Advantages of Using a Block Unstructured Grid in a Casting Scenario

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Abstract. Numerical modeling of heat transfer during solidification has become widespread in the foundry industry. This is because it is possible to investigate the effects of adjustment to the casting variables on final casting quality, without having to do costly trial-and-error experiments. After selecting a suitable mathematical model, one has to choose an appropriate discretization method. If the grid is very fine, each type of method yields the same solution. However, some methods are more suitable to some classes of problems than others. The main objective of this paper is to demonstrate the advantages of using a block unstructured grid in combination with a generalized curvilinear formulation in a casting scenario and compare the performance of two discretization methods, finite differences (FD) and finite volume (FV). The validation of the numerical procedure is done by comparison with measurements which experimental set up is also described. A very good agreement of both numerical methods were verified with a slightly advantage for the finite volume method. Block unstructured grids works well with both discretization methods, allows obtain any physical feature in specific positions of the domain and is suitable for parallel computation; in combination with a generalized curvilinear formulation allows avoid geometric complexities and the development of more efficient algorithms..

Keywords: Casting, block grid, finite difference, finite volume.
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INTRODUCTION

Solidification modelling can be divided into three separate models, where each model is identified by the solution to a separate set of equations: heat transfer modelling which solves the energy equation; fluid-flow modelling which solves the continuity and momentum equations; and free-surface modelling which solves the surface boundary conditions [1]. For a complete description of solidification in a casting, all of these equations need to be solved simultaneously, but under special circumstances, the equations can be decoupled and modelled independently. This is the case for heat-transfer modelling, which has been widely used, and its application has significantly improved casting quality [2-6]. However, heat-transfer modelling has many shortcomings, which must be taken into account whenever predictions are made using the results. These include a poor knowledge of the initial conditions, the fact that fluid flow is not modelled, and that the predictions are based on experimental defect criteria [5]. Improvements can still be made in the area of the thermo-physical data relating to the metal-mold interface, phase changes and the temperature dependence of all thermo-physical data.

One of the majors' challenges of heat transfer modelling of molten metal has been the phase change. To model such a phase change requires the strict imposition of boundary conditions. Normally, this could be achieved with a finite-element that is distorted to fit the interface. Since the solid-liquid phase boundaries are moving the use of level set methods are a recent trend [7]. However, both of these techniques are computationally expensive. The classical fixed mesh is computational less expensive but could not been able to maintain the correct boundary conditions. In this regard Monteiro [10] study the application of the finite difference method to permanent mold casting using generalized curvilinear coordinates. A multi-block grid was applied to a complex geometry and the following boundary conditions: continuity condition to virtual interfaces and convective heat transfer to metal-mold and mold-environment interfaces. The results were compared with experimental data with good agreement. The reproduction of this simulation procedure using the finite volume method was made by Rouboa [9]. The agreement with experimental data was also good. Further developments of this work were made in Eliseu et al. [10] where more reliable initial conditions and two different kinds of boundary conditions were applied with an increase in agreement with the experimental data. In the present work we compare the FD and FV methods in terms of space discretization, boundary conditions definition, and results using a multi-block grid in combination with curvilinear coordinates.

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MATHEMATICAL MODEL

The energy conservation equation (1) states that the rate of gain in energy per unit volume equals the energy gained by any source term, minus the energy lost by conduction, minus the rate of work done on the fluid by pressure and the viscous forces, per unit time. Assuming that: the fluid is isotropic and obeys Fourier's Law; the fluid is incompressible and obeys the continuity equation; the fluid conductivity is constant; viscous heating is negligible, and since the heat capacity of a liquid at constant volume is approximately equal to the heat capacity at constant pressure, then, the internal energy equation is reduced to the familiar heat equation, here shown in curvilinear coordinates [11, 12].

$$J\frac{\partial(\rho c_p T)}{\partial t} = \frac{\partial}{\partial x_j}\left[\frac{K}{J}\left(\frac{\partial T}{\partial \xi_m}B^{mj}\right)\right] + J\dot{q} \qquad (1)$$

where $\beta^{ij} = (-1)^{i+j}\det(J_{ij})$ represents the cofactor in Jacobian J.

The source term $\dot{q}$ can be expressed as a function of effective solid material fraction f_s, metal density ρ and enthalpy variation in phase change Δh_f, called latent heat [11, 12].

$$\dot{q} = \frac{\partial(\rho \Delta h_f f_s)}{\partial t}$$

The continuity condition is considered for the boundaries of the blocks in contact (virtual interfaces): $\left(\frac{\partial T}{\partial n}\right)_{m1} = \left(\frac{\partial T}{\partial n}\right)_{m2}$ and $T_{m1} = T_{m2}$

For the metal-mold interface, convective heat transfer is considered (2). T_m is the mold temperature and T_p is the cast part temperature.

$$k_m\left(\frac{\partial T}{\partial n}\right)_m = h_i\left(T_p - T_m\right) \qquad (2)$$

For the exterior boundary in contact with the environment we have convection and radiation. Is possible to express a mixed convection-radiation boundary condition, as follows :

$$k_m\left(\frac{\partial T}{\partial n}\right)_m = h_{cr}\left(T_m - T_e\right) \qquad (3)$$

Where T_e is the environment temperature and the convection-radiation heat transfer coefficient (h_{cr}) is calculated explicitly as follows:

$$h_{cr} = \left[h_c + \varepsilon_r\sigma_r\left(T_m^3 - T_e^3\right)\right] \qquad (4)$$

where ε_r is emissivity of the material, σ_r the Stefan-Boltzmann constant, and the convective heat transfer coefficient (h_c) was considered equal to 150 W/m^{2o}C [9].

RESULTS AND DISCUSSIONS

In this study, an analysis of heat transfer for the casting process in two dimensions was made for the nonlinear case during solidification taking into account the phase change. The idea was to determine the distribution of temperature, cooling curves in the cast metal, and heating or/and cooling in the mold during the 20 seconds after the beginning of the solidification process. The time step used was 10 3 seconds. The result of the heat transfer is shown in 2D, as well as the cooling curves in different points in the cast metal and mold. The final step consists in solving the problem of heat transfer of the mold – cast metal system, using linearized Eq. (1) and controlled by the convergence criteria (10^{-5} for temperature). The SIP (Strongly Implicit Procedure) solver of Stone [13] was used in this task. Numerical results calculated using FD and FV discretization methods are overlapped with experimental values, measured by the thermocouples T1, T2, T3, T4, T5, T6 and T7, shown in figure 1.

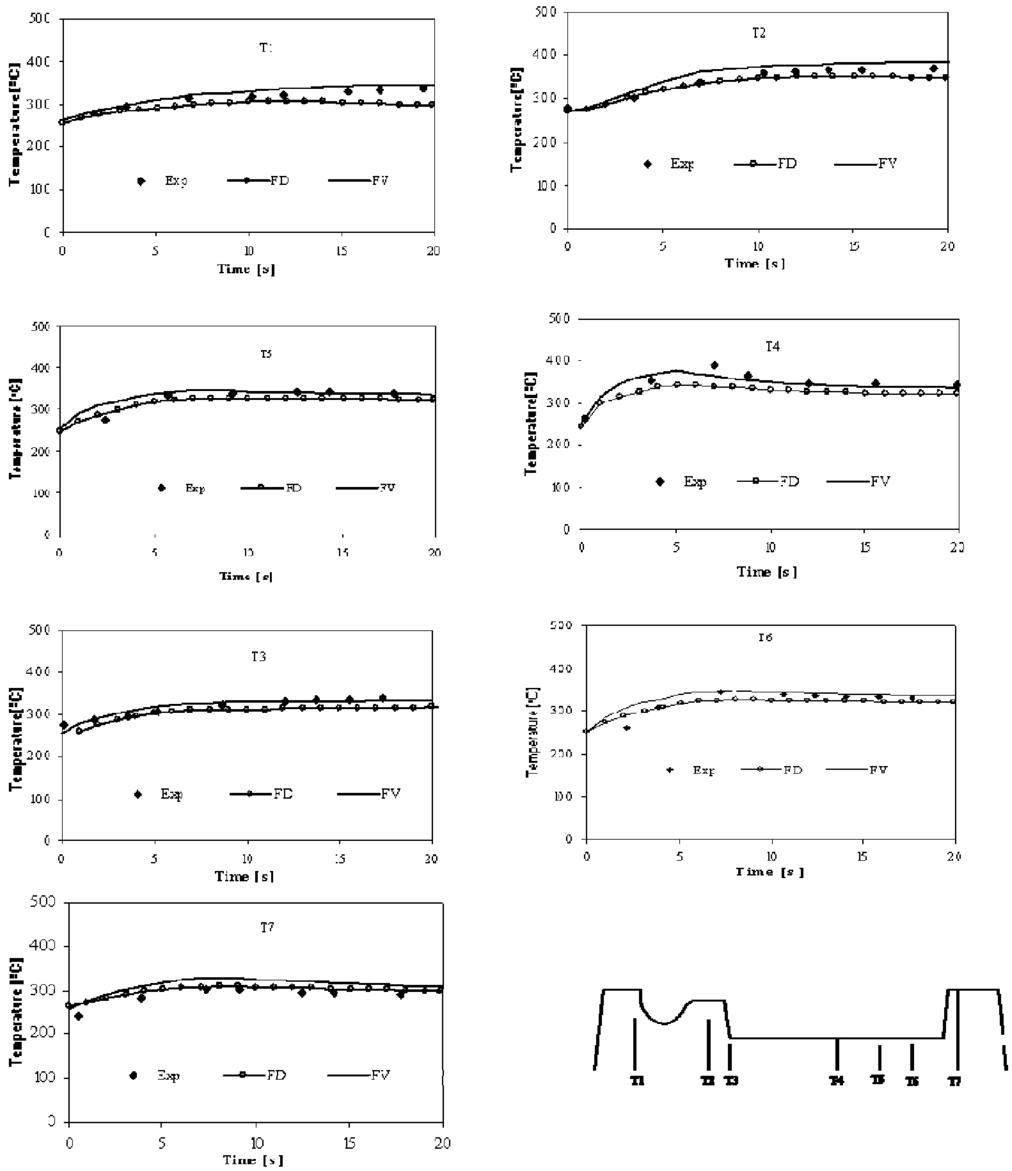

FIGURE 1. Comparison between temperature results in seven points of the mold. Experimental measurement (Exp), finite difference numerical result (FD and finite volume numerical result (FV).

For the thermocouple T1, the temperature increases from 250°C to 335°C during 20 seconds. On the other hand, thermocouples T3, T5 and T6 shows approximately the same behavior because of their similar position in the mold: temperature increases during 20 seconds from 277°C to 338°C for thermocouple T3 and from 273 to 338°C and from 260°C to 330°C for thermocouples T5 and T6, respectively. The highest temperature is verified in the thermocouple T2 because of its location between three high temperature zones. Temperature increases from 280°C to 370°C. Due to the proximities of thermocouple T4 with the metal the temperature behavior is considerably different. In the first 7 seconds the temperature increases from 260°C to 390°C, then decreases gradually to 340°C during 13 seconds.

Thermocouple T7 shows that the temperature increases from 240ºC to 305ºC in 7 seconds then decreases slowly to 292ºC during the following 10 seconds.

A good agreement of both numerical methods was found with a slight advantage for the finite volume discretization method. Finite volume discretization method reveals itself accurate when comparing with experimental results (3.4% variation) and with a previous numerical study based on finite difference discretization method (5% variation) [9].

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