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adaptive controller for robot arms

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# Comparative Experiments with a New Adaptive Controller for Robot Arms

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## Abstract

This paper presents a new adaptive controller and proof of its global asymptotic stability with respect to the standard rigid body model of robot arm dynamics. Experimental data from a study of this and other globally asymptotically stable adaptive controllers on two very different robot arms (i) reconciles several previous contrasting empirical studies (ii) demonstrates and compares their superior tracking performance (iii) examines contexts which compromise their advantage.

## 1 Introduction

Several years ago a flurry of activity amongst robotic control theorists [6, 23, 21, 27, 11] resulted in a new class of adaptive controllers for robot arm manipulators. These algorithms comprised the first in the literature whose stability could be proven rigorously with respect to the highly nonlinear rigid body dynamical model. While many of these authors empirically demonstrated significant performance gains over traditional PD controllers, no systematic empirical comparisons *between* the provably correct rigid body model-based schemes, as applied to various robot plants, seem to have been attempted.

The purpose of the present paper is threefold. First, we wish to present a new rigid body model-based adaptive controller that achieves a slight but potentially significant theoretical advance over past contributions. Second, we offer the first (to the best of our knowledge) empirical comparison within this family of closely related but conceptually and algorithmically distinct adaptive controllers. Finally, we compare the performance of the family when implemented upon a typical industrial manipulator SCARA arm with that achieved upon our new "Yale Bühgler" three degree of freedom direct drive juggling robot. Our data corroborate in part claims made both by the proponents and the detractors of model reference adaptive control for robot arms. In particular they suggest that

1. The tracking performance of rigid body model based controllers is generally superior to conventional PD algorithms.
2. Adaptive model based control algorithms consistently outperform their non-adaptive counterparts. There is only marginal performance distinction between the various adaptive controllers.

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3. Model-based algorithms which feed forward reference trajectory information rather than actual state information yield significant performance benefits *when the controller model is valid*; they fail dramatically (in relative terms) when the actuator model is violated (i.e. under actuator torque saturation).
4. The degree of performance improvement afforded by all model-based algorithms is strictly limited by the accuracy of the plant model employed.

## 2 Background Theory

The multitude of adaptive algorithms that have been proposed over the past decade prohibits our consideration of the literature as a whole. We have therefore restricted the scope of our investigation to those whose asymptotic stability with respect to the commonly accepted exact rigid-body nonlinear robot dynamical model has been established. We were unable to implement any of an entire class of provably correct adaptive algorithms based on exact adaptive linearization as developed in [6, 19, 17] which require instrumentation of joint acceleration.

Adopting the error coordinates,  $e = [e_1, e_2]^T = [r - q, \dot{r} - \dot{q}]^T$ , the control algorithm used almost without exception in every industrial robot available today is the proportional-derivative linear controller, labeled *PD* in the figures of Section 4, and given by  $\tau_{pd} = Ke$ ;  $K = [K_1, K_2]$ .

### 2.1 Model Based Algorithms

#### 2.1.1 ID: Fixed Nonlinear Inverse Dynamics

As of this writing, the most generally discussed algorithm to achieve robot tracking is the "computed torque controller" [9, 15]

$$\tau_{ct} \triangleq g(q_1) + C(q_1, q_2)q_2 + M(q_1)[\ddot{r} + \tau_{pd}] \quad (1)$$

where the matrix  $M(q_1)$  represents joint inertial terms,  $C(q_1, q_2)$  represents coriolis terms, and the vector  $g(q_1)$  represents gravitational terms in the usual model, resulting in asymptotically stable linear time invariant error dynamics, and thus asymptotically exact tracking.

$$\tau_{id} \triangleq g(q_1) + C(q_1, q_2)\dot{r} + M(q_1)\ddot{r} + \tau_{pd} \quad (2)$$

We shall use, instead of (1), a less well known variation, (2), labelled *ID* in the figures of Section 4 which provides for asymptotically exact tracking *without* exact linearization.

We choose this approach because it admits of adaptive extensions which are globally convergent in both state and parameter error, unlike adaptive versions of (1) which have been shown to be globally convergent in plant state error and only locally stable in parameter error [6].

The equation (2) can be equivalently written  $\tau_{id} = W(q_1, q_2, \dot{r}, \ddot{r})\theta^* + \tau_{pd}$  where  $\theta^*$  is a vector of robot inertial parameters and  $W(q_1, q_2, \dot{r}, \ddot{r})$  a nonlinear matrix valued function determined by the robot kinematic parameters. Note that a straightforward use of total energy for the non-autonomous closed loop system resulting from (2) is not a satisfactory Lyapunov function as its time derivative is merely negative semidefinite.

### 2.1.2 IDC: “Critically Damped” Fixed Nonlinear Inverse Dynamics

If the PD component of (2) is chosen in a “critically damped manner,”  $K = K_2[\Lambda, I]$  with respect to this first order system<sup>1</sup> and the following feedback law is employed

$$\tau_{idc} \triangleq \tau_{id} + C\Lambda e_1 + M\Lambda e_2 \quad (3)$$

the resulting closed loop error system has been shown to be globally asymptotically stable with respect to both state and parameter errors [23, 21].

### 2.1.3 IDCA: Adaptive Critically Damped Nonlinear Inverse Dynamics

Following the presentations in [23, 21] define a new reference signal  $\dot{r}' = \dot{r} + \Lambda e_1$ , and notice that IDC (3) may be equivalently written  $\tau_{idc} = W(q_1, q_2, \dot{r}', \ddot{r}')\theta^* + \tau_{pd}$ . Equipped with a stability argument for the case of known parameters, the IDC algorithm (3) an adaptive version of (3), labelled IDCA in the figures to follow is

$$\begin{aligned} \tau_{idca} &= W(q, \dot{q}, \dot{r}', \ddot{r}')\hat{\theta} + \tau_{pd} \\ \dot{\hat{\theta}} &= K_g W(q, \dot{q}, \dot{r}', \ddot{r}')^T K_2^{-1} K e. \end{aligned} \quad (4)$$

### 2.1.4 IDRA: Pure Feedforward Versions

Several researchers [22, 26] have observed that a substitution of reference information for state information in the feedforward portion of (2) can still provide asymptotically exact tracking. This algorithm will be denoted IDR in the figures below. Moreover, a pure feedforward version of the adaptive algorithm, denoted IDRA in the figures below, has been presented in [22].

$$\begin{aligned} \tau_{idra} &= W(r, \dot{r}, \ddot{r})\hat{\theta} + \tau_{pd} + \sigma_n \|e\|^2 K_2^{-1} K e \\ \dot{\hat{\theta}} &= K_g W(r, \dot{r}, \ddot{r})^T K_2^{-1} K e \end{aligned} \quad (5)$$

## 2.2 IDA: A New Adaptive Controller

As matters stand, there is some reason to reconsider the problem of adaptive versions of the ID algorithm (2). A satisfactory theory has been developed for the more specialized

<sup>1</sup>We continue to assume that  $K_2$  has a positive definite symmetric part.

IDC algorithm (3) leading to the adaptive controller IDCA (4) of [21, 23]. In contrast, the absence of a satisfactory stability argument for the ID error system has stalled the development of an adaptive version of ID (2),

$$\tau_{ida} \triangleq W(q, \dot{q}, \dot{r}, \ddot{r})\hat{\theta} + \tau_{pd}, \quad (6)$$

that we shall refer to as IDA in the sequel. The question remains whether some adjustment law can be found for  $\hat{\theta}$  that yields global asymptotic stability in both state and parameter error.

In point of fact, at least three different groups [12, 26, 24] have developed strict Lyapunov functions for the ID error system. Each group independently arrived at (essentially) the same idea (first expounded by Arimoto [24]) of adding to the total energy a cross term that is bilinear in position and velocity. Unfortunately this is not a global Lyapunov function: it requires some a priori known bound (which can be arbitrarily large) on the initial position error magnitude,  $e_1$ . In consequence, the adaptive version of ID (2) offered by Bayard and Wen [27] based upon this Lyapunov function suffers from the requirement of an a priori known bound (which can be arbitrarily large) on the initial parameter error magnitude as well.

Two years ago, Koditschek [13] presented a new strict global Lyapunov function for general mechanical systems that includes in its general purview the ID error dynamics. The specialization of the general idea to the present case amounts to nothing more than a modification of the bilinear cross term of the strict (but local) Lyapunov function [24, 26, 12], as will be seen below. Using this new Lyapunov function, we derive in this section an adaptive law to accompany the IDA controller (6) that yields global asymptotic stability in the state and parameter errors.

### 2.2.1 A New Lyapunov Function

Now consider the modified Lyapunov candidate,

$$\vartheta \triangleq \eta + \epsilon(e_1 e_1^T M(q_1) e_2 = \frac{1}{2} e^T \begin{bmatrix} K_1 & \frac{1}{2}\epsilon M \\ \frac{1}{2}\epsilon M & M \end{bmatrix} e$$

where

$$\epsilon(e_1) \triangleq \frac{\epsilon_0}{1 + \|e_1\|}.$$

Note that a sufficiently small choice of  $\epsilon_0$  guarantees that this is a positive definite function with respect to the error coordinates,  $e_1, e_2$ , for any time varying trajectory,  $q(t)$ . The reader is referred to [29] for the complete proof.

### 2.2.2 Stability of the New Algorithm

Define the adaptive law for (6) to be

$$\dot{\hat{\theta}} = K_g W^T [e_2 + \epsilon e_1]. \quad (7)$$

The scalar valued function

$$v \triangleq \vartheta + \frac{1}{2} \hat{\theta}^T K_g^{-1} \hat{\theta},$$

has a derivative along the motion of the full adaptive system,

$$\begin{aligned} \dot{v} &= -e_2^T (K_2 e_2 + W \hat{\theta}) + \epsilon e_2^T M e_2 - \epsilon e_1^T [C e_2 + K e + W \hat{\theta}] + \\ &\quad \epsilon e_1^T \dot{M} e_2 + \dot{\epsilon} e_1^T M e_2 + \hat{\theta}^T K_g^{-1} \dot{\hat{\theta}} \\ &\leq -\epsilon Q \|e\|^2 - [e_2 + \epsilon e_1]^T W \hat{\theta} + \hat{\theta}^T K_g^{-1} \dot{\hat{\theta}} \\ &\leq -\epsilon Q \|e\|^2, \end{aligned}$$

that is non-positive <sup>2</sup>

It follows that  $v$  is bounded, hence,  $\sqrt{\epsilon}\|e\|$  is an  $\mathcal{L}^2$  function [18]. But an  $\mathcal{L}^2$  function whose derivative is bounded must tend to zero [18], hence  $\|e\| \rightarrow 0$  as desired.

### 3 Experimental Setup

We will study the performance of the controllers implemented on two different robots. The first, the Yale-GMF A-500 Industrial Arm, is a classical industrial manipulator as described in [28]. The second, the three degree of freedom direct drive Yale Bühgler<sup>3</sup>, was designed to support our research program in robot juggling [5, 4, 20]. The very different tasks for which these machines were designed result in two significantly different mechanical systems. It seems valuable to compare the performance of the algorithms described above on these two very different machines.

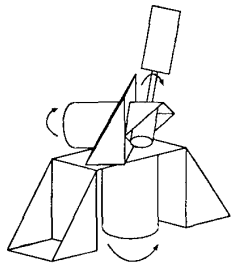


Figure 1: The Yale Bühgler Arm

The computational hardware for these implementations is a network of Yale XP/DCS nodes [8], a distributed real-time controllers based upon the SGS-Thomson INMOS T-800 Transputer 1Mflop floating point microprocessor. The particular architecture used for the experiments discussed here is detailed in [28].

The model-based controller implementations described herein employ the *exact* Lagrangian dynamical equations, for *fully* general link inertial tensors, without omission of a single term. A set of symbolic derivation programs both automatically generated the dynamical equations and translated the resulting symbolic expressions into the high-level language used in the real-time controllers. The control laws were all exactly evaluated at two time scales – the feedback terms at 1KHz and the model-based terms at 400Hz. It should be pointed out that in our present two-node implementation of the high level controller, we can easily run the feedback portion as fast as 2KHz, but the somewhat lower bandwidth of the analog electronics limits the (expected) performance benefits of these higher sample rates.

### 4 Experimental Results

The overall conclusion to be derived from these experiments is probably best summarized by Figure 2. This plot depicts the mean and variance of root mean square position errors

<sup>2</sup>Where  $Q$  is a positive constant depending on known bounds as in [29].

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achieved by each of the seven controllers described in Section 2. The ensemble of runs over which these descriptive statistics are gathered comprise ten very different reference trajectories — different not only in frequency content, but in the region and volume of jointspace they encompass. The results are normalized for convenience with respect to the simple PD controller since all physical significance of the joint angle errors is vitiated by the diversity of trajectories being compared.

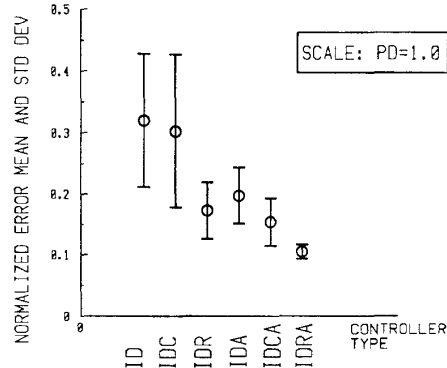


Figure 2: Bühgler Normalized Position Error Norm Ensemble Mean and Standard Deviation Over Ten Different Reference Trajectories for Seven Controllers

Note that the (identical) feedback gain matrices employed for all controllers were considerably lower than the limit dictated by the usual tuning process. We were interested in *comparing* the *relative* performance of the different controllers in an unbiased fashion, and did *not* push gains to the verge of instability to obtain the smallest tracking error magnitude. Higher feedback gains were observed (of course) to provide uniformly smaller steady state tracking errors, but identical relative performance between the various controllers.

#### 4.1 Data Presentation and Repeatability

It has become accepted practice in the robotics community to compare controller performance by the visual examination of tracking error curves as a function of time for a “representative” or “standard” reference trajectory. We wish to compare tracking performance over a variety of reference trajectories. We have employed the scalar valued  $\mathcal{L}^2$  norm —  $\mathcal{L}^2[e(t)] = (\frac{1}{T} \int_{t_0}^t \|e(t)\|^2 dt)^{\frac{1}{2}}$  — an objective numerical measure of tracking performance for an entire error curve. The norm measures the root-mean-square “average” of the tracking error, thus a smaller  $\mathcal{L}^2$  norm represents smaller tracking error — and thus better performance.

#### 4.2 Performance Benefits Due to the Adaptive Algorithms

In general, the performance of each model based controller is improved roughly fifty percent by its adaptive counterpart as shown in Figure 2. Of course, a fair comparison between the fixed and adaptive model based algorithms is complicated

by the issue of where to obtain the necessary parameter estimates for the former class. When hand-measured physical link-mass and link-inertia measurements are used, the adaptive algorithms evince clearly superior performance as shown, for example, in the higher frequency columns of Figure 5. On the other hand, when the fixed model controllers are given parameters resulting from their adaptive counterparts' convergence over a long run, it is not surprising that the adaptive controllers perform little better if at all. Yet, since parameters "optimally tuned" for one reference trajectory are in general "sub-optimal" with respect to any other, the fixed controllers always perform less capably than their adaptive counterparts in any other context. This is reflected not merely in the lower means of Figure 2, but in the comparatively smaller variance of the adaptive algorithms relative to their fixed parameter counterparts.

The reference trajectories were sinusoids, and the error norms are plotted at three different nominal reference trajectory frequencies. The frequency range was chosen to include slow, friction-dominated operation at one end to dynamics-dominated operation at the other. In this plot the slowest frequency corresponds to peak gripper velocity of 0.5 meter/second, and the highest frequency corresponds to peak gripper speeds of 3.0 meters/second. The model based controllers provide tracking performance superior (smaller error norm) to conventional PD control at equal feedback gains.

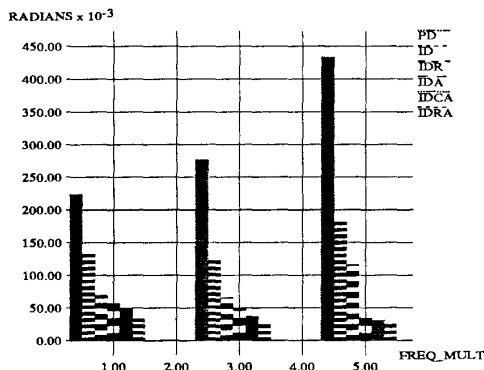


Figure 3: Buhgler Position Error Norm (Radians) vs Reference Trajectory Frequency-Multiplier

Amongst the model based controllers (both fixed and adaptive) the IDR and IDRA (see section 2.1.4) controllers which utilize reference trajectory values in their plant model are uniformly outperforming the controllers which use sensor values in their plant model. We can attribute the poorer performance of the latter algorithms to the noisy velocity signal, obtained via numerical differentiation of encoder position, used in their evaluation, in contrast to the uncorrupted reference trajectory velocity signal used in the former case. As demonstrated in [29], however, these controllers are more easily destabilized by commonplace model defects such as actuator torque saturation.

The IDC and IDCA controllers are seen to marginally outperform the ID and IDA controllers respectively. This consistent difference, due to the differing error feedback structure of the two algorithms, is discussed in [29].

Finally, adaptive controllers were observed to be less robust than the non-adaptive controllers in the presence of certain unmodeled effects such as link vibration modes, actuator saturation, numerical integration, and the like which may occur when the reference trajectories exceed the system's de-

sign capability.

### 4.3 The Effect of Incorrect Parameter Values

It is commonly agreed that effective non-adaptive model-based control relies on the availability of correct model parameter values. A common misconception, however, is that "any model is better than none" – that an "approximately correct" parameter set will result in better tracking than that obtained by PD control alone.

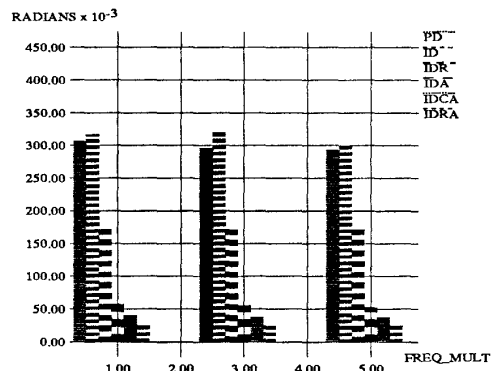


Figure 4: Unmatched Parameters: Buhgler Link 1 Position Error Norm (Radians) vs Reference Trajectory Frequency-Multiplier

In fact, incorrect parameter values may result in *poorer* performance than that of PD control. Figure 4 shows the error norms for link 1 of the Buhgler obtained with the same reference trajectories, controllers, and initial parameters as Figure 3. Here the "gripper payload" was removed from the distal link, and the resulting parameter mismatch degrades the non-adaptive controller performance. The adaptive controllers, in contrast, compensate automatically for the change.

### 4.4 Contrast Between Direct-Drive and Geared Joint Performance

A comparison of the breakdown of typical tracking error for (direct drive) joint 0 and (gear drive) joint 1 for the A-500, as shown in Figure 5 and 6 respectively, reveals striking differences. Figure 5, representing the position error norm for (direct drive) joint 0, shows *all* model based controllers performing *significantly* better than the PD controller — the distinction increasing with nominal reference trajectory frequency. In contrast, Figure 6, representing the position error norm for (gear drive) joint 1, shows only marginal improvement.

We attribute excellent tracking performance of Joint 0 to the relevance of the rigid body model. Conversely, the relatively poor tracking performance of joint 1 under the model based controllers may be attributed to the *completely* unmodeled dynamics of its actuator — consisting of a 47:1 spiroidal gearbox and DC motor. This conjecture is supported by the uniformly superior tracking performance of *all* joints under the model based controllers for totally direct-drive mechanical unit presented in section 4.2. The spiroidal

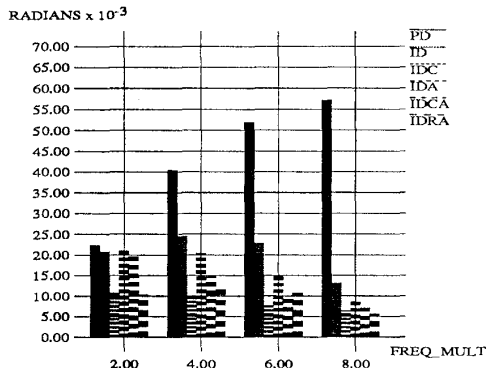


Figure 5: A-500 Joint 0 Position Tracking Error (Rad.) vs. Reference Trajectory Frequency-Multiplier

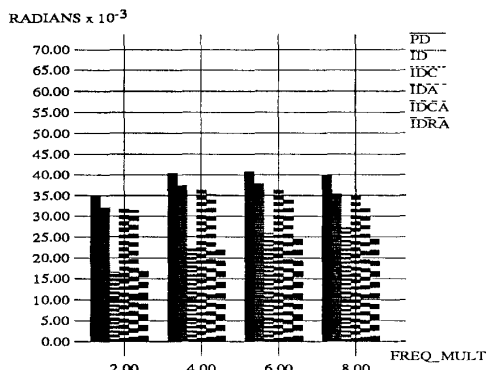


Figure 6: A-500 Joint 1 Position Tracking Error (Rad.) vs. Reference Trajectory Frequency-Multiplier

gear box used in joint 1 of the A-500 is, in fact, a dynamically simple mechanism with only two moving parts, providing very high stiffness and (unfortunately) a noticeable backlash. It is not surprising that other researchers have observed more curious (and even repeatable) performance defects in robots with more complicated actuator systems, e.g. [14].

The relatively poor showing of the model based controllers at low velocities of Figure 6 (in comparison to their excellent showing in Figure 3) occurs because former controller did not incorporate a friction, while the latter included a linearly parameterized model for coulomb and viscous friction. A complete discussion of these effects in [29] demonstrates the necessity of incorporating friction models for accurate low-speed tracking.

#### 4.5 “Long Term Memory” Effects

Recently, a fundamentally different set of “learning” techniques — neural networks [10, 7]; memory-based learning [3]; and repetitive learning [2, 16] methods have challenged the hegemony of Lagrangian model-based methods in robust controller design. The principal advantage of the “learning” control algorithms would be the promise of accurately controlling enormously complicated plants without explicitly modeling the plant’s underlying dynamics. Their disadvantage is the need to repetitively learn each unique task — they

are unable to apply the knowledge of “learned” parameters to any but the original task. While much of the early work in this area was heuristic, recent results directly address stability and robustness [2, 16], thus establishing some of these techniques as theoretically sound alternatives to model based adaptive robot control.

In contrast, the adaptive robot controllers’ “learning” processes (parameter convergence) occur simultaneously with task execution, obviating the need for a separate “learning” phase. Moreover, after achieving parameter convergence, they can (in theory) apply this knowledge to track *any* smooth reference trajectory. In practice, however, these advantages are compromised by the following effects. First, adaptive parameter convergence relies on richness properties of signals within the system [18] that commonplace workplace tasks may fail to produce. Second, we observe in practice that adaptive parameters converge to slightly different “optimal” values for differing reference trajectories, rather than converging to a single value for all rich trajectories, and exhibit (theoretically disallowed) transients when transitioning from one reference to another.<sup>4</sup>

We demonstrate these effects as follows. After running the robot long enough to establish steady-state tracking of a selected trajectory, an analytic “switch” ( $\arctan()$  and its derivatives) smoothly transitioned the controller reference to a second, different, signal. We can thus observe the transient response of the various controllers when transitioning between any two (possibly very different) reference trajectories.

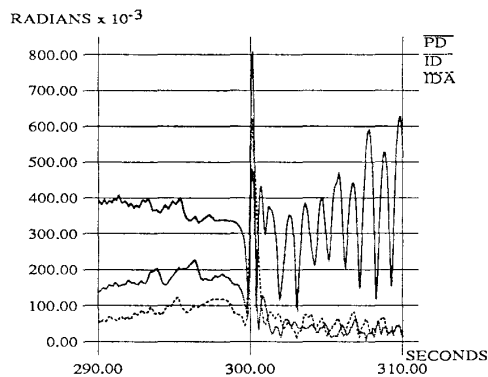


Figure 7: Long Term Memory Effects: Bühgler Instantaneous Position Error Norm (Radians) vs Time (Seconds)

We tried a variety of reference trajectory combinations to find examples evoking a transient response in which the ID controller outperformed its adaptive counterpart.<sup>5</sup> Figure 7, shows the instantaneous position error norm for the transition between extremely slow sinusoids (friction dominates) on the left, and fast sinusoids on the right (dynamics dominates). Given a sufficiently long interval on the slow trajectories (300 seconds in this example), the IDA adaptive parameters drift was sufficient to produce a larger transient excursion than the ID controller. Within a few seconds the

<sup>4</sup>Implementation of representative “learning” controllers falls beyond the scope of the present investigation. Thus this discussion will focus on revealing the nature and extent of the above-mentioned defects in model-based controller performance.

<sup>5</sup>The excellent ID performance in these plots reflects the use of a nearly “optimal” fixed parameter set. Note that it would have been easy to “stack the deck” against ID by giving it a poor parameter set — resulting in arbitrarily poor ID performance.

IDA controller recovers to equal the performance of the ID controller, and it becomes superior at steady state.

We conclude that the performance of model-based, and in particular adaptive model-based, controllers is not seriously compromised by the imperfect parameter convergence mentioned above. The adaptive controller *usually* outperforms both its non-adaptive counterpart as well as PD control. We argue that the worst-case “defects” of the adaptive algorithms (brief transients) are relatively innocuous in comparison to their demonstrated advantages over all of the fixed controllers.

## 5 Conclusion

This paper has reviewed the stability literature for a class of model reference parameter adaptive controllers for robot arm manipulators based upon the ID (2) variant of the popular computed torque algorithm (1). It provides for the first time a rigorous and global stability proof for IDA (6), a member of this class that has heretofore eluded a complete analysis. Comparative experiments of all these variants have been performed on a standard industrial SCARA manipulator and a fast direct drive robot arm developed at the Yale Robotics Laboratory. The highlights of the observations of Section 4 having been previewed in the introduction of the paper, we will only briefly summarize and amplify here.

Fixed model based controllers dramatically outperform the PD controller and their adaptive counterparts perform still better. Thus, if a designer is committed to a computed torque-like controller, since the (computationally intensive)  $W$  terms of (2) must be computed anyway (and since the parameter adaptation integrals represent very minor additional computational burden), the adaptive variant should be preferred.

The degree of performance improvement afforded by model-based algorithms is strictly limited by the accuracy of the plant model employed. For example — the failure of the commonly accepted Lagrangian rigid-body model to represent real-world effects such as joint friction, actuator saturation, gearbox dynamics, and actuator dynamics is shown to result in lackluster performance indistinguishable from conventional PD algorithms. When better models are incorporated, performance benefits are immediate.

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Martin Bühler<sup>6</sup> was responsible in large measure for the original conception and design of the Yale “Bühler” Robot. We thank him for his advice and assistance in pursuing the present study.

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