Robustness in Design and Background Variations in Metamaterial/Plasmonic Cloaking

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Abstract

Here we discuss our recent numerical results concerning the robustness of the scattering cancellation effect produced by a plasmonic cloak with near-zero permittivity and correspondingly negative polarizability. Being based on an integral effect and on an intrinsically non-resonant phenomenon, we show how variations of the geometrical parameters of the design and changes in the background do not sensibly affect the invisibility properties of the plasmonic/metamaterial cloak. Design examples are presented and discussed in order to highlight this robustness and to provide some insights into this cloaking phenomenon, different from other cloaking techniques recently presented in the literature.

1. Introduction

Metamaterial science and technology has received increased attention in the past few years, due to some potential breakthrough technologies and applications based on their anomalous physical properties [*Pendry*, 2000], [*Engheta, Ziolkowski*, 2006]. Also plasmonic materials, often naturally available in the optical and infrared domains [*Bohren, Huffman*, 1983], may be protagonist of several anomalous electromagnetic effects due to their anomalous values of permittivity, which can lead to ideas for some novel devices and components.

As one of such metamaterial applications, cloaking has been recently investigated by several groups worldwide. Transformation-based cloaking with specifically tailored metamaterial cloaks is arguably one of the prominent techniques in this sense, as first envisioned in [*Pendry, et al.*, 2006], [*Leonhardt*, 2006]. Experimental realization, applications at different frequencies, limitations and properties of this cloaking technique have been discussed in subsequent contributions [*Cummer, et al.*, 2006], [*Schurig, et al.*, 2006], [*Schurig, et al.*, 2006b], [*Leonhardt*, 2006b], [*Cai, et al.*, 2007]. A different cloaking technique, based on anomalous localized resonances, has also been recently proposed in [*Milton, Nicorovici*, 2006], [*Milton, et al.*, 2006]. As an alternative possibility, scattering reduction associated with the use of soft and hard surface has been discussed at microwave frequencies [*Kildal, et al.*, 1996]. Intrinsic limitations dictated by causality over these cloaking techniques and the possible use of active materials and sources have also been discussed in a subsequent interesting contribution [*Miller*, 2006].

In our group, we have been interested in the concepts of cloaking and transparency for several years. In 2003, we proposed a mechanism based on the resonance of complementary metamaterials to make planar layers of otherwise opaque materials totally transparent to the impinging radiation [*Alù, Engheta*, 2003]. Later on, we proposed the use of covers with negative polarizability in order to cancel the dominant portion of the scattering from a given object [*Alù*, *Engheta*, 2005]. Some aspects of this mechanism had been suggested under stringent quasi-static assumptions in earlier works [*Kerker*, 1975], [*Chew*, *Kerker*, 1976]. This mechanism is based on the anomalous negative scattering properties that characterize some plasmonic materials.

In [*Alù*, *Engheta*, 2005], [*Alù*, *Engheta*, 2007], we have reported some detailed full-wave analysis of the scattering reduction properties applied to dielectric and metallic objects, underlining the intrinsic robustness of this mechanism, whereas in [*Alù*, *Engheta*, 2007b] we have extended these concepts to collections of particles and larger objects. Moreover, in [*Silveirinha*, *et al.*, 2007], we have shown how similar concepts may be achieved not only with isotropic and homogeneous plasmonic materials at IR and optical frequencies, which may be naturally available, but also with simple parallel-plate metamaterials that effectively possess similar plasmonic properties in a cylindrical geometry.

The mechanism of cloaking on which this scattering cancellation is based on is inherently non-resonant, which represents an important distinction from the other previously mentioned techniques for cloaking that have been recently proposed. This implies that this mechanism, as we discuss in the following, is relatively robust to changes, in frequency, design, shape and electromagnetic parameters of both the cloaked object and the cover itself. Some discussions regarding this robustness have been presented in [*Alù*, *Engheta*, 2007b], where we have shown how the presence of reasonable ohmic losses, shift in the frequency of operation, presence of small defects on the surface of the object to be cloaked do not change the main effect of drastic scattering reduction with respect to the bare case. In the following, we analyze and discuss in more detail the robustness of this cloaking technique with some further numerical simulations and novel physical insights.

2. *Design Robustness*

The visibility of an object at a specific frequency is directly associated with its total scattering. Therefore, the multipole expansion of its scattered fields, in terms of the Mie coefficients, allows obtaining an overall figure of merit for the cloaking of a given object. Using the notation of [*Alù*, *Engheta*, 2005b], we may define the total scattering cross section (SCS) of a given object as:

$$\sigma_s = \frac{2\pi}{{k_0}^2} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} (2n+1) \left(\left| c_{nm}^{TE} \right|^2 + \left| c_{nm}^{TM} \right|^2 \right), \qquad (1)$$

which measures the degree of visibility of a 3D object, independent of the position of the observer and the form of excitation. It is noted, in particular, that substantially reducing σ_s by canceling one or more of the dominant scattering coefficients c is equivalent to cloaking the object to its surrounding. As we have first shown in [*Alù*, *Engheta*, 2005], this may be done by employing covers constituted of plasmonic materials with permittivity near zero. It is interesting to note that the cover design is relatively straightforward when compared with the design of inhomogeneity and anisotropy profiles required for transformation-based cloaking techniques. A homogeneous, isotropic permittivity of about one tenth of the background's permittivity may achieve, under proper conditions, a substantial reduction of scattering from the object, by canceling or reducing one or more of the coefficients corresponding to the dominant *TM* spherical harmonics. If the object is made of conducting material and/or electrically not too small, and portion of its scattering is dominated by *TE* harmonics, a cover with a properly designed positive permeability larger than the permeability of the background is usually sufficient to cancel a good portion of this residual scattering [*Alù*, *Engheta*, 2007].

The mechanism of this type of cloaking, as described in [*Alù, Engheta*, 2005], is based on the fact that the scattered wave from the cloaking material is "oppositely-signed" with respect to that of the object to be cloaked. This implies that a judicious choice of the cloak volume surrounding the object may almost completely cancel the scattering from the system. A corollary of this collective cancellation mechanism is that the small changes in the shape of the cloak and/or of the object do not substantially affect such scattering reduction.

Consider for instance the perfectly electric conducting (PEC) sphere analyzed in [*Alù*, *Engheta*, 2007], with diameter $2a = 0.4\lambda_0$, λ_0 being the background (free-space) wavelength at the design frequency f_0, covered by a suitably designed cover with permittivity $\varepsilon_c = \varepsilon_0\left[1-\omega_p^2/\omega\left(\omega - j\gamma\right)\right]$, with ω_p chosen in such a way to have $\mathrm{Re}\left[\varepsilon_c\right] = 0.1\varepsilon_0$ at frequency f_0, $\gamma = 0.002\,\omega_p$ to consider the possible presence of realistic ohmic losses, permeability $\mu_c = 5.1\mu_0$ and radius $a_c = 1.09\,a$. Here ε_0, μ_0 are permittivity and permeability of free-space that constitutes the background material in this and the following examples. The presence of the properly designed cover reduces the total scattering cross section by more than 99% at the design frequency, when compared to the bare sphere, making it effectively invisible at the design frequency, in all directions and independent on the polarization and form of excitation [*Alù, Engheta*, 2007]. These already exciting results, which show the robustness of this technique and its versatility compared to other cloaking techniques, are even more striking if we start deforming and varying the shape of the original object. In [*Alù*, *Engheta*, 2007], we have added dimples and bumps or cuts on the surface of the PEC sphere, achieving very similar results in terms of total scattering reduction. Here we consider a gradual transformation of the original sphere in a more and more eccentric spheroidal shape, varying the length of two of its three axes.

Figure 1a shows the plot of the peak in the scattering-cross-section radiation pattern versus normalized frequency, evaluated with CST Microwave Studio™, for several different configurations when excited by a plane wave. The solid black line refers to the original cloaked sphere, consistent with the results in [*Alù, Engheta*, 2007]. Compared to the dashed line, which corresponds to the bare sphere, the drastic reduction of scattering caused by the cloak over a relatively broad range of frequencies around f_0 is indeed evident. The other lines refer to different geometries obtained after a perturbation of this spherical shape, for which both the object and the cloak are squeezed in the plane orthogonal to the impinging electric field, making the cloaked object prolate spheroids. The numbers in the inset refer to the ratio between the minor and major axes of each spheroid. It should be noted that the spheroid axis parallel to the impinging electric field (the one that effectively determines the "electrical aperture" of the object), has been kept fixed in all the different geometries, whereas the other two axes have been shortened accordingly to the ratio indicated in the inset of Fig. 1. For better comparison, the scattering peak gain (ratio in dB between the peak in scattering cross section of the bare sphere and that of the cloaked objects) has been reported in Fig. 1b.

It is evident how the cloaking effect is not worsened by a drastic change in the geometry and, even if the object shape is significantly distorted, the cloaking bandwidth and efficiency remain impressively large (even larger when the spheroids are very “prolate, looking” like needles). The reason for this additional increase in bandwidth when prolate spheroids are considered resides in the fact that the object is electrically smaller in one dimension, allowing a reduction of the non-dipolar scattering that is usually more challenging. This makes the whole setup more robust to variations in the geometry and frequency of operation. In this sense, it is worth underlining that the orientation of the spheroids with their major axis parallel to the impinging electric field represents the “worst-case” scenario for this scattering reduction. Any other orientation of the particle would provide even better cloaking results, due to the smaller effective “aperture” of the object.

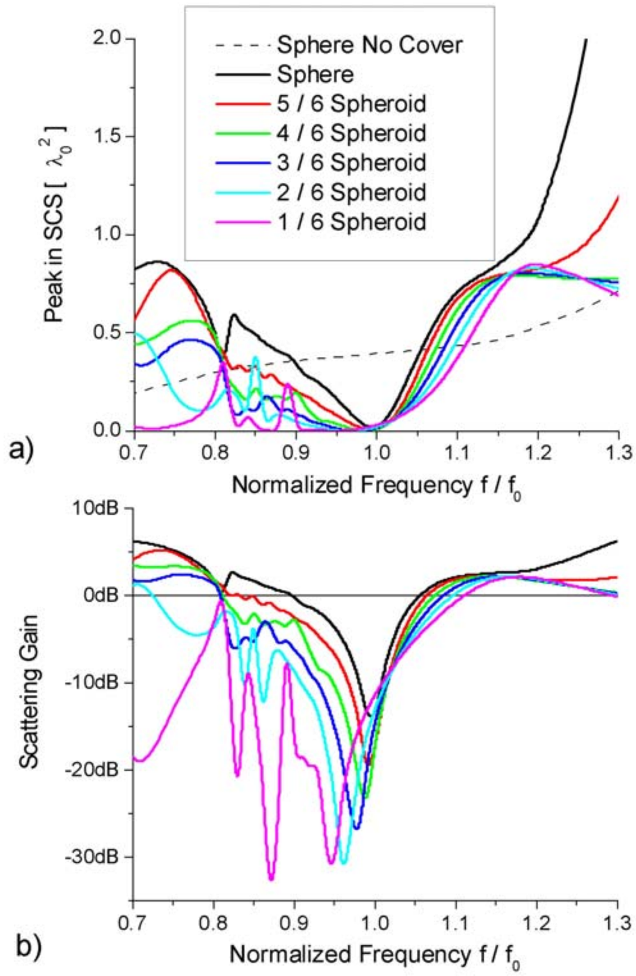

Figure 1 – (a) Peak in the scattering-cross-section pattern versus normalized frequency and (b) scattering peak gain compared to the bare sphere, for several geometries: starting from the cloaked sphere designed in [*Alù, Engheta*, 2007] (black line), the other curves refer to prolate spheroids obtained by squeezing two axes of the original sphere, with a ratio indicated in the inset.

It is important to underline that here the cloaking materials have not been re-optimized in each simulation, but rather the parameters of the original (spherical) cloaking material have been used to design all the other spheroidal cloaks, only deforming both object and cloak with the same aspect ratio. This shows the robustness of the design to changes in shape and design parameters: going from a sphere to a needle, the cloaking effect remains essentially unaffected.

The variation of the frequency dispersion and bandwidth with different cover thicknesses has been analyzed in details in [*Alù, Engheta*, 2007]. Consistent with the results of Fig. 1, the perturbation of the sphere to a spheroidal shape does not notably modify the concepts outlined in [*Alù, Engheta*, 2007], which have shown robustness to the geometrical and electromagnetic parameters of the cloak and a relatively large bandwidth of operation, due to the intrinsically non-resonant properties of this phenomenon. Possible variations in the shape and design of the cloak, due to technological imperfections, like bumps, dimples and cuts have been addressed in [*Alù, Engheta*, 2007b] and they similarly apply to this scenario.

Figures 2 and 3 report the real part of the Poynting vector distribution in the E plane for some of the spheroids simulated in Figure 1 at the design frequency f_0. The plane wave is impinging in all the panels from left to right, with the electric field linearly polarized from top to bottom. Figure 2a refers to the spheroid with ratio between minor and major axes being 5/6, Fig. 2b to ½, Fig. 3a to 1/3 and Fig. 3b to 1/6. It can be seen how the Poynting vector is rerouted around the impenetrable object by the suitably designed plasmonic cover (which is homogeneous and isotropic) and its presence may be hardly detectable by an external observer, even if placed very close to its surface. The overall effect is similar to the one obtainable with a transformation-based cloak, in the sense of rerouting of the energy in the cloak region, but here the cloak has an arguably much simpler geometry and design and it is also thinner. The effect is still unchanged when the object becomes needle-shaped, with a strong aspect ratio, despite its length and shape being close to those of a resonant half-wavelength dipole, which is expected to produce a strong scattering response when uncovered.

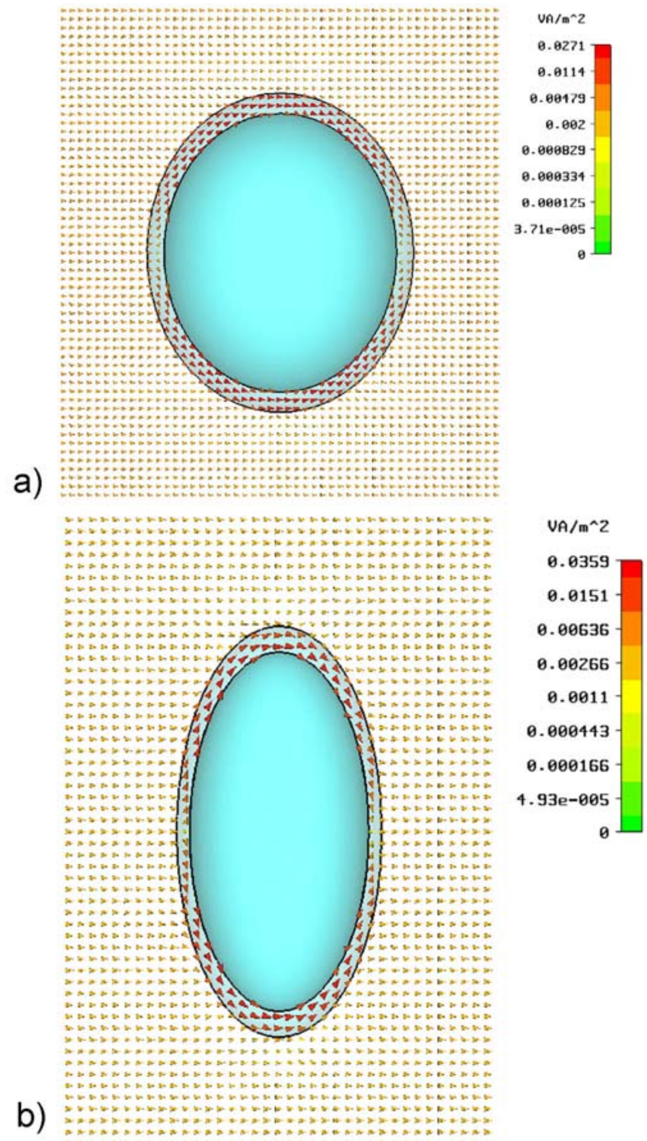

Figure 2 – Real part of Poynitng vector distribution in the E plane at the design frequency f_0 for a cloaked spheroid as in Fig. 1. Panel (a) refers to a ratio between minor and major axis of $5/6$, panel (b) for $1/2$.

It is also noted that here we are dealing with impenetrable objects. In the case of dielectric objects to be cloaked, on the other hand, the wave is not necessarily required to be rerouted around the object, but it can simply pass through it, with the cover simply eliminating the distortion in the outside region. This may allow better bandwidth and cloaking efficiency compared to the PEC scenario, which remains the most challenging for an effective cloak.

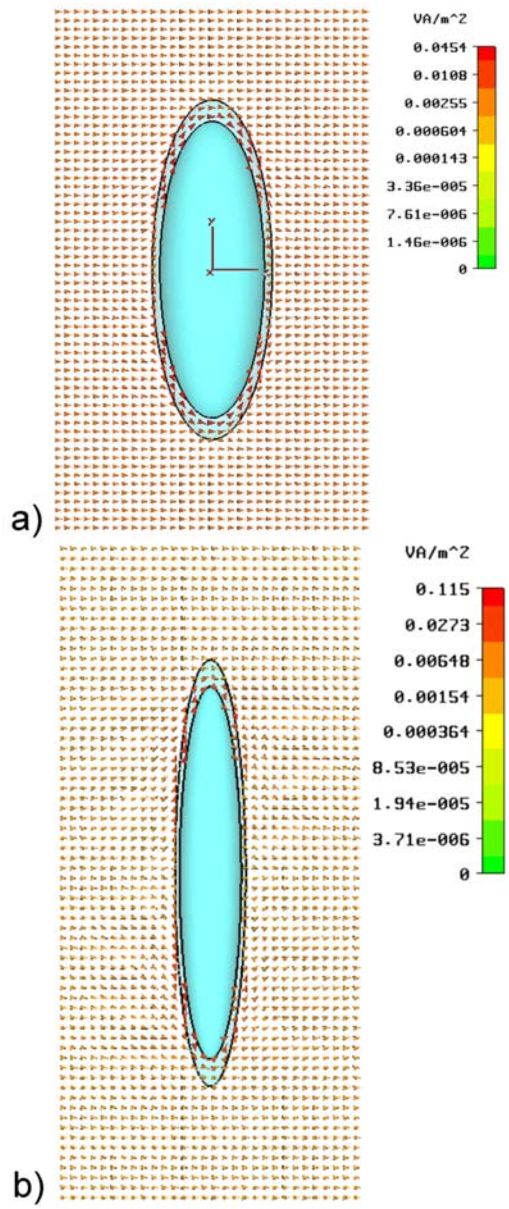

Figure 3 – Similar to Fig. 2, but for a ratio between minor and major axis of: (a) $1/3$; (b) $1/6$.

3. Background Variations

As a different set of simulations that shows the robustness of this cloaking technique to changes in the background environment, we have simulated the original cloaked PEC sphere in Fig. 1 with a nearby conducting (PEC) cubic object. Figure 4 and 5 show the magnetic and electric field distributions on the E and H planes, respectively, for plane wave incidence on such a system. The distance between the PEC cube and the cloaked object is completely arbitrary, and the two objects may be as close as desired, due to the cloaking properties discussed above. In [*Alù, Engheta*, 2007b] we have discussed in details these coupling issues between different neighboring objects in relation with this cloaking setup. In each figure, panel (a) refers to the case in which the sphere is bare, whereas in panel (b) the sphere is cloaked by the properly designed cover described above. One can see how in the first scenario the plane wave impinging on the system is highly perturbed by the presence of the uncloaked PEC sphere and it can barely feel the presence of the cube, which is placed in the sphere "shadow". However, when the sphere is cloaked, the planar phase fronts are essentially restored right outside the sphere, and the plane wave "tunnels" through the sphere and illuminates the cube as if the sphere were effectively not there, even if the cube is placed right behind it. The small scattering perceived by an external observer is just the one coming from the cube, as if the sphere were not there. In other words, an observer placed on the back of the sphere, from where the plane wave originates, would "perceive" the presence of the cube through the impenetrable sphere.

This effect can be seen in both planes of polarization in Fig. 4-5, and it shows how this cloaking effect is indeed achievable in 3D and independent of the polarization of the incident field and the observer's position.

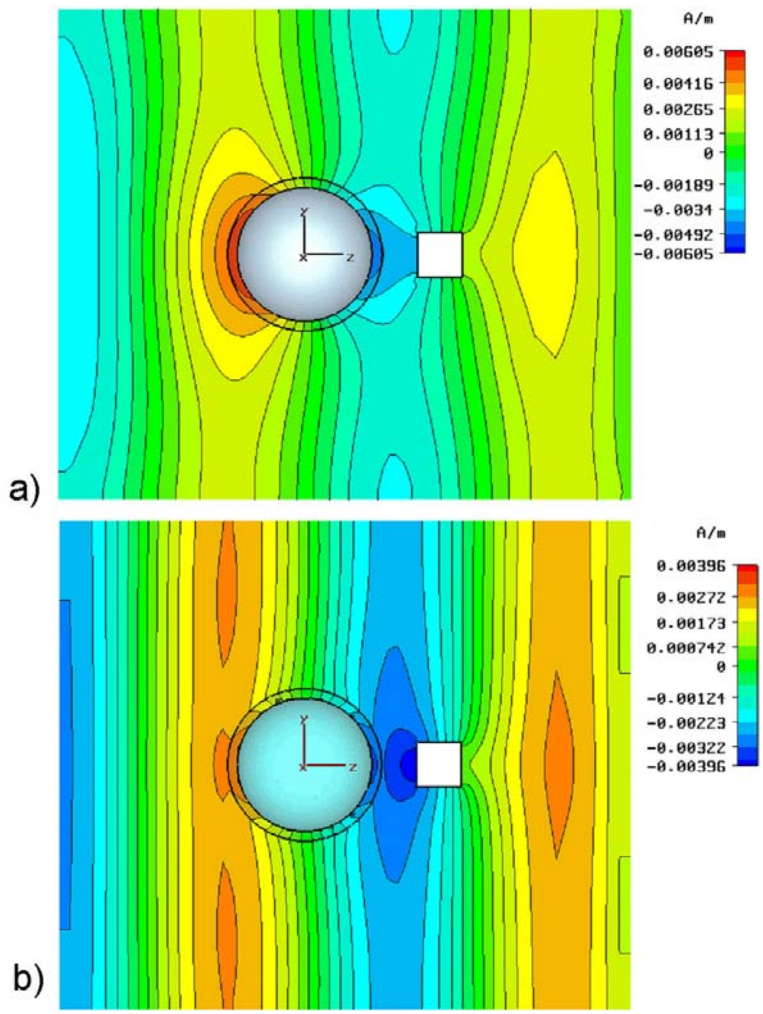

Figure 4 – Magnetic field distribution in the E plane for a system formed by the perfectly electric conducting (PEC) sphere of Fig. 1 and a small conducting cube placed behind it. In panel (a) the sphere is bare, i.e., uncloaked, in (b) it is cloaked with the optimized cloak of Fig. 1.

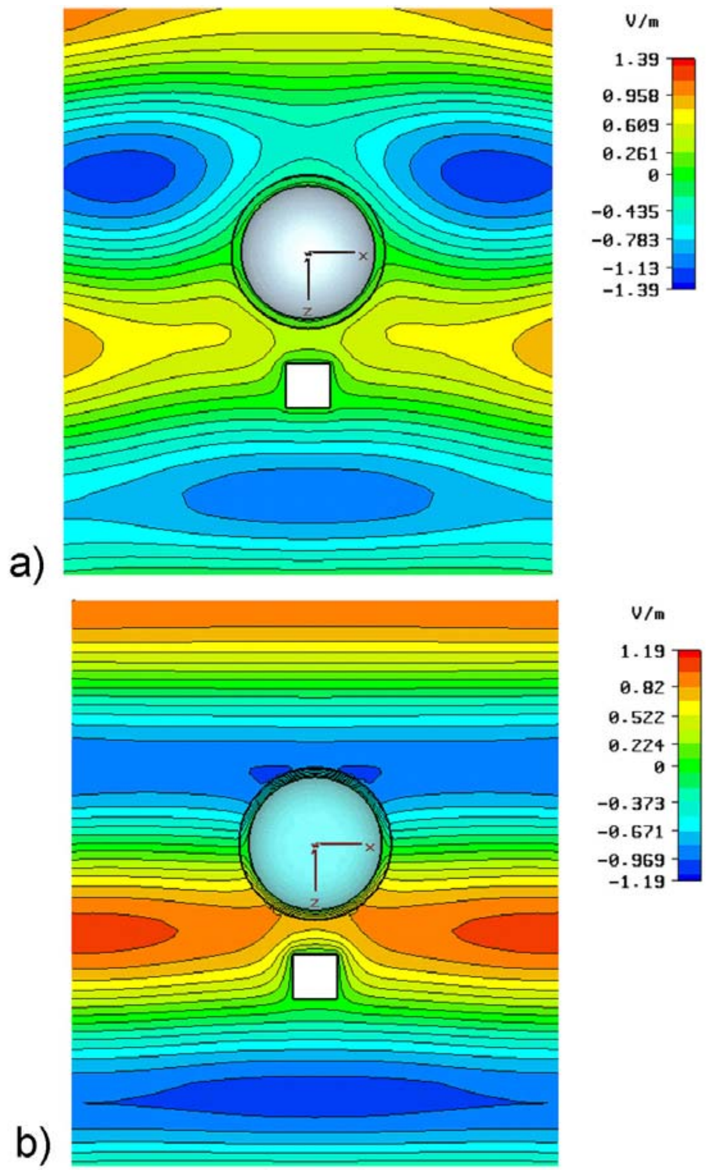

Figure 5 – Electric field distribution in the H plane for the geometry of Fig. 4.

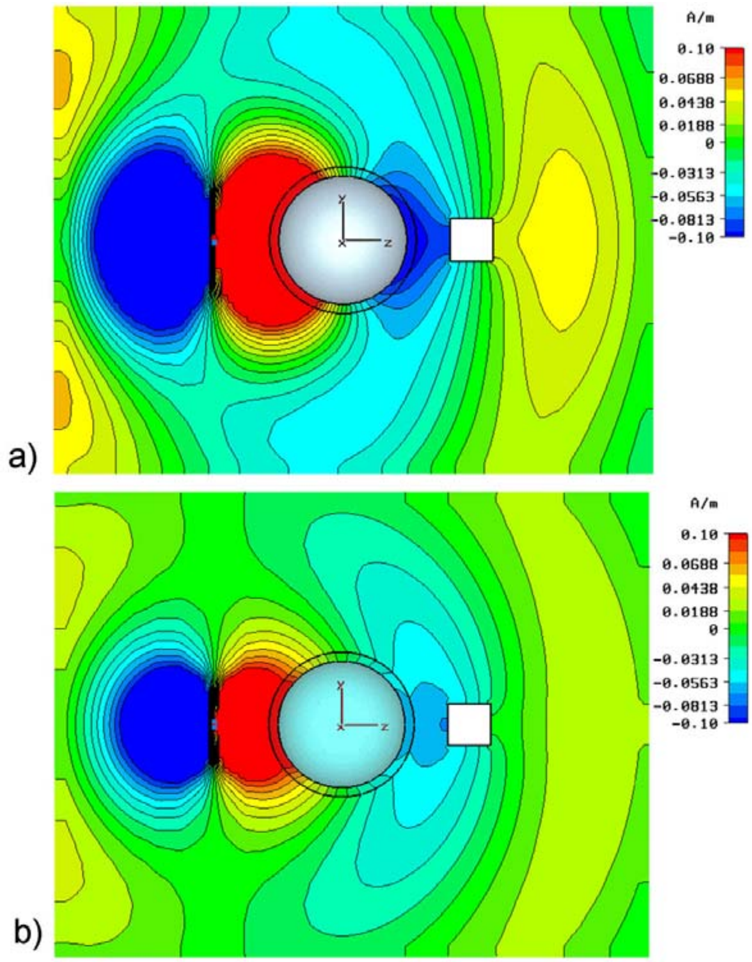

Figure 6 – Magnetic field distribution in the E plane for the same geometry as Fig. 4-5, but for an electric dipole excitation placed very close to the sphere on the back of it with respect to the cube.

Figure 6 and 7 refers to the same geometry as in Fig. 4-5, but now for a finite near-zone excitation in the form of an electric dipole placed behind the sphere. Also in this case, it is striking to see how the cloak may effectively restore the dipolar pattern, isolating the scattering of the small cube on the back of the cloaked object. Once again, an observer placed near the source location would not experience the presence of the cloaked system. The two figures refer to the two polarization planes and confirm how the cloak is robust to changes in the surrounding environment and to the polarization and form of the source, which may be placed in the near as well as in the far-zone region.

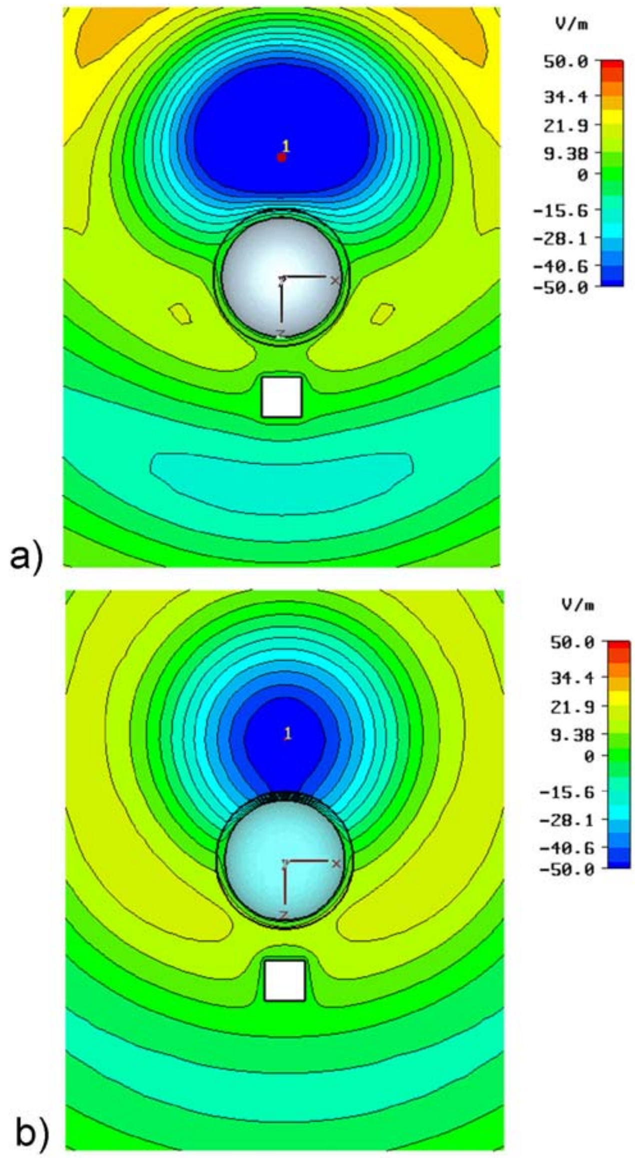

Figure 7 – Electric field distribution in the H plane for the geometry and excitation of Fig. 6.

The functionality of this cloaking technique and the examples in Fig. 4-7 may effectively represent a novel scenario for drastically lowering the field disturbance that a sensor introduces when it is near the structure it measures. If the sphere in the previous examples represents the tip of a sensor (e.g., Near Field Scanning Optical Microscope (NSOM) tip) placed near a small object to be detected/measured, it is clear that the reduction in disturbance that this robust cloaking mechanism may achieve would greatly improve the overall measurement mechanism and possibly its signal-to-noise ratio. This is one of the potential applications we are planning to explore.

Conclusions

Here we have presented some numerical results and full-wave simulations concerning the robustness of the scattering cancellation method in metamaterial cloaking applications. We have shown how this setup may be robust to shape or design variations, even relatively drastic, or to the presence of small objects in the surrounding of the cloaked system. Applications for scattering reduction and non-invasive probing and sensing can be forecasted over different frequency regimes using naturally-available plasmonic materials and/or metamaterials.

References

Alù, A., and N. Engheta (2003), Pairing an epsilon-negative slab with a mu-negative slab: anomalous tunneling and transparency, *IEEE Trans. Antennas Propagat.* **51**, 2558-2570.

Alù, A., and N. Engheta (2005), Achieving transparency with plasmonic and metamaterial coatings, *Phys. Rev. E* **72**, 016623.

Alù, A., and N. Engheta (2005b), Polarizabilities and effective parameters for collections of spherical nano-particles formed by pairs of concentric double-negative (DNG), single-negative (SNG) and/or double-positive (DPS) metamaterial layers, *Journal of Applied Physics* **97**, 094310.

Alù, A., and N. Engheta (2007), Plasmonic materials in transparency and cloaking problems: mechanism, robustness, and physical insights, *Optics Express* **15**, 3318-3332.

Alù, A., and N. Engheta (2007b), Cloaking and transparency for collections of particles with metamaterial and plasmonic covers, *Optics Express* **15**, 7578-7590.

Bohren, C. F., and D. R. Huffman (1983), *Absorption and Scattering of Light by Small Particles*, Wiley, New York, USA.

Cai, W., U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev (2007), Optical cloaking with metamaterials, *Nature Photonics* **1**, 224-227.

Chew, H., and M. Kerker (1976), Abnormally low electromagnetic scattering cross sections, *J. Opt. Soc. Am.* **66**, 445-449.

Cummer, S. A., B. I. Popa, D. Schurig, D. R. Smith, and J. B. Pendry (2006), Full-wave simulations of electromagnetic cloaking structures, *Phys. Rev. E* **74**, 036621.

Engheta, N., and R. W. Ziolkowski (2006), *Metamaterials: Physics and Engineering Explorations*, IEEE Press, John Wiley and Sons, Inc., New York, USA.

Kerker, M. (1975), Invisible bodies, *J. Opt. Soc. Am.* **65**, 376- 379 (1975).

Kildal, P. S., A. A. Kishk, and A. Tengs (1996), Reduction of forward scattering from cylindrical objects using hard surfaces, *IEEE Trans. Antennas Propagat.* **44**, 1509–1520.

Leonhardt, U. (2006), Optical conformal mapping, *Science* **312**, 1777-1780.

Leonhardt, U. (2006b), Notes on conformal invisibility devices, *New Journal of Physics* **8**, 118.

Miller, D. A. B. (2006), *Opt. Expr.* **14**, 12457.

Milton, G. W., and N. A. Nicorovici (2006), On the cloaking effects associated with anomalous localized resonance, *Proc. R. Soc. Lond. A: Math. Phys. Sci.* **462**, 3027–3059.

Milton, G. W., M. Briane, and J. R. Willis (2006), On cloaking for elasticity and physical equations with a transformation invariant form, *New Journal of Physics* **8**, 248.

Nicorovici, N. A. P., G. W. Milton, R. C. McPhedran, and L. C. Botten (2007), Quasistatic cloaking of two-dimensional polarizable discrete systems by anomalous resonance, *Optics Express* **15**, 6314-6323.

Pendry, J. B. (2000), Negative refraction makes a perfect lens, *Phys. Rev. Lett.* **85**, 3966-3969.

Pendry, J. B., D. Schurig, and D. R. Smith (2006), Controlling electromagnetic fields, *Science* **312**, 1780-1782.

Schurig, D., J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith (2006), Metamaterial electromagnetic cloak at microwave frequencies, *Science* **314**, 977-980.

Schurig, D., J. B. Pendry, and D. R. Smith (2006b), Calculation of material properties and ray tracing in transformation media, *Optics Express* **14**, 9794-9804.

Silveirinha, M. G., A. Alù, and N. Engheta (2007), Parallel-plate metamaterials for cloaking structures, *Phys. Rev. E* **75**, 036603.