervention.'

Given any capital structure (D, β) , the financing game proceeds as follows^{††}:

1. The firm learns of the realization of the project, x
2. The firm decides how much to repay its debtors
 - (a) If the firm decides to pay back all of its debt, D , then
 - i. The firm decides how much of the residual earnings to issue as dividends

**This assumption is tautological for debt, and for equity the propositions can be generalized. See Hvide and Leite (2008) for further discussion.

††Note that an intervention always ends the game since the asymmetry disappears.

- ii. The shareholders decide whether or not to intervene
- (b) If the firm decides to default partially or wholly, then
 - i. The debt-holder decides whether to intervene or accept the haircut
 - A. If the debt-holder accepts the haircut, then the firm decides how much of the residual earnings to issue as dividends (absolute priority violation)
 - B. The shareholders decide whether or not to intervene

The game is described visually in Figure 5.C.2. The firm's, debt-holders', and shareholders' strategies are black, red, and blue respectively.

One nice property of this model is its behavior in the absence of financial constraints, as we will see in Proposition 3.3.1.

Proposition 3.3.1 (Modigliani-Miller Equivalence) *If $c_D = c_E = 0$, then the optimal capital structure of the firm is indeterminate.*

The proof is given in 8.C. The intuition for Proposition 3.3.1 is simple. If the firm is entirely financed with debt, then its repayment will be $D = I$ and its payoff will be $Ex - I$. If the firm wishes to start issuing equity to reduce its debt, then the amount by which it reduces its debt obligations will be exactly offset by the requisite dividends it must issue, since shareholders are compensated exactly as the firm is.

3.3.2 Equilibrium in the Financial Game

We use backward induction to characterize the equilibrium.

Equity Holders (No AP Violation)

Suppose that the debt obligations have been repaid in full. This implies that the firm has a value given by $x - D$ and that outside equity holders hold a claim to a fraction β of

these cash flows. We consider an equilibrium in which the firm's strategy is to issue a dividend of size $\tilde{E} = \beta(x - D - c_E)$. The benefit of this strategy is that it always renders the shareholder indifferent between monitoring and not. And so the equity holder's problem does not change from the original H&L model i.e. the monitoring strategy that induces truth-telling on the part of the firm is characterized by a differential equation whose solution takes the form:

$$P_E(\tilde{E}) = e^{\frac{\tilde{E}}{c_E} + \kappa}$$

To choose the constant of integration, we set κ such that the acceptance probability is 1 at $E_H = \beta(x_H - D - c_E)$. This is important for the firm's incentives. To see why, suppose that $P_E(E_H) = \xi < 1$. Then a firm of type x_H could always offer a small $\tilde{E} = E_H + \varepsilon$ for some small ε . Doing so would guarantee acceptance for an arbitrarily small ε since the shareholder would have no incentive to intervene, since the largest claim it can attain by doing so is E_H . Thus, an arbitrarily small ε 'bribe' can induce a discrete change in the probability of acceptance given by $1 - \xi > 0$. We prevent this by choosing κ such that a type E_H firm already faces a probability 1 of acceptance. Doing so yields

$$P_E(\tilde{E}) = e^{\frac{\tilde{E} - E_H}{c_E}}$$

For reasons that will be apparent later, it is convenient to write this expression as

$$P_E(\tilde{E}) = \eta(D, \beta) e^{\frac{\tilde{E} - E_L}{c_E}} \quad (3.1)$$

where $E_L = \beta(c_D - c_E)$ and so

$$\eta(D, \beta) = e^{\frac{-\beta(x_H - D - c_D)}{c_E}} < 1 \quad (3.2)$$

The function $\eta(D, \beta)$ will play an important role in guaranteeing the continuity and convexity of the shareholder's payoff.

The AP Violation Subgame and Weak Perfect Bayesian Equilibrium

The key difference between our equilibrium and H&L lies in the treatment of the subgame in which an absolute priority violation occurs i.e. the firm defaults wholly or partially but nevertheless the creditors accept the payment and shareholders still have some control over the firm.

We assume at this point that the value of the firm in this subgame is c_D , and will verify this with equilibrium behavior in the debt subgame. H&L find a Perfect Bayesian Equilibrium of the game by assuming that shareholders accept a payment of $E_L = \beta(c_D - c_E)$ with probability 1 and any other payment with probability 0. An unfortunate result of doing this is that the debt-holder behavior supporting this behavior generates a discontinuity in the equity payoff function. This results from the necessity of the firm's payoff being continuous at the point $D + c_D$, which requires the debt-holder's monitoring strategy to be discontinuous at this point.

In our equilibrium, we assume that in this subgame shareholders accept a payment E_L not with probability 1 but with probability $\eta(D, \beta) < 1$, where $\eta(D, \beta)$ was derived in the subgame with no AP violation. The shareholder will receive a payment of E_L whether it monitors or not, so it is indifferent between monitoring strategies, and the firm will always be better off having some chance of not being monitored, even if $\eta(D, \beta)$ is arbitrarily small.

The beliefs that support this behavior are not sequentially rational, and so the equilibrium is not Perfect Bayesian, but Weak Perfect Bayesian. We will later discuss this issue in more detail and also elaborate on the manifold benefits that come from our specification relative to H&L.

Debt-Holders

Having characterized the equilibrium in the latter two subgames, we now consider the debt subgame. We assert that the strategy of the firm is to either repay or offer $\tilde{D} = x - c_D$. The advantage of this strategy is that, as in the equity subgames, the incentives of the bank to monitor are trivial. In the case of full repayment, D , the bank will never monitor since it can claim at most D by doing so. In the case of default, the bank is rendered indifferent between monitoring and not monitoring, since if it monitors it will get $x - c_D$, which is exactly the payment it receives.

We now simply need to find a strategy on the part of the bank that gives the firm the appropriate incentives. Let $P_D(\tilde{D})$ be the probability that the bank accepts a default offer \tilde{D} . Then this function must induce the firm to reveal truthfully, i.e. the problem

$$\max_{\tilde{D}} P_D(\tilde{D}) [\eta(x - \tilde{D} - \beta(c_D - c_E)) + (1 - \eta)(1 - \beta)(x - \tilde{D} - c_E)] + (1 - P_D(\tilde{D})) \times 0$$

must yield as it's unique solution $\tilde{D} = x - c_D$. A FOC(\tilde{D}) along with this substitution yields a differential equation with a solution characterized by

$$\rightarrow P_D(\tilde{D}) = e^{\frac{1-\beta+\eta\beta}{[\eta c_E + (1-\beta)(c_D - c_E)]} \tilde{D} + \kappa}$$

We choose the constant of integration such that if the firm repays the full debt, D , the debtholder accepts with probability 1 to prevent the sort of 'bribing' behavior described in the equity subgame. Thus the bank's monitoring strategy is

$$P_D(\tilde{D}) = e^{\frac{1-\beta+\eta\beta}{[\eta c_E + (1-\beta)(c_D - c_E)]} (\tilde{D} - D)} \quad (3.3)$$

At the left-tail of the cash flow distribution, x_L , we specify that $P_D(\tilde{D}) = 0 \forall \tilde{D} < x_L - c_D$.

Thus, if a firm plays the equilibrium strategy at x_L , its payoff will be $c_D P_D(x_L - c_D) > 0$, which is the payoff from deviation to no payment.

Results

Having described the equilibrium of interest, we now provide some interesting results.

Theorem 3.3.2 *For any mixed capital package (D, β) , there exists an \hat{x} such that if $x_H > \hat{x}$, then a unique, weak perfect Bayesian equilibrium (WPBE) exists of the following type.*

η is a constant given by

$$\eta = e^{-\frac{\beta(x_H - D - c_D)}{c_E}}$$

In the debt subgame, the strategies are:

$$\tilde{D}(x) = \begin{cases} x - c_D, & x \leq D + c_D \\ D, & x > D + c_D \end{cases}$$

$$P_D(\tilde{D}) = \begin{cases} 0, & \tilde{D} < x_L - c_D \\ e^{\frac{1 - \beta + \eta\beta}{[\eta c_E + (1 - \beta)(c_D - c_E)]}(\tilde{D} - D)}, & x_L - c_D \leq \tilde{D} < D \\ 1, & D \leq \tilde{D} \end{cases}$$

In the equity (no AP violation) subgame, the strategies are:

$$\tilde{E}(x) = \beta(x - D - c_E)$$

$$P_E(\tilde{E}) = \begin{cases} 0, & \tilde{E} < E_L \\ \eta e^{\frac{\tilde{E} - E_L}{c_E}}, & E_L \leq \tilde{E} < E_H \\ 1, & E_H \leq \tilde{E} \end{cases}$$

In the equity (AP violation) subgame, the strategies are:

$$\tilde{E} = \beta(c_D - c_E)$$

$$H_E(\tilde{E}) = \begin{cases} \eta, & \tilde{E} = \beta(c_D - c_E) \\ 0, & o/w \end{cases}$$

Again, the proof is given in 8.C. Before we discuss this theorem, we can infer a trivial corollary demonstrating the risk-aversion of the debt-holders.

Corollary 3.3.3 *The debt payoff function is concave in the cash flows.*

Notice that, unlike H&L, our equilibrium is *not* sequentially rational (thus, the equilibrium found in H&L is unique in its class, as they claim). This is because of the assumption made on the off-the-equilibrium-path beliefs in the AP violation subgame. If the firm deviates from paying the dividend specified by the equilibrium during an AP violation and instead tries to offer ε more to guarantee acceptance, the shareholder interprets this as an enormously successful firm and slams it with an intervention.

The equilibrium described in Theorem 3.3.2 thus requires the existence of potentially large realizations, but fortunately there is no requirement on their relative likelihood. Since the beliefs are off the equilibrium path, they could even have probability 0 provided they are in the domain since they need not be reinforced by Bayes' rule.

This belief system, however, is not consistent with a sequence of 'nearby' mixed strategies. If, from time to time, the firm in this subgame randomly paid a dividend slightly higher than the one specified in the equilibrium, then the shareholder would accept for certain, knowing that the firm's type hasn't changed and thus he's getting a slightly higher payoff by not monitoring.

There are significant benefits to weakening our equilibrium concept relative to H&L. Though in a theoretical sense the shareholder's strategy is myopic but unstable, the prac-

tical implication is that the shareholder intervenes in times of financial distress. In a more intricate theoretical model with an explicit contracting problem between the owner and the firm manager, we would certainly expect this and our equilibrium can proxy for this sort of behavior. Further, there is empirical support for shareholder intervention during financial distress. Gilson (1989) demonstrates that, during this time period, 52% of financially distressed firms experienced senior-level management change^{††}, and only direct intervention by bank lenders only accounts for 21% of these changes. Wruck (1990) demonstrates that financial distress provides a mechanism to initiate top-management changes.

Further, in our equilibrium, unlike H&L, we can demonstrate the following proposition using a fairly benign assumption.

Proposition 3.3.4 *If $c_E \in [c_D/2, c_D)$, then the equity payoff function is continuous and convex.*

Once again, the proof is provided in the 8.C. The reason why the lower bound on the equity monitoring costs are needed is to ensure that the marginal returns to equity in the AP Violation subgame are not excessively high at the point of repayment. High marginal returns here would induce a kink with a ‘tent’ shape in the equity payoff function, which would clearly not be convex. Figure 5.C.3 shows graphically the payoff to equity under the assumptions given in Proposition 3.3.4. From this figure it is also trivial that the debt payoff function is concave, since it is linear (and upsloping) in the region of default and fixed in the region of repayment. The curvature of these payoff functions has significant implications for capital structure choice as a function of risk.

Notice further that our separating equilibrium satisfies the single-crossing condition Spence (1973) put forth as necessary for a separating equilibrium to exist. It can be shown that

^{††}This can be compared to only 19% when the firm is not financially distressed.

$$\frac{\partial^2 \hat{x}}{\partial \tilde{E} \partial x} \propto -\frac{\beta}{c_E + \beta(x - D - c_E) - \tilde{E}} < 0$$

which is a sufficient condition for single-crossing i.e. the indifference curves in *Dividend* \times *Perceived Type* space intersect only once and grow flatter as types increase. Intuitively this means that it is less costly for high-type firms to issue larger dividends to make their types known than it is for low-type firms. The indifference curves in the debt subgame look similar, since the beliefs are also linear.

3.3.3 Firm's Optimal Capital Structure Choice

Having characterized the financing game, we now explore the firm's problem. The risk-neutral firm maximizes its expected payoff, which are cash flows net of debt and dividend payments, subject to the constraint that the expected payoff of the investors is zero. Thus, in the absence of taxation, the firm's problem is given by the following program:

$$\begin{aligned} \max_{\beta \in [0,1], D \geq 0} & \int_{x_l}^{D+c_D} [\eta(D, \beta)c_E + (1 - \beta)(c_D - c_E)] e^{\frac{1-\beta(1-\eta(D,\beta))}{\eta(D,\beta)c_E+(1-\beta)(c_D-c_E)}[x-c_D-D]} f(x) dx \\ & + \int_{D+c_D}^{x_H} \left[(1 - \beta)(x - D - c_E) + \eta(D, \beta) e^{\frac{\beta(x-D-c_D)}{c_E}} c_E \right] f(x) dx \end{aligned}$$

s.t.

$$\underbrace{\int_{x_L}^{D+c_D} \beta(c_D - c_E) e^{\frac{1-\beta(1-\eta(D,\beta))}{\eta(D,\beta)c_E+(1-\beta)(c_D-c_E)}[x-c_D-D]} f(x) dx}_{\text{Default Equity Payoff}} + \underbrace{\int_{D+c_D}^{x_H} \beta(x - D - c_E) f(x) dx}_{\text{Repayment Equity Payoff}} +$$

Equity Payoff

$$\underbrace{\int_{x_L}^{D+c_D} (x - c_D) f(x) dx}_{\text{Default Debt Payoff}} + \underbrace{\int_{D+c_D}^{x_H} D f(x) dx}_{\text{Repayment Debt Payoff}} \geq \underbrace{I}_{\text{Investment Size}}$$

$$\underbrace{\hspace{10em}}_{\text{Debt Payoff}}$$

$$\eta(D, \beta) = e^{-\frac{\beta(x_H - D - c_D)}{c_E}}$$

It is important to note that we are not technically guaranteed a solution to this problem. Theorem 3.3.2 demonstrated that for any capital structure, there was a sufficiently high cash flow realization such that the equilibrium of the finance game existed. If we take x_H as a non-changing fundamental though, then there may be very small levels of β for which the shareholders' beliefs in the subgame with an AP violation can never be justified i.e. the shareholders have a claim to such small fraction of the cash flows that even if the cash flows were enormous it is not worth it to intervene when a high dividend 'bribe' is offered to guarantee acceptance.

In the quantitative work, as we will show, this is never a problem. It is never optimal for the firm to issue such minute quantities of external equity. We will show that this model tends to generate IPOs quite naturally. Firms prefer debt finance for safer projects since they can avoid paying the dilution costs associated with equity, but at a certain point the risk-seeking equity investors become so willing to invest in this project that they pay a large premium and the firm issues a massive amount of external equity all at once while simultaneously deleveraging.

Note also that we abstract from explicitly incorporating tax policy into the firm's problem. During the period of interest there were three main tax changes: The corporate tax rate fell, the effective marginal dividend tax rate fell, and, substantially later, the capital gains tax fell. We will interpret tax changes over this time as simply a reduction in the cost of equity issuance, c_E , since they will all work to simply increase the supply of private equity since they increase the post-tax returns to equity investment. This interpretation only grows

more complicated if we allow for a tax advantage to debt, since we would need to account for this policy explicitly in the firm's incentives and this becomes difficult quickly. Thus, we ignore for simplicity this possibility, since it would only strengthen our results anyway by increasing the relative cost of debt to equity.

Characterizing the Optimal Solution

Though this problem is quite complicated, there is some simple intuition regarding its solution. The solution as a function of the risk of the project, σ will be of the form

$$V(\sigma) = \max\{V_c(\sigma), V_i(\sigma)\}$$

where $V_c(\sigma)$ is the level of utility when no external equity is issued i.e. the corner solution and $V_i(\sigma)$ is the level of utility when the marginal benefit of external equity issuance is equated to its marginal cost i.e. the interior solution. We can rest assured that there will not be a corner solution when $\beta = 1$ since debt contracts minimize monitoring costs (See Townsend (1979)) and so at least *some* debt will be used in any optimal solution.

The corner problem is quite easy to characterize, since it simply becomes

$$\begin{aligned} \max_{D \geq 0} \quad & \int_{x_L}^{D+c_D} c_D e^{\frac{x-D-c_D}{c_D}} f(x) dx + \int_{D+c_D}^{x_H} (x-D) f(x) dx \\ \text{s.t.} \quad & \int_{x_L}^{D+c_D} (x-c_D) f(x) dx + \int_{D+c_D}^{x_H} D f(x) dx \geq I \end{aligned}$$

The left-hand side of the budget constraint is strictly increasing in D , and so if a solution to this problem exists i.e. if I is not too large, then it is unique and given by solving the participation constraint. Let the solution to this problem be $D^*(\sigma)$ for a given level of risk, σ . We can establish a simple proposition by starting with the following lemma:

Lemma 3.3.5 *Suppose a solution to the corner problem exists. Then $D^*(\sigma)$ is strictly increasing in σ .*

Proof The bank's payoff is both *strictly increasing* in D and *strictly concave* in x . Increasing the risk thus *strictly decreases* the bank's payoff, and so D must strictly increase to meet the participation constraint. ■

Before we consider how the value function responds to risk changes, we must establish one more lemma. First, we will define carefully how we will explore risk. Let F be a distribution over the cash flows with a mean, \bar{x} . We say that the distribution G is **symmetrically riskier** than F if and only if

$$\begin{cases} F(x) < G(x), & x < \bar{x} \\ F(x) > G(x), & x > \bar{x} \end{cases}$$

where \bar{x} is the mean of both distributions. Note that this is the same definition of risk that is used in Dewatripont and Tirole (1994). Now, let us define

$$\hat{F}(x) = e^{\frac{x-D-c_D}{c_D}} F(x)$$

where D is some arbitrary positive number. The following is trivial:

Lemma 3.3.6 *$G(x)$ is symmetrically riskier than $F(x)$ if and only if $\hat{G}(x)$ is symmetrically riskier than $\hat{F}(x)$.*

And the following follows from the definition of second-order stochastic dominance^{§§}.

Lemma 3.3.7 *If $\hat{G}(x)$ is symmetrically riskier than $\hat{F}(x)$, then $\hat{F}(x)$ second-order stochastically dominates $\hat{G}(x)$.*

^{§§}In particular, it follows from the fact that if the mean is preserved between the two distributions, a risk-averse agent will strictly prefer the distribution with less risk.

With these lemmas in hand, we now consider how the value function will respond to risk changes. We can now establish the following proposition:

Theorem 3.3.8 *Suppose that G is symmetrically riskier than F . Then $V_c(\sigma_G) < V_c(\sigma_F)$.*

Proof First, we note that we can substitute in the participation constraint to derive the following expression for the value function.

$$V_c(F) = Ex - I - \int_{x_L}^{D_F+c_D} c_D \left(1 - e^{-\frac{x-D_F-c_D}{c_D}}\right) f(x) dx$$

In words, since the bank receives a payoff of zero in expectation, the firm essentially pays the investment plus any monitoring costs incurred by the bank. Now we can consider the difference in value functions below:

$$\begin{aligned} V_c(F) - V_c(G) &= - \int_{x_L}^{D_F+c_D} c_D \left(1 - e^{-\frac{x-D_F-c_D}{c_D}}\right) f(x) dx + \int_{x_L}^{D_G+c_D} c_D \left(1 - e^{-\frac{x-D_G-c_D}{c_D}}\right) g(x) dx \\ &= c_D \left[\int_{D_F+c_D}^{D_G+c_D} \left(1 - e^{-\frac{x-D_G-c_D}{c_D}}\right) g(x) dx + \right. \\ &\quad \left. \int_{x_L}^{D_F+c_D} \left[\left(1 - e^{-\frac{x-D_G-c_D}{c_D}}\right) g(x) - \left(1 - e^{-\frac{x-D_F-c_D}{c_D}}\right) f(x) \right] dx \right] \\ &> c_D \left[\int_{x_L}^{D_F+c_D} \left[\left(1 - e^{-\frac{x-D_F-c_D}{c_D}}\right) g(x) - \left(1 - e^{-\frac{x-D_F-c_D}{c_D}}\right) f(x) \right] dx \right] \\ &= \int_{x_L}^{D_F+c_D} (\hat{G}(x) - \hat{F}(x)) dx \geq 0 \end{aligned}$$

The second to last line follows because $D_F < D_G$. The last line follows from integration by parts and the last inequality follows from second-order stochastic dominance. ■

Theorem 3.3.8 is significant but not a new notion in the finance literature. It says that the debt-only value function is strictly decreasing in risk. This is because riskier projects

will place more weight on the tail realizations where monitoring is necessary, and since monitoring costs drive the real interest rate that comes out of these debt contracts, riskier projects will imply higher interest rates and ultimately lower payoffs.

This stands in sharp contrast to the case of a mixed capital structure. Although we have not been able to formulate a theoretical proof, quantitative results suggest that risk has the *opposite* effect on firm payoff in the case of mixed capital structure. That is, as the risk increases, the firm's payoff increases. This is because the equity holders are willing to pay a premium for the risk and so, for any given fraction of external equity issued, the firm does not need to issue as much debt to cover the costs of investment. This effect is strong enough to counteract the reduced payoff of the debt-holders that both comes from a lower D and from greater risk.

Given these trends, we can begin to characterize finance behavior as a function of risk. For very low levels of risk, the firm gains little from the risk premium coming from the equity holders; as such it is better off by choosing the debt-only solution, which minimizes monitoring costs. As the risk increases, however, the payoff from using only debt falls and the payoff from mixing rises. Thus, at some point the firm IPOs and issues away, all at once, a substantial fraction of its equity. This can be seen in the quantitative results that we will see in Section 3.5, particularly in Figures 5.C.7 and 5.C.8.

These theoretical results support a 'pecking-order' theory of finance as a function of risk. Firms will first exhaust any retained earnings or net worth when they require funds for investment, since there is no costly informational friction associated with this. If the project is only modestly risky, then the firm will fund the project using debt alone, since this minimizes expected monitoring costs. It is only for riskier projects that equity begins to be used, and even when it is, debt is used as well.

3.4 Empirics

In this section, we investigate the trends in the data that will be of relevance in analyzing the implications of the theoretical model just described.

To conduct the empirical analysis, we construct a dataset consisting of an unbalanced panel of public manufacturing firms between the years 1976-2005, drawing information from two sources: Standard and Poors Compustat and the NBER Patent Database. Table 8.A.1 in 8.A provides some of the summary statistics for the dataset. All nominal variables are adjusted by the GDP deflator in the corresponding year and expressed in millions of 2005 US dollars. Definitions of the data items used in the empirical analysis are outlined in 8.A. What follows is a description of these data sources.

Standard and Poor's Compustat Database: The S&P Compustat North America files contain financial statements for publicly traded firms from the 1950's until the present day. We restrict the analysis to domestic^{¶¶} manufacturing firms^{***} in the dataset between 1976-2005^{†††}. From this database, we use firms' annual income statements, balance sheet, and cash flow statements to gather information on industry, firm size, R&D and capital expenditures, and capital structure. We drop observations for which sales are nonpositive or total stockholders' equity is negative.

One point that is worth discussing is that the sample of Compustat firms is one subject to selection bias and thus has changed significantly over the period in focus. Figure 8.A.1 in 8.A illustrates this point. The first graph shows that the number of firms has varied throughout the sample period. Of particular interest is the surge in IPO activity in the

^{¶¶}Domestic is defined as having headquarters in the United States.

^{***}Manufacturing firms are defined as those with 2-digit industry SIC codes between 20 and 39.

^{†††}We start the sample in 1976 because that is the starting year for the NBER patent data. In addition, we ensure consistency in the accounting reporting of R&D as the Financial Accounting Standards Board issued a uniform standard for reporting R&D expenditures in June 1974. We end our analysis in 2005 as we would like to abstract from effects of the financial crisis in 2008.

early 1990's and its subsequent drop^{†††}. The second graph in this figure illustrates how the composition of firms have changed over time across a subset of the most prominent sectors in the sample. An example of such a shift is the sharp increase in the share of firms in the Chemicals industry, mostly driven by the rise of Drugs (SIC 283) firms. The last panel shows how the distribution of size changed over time. The distribution of sales seems to have gotten more dispersed, with a higher fraction of small firms in the sample. Throughout the empirical analysis section, we will explore how these compositional changes affect the capital structure and innovation trends.

NBER Patent Database: The NBER Patent Database contains information on all utility patents granted by the United States Patent and Trademark Office (USPTO) in the period 1976-2006. A patent is a type of intellectual property which gives its owner monopoly rights to commercialize an invention in exchange for public disclosure. A patent in the database includes information about characteristics of the inventor, the technology class of the innovation, and the legal claims which are covered by the patent. In addition, it is possible to link patents through citations and for each patent to compute statistics regarding forward citations, which can be used to proxy for patent quality. A detailed description of this dataset can be found in Hall et al. (2001). We use the application year as the relevant date for the patent as it corresponds more closely to when the innovation was actually created and developed. Since there is usually a time lag between when the patent is applied for and when it is granted, there is a truncation issue in the sample of granted patents. An example of this could a patent which was applied for in 2004 but not granted until 2007. Although this patent should be included when in the empirical analysis, it is missing from the patent data. In addition, forward citations of patents granted later on in the period are subject to the truncation bias described in Hall et al. (2001). To mitigate these issues, we

^{†††}[Reference something here with regard to this].

restrict the data to the period 1976-1997 when looking at patent statistics.

A crucial feature of this database is the ability to match to the Compustat firms, which allows us to construct patent portfolios for those public manufacturing firms that were granted a patent in this period. We use the subset of observations from this database which are matched to the sample of Compustat firms.

In the rest of the empirical section, we will isolate trends in the data that will be useful when we return to quantitatively characterize the theoretical model.

3.4.1 R&D Trends

In the introduction, we showed that trends in firm capital structure diverge by innovative behavior. In this section, we outline the facts associated with the rise in the undertaking of innovative activity over the period among the firms in our sample. The first panel of Figure 5.C.4 shows the *intensive* margin of these trends. In particular, the mean and median R&D composition of investment among all of the firms who reported positive R&D in this sample undoubtedly rose. The second panel of Figure 5.C.4 demonstrates that there has also been an increase in the share of firms reporting positive R&D expenditures in a particular year i.e. the *extensive* margin, rising from 55 percent to almost 75 percent. This is a key empirical fact that will be quite important when we analyze the model's predictions.

In the third panel of Figure 5.C.4, we show further evidence that the changes in innovation investment occurred at both margins. This panel shows that the upward trend in R&D composition of investment has been prevalent in each cohort within our sample, even for those firms which had already become public by 1976. Notice, though, that younger cohorts invest relatively more in innovation than their older counterparts.

Thus, the data suggest that whatever structural changes took place in the innovative (or financial) sector increased innovative behavior at *both* the intensive and extensive margins.

This is the relevant take-away for the analysis of the model’s implications.

3.4.2 Innovation Risk Trends

In this section, we demonstrate that the kind of innovation that these firms undertook has also changed. In particular, we document an increase in the risk associated with investing in innovation.

We first note that there has been an increase in the dispersion of sales growth among innovative firms. In the second panel of Figure 5.C.5, we plot the median standard deviation of sales growth within the firm. In order to construct this metric, we follow Comin and Philippon (2005) and define within firm volatility as:

$$\hat{\sigma}_{i,t} = \left[\frac{1}{5} \sum_{\tau=-2}^{\tau=2} (\gamma_{i,t+\tau} - \bar{\gamma}_{i,t})^2 \right]^{\frac{1}{2}}$$

where $\gamma_{i,t+\tau}$ is firm i ’s sales growth in period t and $\bar{\gamma}_{i,t}$ is the 5-year rolling average of sales growth for this firm. The resulting trends illustrate a steep increase in the idiosyncratic volatility of innovative firms over their counterparts. We also verify in 8.A that these trends are present when we look at the growth in labor productivity as opposed to sales.

The first panel of Figure 5.C.5 displays the dispersion of sales growth in the cross-section. We compute the standard deviation of sales growth for each year among innovative and non-innovative firms separately^{§§§}. Although there seems to be a rise in volatility for all firms, the increase is much sharper if we restrict the sample to only innovative firms. Even though this trend is not as strong as the within-firm measure, it is useful for our purposes because it is a measure we will be able to map into our theoretical model directly^{¶¶¶}.

Next, we look at the patents granted to the firms in the sample to verify that there has

^{§§§}In the first panel of Figure 5.C.5, we exclude top and bottom 5 percent of sales growth observations in each year.

^{¶¶¶}Since firms in our model are short-lived, we have no way of computing a similar measure of within-firm volatility, although we will certainly show that the volatility of projects undertaken increases on average.

been an increase in dispersion associated with the value of these patents. We employ two metrics of quality: The first is the number of forward citations that each patent receives and the second is the number of patent claims. Hall et al. (2001) and Lanjouw and Schankerman (2004) show that patent citations and claims act as good proxies for innovative quality and economic importance. The first and last panels in Figure 5.C.6 show that the standard deviation of both measures of patent quality has increased over the from 1976-1997. The middle panel shows that the share of patents receiving no citations has also increased. In order to verify that the increase in the standard deviation in patent citations is not driven by the surge in patenting activity in the 80's and 90's, we adjust the standard deviation in patent citations by the average patent citations in each year and report the results in 8.A. These facts suggest public manufacturing firms have been investing in progressively riskier innovation projects over this time period.

At this point, we do not take a stance on causality. Seeing this increase in the risk could be explained by either of our two stories: We could take it as an indication that the degree of idiosyncratic risk at the research frontier is increasing as the frontier grows more 'complex.' In this paradigm, the data are evidence of the demand-side story we told in the introduction. However, it could also be the case that a greater capacity for financing risky projects allowed for riskier projects to be financed. According to this story, the research frontier did not change in any fundamental way, but its riskier parts were simply unreachable before. The data alone would not be able to distinguish between these two theories.

3.5 Quantitative Results and Theory Tests

Having identified some key patterns of the data, we now turn to our model's implications and how they line up with these patterns.

The firm's problem described at the end of Section 3.3 is quite complicated, but it is not

difficult to solve numerical examples, which is sufficient for our purposes. We have solved the model for a simple parameterization described here: $I = 1.75, x_L = 1.2, x_H = 4.8, c_D = x_L, c_E = .7c_D$. We assume the cash flow is a symmetric truncated normal in the region of the cash flows and explore the optimal capital structure in response to different levels of project risk, as measured by the standard deviation of the cash flow distribution.

3.5.1 Policy Functions and Payoffs

We examine how capital structure differs across project risk, and how this optimal choice is affected by a decrease in c_E i.e. we can explore implications from both the supply-side changes and demand-side changes. The results are plotted in Figure 5.C.7.

Our numerical work shows that indeed an interior capital structure choices exist for this reasonable parameterization¹⁷. Figure 5.C.7 shows us quite clearly that, as we predicted, an exogenous reduction in equity monitoring costs has a differential impact on capital structure based on project risk. Further, just as was predicted in our theoretical section, there exists some threshold level of risk below which projects are entirely funded by debt and above which projects are funded with a mixed capital structure.

This is readily apparent in the policy functions given by Figure 5.C.8. For low equity costs, some equity issuance is optimal even at low levels of risk, but there is a level of risk above which the firm's optimal choice dictates a discrete drop in leverage, which entails both a significant fall in debt levels and a significant rise in external equity issuance. For higher equity costs, this low-risk state simply implies that the firm remains debt-financed; once the risk becomes large enough, the firm issues away a significant fraction of its ownership while cutting back substantially on its debt levels.

¹⁷Refer to 8.B for details describing our solution method.

3.5.2 Quantitative Analysis

With an understanding of both the data and the basic quantitative properties of our model, we now turn to analyze how our proposed theories impact this economy. Recall that the firms are heterogenous along two dimensions: They have varying relative productivities of the safe project, γ_i , and they have varying risk levels, σ_j . Their expected payoff from the safe project is simply $\gamma_i Ex - I$ because there are no intervention costs since there is no asymmetry of information.

Since there is no risk involved in the safe project, the debt and equity contract are really the same thing. Each specifies a non-state-contingent payment of I once the project is realized. We will assume, however that the safe project is financed with entirely debt because Figure 5.C.7 shows that this is the clear optimal choice of risky projects as we take the risk to zero in the limit. We define firms that undertake the risky projects as innovative.

Supply-Side Changes

We begin our analysis of the quantitative results by examining how the economy responds to a reduction in the cost of the equity intervention from $.7c_D$ to $.6c_D$. At the present moment we will interpret historical tax changes in this way as well (though the next step in our project is to solve for them explicitly), as these tax changes had essentially the same effect: They reduced the effective cost of equity finance relative to debt. Figure 5.C.9 shows how both optimal project choice and capital structure respond to this change.

From Figure 5.C.9 a few trends are clear. First, the decision to undertake the risky project is non-monotonic because of the optimal financial instruments. For low levels of risk, risky projects are only undertaken when the safe project performs poorly i.e. the risky project has a very low opportunity cost. These projects are funded with debt alone, which minimizes monitoring costs and avoids dilution costs as in the CSV paradigm.

However, once the project becomes sufficiently risky, then the risk preference of po-

tential equity-holders kicks in; this preference is strong enough to overcome the larger monitoring costs associated with a contractually sub-optimal equity scheme. As the risk grows, leverage ratios fall as we would expect. Further, we see that as the project risk increases, some firms begin to undertake risky projects *given the same opportunity cost*. The risk ‘discount’ taken by shareholders increases with the project risk and so the risky project becomes more appealing as risk increases.

We can see that the changes in the effective cost of equity does not affect most debt-funded firms i.e. firms on the far left of the graph are essentially unchanged. However, the marginal degree of risk for which a mixed capital structure becomes optimal shrinks. Further, in the region of risk where a mixed capital structure can be optimal, more firms choose the risky project. Lastly, the non-monotonicity that results from these two distinct financing schemes becomes more pronounced as the risk ‘discount’ taken by the shareholders increases as c_E falls.

The first set of results from this exercise is given in Table 4.B.1. We see that as equity finance becomes more accessible we see a large increase in the measure of innovative firms. This is the *extensive* margin discussed in the introduction i.e. more firms now find it optimal to undertake innovation projects. The measure more than doubles, in fact. At the same time, we can explain another historical fact of this period, which is falling interest rates. This framework suggests one reason why post-war interest rates fell from the 1970’s until the recent financial crisis is that outside equity became a more preferred means of finance. This drove down the demand for debt finance and its concomitant price, the interest rate.

In the next set of results, given by Table 4.B.2, we see that the capital structure of innovative firms falls as we reduce equity monitoring costs. In fact, it falls quite substantially, by a factor of less than one-third. Even though this does not happen with non-innovative firms by assumption, since they are entirely debt-financed, the innovative firms pull down the aggregate leverage figures in this economy, which occurred in the data as well.

The final set of results explains the *intensive* margin. Table 4.B.3 shows how the cross-sectional dispersion of project outcomes increases for innovative firms and decreases for safe project firms i.e. firms that were undertaking risky projects will now take on even riskier projects as a result of these financial innovations. The opposite effect is true for the safe project firms; since more firms are undertaking risky projects, only safe projects of high relative productivity are being undertaken instead of a larger range of such projects, and so their cross-sectional dispersion falls.

Demand-Side Changes

We now consider the effects of a demand-side change i.e. an exogenous increase in the risk-frontier for innovative projects. Here, we simply increase the risk of every type of project uniformly while keeping the return structure otherwise identical. We leave the monitoring structure the same.

We interpret this change not simply as an increase in idiosyncratic risk that one might expect from a growing economy as in a quality-ladder growth model such as Aghion and Howitt (1990) or Klette and Kortum (2004). In these models, the returns to innovation are constant, but the resources required to innovate grow over time, implying an increasing idiosyncratic risk. Since innovating firms are risk-neutral, or innovating households are perfectly diversified, this is never an issue in those models. But this natural increase is not what we have in mind in our demand-side theory, since it would follow nearly trivially that financial markets need to grow in efficiency at the same rate to finance these projects and sustain a balanced growth path.

Rather, we are interested in the notion that the research frontier itself may be fundamentally changing, perhaps as a result of the ‘knowledge production function’ described in Lanjouw and Schankerman (2004) or Jones (2009). One could think of all of the ‘low-hanging fruit,’ on the research frontier as disappearing over time, leaving only projects

requiring increasingly intensive expenditures on human capital. Greater levels of human capital will imply greater adverse selection problems and, from the investor's point of view, greater risk.

Figure 5.C.10 shows the economy's response to such a change. The two graphs on the far left are identical to their counterparts in Figure 5.C.9. As the risk of the projects increases overall, we see that the marginal degree of risk required for a mixed capital structure falls, as with the supply-side changes, but it does not seem that, contingent on sufficient risk for a mixed capital structure project, *more* firms are choosing the risky project as the project risk increases.

Instead the major effects seem to be coming from those risky projects financed entirely with debt. In particular, fewer and fewer such projects are being undertaken. Why is this? In the Townsend CSV paradigm, expected monitoring costs are increasing in the degree of risk. Since there is no monitoring associated with the safe project and because the expected monitoring costs are increasing as the risk-frontier increases, the mass of risky projects chosen falls substantially in this region.

Table 4.B.4 gives the first set of results, which is again the extensive margin and the interest rate. In this table we see that, as with the supply-side changes, the interest rate falls. Unlike the supply-side changes, though, the extensive margin actually *falls* i.e. as the risk-frontier increases, less firms begin to innovate. The intuition for this is simple: As the projects become riskier, the expected intervention/monitoring costs increase as in the typical Townsend (1979) paradigm, driving down the value of the project. If there is no fundamental change in the monitoring technology or supply of finance, then we would expect less firms to undertake these risky projects due to the high monitoring costs.

The next set of results is given in Table 4.B.5 and gives the capital structures under the changes. These results are isomorphic to their supply-side counterparts but they are not as drastic. Safe projects remain debt-financed by assumption and risky projects decrease their

leverage as the level of risk increases. This tends to drive down aggregate leverage ratios, but by not nearly as much as the supply-side changes since the measure of innovative firms is also decreasing.

The last set of results regards the intensive margin and is given in Table 4.B.6. The results from the demand-side change are essentially the same as the supply-side change. Safe project dispersion falls only slightly while risky project dispersion increases substantially, which causes overall project risk to increase even though their mass is falling.

3.5.3 Estimation Strategy

In the previous section we derived intuition regarding how our model reacts to supply-side and demand-side changes quantitatively. In this section, we attempt to take our model to the data in a serious way to discover by *how much* each of these changes must have occurred to produce the observed leverage trends. For a detailed description of our solution method, refer to 8.B.

In a preliminary calibration, we begin by assuming that the cash flows follow a symmetric, truncated normal distribution. This is convenient since it will imply that our model is *scale-invariant*, and thus we only need to target relevant ratios. We normalize the required investment, I , to one. Table 4.B.7 describes the rest of the parameterization.

Note that the values in Table 4.B.7 are computed using COMPUSTAT data from 1975-1986. As is made clear in Table 4.B.7, much of the calibration can be done without simulation. Only two parameters need to be estimated: The effective cost of equity, c_E , which captures supply-side changes, and the degree of risk, σ_{max} , which captures the demand-side changes. It is assumed that $\sigma_{min} = 0$ i.e. there exist ‘risky’ projects that have effectively no risk. How risky they become will be a target for estimation.

Table 4.B.8 describes the identification strategy used in simulated method of moments. In particular, we use the median leverage of innovative firms to identify the effective cost

of equity issuance and we use the fraction of innovative firms over total firms to identify the degree of risk. Both parameters will have the same substantial effect on leverage, but they will tend to have the opposite effects on the measure of innovative firms. We exploit this orthogonality to identify these parameters.

Lastly, it should be noted that we relax the assumption that $c_E \in [c_D/2, c_D)$. While we lose explicit convexity of the equity payoff function outside of these bounds, the relationship with risk is still there quantitatively, and it can fit the data better. This will likely change in a more nuanced calibration that we plan to pursue later.

It should be noted before we proceed with the discussion of the results how we interpret our model empirically. In our present calibration, we take a firm of type ij to be a ‘firm’ as we see it in the data. This is a bit problematic, given that our analysis in Section 3.3 suggests that a mass of firms will have interior leverage choices and a mass will be entirely debt-financed. This implies that our model will generate some firms that are entirely debt-financed, which is not consistent with our sample of COMPUSTAT firms, all of which have issued an IPO.

A more detailed calibration is indeed in order, and likely one that interprets a firm as a ‘project’ and not as a firm in an empirical sense. Thus, a publicly traded firm will consist of a portfolio of projects, some of which as financed with a mixed capital structure and others which are financed with debt alone. The end result will be some more plausible capital structure. However, this will require us to change our definition of ‘innovative,’ to one in which, for example, a firm is innovative if its innovation intensity is greater than one half. The empirical trends still hold for this definition, and it is simply a matter of re-working the empirical and quantitative exercise to derive a more precise estimate.

It should also be noted that our measure of ‘return on assets’ may be on the small side. This is because we assume that the firm finances its projects entirely with external funds i.e. it has no net worth or retained earnings. In reality, firms, especially large firms, often

finance many projects with retained earnings. We can also account for this by explicitly allowing for some net worth, which we plan to do in future work.

With these qualifications in mind, we now turn to the key results. We can see that under the assumption that only the two theories discussed in this paper are responsible for changing the leverage ratios, that equity issuance costs must have fallen by 31.2% while the exogenous level of risk must have risen by only 4.7%. The intuition for these changes comes largely from the *external* margin of entry into innovation, as was discussed before. Both effects will reduce leverage and interest rates, but equity issuance costs will *increase* the mass of firms engaging in innovation by making it optimal more often to mix capital structures and thus pursue the risky project, while raising the level of risk will *increase* the cost of debt-only projects, which remain a substantive fraction of overall projects, since the debt contract does minimize monitoring costs.

In a more nuanced and careful calibration we expect that these key results will change slightly in magnitude, but certainly not in direction. The intuition will still remain even if the identification strategy changes. Our results suggest that the most relevant change for innovative firms over the past 40 years or so is not in the *type* of projects that they undertake, but rather their capacity to raise *funds* for those projects by issuing outside equity. Thus, our results support Greenwood and Jovanovic (1990) and suggest that the ‘research frontier’ and the level of financial development in any country may be much more endogenous and interconnected than they at first appear.

One last comment regarding the calibration is in order. It should be noted that we have assumed that c_D , the stationary monitoring cost of debt, has remained constant over time. We justify this by referring to work by Philippon (2012), which demonstrates that by value added measures, the financial sector has not become more efficient over time. We plan to make this more explicit by perhaps constructing a measure of financial efficiency that is some function $f(c_D, c_E)$, which is decreasing in both arguments. We could then calibrate

the model such that this measure remains constant. Our results would almost surely suggest that c_D has increased over time while c_E has fallen¹⁸, which would flesh out in more detail the story of financial development over the last forty years or so. Even though there have been innovations to debt funding, such as the acceptance of Asset-Backed Securities and the collateralizability of intangibles, these innovations have not been able to keep pace with the more complicated nature projects they are used to fund. On the other hand, deregulations and development of private equity markets have more than kept pace with the advancement of these projects.

3.6 Conclusion

In this paper, we demonstrated empirically that leverage ratios diverged for innovative and non-innovative firms over the latter half of the post-war era in the US and presented a tractable model for parsing apart potential theories to explain this. In particular, we demonstrated a strong negative relationship between innovation and leverage both at the intensive and extensive margins and demonstrated that this trend grew over time.

Conditional on risk being the finance-relevant aspect of innovative projects, we developed a model in the spirit of Townsend (1979) following most closely Hvide and Leite (2008) to parse apart potential explanations for this leverage phenomenon. The model generated a ‘pecking-order’ hierarchy for finance as a function of risk, in which retained earnings are employed first, then debt, then equity as the project grows riskier. The calibrated model suggested that equity issuance costs must have fallen by 31.2% while idiosyncratic project risk must have risen by only 4.7%.

Our results underscore the importance supply-side changes in the financial markets. Both supply-side and demand-side effects work to decrease leverage ratios, lower real interest rates, and increase innovation at the intensive margin, but supply-side changes also

¹⁸Remember that both are interpreted as stationarized.

increased innovation at the extensive margin whereas demand-side changes decreased innovation at this margin. The intuition was that a large mass of innovative projects are optimally financed entirely with debt, and as risk increases the expected monitoring costs increase, rendering these projects inferior to their safer counterparts with no associated monitoring costs.

Our work supports claims by Rajan and Zingales (1998), Brown et al. (2009), and others who claim that financial considerations are relevant for investment decisions in contrast to the result of Modigliani and Miller (1958). Further, our work demonstrates that either tax policy, deregulation, or substantial development of private equity markets must be present to justify patterns seen in the data, and thus complements Itenberg (2013).

This paper could serve as a launchpad for several potential avenues of future research. First, the model predicts an intimate relationship between the undertaking of innovative projects and equity financing. One could test these predictions using cross-country data on economic development and the efficiency of private equity markets. In fact, one could potentially calibrate our model more appropriately using this cross-country data and perhaps use it to construct a new measure of the efficiency of equity markets.

On the other hand, one could explore in more theoretical depth the implications of the discrete jumps in our model. In our model, as projects become riskier, there exists some risk threshold beyond which firms discretely delever and issue away a large fraction of their firm to outside investors. This may be one potential explanation for the ‘leverage collapse’ observed in the recent financial crisis, which was commonly associated with an increase in risk.

Appendix

Chapter 4

List of Tables

4.A Tables for Chapter 2

Parameter	Distribution	Parameter 1: a	Parameter 2: b	Implied Mode
γ_b	Beta	3.0	3.0	0.5
γ_g	Beta	3.0	3.0	0.5
γ_R	Beta	3.0	3.0	0.5
η	Beta	1.010	3.0	0.01
p_L	Beta	1.002	3.0	0.10

Table 4.A.1: Prior Distributions

Parameter	Median (CW)	Mean (CW)	Credible Set (CW)
γ_b	0.0584	0.0616	[0.0267, 0.1109]
γ_g	0.5095	0.5064	[0.4037, 0.5987]
γ_r	0.6441	0.6327	[0.3402, 0.8816]
η	0.0339	0.0472	[0.0080, 0.1394]
p_L	0.2204	0.2710	[0.0744, 0.6336]

Table 4.A.2: Posterior Statistics and 90% Credible Sets

4.B Tables for Chapter 3

Table 4.B.1: Supply-Side Change: Extensive Margin and Interest Rate

	$c_E = 0.70c_D$	$c_E = 0.65c_D$	$c_E = 0.60c_D$
Measure of Innovative Firms	0.199	0.355	0.552
Interest Rate	0.079	0.058	0.038

Table 4.B.2: Supply-Side Change: Capital Structures

Mean α	$c_E = 0.70c_D$	$c_E = 0.65c_D$	$c_E = 0.60c_D$
Safe Project Firms	1.000	1.000	1.000
Risky Project Firms	0.723	0.350	0.200
All Firms	0.945	0.769	0.558

Table 4.B.3: Supply-Side Change: Intensive Margin

S.D. Project Realization	$c_E = 0.70c_D$	$c_E = 0.65c_D$	$c_E = 0.60c_D$
Safe Project Firms	0.038	0.030	0.020
Risky Project Firms	0.875	0.919	0.934
All Firms	0.449	0.604	0.741

Table 4.B.4: Demand-Side Change: Extensive Margin and Interest Rate

	<i>Base Risk</i>	$1.25 \times \textit{Base Risk}$	$1.5 \times \textit{Base Risk}$
Measure of Innovative Firms	0.199	0.1516	0.1406
Interest Rate	0.079	0.0382	0.0190

Table 4.B.5: Demand-Side Change: Capital Structures

	<i>Base Risk</i>	$1.25 \times \textit{Base Risk}$	$1.5 \times \textit{Base Risk}$
Safe Project Firms	1.000	1.000	1.000
Risky Project Firms	0.723	0.4457	0.2504
All Firms	0.945	0.9160	0.8946

Table 4.B.6: Demand-Side Change: Intensive Margin

	<i>Base Risk</i>	$1.25 \times \textit{Base Risk}$	$1.5 \times \textit{Base Risk}$
Safe Project Firms	0.0127	0.0126	0.0126
Risky Project Firms	1.3256	1.9220	2.4096
All Firms	0.2739	0.3021	0.3609

Table 4.B.7: Calibration

Parameter	Value	Target	Reasoning
I	1	1	Normalization
\bar{x}/I	1.182	1.182	Innovative Firm Average Return on Assets
$[\gamma_L, \gamma_H]$	[.95, 1]	1.129	Non-Innovative Firm Average Return on Assets
c_D	.05	5% <i>Assets</i>	Boyd and Smith (1994)
x_L	.05	c_D	Lowest Possible
x_H	2.314	$2\bar{x} - c_D$	Symmetry of Truncated Normal

Table 4.B.8: Estimation (SMOM)

Parameter	Value	Target (Data)	Target (Model)	Reasoning
c_E (Before)	$0.562c_D$	0.211	0.211	Median Leverage
σ_{max} (Before)	3.113	0.570	0.570	Measure of Innovative Firms
c_E (After)	$0.387c_D$	0.122	0.122	Median Leverage
σ_{max} (After)	3.259	0.693	0.693	Measure of Innovative Firms

Chapter 5

List of Illustrations

5.A Figures for Chapter 1

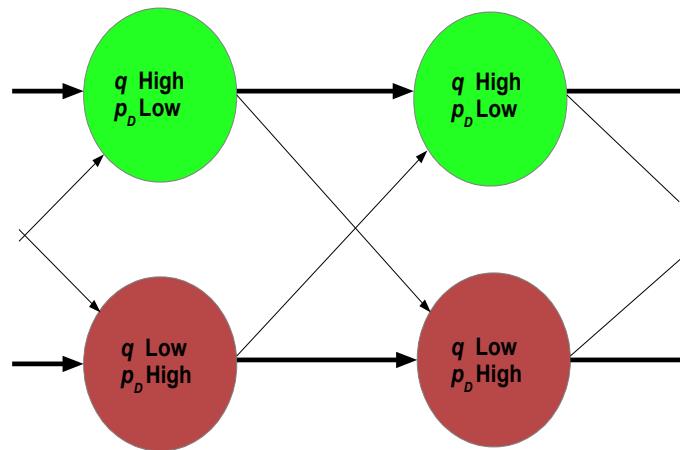


Figure 5.A.1: Dynamic Panics: A Simple Illustration

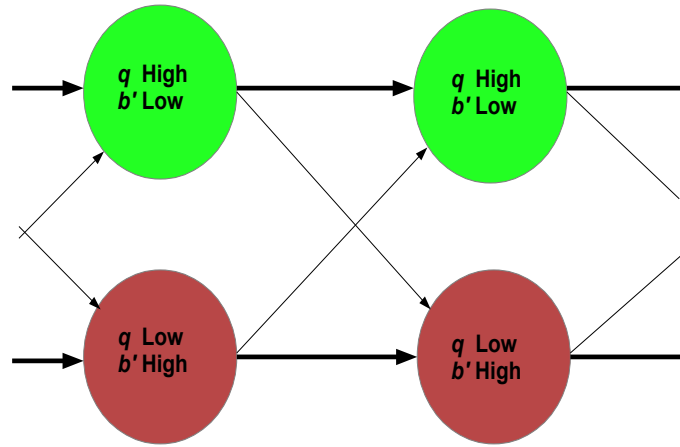


Figure 5.A.2: Non-Default-Relevant Dynamic Panics: An Illustration

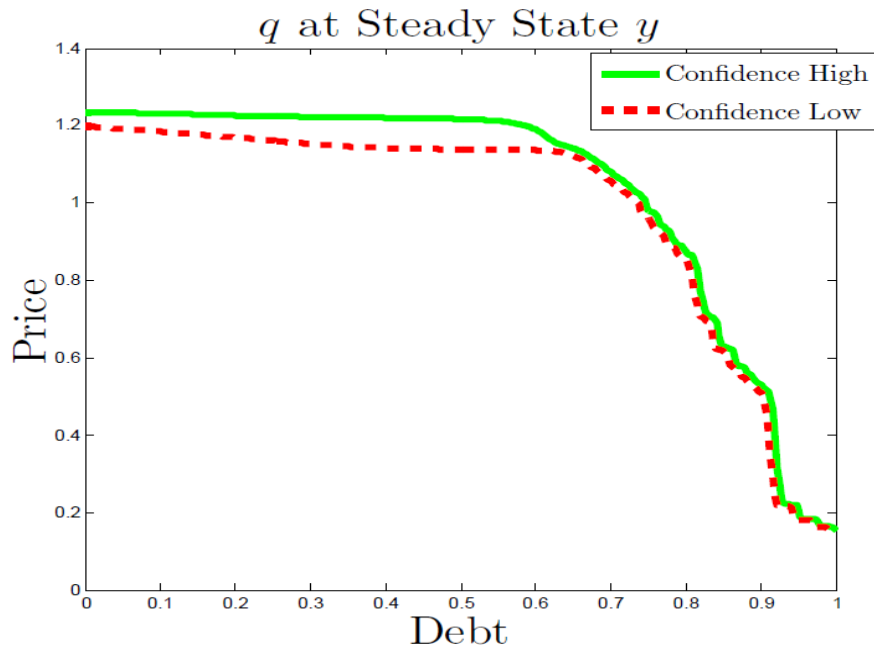


Figure 5.A.3: Demand Functions in a Long-Term Dynamic Panic (Non-Default-Relevant)

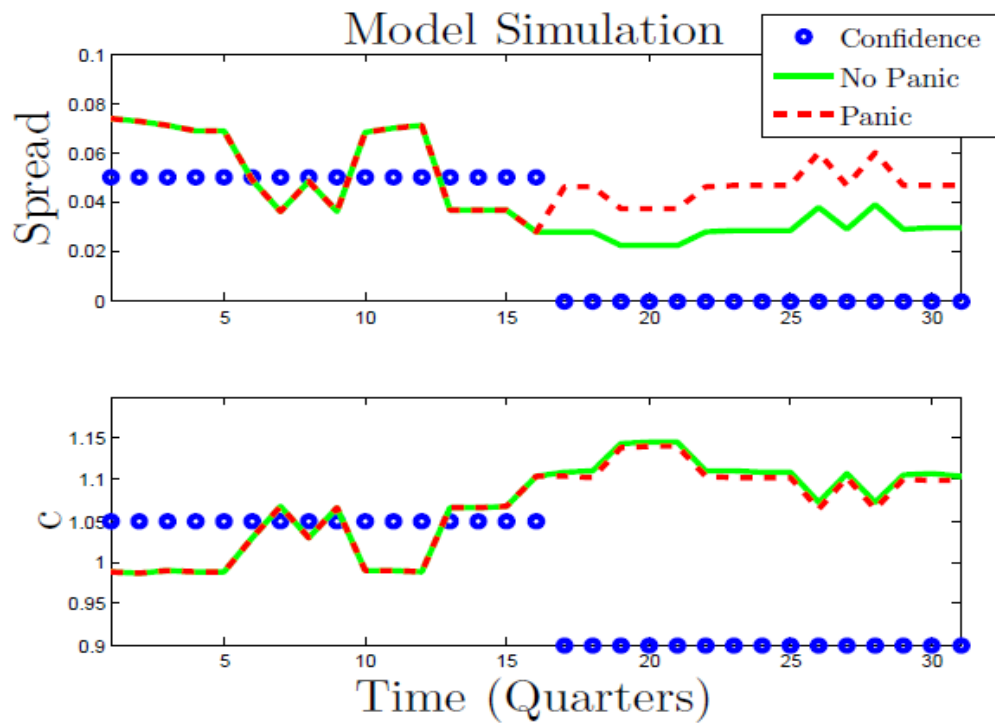


Figure 5.A.4: Simulation of a Long-Term Dynamic Panic (Non-Default-Relevant)

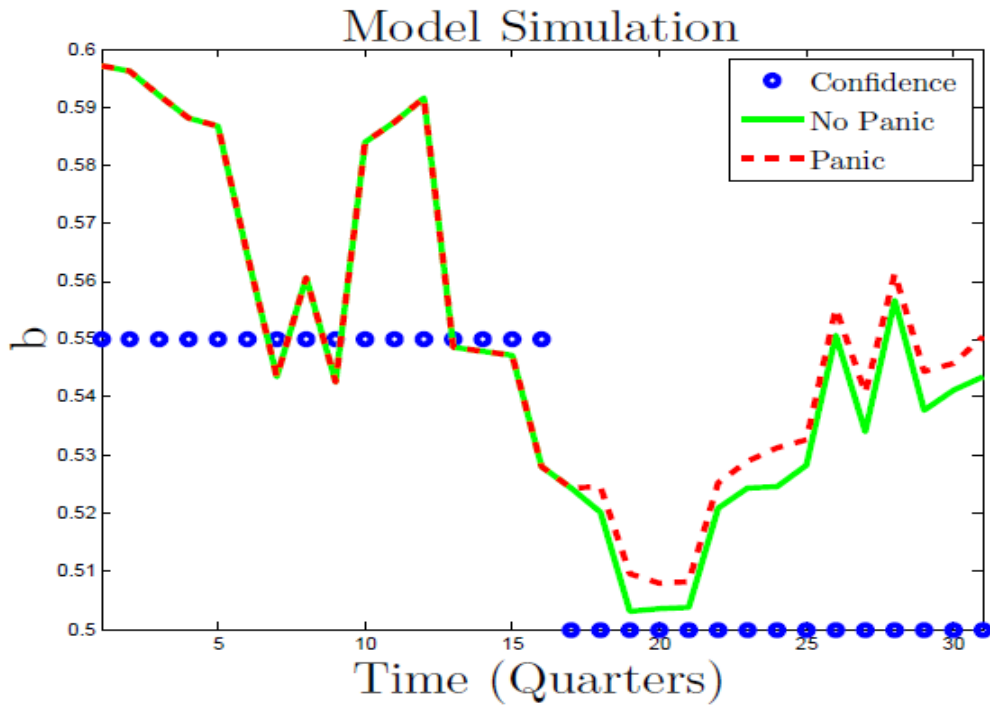


Figure 5.A.5: Simulation of a Long-Term Dynamic Panic (Non-Default-Relevant)

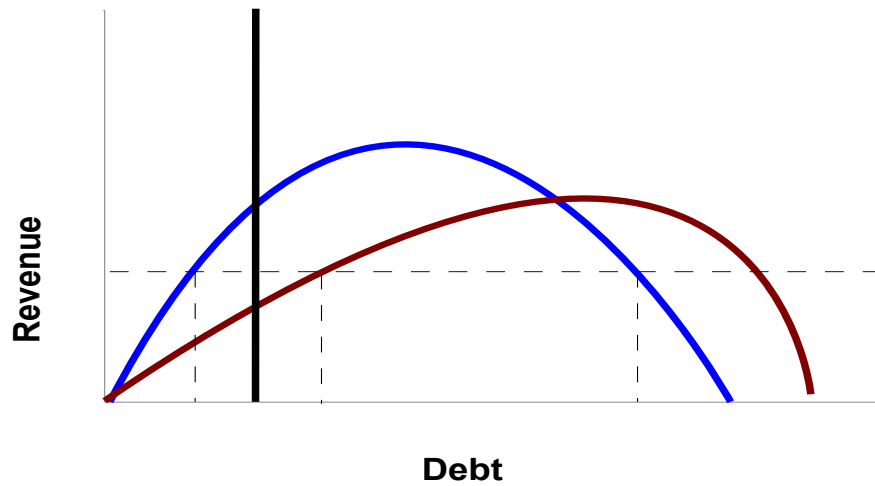


Figure 5.A.6: Laffer-Curves: Non-crisis (Blue) and Crisis (Red)

5.B Figures for Chapter 2

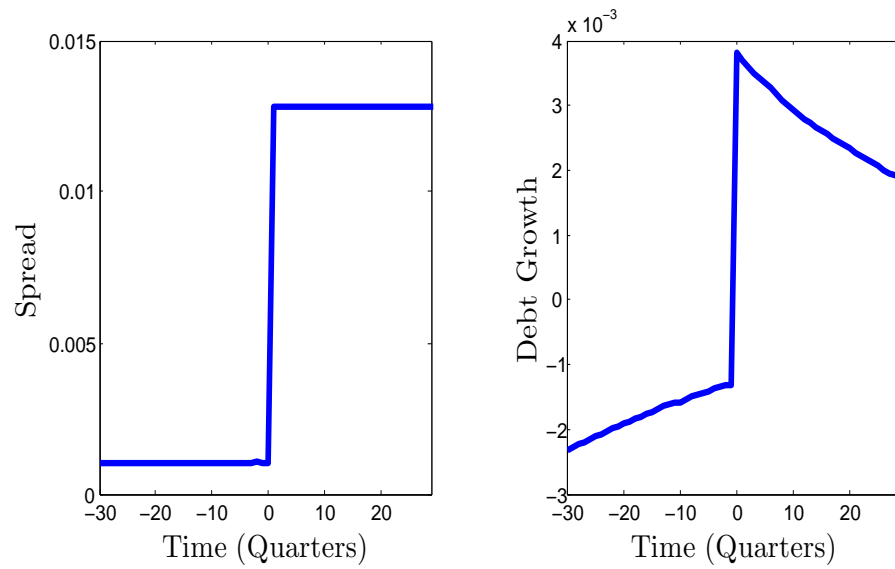


Figure 5.B.1: Mean Trajectories: Long-Term Dynamic Panic at $t = 0$

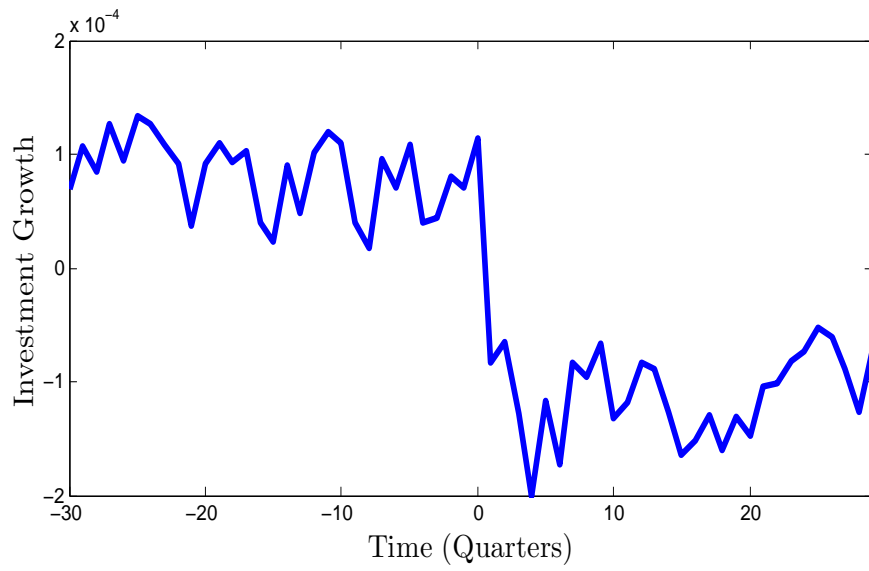


Figure 5.B.2: Mean Trajectory: Long-Term Dynamic Panic at $t = 0$

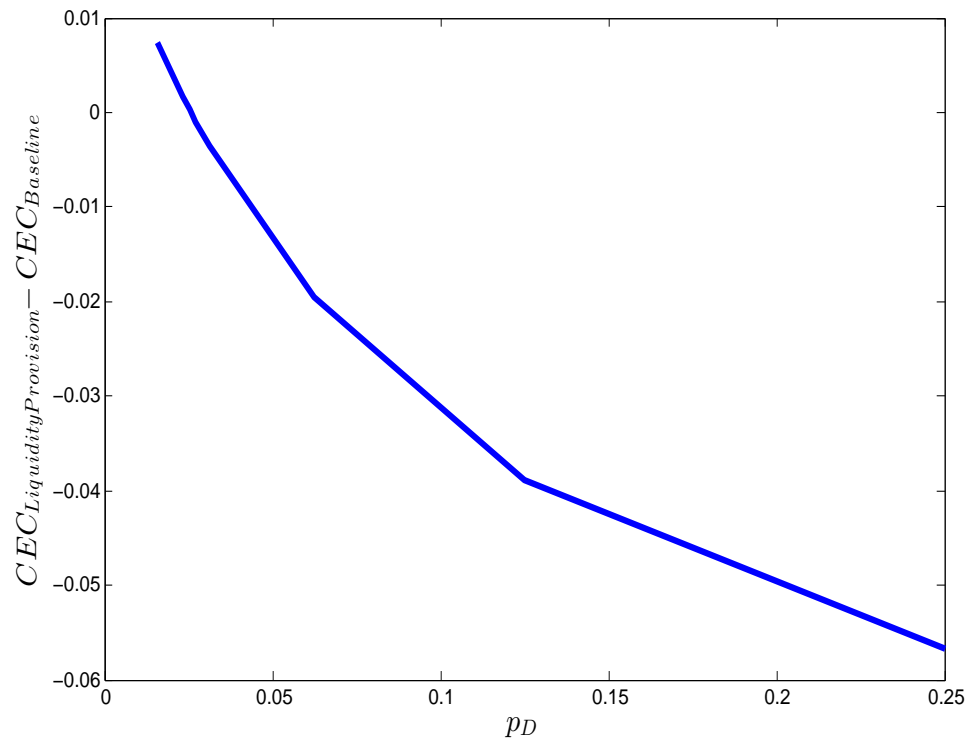


Figure 5.B.3: Welfare Difference: Liquidity Provision and Baseline Model

5.C Figures for Chapter 3

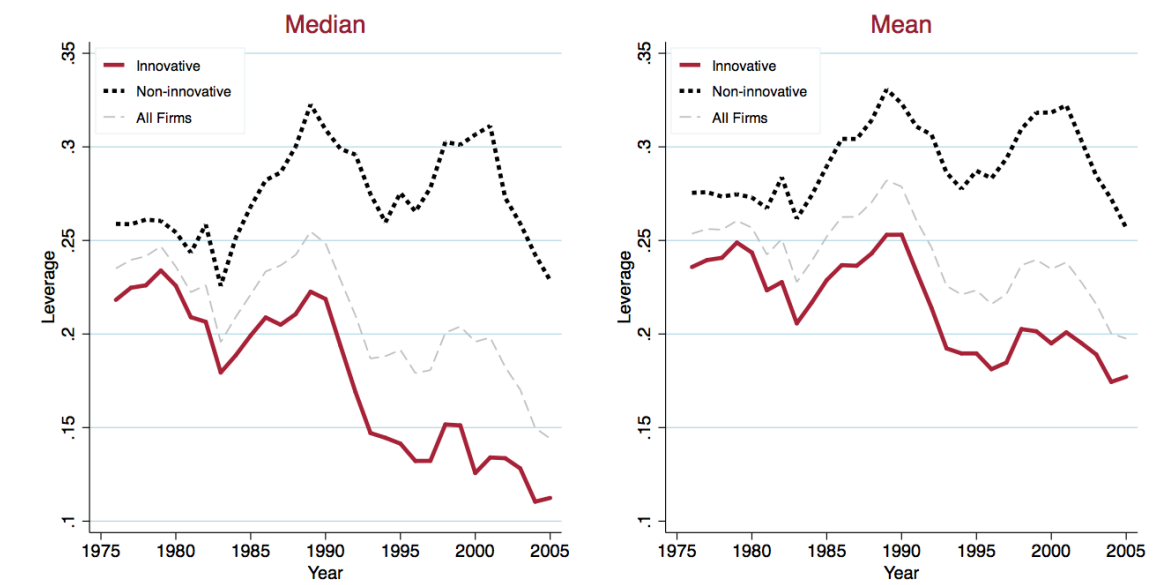


Figure 5.C.1: Leverage Ratios Over Time

Source: WRDS COMPUSTAT Database. Innovative firms invest positively in R&D in that year.

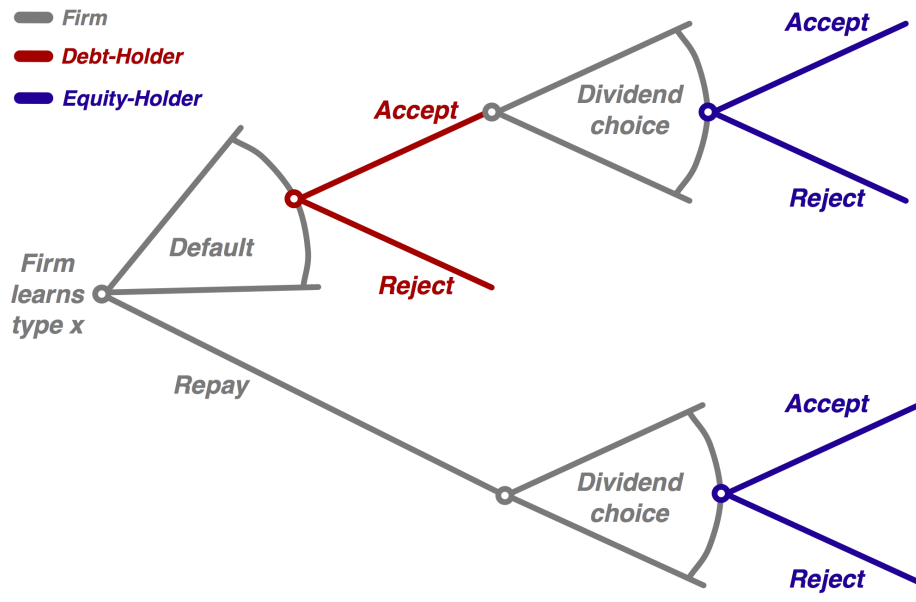


Figure 5.C.2: Finance Game

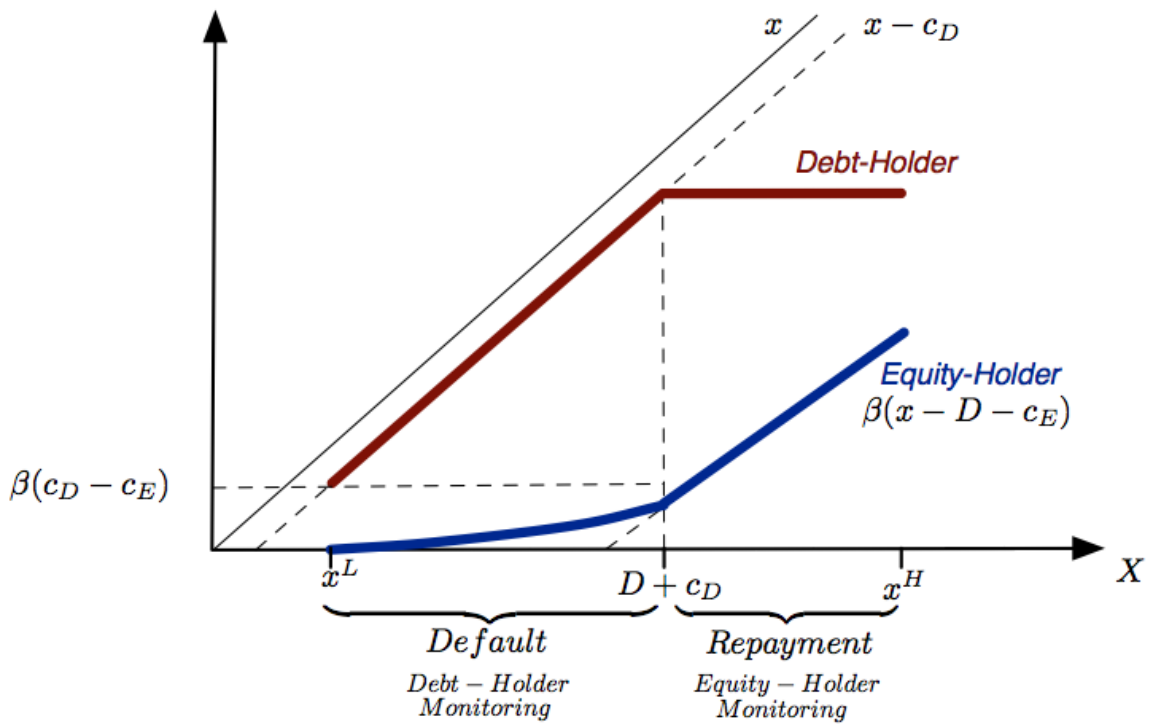


Figure 5.C.3: Payoff Functions

Figure 5.C.4: R&D Trends

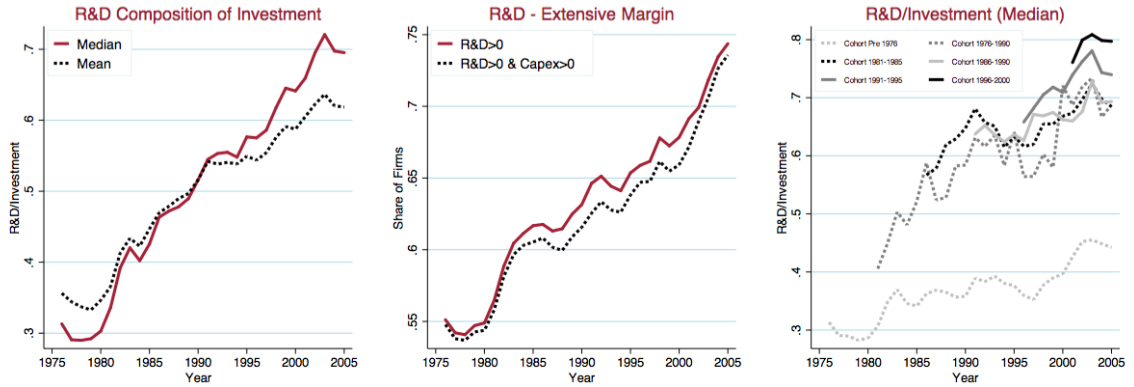


Figure 5.C.5: Sales Growth Volatility Trends

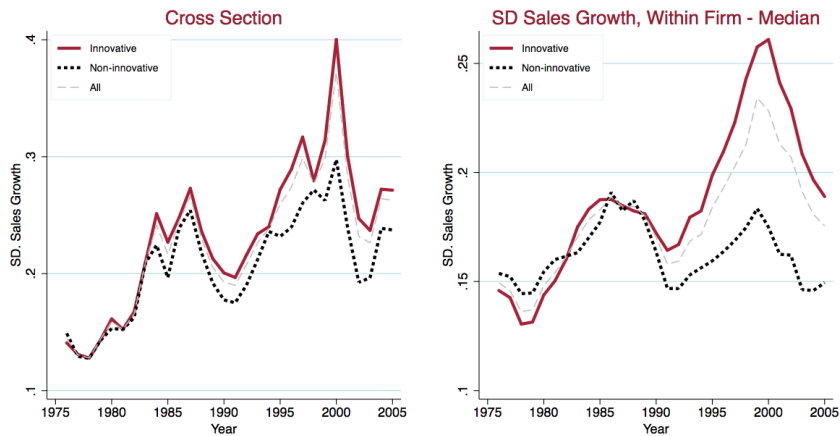
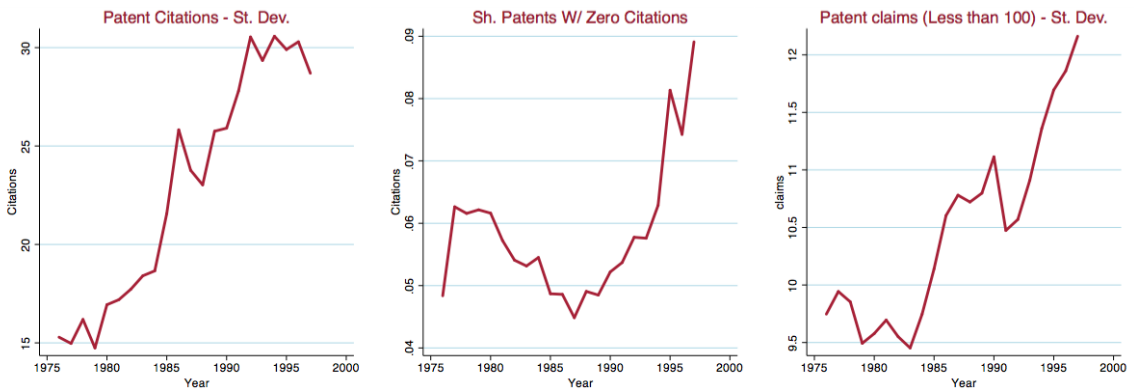


Figure 5.C.6: Patent Forward Citations and Claims



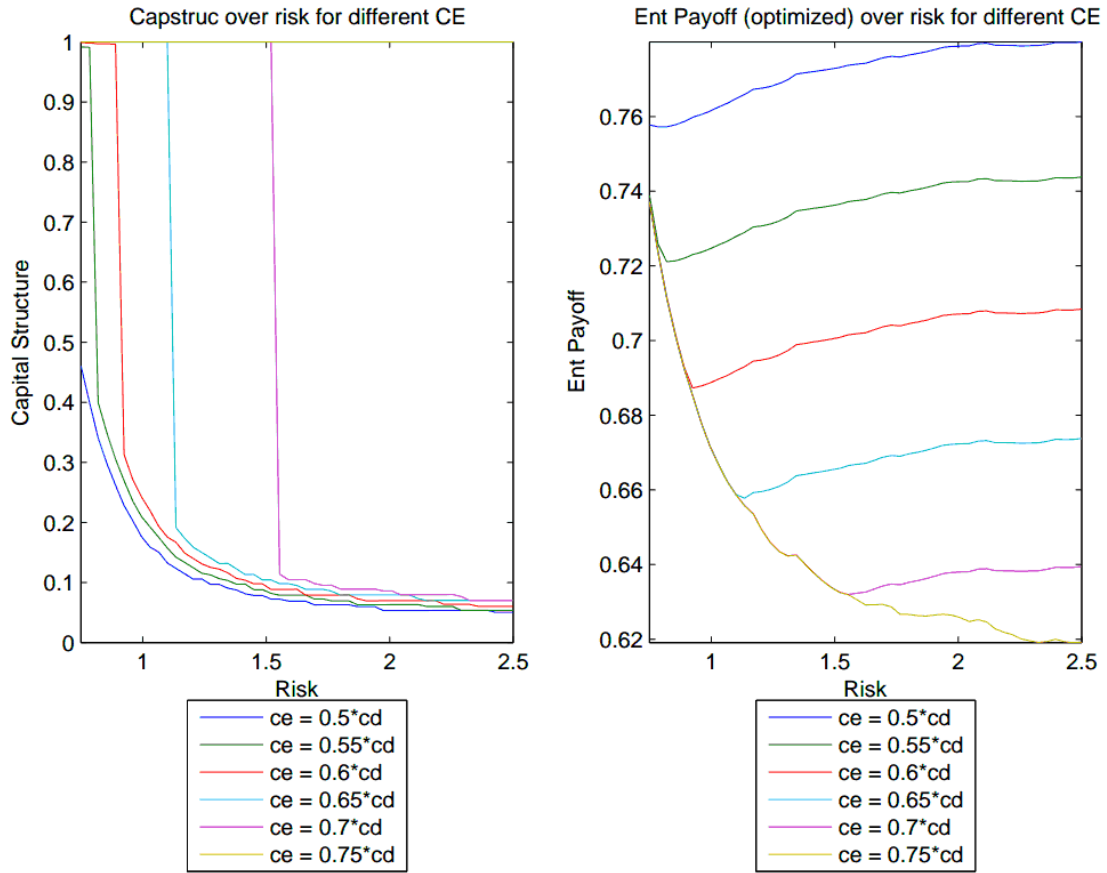


Figure 5.C.7: Optimal Capital Structure Across Risk and Equity Efficiency

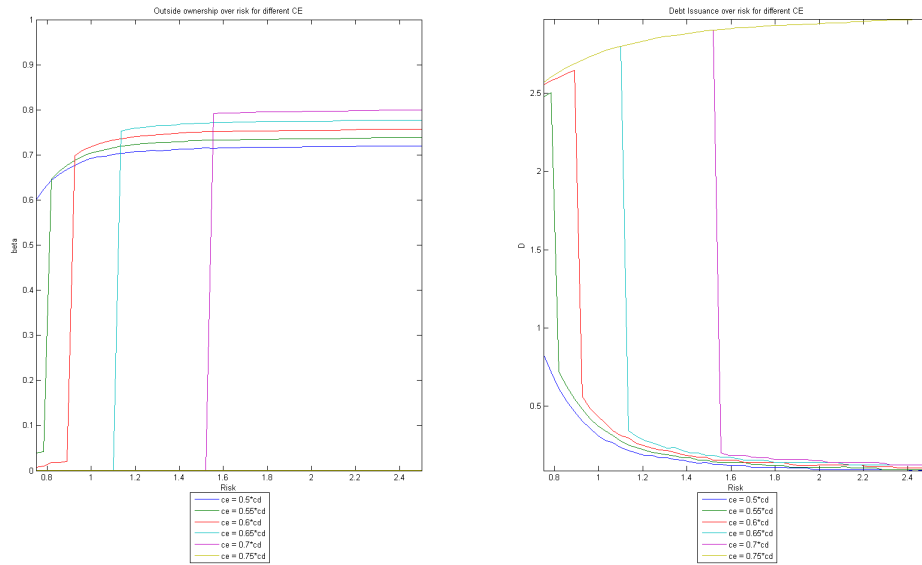


Figure 5.C.8: Policy Functions Across Risk

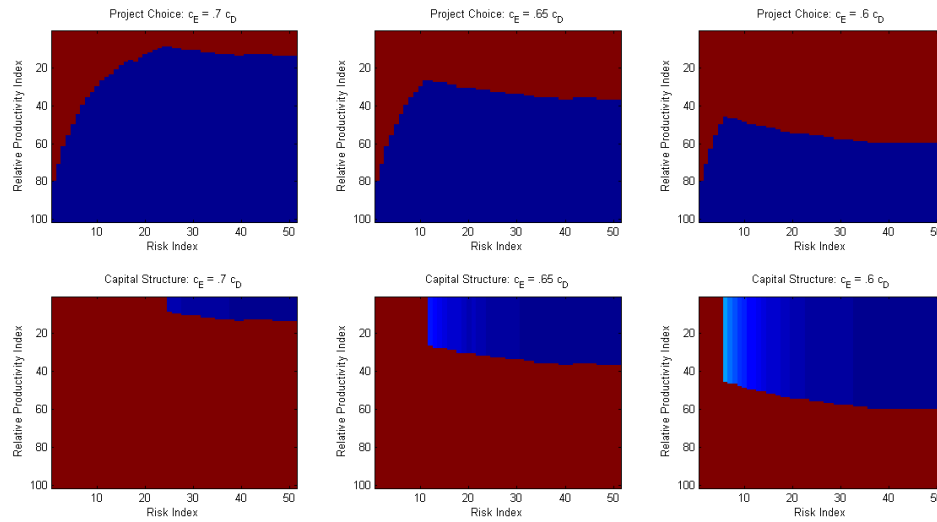


Figure 5.C.9: Economy's Response to a Reduction in c_E

For project choice, safe projects are in blue and risky (innovative) projects are in red. For capital structure, red is entirely debt-funded and blue is a mixed capital structure; deeper shades of blue signify lower leverage.

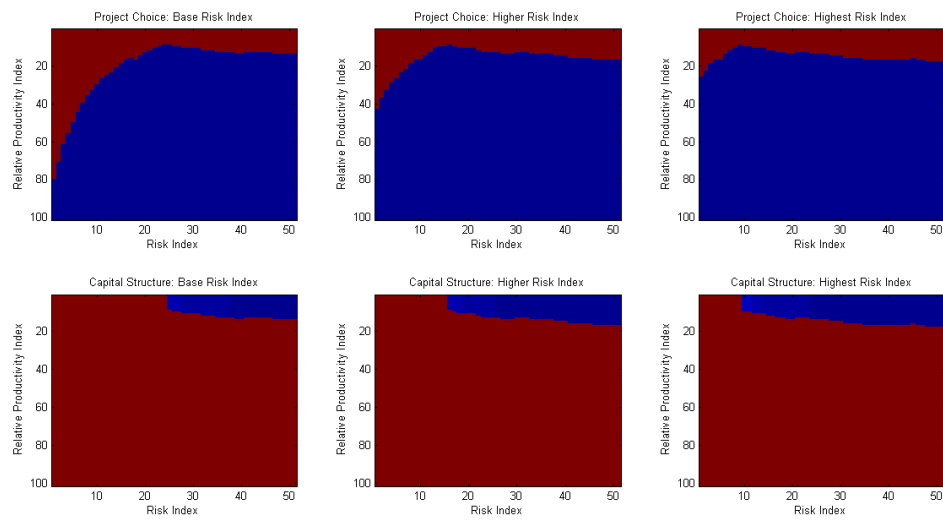


Figure 5.C.10: Economy’s Response to an Increase in the Risk Frontier
For project choice, safe projects are in blue and risky (innovative) projects are in red. For capital structure, red is entirely debt-funded and blue is a mixed capital structure; deeper shades of blue signify lower leverage.

Chapter 6

Appendix for Chapter 1

6.A Theoretical Proofs

6.A.1 Proof of Theorem 1.3.1

To characterize the set of Confidence-Waves Equilibria, note first that there is only one way that ξ can have real effects: It must induce the sovereign to default in one state and repay in the other. If the sovereign defaults in both states, then confidence has no effects; the same is true if the sovereign repays in both states. Without loss of generality, let us search for an equilibrium in which the sovereign repays in ξ_H and defaults in ξ_L .

If this is the case, then the equilibrium pricing function must be given as follows:

$$\begin{aligned}q(\xi_L) &= \frac{\eta}{R} \\q(\xi_H) &= \frac{1-\eta}{R}\end{aligned}$$

If we impose the default strategy in the continuation value of the sovereign, then we

can write the Bellman of the sovereign conditional on repayment in ξ_H as follows:

$$V(\xi_H) = u\left(y - b + \frac{1 - \eta}{R}b\right) + \beta[\eta X + (1 - \eta)V(\xi_H)]$$

Notice that if we difference this expression with the value of default and call this object $M(\xi_H) = V(\xi_H) - X$, then we have

$$\begin{aligned} M(\xi_H) &= u\left(y - \frac{R - 1 + \eta}{R}b\right) - u(y - \phi(y)) + \beta(1 - \eta)M(\xi_H) \\ \rightarrow M(\xi_H) &= \frac{u\left(y - \frac{R - 1 + \eta}{R}b\right) - u(y - \phi(y))}{1 - \beta(1 - \eta)} \end{aligned}$$

In order for default to be the optimal response, it will be both necessary and sufficient that $M(\xi_H) \geq 0$. This condition is precisely the first assumption under the assumption of an increasing utility function.

We will also require that $V(\xi_L) - X = M(\xi_L) < 0$ i.e. default is optimal in the low-confidence state. This Bellman can be written as

$$V(\xi_L) = u\left(y - b + \frac{\eta}{R}b\right) + \beta[(1 - \eta)X + \eta V(\xi_H)]$$

We can again take the difference with X to define $M(\xi_L)$:

$$\begin{aligned} M(\xi_L) &= u\left(y - \frac{R - \eta}{R}b\right) - u(y - \phi(y)) + \beta\eta M(\xi_H) \\ \rightarrow M(\xi_L) &= u\left(y - \frac{R - \eta}{R}b\right) - u(y - \phi(y)) + \\ &\quad \frac{\beta\eta}{1 - \beta(1 - \eta)} \left[u\left(y - \frac{R - 1 + \eta}{R}b\right) - u(y - \phi(y)) \right] \end{aligned}$$

This last expression will be strictly less than zero if and only if the second assumption holds. In other words, the flow difference must be largely negative; enough so to

compensate for the smaller but positive continuation value difference.

The intuition here is that the cost of debt service is greater than the default costs when confidence is low since debt prices are also very low. As such, it is no longer worthwhile to service the debt and default becomes optimal. However, when confidence is high so are debt prices and so the cost of debt service is now lower than default costs.

Thus, the two conditions in the theorem are both necessary and sufficient for the existence of Confidence-Waves Equilibrium. ■

Proof of Corollary 1.3.2

To see why this persistence holds, note that the second assumption requires the following to be true

$$u(y - \phi(y)) - u\left(y - \frac{R - \eta}{R}b\right) > 0$$

$$\rightarrow \frac{R - \eta}{R}b - \phi(y) > 0$$

i.e. the cost of debt service in the low confidence state is strictly greater than the default costs. If we take the difference of this expression with the first assumption, we arrive at the result:

$$\frac{R - 1 + \eta}{R}b - \frac{R - \eta}{R}b < 0$$

$$R - 1 + \eta - R + \eta < 0$$

$$\eta < 1/2$$

■

Proof of Corollary 1.3.3

Notice first that in any Confidence-Waves Equilibrium, it must be that $\frac{R-1+\eta}{R}b \leq \phi(y) < \frac{R-\eta}{R}b$. If we increase the concavity, then the utility difference between the first and second of these terms will increase more than the utility difference between the utility difference between the second and third terms. But this will imply that the second condition of Theorem 1.3.1 will continue to hold.

Notice, however, that this result does not hold if we make u more convex. In fact, some Confidence-Waves Equilibria can disappear when this happens. ■

Proof of Corollary 1.3.5

First, let us find the conditions that govern the two non-sunspot equilibria: Full default and full repayment. Under the assumption of full-default, the price must be 0 in equilibrium. If we insert default as the optimal strategy in the continuation value, we derive the following Bellman:

$$V = u(y - b) + \beta X$$

If we take the difference of this value with X we arrive at

$$M = u(y - b) - u(y - \phi(y))$$

We require that $M < 0$ in order for this to be an equilibrium, which will be true provided $b > \phi(y)$. Notice that this is implied by our second assumption, which requires that

$$\begin{aligned} \frac{R-\eta}{R}b - \phi(y) &> 0 \\ \rightarrow b - \phi(y) &> 0 \end{aligned}$$

Thus, if a Confidence-Waves Equilibrium exists, so too does the full-default equilibrium.

To verify the full-repayment equilibrium, the procedure is the same. Notice that the price here is $\frac{1}{R}$:

$$\begin{aligned} V &= u\left(y - b + \frac{1}{R}b\right) + \beta V \\ \rightarrow M &= u\left(y - \frac{R-1}{R}b\right) - u(y - \phi(y)) + \beta M \\ \rightarrow M &= \frac{u\left(y - \frac{R-1}{R}b\right) - u(y - \phi(y))}{1 - \beta} \end{aligned}$$

We require that $M \geq 0$ in order for this to be an equilibrium, which is true provided $\frac{R-1}{R}b \leq \phi(y)$. But this follows direction from the first assumption which states that $\frac{R-1+\eta}{R}b \leq \phi(y)$. Thus, if a Confidence-Waves Equilibrium exists, so too does the full-repayment equilibrium. ■

Sunspots Randomizing over Non-Equilibrium Pricing Schedules: An Example

I provide a simple example here of a case in which sunspot activity can randomize over non-equilibrium pricing schedules. This example demonstrates that the regimes to which sunspots transition may not themselves be sustainable, which has important policy implications.

Consider again the simple model, but now suppose that $u(c) = c$ and that there is some simple intrinsic uncertainty as well. In particular, $y \in \{y_1, y_2\}$ and changes regimes with probability p . Continue to suppose that default costs can depend on y .

I now outline the conditions that define several non-sunspots equilibria. Using the techniques outlined in the proof of Theorem 1.3.1, it can be shown that a full repayment equilibrium exists if and only if the following condition holds:

Assumption FR: $(\phi(y_i) - \frac{r}{1+r}b) + \frac{\beta p}{1-\beta(1-p)} (\phi(y_{-i}) - \frac{r}{1+r}b) \geq 0$ for both states, i .

We can also find conditions under which an alternative equilibrium exists: One in which repayment occurs in y_2 but default occurs in y_1 . This equilibrium can exist if and only if the following conditions hold:

Assumption INT1: $\phi(y_2) \geq \frac{p+r}{1+r}b$

Assumption INT2: $(\frac{1+r-p}{1+r} + \frac{\beta p}{1-\beta(1-p)} \frac{p+r}{1+r}) b > \phi(y_1) + \frac{\beta p}{1-\beta(1-p)} \phi(y_2)$

Now that we have outlined tightly how two non-sunspots equilibria can arise(or not arise), I will show how a sunspots equilibrium can randomize over these pricing schedules even if one of them is not an equilibrium. In particular, we will search for a Confidence-Waves Equilibrium in which the following holds:

$$V(\xi_1, y_1) < X(y_1) \leq V(\xi_2, y_1)$$

$$X(y_2) \leq V(\xi_1, y_2) \leq V(\xi_2, y_2)$$

Thus, we are searching for a sunspots equilibrium in which the sunspot randomizes over the RF default strategy and the INT default strategy. This pattern of default and repayment will suggest the following pricing schedule:

$$q(\xi_1, y_1) = \frac{p + \eta - \eta p}{1 + r}$$

$$q(\xi_2, y_1) = \frac{1 - \eta + \eta p}{1 + r}$$

$$q(\xi_1, y_2) = \frac{1 - p + \eta p}{1 + r}$$

$$q(\xi_2, y_2) = \frac{1 - \eta p}{1 + r}$$

Notice that, under sunspot persistence, the following is true:

$$\begin{aligned} q(y_1) &< q(\xi_1, y_1) < q(\xi_2, y_1) < \frac{1}{1+r} \\ q(y_2) &< q(\xi_1, y_2) < q(\xi_2, y_2) < \frac{1}{1+r} \end{aligned}$$

where $q(y_i)$ is the price of an INT equilibrium (if it exists, which it may not). Thus the sunspot is randomizing over these two regimes in which the sovereign follows either FR or INT.

In words, if this pricing schedule is an equilibrium, then it implies a welfare improvement over the interior solution without confidence waves. This is because the overall probability of default has fallen, since the sovereign does not always default in fundamental state 1. Even though it appears as if this might be a convexification over the risk-free equilibrium and the interior one, we will show that it is not in a moment.

To determine whether an sunspots equilibrium exists, define $M_k^{ij} = V(\xi_i, y_j) - X(y_k)$. It is sufficient for the above equilibrium to exist provided that $M_1^{11} < 0$ and $M_1^{21}, M_2^{12}, M_2^{22} \geq 0$. Assuming the appropriate default/repayment scheme in the continuation value, we can difference the Bellmans of the value and default functions to derive these objects as functions of themselves as follows:

$$\begin{aligned} M_1^{11} &= \phi(y_1) - \frac{1+r-p-\eta+\eta p}{1+r}b + \beta\eta(1-p)M_1^{21} + \beta p[(1-\eta)M_2^{12} + \eta M_2^{22}] \\ M_1^{21} &= \phi(y_1) - \frac{r+\eta-\eta p}{1+r}b + \beta(1-\eta)(1-p)M_1^{21} + \beta p[\eta M_2^{12} + (1-\eta)M_2^{22}] \\ M_2^{12} &= \phi(y_2) - \frac{r+p-\eta p}{1+r}b + \beta\eta p M_1^{21} + \beta(1-p)[(1-\eta)M_2^{12} + \eta M_2^{22}] \\ M_2^{22} &= \phi(y_2) - \frac{r+\eta p}{1+r}b + \beta(1-\eta)p M_2^{21} + \beta(1-p)[\eta M_2^{12} + (1-\eta)M_2^{22}] \end{aligned}$$

The above is a linear system of four equations in four unknowns. The analytic solution to

this system is quite complicated and difficult to characterize (though feasible to find), but it is quite easy to determine computationally the solution for simple parameter values.

Consider the following parameterization, but which satisfies Assumptions FR and INT2, but which violates INT1. Thus, for these parameters, the full-repayment scheme is an equilibrium but the interior one is not.

$$\beta = 0.98$$

$$p = 0.01$$

$$r = 0.02$$

$$b = 1.0$$

$$\phi(y_1) = 0.020$$

$$\phi(y_2) = 0.029$$

For varying values of η (non-fundamental persistence), we get the following differences.

Recall that an equilibrium exists if $M_1^{11} < 0$ and the rest are positive.

Value of η	0.0	0.002	0.05
M_1^{11}	-0.9703	-0.9681	-0.9507
M_1^{21}	0.1310	0.0468	-0.6068
M_2^{12}	-0.0138	0.0066	0.0372
M_2^{22}	0.3582	0.3101	0.0646

We can see that, as assumed, when $\eta = 0$ and there is no switching, there is no equilibrium. Also, if η is too large, we lose the equilibrium as well. However, for some persistent process e.g. $\eta = 0.002$, a CW equilibrium does exist. Thus, in a sunspots equilibrium we can sustain a temporary pattern of default in state 1 and repayment in state 2 even when this pattern of default is not itself an equilibrium.

This is possible because INT is close enough to satisfying the equilibrium conditions i.e. it is a ‘potential’ equilibrium. In particular, the value of repayment in the high state is just a bit too small to justify repayment on its own. However, once we insert a sunspot that gives us the opportunity to jump back into a more beneficial regime, the price of repayment in y_2 increases enough to suddenly make repayment worthwhile. Thus the non-equilibrium INT regime is a potential equilibrium and thus can be randomized over in a sunspots equilibrium.

6.A.2 Proof of Proposition 1.4.2

When the equilibrium is not default relevant, then for any (y, m, b) , we will have either $V(y, \xi, m, b) \geq X(y)$ or $X(y) > V(y, \xi, m, b)$ for all $\xi \in \Xi$. Further, if the debt is short-term, then we will have

$$\begin{aligned} q(y, \xi_1, b') &= \frac{1}{R} E_{(\tilde{y}, \tilde{\xi}, \tilde{m})|(y, \xi_1)} \left[1\{V(\tilde{y}, \tilde{\xi}, \tilde{m}, b') \geq X(\tilde{y})\} \right] = \\ &\quad \frac{1}{R} E_{(\tilde{y}, \tilde{\xi}, \tilde{m})|(y, \xi_0)} \left[1\{V(\tilde{y}, \tilde{\xi}, \tilde{m}, b') \geq X(\tilde{y})\} \right] \\ &\rightarrow q(y, \xi_1, b') = q(y, \xi_0, b') = q(y, b') \end{aligned}$$

and so the sunspot does not affect the price. ■

6.A.3 Proof of Proposition 1.4.6

This result follows because, ignoring the non-fundamental ξ , the model becomes isomorphic to the model of Chatterjee and Eyigungor (2012). Thus, the existence result they provide for long-term debt *without* confidence fluctuations still holds. Denote this equilibrium price of debt to be $q(y, b')$.

The European Central Bank can pledge liquidity by guaranteeing to purchase debt at a schedule $q(y, b')$ for the foreseeable future. If it does so, it will induce the sovereign to

adopt the policy rules from the equilibrium free of confidence shifts. When this happens, investors will lend to the sovereign at the price $q(y, b')$, since it is in fact an equilibrium price, and the ECB never actually has to purchase the debt. ■

Chapter 7

Appendix for Chapter 2

7.A Computation and Estimation

7.A.1 Model Solution

We seek a solution of the form:

$$y_t = g(x_{t-1}, \varepsilon_t, \chi, s_t), y_{t+1} = g(x_t, \chi \varepsilon_t, \chi, s_{t+1}), x_t = h(x_{t-1}, \varepsilon_t, \chi, s_t) \quad (7.1)$$

Foerster et al. (2013) demonstrate that a first-order approximation to the solutions g and f can be obtained in two steps. The first step entails solving the following quadratic system for $\{D_{1,n_x} g_{ss}(s_t), D_{1,n_x} h_{ss}(s_t)\}_{s_t=1}^{n_s}$:

$$A(s_t) \begin{bmatrix} \mathbf{I}_{n_x} \\ D_{1,n_x} g_{ss}(1) \\ \dots \\ D_{1,n_x} g_{ss}(n_s) \end{bmatrix} D_{1,n_x} h_{ss}(s_t) = B(s_t) \begin{bmatrix} \mathbf{I}_{n_x} \\ D_{1,n_x} g_{ss}(s_t) \end{bmatrix} \quad (7.2)$$

for all s_t . Where $A(s_t)$ is an $(n_x + n_y) \times (n_x + n_s n_y)$ matrix given by

$$A(s_t) = \left[\sum_{s'=1}^{n_s} p_{s_t, s'} D_{2n_y+1, 2n_y+n_x} f_{ss}(s', s_t), p_{s_t, 1} D_{1, n_y} f_{ss}(1, s_t), \dots, p_{s_t, n_s} D_{1, n_y} f_{ss}(n_s, s_t) \right] \quad (7.3)$$

And $B(s_t)$ is an $(n_x + n_y) \times (n_x + n_y)$ matrix given by:

$$B(s_t) = - \sum_{s'=1}^{n_s} p_{s_t, s'} \left[D_{2n_y+n_x+1, 2(n_y+n_x)} f_{ss}(s', s_t), D_{n_y+1, 2n_y} f_{ss}(s', s_t) \right] \quad (7.4)$$

The second step uses the result from the first step and the involves solving two linear systems, the first being

$$\begin{bmatrix} D_{n_x+1, n_x+n_\varepsilon} g_{ss}(1) \\ \dots \\ D_{n_x+1, n_x+n_\varepsilon} g_{ss}(n_s) \\ D_{n_x+1, n_x+n_\varepsilon} h_{ss}(1) \\ \dots \\ D_{n_x+1, n_x+n_\varepsilon} h_{ss}(n_s) \end{bmatrix} = [\Theta_\varepsilon, \Phi_\varepsilon]^{-1} \Psi_\varepsilon \quad (7.5)$$

and the second being

$$\begin{bmatrix} D_{n_x+n_\varepsilon+1, n_x+n_\varepsilon+1} g_{ss}(1) \\ \dots \\ D_{n_x+n_\varepsilon+1, n_x+n_\varepsilon+1} g_{ss}(n_s) \\ D_{n_x+n_\varepsilon+1, n_x+n_\varepsilon+1} h_{ss}(1) \\ \dots \\ D_{n_x+n_\varepsilon+1, n_x+n_\varepsilon+1} h_{ss}(n_s) \end{bmatrix} = [\Theta_\chi, \Phi_\chi]^{-1} \Psi_\chi \quad (7.6)$$

The matrices $\{\Theta_\varepsilon, \Phi_\varepsilon, \Psi_\varepsilon, \Theta_\chi, \Phi_\chi, \Psi_\chi\}$ can be constructed from the solutions to the quadratic system and selected derivatives of the function f . See Foerster et al. (2013) for more details.

The objects from the model that I match to the data are as follows:

- Output: y_t
- Public Current Account: $-\frac{b_t - (1-\lambda)b_{t-1}}{y_t}$
- Spread: $\frac{\lambda + (1-\lambda)(\kappa + q_t)}{q_t} - R_t$

Note that I compute the spread in the same way as Chatterjee and Eyigungor (2012), by assuming that the price tomorrow is expected to be the same as the price today.

7.A.2 Proof of Theorem 2.3.1

For the purposes of the following results, I denote the i th row and the j th column of $h_{ss}(k)$ to be h_{ij}^k . The same notation applies to the coefficients g_{ij}^k . I consider all rows of these matrices but *only* the first n_x columns, since this is all that is required in the quadratic system. We can reduce the dimensionality of the quadratic problem over a series of three propositions.

(Proposition) 7.A.1 *The coefficients governing the stochastic process are predetermined. If row i is an exogenous stochastic process, then we will have that $h_{ii}^k = \rho(i)$ for all states k , where $\rho(i)$ is the degree of persistence of the process in row i . All other elements in row i must be zero*

Proof This follows mechanically since exogenous stochastic processes are, by definition, not affected by any of the other equilibrium objects. Further, an AR(1) process itself is already linear in its past values, with a coefficient equal to the persistence, so any valid approximation must reflect this. ■

This simple and intuitive step reduces the dimensionality of the problem by $n_s n_x n_{exo}$, where n_{exo} is the number of exogenous stochastic processes. In our case, the dimensionality of the problem drops by 45, which is a substantial improvement but nowhere near large enough yet. We can continue the reduction, though, with another result, which follows from exogeneity of the fiscal rule:

(Proposition) 7.A.2 *The coefficients governing the evolution of government debt process are predetermined.*

Proof This follows from the exogeneity of the fiscal rule. The impact of interest rates, productivity, and policy shocks as well as the impact of past debt can be determined via the relevant derivatives of this fiscal rule. ■

This step reduces the dimensionality by $n_x n_s = 15$. With the exogenous processes predetermined, I now turn to the consumption-saving decision, which is at the crux of the model solution:

(Proposition) 7.A.3 *Given coefficients on investment movements, the coefficients on capital and consumption movements are uniquely determined. Coefficients on capital movements will be the same as the coefficients on investment movements with the exception of capital itself, which requires an additional $1 - \delta$. If row i' corresponds to consumption and row i corresponds to capital, then $g_{i'j}^k = f_j^k - h_{ij}^k$. f_j^k will either be a known constant or some linear combination of other unknowns of the matrices h^k and g^k .*

Proof This result comes from the fact that the problem faced by the household is the marginal allocation of additional income to capital. Thus, given a positive shock to the

budget set of the household, if we know how much of that additional income was allocated to capital, then we simply subtract that amount from the size of the shock to determine how much was allocated to consumption. Therefore, the constants f_j^k are simply the impact on the budget constraint in state k from a unit shock to the state variable in column j . ■

In my case, the set f_j^k are known constants. This result, upon implementation, reduces the dimensionality by $2 \times n_s n_x$, which in our case reduces the dimensionality by 30. This proposition is useful because it is extremely applicable. The consumption-saving decision is the cornerstone of modern macroeconomics and nearly every recursive problem entails this decision at some level. Thus, this technique could have near universal applicability for those seeking to implement the method of Foerster et al. (2013) to derive an approximation to a given equilibrium.

We can reduce the dimensionality once more before we actually solve for the approximation with the following proposition:

(Proposition) 7.A.4 *Suppose that the consumption coefficients have been removed from the system using the techniques described thus far. If i is the row containing the investment coefficient, a solution to a quadratic system of size k entailing only $\{h_{ii}^k\}_{k=1, n_s}$ is both necessary and sufficient to solve the entire quadratic system.*

To see this result, note that after the requirements thus far that have been imposed will imply a quadratic system of 15 equations in 15 unknowns: These unknowns are the linear response of investment to the 5 states variables in each of the 3 states. This system can be stated as follows after some tedious algebra (or, more easily, by use of symbolic engine)

for a set of parameter-determined constants $\{c_{i,j}\}$:

$$\begin{aligned}
0 &= c_{1,0} + c_{1,1}i_{k,1}^2 + c_{1,2}i_{k,1}i_{k,2} + c_{1,3}i_{k,1}i_{k,3} + \sum_{i=1}^3 c_{1,i+3}i_{k,i} \\
0 &= c_{2,0} + c_{2,1}i_{k,2}^2 + c_{2,2}i_{k,1}i_{k,2} + c_{2,3}i_{k,2}i_{k,3} + \sum_{i=1}^3 c_{2,i+3}i_{k,i} \\
0 &= c_{3,0} + c_{3,1}i_{k,3}^2 + c_{3,2}i_{k,1}i_{k,3} + c_{3,3}i_{k,2}i_{k,3} + \sum_{i=1}^3 c_{3,i+3}i_{k,i} \\
0 &= c_{4,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{4,(i-1)*3+j}i_{b,i}i_{k,j} + \sum_{i=1}^3 c_{4,i+9}i_{b,i} + \sum_{i=1}^3 c_{4,i+12}i_{k,i} \\
0 &= c_{5,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{5,(i-1)*3+j}i_{b,i}i_{k,j} + \sum_{i=1}^3 c_{5,i+9}i_{b,i} + \sum_{i=1}^3 c_{5,i+12}i_{k,i} \\
0 &= c_{6,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{6,(i-1)*3+j}i_{b,i}i_{k,j} + \sum_{i=1}^3 c_{6,i+9}i_{b,i} + \sum_{i=1}^3 c_{6,i+12}i_{k,i} \\
0 &= c_{7,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{7,(i-1)*3+j}i_{R,i}i_{k,j} + \sum_{i=1}^3 c_{7,i+9}i_{b,i} + \sum_{i=1}^3 c_{7,i+12}i_{R,i} \\
0 &= c_{8,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{8,(i-1)*3+j}i_{R,i}i_{k,j} + \sum_{i=1}^3 c_{8,i+9}i_{b,i} + \sum_{i=1}^3 c_{8,i+12}i_{R,i} \\
0 &= c_{9,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{9,(i-1)*3+j}i_{R,i}i_{k,j} + \sum_{i=1}^3 c_{9,i+9}i_{b,i} + \sum_{i=1}^3 c_{9,i+12}i_{R,i} \\
0 &= c_{10,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{10,(i-1)*3+j}i_{g,i}i_{k,j} + \sum_{i=1}^3 c_{10,i+9}i_{b,i} + \sum_{i=1}^3 c_{10,i+12}i_{g,i} \\
0 &= c_{11,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{11,(i-1)*3+j}i_{g,i}i_{k,j} + \sum_{i=1}^3 c_{11,i+9}i_{b,i} + \sum_{i=1}^3 c_{11,i+12}i_{g,i} \\
0 &= c_{12,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{12,(i-1)*3+j}i_{g,i}i_{k,j} + \sum_{i=1}^3 c_{12,i+9}i_{b,i} + \sum_{i=1}^3 c_{12,i+12}i_{g,i} \\
0 &= c_{13,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{13,(i-1)*3+j}i_{z,i}i_{k,j} + \sum_{i=1}^3 c_{13,i+9}i_{z,i} \\
0 &= c_{14,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{14,(i-1)*3+j}i_{z,i}i_{k,j} + \sum_{i=1}^3 c_{14,i+9}i_{z,i} \\
0 &= c_{15,0} + \sum_{i=1}^3 \sum_{j=1}^3 c_{15,(i-1)*3+j}i_{z,i}i_{k,j} + \sum_{i=1}^3 c_{15,i+9}i_{z,i}
\end{aligned}$$

Let us denote a solution to this system by $I^n = \{i_{j,i}^n\}_{j \in \{k,b,R,g,z\}, i=1,3}$, where n indexes the solution from a possible set of many solutions. We can glean from this system that the first three equations are in fact an isolated quadratic system i.e. they contain 3 equations in 3 unknowns: $i_{k,1}, i_{k,2}, i_{k,3}$. None of the other coefficients enter into this system. Further, conditional on having a solution to this system, the remaining 12 equations are linear in their 12 unknowns.

Thus, we can find a solution to this large system as follows:

1. Solve the subsystem given by the first three equations for all solutions,

$\{i_{k,1}^n, i_{k,2}^n, i_{k,3}^n\}_{n=1,N}$, where there are N determinate solutions.

2. For each solution, $n \leq N$, fix $\{i_{k,1}^n, i_{k,2}^n, i_{k,3}^n\}$ as constants and solve Equations 4-15 as a linear system. This will, of course, yield either no solution, a unique solution, or a continuum of solutions for each n .

Let $\hat{N} \leq N$ be number of total solutions for which the linear system for which Step 2 yields a unique solution.* If at n the linear system had no solution, then $\{i_{k,1}^n, i_{k,2}^n, i_{k,3}^n\}$ could not have formed the basis of a solution in the first place, and if at n the linear system had a continuum of solutions, then the approximation (not the equilibrium) would be indeterminate and thus of no use.

Now, I argue that this procedure yields all determinate approximations to the entire system and only those determinate approximations. The latter claim is easy to understand: By construction, any solution constructed with this procedure must be a determinate equilibrium.

The former is also fairly trivial: Suppose that there was another solution, $I^{\hat{n}}$, to the entire system that was not found via this procedure. Then we could isolate the terms $\{i_{k,1}^{\hat{n}}, i_{k,2}^{\hat{n}}, i_{k,3}^{\hat{n}}\}$ from this solution and apply them to the first three equations. Because the procedure did not find them, this subsystem will not be satisfied. But this contradicts the fact that \hat{n} was

*In practice, both N and \hat{N} almost invariably equal 8.

indeed a solution to the system, since all conditions do not hold with equality. Thus, our procedure must find *all* valid solutions and *only* valid solutions. ■

7.A.3 Fixed Parameters

Parameter	Value	Rationale
σ	2	RBC Standard
α	0.33	RBC Standard
β	0.999	RBC Standard
δ	0.005	RBC Standard
z_H	1.0	RBC Standard
Φ''	0.455	RBC Standard
χ	0.4	RBC Standard
ψ	1.5	RBC Standard
R^*	1.01	RBC Standard
z_L	0.95	Mendoza and Yue (2012)
b^*	$-0.19653 \times y_{ss}$	Spanish Data (ECB SWD)
g^*	$0.375289 \times y_{ss}$	Spanish Data (ECB SWD)
ρ_z	0.95	Arias et al. (2007)
σ_z	0.0031	Arias et al. (2007)
σ_g	0.0077	Arias et al. (2007)
ρ_g	0.98	Arias et al. (2007)
σ_r	0.0063	Neumeyer and Perri (2005)
ρ_r	0.81	Neumeyer and Perri (2005)
π_{RE}	0.125	Match Grecian Exclusion
λ	0.0385	Match Average Maturity
κ	0.04	Match Average Coupon
$\hat{\delta}$	0.7861	Equation 2.10
p_H	0.0	Calibrated (Identification)

Table 7.A.1: Fixed Parameters

7.A.4 Kalman Filter: Observables

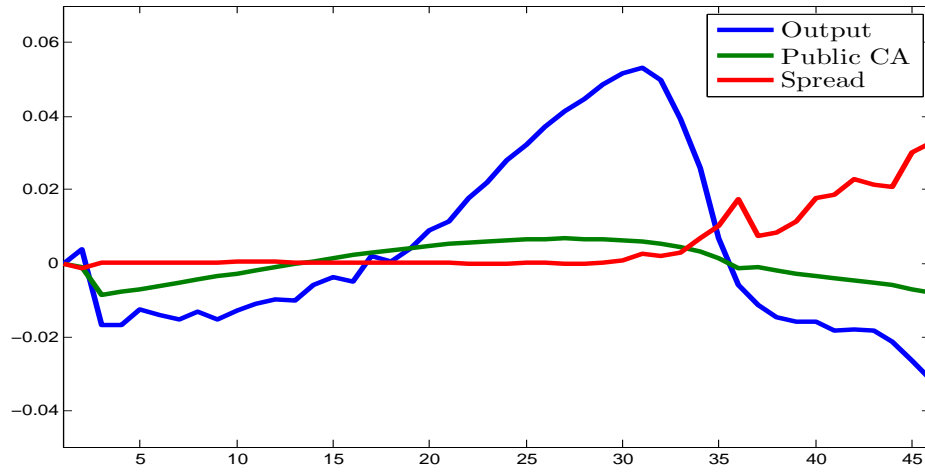


Figure 7.A.1: Model-Implied Observables: Data

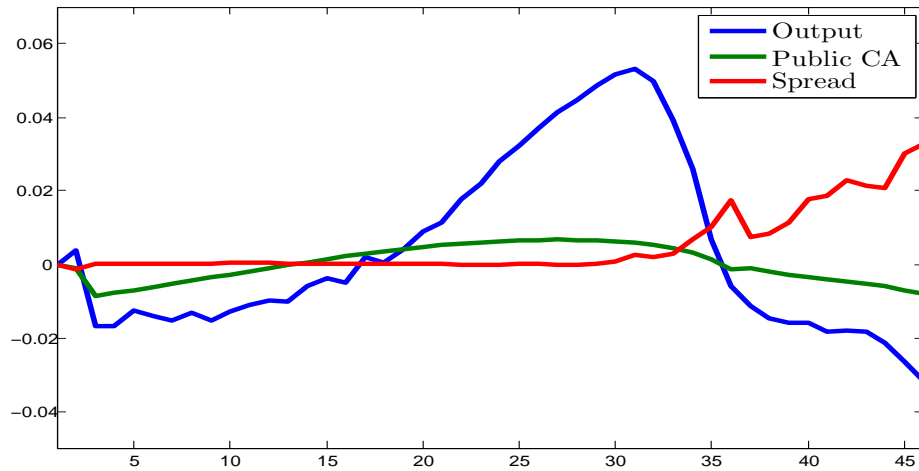


Figure 7.A.2: Model-Implied Observables: Baseline Kalman Filter Predictions

7.A.5 Kalman Filter: Model Components

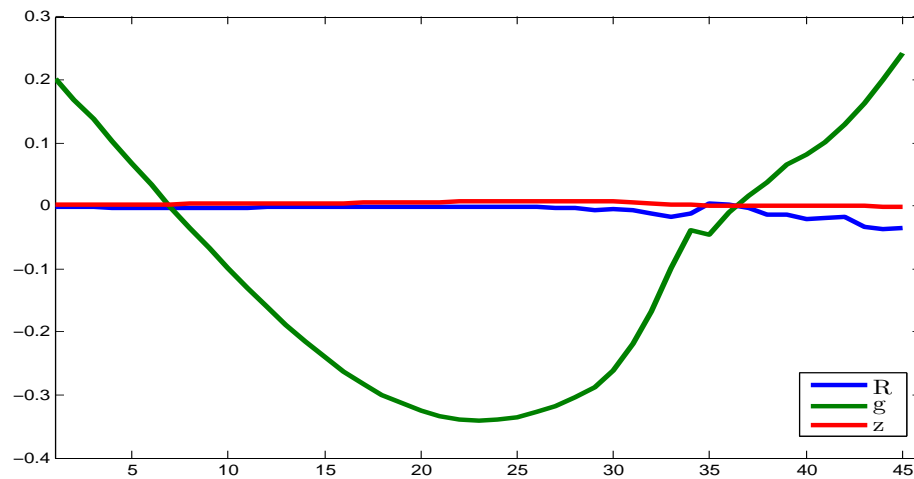


Figure 7.A.3: Exogenous Shocks: Deviations from Steady State

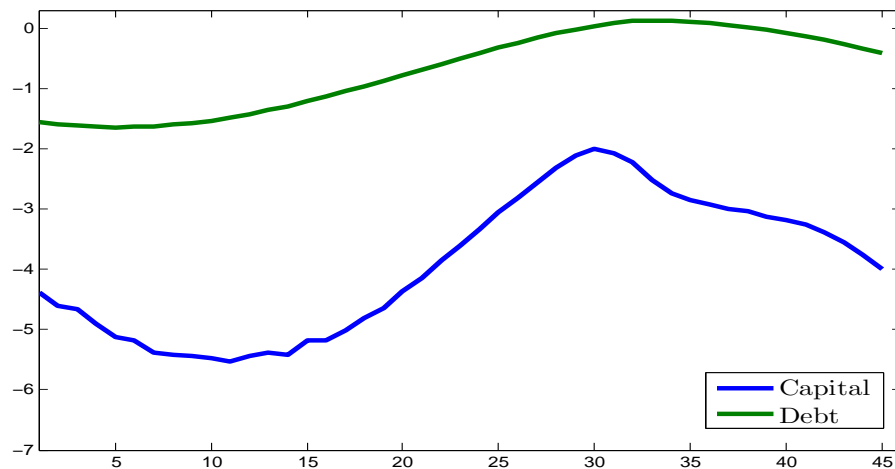


Figure 7.A.4: Endogenous State Variables: Deviations from Steady State

7.A.6 Posterior Distribution

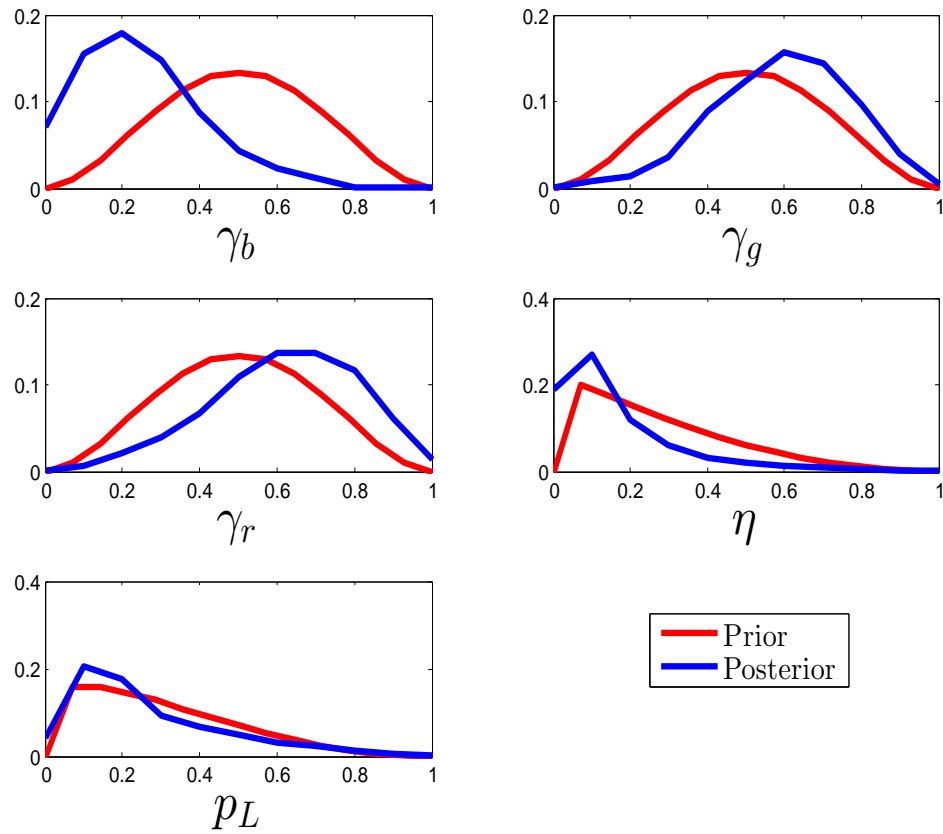


Figure 7.A.5: Posterior Distribution of Baseline Model

7.B Robustness and Model Fit

I present in this section several tests of robustness and model fit for my empirical specification. I find that along all relevant dimensions, the data largely favor my specification.

7.B.1 Posterior Predictive Distributions

I explore here the predictions of my estimated model against the data itself with regards to several key moments. To do this, I follow techniques outlined in Geweke (2005) and

discussed in Geweke (2007). In particular, I do the following:

1. For every draw from the posterior distribution $\theta^{(i)}$, simulate the model for a long time i.e. $T = 100000$ and record key moments in a vector $z^{(i)}$
2. Compare the distribution of $z^{(i)}$ against the same moments computed from the data itself, z

The results of this experiment can be found in Figure 7.B.1. The moments I choose are the mean and volatility of the public current account deficit (PCA) as well as the spread. I also look at the cyclicality of the public current account as well as the correlation between the public current account and the spread. I compute all of these statistics both conditional on a non-crisis state and conditional on a crisis state. The 90% credible sets are outlined in blue, the red line gives the median, and the black lines denote the quartiles. The green dots represent these statistics computed from the data itself.

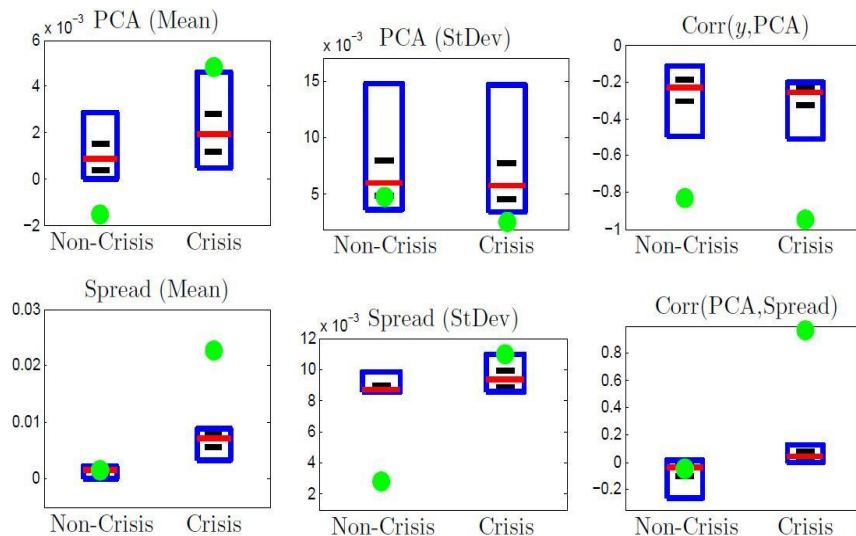


Figure 7.B.1: Posterior Predictive Distributions of Baseline Model

Before I discuss the results, note first that the moments computed from the data, especially during the crisis, are computed from relatively few observations. I only have 9 quarters of crisis observations. Thus, in a frequentist sense, the moments taken from the data are likely not ‘accurate’. Nevertheless, we can see that the model and the data follow the same trends along each dimension and in many cases the model accurately predicts these moments.

First, note that in all cases the model gets the direction correct. For example, during a crisis the public current account deficit is in expectation higher but its volatility is lower; spreads, on the other hand, are higher and more volatile during a crisis. Both of these are true in the estimated model as well. We can see further that the model does a very good job of matching spreads *before* the crisis, which is precisely the parameter used to identify η . Thus, we can rest assured that our estimate of η is likely quite consistent with the actual data, even if the estimated default frequency during a crisis is not.

We also see that virulent spreads tend to be more correlated with the deficit during crisis times and that the model predicts that the public current account is more counter-cyclical during a crisis, which is true in the data as well. This last feature of the model is particularly desirable. It implies that during non-crisis times the economy behaves more like a developed economy, driven largely by TFP and government spending shocks and with the government tending toward consumption smoothing. During a crisis, however, it begins to look more like a developing economy: The risk-free rate shock begins to play a more crucial role since the higher spreads magnify these shocks. This in turn creates a strongly counter-cyclical current account, which accords with the data (see Aguiar and Gopinath (2006) or Neumeyer and Perri (2005)).

7.B.2 Alternative Model Specifications

To determine if my model captures the data accurately, I now estimate a pair of competing models. I will call the baseline model M_A . First, I consider a fairly trivial model, which is simply the baseline model without regime-shifts. In other words, I impose that $p_L = p_H$ and that $\eta = 0.0$. I will call this model M_B .

Next, I consider a variant of the model that allows for bailouts. I call this model M_C . In particular, I assume that there is a constant probability of default, p_D , over the entire horizon. However, during the mid-2000s investors anticipated a full bailout upon a default i.e. $\hat{\delta} = 0$ and only 1 period exclusion from credit markets. The crisis was then a period in which investors suddenly and unexpectedly ceased to expect a bailout. In other words, there is some new, non-zero $\hat{\delta}_c$ that I will estimate and which is followed by a longer period of credit market exclusion.

To solve model C, I consider 4 regimes instead of 3. There are two default regimes, one with a bailout and one without, and two non-default regimes, one that expects a bailout during a default and one that does not. I will interpret the crisis to be switch from anticipating bailouts to not.

The estimates and 90% credible sets for all three models are given in Table 7.B.1. We can glean from casual inspection that none of the fiscal rules differ from one another statistically; they are all within one another's credible sets. Nevertheless, the models differ starkly from one another in terms of their overall fit. The last row in the table contains the posterior odds ratio of the three models, which is the ratio of the model likelihoods integrating out the uncertainty with respect to the parameters in each case. I normalize these figures such that they sum to one. We can see that this figure is .9754 for my specification, which clearly and necessarily dominates both of the other two specifications. This implies that the data strongly favors my baseline specification to these plausible alternatives.

It is not surprising that M_B is rejected, since there are fewer parameters estimated and

Parameter	Median (M_A)	Credible Set (M_A)	Median (M_B)	Credible Set (M_B)	Median (M_C)	Credible Set (M_C)
γ_b	0.0584	[0.0267, 0.1109]	0.0646	[0.0280, 0.1330]	0.0418	[0.0198, 0.0846]
γ_g	0.5095	[0.4037, 0.5987]	0.4869	[0.3579, 0.5813]	0.5349	[0.4353, 0.6172]
γ_r	0.6441	[0.3402, 0.8816]	0.6319	[0.3050, 0.8626]	0.6115	[0.3073, 0.8534]
$\eta(\delta)$	0.0339	[0.0080, 0.1394]	N/A	N/A	0.6695	[0.3894, 0.8829]
$p_L(p_D)$	0.2204	[0.0744, 0.6336]	0.0297	[0.0072, 0.0556]	0.0046	[0.0007, 0.0141]
$\frac{p(M_A y)}{\sum_j p(M_j y)}$	0.9754		0.0228		0.0017	

Table 7.B.1: Posterior Statistics and 90% Credible Sets

therefore the model imposes more restrictions on the empirical inference. What is more surprising is that M_C is strongly rejected as well, since there are as many estimated parameters in this model as in M_A . The reason for the rejection is that, while M_C is able to replicate well the spread dynamics of the data, the response of the real economy is required to be homogeneous before and during the crisis, since the real probability of default never changed. To the contrary, the data suggest a strong, negative response from the domestic economy in response to the crisis. This response can never be captured in M_C , which only features bailout dynamics.

Finally, notice that estimated probability of default is significantly higher in the model with dynamic panics. This result is the least surprising, since the dynamic panics model has the ability to fit two different default probabilities while the other models are required to fit only one. The model without such panics thus places the uniform default probability somewhere between the crisis and non-crisis estimates.

7.B.3 Alternative Example: Italy

The only other country in the Peripheral Eurozone that experienced in a substantial way the sovereign debt crisis but did not receive any kind of direct bailout package during the crisis was Italy. Therefore, as a robustness exercise, I estimate the model on the same three time series from Italy. The results are given in Table 7.B.2.

There are two primary difference between the Spanish and Italian estimates. First, Italian debtholders seemed to expect such crises more frequently than Spanish debtholders.

Parameter	Median	Mean	Credible Set
γ_b	0.1682	0.1736	[0.0829, 0.2814]
γ_g	0.8015	0.7989	[0.7532, 0.8357]
γ_R	0.7069	0.7002	[0.5285, 0.8605]
η	0.0744	0.1083	[0.0051, 0.3121]
ρ_H	0.0515	0.0594	[0.0160, 0.1250]

Table 7.B.2: Italy: Posterior Estimates and 90% Credible Sets

The average quarterly frequency of a confidence crisis is estimated to be about 7.44%, or once every 3.36 years. Again, the mean estimate is even more frequent than this. One can see, however, that the credible set is substantially larger than its Spanish counterpart. This is due to the fact there was more variation in the pre-crisis spreads for Italy than for Spain. The opposite was true of crisis spreads.

Second, the estimated γ_b , γ_g , and γ_R are higher for Italy i.e. the model infers that Italy responded more aggressively to adverse shocks with fiscal consolidation than Spain. This is due to the fact that Italy started with much larger debt burdens and, while those burdens increased mildly throughout the crisis, the magnitude of this fiscal worsening was not nearly as large as it was for Spain.

Chapter 8

Appendix for Chapter 3

8.A Empirics

8.A.1 Summary Statistics

Summary statistics for the sample of public manufacturing firms are in Table 8.A.1.

8.A.2 Definition of Data Items

- $Assets_t$: Book assets (item 6) in period t .
- ROA_t : Return on assets in period t is defined as operating income before depreciation (item 13) divided by book assets in period t (item 6).
- $Levg_t$: Leverage is defined as debt in current liabilities (item 34) plus long-term debt (item 9) divided by book assets (item 6) in period t . Alternative definitions of leverage are as follows: a) long-term debt (item 9) divided by book assets (item 6) in period t ; b) long-term debt (item 9) divided by total stockholders' equity (item 216) in period t ; c) debt in current liabilities (item 34) plus long-term debt (item 9) divided

Table 8.A.1: Summary Statistics

Dataset	Variable	Obs	Mean	St Dev
S&P Compustat	Firm Size			
	<i>Log(Assets)</i>	94,972	4.687	2.317
	<i>Log(Sales)</i>	95,027	4.643	2.565
	<i>Log(Employment)</i>	86,418	-0.306	2.207
	Investment			
	<i>R&D</i>	60,505	77.072	403.028
	<i>Capx</i>	92,689	116.501	761.145
	<i>R&D Share of Inv</i>	94,055	0.331	0.340
	Financing			
	<i>Leverage</i>	94,747	0.241	0.206
	<i>Cash Flows</i>	94,709	204.947	1223.079
	<i>Equity Financing</i>	93,905	-1.883	217.182
	<i>Debt Financing</i>	93,667	19.996	460.257
	Other			
<i>Firm Age</i>	95,027	14.266	12.206	
<i>Unique Firms</i>	9,154	—	—	
NBER Patent Database (matched)	Citations	445,351	17.229	25.486
	Claims	445,351	15.528	12.828
	Unique Firms	3,843	—	—

Note: All nominal variables are adjusted by the GDP deflator in the corresponding year and expressed in millions of 2005 U.S. dollars.

by the sum of debt in current liabilities (item 34), long-term debt (item 9) and total stockholders' equity (item 216) in period t .

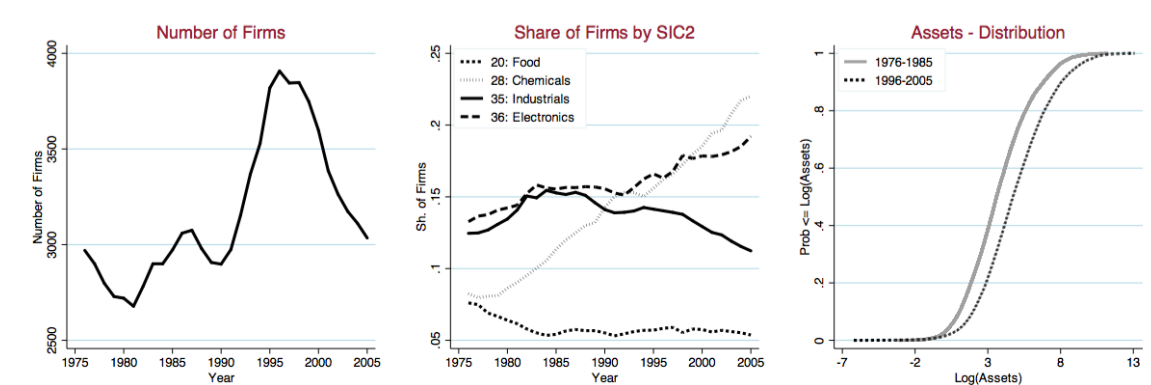
- $R\&D_t$: Research and development expenditures (item 46) in period t .
- $Capx_t$: Capital expenditures (item 128) in period t .
- $CashFlows_t$: Gross cash flows in period t are defined as (after-tax) income before extraordinary items (item 18) in period t plus depreciation and amortization (item 14) in period t plus research and development expenses (item 46) in period t .
- $EquityFin_t$: Equity financing in period t is defined as the sale of common and preferred stock (item 108) in period t minus the purchase of common and preferred stock (item 115) in period t . Alternative definition of equity financing used is the change in the book value of equity (item 216) between period t and $t - 1$ minus change in the balance-sheet item for (accumulated) retained earnings (item 36) between period t and $t - 1$.
- $DebtFin_t$: Debt financing in period t is defined as the long-term debt issuance (item 111) in period t minus long-term debt reduction (item 115) in period t . Alternative definition of debt financing used is the change in total liabilities (item 181) between period t and $t - 1$.
- $SalesGr_t$: Sales growth in period t is defined as sales (item 12) in period t minus sales in period $t - 1$ divided by sales in period $t - 1$.
- $LPGr_t$: Labor productivity growth is defined as labor productivity in period t minus labor productivity in period $t - 1$ divided by labor productivity in period $t - 1$. Labor productivity in period t defined as sales (item 12) in period t divided by employment (item 29) in period t .

- $FirmAge_t$: Firm age in period t is defined as the number of years from when the firm first appeared in the Compustat to period t .
- $Cite_t$: Forward citation for a particular patent are defined as the number of future patents citing it. The citation count is adjusted by a weight to correct for truncation as derived in Hall, et al (2001).
- $Claim_t$: The claims in the patent specification outline the property rights protected by the patent and define the novel features of the invention.
- $Innovative$: A firm is defined as innovative in period t if they report positive R&D expenditures in period t . Alternative definitions used rely on the cutoff for R&D share of investment and average reporting of R&D throughout the firm history.

8.A.3 Composition of Firms in the Compustat

Figure 8.A.1 shows the trends in the number of firms in the sample, share accounted for by the most prominent industries, and the size distribution change over time.

Figure 8.A.1: Compustat Sample Composition Change Over Time

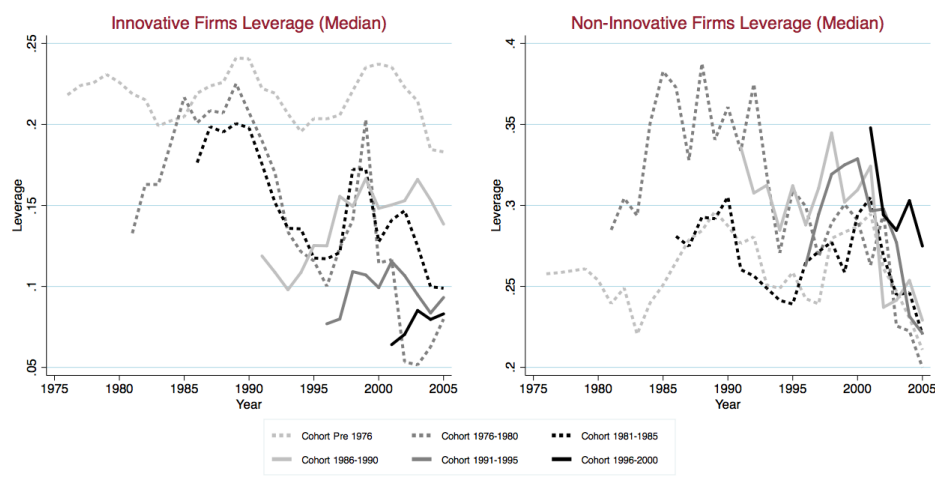


8.A.4 Additional Capital Structure Trends

In this section, we present additional facts regarding capital structure and firm financing.

Cohort Analysis: Although median and mean leverage decreased over the period of 1976-2005, it is not yet clear whether this change was due to differences in the financing behavior of incumbent firms or differences in the characteristics of firms that made an IPO within that period. To shed light on this, we break our sample into 6 cohorts depending on when the firms first show up in the sample and look at their respective leverage trends separately. The results of this experiment are demonstrated in Figure 8.A.2. The leverage trends for innovative firms appear to be a combination of two factors: a decrease in leverage over time for older cohorts and a lower initial level of leverage for each entering cohort. The decrease of leverage within the cohort is more pronounced for the firms which entered the sample between 1976-1980 and 1981-1985. The observation of lower leverage of entering cohorts is prevalent for each innovative cohort throughout this period. We do not observe similar trends for the sample of those firms which do not innovate. More specifically, although there does seem to be a slight decrease in leverage in the last decade for the older cohorts, some entrants appeared in the sample with higher levels of leverage than the incumbent firms.

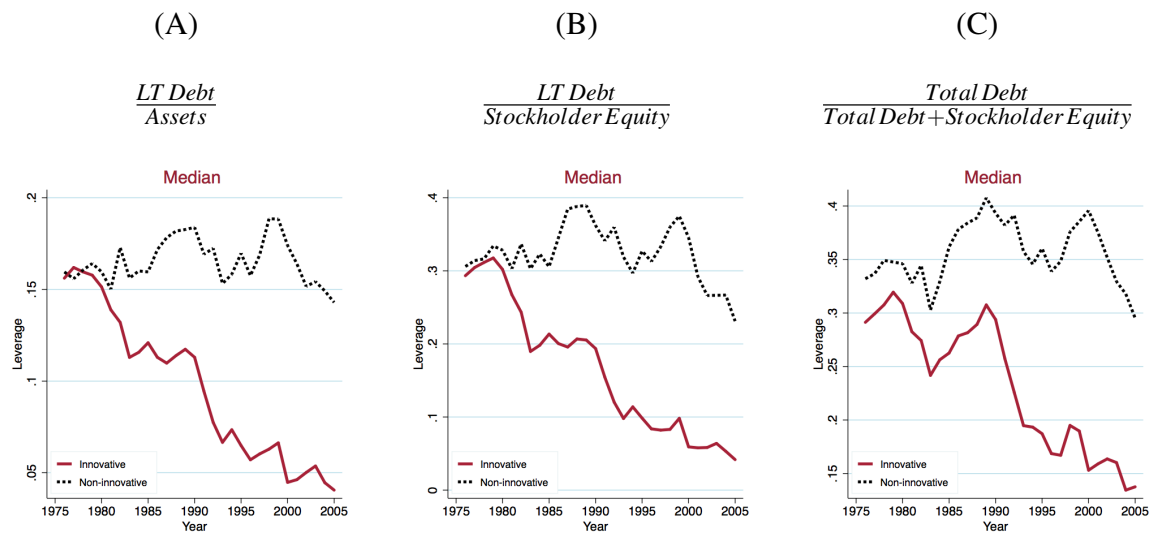
Figure 8.A.2: Leverage Cohort Trends



Alternative Definitions of Leverage: The observed leverage trends are robust to dif-

ferent definitions of leverage. Using three alternative definitions, we demonstrate that there has been a decrease in leverage among innovative firms, but not their counterparts. For more details regarding alternative definitions of leverage as well as the advantages and disadvantages of each specification, we refer the readers to Rajan and Zingales (1995) and Frank and Goyal (2009).

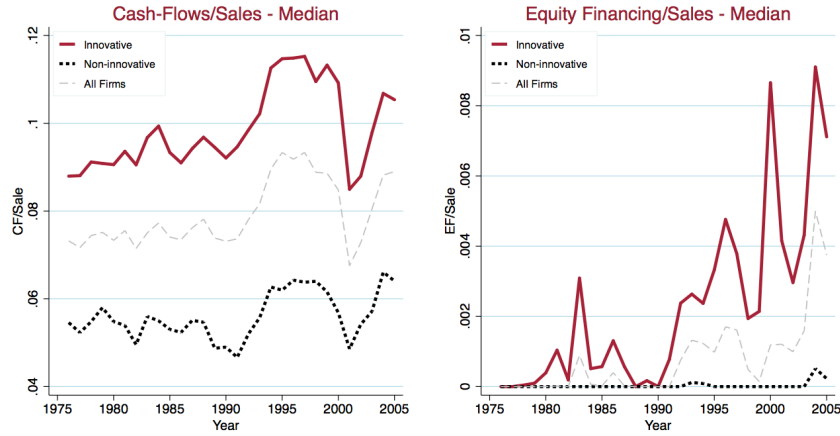
Figure 8.A.3: Leverage (Alternative Definitions) Trends



Firm Internal and External Financing: In Figure 8.A.4, we demonstrate how the availability of internal financing and external equity financing scaled by sales has evolved over the period for innovative and non-innovative firms. There do not seem to be pronounced trends in the availability of internal financing (cash-flows) for either type of firm. On the other hand, there is a striking increase in the use of external equity financing among innovative firms beginning in the 1990's. This result further confirms our interpretation of leverage trends among innovative firms resulting from greater use of equity as opposed to debt to finance investment. This observation is also consistent with the finding in Brown et al. (2009) of increased equity financing for high-tech public firms over the period since a significant fraction of innovation activity is concentrated among these firms.

Leverage Regressions: In order to explore the relationship between capital structure

Figure 8.A.4: Internal and External Financing



and innovation controlling for other firm observables, we run the following regression for firm-year observations:

$$Lev_{i,t} = \beta_0 + \beta_1(R\&D_{i,t}/Inv_{i,t}) + \beta_3X_{i,t-1} + \delta_j + \delta_t + \varepsilon_{i,t}$$

where $X_{i,t-1}$ contains a measure of firm size, $Log(Assets)$, and profitability, ROA . Industry and year fixed effects are denoted by δ_j and δ_t respectively. The results of this regression are presented in Table 8.A.2. The negative relationship between leverage and firm profitability and the positive relationship between leverage and firm size is consistent with findings by Fama and French (2002) and Frank and Goyal (2009). In addition to these observations, we find that leverage is negatively related to a measure of firm-level innovation intensity, R&D share of investment. This holds true even when firm fixed effects are included in the regression.

Next, we explore how these relationships changed over time. We modify the specification above to include an indicator for the years 1996-2005 and run the following regression for the data containing only periods 1976-1985 and 1996-2005:

Table 8.A.2: Leverage and Firm Size Regressions (Variables Winsorized)

	(1)	(2)	(3)	(4)	(5)
	OLS	OLS	OLS	OLS	FE
$\text{Log}(\text{Assets}_{i,t-1})$	0.00331*** (0.000325)	0.00611*** (0.000339)	0.00664*** (0.000337)	0.00793*** (0.000354)	0.0269*** (0.00104)
$\text{ROA}_{i,t-1}$		-0.0635*** (0.00358)	-0.101*** (0.00375)	-0.104*** (0.00380)	-0.121*** (0.00474)
$\text{R\&D}_{i,t}/\text{Inv}_{i,t}$			-0.127*** (0.00304)	-0.101*** (0.00338)	-0.0212*** (0.00412)
Constant	0.216*** (0.00823)	0.199*** (0.00846)	0.231*** (0.00840)	0.216*** (0.00873)	0.0943*** (0.00681)
Year dum	Yes	Yes	Yes	Yes	Yes
Industry dum	Yes	Yes	Yes	Yes	No
Cohort dum	No	No	No	Yes	No
Observations	84709	84502	83814	83814	83814
R^2	0.089	0.094	0.118	0.125	0.613
Adjusted R^2	0.087	0.092	0.116	0.123	0.569

Robust standard errors in parentheses.

Source: COMPUSTAT 1976-2005

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

$$\text{Levg}_{i,t} = \beta_0 + \beta_1(\text{R\&D}_{i,t}/\text{Inv}_{i,t}) + \beta_3\text{Present} + \beta_4\text{Present} \times (\text{R\&D}_{i,t}/\text{Inv}_{i,t}) + \beta_5X_{i,t-1} + \delta_j + \delta_t + \varepsilon_{i,t}$$

The *Present* indicator captures the average trend in leverage controlling for the other observables and interacting this indicator with R&D share of investment captures the changes in the relationship between innovation and leverage over time. Therefore, $\beta_1 < 0$ and $\beta_4 < 0$ denotes that the negative relationship between leverage and innovation has become even more so over time. Table 8.A.3 shows that this is indeed the case even when fixed effects are included.

Table 8.A.3: Leverage and Firm Size Trend Regressions (Variables Winsorized)

	(1)	(2)	(3)	(4)
	OLS	OLS	OLS	FE
<i>Present</i>	-0.0415*** (0.00505)	0.00836 (0.00537)	0.00657 (0.00538)	-0.0238*** (0.00497)
$R\&D_{i,t}/Inv_{i,t}$		-0.0344*** (0.00523)	-0.0494*** (0.00554)	-0.0196** (0.00692)
$R\&D_{i,t}/Inv_{i,t} \times Present$		-0.111*** (0.00553)	-0.121*** (0.00583)	-0.0143* (0.00714)
$Log(Assets_{i,t-1})$			0.00651*** (0.000409)	0.0233*** (0.00133)
$ROA_{i,t-1}$			-0.101*** (0.00458)	-0.116*** (0.00600)
Constant	0.296*** (0.00886)	0.288*** (0.00873)	0.245*** (0.00910)	0.140*** (0.00683)
Year dum	Yes	Yes	Yes	Yes
Industry dum	Yes	Yes	Yes	Yes
Observations	63083	62583	55550	55550
R^2	0.081	0.107	0.128	0.667
Adjusted R^2	0.079	0.105	0.126	0.612

Robust standard errors in parentheses

Source: COMPUSTAT 1976-2005.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

8.A.5 Additional Innovation Trends

In this section, we present additional facts regarding firm-level risk and innovation. Figure 8.A.5 shows an increase in the cross-sectional and within firm dispersion in labor productivity growth. Figure 8.A.6 instead shows the increase in patent citations scaled by the mean patent citations in each year.

Figure 8.A.5: Sales Growth Volatility Trends

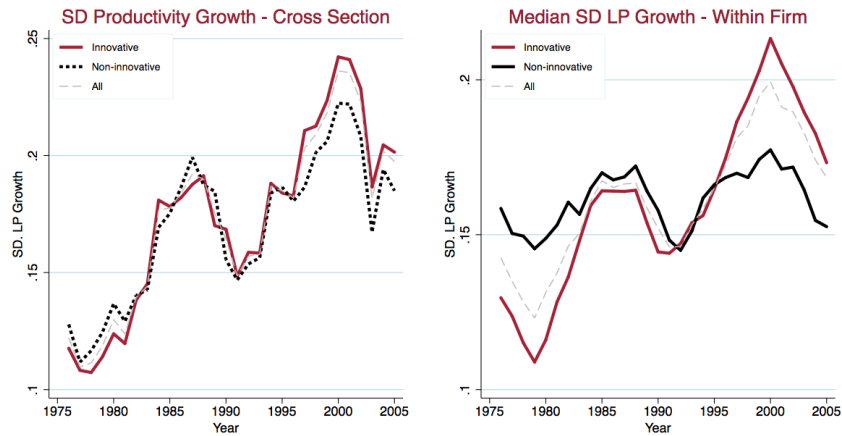
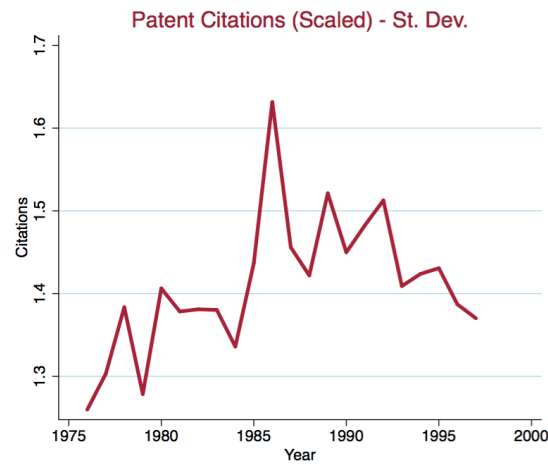


Figure 8.A.6: Patent Citations Trends



8.B Solution Method

We know that the firm's optimal capital structure will either be debt-only or a mixture of the two. It will never use equity alone since equity-financing a sub-optimal contract and the firm has access to the optimal contract, so it will choose to finance at least a portion of the project with debt.

The debt-only solution is pinned down uniquely by the lender's participation constraint:

$$\int_{x_L}^{D+c_D} (x - c_D) f(x) dx + \int_{D+c_D}^{x_H} D f(x) dx = I$$

It is easy to show that the LHS is *concave* and *strictly increasing* in D , so long as there is positive mass on the entire domain of x .

$$FOC(D) = \int_{D+c_D}^{x_H} f(x) dx > 0$$

$$SOC(D) = -f(D + c_D) < 0$$

Thus, if there is a feasible repayment scheme i.e. I is not too large, then then that repayment scheme is unique and we can simply solve this lender's participation constraint to obtain the debt-only solution.

The mixed capital solution will involve a trade-off of debt and equity at the margin, thus the FOC of the Lagrangian optimization problem will hold (it is not sufficient for an overall maximum, but it is necessary for an interior maximum). To estimate the model, we solve for both types of solutions (debt alone and mixed capital structure), and then determine which yields the higher payoff. After estimation, we verify that the interior solutions found are indeed optimal with a grid-search algorithm, since there can be multiple local maxima (and indeed, some local minima).

8.C Theoretical Proofs

Proof of Proposition 3.3.1: Given $c_D = c_E = 0$, the investors will choose to monitor with probability 1 and the asymmetry of information disappears. Thus, the firm solves the problem:

$$\begin{aligned} & \max_{D \geq 0, \beta \in [0,1]} (1 - \beta) \int_D^{x_H} (x - D) f(x) dx \\ \text{s.t. } & \int_{x_L}^D x f(x) dx + \int_D^{x_H} D f(x) dx + \int_D^{x_H} \beta (x - D) f(x) dx \geq I \end{aligned}$$

We can substitute out for β from the constraint and simplify the integrals, giving the following problem

$$\max_{D \geq 0} \left(1 - \frac{I - \int_{x_L}^D x f(x) dx - D[1 - F(D)]}{\int_D^{x_H} x f(x) dx - D[1 - F(D)]} \right) \left(\int_D^{x_H} x f(x) dx - D[1 - F(D)] \right)$$

Which simplifies to

$$\max_{D \geq 0} Ex - I$$

Regardless of the choice of D (and concomitant β , via the lender participation constraint), the firm's payoff is the same, $Ex - I$. Thus, the firm is completely indifferent to the choice of capital structure. ■

Proof of Theorem 3.3.2: In the debt subgame, supporting beliefs on the part of the lender are given by $\tilde{D}^{-1}(\tilde{D})$ in the region of partial default. When $\tilde{D} < x_L - c_D$, any beliefs will support the action of monitoring; the same is true of acceptance when $\tilde{D} \geq D$.

In the equity subgame with no absolute priority violation, supporting beliefs are given by $\tilde{E}^{-1}(\tilde{E})$ in the region of equilibrium repayment. If $\tilde{E} < E_L$, any beliefs support intervention; the same is true for $\tilde{E} \geq E_H$ to support acceptance.

In the equity subgame with absolute priority violation, supporting beliefs are the same

as the debt sub-game on the equilibrium path i.e. $\tilde{D}^{-1}(\tilde{D})$, since the equity holder observes the recovery rate by assumption. *Off* the equilibrium path, if $\tilde{E} > E_L$, then equity-holders believe the type to be x_H with probability 1, implying that the expected payoff from intervention is E_H . Thus, so long as $E_H = \beta(x_H - D - c_E) > c_D$, which is the largest bribe the firm can offer, these beliefs can sustain an equilibrium. The threshold \hat{x} is thus given by

$$\hat{x} = \frac{c_D}{\beta} + D + c_E$$

If $\tilde{E} < E_L$, then any beliefs will support intervention.

Lastly, the firm's payoffs need to be continuous at the point $x = D + c_D$ i.e. this threshold firm needs to be indifferent between joining the default subgame and the repayment subgame. In the default subgame, a payment of D would be accepted for certain by the creditor, and so the firm would immediately enter the AP violation subgame.

$$\eta(c_D - \beta(c_D - c_E)) + (1 - \eta)((1 - \beta)(c_D - c_E)) =$$

$$P_E(\beta(c_D - c_E))(c_D - \beta(c_D - c_E)) + (1 - P_E(\beta(c_D - c_E)))(1 - \beta)(c_D - c_E)$$

Since $P(\beta(c_D - c_E)) = \eta e^{\frac{\tilde{E}_L - \tilde{E}_L}{c_E}} = \eta$, the above expression always holds.

The equilibrium is unique because the value of η given in the theorem is the only value that simultaneously satisfies the firm's indifference requirements at the threshold $x = D + c_D$ and at the boundary point $x = x_H$ in the equity subgame with full debt repayment. ■

Proof of Proposition 3.3.4: We know that in the region of default the equity-holders' payoff function from a realization x will be E_L conditional on the debt-holders accepting a default. Thus, their unconditional payoff is given by $P_D(\tilde{D}(x))E_L$, which is a continuous function of x . At the point $\hat{x} = D + c_D$, we will have $\tilde{D} = D$ and thus the left-hand side of the payoff function at \hat{x} converges to E_L . Since the equity subgame to the right of \hat{x} simply

provides a linear payoff from E_L to E_H , we know that the right-hand side of the payoff function is continuous and converges to E_L . Thus, the equity payoff function is continuous.

We know that the payoff function is exponential in the range of default, and therefore convex. It is linear in the range of repayment. We need only show that the slope at $D + c_D$ is greater in the case of repayment than default.

$$\begin{aligned}
 FOC(RHS) &= \beta \quad \forall x \\
 FOC(LHS) &= \beta(c_D - c_E) \frac{1 - \beta + \eta\beta}{[\eta c_E + (1 - \beta)(c_D - c_E)]} e^{\frac{1 - \beta + \eta\beta}{[\eta c_E + (1 - \beta)(c_D - c_E)]}(x - c_D - D)} \\
 FOC(LHS)|_{x=D+c_D} &= \beta(c_D - c_E) \frac{1 - \beta + \eta\beta}{[\eta c_E + (1 - \beta)(c_D - c_E)]} = \beta \left[\frac{1 - \beta + \eta\beta}{1 - \beta + \eta \left(\frac{c_E}{c_D - c_E} \right)} \right]
 \end{aligned}$$

Therefore, the LHS will be smaller than the RHS iff

$$\begin{aligned}
 \beta &\leq \frac{c_E}{c_D - c_E} \\
 1/\beta &\geq \frac{c_D}{c_E} - 1
 \end{aligned}$$

We know that $1/\beta \geq 1$ and under the assumption, the largest that c_D/c_E can get is 2. Therefore, this inequality always holds in equilibrium and the function is convex. ■

References

- Aghion, Philippe and Patrick Bolton**, “An Incomplete Contracts Approach to Financial Contracting,” *Review of Economic Studies*, 1992, 59 (3), 473–494.
- and **Peter Howitt**, “A Model of Growth Through Creative Destruction,” *NBER Working Paper No. 3223*, 1990.
- , **George-Marios Angeletos, Abhijit Banerjee, and Kalina Manova**, “Volatility and Growth: Credit Constraints and Productivity-Enhancing Investment,” *NBER Working Paper No. 11349*, 2005.
- , **John Van Reenen, and Rajan Zingales**, “Innovation and Institutional Ownership,” *American Economic Review*, 2013, 103 (1), 277–304.
- Aguiar, Mark and Gita Gopinath**, “Defaultable Debt, Interest Rates, and the Current Account,” *Journal of International Economics*, 2006, 69 (1), 64–83.
- and **Manuel Amador**, “Sovereign Debt,” *Handbook of International Economics*, 2014, 4.
- , **Satyajit Chatterjee, Harold Cole, and Zachary R. Stangebye**, “Sovereign Debt,” *Handbook of Macroeconomics, Volume 2 (Forthcoming)*, 2015.

- Aizenman, Joshua, Michael Hutchison, and Yothin Jinjarak**, “What is the Risk of Sovereign European Debt Defaults? Fiscal Space, CDS Spreads, and Market Pricing of Risk,” *Journal of International Money and Finance*, 2013, 34, 37–59.
- Arellano, Cristina**, “Default Risk and Income Fluctuations in Emerging Economies,” *American Economic Review*, 2008, 98 (3), 690–712.
- and **Ananth Ramanarayanan**, “Default and Maturity Structure in Sovereign Bonds,” *Journal of Political Economy*, 2012, 120 (2), 187–232.
- Arias, Andres, Gary D. Hansen, and Lee E. Ohanian**, “Why Have Business Cycle Fluctuations Become Less Volatile?,” *Economic Theory*, 2007, 32 (1), 43–58.
- Armenter, Roc and Viktoria Hnatkovska**, “The Macroeconomics of Firms’ Savings,” *Philadelphia Federal Reserve Working Paper No. 12-1*, 2012.
- Auclert, Adrien and Matthew Rognlie**, “Unique Equilibrium in the Eaton-Gersovitz Model of Sovereign Debt,” *MIT Department of Economics Graduate Student Research Paper No. 14-01*, 2014.
- Azariadis, Costas**, “Self-Fulfilling Prophecies,” *Journal of Economic Theory*, 1981, 25 (3), 380–396.
- Bajaj, Mukesh, Yuk-Shee Chan, and Sudipto Dasgupta**, “The Relationship between Ownership, Financing Decisions, and Firm Performance: A Signaling Model,” *International Economic Review*, 1998, 39 (3), 723–744.
- Barsky, Robert B. and Eric R. Sims**, “Information, Animal Spirits, and the Meaning of Innovations in Consumer Confidence,” *NBER Working Paper No. 15049*, 2009.

- Bernanke, Ben S., Mark Gertler, and Simon Gilchrist**, “The Financial Accelerator in a Quantitative Business Cycle Framework,” *Handbook of Macroeconomics*, 1999, 1 (C), 1341–1393.
- Bi, Huixin and Nora Traum**, “Estimating Sovereign Default Risk,” *American Economic Review*, 2012, 102 (3), 161–166.
- Bianchi, Francesco**, “Regime Switches, Agents’ Beliefs, and Post-World War II U.S. Macroeconomic Dynamics,” *The Review of Economic Studies*, 2013, 80 (2), 463–490.
- Blackburn, Keith, Niloy Rose, and Salvatore Capasso**, “Financial Development, Financing Choice, and Economic Growth,” *Review of Development Economics*, 2005, 9 (2), 135–149.
- Bocola, Luigi**, “The Pass-Through of Sovereign Risk,” *Job Market Paper*, 2014.
- Boyd, John H. and Bruce D. Smith**, “How Good Are Standard Debt Contracts? Stochastic Versus Nonstochastic Monitoring in a Costly State Verification Environment,” *The Journal of Business*, 1994, 67 (4), 539–561.
- ___ and ___, “The Evolution of Debt and Equity Markets in Economic Development,” *Economic Theory*, 1998, 12 (3), 519–560.
- ___ and ___, “The Use of Debt and Equity in Optimal Financial Contracts,” *Journal of Financial Intermediation*, 1999, 8 (4), 270–316.
- Broner, Fernando, Aitor Erce, Alberto Martin, and Jaume Ventura**, “Sovereign Debt Markets in Turbulent Times: Creditor Discrimination and Crowding-Out Effects,” *Journal of Monetary Economics*, 2014, 61, 114–142.

- Brown, James R., Steven M. Fazzari, and Bruce C. Petersen**, “Financing Innovation and Growth: Cash Flow, External Equity, and the 1990’s R&D Boom,” *The Journal of Finance*, 2009, 64 (1), 151–185.
- Calvo, Guillermo A.**, “Servicing the Public Debt: The Role of Expectations,” *American Economic Review*, 1988, 78 (4), 647–661.
- Cass, David and Karl Shell**, “Do Sunspots Matter?,” *Journal of Political Economy*, 1983, 91 (2), 193–227.
- Chatterjee, Satyajit and Burcu Eyigungor**, “Maturity, Indebtedness, and Default Risk,” *American Economic Review*, 2012, 102 (6), 2674–2699.
- Cole, Harold L. and Timothy J. Kehoe**, “A Self-Fulfilling Model of Mexico’s 1994-1995 Debt Crisis,” *Journal of International Economics*, 1996, 41 (3-4), 309–330.
- Comin, Diego and Thomas Philippon**, “The Rise in Firm-level Volatility: Causes and Consequences,” *NBER Macroeconomics Annual 2005*, 2005.
- Conesa, Juan Carlos and Timothy J. Kehoe**, “Gambling for Redemption and Self-Fulfilling Debt Crises,” *Federal Reserve Bank of Minneapolis Research Department Staff Report 465*, 2012.
- Corsetti, Giancarlo and Luca Dedola**, “The Mystery of the Printing Press: Self-Fulfilling Debt Crises and Monetary Sovereignty,” *Working Paper*, 2013.
- Covas, Francisco and Wouter J. Den Haan**, “The Role of Debt and Equity Finance over the Business Cycle,” *The Economic Journal*, 2012, 122 (565), 1262–1286.
- De Grauwe, Paul**, “The European Central Bank: Lender of Last Resort in the Government Bond Markets?,” *CESifo Working Paper Series 3569*, *CESifo Group Munich*, 2011.

- and **Yuemei Ji**, “Self-fulfilling Crises in the Eurozone: An Empirical Test,” *Journal of International Money and Finance*, 2013, 34, 15–36.
- Dewatripont, Mathias and Jean Tirole**, “A Theory of Debt and Equity: Diversity of Securities and Manager-Shareholder Congruence,” *Quarterly Journal of Economics*, 1994, 109 (4), 1027–1054.
- Diamond, Douglas W. and Philip H. Dybvig**, “Bank Runs, Deposit Insurance, and Liquidity,” *Journal of Political Economy*, 1983, 91 (3), 401–419.
- Eaton, Jonathon and Mark Gersovitz**, “Debt with Potential Repudiation: Theoretical and Empirical Analysis,” *Review of Economic Studies*, 1981, 48 (2), 289–309.
- Fama, Eugene F. and Kenneth R. French**, “Testing Trade-Off and Pecking Order Predictions About Dividends and Debt,” *Review of Financial Studies*, 2002, 15 (1), 1–33.
- Fernandez-Villaverde, Jesus and Juan Rubio-Ramirez**, “Estimating Macroeconomic Models: A Likelihood Approach,” *Review of Economic Studies*, 2007, 74 (4), 1059–1087.
- Foerster, Andrew, Juan Rubio-Ramirez, Dan Waggoner, and Tao Zha**, “Perturbation Methods for Markov-Switching DSGE Models,” *Atlanta Fed Working Paper 2013-1*, 2013.
- Frank, Murray Z. and Vidhan K. Goyal**, “Capital Structure Decisions: Which Factors Are Reliably Important?,” *Financial Management*, 2009, 38 (1), 1–37.
- Gale, Douglas and Martin Hellwig**, “Incentive-Compatible Debt Contracts: The One-Period Problem,” *Review of Economic Studies*, 1985, 52 (4), 647–663.
- and — , “Repudiation and Renegotiation: The Case of Sovereign Debt,” *International Economic Review*, 1989, 30 (1), 3–31.

- Geweke, John**, *Contemporary Bayesian Econometrics and Statistics*, Hoboken, NJ: Wiley-Interscience, 2005.
- , “Bayesian Model Comparison and Validation,” *American Economic Review*, 2007, 97 (2), 60–64.
- Gilson, Stuart**, “Management Turnover and Financial Distress,” *Journal of Financial Economics*, 1989, 25 (2), 241–262.
- Gornemann, Nils**, “Sovereign Default, Private Investment, and Economic Growth,” *Job Market Paper*, 2014.
- Gottardi, Piero and Atsushi Kajii**, “The Structure of Sunspot Equilibria: The Role of Multiplicity,” *Review of Economic Studies*, 1999, 66 (3), 713–732.
- Greenwood, Jeremy and Boyan Jovanovic**, “Financial Development, Growth, and the Distribution of Income,” *Journal of Political Economy*, 1990, 98 (5), 1076–1107.
- and **Bruce Smith**, “Financial Markets in Development and the Development of Financial Markets,” *Journal of Economic Dynamics and Control*, 1997, 21 (1), 147–181.
- , **Zvi Hercowitz, and Gregory W. Huffman**, “Investment, Capacity Utilization, and the Real Business Cycle,” *American Economic Review*, 1988, 78 (3), 402–417.
- Grinblatt, Mark and Chuan Yang Hwang**, “Signalling and the Pricing of New Issues,” *The Journal of Finance*, 1989, 44 (2), 393–420.
- Hall, Bronwyn H.**, “The Financing of Research and Development,” *NBER Working Paper No. 8773*, 2002.
- , **Adam B. Jaffe, and Manuel Trajtenberg**, “The NBER Patent Citation Data File: Lessons, Insights, and Methodological Tools,” *NBER Working Paper No. 8498*, 2001.

- Hamilton, James D.**, “A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle,” *Econometrica*, 1989, 57 (2), 357–384.
- Hatchondo, Juan Carlos and Leonardo Martinez**, “Long-Duration Bonds and Sovereign Default,” *Journal of International Economics*, 2009, 79 (1), 117–125.
- Hodrick, Robert J. and Edward C. Prescott**, “Postwar U.S. Business Cycles: An Empirical Investigation,” *Journal of Money, Credit, and Banking*, 1997, 29 (1), 1–16.
- Hoelle, Matthew**, “The Relation Between Sunspot Effects and Multiplicity in Incomplete Markets Models with Numeraire Assets,” *Journal of Mathematical Economics*, 2014, 50, 119–127.
- Hvide, Hans K. and Tore Leite**, “Capital Structure Under Costly Enforcement,” *Scandinavian Journal of Economics*, 2008, 110 (3), 543–565.
- Issing, Otmar**, “Slithering to the Wrong Kind of Union,” *Financial Times*. August 8, 2011, 2011, pp. Retrieved from www.ft.com/intl/cms/s/0/c4159b34-cla8-11e0-acb3-00144feabdc0.html.
- Itenberg, Olga**, “Firm Size, Equity Financing, and Innovation Activity,” *Working Paper*, 2013.
- Jones, Benjamin F.**, “The Burden of Knowledge and the “Death of the Renaissance Man”: Is Innovation Getting Harder?,” *Review of Economic Studies*, 2009, 76 (1), 283–317.
- Karabarbounis, Loukas and Brent Neiman**, “Declining Labor Shares and the Global Rise of Corporate Saving,” *NBER Working Paper No. 18154*, 2012.
- King, Robert G. and Ross Levine**, “Finance and Growth: Schumpeter Might Be Right,” *Quarterly Journal of Economics*, 1993, 108 (3), 717–737.

- and — , “Finance, Entrepreneurship, and Growth: Theory and Evidence,” *Journal of Monetary Economics*, 1993, 32 (3), 513–542.
- Klette, Tor Jakob and Samuel Kortum**, “Innovating Firms and Aggregate Innovation,” *Journal of Political Economy*, 2004, 112 (5), 986–1018.
- Lane, Philip R.**, “The European Sovereign Debt Crisis,” *Journal of Economic Perspectives*, 2012, 26 (3), 49–68.
- Lanjouw, Jean O. and Mark Schankerman**, “Patent Quality and Research Productivity: Measuring Innovation with Multiple Indicators,” *The Economic Journal*, 2004, 114 (495), 441–465.
- Leeper, Eric M.**, “Equilibria Under ‘Active’ and ‘Passive’ Monetary and Fiscal Policies,” *Journal of Monetary Economics*, 1991, 27 (1), 129–147.
- Leland, Hayne E. and David H. Pyle**, “Informational Asymmetries, Financial Structure, and Financial Intermediation,” *The Journal of Finance*, 1977, 32 (2), 371–387.
- Levine, Ross and Sara Zervos**, “Stock Markets, Banks, and Economic Growth,” *American Economic Review*, 1998, 88 (3), 537–558.
- Lorenzoni, Guido and Ivan Werning**, “Slow Moving Debt Crises,” *NBER Working Paper No. 19228*, 2013.
- Lubik, Thomas A. and Frank Schorfheide**, “Testing for Indeterminacy: An Application to U.S. Monetary Policy,” *American Economic Review*, 2004, 94 (1), 190–217.
- McGrattan, Ellen R. and Edward C. Prescott**, “Average Debt and Equity Returns: Puzzling?,” *American Economic Review*, 2003, 93 (2), 392–397.

- Mendoza, Enrique G. and Vivian C. Yue**, “A General Equilibrium Model of Sovereign Default and Business Cycles,” *Quarterly Journal of Economics*, 2012, 127 (2), 889–946.
- Modigliani, Franco and Merton H. Miller**, “The Cost of Capital, Corporation Finance, and the Theory of Investment,” *American Economic Review*, 1958, 48 (3), 261–297.
- Na, Seunghoon, Martin Uribe, Stephanie Schmitt-Grohe, and Vivian Z. Yue**, “A Model of the Twin Ds: Optimal Default and Devaluation,” *NBER Working Paper No. 20314*, 2014.
- Neumeyer, Pablo A. and Fabrizio Perri**, “Business Cycles in Emerging Economies: The Role of Interest Rates,” *Journal of Monetary Economics*, 2005, 52 (2), 345–380.
- Obstfeld, Maurice**, “Models of Currency Crises with Self-Fulfilling Features,” *European Economic Review*, 1996, 40 (3-5), 1037–1047.
- Passadore, Juan and Juan Pablo Xandri**, “Robust Conditional Predictions in Dynamic Games: An Application to Sovereign Debt,” *Job Market Paper*, 2014.
- Philippon, Thomas**, “Has the US Finance Industry Become Less Efficient? On the Theory and Measurement of Financial Intermediation,” *NBER Working Paper No. 18077*, 2012.
- Puch, Luis A. and Omar Licandro**, “Are There Any Special Features in the Spanish Business Cycle?,” *Investigaciones Economicas*, 1997, 21 (2), 361–394.
- Rajan, Raghuram G. and Luigi Zingales**, “What Do We Know About Capital Structure? Some Evidence from International Data,” *The Journal of Finance*, 1995, 50 (5), 1421–1460.

- and — , “Financial Dependence and Growth,” *American Economic Review*, 1998, 88 (3), 559–586.
- Reinhart, Carmen M. and Kenneth S. Rogoff**, “From Financial Crash to Debt Crisis,” *NBER Working Paper No. 15795*, 2010.
- Rodrik, Dani and Andres Velasco**, “Short-Term Capital Flows,” *NBER Working Paper No. 7364*, 1999.
- Romer, Paul M.**, “Increasing Returns and Long-Run Growth,” *Journal of Political Economy*, 1986, 94 (5), 1002–1037.
- Rousseau, P.L. and P. Wachtel**, “Equity Markets and Growth: Cross-country evidence on timing and outcomes, 1980-1995,” *Journal of Banking and Finance*, 2000, 24 (12), 1933–1957.
- Salomao, Julianne**, “Sovereign Debt Renegotiation and Credit Default Swaps,” *Job Market Paper*, 2014.
- Schmitt-Grohé, Stephanie and Martín Uribe**, “Optimal Simple and Implementable Monetary and Fiscal Rules,” *Journal of Monetary Economics*, 2007, 54 (6), 1702–1725.
- Schorfheide, Frank and Sungbae An**, “Bayesian Analysis of DSGE Models,” *Econometric Reviews*, 2007, 26 (2-4), 113–172.
- Shell, Karl**, “Sunspot Equilibrium,” in “The New Palgrave Dictionary of Economics, 2nd Edition,” New York: Palgrave MacMillan, 2008.
- Spence, Michael**, “Job Market Signaling,” *Quarterly Journal of Economics*, 1973, 87 (3), 355–374.

Townsend, Robert M., “Optimal Contracts and Competitive Markets with Costly State Verification,” *Journal of Economic Theory*, 1979, 21, 265–293.

Williamson, Stephen D., “Costly Monitoring, Financial Intermediation, and Equilibrium Credit Rationing,” *Journal of Monetary Economics*, 1986, 18 (2), 159–179.

Wolf, Martin, *The Shifts and the Shocks: What We’ve Learned-And Still Have to Learn-From the Financial Crisis*, New York, NY: Penguin Press, 2014.

Wruck, Karen Hopper, “Financial Distress, Reorganization, and Organizational Efficiency,” *Journal of Financial Economics*, 1990, 27 (2), 419–444.