

TWO-SIDED SEARCH AND MATCHING:
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to my family

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ABSTRACT

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Frictions are a potentially important feature of many two-sided settings, for example, in the marriage market. My dissertation develops and estimates two-sided search and matching models and uses them to assess the importance of frictions in explaining observed marriage patterns. In the models, unmatched individuals search for long-term partners. Opportunities to meet potential partners arrive over time at uncertain intervals. Individuals are of different observable discrete types (e.g., gender and race/ethnicity) and types vary in their proportions within the population. Types may also differ in their type-pair specific utilities of marrying and their likelihood of meeting certain types of potential spouses.

The first chapter in my dissertation proposes a new identification approach to separately estimate type-specific preferences and opportunities. I implement the technique to understand the marital patterns of racial and ethnic groups in the United States and to analyze the reasons for a high degree of same-type marriages. The second chapter theoretically analyzes the implications of the assumption that people prefer same-type partners. I show that frameworks with and without search frictions deliver different insights about the effects of group size on matching outcomes.

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Chapter 1

Preferences vs. Opportunities:

Racial/Ethnic Intermarriage in the United States

This chapter develops and implements a new approach for separately identifying preference and opportunity parameters of a two-sided search and matching model in the absence of data on choice sets. This approach exploits information on the dynamics of matches: how long it takes for singles to form matches, what types of matches they form, and how long the matches last. Willingness to accept a certain type of partner can be revealed through the dissolution of matches. Given recovered acceptance rules, the rates at which singles meet different types are inferred from the observed transitions from singlehood to matches. Imposing equilibrium conditions links acceptance rules and arrival rates to underlying preference and opportunity parameters. Using the Panel Study of Income Dynamics, I apply this method to examine the marriage patterns of non-Hispanic whites, non-Hispanic blacks and Hispanics in the United States. Results indicate that the observed infrequency of intermarriage is primarily attributable to a low incidence of interracial/interethnic meetings rather than same-race/ethnicity preferences. Simulations based on the estimated model predict the effects of demographic changes on marital patterns.

1.1 Introduction

“People can have the Model T in any color they want – as long as it’s black.”

(Henry Ford)

As Henry Ford observed, people make choices based on the opportunities they have.¹ Unless the limitations on choice sets are carefully considered, interpretation of preferences based on final choices could be misleading. In some cases, however, researchers are unable to observe the set from which the final option was chosen. For example, in contexts where individuals experience match opportunities (meetings) and decide which matches to form, what is observed in the data is usually final matches as opposed to all meetings. Confronting this challenge, I develop a new approach for separately identifying preference and opportunity parameters of a two-sided search and matching model in the absence of data on meetings. This identification strategy exploits information on the dynamics, formation and dissolution, of final matches. I implement this method to study the marriage patterns in the United States, which are characterized by high frequencies of same-race marriages (racial homogamy).

In 2011, 93.8% of married whites, 88.6% of married blacks, and 80.2% of married Hispanics had spouses of the same racial group, whereas their shares of the U.S. population were 64.5%, 12.8%, and 16.3%.² This marriage pattern has received widespread scholarly and public attention concerning the causes and consequences of within-group and between-group marriages in relation to the salience of cultural distinctions and atti-

¹The original remark found in his biography was “Any customer can have a car painted any colour that he wants so long as it is black.” (Ford and Crowther 1922) Since the color black was the only available option for customers who bought a Model T produced by the Ford Motor Company from 1908 and 1927, this color choice by consumers could not reflect their preferences.

²Source: PSID 2011 for homogamous marriage rates and Current Population Survey 2011 for population shares. Throughout the paper, whites and blacks refer to non-Hispanic whites and non-Hispanic blacks. Race refers to racial and ethnic distinctions.

tudes towards other groups.³ This paper focuses on two potential sources of homogamy. First, homogamy may stem from people's preferences for spouses of the same type.⁴ Second, homogamy may reflect people's more frequent opportunities to meet members of their own group, which may be a consequence of a high degree of residential and school segregation.⁵ Although the two factors, preferences and opportunities, have distinct roles in marital decisions, little is known about the relative strength of each channel.

In this paper, I construct and estimate a dynamic equilibrium model of the marriage market with transferable utility and search frictions, building on Shimer and Smith (2000) and Jacquemet and Robin (2013). Individuals, who differ by gender and race (type), are either single or married. Singles meet different types of potential spouses and jointly decide whether to marry based on type-specific marital utilities and idiosyncratic match quality. Couples may receive new match quality and decide whether to divorce. The model allows marital preferences and opportunities to differ for all possible combinations of male and female types.⁶ For example, black male - black female marriages may generate different utility than white male - black female marriages. The rate at which males meet a black female as a potential spouse (the arrival rate of black females) depends not only on the percentage of black females among singles, but also on male types through opportunity parameters that I introduce.

I estimate the model using data from the Panel Study of Income Dynamics (PSID). The data include longitudinal information on individuals' marriage transitions from 1968 to 2011: how long individuals stay single, whom they marry, and how long their marriages last. The estimation of the model is done by three steps which entail technical innovations.

³See Kalmijn (1998), Fryer (2007), Bisin and Verdier (2011) and Schwartz (2013) for an overview of this topic.

⁴Obstacles to interracial marriages have been cited as differences in cultural backgrounds and the negative social responses from family and community (Dalmage 2000, Childs 2005, Bratter and King 2008).

⁵See Cutler, Glaeser, and Vigdor (1999), Easterly (2009) and Orfield, Kuscera, and Siegel-Hawley (2012) for evidence on the high degree of residential and school segregation.

⁶The model is silent on how types, and their preferences and opportunities, are determined. (See Bisin, Topa, and Verdier 2004 for an alternative setting.) I focus on the role of preferences and opportunities in explaining individual marriage histories, whereas Bisin et al. focus on cultural transmission and evolution of types across generations. Evidence for exogenous meeting opportunities is provided in Section 1.5.4.

In the first step, hazard rates of marital formation and dissolution are estimated based on duration analysis. In the second step, given the assumption that the evolution of match quality does not depend on marriage types, willingness to accept a spouse of a different type can be revealed through marital dissolution. Once acceptance rules are recovered, the rates at which singles meet different types are inferred from data on marital formation. Finally, imposing equilibrium conditions links acceptance rules and arrival rates to underlying preference and opportunity parameters for all possible pairings of whites, blacks, and Hispanics in the United States.

As a brief intuition of the identification strategy, consider the observation that most black females marry blacks rather than whites. If this pattern is explained by that people find black - black (BB) marriages more acceptable than white male - black female (WB) marriages, we would expect that BB marriages to be more persistent. However, WB marriages are estimated to be more stable than BB marriages, which signals that people may accept WB marriages relatively easily if they meet. Then, the only way to explain the infrequent formation of WB marriages is the opportunity channel, which means WB meetings are rare. In other words, while marriage formation is affected both by type-specific acceptance and arrivals, divorce is affected only by the degree of acceptance. Thus, by examining formation and dissolution of matches simultaneously, I can infer frequencies of different meetings without directly observing how people meet.

The estimates reveal significant differences among racial groups in their structural parameters. Hispanics value their homogamous marriage more than whites do, while blacks value it less. Same-race preferences are shown in the sense that two racial groups find forming two types of homogamous marriages more beneficial than forming interracial marriages. The sum of utilities of white - white and Hispanic - Hispanic marriages is higher than the total utilities from white - Hispanic and Hispanic - white marriages ($WW + HH \succ WH + HW$), which is also the case between blacks and Hispanics.⁷ In addition, singles

⁷This inequality between whites and blacks, $WW + BB \succ WB + BW$, is not statistically significant.

receive greater opportunities to meet members of their own groups than the rates implied by uniformly random meeting.

The estimated parameters imply that both preferences and opportunities play important roles in explaining observed marital patterns. Because of same-race preferences, people accept same-race meetings more easily than interracial meetings, with the ratio of 0.81. Homogamous marriages are, on average, more stable than interracial unions, but marital stability varies by the racial composition of the couple. Moreover, differences in marital stability are not dramatic enough to explain all of the observed infrequency of interracial marriages, suggesting the role of opportunities, that is, meetings are also subject to the same-race bias. If meetings were random, 36.3% of meetings would be interracial. In contrast, since individuals tend to meet members of the same group more frequently, only 8.0% of meetings are estimated to be across races. Hispanics, although they face more interracial meeting opportunities, more selectively accept homogamous meetings than other groups do.

To compare the relative importance of preferences and opportunities in explaining racial homogamy, I use the estimated model to perform counterfactual experiments. Eliminating preference heterogeneity has only a limited effect on the homogamy patterns, while eliminating differences in meeting frequency significantly increases the incidence of interracial marriages. Across racial types, opportunities explain 91.6%, 94.6%, and 80.7% of homogamy patterns of whites, blacks and Hispanics. The results suggest that the high probabilities of individuals marrying spouses of the same race are primarily attributable to frequent incidences of intragroup meetings rather than same-race preferences.

The model further offers predictions on how marital outcomes correspond to demographic changes. In the hypothetical situation where, for example, the gender ratio of blacks becomes more balanced, which could potentially occur if the incarceration and mortality rates of black males decreased, the marriage rate for black females increases by 6.6-10.0%. However, only 16-22% of the white-black gap in marriage rates would disap-

pear after the increased availability of potential spouses. Lastly, I use the model to predict the effects of an increasing Hispanic population. Analysis based on expected population distribution in year 2050 projects that Hispanics will meet their same-race spouses more easily at that time, leading to an increase in Hispanic intramarriages and a higher degree of racial homogamy.

This paper contributes to three bodies of literature. The first is the literature that structurally estimates equilibrium models of the marriage market. Following pioneering work by Becker (1973, 1974), economists have investigated the marriage market. Because marriage is the foundation of the family, where major economic decisions (e.g., consumption, labor supply, and human capital investment) are made and children learn values and virtues, understanding how marriages are formed and function is crucial for economics and society.⁸ Choo and Siow (2006b) first propose and estimate a frictionless marriage market model, which is extended by Chiappori, Salanié, and Weiss (2012), and Dupuy and Galichon (2012) among many others. In contrast, Jacquemet and Robin (2013) and Goussé (2012) use frictional models to measure the gains of marriages between agents who differ in traits such as income.⁹ Most related papers by Wong (2003b) and Bisin, Topa, and Verdier (2004) focus on racial and religious dimensions respectively, estimating intolerance to intercultural marriages.¹⁰ However, these papers do not address factors involving meeting opportunities in marriage, either because they adopt a frictionless setting where all options are always available or frictional settings with random meetings where the arrival rates of different spouses are type-independent.¹¹ Since the observed proportions of final

⁸There is a large literature on the marriage market. The literature covers broader meaning of marriage, that is, how people are matched with one another and form households. Extensive reviews of the literature can be found in Becker (1993), Stevenson and Wolfers (2007), Browning, Chiappori, and Weiss (2011).

⁹Jacquemet and Robin (2013) rigorously examine the intra-household bargaining based labor supply and home production decisions and Goussé (2012) incorporates several traits including income, education and physical attractiveness.

¹⁰Wong (2003b) proposes a nontransferable utility model with uniformly random meeting. Bisin, Topa, and Verdier (2004) allow agents to make costly decisions on whom they meet and marry. However, the underlying search technology is assumed to be type-independent.

¹¹Sietz (2009) and Keane and Wolpin (2010) also build a model that can account for racial differences in marital patterns. These models explain racial differences both in the marriage market and the labor market. Keane and Wolpin (2010) only consider the female side of the market, and are silent on the racial identity

matches are directly mapped into underlying preferences, the difference in marriage shares may overstate the difference in marital preferences. Thus, estimated preferences for groups who meet each other more frequently would tend to be upwardly biased.¹²

Second, mate preferences have previously been estimated in some settings where researchers can observe both available options and choices. Fisman et al. (2008) designed and conducted speed-dating experiments with voluntary participants. Hitsch, Hortaçsu, and Ariely (2010a) and Lee (2009) and Banergee et al. (2013) use data from an online dating site, a matchmaking company and newspaper matrimonial advertisements, respectively. The results from these papers, however, may not be generalizable to the marriage context of the whole population. In contrast, my estimation is based on population data and actual marital decisions. Moreover, I identify heterogeneity in opportunities as well as differences in preferences. With recovered preferences and opportunities, I can further explore and quantify key mechanisms that generate marital patterns in terms of both formation and dissolution.¹³

Third, Currarini, Jackson, and Pin (2009) propose and estimate a model to identify the roles of meetings and preferences in friendship, characterized by the high frequency of same-race friends. In their framework, because there is no rejection upon meeting, the fraction of same-type friends recovers the meeting factor. They then recover preferences based on variations in the number of friendships. Despite its novel features, the model they develop is not appropriate for monogamous marriages. My identification strategy is general enough to be applicable to any matching models with search frictions, such as two-sided

of spouse. Sietz (2009) assumes that marriage markets are segmented by race. Several papers consider the marital opportunities only in a restrictive sense (for examples, as the availability related to gender ratio as in Choo and Siow 2006a, Abramitzky, Delavande, and Vasconcelos 2011). In the work of Logan, Hoff, and Newton (2008), opportunities are restricted only by the preferences of the opposite side.

¹²For example, Wong (2003b) quantifies the effect of racial taboos and that of differences in human capital endowments on racial homogamy, finding that racial taboos explain 74% of intermarriage patterns. However, since she does not control for opportunity differences, the role of same-race preferences may be overestimated.

¹³It is worth emphasizing that my model predictions are broadly consistent with the findings from a randomized controlled experiment that shows more interracial matches under random arrivals. Fisman et al. (2008) report that the acceptance probability of interracial meetings is 78% of the intrameeting acceptance rate which is comparable to my findings.

markets with workers and firms and buyers and sellers, and many-to-many sided markets, where data on the dynamics of matches are available.

The rest of the paper is organized as follows. Section 1.2 introduces the two-sided search and matching model. After describing the data in Section 1.3, I demonstrate the methods of recovering primitives and the implementation of the techniques in Section 1.4. The estimated parameters of the model and results from counterfactual experiments are presented in Section 1.5. Section 1.6 compares my framework to alternative frictionless settings and discusses possible applications of my model. Section 1.7 provides a conclusion.

1.2 The Model

In this section I construct a dynamic equilibrium model of marriage formation and dissolution. The model presented here is an extension of the model of Jacquemet and Robin (2013). Some model assumptions are driven by a desire to estimate not only marital preferences but also meeting opportunities.¹⁴

1.2.1 Environment

The marriage market is populated by men and women with the finite and discrete types, $i \in \{1, 2, \dots, I\}$ and $j \in \{1, 2, \dots, J\}$. The measure of each type among population is exogenously given by $g^m = (g_1^m, g_2^m, \dots, g_I^m)$ for males and $g^f = (g_1^f, g_2^f, \dots, g_J^f)$ for females, with the measure of females normalized to 1. Types represent social categories that determine the preferences and meeting environments of members of the group. Furthermore, types are assumed to be exogenously given, invariant, and observed to other agents and the

¹⁴Three major extensions are made in addition to the settings of Jacquemet and Robin (2013): 1) endogenized divorce, 2) type-specific meetings as apposed to random meetings, and 3) discrete types. The first extension was also made by Goussé (2012), while her model can be overidentified with her assumption of random meetings. The discretized type space enables us to identify preference and opportunity parameters in a new and straightforward way as introduced in Section 1.4.

econometrician.¹⁵

Consider an infinite horizon and stationary environment with continuous time. Individuals, who discount the future with rate r , are either single or married. Singles and married agents are ageless, but they face the risk of death that occurs according to the Poisson parameter of δ . Couples are assumed to die together. New individuals are born as single with the rate of δ , where the types of new agents are randomly drawn from the distribution (g^m, g^f) . The measure of types among single is denoted by $s^m = (s_1^m, s_2^m, \dots, s_I^m)$ for males and $s^f = (s_1^f, s_2^f, \dots, s_J^f)$ for females with $S^m = \sum_i s_i^m$ and $S^f = \sum_j s_j^f$. These measures (s^m, s^f) are endogenously determined as one of the steady-state equilibrium objects.

Only singles search and meet potential spouses in the marriage market. Suppose the number of meetings between male i and female j taking place per unit time is given by $M_{ij} = \mu_{ij} M(S^m, S^f) \frac{s_i^m s_j^f}{S^m S^f}$. The function $M(S^m, S^f)$ is assumed to be increasing in both arguments, homogeneous of degree one and concave.¹⁶ The arrival rate of female j faced by male i , which is the rate at which male i meets potential spouses of female j , is then given by $\frac{M_{ij}}{s_i^m} = \mu_{ij} \frac{M(S^m, S^f)}{S^m} \frac{s_j^f}{S^f} \equiv \alpha_{ij}^m(s^m, s^f)$. The Poisson rate of meeting depends not only on the gender ratio and the type of potential spouse but also on their own type through parameters μ_{ij} . I thus allow pair-specific arrivals to capture the case where one type meets certain types more frequently. For example, the arrival rate of black females, which depends on the percentage of blacks in female singles, may differ for white and black males.

When male i and female j meet, they sample a match quality ε which is defined as an independently and identically distributed (i.i.d.) random draw from a known distribution $F(\varepsilon)$. $F(\varepsilon)$ is assumed to be twice differentiable and strictly increasing over the interval $(-\infty, \infty)$. ε is a stochastic and idiosyncratic component of marital utility and is assumed

¹⁵ i type could be, for example, a white male who has above average height. Introducing additional traits increases the dimension of the type space. $I \neq J$ refers to the case where males and females have different spouse evaluations with different traits that matter.

¹⁶Despite its notation that conventionally denotes the total measure of meetings in search literature, $M(S^m, S^f)$ is not the total measure of meetings in my setting. The total measure of meetings is given by $\sum_i \sum_j M_{ij} = \sum_i \sum_j \mu_{ij} M(S^m, S^f) \frac{s_i^m s_j^f}{S^m S^f}$. I thus allow that the total number of meetings at a given time depends not only on the total number of male and female singles, but also on how types are distributed among singles.

to be independent of the types of both sides. Given ε , the two singles first decide whether to form a match or not. Therefore, meeting, which reflects search frictions in the marriage market, is understood as a significant encounter between one male and one female that involves the following: 1) a male and a female come together and consider each other as potential spouses (availability); 2) they draw a match quality (learning); and 3) they jointly decide whether to marry (decision).

If they decide to form a marriage, couples also determine how to divide the marital surplus. They determine the amount of the transfer $t_{ij}(\varepsilon)$ to share the surplus from a match in fixed proportion with the female's share β .¹⁷ The division of marital surplus may take several forms. For example, married couples may decide who does household chores, or how to allocate household resources. In these instances, the increase in one's utility often accompanies the decrease in the utility of his or her spouse. I further assume that utilities are perfectly transferable.¹⁸

In marriages between male i and female j , the husband i gets flow utility of $u_{ij}^m + \sigma \frac{\varepsilon}{2} - t_{ij}(\varepsilon)$ and wife j gets $u_{ij}^f + \sigma \frac{\varepsilon}{2} + t_{ij}(\varepsilon)$. u_{ij}^m and u_{ij}^f are deterministic components of utilities from marriage ij evaluated by male i and female j . These terms capture heterogeneity in preferences for different marriages. σ denotes the weight on the stochastic component ε compared to the deterministic components, u_{ij}^m and u_{ij}^f , which is also related to the dispersion of match quality draws. With the introduction of (u^m, u^f) and σ , $F(\varepsilon)$ can be normalized to have mean 0 and variance 1 without loss of generality.¹⁹ In this model,

¹⁷In other words, they decide how to divide their marital surplus by determining the transfer $t_{ij}(\varepsilon)$ based on the generalized Nash Bargaining with female bargaining weight β . The Nash Bargaining solution can be achieved as a unique sub-game perfect equilibrium of a properly designed non-cooperative bargaining game under certain conditions (Rubinstein and Wolinsky 1985). Note that under Nash bargaining, the decisions of the male and the female always agree. They form a match with positive aggregate surplus. It can be easily shown that $U_{ij}(\varepsilon) - U_i \geq 0$ and $V_{ij}(\varepsilon) - V_j \geq 0$ if and only if $Sur_{ij}(\varepsilon) \geq 0$.

¹⁸There are more advantages of transferable utility compared to non-transferable utility: 1) it is not necessary to estimate separate male and female utilities; and 2) the absence of data on transfers and data on which side initiates divorce may not be a problem. In this setting, individual marital surpluses are proportional to the total surplus, and the decisions of the husband and wife always agree, which may be consistent with the altruistic nature of becoming one flesh. These features are more important than transferability itself, warranting further development and implementation of the model.

¹⁹Type-specific flow utility as single may be assumed, for male i , u_i^m and for female j , u_j^f . However, what matters and can be identified from observables is the gap between marital utility and singlehood utility,

possible emotional and economic gains of marriage, such as commitment and fidelity, rearing children, joint production and consumption, sharing of public goods, and risk pooling, are non-parametrically represented by utility terms, (u^m, u^f) . The stochastic component, ε , is then understood as a time-varying and match-specific disturbance from the mean.

Married couples are subject to the risk of divorce. With a Poisson process with rate λ , couples draw a new match quality ε from the same distribution $F(\varepsilon)$. Based upon this new quality, couples decide to separate and return to the marriage market as a single person or to stay married. I assume that λ , how frequently match qualities change, and $F(\cdot)$, the distribution of match qualities, are common to all pairs.

To summarize, this is a dynamic model of the marriage market in which singles can make decisions on marital formation upon meeting, and couples can make decisions on marital separation upon updating their match quality. Key parameters are the marriage opportunity parameters, μ_{ij} , which describe the bias of meeting frequencies from random matching; and the marriage preference parameters, u_{ij}^m, u_{ij}^f , which represent gains from marriage i, j evaluated by i type husband and j type wife. Discussions on how certain assumptions play their role in estimation and how I can relax them will be investigated in Section 1.4.

1.2.2 Equilibrium

Optimal cutoff strategies

Assume first that the distributions (s^m, s^f) , and thus arrivals $\alpha^m(s^m, s^f), \alpha^f(s^m, s^f)$ are given. Let U_i^m and U_j^f denote the expected lifetime value of staying single as a type i

$u_{ij}^m - u_i^m$ or $u_{ij}^f - u_j^f$. Thus, the flow utility in the single state is normalized to 0 and the marital utilities capture payoff surplus compared to being single.

male and as a type j female where expectation \mathbb{E} is over $\tilde{\varepsilon}$.²⁰

$$(r + \delta)U_i^m = \sum_l \alpha_{il}^m \mathbb{E} \max[U_{il}^m(\tilde{\varepsilon}) - U_i^m, 0] \quad (1.1)$$

$$(r + \delta)U_j^f = \sum_k \alpha_{kj}^f \mathbb{E} \max[U_{kj}^f(\tilde{\varepsilon}) - U_j^f, 0] \quad (1.2)$$

$U_{ij}^m(\varepsilon)$ and $U_{ij}^f(\varepsilon)$ are given as the expected lifetime value of being a husband of type i and a wife of type j in their ij marriages with match quality ε .

$$(r + \delta)U_{ij}^m(\varepsilon) = u_{ij}^m + \sigma \frac{\varepsilon}{2} - t_{ij}(\varepsilon) + \lambda \mathbb{E} \max[U_{ij}^m(\tilde{\varepsilon}) - U_{ij}^m(\varepsilon), U_i^m - U_{ij}^m(\varepsilon)] \quad (1.3)$$

$$(r + \delta)U_{ij}^f(\varepsilon) = u_{ij}^f + \sigma \frac{\varepsilon}{2} + t_{ij}(\varepsilon) + \lambda \mathbb{E} \max[U_{ij}^f(\tilde{\varepsilon}) - U_{ij}^f(\varepsilon), U_j^f - U_{ij}^f(\varepsilon)] \quad (1.4)$$

Let $Z_{ij}(\varepsilon)$ denote the expected lifetime value of marital surplus between male i and female j . With transferable utility, both agents' decision on marriage or divorce agree, and they split their surplus based on the female bargaining weight β as follows.²¹

$$Z_{ij}(\varepsilon) \equiv U_{ij}^m(\varepsilon) + U_{ij}^f(\varepsilon) - U_i^m - U_j^f \quad (1.6)$$

$$U_{ij}^m(\varepsilon) - U_i^m = (1 - \beta) Z_{ij}(\varepsilon) \quad (1.7)$$

$$U_{ij}^f(\varepsilon) - U_j^f = \beta Z_{ij}(\varepsilon) \quad (1.8)$$

$Z_{ij}(\varepsilon)$ is also written as

$$(r + \delta + \lambda)Z_{ij}(\varepsilon) = u_{ij}^m + u_{ij}^f + \sigma \varepsilon + \lambda \mathbb{E} \max[Z_{ij}(\tilde{\varepsilon}), 0] \\ - (1 - \beta) \sum_l \alpha_{il}^m \mathbb{E} \max[Z_{il}(\tilde{\varepsilon}), 0] - \beta \sum_k \alpha_{kj}^f \mathbb{E} \max[Z_{kj}(\tilde{\varepsilon}), 0] \quad (1.9)$$

²⁰See the Appendix 3.1.3 for a derivation.

²¹This is obtained based on the generalized Nash Bargaining by finding the first order condition of the equation

$$\max_{t_{ij}(\varepsilon)} (U_{ij}^m(\varepsilon) - U_i^m)^{1-\beta} (U_{ij}^f(\varepsilon) - U_j^f)^\beta. \quad (1.5)$$

Because $Z_{ij}(\varepsilon)$ increases in ε as in equation (1.9), agents use the cutoff strategy ε_{ij}^* such that $Z_{ij}(\varepsilon_{ij}^*) = 0$ in optimal decisions. Under this stationary environment, the reservation match quality ε_{ij}^* governs both formation and separation. If and only if ε is higher than the reservation match quality ε_{ij}^* , singles form the potential match or couples continue their marriage. These cutoff strategies yield simpler representations of $Z_{ij}(\varepsilon_{ij}^*)$, U_i^m and U_j^f where we define $\varphi(\varepsilon^*) \equiv \int_{\varepsilon^*}^{\infty} [1 - F(x)] dx$ for the expected match surplus in a flow term when ε^* is the reservation match quality.²²

$$(r + \delta)Z_{ij}(\varepsilon_{ij}^*) = u_{ij}^m + u_{ij}^f + \sigma \varepsilon_{ij}^* + \frac{\sigma \lambda}{r + \delta + \lambda} \varphi(\varepsilon_{ij}^*) - (r + \delta)U_i^m - (r + \delta)U_j^f = 0 \quad (1.10)$$

$$(r + \delta)U_i^m = \frac{\sigma(1 - \beta)}{r + \delta + \lambda} \sum_l \alpha_{il}^m \varphi(\varepsilon_{il}^*) \quad (1.11)$$

$$(r + \delta)U_j^f = \frac{\sigma\beta}{r + \delta + \lambda} \sum_k \alpha_{kj}^f \varphi(\varepsilon_{kj}^*) \quad (1.12)$$

Finally, the following system of equations with $I \times J$ number of unknowns ε_{ij}^* constitutes the marriage market equilibrium given arrival rates. The reservation match quality equates the value from being married (the left side of equation (1.13), $\frac{r + \delta}{\sigma} [U_{ij}^m(\varepsilon_{ij}^*) + U_{ij}^f(\varepsilon_{ij}^*)]$) to the value from being single (the right side of equation (1.13), $\frac{r + \delta}{\sigma} [U_i^m + U_j^f]$).

$$\varepsilon_{ij}^* + \frac{\lambda}{r + \delta + \lambda} \varphi(\varepsilon_{ij}^*) + \frac{u_{ij}^m + u_{ij}^f}{\sigma} = \frac{1}{r + \delta + \lambda} \left\{ (1 - \beta) \sum_l \alpha_{il}^m(s^m, s^f) \varphi(\varepsilon_{il}^*) + \beta \sum_k \alpha_{kj}^f(s^m, s^f) \varphi(\varepsilon_{kj}^*) \right\} \quad (1.13)$$

Proposition 1 (The existence and uniqueness of equilibrium reservation strategies ε^*)

Given $\{u_{ij}^m, u_{ij}^f, F(\cdot), \sigma, M(\cdot, \cdot), \mu_{ij}, \lambda, \beta, r, \delta\}$ and distributions (s^m, s^f) , the solution ε^* to equation (1.13) exists and is unique.

²²Note that $\mathbb{E}[Z_{ij}(\tilde{\varepsilon}), 0] = \int_{\varepsilon_{ij}^*}^{\infty} Z_{ij}(\varepsilon) dF(\varepsilon) = \frac{\sigma}{r + \delta + \lambda} \int_{\varepsilon_{ij}^*}^{\infty} [\varepsilon - \varepsilon_{ij}^*] dF(\varepsilon) = \frac{\sigma}{r + \delta + \lambda} \varphi(\varepsilon_{ij}^*)$, and the third equality results from integration by part and the fact that $\frac{\partial Z_{ij}(\varepsilon)}{\partial \varepsilon} = \frac{\sigma}{r + \delta + \lambda}$. $\mathbb{E}[Z_{ij}(\tilde{\varepsilon}), 0]$ is also understood as the expected surplus from marriage i, j , which is $\mathbb{E}[Z_{ij}(\tilde{\varepsilon}), 0] = [1 - F(\varepsilon_{ij}^*)] \mathbb{E}[Z_{ij}(\tilde{\varepsilon}) | \tilde{\varepsilon} \geq \varepsilon_{ij}^*]$, the probability of acceptance of i, j marriages, $(1 - F(\varepsilon_{ij}^*))$, multiplied by the expected surplus conditional on acceptance, $(\mathbb{E}[Z_{ij}(\tilde{\varepsilon}) | \tilde{\varepsilon} \geq \varepsilon_{ij}^*])$.

Proof. Implied by the Banach Fixed-point Theorem. See Appendix 3.1.1. \square

At the equilibrium, transfers $t_{ij}(\varepsilon)$ are made from male i to female j as follows:

$$u_{ij}^f + \sigma \frac{\varepsilon}{2} + t_{ij}(\varepsilon) = \beta(u_{ij}^m + u_{ij}^f + \sigma \varepsilon) + (1 - \beta)(r + \delta)U_j^f - \beta(r + \delta)U_i^m \quad (1.14)$$

$$t_{ij}(\varepsilon) = \beta u_{ij}^m - (1 - \beta)u_{ij}^f + \sigma \left(\beta - \frac{1}{2}\right) \varepsilon - \frac{\sigma \beta (1 - \beta)}{r + \delta + \lambda} \left\{ \sum_l \alpha_{il}^m \varphi(\varepsilon_{il}^*) - \sum_k \alpha_{kj}^f \varphi(\varepsilon_{kj}^*) \right\}. \quad (1.15)$$

Note that the transfer to female j increases in the outside option of female j , U_j^f , and decreases in the outside option of male i , U_i^m . As shown in equation(1.15), higher marriage market prospects of wife j summarized in the term, $\sum_k \alpha_{kj}^f \varphi(\varepsilon_{kj}^*)$ leads to higher transfers to her.

Measure of singles

The equilibrium requires that the optimal strategies ε^* are consistent with the assumed distribution of (s^m, s^f) . Let η_{ij} denote the steady-state stock of marriages between i type husbands and j type wives. Then, the inflow to this stock will be the meetings that draw match qualities that exceed the threshold, given by $M_{ij}[1 - F(\varepsilon_{ij}^*)] = \mu_{ij}M(S^m, S^f) \frac{s_i^m s_j^f}{S^m S^f} [1 - F(\varepsilon_{ij}^*)]$. The outflow from the i, j marriages will be $\eta_{ij}[\lambda F(\varepsilon_{ij}^*) + \delta]$; that is, the couples whose updated match qualities are lower than the threshold, given by $\eta_{ij}\lambda F(\varepsilon_{ij}^*)$, plus the couples who die, given by $\eta_{ij}\delta$. Thus, at the steady state where the inflow equals to the outflow, η_{ij} is given as follows.

$$\eta_{ij} = \frac{M_{ij}[1 - F(\varepsilon_{ij}^*)]}{\delta + \lambda F(\varepsilon_{ij}^*)} = \frac{1 - F(\varepsilon_{ij}^*)}{\delta + \lambda F(\varepsilon_{ij}^*)} \alpha_{ij}^m s_i^m = \frac{1 - F(\varepsilon_{ij}^*)}{\delta + \lambda F(\varepsilon_{ij}^*)} \alpha_{ij}^f s_j^f \quad (1.16)$$

This equilibrium outcomes should be consistent with exogenous distribution of types (g^m, g^f) , which leads to the following conditions about the measure of singles (s^m, s^f) .

$$g_i^m = s_i^m + \sum_l \eta_{il} = s^m + \sum_l \frac{\alpha_{il}^m [1 - F(\varepsilon_{il}^*)]}{\delta + \lambda F(\varepsilon_{il}^*)} s_i^m \quad (1.17)$$

$$g_j^f = s_j^f + \sum_k \eta_{kj} = s_j^f + \sum_k \frac{\alpha_{kj}^f [1 - F(\varepsilon_{kj}^*)]}{\delta + \lambda F(\varepsilon_{kj}^*)} s_j^f \quad (1.18)$$

$$s_i^m = \frac{g_i^m}{1 + \sum_l \frac{\alpha_{il}^m(s^m, s^f) [1 - F(\varepsilon_{il}^*)]}{\delta + \lambda F(\varepsilon_{il}^*)}} \quad (1.19)$$

$$s_j^f = \frac{g_j^f}{1 + \sum_k \frac{\alpha_{kj}^f(s^m, s^f) [1 - F(\varepsilon_{kj}^*)]}{\delta + \lambda F(\varepsilon_{kj}^*)}} \quad (1.20)$$

Finally, the marriage market equilibrium $\{\varepsilon^*, (s^m, s^f)\}$ is defined as follows.

Definition 1 (Marriage Market Equilibrium) For given primitives $\{u_{ij}^m, u_{ij}^f, F(\cdot), \sigma, M(\cdot, \cdot), \mu_{ij}, \lambda, \beta, r, \delta\}$ and exogenous distributions (g^m, g^f) , an equilibrium consists of a pair $\{\varepsilon^*, (s^m, s^f)\}$ such that

1. given (s^m, s^f) , reservation match qualities ε^* , which characterize individually optimal acceptance strategies, are the unique solution of equation (1.13), and
2. these individuals' decisions, ε^* , produce the same steady-state singles distributions (s^m, s^f) according to equations (1.19), (1.20).

Proposition 2 (The existence of a marriage market equilibrium $\{\varepsilon^*, (s^m, s^f)\}$) For any primitives $\{u_{ij}^m, u_{ij}^f, F(\cdot), \sigma, M(\cdot, \cdot), \mu_{ij}, \lambda, \beta, r, \delta\}$ and exogenous distributions (g^m, g^f) , a marriage market equilibrium exists.

Proof. Implied by the Brouwer Fixed-point Theorem. See Appendix 3.1.2 □

Two features of the model, discrete type spaces and match quality shocks, are empirically relevant and simplify theoretical analysis as well. First, cultural types, such as

race, ethnicity and religion are indeed discrete. Although some other traits, such as wealth, are continuous in nature, they may be categorized (e.g. very rich, rich, average, poor, very poor) when one evaluates the traits of others. With these discrete types, an equilibrium consists of discrete real numbers rather than a set of real functions over continuous domains. In addition, this specification can incorporate multiple attributes by defining types differently. Unlike previous marriage market models in which marital utilities are defined as a parametric function of traits of husbands and wives, I adopt non-parametric specification of preferences for every type of pairs. This feature yields explicit equilibrium conditions and identification of further parameters (opportunity parameters).²³ Lastly, the beauty of love, approximated by a random draw of match quality ε , is that it grants smoothness (continuity) to this discrete world.²⁴

Even though the existence of equilibrium is established, the uniqueness of equilibrium is not guaranteed. I accordingly propose an identification technique that is robust to multiplicity.

1.3 Data

The Panel Study of Income Dynamics (PSID 1968-2011) includes longitudinal information on individual marriage transitions.²⁵ The study starts in 1968 and includes retrospective questions on people's first and most recent marriages that began before 1968. However, I only use information after 1968, the year in which laws prohibiting interracial marriage were invalidated.²⁶

²³The limitation of this approach is that it does not allow for substantial heterogeneity of agents, because adding one trait increases the number of parameters exponentially and some pairs may have limited number of observations. I will return to the issue of missing dimensions in the following section.

²⁴Match specific heterogeneity as a form of random shock has long been employed in the search literature. See survey papers for the labor market (Rogerson, Shimer, and Wright 2005) and for the marriage market (Burdett and Coles 1999). See the analysis on existence of equilibrium with a continuum of types and transferable utility (Shimer and Smith 2000).

²⁵See details on data construction in Appendix 3.2.

²⁶*Loving v. Virginia* (388 U.S. 1 1967) is the decision of the United States Supreme Court in 1967 which invalidated laws prohibiting interracial marriages, leading to the repeal of anti-miscegenation laws in 16 Southern states.

I consider racial/ethnic types (white, black and Hispanic) of individuals for the main analysis.²⁷ I use information on self-reported race and ethnicity. Regardless of their race, if individuals report themselves as Hispanic descent in response to the question on ethnicity, they are categorized as Hispanic. Non-Hispanic others are excluded from the analysis due to their small sample size.

The following tables document the observed patterns of duration of singlehood and marriage and transition between the two states.

Table 1.1: Duration of singlehood

	white	black	Hispanic
male	8.26 [0.76]	10.83 [0.60]	8.48 [0.70]
female	7.33 [0.74]	11.11 [0.51]	7.48 [0.64]

Notes: Mean duration of all observed singlehood spells in year and fraction of completed spells in brackets.

Table 1.1 shows the mean duration of singlehood including both completed and censored spells and the fraction of completed spells in brackets. Since observations may face different censoring points, it is not straightforward to directly compare different cells in Table 1.1.²⁸ Despite this limitation, this table shows that blacks seem to wait longer than whites and Hispanics before they marry in terms both of observed mean durations and fractions of completed spells.

²⁷ W, B, H denote non-Hispanic white, non-Hispanic black, and Hispanic respectively. ij marriage denotes marriage between i type husband and j type wife with 9 possible combinations.

²⁸The estimates of the hazard rate of divorce that enable comparison of singlehood durations among types will be reported in Table 1.7, Section 1.4.

Table 1.2: Transition from singlehood to marriages

	spouse type					
	white		black		Hispanic	
white male	0.948	(5,156)	0.008	(45)	0.044	(245)
white female	0.942	(5,441)	0.022	(129)	0.036	(216)
black male	0.053	(119)	0.913	(2,051)	0.034	(75)
black female	0.017	(40)	0.951	(2,189)	0.032	(76)
Hispanic male	0.167	(283)	0.048	(81)	0.784	(1,331)
Hispanic female	0.193	(340)	0.056	(96)	0.751	(1,311)

Notes: Fraction of marriages with column type spouses in all recorded marriages of row types between 1968-2011. Number of observations in parentheses.

Table 1.2 presents the transitions to marriage. Its salient feature is the high frequencies of marriages within race. Gender asymmetry exists in marital patterns. There are more transitions to *BW* marriages than to *WB* marriages.²⁹ On the other hand, there are more transitions to *WH* marriages than to *HW* marriages. It should be noted that there are very few intermarriages between blacks and other groups (e.g., only 85 marriages between white males and black females). Even though the model itself is flexible to the introduction of several dimensions (e.g., race, religion, education), the paucity of intermarriages in the data prevents me from doing an analysis with detailed type specifications. Moreover, I need to pool all observations who may participate in the marriage market in different areas and times, although I acknowledge that it becomes difficult to defend the assumption of time-invariant parameters over 40 years.³⁰

²⁹White females marry blacks more than white males do and black males marry whites more than black females do.

³⁰Missing dimensions may cause problems in estimation regarding two hypotheses: 1) by-product hypothesis: Homogamy in racial dimension may be the by-product of sorting along other dimensions. However, it is shown by Wong (2003b) that differences in racial identities matter much more (explaining 74% of the low intermarriage rate) than differences in endowments (wage and education). 2) exchange hypothesis: This hypothesis argues that members of groups whose prestige in society is low would have better chance of marrying outside their group, if these members offered a high socioeconomic status in return. While mixed evidence has been reported for the patterns of exchange in interracial marriage (findings summarized by Schwartz 2013), this hypothesis may cause bias if, for example, the stability of *WB* marriages is not the result of their racial types but that of other dimensions. To partially address this concern, I estimate hazard rate of divorce after controlling for other dimensions (education and body mass index of the husband and wife, and the year of marriage). The main results regarding the relative strength of preferences and opportunities are unaffected. (Results are available upon request.) However, a thorough analysis with multiple traits can be done with models in which individual types are newly defined (ex: blacks above and below average education). Differences in preferences and opportunities of the same racial individuals with different endowments would be an interesting topic of investigation.

Table 1.3: Duration of marriages

male \ female	white	black	Hispanic
white	13.43 [0.21]	10.15 [0.13]	10.54 [0.18]
black	8.37 [0.20]	11.78 [0.22]	11.13 [0.23]
Hispanic	8.98 [0.13]	9.52 [0.17]	11.62 [0.06]

Notes: Mean duration of all observed marriages in year and fraction of completed spells in brackets. Each cell represents ij marriage between male type i (in the row) and female type j (in the column).

Table 1.3 documents the observed mean duration and the fraction of completed durations (marriages ended by divorce) in brackets. As discussed in Table 1.1, direct comparisons cannot be made based on this table due to different fractions of censoring and different censoring points. Based on the comparisons among the cell with similar fractions of censored spells, it is shown that WW marriages may be more stable than BW , BB , and BH marriages. Considering that only 6% of their marriages are ended in divorce, HH marriages also seem to last longer than other marriages.

Table 1.4: Type distribution in population

	white	black	Hispanic
male	0.743	0.118	0.119
female	0.749	0.134	0.117

Notes: Measure of each type when the total number of females is normalized to 1. Gender ratio is 0.980.

Lastly, as in Table 1.4, type distributions (g^m, g^f) are constructed based on the resident population in 2000 (age 16-64).³¹

³¹Source: Projections of the Resident Population by Age, Sex, Race and Hispanic Origin, Population Projections Program (U.S. Census Bureau 2000). Including non-Hispanic others (denoted as O), the fractions of each type in the Census are $\{W, B, H, O\} = \{0.709, 0.113, 0.114, 0.045\}$ for males and $\{0.712, 0.127, 0.111, 0.049\}$ for females.

1.4 Identification and Estimation

This section introduces identification techniques for recovering model primitives from observables. The model has the following primitives:

preferences $u_{ij}^m(u_{ij}^f)$: deterministic flow utility from marriage i, j for husband i (wife j); $F(\varepsilon)$: the distribution of match quality; σ : the weight on or dispersion of the stochastic component of marital utility

meeting technology μ_{ij} : the bias of meetings between i males and j females compared to random meeting; $M(S^m, S^f)$: the part in meeting specification that depends on the total numbers of male and female singles (which will be normalized to be the total number of meetings at the steady state)

other primitives λ : the Poisson arrival rate of new draws of match quality; β : the female bargaining weight; r : the discount rate; δ : the Poisson rate of death

Assume we have the following data on the environment and outcomes of the marriage market.

type distributions g_i^m, g_j^f : the type distribution in male and female populations.

individual marriage histories d_{0c} : duration of singlehood in cycle c ; γ_{0c} : 1 if the singlehood spell in cycle c is completed and 0 if the spell is censored; $\mathbb{1}_c^j$: 1 if the type of spouse in cycle c is a type j female; d_{1c} : duration of marriage in cycle c ; γ_{1c} : 1 if the marriage spell in cycle c is completed and 0 if censored.

Parameters that can be identified with given data are preference parameters, $\omega_{ij} \equiv \frac{u_{ij}^m + u_{ij}^f}{\sigma}$, (the total deterministic part of utilities from i, j marriages evaluated by husband i and wife j , divided by the dispersion parameter σ) and opportunity parameters, μ_{ij} , (the bias of meetings between i and j compared to uniformly random encounters). I calibrate other parameters based on different sources as in Table 1.5. Since those parameters in Table 1.5 affect all types of individuals and marriages at the same time, my main results about relative differences in preference and opportunity parameters across nine pairings and especially rankings among estimates are overall robust to the choice of calibrated parameters.

Even though the choice of λ has different implications on the relative strength of preferences and opportunities, the main results are also robust over the reasonable range of λ . The discussion on this aspect follows in Appendix 3.3.

Table 1.5: Parameters calibrated

parameter	description	value	source
β	bargaining weight	0.50	gender symmetry
$M(S^m, S^f)$	part of meeting process	$\sqrt{S^m, S^f}$	gender symmetry
$F(\varepsilon)$	match quality distribution	$\Phi(\varepsilon)$	standard normal
r	discount rate	0.04	standard
δ	death rate	0.0159	the average remaining number of years expected at age 16 ³²
λ	arrival rate of new match quality shock to couples	0.03	See Appendix 3.3

Notes: This table describes the baseline values of calibrated parameters and how I set this level to fit related information.

I develop a three-step estimation procedure to recover preference and opportunity parameters of the marital search model. The first step estimates the hazard rates of transitions, from singlehood to different marriages and from different marriages to singlehood, using the data on durations to marriages and durations to divorce and how duration spells are completed. In the second step, I decompose the estimated hazard rates into two parts: arrival components (arrival rates of meetings) and decision components (acceptance probabilities upon meeting). The last step maps the recovered decision rules and arrival rates to model parameters, based on the conditions that the marriage market equilibrium should satisfy.

This sequential identification establishes intuitive one-to-one mappings from observations to model outcomes, and from model outcomes to primitives, which are in the reverse order of the logic of the model. Each stage uses the results of the preceding one in an explicit manner. Estimation errors are thus computed based on the delta method. One of the strengths of this method is its robustness to multiplicity of equilibria. Since sorting externality may cause multiple equilibria, the uniqueness of equilibrium, a one-to-one mapping from primitives to model outcomes, is not guaranteed. However, there is a one-to-one

³²Source: USA Social Security Administration 2009. male: 60.64 years, female: 65.45. $0.0159 = \frac{1}{63}$

mapping from realized model outcomes suggested by the data to primitives.

1.4.1 Step 1: estimating hazard rates

This first step estimates a four-state hazard model (single, or married with a white, black, or Hispanic partner) using maximum likelihood estimation. The hazard rates of marriage formation and dissolution are recovered from observed patterns of duration and transition.

Let h_{ij}^1 denote the hazard rate of divorce of marriage ij . Let h_{ij}^{0m} (h_{ij}^{0f}) denote the hazard rate of transition to marriage ij of male i (female j) from his (her) singlehood.³³

The duration of marriage ij , d_{ij}^1 , has the density given by

$$d_{ij}^1 \sim h_{ij}^1 \exp\{-h_{ij}^1 d_{ij}^1\}. \quad (1.21)$$

The singlehood duration of male i ending in transition to ij marriages, d_{ij}^{0m} follows the density as follows.³⁴

$$d_{ij}^{0m} \sim h_{ij}^{0m} \exp\{-\sum_k h_{ik}^{0m} d_{ij}^{0m}\} \quad (1.22)$$

Taking into account censoring, the likelihood contribution for a given individual who is categorized as i type male can be written as

$$l_i^m(h_{ik}^{0m}, h_{ik}^1 \forall k; d_{0c}, \gamma_{0c}, \mathbb{1}_c^k \forall k, d_{1c}, \gamma_{1c}) = \prod_c \exp\{-\sum_k h_{ik}^{0m} d_{0c}\} [\prod_k (h_{ik}^{0m})^{\mathbb{1}_c^k}] \gamma_{0c} \prod_k [\exp\{-h_{ik}^1 d_{1c}\} (h_{ik}^1)^{\gamma_{1c}}]^{\mathbb{1}_c^k}. \quad (1.23)$$

Estimated hazard rates of marriage transition and divorce and their standard errors

³³Technically, the Poisson rate of death, δ , can also be estimated in this step. However, some reports of one's death appear inconsistent (indistinguishable from censoring of the data), leading to an unreasonably low death rate. Thus, recover the death rate directly from other data sources as shown in Table 1.5 and I ignore observed death when I construct the data on duration and transition. Marriages ended by widowhood are considered as incomplete.

³⁴This is the estimation of a competing risks model. Several hazard rates associated with various exits (all possible marriage types male i may engage) govern the duration of his singlehood.

are reported in Tables 1.6 and 1.7.

Table 1.6: Estimated hazard rates of marriage

	spouse type		
	white	black	Hispanic
white male	0.0806 (0.0012)	0.0007 (0.0001)	0.0038 (0.0003)
white female	0.0876 (0.0013)	0.0022 (0.0002)	0.0038 (0.0003)
black male	0.0030 (0.0003)	0.0452 (0.0011)	0.0018 (0.0002)
black female	0.0008 (0.0001)	0.0392 (0.0009)	0.0015 (0.0002)
Hispanic male	0.0148 (0.0010)	0.0039 (0.0005)	0.0565 (0.0019)
Hispanic female	0.0159 (0.0009)	0.0041 (0.0005)	0.0545 (0.0017)

Notes: MLE estimates, annual values, standard errors in parentheses. Each cell represents the exit of singles (in the row) to marriages with different spousal types (in the column).

Table 1.6 presents the estimated hazard rates of marriage formation, h^{0m} , h^{0f} , and standard errors in parentheses. The estimates show high frequencies of exits to intra-racial marriage for all types, which are associated with patterns in distributions in marital flows (Table 1.2). The overall levels of hazard rates are low for black males and females, related to their long singlehood duration shown in Table 1.1.

Table 1.7: Estimated hazard rates of divorce

male \ female	white	black	Hispanic
white	0.0154 (0.0003)	0.0128 (0.0038)	0.0174 (0.0017)
black	0.0236 (0.0034)	0.0186 (0.0006)	0.0205 (0.0033)
Hispanic	0.0145 (0.0018)	0.0181 (0.0035)	0.0053 (0.0004)

Notes: MLE estimates, annual values, standard errors in parentheses. Each cell represents ij marriage between male type i (in the row) and female type j (in the column).

Table 1.7 presents the estimated hazard rates of marriage dissolution, h^1 , and standard errors in parentheses. It first shows the stability of HH marriages while the most unstable marriages are between black males and white females. Marriages involving white males are relatively stable. On the other hand, marriages of black males are overall less persistent, although for black females' marriages to white males are estimated to be stable.³⁵

³⁵Bratter and King (2008) document the similar patterns in marital stability of different pairings based on

1.4.2 Step 2: recovering model outcomes

In the second step, I recover equilibrium outcomes of the model, arrival rates of different meetings (marriage opportunities), and decision rules of singles and couples. Note that a hazard rate of transition has two parts, the arrival component and the decision component. The hazard rate of divorce of marriage ij , h_{ij}^1 , is given by the following.

$$h_{ij}^1 = \lambda F(\varepsilon_{ij}^*) \quad (1.24)$$

Given the calibrated value of λ , the rate of updating match quality, $F(\varepsilon_{ij}^*)$, the probability of rejecting marriage ij , is recovered. The crucial identifying assumption is that $F(\cdot)$ and λ , how match qualities of couples are distributed and evolve, do not depend on types. Different types experience the same process of stochastic shocks after marriage. Therefore, differences in observed marriage stability across different type pairings can be attributed to differences in the time-invariant willingness to accept this type of marriage represented by $F(\varepsilon_{ij}^*)$.³⁶

After recovering the acceptance rules, across-type variation in hazard rates of mar-

the National Survey of Family Growth (NSFG) 2002. They also find the likelihood of divorce is race- and gender-specific, and the differences among *WW*, *WB*, *BW* marriages are statistically significant. Black-white marriages were twice (2.26) as likely to result in divorce in 10 years after marriage as white-white couples, while white-black marriages were less likely (0.67). Considering the patterns in dimensions other than race (e.g., Blacks who marry whites may be more educated than blacks in *BB* marriages), they include controls (age cohort, age at marriage, education of respondent, status of parents' marriage, status of parents' marriage when respondent was age 14, age differences between spouses, premarital cohabitation, and premarital births). The ratio (calculated based on the coefficient on racial dummies) from the adjusted model are 2.09 and 0.56, which are similar to the results without controls. This evidence supports that the differences in marital stability across racial pairings in Table 1.7 are not systematically affected by small sample bias or by missing dimensions through the exchange hypothesis.

³⁶Linking higher marital stability to the higher degree of acceptance of that marriage is one of the most significant features and major contributions of my model. I will discuss how relaxing of some model assumptions affects this link and the estimated results:

1) First to consider is the type-dependent λ , such as λ_{ij} . Some types of couples may update their match quality more frequently than others. It then becomes impossible to find the degree of acceptance $F(\varepsilon_{ij}^*)$ from the information on marital stability. However, it is likely that interracial marriages have higher λ than same-race marriages, $\lambda_{ij} > \lambda_{ii} \forall i, j, i \neq j$. In this case the rejection probabilities for interracial marriages would be lower than estimated ones while the rejection probabilities for same-race marriages would be higher. The gap in acceptance between same-race and interracial marriages would be smaller than what is estimated. Thus, the role of meetings, which turns out to be a critical factor in racial marital patterns, would be underestimated rather than overestimated.

riage transition (variation in singlehood durations and transitions into different marriages) can be used to identify how frequently singles receive marriage opportunities with different potential spouses. The hazard rate of marriage ij to male i , h_{ij}^{0m} is given by

$$h_{ij}^{0m} = \alpha_{ij}^m [1 - F(\varepsilon_{ij}^*)]. \quad (1.26)$$

Given that the decision rule, the acceptance probability $[1 - F(\varepsilon_{ij}^*)]$, is recovered from the persistence of marriage, the arrival component, α_{ij}^m is now recovered in equation (1.26). All information on marital and singlehood durations, and transitions to different marriages, contributes to the identification of arrival rates. The mean duration of marriage ij is $\mathbb{E}(d_{ij}^1) = \frac{1}{h_{ij}^1} = \frac{1}{\lambda F(\varepsilon_{ij}^*)}$ and the mean duration of i 's singlehood is given by $\mathbb{E}(d_i^{0m}) = \frac{1}{\sum_l h_{il}^{0m}} = \frac{1}{\sum_l \alpha_{il}^m [1 - F(\varepsilon_{il}^*)]}$. If p_{ij}^m denotes the probability that male i marries female j conditional that he marries someone, which is given by $p_{ij}^m = \frac{h_{ij}^{0m}}{\sum_l h_{il}^{0m}} = \frac{\alpha_{ij}^m [1 - F(\varepsilon_{ij}^*)]}{\sum_l \alpha_{il}^m [1 - F(\varepsilon_{il}^*)]}$, then rearranging equation

2) Secondly, the distribution of match quality shock $F(\varepsilon)$ can also be type-specific, such as $F_{ij}(\varepsilon)$. Different variances σ_{ij} prevent us from recovering ε_{ij}^* and σ_{ij} separately (only $F_{ij}(\varepsilon_{ij}^*)$, and then $\frac{\varepsilon_{ij}^*}{\sigma_{ij}}$ can be recovered under normal distribution.) The equilibrium condition in Equation 1.13 cannot be used to recover marital utilities (step 3, first part), since equilibrium reservation strategies ε^* are now governed by the following equation.

$$\sigma_{ij} \varepsilon_{ij}^* + \frac{\lambda \sigma_{ij}}{r + \delta + \lambda} \varphi(\varepsilon_{ij}^*) + u_{ij}^m + u_{ij}^f = \frac{1}{r + \delta + \lambda} \left\{ (1 - \beta) \sum_l \alpha_{il}^m \sigma_{il} \varphi(\varepsilon_{il}^*) + \beta \sum_k \alpha_{kj}^f \sigma_{kj} \varphi(\varepsilon_{kj}^*) \right\} \quad (1.25)$$

Nevertheless, one can still recover opportunity components, arrival rates $\alpha_{ij}^m, \alpha_{ij}^f$ (step 2, second part) and meeting frequency parameters μ_{ij} (step 3, second part), only based on recovered $F_{ij}(\varepsilon_{ij}^*)$. However, possible systematic differences in σ may cause biases in estimation of preference components. Consider the case where intermarriages draw match quality from a distribution with higher variance, $\sigma_{ij} > \sigma_{ii} \forall i, j, i \neq j$. Cutoffs for interracial marriages would then be underestimated, whereas cutoffs for same-race marriages would be overestimated. On the contrary to type-specific λ_{ij} , this may lead to the overestimation of the importance of meeting factors compared to preference factors.

3) Lastly, allowing the standard form of persistency in the evolution of match quality ε (e.g., new match quality ε' is $\varepsilon' = \varepsilon_0 + v$ while v is drawn from a common distribution and independent of ε_0 , the initial match quality) still preserves the link. It is true that the type of marriage with a higher match quality cutoff (ε_{ij}^*) has a higher level of match quality draw than other marriages with lower cutoffs on average. However, what matters in the stability of marriage is not the absolute level of match quality but the distance between the current match quality draw and the cutoff. Thus, my model says that marriages that survive longer are the ones with high time-invariant utilities, $u_{ij}^m + u_{ij}^f$, rather than the ones with passionate but possibly fleeting emotion, ε .

(1.26) then yields

$$\alpha_{ij}^m = \frac{p_{ij}^m}{\mathbb{E}(d_i^{0m})} \frac{1}{1 - \frac{1}{\lambda \mathbb{E}(d_{ij}^1)}}. \quad (1.27)$$

Shown in equation (1.27), if the share of ij marriage in all marriage of male i is small (low p_{ij}^m), regardless of the stability of ij marriages (high $\mathbb{E}(d_{ij}^1)$), this pattern is explained by the infrequent arrivals of ij meetings (low α_{ij}^m). In addition, if he stays single for a long period of time (high $\mathbb{E}(d_i^{0m})$), this also lowers his overall arrival rates.

Tables 1.8 and 1.9 document the results from this second step.

Table 1.8: Estimated rejection probabilities

male \ female	white	black	Hispanic
white	0.513 (0.011)	0.426 (0.128)	0.579 (0.056)
black	0.787 (0.112)	0.621 (0.020)	0.683 (0.109)
Hispanic	0.483 (0.060)	0.602 (0.116)	0.176 (0.014)

Notes: Estimated rejection probability of marriage upon meeting or upon updating of match quality, $F(\varepsilon_{ij}^*)$, for male i as a row player and female j as a column player. Continuous mapping from MLE estimates. Standard errors in parentheses, computed based on the delta method.

Table 1.8 shows the rejection probability of each meeting based on a one-to-one mapping from the estimated hazard rate of divorce as documented in Table 1.7. The lowest rejection probability (or the highest acceptance probability) is found in Hispanic intra-marriages, which is linked to observations in marital stability.

Table 1.9: Estimated arrival rates

	spouse type		
	white	black	Hispanic
white male	0.1654 (0.0045)	0.0012 (0.0003)	0.0090 (0.0014)
white female	0.1797 (0.0048)	0.0104 (0.0057)	0.0074 (0.0010)
black male	0.0139 (0.0076)	0.1193 (0.0071)	0.0055 (0.0020)
black female	0.0014 (0.0004)	0.1033 (0.0060)	0.0037 (0.0012)
Hispanic male	0.0286 (0.0038)	0.0097 (0.0031)	0.0686 (0.0025)
Hispanic female	0.0377 (0.0055)	0.0131 (0.0048)	0.0662 (0.0024)

Notes: Estimated arrival rate of each meeting, $\alpha_{ij}^m(\alpha_{ij}^f)$, for male i (female j) with column type females $\forall j$ ($\forall i$). Continuous mapping from MLE estimates. Standard errors in parentheses, computed based on the delta method.

Table 1.9 shows the recovered arrival rates. Since the differences in acceptance probabilities are not substantial, these arrival rates mimic the hazard rates of marriage formation in Table 1.6 and are characterized by high arrival rates of intra-racial meetings. It has been shown that some intramarriages (e.g., *WB* marriages) are stable despite their low frequencies. This further lowers inferred arrivals of these types of meetings. In addition, the arrival rates of meetings between non-Hispanics and Hispanics are higher than the rates for meetings between blacks and whites.

1.4.3 Step 3: recovering model parameters

Using the recovered decision rules and arrival rates, the last step employs the equilibrium condition to identify underlying parameters. Note first that what is recovered in the second step is reduced-form parameters that depend on equilibrium outcomes, instead of structural parameters that are invariant to environment. The counterfactual experiments can be done after finding the structural parameters that govern underlying preferences and restrictions on individuals' decision making.³⁷

First, I use the condition for optimal threshold, ε^* , to identify underlying preferences. As stated in Section 1.2, equation (1.13), the optimal stopping rules are determined at the point where two individuals are indifferent between being single and being married with the threshold match quality.

$$\frac{u_{ij}^m + u_{ij}^f}{\sigma} + \varepsilon_{ij}^* + \frac{\lambda \varphi(\varepsilon_{ij}^*)}{r + \delta + \lambda} = \frac{r + \delta}{\sigma} U_i^m + \frac{r + \delta}{\sigma} U_j^f \quad (1.28)$$

$$\frac{r + \delta}{\sigma} U_i^m = \frac{(1 - \beta) \sum_l \alpha_{il}^m \varphi(\varepsilon_{il}^*)}{r + \delta + \lambda} \quad (1.29)$$

$$\frac{r + \delta}{\sigma} U_j^f = \frac{\beta \sum_k \alpha_{kj}^f \varphi(\varepsilon_{kj}^*)}{r + \delta + \lambda} \quad (1.30)$$

³⁷For example, the acceptance rules represented by ε_{ij}^* (often recovered by revealed preference argument based on choice-set data) involve values in marriage $U_{ij}^m + U_{ij}^f$ and values in singlehood $U_i^m + U_j^f$, which are equilibrium objects.

I first recover the outside option of each type, $\frac{r+\delta}{\sigma}U_i^m, \frac{r+\delta}{\sigma}U_j^f \forall i, j$. The discounted lifetime value of being single of male type i in flow utility term, $\frac{r+\delta}{\sigma}U_i^m$, is related to his marital prospects. These marital prospects include two factors: first, how frequently he gets marriage opportunities, $\alpha_{ij}^m \forall j$; and second, how much expected utility he gets from each marriage, $\varphi(\varepsilon_{ij}^*) \forall j$.³⁸

At the threshold match quality, ε_{ij}^* , the utility of being married with this match quality (the left hand side of equation (1.28)) should be equal to the utility of staying single (the right hand side of equation (1.28)). This leads to the identification of marriage utilities between male i and female j , $\omega_{ij} \equiv \frac{u_{ij}^m + u_{ij}^f}{\sigma}$. This exercise shows that high marital utility (high ω_{ij}) is related to two factors: 1) the stability of this type of marriage (high ε_{ij}^*), and 2) male i and female j 's high outside options (high U_i^m, U_j^f) that are affected by preferences and opportunities of all marriages that male i and female j may engage in.

Next, I use the conditions for steady states and gender consistency in meetings to identify opportunity parameters that govern the meeting process, μ_{ij} . Recall the specification of meetings between i and j , M_{ij} , and their relationship with arrival rates.

$$M_{ij} = \mu_{ij} \sqrt{S^m S^f} \frac{s_i^m s_j^f}{S^m S^f} = \alpha_{ij}^m s_i^m = \alpha_{ij}^f s_j^f \quad (1.31)$$

Given the steady state conditions, the measure of singles, s_i^m, s_j^f , are recovered from esti-

³⁸Recall that $\varphi(\varepsilon_{ij}^*) = \frac{r+\delta+\lambda}{\sigma} \mathbb{E} \max[Z_{ij}(\tilde{\varepsilon}), 0] = \int_{\varepsilon_{ij}^*}^{\infty} [1 - F(\varepsilon_{ij})]$, thus $\varphi(\varepsilon_{ij}^*)$ measures the expected marital surplus evaluated before match quality is realized. Given recovered ε_{ij}^* and the assumption on the functional form of $F(\cdot)$, $\varphi(\varepsilon_{ij}^*)$ is recovered.

mated hazard rates, h_{ij}^1, h_{ij}^{0m} and h_{ij}^{0f} , as follows: ³⁹

$$s_i^m = \frac{g_i^m}{1 + \sum_l \frac{\alpha_{il}^m [1 - F(\varepsilon_{il}^*)]}{\delta + \lambda F(\varepsilon_{il}^*)}} = \frac{g_i^m}{1 + \sum_l \frac{h_{il}^{0m}}{\delta + h_{il}^1}} \quad (1.32)$$

$$s_j^f = \frac{g_j^f}{1 + \sum_k \frac{\alpha_{kj}^f [1 - F(\varepsilon_{kj}^*)]}{\delta + \lambda F(\varepsilon_{kj}^*)}} = \frac{g_j^f}{1 + \sum_k \frac{h_{kj}^{0f}}{\delta + h_{kj}^1}} \quad (1.33)$$

By plugging these into equation (1.31), I can find $\hat{\mu}_{ij}^m, \hat{\mu}_{ij}^f$ to fit the estimated arrival rates $(\alpha_{ij}^m, \alpha_{ij}^f)$, from Step 2). Note that the value obtained from the male side can be different from the value obtained from the female side.⁴⁰ To find one $\hat{\mu}_{ij}$, I efficiently combine these two pieces of information based on the minimum distance estimation as follows where \hat{V} denotes the estimated variance of estimators.

$$\hat{\mu}_{ij} = \hat{\mu}_{ij}^m \frac{\hat{V}(\hat{\mu}_{ij}^f)}{\hat{V}(\hat{\mu}_{ij}^m) + \hat{V}(\hat{\mu}_{ij}^f)} + \hat{\mu}_{ij}^f \frac{\hat{V}(\hat{\mu}_{ij}^m)}{\hat{V}(\hat{\mu}_{ij}^m) + \hat{V}(\hat{\mu}_{ij}^f)} \quad (1.34)$$

The results, the estimated preference and opportunity parameters, ω_{ij}, μ_{ij} , from this last step will be presented in the subsequent section.

³⁹The comparison of the fraction of singles with the data counterpart will be presented in Section 1.5. My model can fit these untargetted measures.

⁴⁰The reason is that the samples do not contain all agents in the marriage market. In addition, observations are from different periods of time.

1.5 Results

1.5.1 Estimated parameters

Table 1.10: Estimated marital preferences

male \ female	white	black	Hispanic
$[\frac{r+\delta}{\sigma}U_i^m] \setminus [\frac{r+\delta}{\sigma}U_j^f]$	[0.427]	[0.169]	[0.478]
white	0.650	0.571	0.562
[0.389]	(0.034)	(0.263)	(0.129)
black	-0.212	-0.032	0.130
[0.200]	(0.363)	(0.055)	(0.277)
Hispanic	0.818	0.307	1.547
[0.496]	(0.131)	(0.263)	(0.063)

Notes: Estimated preferences of marriage ij , $\omega_{ij} = \frac{u_{ij}^m + u_{ij}^f}{\sigma}$, between male i (row player) and female j (column player). The value of being single of each type in flow terms, $\frac{r+\delta}{\sigma}U_i^m$, $\frac{r+\delta}{\sigma}U_j^f$, at the margin. Continuous mapping from MLE estimates, standard errors in parentheses computed based on the delta method.

In Table 1.10, the number at the margin is the value of being single for each type, $\frac{r+\delta}{\sigma}U_i^m$, $\frac{r+\delta}{\sigma}U_j^f$, that represents marital prospects. Due to both their stable marriages and frequent arrivals of HH marriage opportunities, it is estimated that Hispanic males and females have the highest marital prospects and black males and females have the lowest.⁴¹

Each cell in Table 1.10 presents the estimated marital preference, $\omega_{ij} = \frac{u_{ij}^m + u_{ij}^f}{\sigma}$, the aggregate gains from ij marriage evaluated by i husband and j wife apart from match quality.⁴² Comparisons across different marriage types find that the marital utilities associated with white males are high overall, whereas the marital utilities associated with black males are generally low. Two channels may explain this. First, u_{Wj}^m is high for a certain female j ; white males get high utilities from marriage with wife j . Second, u_{Wj}^f is high for a certain j ; white males are desirable spouses for their wives of type j . Since I cannot separate u_{ij}^m and u_{ij}^f without information on intra-household transfers, it is impossible to distinguish

⁴¹Under the normalization of singlehood flow utilities to zero, this value of being a different type captures only marital prospects. The value of being a different race in other aspects (e.g., labor market prospects) cannot be measured in my setting.

⁴²Even though some values of deterministic components of utility are negative, due to the existence of the idiosyncratic component (match quality), expected marital surpluses of all marriage types are positive. Note that $\varphi(\varepsilon^*) = \int_{\varepsilon^*}^{\infty} [1 - F(\varepsilon)] d\varepsilon > 0$ for all $-\infty < \varepsilon^* < \infty$.

between the two explanations.⁴³

Due to this inseparability of utilities, $u_{ij}^m + u_{ij}^f$, preferences for the same-race marriages cannot be tested according to individual perspectives (for example, whether $u_{ii}^m > u_{ij}^m \forall j \neq i$ holds for male i). However, I can still compare two homogamous marriages with two intermarriages and see, for example, whether $\omega_{WW} + \omega_{HH} > \omega_{WH} + \omega_{HW}$ holds. If this inequality holds, two groups may generate more marital utilities by segregating instead of mixing, even though this does not guarantee that all four individuals involved prefer the same-type spouses. These inequalities are statistically significant between whites and Hispanics (p-value: 0.000), and between blacks and Hispanics (p-value: 0.005), but not between whites and blacks (p-value: 0.56).

By comparing utilities of different homogamous marriages, I find $\omega_{HH} > \omega_{WW}$ (p-value: 0.000) and $\omega_{WW} > \omega_{BB}$ (p-value: 0.000). Hispanic couples value marriage more than white couples, whereas black couples value it less. Lastly, some intermarriages show gender asymmetry in their utility levels, $\omega_{WB} > \omega_{BW}$ (p-value: 0.08), whereas these inequalities are not significant for the rest of the pairs, $\omega_{WH} \simeq \omega_{HW}$ (p-value: 0.16), $\omega_{BH} \simeq \omega_{HB}$ (p-value: 0.64).

Table 1.11: Estimated marital opportunities

male \ female	white	black	Hispanic
white	1.504 (0.029)	0.014 (0.006)	0.375 (0.059)
black	0.150 (0.127)	3.452 (0.175)	0.376 (0.138)
Hispanic	0.305 (0.043)	0.245 (0.075)	4.074 (0.173)

Notes: Estimated opportunities of marriage ij , $\tilde{\mu}_{ij}$, between male i (row player) and female j (column player), illustrating the bias from random matching, $\tilde{\mu}_{ij} =$ (estimated measure of ij meetings) / (total meetings $\times \frac{s_i^m}{S^m} \times \frac{s_j^f}{S^f}$). Estimates are obtained based on the minimum distance estimation.

Table 1.11 shows the estimated marital opportunities, where the Poisson parameter

⁴³See Appendix 3.4 on the separate identification of husbands' and wives' utilities upon availability of transfer data.

that governs the frequency of ij meetings is specified as follows:

$$M_{ij} = \mu_{ij} \sqrt{S^m S^f} \frac{s_i^m s_j^f}{S^m S^f} = \tilde{\mu}_{ij} \bar{\mu} \sqrt{S^m S^f} \frac{s_i^m s_j^f}{S^m S^f}. \quad (1.35)$$

I further separate μ_{ij} into two parts, $\tilde{\mu}_{ij}$ and $\bar{\mu}$. $\bar{\mu}$, a common part for all μ_{ij} , is normalized so that $\bar{\mu} \sqrt{S^m S^f} = \sum_{i,j} M_{ij}$ (total meetings).⁴⁴ Upon this normalization, the pair-specific part $\tilde{\mu}_{ij}$ now represents the bias from random matching, since this is the ratio between the estimated measure of ij meetings (M_{ij}) and the measure of ij meetings implied by the uniform random process (total meetings $\times \frac{s_i^m}{S^m} \times \frac{s_j^f}{S^f}$).

Results show that there are more same-race meetings implied by the uniform random process, and fewer interracial meetings. The bias toward meetings among Hispanics is the highest, and WW the lowest. This helps minorities increase the likelihood of within-group meetings, which would be rare under a random process due to their small population shares. Regardless of this effect, however, the arrival rates, taking into account the shares in the population, are favorable for majority groups, $\alpha_{WW} > \alpha_{BB} > \alpha_{HH}$ (Table 1.9). Gender asymmetry is found but is statistically insignificant.⁴⁵

1.5.2 Model fit

With the estimated parameters, I can solve the equilibrium.⁴⁶ The patterns predicted by the model mimic those observed in the data, suggesting that the model is properly specified.

⁴⁴ $\bar{\mu}$ is estimated to be 0.164 with standard error 0.004.

⁴⁵ $\mu_{BW} \simeq \mu_{WB}$ (p-value: 0.28), $\mu_{WH} \simeq \mu_{HW}$ (p-value: 0.34), and $\mu_{BH} \simeq \mu_{HB}$ (p-value: 0.41). However, it is shown in Table 1.10 that preference components are unable to drive gender asymmetry in marital patterns. (e.g., more BW than WB marriages are observed but utilities from WB marriages are higher than those from BW marriages.) One can conclude that meetings, not preferences, primary cause gender asymmetric marital patterns.

⁴⁶ Solving for equilibrium $\{\varepsilon^*, (s^m, s^f)\}$ given parameters is finding a fixed-point. I begin t^{th} iteration with type distributions among singles (s_t^m, s_t^f) . Given these type distributions and thus arrival rates, optimal thresholds ε_t^* are obtained by solving equations (1.13). Based on equations (1.19) and (1.20), I can update the type distributions (s_{t+1}^m, s_{t+1}^f) and continue this procedure until convergence is achieved. I always obtain the same equilibrium outcome regardless of the choice of initial guesses.

Table 1.12: Model fit: hazard rates of marriage formation

	Model			Data		
	W	B	H	W	B	H
white male	0.0853	0.0003	0.0026	0.0806	0.0007	0.0038
white female	0.0845	0.0009	0.0026	0.0876	0.0022	0.0038
black male	0.0037	0.0490	0.0019	0.0030	0.0452	0.0018
black female	0.0009	0.0353	0.0016	0.0008	0.0392	0.0015
Hispanic male	0.0185	0.0037	0.0553	0.0148	0.0039	0.0565
Hispanic female	0.0178	0.0032	0.0542	0.0159	0.0041	0.0545

Notes: Each cell represents the hazard rate of marriage formation for the type in the row. The column represents the type of spouses (W:white, B:black, H:Hispanic). Model: model predicted hazard rates given estimated parameters. Data: MLE estimates of hazard rates from Step 1 (Table 1.6).

Table 1.13: Model fit: hazard rates of divorce

male\female	Model			Data		
	W	B	H	W	B	H
white	0.0153	0.0128	0.0176	0.0154	0.0128	0.0174
black	0.0236	0.0187	0.0207	0.0236	0.0186	0.0205
Hispanic	0.0143	0.0180	0.0054	0.0145	0.0181	0.0053

Notes: Each cell represents the hazard rate of divorce. Husband types are in the row and wife types (W:white, B:black, H:Hispanic) are in the column. Model: model predicted hazard rates given estimated parameters. Data: MLE estimates of hazard rates from Step 1 (Table 1.6).

Tables 1.12 and 1.13 compare the fitted hazard rates given estimated parameters (baseline hazard rates) with the observed hazard rates that are estimated in Step 1. The baseline model fits the estimated hazard rates and thus marital transition patterns in the data.

Table 1.14: Model fit: distribution of marriage flows

	Model			Data		
	W	B	H	W	B	H
white male	0.967	0.003	0.029	0.948	0.008	0.044
white female	0.961	0.010	0.030	0.942	0.022	0.036
black male	0.068	0.897	0.035	0.053	0.913	0.034
black female	0.024	0.934	0.042	0.017	0.951	0.032
Hispanic male	0.239	0.048	0.714	0.167	0.048	0.784
Hispanic female	0.237	0.042	0.721	0.193	0.056	0.751

Notes: Fraction of marriages with column type spouses (W:white, B:black, H:Hispanic) among all marriage flows of row types in a given period. Model: fraction in flows at a fixed time period. Data: fraction in all marriages of sample individuals ever recorded.

Table 1.15: Model fit: distribution of marriage and single stocks

	Model				Data			
	W	B	H	S	W	B	H	S
white male	0.716	0.003	0.020	0.261	0.723	0.007	0.034	0.236
white female	0.710	0.006	0.023	0.262	0.704	0.018	0.032	0.247
black male	0.037	0.553	0.021	0.390	0.032	0.545	0.019	0.404
black female	0.015	0.487	0.022	0.476	0.011	0.464	0.018	0.507
Hispanic male	0.142	0.025	0.602	0.231	0.119	0.035	0.601	0.245
Hispanic female	0.128	0.021	0.612	0.240	0.127	0.035	0.560	0.279

Notes: Fraction in each state: being married to column type spouses (W:white, B:black, H:Hispanic), or being single (S:single). Model: at the steady state. Data: when the last information is collected.

Table 1.14 presents the matching outcomes in flows of consummated marriages at a given period of time. The values are related to the hazard rates of marriage formation (Table 1.12), thus indirectly used for my estimation. Table 1.15 presents the fraction of each type in different states at the steady state.⁴⁷ Even though I do not use this information (distributions of marriages and singles) for my estimation, the observed data are similar to the model predictions. Given the fact that these model predictions of type distribution on stocks are based on model specifications (steady-state conditions and description of hazard rates), I can conclude that the model is sufficiently well specified and reasonably estimated enough to be used for counterfactual analyses.⁴⁸

1.5.3 Same-race biases in acceptances and meetings

Several analyses with the baseline model show biases in acceptances and meetings that potentially lead to racial homogamy.

⁴⁷The data present the last-observed status of sample individuals.

⁴⁸Some discrepancies in Tables 1.12, 1.14 and 1.15 are caused by gender consistency restrictions of the model, where I get μ_{ij} from male side estimates, μ_{ij}^m , and from female side estimates, μ_{ij}^f . For example, if I use male side information only, the fit for white males' exit to WH marriages improves, but the Hispanic females' exit to WH marriages worsens. If I use female side information, the opposite holds. This inconsistency is attributable to the pooling of all samples that may participate in different marriage markets during different time periods.

Table 1.16: Same-race biases in acceptances

	probability of acceptance		
	intra-meeting	inter-meeting	ratio ($\frac{\text{inter}}{\text{intra}}$)
white male	0.491	0.427	0.870
white female	0.491	0.385	0.784
black male	0.378	0.240	0.635
black female	0.378	0.451	1.194
Hispanic male	0.822	0.499	0.608
Hispanic female	0.822	0.394	0.480
Total	0.489	0.395	0.808

Notes: Probabilities of accepting intra- and inter- meetings and their ratios are presented.

The first two columns in Table 1.16 present the probability of accepting intraracial and interracial meetings. Except for black females, individuals accept (and are accepted at) same-race meetings more easily than meetings across racial borders. Overall, the acceptance rate of interracial meetings is 80.8% of the acceptance rate of same-race meetings, indicating the same-race preferences in accepting behaviors. These ratios are the lowest among Hispanics, indicating that the same-race preferences are highest for Hispanics.

Table 1.17: Same-race biases in meetings

	fraction of inter-meetings		
	baseline	shuffled	index
white male	0.037	0.214	0.826
white female	0.050	0.224	0.778
black male	0.154	0.860	0.821
black female	0.056	0.844	0.934
Hispanic male	0.398	0.926	0.571
Hispanic female	0.446	0.932	0.522
Total	0.080	0.363	0.780

Notes: The fractions of inter-meetings in the baseline model and in the uniformly random process are presented. The third column (segregation index) measures $(\text{shuffled-baseline})/(\text{shuffled})$.

The first column of Table 1.17 documents the fraction of interracial meetings among all meetings for each type. Among all meetings of white males, only 3.7% of them are interracial (meetings with a black or Hispanic potential spouse), whereas the share of blacks and Hispanics is 21.4% among female singles in the marriage market. This gap is summa-

rized as a homogamy index of 0.826 in the third column.⁴⁹ This index summarizes how the current rate of intermeeting is far from the rate under random meeting, which is also connected to the odd ratio (1-homogamy index). White males are 0.174(= 1 – 0.826) times less likely to meet outside their race than they would under random meeting. It also shows that the meeting opportunities of Hispanics are close to the racial mix of the marriage market. The opportunities for black females tend toward same-race meetings. Overall, only 8.0% of all meetings are interracial, whereas 36.3% is expected under a uniformly random process.⁵⁰ There is a great distance between the current meeting distribution and the shuffled distribution, with a homogamy index of 0.78. People are 0.22 times less likely to meet across their racial boundaries than they would under uniformly random meeting.

Table 1.18: Same-race biases in marriage stocks

	fraction of inter-marriages		
	baseline	shuffled	index
white male	0.031	0.224	0.862
white female	0.038	0.230	0.833
black male	0.094	0.901	0.896
black female	0.072	0.899	0.920
Hispanic male	0.217	0.875	0.752
Hispanic female	0.195	0.872	0.776
Total	0.061	0.376	0.837

Notes: The fractions of inter-marriages in the baseline model and in uniformly random process are presented. The third column (homogamy index) measures (shuffled-baseline)/(shuffled).

Table 1.18 shows the distribution of marriage stocks and how it differs from uniformly random matching. Since marriages are meetings that are accepted, the marriage distribution in Table 1.18 summarizes the features of acceptance behaviors (Table 1.16) and

⁴⁹For example, for white males, the homogamy index is computed as $(0.214 - 0.037)/0.214$. This index is also known as the homophily index suggested by Coleman (1959) and used also by Currarini, Jackson, and Pin (2010): $\text{homogamy index}_i^m = ((1 - \text{share}_i^m) - (1 - \text{intra}_i^m))/(1 - \text{share}_i^m) = (\text{intra}_i^m - \text{share}_i^m)/(1 - \text{share}_i^m)$. An index of 0 indicates that this individual's meeting (or marriage) is distributed according to the racial mix of the market, whereas an index of 1 indicates that this individual only meets (or marries) a spouse of his or her own race (perfect segregation). One minus this index gives the odd ratio of intermarriage, $1 - \text{homogamy index}_i^m = (1 - \text{intra}_i^m)/(1 - \text{share}_i^m)$.

⁵⁰Shuffled distributions are specific to initial distributions. Given an initial distribution, take the total number of meetings and the share of each male and female type. Then, the meeting between i, j in the shuffled distribution is calculated by 'shuffled meeting $_{ij} = \text{total meetings} \times \text{share}_i^m \times \text{share}_j^f$.'

of meeting opportunities (Table 1.17). In addition, how long marriage lasts contributes to the measure of steady-state stock of a certain marriage type. Because singles accept intra-meetings more easily than inter-meetings and same-race marriages are on average more stable than interracial marriages, the degree of segregation increases to 0.837 compared to the homogamy index in meetings (0.780 in Table 1.17). Only 6.1% of all marriages are interracial, which is above 37.6%, the level expected in random matching.⁵¹

1.5.4 Decomposition: preferences vs. opportunities

In this subsection, I decompose the role of opportunities and preferences in explaining racial homogamy. It should first be reiterated that the findings of this exercise depend on the assumption that preferences and opportunities are taken as initial conditions to individuals in the marriage market.⁵² If there are interactions between preferences and opportunities, for example, if meetings are affected by mate preferences (if preferences are shaped by meetings), the following counterfactual analysis that does not capture this channel may underestimate the role of preferences (opportunities). My model has the limitation of treating meetings and preferences as essentially exogenous, its advantage being its ability to identify and quantify the separate role of each channel.

The following arguments may validate the assumption that intraracial and interracial meeting opportunities can be predetermined to each individual in the marriage market, which justifies the proceeding counterfactual analysis to quantify the role of each channel in shaping marital outcomes.

- 1. Residential and school segregation:** In 2000, the median black lived in a neighborhood that was 52 percent black (Easterly 2009). In 2001, 80% of Latino students and

⁵¹These numbers also fit the data. In the observed data as given in Table 1.15, the intermarriage rate is given by 7.9%, where 40.2% is expected under uniformly random matching, which leads to the homogamy index of 0.803.

⁵²The parameter estimates given in Tables 1.10 and 1.11 are correctly estimated under the less restrictive assumption that meetings and preferences are determined before individuals play in the marriage market where they respond to presented opportunities by rejecting or accepting.

74% of black students attended schools where whites were not the majority. 43% of Latinos and 38% of blacks attended intensely segregated schools (less than 10% of whites students) across the nation (Orfield, Kuscera, and Siegel-Hawley 2012). Since the area where individuals reside and the school which individuals attend are mainly determined by their parents, their earlier year experiences with intense intraracial contacts in neighborhood and schools are exogenous to young adults. This may have continual effects on their networks – whom they get along with and whom they meet as potential spouses.

2. **Institutional settings and the role of third parties:** Laumann et al. (2000) document that 23%, 15%, 10%, 8%, and 4% of surveyed married couples met partners at school, work, private parties, church, and gym/social clubs respectively. These institutional/preselected meeting settings account for a sizable portion (60%) of the places where partners have met. These researchers argue that meetings occur frequently in work and school because of the ubiquity of these in people's lives. This also shows that gatherings occur and draw people to them for many reasons other than meeting potential partners, including religious worship, sharing common interests, and enjoying the experience of being in a group. The same study also shows that 66% percent of all marriages involved an introduction by some third party (e.g., family members, friends, coworkers and classmates). This also indicates the importance of a person's social network in determining his or her set of potential partners.⁵³
3. **Role of randomness:** Anecdotal evidence witnesses the role of randomness (God, Cupid, karma, serendipity, or whatever it is called) in the beginning and development of romantic relationship. Unintended circumstances often bind one and his and her

⁵³This finding is based on a 1992 nationwide survey of 3432 American men and women between the ages of 18 and 59. Some may argue that singles may control their choice sets especially in special settings such as online dating sites. For example, users of those sites may begin search by sorting potential partners based on their races. However, according to more recent analysis by Cacioppo et al. (2013) (an online survey with voluntary nature of sampling), only 15.7% of those married between 2005 and 2012 met on online dating sites. Even in this survey population, meetings at offline preselected settings (work, friend, school, family, place of worship, social gathering, grew up together, and blind date) still account for 53.7%.

spouse together.

I conduct counterfactual experiments by solving the model with hypothetical parameters (no differences in opportunities, no differences in preferences). Hypothetical parameters are set to keep the number of total marriages constant.

Table 1.19: Experiment: eliminating opportunity differences

	w/o opportunity differences					baseline				
	W	B	H	S	index	W	B	H	S	index
white male	0.572	0.101	0.071	0.258	0.076	0.716	0.003	0.020	0.261	0.862
white female	0.567	0.059	0.087	0.287	0.088	0.710	0.006	0.023	0.262	0.833
black male	0.373	0.100	0.091	0.436	0.054	0.037	0.553	0.021	0.390	0.896
black female	0.562	0.088	0.045	0.305	0.037	0.015	0.487	0.022	0.476	0.920
Hispanic male	0.545	0.051	0.183	0.221	0.131	0.142	0.025	0.602	0.231	0.752
Hispanic female	0.450	0.092	0.186	0.272	0.145	0.128	0.021	0.612	0.240	0.776
Total	(inter marriage=35.6%)				0.087	(inter marriage=6.1%)				0.837

Notes: Predicted type distribution in steady-state stocks. Fraction in each state: being married to column type spouses (W:white, B:black, H:Hispanic), or being single (S:single). Without opportunity differences: model outcomes with type-independent (hypothetical) opportunity parameters and estimated preference parameters. Baseline: model outcomes with estimated opportunity and preference parameters.

Table 1.19 shows that without differences in opportunities, there are substantial increases in interracial marriages. For instance, 37% of black males marry white females as opposed to 4% in the baseline model. Even under random meetings, Hispanics, who exhibit the strongest same-race preferences, selectively accept intraracial meetings, resulting in the highest homogamy index among all groups. Overall, 35.6% of marriages are across races, which is very close to the 39.0%, that is implied by random matching.⁵⁴ The homogamy index decreases to 0.087.

These outcomes from counterfactual experiment with hypothetical, type-independent meeting opportunities are compared to the findings from randomized controlled meetings (speed-dating experiments) by Fisman et al. (2008). In their experiments, subjects meet a number of potential mates for four minutes each, and have the opportunity to accept or reject each partner. In these speed dating experiments, 47% of all matches are interracial,

⁵⁴This number, the rate of intermarriages in the random matching process, varies across experiments, since the proportion of singles in each type is changed upon different counterfactual experiments.

which is comparable to the 53% that would be expected under random matching. The implied homogamy index is $(0.53 - 0.47)/0.53 = 0.113$, which is below the index in observed marriages (0.837) and is closer to my first counterfactual experiment without meeting differences (0.087). Both results show that under random meeting, there would be more intermatches. These similar findings may validate my model assumptions for identification, confirming the importance of meeting differences in mating outcomes of individuals.

Table 1.20: Experiment: eliminating preference differences

	w/o preference differences					baseline				
	W	B	H	S	index	W	B	H	S	index
white male	0.699	0.002	0.033	0.266	0.792	0.716	0.003	0.020	0.261	0.862
white female	0.693	0.012	0.029	0.266	0.762	0.710	0.006	0.023	0.262	0.833
black male	0.073	0.610	0.035	0.280	0.827	0.037	0.553	0.021	0.390	0.896
black female	0.013	0.537	0.046	0.405	0.889	0.015	0.487	0.022	0.476	0.920
Hispanic male	0.183	0.052	0.455	0.310	0.616	0.142	0.025	0.602	0.231	0.752
Hispanic female	0.206	0.036	0.463	0.295	0.612	0.128	0.021	0.612	0.240	0.776
Total	(inter marriage=9.38%)				0.755	(inter marriage=6.1%)				0.837

Notes: Predicted type distribution in steady-state stocks. Fraction in each state: being married to column type spouses (W:white, B:black, H:Hispanic), or being single (S:single). Without preference differences: model outcomes with type-independent (hypothetical) preference parameters and estimated opportunity parameters. Baseline: model outcomes with estimated preference and opportunity parameters.

With the current differences in meeting and without differences in preferences, the change in intermarriages is limited as shown in Table 1.20. Even in the absence of same-race preferences, only 9.4% of marriages are across races due to significantly frequent meeting opportunities within racial groups as shown in Table 1.17. This predicted rate of intermarriage is still far less than the case with random matching, 38.2%. The homogamy index decreases slightly to 0.755 from the baseline case of 0.837.⁵⁵

⁵⁵Learning match quality is one of the characteristics of meeting and it may take time (e.g., several dates). This may suggest that the second experiment may underestimate the role of preferences by neglecting the channel that preferences have impacts on meetings. In this case, the role of preferences compared to meetings in Table 1.21 may be considered as a lower bound. Despite this possible limitation, the results from the first experiment alone can validate that much of observed differences in marital behavior is attributed to differences in meetings. In addition, the anatomy of preferences and availability (more exogenous force in meeting) can be done only by analyzing how meetings occur and develop, which may be difficult to observe. The interplay between preferences and availability may further complicate the decomposition. I contribute the literature of the marriage market by first incorporating the exogenous and type-specific restrictions in marital opportunities, compared to previous work that assumes ‘whom to choose’ is under complete control of agents or is identically given to all individuals.

Table 1.21: Role of opportunity differences

	white	black	Hispanic	
male	0.918	0.925	0.820	
female	0.913	0.966	0.794	
total	0.916	0.946	0.807	0.901

Notes: The share in a homogamy index explained by differences in meetings as opposed to differences in preferences.

In sum, I find that the high probabilities for individuals marrying within their racial groups are primarily attributable to high incidences of intraracial meetings rather than same-race preferences. In terms of a homogamy index, overall 90.1% of racial homogamy is explained by opportunity differences.⁵⁶ Therefore, my finding concludes that marital outcomes of individuals hugely depend on whom they meet rather than whom they prefer.

Across races, the opportunity differences explain 91.8%, 92.5% and 82.0% of the homogamy behavior of white, black, and Hispanic males respectively and are 91.3%, 96.6%, and 79.4% for female counterparts. Opportunities play the greatest role in marital patterns of black females. In contrast, preferences play a greater role in Hispanic marital outcomes as opposed to those of whites and blacks.

1.5.5 Predictions upon demographic changes

The marriage market model that I develop can provide a better understanding of how marital patterns respond to demographic changes. I first consider the relationship between an unbalanced gender ratio for blacks and the high rate of being single for black females, which is potentially related to the high rate of unmarried births.⁵⁷

⁵⁶The number is calculated by $0.901 = (0.837 - 0.087) / (0.837 - 0.087 + 0.837 - 0.755)$.

⁵⁷In 2010, 40.8 % of all births were to unmarried women. These proportions were 29% for whites, 53% for Hispanics and 73% for black births. Source: National Vital Statistics Reports, 2012 (U.S. Department of Health and Human Services)

Table 1.22: Experiment: balanced gender ratio for blacks

	balanced black gender ratio					baseline				
	W	B	H	S	index	W	B	H	S	index
white male	0.715	0.003	0.020	0.263	0.868	0.716	0.003	0.020	0.261	0.862
white female	0.709	0.007	0.023	0.261	0.830	0.710	0.006	0.023	0.262	0.833
black male	0.039	0.529	0.022	0.410	0.885	0.037	0.553	0.021	0.390	0.896
black female	0.014	0.525	0.020	0.441	0.932	0.015	0.487	0.022	0.476	0.920
Hispanic male	0.143	0.023	0.601	0.234	0.753	0.142	0.025	0.602	0.231	0.752
Hispanic female	0.126	0.025	0.611	0.238	0.773	0.128	0.021	0.612	0.240	0.776
Total	(inter marriage=6.2%)				0.839	(inter marriage=6.1%)				0.837

Notes: Predicted type distribution in steady-state stocks. Fraction in each state: being married to column type spouses (W:white, B:black, H:Hispanic), or being single (S:single). Balanced black gender ratio: model outcomes under the increased measure of black males from the type distribution in 2000. Baseline: model outcomes under the type distribution of resident population in 2000.

The third experiment examines that the hypothetical population distribution with the balanced gender ratio of blacks. The gender ratio of blacks in the baseline model is 0.881. I replace this gender ratio with the white counterpart (0.992) by increasing the number of black males.⁵⁸ The results from the model with the new population distributions are documented in Table 1.22. Since the marriage market is segregated with low occurrence of interracial meetings, the change in black males has negligible effects on marital patterns of other races. What is noteworthy is the increase in the marriage rate of black females, from 52.4% to 55.9% by 6.6%. This result indicates that the high rate of black female singles is attributable to the imbalanced gender ratio for blacks. However, compared to white females with the marriage rate of (73.9%), black females still marry less (only 16.0% of white-black gap in marriage rate has diminished.)

Since the distribution I use is fractions among all residents, I cannot deal with the critical issue of high incarceration rates of black males.⁵⁹ If people in prisons and jails are excluded from the marriage market, the distribution of this non-institutional population becomes $\{W, B, H\} = \{0.750, 0.133, 0.117\}$, $\{0.737, 0.110, 0.116\}$ for males and females

⁵⁸New population distribution becomes $\{W, B, H\} = \{0.742, 0.131, 0.117\}$, $\{0.751, 0.133, 0.116\}$ for males and females.

⁵⁹What is only captured in resident populations is higher mortality rates of black males. Meanwhile, an estimated 11.8% of black males in their twenties and early thirties (20-34) were in prison or jail in 2000, compared to 1.7% and 3.4% of males for white and Hispanic counterparts. For females aged 20-34, the fraction of inmates in population is $\{0.18\%, 0.98\%, 0.28\%\}$ for whites, blacks and Hispanics. (Source: Prison and Jail Inmates at Midyear 2000, U.S. Department of Justice)

with gender ratio for each group {0.983, 0.827, 0.992}. If I conduct analyses based with the non-institutional population, the balanced gender ratio increases the marriage rate of black females from 50.7% to 55.7% by 10.0%. However, the increased marriage rate is still below the marriage rate for white females (73.4%). More balanced gender ratio among blacks decreases white-black marriage rate gap only by 22.0%.⁶⁰

A greater increase of marriage rate for blacks can happen once marital preferences and opportunities change. The previous experiment (Table 1.19) shows that if meetings become uniformly random, the marriage rate of black females raises to 69.5% (increased by 32.5%). The gap in the marriage rate between black and white females decreases by 91.7%. In addition, as shown in Table 1.20, if all couples get the same marital preferences, the marriage rate of black males becomes 72.0% (increased by 18.1%), and the gap between white and black males is eliminated by 89.7%. These exercises suggest that rare opportunities of interracial marriages and low marital utilities of black males may explain the greater portion of low marriage rates for blacks than the unbalanced gender ratio.⁶¹

Table 1.23: Experiment: population 2050

	year 2050					baseline				
	W	B	H	S	index	W	B	H	S	index
white male	0.682	0.004	0.035	0.279	0.886	0.716	0.003	0.020	0.261	0.862
white female	0.682	0.007	0.040	0.271	0.850	0.710	0.006	0.023	0.262	0.833
black male	0.029	0.573	0.029	0.369	0.896	0.037	0.553	0.021	0.390	0.896
black female	0.012	0.500	0.034	0.454	0.906	0.015	0.487	0.022	0.476	0.920
Hispanic male	0.083	0.019	0.708	0.190	0.819	0.142	0.025	0.602	0.231	0.752
Hispanic female	0.071	0.015	0.711	0.203	0.843	0.128	0.021	0.612	0.240	0.776
Total	(inter marriage=8.1%)				0.858	(inter marriage=6.1%)				0.837

Notes: Predicted type distribution in steady-state stocks. Fraction in each state: being married to column type spouses (W:white, B:black, H:Hispanic), or being single (S:single). Balanced black gender ratio: model outcomes under the type distribution expected in 2050. Baseline: model outcomes under the type distribution in 2000.

The last experiment predicts marital outcomes for a population in 2050 where there are substantial increases in the number of minorities. The population share of whites,

⁶⁰For detailed results, see Table 3.1 in Appendix 3.6.

⁶¹These outcomes are consistent with findings from previous papers that link the decline in the marriage rate of black women to a fall in the pool of ‘marriagable’ black men. Examples are Wilson (1987), Brien (1997), Sietz (2009) and Keane and Wolpin (2010).

blacks, Hispanics and others is expected to change from $\{0.718, 0.121, 0.114, 0.047\}$ in 2000 to $\{0.512, 0.136, 0.248, 0.104\}$ in 2050.⁶² Under the new distribution, there is an increase of interracial marriages, from 6.1% to 8.1%. However, the fact that a more balanced type distribution itself can promote intermarriages should be taken into account.⁶³ If I control for the effect of distribution by computing a homogamy index, I find a higher degree of segregation in year 2050 compared to year 2000. The homogamy index increases from 0.837 to 0.858. This rise is mainly due to the increase in Hispanic homogamous marriages from 60-61% to 71%. Because of greater numbers of their own type, Hispanics now more easily meet and marry spouses of the same race, leading to a higher degree of relative racial homogamy in 2050.⁶⁴

1.6 Discussion

This section compares the implications of my model with those of frictionless settings. I then discuss how my framework can be applied to other frictional frameworks.

1.6.1 Comparisons

Frictionless models of the marriage market

I have discussed how the search theoretic framework can contribute to the estimation of the equilibrium model of the marriage market. While previous studies have done estimations of the marriage market, many of them are based on frictionless settings, employing estimation methods for discrete choice models.⁶⁵ The comparison below shows that frictionless

⁶²Source: Projections of the Resident Population by Age, Sex, Race and Hispanic Origin 1999 to 2100, Population Projections Program (U.S. Census Bureau 2000). I keep the gender ratios within each race the same as 2000 levels.

⁶³If there are two groups with shares $(g, 1 - g)$, the fraction of intermarriage is $2g(1 - g)$ which is maximized when $g = 0.5$.

⁶⁴A decline in Hispanic intermarriage from the 1990s has also been reported and investigated by Lichter, Carmalt, and Qian (2011) and Qian and Lichter (2011).

⁶⁵The baseline frictionless model introduced in this section is from Choo and Siow (2006b). They first applied the random utility model by McFadden (1973) to the marriage market context. See the survey by

settings and frictional settings may differently interpret the same observed marital patterns, requiring a careful choice of information structure when analyzing two-sided markets.

Let us first summarize the model without search frictions. The model is derived in a random utility model framework. A decision maker, a single male m (female f) of type i (j) faces a choice among J (I) potential spouse types. The utility that male m of type i (female f of type j) obtains from marrying female type j (male type i) is U_{ijm} (V_{ijf}). One of the possible options is staying single, which gives agents utilities of U_{i0m} for males and V_{0jf} for females.

$$U_{ijm} = u_{ij} - t_{ij} + \varepsilon_{ijm} \quad (1.36)$$

$$V_{ijf} = v_{ij} + t_{ij} + \varepsilon_{ijf} \quad (1.37)$$

$$U_{i0m} = u_{i0} + \varepsilon_{i0m} \quad (1.38)$$

$$V_{0jf} = v_{0j} + \varepsilon_{0jf} \quad (1.39)$$

The utility specification includes the systematic component of the base utility u_{ij}, v_{ij} , the transfer made from the husband to his wife t_{ij} , and the stochastic component denoted as ε_{ijm} and ε_{ijf} . Each ε is distributed i.i.d. extreme value.

Male m of i will choose the female type according to

$$U_{im} = \max\{U_{i0m}, \dots, U_{ijm}, \dots, U_{iJm}\}. \quad (1.40)$$

The number of i, j marriages demanded by male type i (supplied by female type j), denoted by $\eta_{ij}^d(\eta_{ij}^s)$ is given as follows in this Logit model:

$$\ln \eta_{ij}^d = \ln \eta_{i0}^d + u_{ij} - u_{i0} - t_{ij} \quad (1.41)$$

$$\ln \eta_{ij}^s = \ln \eta_{0j}^s + v_{ij} - v_{0j} + t_{ij} \quad (1.42)$$

Siow (2008) about the applications and extensions of Choo and Siow's framework.

The market clearing condition $\eta_{ij}^d = \eta_{ij}^s$ adjusts the values of transfers and yields the following mapping, allowing the identification of preference parameters from observed data. This equation (1.43) states that the ratio of the number of i, j marriages to the geometric average of those types who are unmarried is directly correlated with total marriage utility.

$$\ln\left(\frac{\eta_{ij}}{\sqrt{\eta_{i0}\eta_{0j}}}\right) = \frac{u_{ij} + v_{ij} - u_{i0} - v_{0j}}{2} \quad (1.43)$$

The two settings with and without search frictions may result in different interpretations on preferences and the marriage market environment.

Comparison 1. Perfect information on the location of types

This frictionless model depends on the assumption that any individual can demand any type of the opposite gender because they know where each type is located. This is the major difference from the frictional models in which meetings arrive exogenously to agents who know the distribution of types but not the location. These differences become clear when two equations (1.43) and (1.44) are compared. The counterpart of equation (1.43) can be computed based on equation (1.16) from the frictional model as follows.

$$\ln\left(\frac{\eta_{ij}}{\sqrt{s_i^m s_j^f}}\right) = \ln\left(\frac{1 - F(\varepsilon_{ij}^*)}{\delta + \lambda F(\varepsilon_{ij}^*)}\right) + \ln(\mu_{ij}) + \frac{1}{2} \ln\left(\frac{s_i^m s_j^f}{S^m S^f}\right) \quad (1.44)$$

As shown in equation (1.44), the ratio in equation (1.43) does contain information on the willingness to accept ij matches through the term, $\ln\left(\frac{1 - F(\varepsilon_{ij}^*)}{\delta + \lambda F(\varepsilon_{ij}^*)}\right)$. However, it may also be positively correlated with the fraction of these types among singles, $\frac{s_i^m}{S^m}, \frac{s_j^f}{S^f}$, and bias in opportunity, μ_{ij} . As a result, if equation (1.43) is used for recovering preference parameters of frictional settings, it tends to overestimate the marriage payoff to the two types whose shares are large in the marriage market or two types who meet each other more frequently. This bias may stem from the specification of information and the matching process in the

frictionless model.

Comparison 2. Singlehood as a choice option

The frictionless model treats staying single as one of the choice alternatives, together with marrying all possible types. In contrast, in the frictional framework, agents choose between acceptance and rejection upon a meeting that is randomly arrived at with a certain partner type and a specific value of match quality. Thus, staying in a singlehood state is related to some involuntary features such as an unlucky history of bad arrivals and draws. This issue has further implications in the estimated results. The payoffs attached to singlehood in the frictionless model u_{i0} and v_{0j} in equations (1.38) (1.39) are comparable to equilibrium objects U_i^m and U_j^f in flow term,

$$(r + \delta)U_i^m = \frac{\sigma(1 - \beta) \sum_l \alpha_{il}^m \int_{\varepsilon_{il}^*}^{\infty} [1 - F(x)] dx}{r + \delta + \lambda}. \quad (1.45)$$

According to this equation, u_{i0} , v_{0j} in the frictionless models may depend on all structural parameters in a highly nonlinear fashion. Treating those values as primitive may, consequently, cause problems in counterfactual analysis and policy evaluations. This issue is more crucial in the model in which payoffs to singlehood are the baseline utility to all other payoffs.⁶⁶

Comparison 3. idiosyncratic taste shocks to each type

The two models differ in their specification of idiosyncratic taste shocks. In the frictionless model, each shock is attached to each option, that is, each type of potential spouses. Different individuals can be viewed as identical if they are classified in the same group. In contrast, in the frictional framework, if a meeting occurs, the person one meets is unique in terms of availability and pair-specific match quality (ε), even though all persons in the

⁶⁶This difference in the interpretation of singlehood partially depends on the static nature of Choo and Siow's baseline model. This issue will be addressed by Choo and Siow (2007) in the dynamic extension of Choo and Siow (2006b)'s baseline framework.

same type are ex-ante treated as the same. Different theoretical properties can be derived from two distinct specifications.⁶⁷

Stable matching

Starting with the seminal work of Gale and Shapley (1962), stable matching has been a key concept in matching markets including marriage markets. In a stable matching, no matched pair prefers to be single or no unmatched pair can later find out that they can both do better by abandoning their current partners and matching each other.⁶⁸

In their studies of mating preferences, Hitsch, Hortaçsu, and Ariely (2010a) predict matching outcomes based on this Gale-Shapley algorithm and the estimated mate preferences. Preferences are estimated based on the revealed preference argument using the data from online dating sites where users' browsing behaviors (choice-sets) and contacts (one-sided acceptances) are directly observed. These researchers then use the Gale-Shapley deferred acceptance algorithm to simulate match outcomes for a general population based on recovered preference parameters.⁶⁹ They argue that since the predicted mating patterns resemble observed matching patterns, preferences, rather than search frictions or opportunities, are the main cause of sorting in marriages.

⁶⁷Chapter 2 of this dissertation shows that this option-specific shock specification of frictionless models leads to negative group size effects. As the share of one type increases, the marital prospects to individuals of this type decrease. The reason is the following. Some people, depending on their shock realizations, demand intermarriages and the size of this demand is larger when coming from majorities rather than from minorities. Because market clearing prices become favorable for minorities, minorities enjoy better matching prospects than majorities in this frictionless setting. Meanwhile, under the particular assumption of matching preferences that people prefer the same-type, the frictional model with uniformly random meetings may provide better matching outcomes to majorities who have more opportunities for intrameetings compared to minorities.

⁶⁸See Abdulkadiroglu and Sonmez (2011) and Roth (2008) for an overview of stable matching and deferred acceptance algorithms, and Browning, Chiappori, and Weiss (2011) for applications of stable matching in the context of the marriage market. See also Fox (2009) for a survey on structural empirical work using two-sided matching games in various applications.

⁶⁹The deferred acceptance algorithm works in the following manner: the one side of the market makes proposals to agents on the opposite side based on the order reflecting their preferences. Those who receive more proposals than they accept hold their best preferred proposal and reject the others. In the next round, agents who have been rejected make new proposals. The process continues until there are no rejected agents who wish to make further proposals. At this point, all proposals held are finally accepted to constitute a matching.

However, in the absence of a centralized marriage market, there is no explicit mapping between this deferred acceptance algorithm and the process of forming marriages. Furthermore, search frictions allow the conditions of stable matching to be violated. In other words, it is possible that the marriage market equilibrium has two pairs (for example, $AB + CD$) whose members of those pairs find switching their spouses would increase their utilities ($AD + CB \succ AB + CD$). The alternative and possibly profitable matches ($AD + CB$) would not occur if there were no such meetings.

Accordingly, the coincidence of stable matching outcomes mimicking the observed outcomes may not guarantee the dominant role of preferences. According to my preferences estimates in Table 1.10, the unique stable matching is perfect homogamy.⁷⁰ This result is in sharp contrast with my counterfactual experiment without preference differences in Table 1.20 that shows limited effects of preferences. Since agents differ in meeting opportunities with different potential spouses, type-independent preferences alone cannot generate fewer homogamous marriages. Observed homogamy patterns can be obtained once heterogeneity in meeting opportunities are considered.

My results, on the contrary, suggest that search frictions (limited meeting opportunities) play a dominant role in observed marriage patterns. To quantify the direction and the strength of search frictions is the principal contribution of this paper. The results show that search frictions do not necessary scatter sorting patterns or preclude optimal assignments as in standard matching models. The existence of search frictions or the process of meetings may enhance efficient sorting patterns. By explicitly incorporating and estimating how search frictions play a role in the process of match formation, my model can contribute to a better understanding of preferences and opportunities in the marriage market.

⁷⁰The standard Gale-Shapley mechanism is based on nontransferable utility. In transferable utility models, the marriage market equilibrium is obtained where the sum of marital output in society is maximized (Shapley and Shubik 1972). I ignore idiosyncratic shocks for this prediction. All white males (of measure 0.743) marry their female counterparts, and all Hispanic females (0.117) marry Hispanic males. There are *HW* intermarriages (0.002). Some white females (0.004) and all black males (0.118) and females (0.134) remain single.

1.6.2 Applications

Frictional models of the labor market

My model of the marriage market can be compared to the labor market with firms and workers. However, there are several obstacles for this model being applied to the labor market settings. The main difference between labor models and marriage models are the symmetric features of the marriage market. Agents in the marriage market, males and females, face an identical environment with respect to their strategies and information whereas firms and workers in the labor market solve different optimization problems (e.g., firms may make take-it-or-leave-it offers to workers, or inflows of firms may be determined by the zero profit condition.) These asymmetric features become crucial also in terms of data availability. In the marriage market, major traits of both married or single males and females are observed by econometricians, allowing them to identify relevant parameters. In contrast, the labor search estimation suffers from the problems involving unobserved firm characteristics even though researchers have better data on transfers (wage). As a result, to the best of my knowledge, there is no paper that estimates type-independent meetings (wage offer arrivals) in the labor market.⁷¹

However, there are still questions to be investigated using my frameworks in the context of the labor market. For example, regarding the patterns that women tend to work for lower paying jobs, two potential explanations can be compared: 1) women get non-pecuniary benefits from those jobs (preferences), and 2) women receive limited opportunities. This question can be answered even based on one-sided (worker-sided) data, where firm types and durations involving match formation and dissolution are recorded.⁷²

⁷¹See Eeckhout and Kircher (2011) and Hagedorn, Law, and Manovskii (2012) for further discussion on recovering preferences (production functions) in the labor market settings.

⁷²See Appendix 3.5 for how one-sided model can be constructed and identified as an application of my framework. The questions on gender/race discrimination have been analyzed by many researchers including Bowlus and Eckstein (2002), Flabbi (2010), and Flabbi and Moro (2012). However, different preferences and opportunities regarding observed characteristics of jobs have not been yet simultaneously addressed.

Frictional discrete choice models of consumers

My framework can be applied to the analysis of discrete choice models where one side of the model is sellers and the other side buyers, or one-sided models of buyers. Researchers can separately identify preferences (consumer i 's utility from consuming good j) and opportunities (the arrival rate of good j to consumer i) based on duration and transition data.⁷³ Understanding these differential effects of opportunity and preference may be crucial in the analysis of frictional markets where all options are not always available. Possible applications include preference and opportunity effects of regulations and subsidies, the introduction of new products, and marketing strategies.

My paper can be related, for example, to Akerberg (2001, 2003) on the effects of advertisements. He compares differential effects of advertisements on experienced and inexperienced users to distinguish between informative effects and prestige effects of advertising in the yogurt market. Informative effects lead consumers to be informed of the existence and characteristics of advertised goods, and Akerberg assumes that only inexperienced consumers are affected by this effect. In contrast, prestige effects increase a consumer's utility of consuming advertised goods, affecting both experienced and inexperienced consumers.⁷⁴

Unlike Akerberg's focus on non-durable goods, my framework is suitable for durable goods. Using information about which good consumers choose, how long it takes to make this choice, and how long they use this durable good, researchers may be able to disentangle preference effects (prestige effects) from opportunity effects (stimulative effects) of advertising. The opportunity effects take place by reminding consumers of the advertised good (arrivals of this good to consumers as a choice option), and potentially differ from the informative effects. I state that search frictions may exist even in consumer choice settings that appear frictionless. Even though options are always available, consumers do not

⁷³The one-sided model introduced in Appendix 3.5 is again applicable in this setting.

⁷⁴By observing that advertisements primarily affected inexperienced users, he concludes the primary effect was that of informing consumers.

mentally process all options all the time, leaving room for arrivals and stimulation.

Network economics

Lastly, my model is related to the work of Currarini, Jackson, and Pin (2009, 2010) on preferences vs. opportunities in racial homophily (high frequencies of same-race friendship) in the many-sided markets (network formation). The model proposed by these researchers do not allow rejection of meetings, recovering opportunity differences directly from observed frequencies of friendships. They then use the variation in the number of friendships to infer preference differences. For example, if individuals have strong same-race preferences and meet the same-race persons often, they will have a large number of friends.

Instead of using variations in the number of friends as they do, one can use the data on how long friendships last to disentangle preferences from opportunities. Using dynamics of match formation is more appropriate for the marriage market where all unions are monogamous. I can further distinguish white' taste for blacks verses white' taste for Hispanics whereas the approach by Curraini et al. can only estimate taste for own type compared to other types combined. In addition, the approach I propose not only can recover the differences in preferences and opportunities for all possible pairings, but also can quantify the relative strength of each channel. Therefore, my identification method can shed light on match/relationship formation in various settings.

1.7 Conclusion

The equilibrium marriage market model developed in this paper analyzes the relative importance of preferences and opportunities as potential sources of racial homogamy. In the model, individuals are heterogeneous in their marital opportunities (meetings) and preferences. Married couples differ in match qualities, which are updated at a Poisson rate that is common across marriage types. Under this assumption, variations in the duration of

marriages can be used to identify the willingness to accept certain types of marriage separately from arrival rates, even in the absence of data on choice sets. I estimate the structural parameters via a three-step estimation procedure. The estimated model closely replicates the patterns of marital formation and dissolution in the data. My results show that even though same-race preferences exist, the racial homogamy observed is primarily caused by the rarity of interracial meetings in the United States.

In addition, the model is also used to simulate the effect of an increased share of black males in the population relative to their current share. Empirical results indicate that the high rate of singlehood for black women is partly attributable to a shortage of black males. Lastly, I use the model to predict the effects of an increasing Hispanic population, showing that the increase will lead to more Hispanic intramarriages and a higher degree of homogamy.

The significant role of meetings in matching patterns that this research finds may further motivate studies on social interactions and networks in broader settings.⁷⁵ In these studies, the idea that choice opportunities are sometimes granted to individuals who are restricted by their environment should be carefully considered. Search frictional frameworks proposed in this paper may be helpful for studies of social interactions and network formation.

There are several ways to extend my model. Household bargaining information can be incorporated to identify husbands' and wives' individual gains from different marriages. Building on Wong (2003b), Chiappori, Oreffice, and Quintana-Domeque (2012), and Dupuy and Galichon (2012), one can study marriage markets with multiple attributes where people are differentiated by their race and human capital.

To address recent changes in interracial marriage patterns and attitudes and in search technologies (e.g., online dating sites), the model can be extended to have time-varying preferences and opportunities. Building on Bisin, Topa, and Verdier (2004), determinants

⁷⁵See McPherson, Smith-Lovin, and Cook (2001) and Jackson (2011) for an overview.

and generational evolution of types, preferences, and opportunities can be considered. Estimation of such models would require enough samples of interracial marriages and additional information on how meetings take place.

Lastly, the methods developed in this paper are applicable to other frictional search settings, such as two-sided markets with firms and workers, or buyers and sellers, and one-sided or many-to-many markets where researchers can observe the dynamics of matches.

Chapter 2

Matching along Horizontal

Differentiations

This chapter analyzes the effects of horizontal differentiations and search frictions in two-sided markets with transferable utility. There are two types of individuals (a majority type and a minority type) and agents prefer same-type partners. In a model with search frictions where meetings are random and transfers are determined via Nash bargaining, minorities seldom meet same-type individuals and suffer from poor outside options. In contrast, I show that minorities have better matching prospects than majority members in frictionless frameworks where locations are known and transfers are collectively determined by market clearing conditions.

2.1 Introduction

People often prefer partners of the same type. In the marriage market, for example, singles may wish to marry a spouse of the same ethnicity or religion to share their cultural heritage with family members. In the labor market and academia, workers and scholars may want to collaborate with partners in the same field. In these cases, agents are horizontally differentiated with respects to their qualitative traits. The criterion behind evaluating horizontal traits is ‘the more similar, the better.’

These horizontal differentiations can be compared to vertical differentiations. With respect to some quantitative traits, such as income and physical attractiveness, the evaluation criterion is ‘the more, the better.’ According to this standard, people are vertically ranked. Whereas matching along vertical dimensions has been extensively investigated in the literature in economics, little is known about matching along horizontal differentiations.¹ Previous studies which do not address horizontal differentiations often assume that the overall market is decomposed into various sub-markets for each horizontal type. However, as matches across types are empirically reported, there is a need for careful analysis of the interactions among horizontally differentiated agents.

This paper studies the implications of horizontal differentiations in two-sided matching models with and without search frictions. In both settings, there are two types of individuals (a majority type and a minority one). Agents prefer partners of the same type and matching gains are transferable. Frictional and frictionless settings demonstrate contrasting matching outcomes for two groups. In frictional settings, majorities have better matching prospects than minorities. Inversely, matching outcomes are better for minorities com-

¹Studies on this assortative matching are motivated by Becker (1973). Search and matching along vertical dimensions are investigated by Burdett and Coles (1997) in non-transferable utility frameworks, and by Shimer and Smith (2000) in transferable utility frameworks. For more surveys of search and matching models in the context of the labor market and the marriage market, see Rogerson, Shimer, and Wright (2005) and Burdett and Coles (1999).

pared to majorities in frictional settings. This analysis, therefore, sheds light not only on aspects of identities and diversity, but also on different matching and price-determination mechanisms of frictional and frictionless frameworks.

I first develop a two-sided random search model which builds on Pissarides (2000) and Shimer and Smith (2000). Singles wait for meeting opportunities that occur randomly depending on the type distribution of the opposite side of the market. Upon meeting, a male and a female agree to form a match with a positive surplus. This matching surplus is then divided via Nash Bargaining. In this setting, minorities have a poor outside option (or threat point) since they have less frequent meetings with members of their own group. This will reduce the transfer to minorities from majorities in their intermarriages and lower matching prospects of minorities.

The second setting adopts the frictionless framework of Choo and Siow (2006b). Individuals can demand any type of potential spouse based on their match preferences and idiosyncratic taste shocks attached to each type of the opposite side. Match transfers are determined by market clearing conditions. Depending on shock realizations, some individuals prefer inter-type matches. The number of majority type individuals preferring an intermarriage will be greater than the number of minorities that prefer marrying across types, which generates a higher relative demand and eventually higher prices (transfers) for minorities. For this reason, minorities enjoy better matching outcomes than majorities in a frictionless setting where their locations are known and prices are collectively determined to clear the market. Consequently, the group size effects have different signs in two settings. Depending on model assumptions on information and price-determination, being unique may or may not be appreciated.

I further generalize the model, analyzing two different criteria of evaluating potential partners; horizontal differentiations and vertical differentiations. Allowing the interactions of two differentiations, the model obtains implications on matching assortativeness in horizontal and vertical dimensions. Depending on the degree of modularity, high types may or

may not be engaged more in inter-horizontal marriages than low types.

While my research is related to a few papers that study preferences for the similar, it still differs from those studies. Bisin and Verdier (2000) investigate the implications of these preferences on cultural transmission and evolution of types, whereas my work focuses more on individual matching outcomes. Alpern and Reyniers (1999) in biology, and Gautier and Teulings (2012) in labor economics analyzes models with preferences for partners who are close to oneself. However, due to their assumption that types are continuous and uniformly distributed, these papers are unable to study the group size effects, the analysis of which constitutes the main contribution of my paper.

The rest of the paper proceeds as follows. A frictional model and its properties are discussed in Section 2.2. Section 2.3 explores a frictionless framework which results in the opposite implications from those of the frictional model. Extensions and conclusions are presented in Section 2.4 and Section 2.5.

2.2 The Model with Search Frictions

There is a unit measure of infinitely-lived males and females who are interested in forming bilateral relationship. Males and females can represent each side of any two-sided markets with frictions where agents look for long-term partners, such as markets with firms and workers or buyers and sellers. There are two types of individuals, A and B . Let g denote the share of A types and $1 - g$ denote the share of B types in each side.

Time is continuous and individual discount the future at the rate of r . Suppose also that there is no divorce, thus marriage is an absorbing stage. Marriage ij denotes a match between a husband of type i and a wife of type j . For marital preferences, assume $Q > 0$ and $\tau > 0$. Let us assume that the discounted lifetime value of marriages between same types (AA, BB) is $2Q$, while the discounted lifetime value of intermarriage (AB, BA) is $2(Q - \tau)$. Since it is assumed that $Q > 0$, individuals prefer the state of same-type marriage

compared to the state of singlehood where flow payoff from being single is normalized to 0. τ measures a penalty for intermarriage. Assuming $\tau > 0$ induces individuals to prefer to marry within their own group.

Individuals meet the potential spouse of the opposite side with a Poisson rate of α . Meeting is random, thus conditional on meeting, the probability that the potential spouse of a meeting is type A (type B) is given by g ($1 - g$). Once two singles meet, they decide to marry or not. When couples decide to marry, they determine how to divide the matching surplus, the gap between value of being married and being single. I assume the symmetric Nash Bargaining where two parties divide the surplus equally. The outside option in this bargaining procedure is the value of staying single as each type.

I further adopt the cloning assumption. If two singles are matched and exit the marriage market, two new singles with the identical types, who could be understood as their offspring, enter and replace their predecessors. Therefore, the distribution of type among male and female singles ($g, 1 - g$) and the measure of singles 1 stay constant.

For simplicity, I focus only on symmetric pure strategy equilibria. Mixed strategy equilibria or the case husbands and wives strategies regarding the same marriage differ are not considered. Under this setting, males and females both agree to form a match with a positive match surplus. Let β_{ij} be an indicator of whether a match forms: $\beta_{ij} = 1$ if i and j form a match upon meeting and then exit the marriage market; and $\beta_{ij} = 0$ if i and j do not form a match upon meeting and return to the marriage market as singles.

Let V_{ij} denote the discounted lifetime value of type i being married to spouse j , which is given by the following.

$$2V_{AA} = 2V_{BB} = 2Q \tag{2.1}$$

$$(V_{AB} + V_{BA}) = 2(Q - \tau) \tag{2.2}$$

V_i denotes the discounted lifetime value of type i , which will be derived by the fol-

lowing two equations.

$$rV_A = \alpha g \beta_{AA}(V_{AA} - V_A) + \alpha(1 - g)\beta_{AB}(V_{AB} - V_A) \quad (2.3)$$

$$rV_B = \alpha g \beta_{BA}(V_{BA} - V_B) + \alpha(1 - g)\beta_{BB}(V_{BB} - V_B). \quad (2.4)$$

Lemma 1. Symmetric Nash Bargaining Solution. Assuming symmetric Nash Bargaining implies the following:

1. $\beta_{AA} = 1$ if and only if $Q \geq V_A$, $V_{AA} = Q$.
2. $\beta_{BB} = 1$ if and only if $Q \geq V_B$, $V_{BB} = Q$.
3. $\beta_{AB} = \beta_{BA} = 1$ if and only if $2(Q - \tau) \geq V_A + V_B$. They divide surpluses such that $V_{AB} = V_A + \frac{2(Q - \tau) - V_A - V_B}{2}$ and $V_{BA} = V_B + \frac{2(Q - \tau) - V_A - V_B}{2}$.

Lemma 1 implies that meetings lead to marriages when surpluses are positive. The same type husbands and wives get the same discounted utility value in their homogamous marriages. In intermarriages, marital utilities of each depend on the outside option, being a single of each type.

Under the Nash Bargaining solution, equations (2.3) and (2.4) become

$$rV_A = \alpha g \beta_{AA}(Q - V_A) + \alpha(1 - g)\beta_{AB} \frac{2(Q - \tau) - V_A - V_B}{2} \quad (2.5)$$

$$rV_B = \alpha g \beta_{BA} \frac{2(Q - \tau) - V_A - V_B}{2} + \alpha(1 - g)\beta_{BB}(Q - V_B). \quad (2.6)$$

The following proposition demonstrates different types of equilibria depending on the parameter values. Several crucial parameters are τ , the penalty for intermarriage, and $x \equiv \frac{r}{\alpha}$, which captures the degree of search frictions where high x indicates severe search frictions.

Proposition 1. Two types of pure strategy equilibria. *Under the Nash Bargaining solution, there are two types of pure strategy equilibria. If the penalty for intermarriage is*

high, $\tau \geq \tau^* \equiv \frac{x(1+2x)Q}{2(x+g)(x+(1-g))}$, only homogamous marriages (AA, BB) are formed in the equilibrium (the homogamous equilibrium). If $\tau < \tau^*$, all matches (AA, BB, AB, BA) are formed in the equilibrium (the all-matches equilibrium).

Proof. See Appendix 4.1.1. □

It is noteworthy that in both equilibria, people always accept homogamous matches. This is in contrast with the matching outcomes with vertical differentiations with high (H) and low (L) types analyzed in Burdett and Coles (1999). They assume that H types contribute more to matches than L types. Match utilities are higher in HH marriages and lower in LL marriages than in HL and LH marriages. Under some parameters, high types receive enough marital gains from their marriages with low types and reject the other high types (no HH marriages formed). This happens when high types are minorities, and their HL matches generate utilities that are sufficiently higher than those from LL matches and close to utilities from HH matches. However, this result (no AA or BB marriages) cannot be obtained with horizontal differentiations, where the group size has important implications for matching prospects. For type i to have enough intermarriage opportunities, the share of type i needs to decrease. This decrease in type i 's share, with a corresponding increase in increase in the other type's share, will lead to a higher threat point of the opposite type, which prevents type i from collecting enough matching surplus from intermarriage. Accordingly, by only accepting intermarriages and receiving higher transfers from the other party, singles cannot gain enough matching value that exceeds the surplus from the most desirable marriages.

Examining the equilibrium threshold of the intermarriage penalty, $\tau^* \equiv \frac{x(1+2x)Q}{2(x+g)(x+(1-g))}$, leads to the following claims.

Claim 1. *As frictions become severe, singles are more likely to accept intermarriages.*

Note that $\frac{\partial \tau^*}{\partial x} = \frac{Q(g(1-g)(1+4x)+x^2)}{2(g-g^2+x+x^2)^2} \geq 0$. As $x = \frac{r}{\alpha}$ increases because waiting becomes more painful (people become impatient, $r \uparrow$, or meetings arrives less frequently, $\alpha \downarrow$), then singles

become more willing to accept intermarriages. If frictions vanish, $x = 0$ (no time discount, $r = 0$, or instant arrivals, $\alpha \rightarrow \infty$), then $\tau^* = 0$, meaning that people always reject intermarriages. This outcome at the limit ($x = 0$) is consistent with the results expected by the standard matching model where stability conditions govern equilibrium outcomes. It is also noteworthy that intermarriages violate the stability condition. If there are JK, KJ matches, agents will find that switching partners to have JJ, KK matches is a Pareto-improvement.

Claim 2. *The more balanced the distribution of types, the less chance of forming intermarriages.*

Note that $\frac{\partial \tau^*}{\partial g} = \frac{-(1-2g)Q(1+2x)x}{2(g-g^2+x+x^2)^2} < 0$ if $g < \frac{1}{2}$, and ≥ 0 if $g \geq \frac{1}{2}$. When g approaches 0 or 1, the all-matches equilibrium is obtained in a larger set of parameter values. When the distribution of types becomes lopsided, minorities who face lower marriage prospects (lower value of being singles) make the condition $2(Q - \tau) \geq V_A + V_B$ less restrictive. In contrast, when g becomes closer to $\frac{1}{2}$ (when the shares of two groups become even), staying single to wait for the ideal type becomes more affordable, thereby making both sides more selective in evaluating intermarriages.

Note also that two claims are about the possibility that the economy is at *the all-matches equilibrium* as opposed to *the homogamous equilibrium*. The claim is not about the measure of intermarriages. Once the economy is at *the all-matches equilibrium* in all ranges of g , which happens when $\tau < \frac{2Qx}{1+2x}$, the fraction of intermarriages among all marriages is determined as $2g(1 - g)$, which is maximized when the distribution is balanced, $g = \frac{1}{2}$. When singles accept all marriages (including intermarriages), fraction g of all types marry A type spouses, and fraction $1 - g$ of all types marry B type spouses.²

Figure 2.1 demonstrates the partition of equilibria over (g, τ) space where other parameters are set to $Q = 1$, $\alpha = 0.25$, and $r = 0.04$. Line $\tau^* \equiv \frac{x(1+2x)Q}{2(x+g)(x+(1-g))} = \frac{0.1056}{0.1856+g-g^2}$

²In other word, in *the all-matches equilibrium*, people accept intermarriage as likely as homogamous marriage. This result changes when the assumption on deterministic match utilities is relaxed. By introducing match quality shocks as in Chapter 1 of this dissertation, the outcomes where people accept homogamous marriages more easily than intermarriage can be demonstrated.

divides the parameter set regions into areas involving two different equilibria: *the homogamous equilibrium* with only AA, BB and *the all-matches equilibrium* with all AA, BB, AB, BA . Claim 1 is related to potential upward shifts of τ^* upon higher values of x , which will lead to wider regions for *all-matches equilibrium*. Claim 2 is reflected in the U shape of τ^* which obtains its minimum at $g = \frac{1}{2}$, where the parameter region for *the all-matches equilibrium* is minimized.

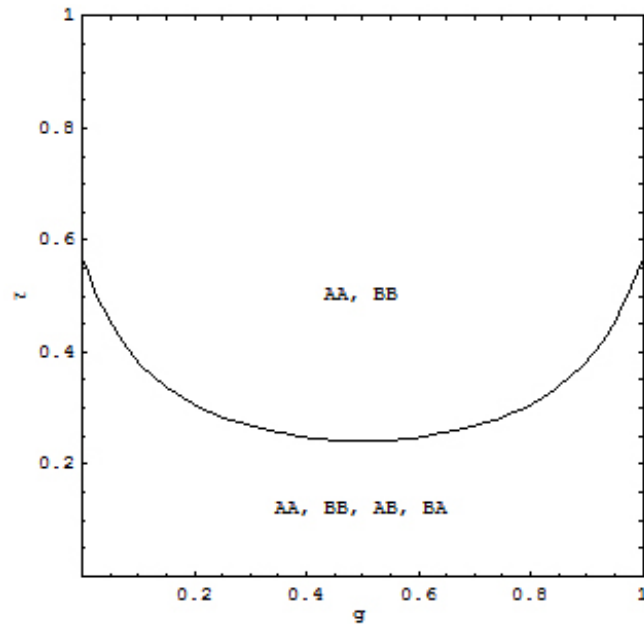


Figure 2.1: Equilibrium set in (g, τ) space

This model also provides several implications regarding the group size effects on matching outcomes.

Proposition 2. Positive group size effects. *Under same-type preferences ($\tau > 0$), the following holds.*

- i In all equilibria, V_i is strictly increasing in the fraction of singles with trait i .*
- ii In all-matches equilibrium, transfer to i in his or her intermarriage is strictly increasing in the fraction of i .*

Proof. Under the homogamous equilibrium, the value of being single as type i and its first

derivative with respect of the fraction of type i , g_i are given by

$$V_i^{homo} = \frac{g_i Q}{g_i + x} \quad (2.7)$$

$$\frac{\partial V_i^{homo}}{\partial g_i} = \frac{Qx}{(Q+x)^2} > 0. \quad (2.8)$$

The outcomes under *the all-matches equilibrium* are given by the following:

$$V_i^{all} = \frac{Q(1+2x) - 2(1-g_i)(1-g_i+x)\tau}{1+3x+2x^2} \quad (2.9)$$

$$\frac{\partial V_i^{all}}{\partial g_i} = \frac{2(2(1-g_i+x)\tau)}{1+3x+2x^2} > 0. \quad (2.10)$$

The transfer that type i gets from his or her spouse j in their intermarriage, $t_{j \rightarrow i}$, is given by the following and is strictly increasing in g_i .

$$t_{j \rightarrow i} = \frac{1}{2}(V_i - V_j) = \frac{-(1-2g_i)\tau}{1+2x} \quad (2.11)$$

$$\frac{\partial t_{j \rightarrow i}}{\partial g_i} = \frac{2\tau}{1+2x} > 0. \quad (2.12)$$

□

Although horizontally differentiated agents find homogamous marriages the most desirable, they are nevertheless subject to search frictions. Under random arrivals, the opportunities of homogamous marriages occur at a lower rate to minorities than majorities, which worsens the matching prospects of minorities. Therefore, when a minority and a majority type form an intermarriage, more transfers go to the majority member due to his or her higher threat point in bargaining. This positive sign of group size effects contrasts with the outcome from frictionless frameworks introduced in the next section.

2.3 The Model without Search Frictions

Major differences between frictional and frictionless frameworks are their assumptions about information and price-determination. Whereas the frictional framework in Section 2.2 assumes that forming a match with type j is available only upon a meeting with type j that occurs in uncertain intervals, frictionless frameworks assume that people can demand any type if they want to marry that type. While the model in Section 2.2 supposes that transfers are determined individually via Nash bargaining, transfers in the frictionless frameworks are determined collectively based on market clearing conditions. However, as I discuss earlier in Claim 1, the standard frictionless model cannot generate intermarriages that violate the stability conditions. I therefore need to introduce some types of disturbances to allow intermarriages in the economy. The framework introduced by Choo and Siow (2006b) provides a suitable model to explore the marriage market where intermarriages are present. In this Section, two models (frictional and frictionless) are compared with respect to their opposite conclusions about the group size effects on matching outcomes under horizontal differentiations.

This frictionless model is based on a random utility model framework. A decision maker, a single male m (female f) of type $i(j)$ faces a choice among $J(I)$ potential spouse types. The utility that male i (female j) obtains from marrying $j(i)$ is $U_{ijm}(V_{ijf})$. One of the options to be chosen is staying single which gives male i and female j utilities of U_{i0m} and V_{0jf} .

$$U_{ijm} = u_{ij} - t_{ij} + \varepsilon_{ijm} \quad (2.13)$$

$$V_{ijf} = v_{ij} + t_{ij} + \varepsilon_{ijf} \quad (2.14)$$

$$U_{i0m} = u_{i0} + \varepsilon_{i0m} \quad (2.15)$$

$$V_{0jf} = v_{0j} + \varepsilon_{0jf} \quad (2.16)$$

This utility specification has the systematic component of the base utility u_{ij}, v_{ij} and trans-

fer made from the husband to his wife t_{ij} , and the stochastic component denoted as ε_{ijm} and ε_{ijf} . Each ε is an independently and identically distributed random variable from a type I extreme-value distribution. These shocks are specific to each available type in the opposite side of the market.

Male m of i will choose the female type according to

$$U_{im} = \max\{U_{i0m}, \dots, U_{ijm}, \dots, U_{iJm}\}. \quad (2.17)$$

The number of i, j marriages demanded by male type i (supplied by female type j), denoted by $\mu_{ij}^d(\mu_{ij}^s)$ is given as follows in this Logit model.

$$\ln \mu_{ij}^d = \ln \mu_{i0}^d + u_{ij} - u_{i0} - t_{ij} \quad (2.18)$$

$$\ln \mu_{ij}^s = \ln \mu_{0j}^s + v_{ij} - v_{0j} + t_{ij} \quad (2.19)$$

The market clearing condition $\mu_{ij}^d = \mu_{ij}^s$ adjusts the values of transfers and yields the following mapping, allowing us to link underlying preference parameters to matching outcomes. The following equation (2.20) suggests that the ratio of the number of i, j marriages to the geometric average of those types who are unmarried is directly correlated with total marriage utility.

$$\ln \left(\frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} \right) = \frac{u_{ij} + v_{ij} - u_{i0} - v_{0j}}{2} \quad (2.20)$$

Let us now consider a simple gender-symmetric case where there are two types $i \in A, B$ of males and females. When agents marry a spouse of the same race, type i gets $u_{ii} = v_{ii}, \forall i$. When they marry a spouse of the other type (AB or BA marriages), type A males and females get $u_{AB} = v_{BA}$ and B types get $u_{BA} = v_{AB}$. g_i is the fraction of i type singles in males and females, and g denotes g_A . Utilities to the option of being single are normalized to zero, $u_{i0} = v_{0i} = 0 \forall i$. For this simple case, closed-form solutions of

equilibrium outcomes can be obtained.

Proposition 3. Existence and uniqueness of the frictionless market equilibrium. *Under symmetric frictionless settings with two types, there exists a unique marriage market equilibrium.*

Proof. See Appendix 4.1.2 for the complete characterization of the equilibrium. \square

Proposition 4. Negative group size effects. *Regardless of utility specifications, the group size effect is always negative. In other words, as the fraction of i increases, the marital prospects of type i decrease.*

Proof. Marital prospects of type A are represented by $\mathbb{E}U_A$, the utility of being a type A before seeing the realization of shocks in equation (2.21). $\mathbb{E}U_A$ decreases in the fraction of type A , g . As shown in equation (2.22), this property does not depend on preference parameters. See further development of this proof in Appendix 4.1.2.

$$\mathbb{E}U_A = c + \ln \left[1 + e^{u_{AA}} + \frac{e^{u_{AB}+u_{BA}}(1-2g) + \sqrt{e^{u_{AB}+u_{BA}}(e^{u_{AB}+u_{BA}}(1-2g)^2 + 4(1+e^{u_{AA}})(1+e^{u_{BB}})(1-g)g)}}{2(1+e^{u_{BB}})g} \right] \quad (2.21)$$

$$\frac{\partial \mathbb{E}U_A}{\partial g} = - \frac{e^{u_{AB}+u_{BA}}}{g \sqrt{e^{u_{AB}+u_{BA}}(e^{u_{AB}+u_{BA}}(1-2g)^2 + 4(1+e^{u_{AA}})(1+e^{u_{BB}})(1-g)g)}} \quad (2.22)$$

\square

Proposition 3 states that, given shock realizations, market clearing prices (transfers), that make supply and demand consistent, are always found. Proposition 4 demonstrates the implications of the negative group size effects. Depending on shock realizations, some individuals prefer inter-type matches. The number of majority individuals preferring an intermarriage will be greater than the number of minorities that prefer intermarriage, generating a higher relative demand for minorities. For this demand to be cleared, it is necessary that a higher fraction of minorities choose to marry majority members. Therefore,

this force increases transfers from majorities to minorities, and ultimately leads to better prospects to minorities due to their scarcity.

Therefore, under horizontal differentiations where singles prefer same-type spouses, frictional and frictionless models result in the opposite implications for the matching outcomes. The negative group size effects in frictionless models result from the market clearing condition (demand= supply) together with the free information assumption. When these conditions are replaced by Nash bargaining and random arrivals respectively as given in the frictional model, the model predicts positive group size effects where majorities enjoy better matching outcomes because of their more frequent same-type meeting opportunities.

The following simulations show the expected values of being type A over $g \in [0, 1]$ in frictional and frictionless cases, where the parameters are set as $Q = 1, \tau = 0.2$ (for the frictional model, $x \equiv \frac{r}{\alpha} = \frac{0.04}{0.25}$).

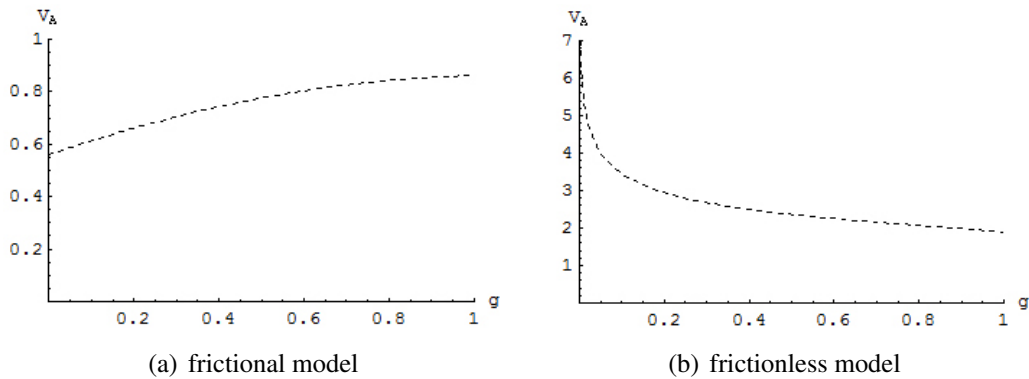


Figure 2.2: Expected utility of being single of type A

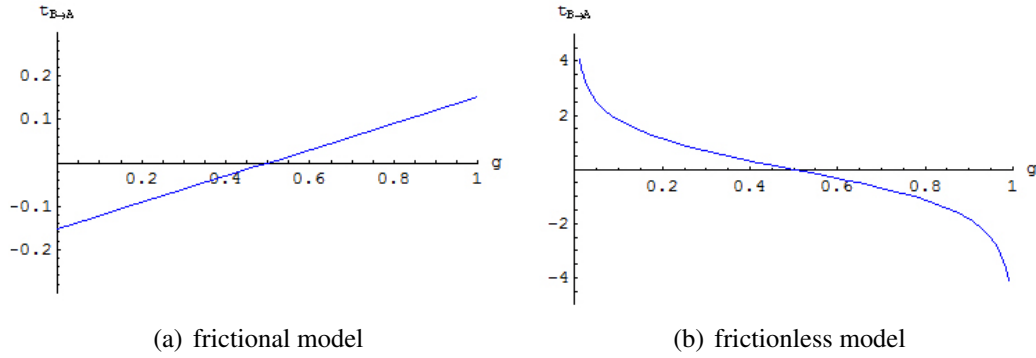


Figure 2.3: Transfer from type B to type A

Figures 2.2 and 2.3 suggest that, in terms of matching prospects and transfers in intermarriages, being unique would be undesirable in frictional settings and desirable in frictionless settings.³

2.4 Horizontal and Vertical Differentiations

Now I apply the framework developed in Section 2.2 to investigate the different types of preferences in matching. More specifically, this subsection investigates the marital search and matching equilibrium along two dimensions: vertical and horizontal differentiation. With vertical differentiation, individuals are ranked and this ranking is universal according to the standard, ‘the more, the better’, with respect to wealth and beauty. Regardless of their own beauty, singles may prefer good looking spouses to less attractive ones. In contrast, horizontal differentiation gives individuals religious, ethnic or racial identity, making them prefer a partner with the same cultural background, which means, ‘the more similar, the better.’ Overall, this model can analyze two driving forces under positive assortative matching, producing implications on intercultural marriages and diversity.

Assume further that we have 4 types denoted by $\{JH, JL, KH, KL\}$. We have two

³Even though the two cases have the same values for Q and τ , the level of utilities cannot be compared. Singles get higher utilities in the frictional case, not only because there is neither search frictions nor waiting time, but also because there are additional stochastic utility terms represented by ε .

horizontal types $\{J, K\}$ with the share of J given by q and two vertical types $\{H, L\}$ with the share of H given by π . The utility from marriages between types is given by Table 2.1.

Table 2.1: Flow Utilities from Marriages

	JH	JL	KH	KL
JH	Q^H	Q^M	$Q^H - \tau$	$Q^M - \tau$
JL		Q^L	$Q^M - \tau$	$Q^L - \tau$
KH			Q^H	Q^M
KL				Q^L

The parameters Q is related to the vertical dimension, assumed to be $Q^H \geq Q^M \geq Q^L$. It reflects ‘The more, the better’ criteria, having more of H types in an union increases the payoff. The parameter τ measures the penalty to inter-cultural marriages $\tau \geq 0$, making singles prefer to marry the one with the same identity. That is, ‘The more similar, the better’ criteria. It is also assumed that they get no flow utility when single, $u = 0$.

2.4.1 Vertical Differentiation

Along the vertical differentiation such as wealth, beauty and education, what matters most is the the gap between different types of marriages. Let Δ_{HM}, Δ_{ML} denote $Q^H - Q^M$ and $Q^M - Q^L$, respectively. The patterns of the different equilibria also depend on the share of H types, π . Figure 2.4 (Figure 2.5) shows the case where π is sufficiently low, $\pi < \pi^*$ (high, $\pi > \pi^*$).⁴

⁴ $\pi^* = \frac{1}{2} - \frac{r}{\alpha}$. Other parameters take values $Q^L = 1, \tau = 0, \pi = .25, \alpha = .25, r = (0 \text{ or } .04)$.

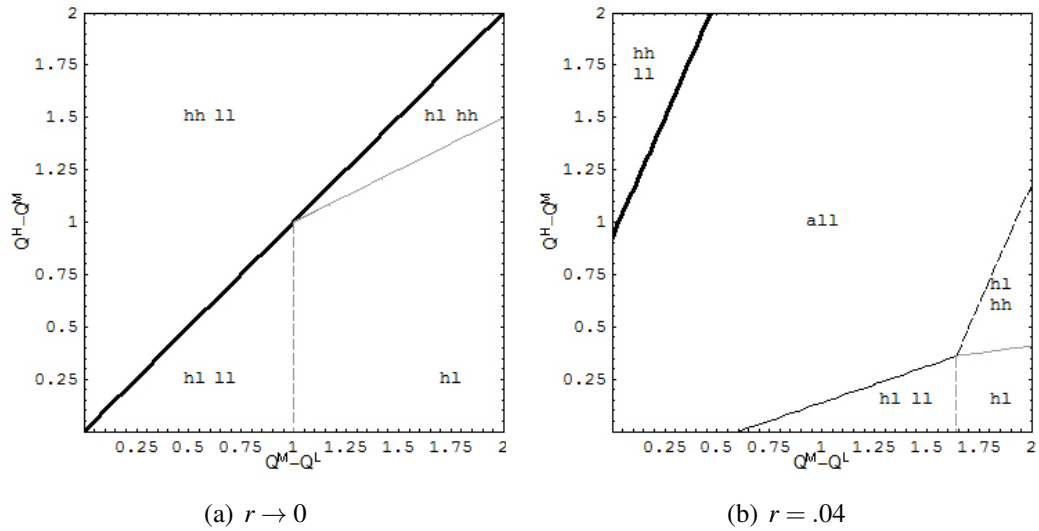


Figure 2.4: Equilibrium set in $(\Delta_{ML}, \Delta_{HM})$ space

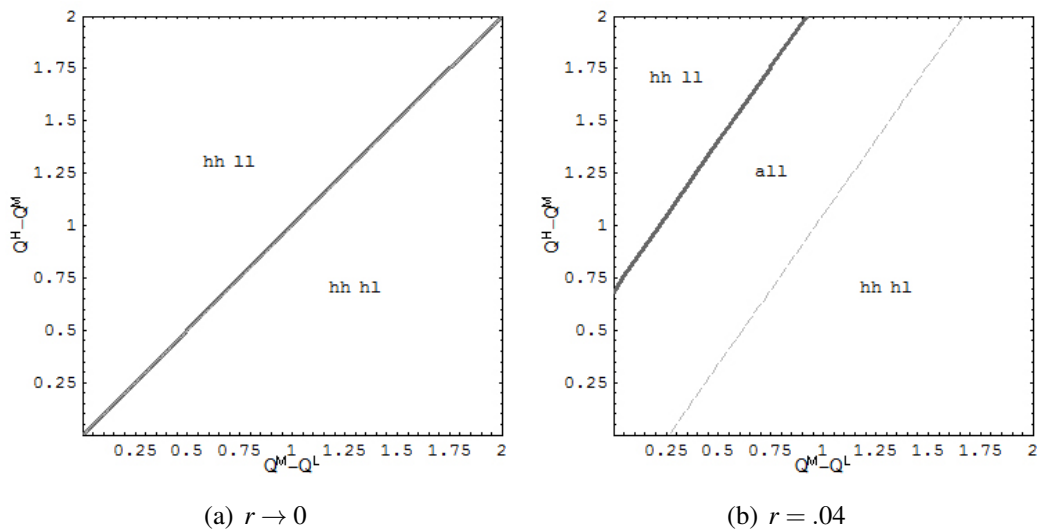


Figure 2.5: Equilibrium set in $(\Delta_{ML}, \Delta_{HM})$ space

The important feature regarding vertical differentiations is the modularity of the utility function. In other words, the type of equilibria responds to the modularity. When $r \rightarrow 0$ and $\Delta_{HM} > \Delta_{ML} \Leftrightarrow Q^H + Q^L > 2Q^M$ (supermodularity of household production function), the economy does not have *HL* marriages which are not efficient according to the criteria by Becker (1973)Becker (1973). In the area of $\Delta_{HM} < \Delta_{ML}$ (submodularity), we may

have the equilibrium in which high types reject each other. This arises when Q^H is not sufficiently higher than Q^M and the share of high types π is low. Under these circumstances, H types can collect enough surplus from their marriages with L types, thus rejecting the other H types.⁵

As shown in Figure 2.4.(b) and 2.5.(b), when there are search frictions, the acceptance strategies of singles are not as efficient as the case without frictions (Figure 2.4,2.5 (a) in which r converges to 0) and the mixed marriage equilibrium with all marriage types may arise in a large set of parameters.

2.4.2 Interaction

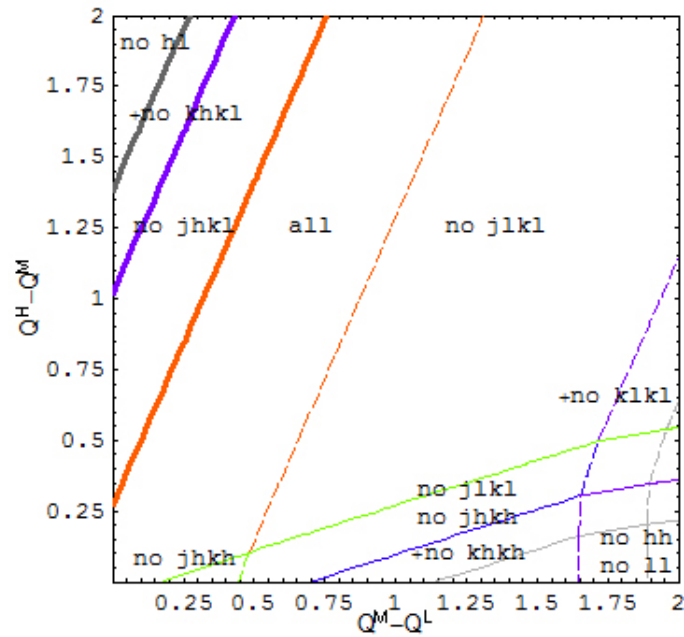


Figure 2.6: Equilibrium set in $(\Delta_{ML}, \Delta_{HM})$ space

The interaction between horizontal and vertical differentiations also depends on the modularity of the household production function. Figure 2.6 presents the different equilibria in $(\Delta_{ML}, \Delta_{HM})$ space where $Q^L = 1, \tau = .2, \pi = .25, \alpha = .25, r = .04, q = .25$. Due to the

⁵The horizontal differentiation imposes supermodularity of the household production, thus the similar pattern (e.g., majorities reject each other) will never happen.

disutility τ , inter-cultural unions are the most responsive to inefficiency in vertical dimensions. See also that the in-cultural unions between two majority types are more responsive in inefficiency than two minority types. This is because that majority types with the higher value of staying single are more willing to wait for vertically efficient matches. For example in Figure 2.6, when *HL* marriages become less desirable (the parameters move to the upper left corner, supermodularity), people stop *JHKL* and *JLKH* marriages first and then *KHKL* marriage, then finally *JHJL* marriages. In summary, people accept inter-horizontal marriages when they can enjoy high surplus gains in their combination of vertical dimensions. Depending on parameter values, high types may or may not engage more in inter-horizontal marriages than low types.⁶

2.5 Conclusion

This paper has analyzed two-sided matching models with and without search frictions in order to study the effects of horizontal differentiations. Unlike vertical differentiations where agents prefer partners of better types, horizontally differentiated agents prefer partners of the same type. Under horizontal differentiations, frameworks with and without search frictions result in opposite implications for matching outcomes of majorities and minorities.

The frictional setting provides better matching prospects to majority members who frequently meet potential partners of their own type and thus have a better outside option in intermarriage bargaining. On the other hand, minorities expect better matching outcomes in the frictionless setting where agents can demand any type of potential spouses depending on taste shocks and the division of matching surplus is adjusted by the market clearing condition. In conclusion, my analysis indicates that the specification of frictions and price-

⁶For example, if we are in the equilibrium without *JLKL* marriages, high types engage all marriages while low types do not inter-horizontally marry with low type spouses. Thus we observe that high types marry more inter-culturally than low types. In contrast, if we are in the equilibrium without *JHKL* marriages (at the bottom left corner of Figure 2.6), low types are observed to inter-horizontally marry more than high types. The intermarriage premium, the pattern that high types intermarry more than low types since they can offset the disutility cause by their intermarriages, analyzed in some literature (Wong 2003 Wong (2003a)), is not a universal phenomenon according to my model.

determination should be carefully considered in an investigation of two-sided markets with horizontally differentiated agents.

There are a number of potentially interesting applications and extensions of the analysis. The model can be extended to more than two types to represent some relevant settings. The interactions between horizontal and vertical differentiations lead to interesting findings on matching outcomes and group behaviors. The assortativeness along vertical differentiations (e.g., positive correlations of husbands' and wives' income) can be affected by horizontal dimensions of couples. Minorities and majorities may exhibit different tendencies to match along vertical traits, which may also lead to new insights on distribution of resources across groups. Also, one could consider endogenizing types by modeling how types of entrants are determined either by self-investment in vertical traits as in Burdett and Coles (2001) or by parents' cultural transmission in horizontal traits as in Bisin and Verdier (2000).

Finally, the framework and the questions proposed in this question may be useful for addressing a variety of empirical and policy related issues in two-sided matching markets. Horizontal differentiations and search frictions may particularly matter when researchers study resource allocation outcomes inside matches (households or worker-firm units). The contrasting group size effects depending on settings with and without search frictions should be considered when policy makers aim to subsidize disadvantaged groups in the economy.

Chapter 3

Appendix to Chapter 1

3.1 Proofs and derivations

3.1.1 Proof of Proposition 1

Proposition 1 (The existence and uniqueness of equilibrium reservation strategies ε^*)
 Given $\{u_{ij}^m, u_{ij}^f, F(\cdot), \sigma, M(\cdot, \cdot), \mu_{ij}, \lambda, \beta, r, \delta\}$ and distributions (s^m, s^f) , the solution ε^* to equation (1.13) exists and is unique.

$$\varepsilon_{ij}^* + \frac{\lambda}{r + \delta + \lambda} \varphi(\varepsilon_{ij}^*) + \frac{u_{ij}^m + u_{ij}^f}{\sigma} = \frac{1}{r + \delta + \lambda} \left\{ (1 - \beta) \sum_l \alpha_{il}^m(s^m, s^f) \varphi(\varepsilon_{il}^*) + \beta \sum_k \alpha_{kj}^f(s^m, s^f) \varphi(\varepsilon_{kj}^*) \right\} \quad (1.13)$$

Proof.

Step 1: Existence

Note that given (s^m, s^f) , arrivals $(\alpha_{ij}^m, \alpha_{ij}^f)$ are also treated as given. Let $\psi(\varepsilon) = \varepsilon + \frac{\lambda}{r + \delta + \lambda} \varphi(\varepsilon)$. Define $\check{\varepsilon}_{ij}(\varepsilon)$ as follows.

$$\check{\varepsilon}_{ij}(\varepsilon) = \psi^{-1} \left(-\frac{u_{ij}^m + u_{ij}^f}{\sigma} + \frac{1}{r + \delta + \lambda} \left\{ (1 - \beta) \sum_l \alpha_{il}^m \varphi(\varepsilon_{il}) + \beta \sum_k \alpha_{kj}^f \varphi(\varepsilon_{kj}) \right\} \right) \quad (3.1)$$

It has the following properties.

$$\frac{\partial \check{\varepsilon}_{ij}}{\partial \varepsilon_{ij}} = -\frac{\{(1 - \beta) \alpha_{ij}^m + \beta \alpha_{ij}^f\} [1 - F(\varepsilon_{ij})]}{r + \delta + \lambda F(\varepsilon_{ij})} < 0 \quad (3.2)$$

$$\frac{\partial \check{\varepsilon}_{ij}}{\partial \varepsilon_{il}} = -\frac{(1 - \beta) \alpha_{il}^m [1 - F(\varepsilon_{il})]}{r + \delta + \lambda F(\varepsilon_{ij})} < 0 \text{ if } l \neq j \quad (3.3)$$

$$\frac{\partial \check{\varepsilon}_{ij}}{\partial \varepsilon_{kj}} = -\frac{\beta \alpha_{kj}^f [1 - F(\varepsilon_{kj})]}{r + \delta + \lambda F(\varepsilon_{ij})} < 0 \text{ if } k \neq i \quad (3.4)$$

$$\frac{\partial \check{\varepsilon}_{ij}}{\partial \varepsilon_{kl}} = 0 \text{ if } k \neq i \text{ and } l \neq j \quad (3.5)$$

Define a vector valued function, $\check{\varepsilon} = (\check{\varepsilon}_{11}, \dots, \check{\varepsilon}_{JJ})$. The solution of the problem ε^* satisfies $\varepsilon^* = \check{\varepsilon}(\varepsilon^*)$, ε^* as a fixed point of $\check{\varepsilon}(\cdot)$. We can find upper and lower bound of $\check{\varepsilon}_{ij}$ by setting $\underline{\varepsilon}_{ij} = \check{\varepsilon}_{ij}(\{\infty\}^{I \times J})$ and $\bar{\varepsilon}_{ij} = \check{\varepsilon}_{ij}(\{\underline{\varepsilon}\}^{I \times J})$.¹ The lower and upper bounds, $\underline{\varepsilon}$ and $\bar{\varepsilon}$, for an appropriate domain can be picked, $\underline{\varepsilon} = \min\{\underline{\varepsilon}_{ij}\}$ and $\bar{\varepsilon} = \max\{\bar{\varepsilon}_{ij}\}$ for all i, j . Then, $\check{\varepsilon}$ is a continuous function which maps $[\underline{\varepsilon}, \bar{\varepsilon}]^{I \times J}$ into itself. Note that $[\underline{\varepsilon}, \bar{\varepsilon}]^{I \times J}$ is a nonempty,

¹Note that $\varphi(\varepsilon)$ is bounded above and unbounded below.

closed, convex subset of a finite-dimensional real vector. Existence is established by applying the Brouwer's Fixed Point Theorem.²

Step 2: Uniqueness

Recall the equations (1.10)-(1.12) that constitute the equilibrium cutoff strategies ε_{ij}^* . For this proof, U denotes male value function, U^m , and V denotes female value functions, U^f for simplicity.

$$(r + \delta)Z_{ij}(\varepsilon_{ij}^*) = u_{ij}^m + u_{ij}^f + \sigma \varepsilon_{ij}^* + \frac{\sigma \lambda}{r + \delta + \lambda} \varphi(\varepsilon_{ij}^*) - (r + \delta)U_i - (r + \delta)V_j = 0 \quad (1.10)$$

$$(r + \delta)U_i = \frac{\sigma(1 - \beta)}{r + \delta + \lambda} \sum_l \alpha_{il}^m \varphi(\varepsilon_{il}^*) \quad (1.11)$$

$$(r + \delta)V_j = \frac{\sigma\beta}{r + \delta + \lambda} \sum_k \alpha_{kj}^f \varphi(\varepsilon_{kj}^*) \quad (1.12)$$

From equation (1.10), find function for $\hat{\varepsilon}$ as a function of (U_i, V_j) where $\psi(\varepsilon)$ is defined as $\psi(\varepsilon) \equiv \varepsilon + \frac{\lambda}{r + \delta + \lambda} \varphi(\varepsilon)$.

$$\varepsilon_{ij}^* = \hat{\varepsilon}(U_i, V_j) = \psi^{-1}\left(\frac{r + \delta}{\sigma}U_i + \frac{r + \delta}{\sigma}V_j - \frac{u_{ij}^m + u_{ij}^f}{\sigma}\right). \quad (3.6)$$

Then plug $\hat{\varepsilon}_{ij}$ to (1.11) and (1.12) to find the following.

$$(r + \delta)U_i = \frac{\sigma(1 - \beta)}{r + \delta + \lambda} \sum_l \alpha_{il}^m \varphi(\hat{\varepsilon}_{il}(U_i, V_l)) \quad (3.7)$$

$$(r + \delta)V_j = \frac{\sigma\beta}{r + \delta + \lambda} \sum_k \alpha_{kj}^f \varphi(\hat{\varepsilon}_{kj}(U_k, V_j)) \quad (3.8)$$

We can find \hat{U}_i and \hat{V}_j that satisfy the equations (3.7) (3.8) as a function of all U_j^f and U_i^m respectively.

$$F_{U_i}(\hat{U}_i(\mathbf{V}), \mathbf{V}) = \hat{U}_i - \frac{\sigma}{r + \delta} \frac{(1 - \beta)}{r + \delta + \lambda} \sum_l \alpha_{il}^m \varphi(\hat{\varepsilon}_{il}(\hat{U}_i, V_l)) \quad (3.9)$$

$$F_{V_j}(\hat{V}_j(\mathbf{U}), \mathbf{U}) = \hat{V}_j - \frac{\sigma}{r + \delta} \frac{\beta}{r + \delta + \lambda} \sum_k \alpha_{kj}^f \varphi(\hat{\varepsilon}_{kj}(U_k, \hat{V}_j)) \quad (3.10)$$

Since

$$\frac{\partial \hat{U}_i}{\partial V_j} = - \frac{\frac{\partial F_{U_i}}{\partial V_j}}{\frac{\partial F_{U_i}}{\partial U_i}} = - \frac{(1 - \beta) \frac{\alpha_{ij}^m [1 - F(\varepsilon_{ij})]}{r + \delta + \lambda F(\varepsilon_{ij})}}{1 + (1 - \beta) \sum_l \frac{\alpha_{il}^m [1 - F(\varepsilon_{il})]}{r + \delta + \lambda F(\varepsilon_{il})}}, \quad (3.11)$$

²Since the following proof, step2, shows a unique fixed point, I do not need the separate existence proof. However, I state this proof to find the lower bound of ε , $\underline{\varepsilon}$, for Proof 3.1.2

this leads to the following properties of functions (3.9) and (3.10):

$$\sum_j \left| \frac{\partial \hat{U}_i}{\partial \hat{V}_j} \right| < 1 \quad (3.12)$$

$$\sum_i \left| \frac{\partial \hat{V}_j}{\partial U_i} \right| < 1 \quad (3.13)$$

Then finding an equilibrium involves finding a fixed point of $\mathbf{U}^* = \{U_1^*, \dots, U_i^*, \dots, U_I^*\}$ of the following mapping \mathbf{T} ,

$$\mathbf{T}(\mathbf{U}) = \{\hat{U}_1(\hat{V}_1(\mathbf{U}), \dots, \hat{V}_j(\mathbf{U}), \dots, \hat{V}_J(\mathbf{U})), \dots, \hat{U}_i(\hat{V}_1(\mathbf{U}), \dots, \hat{V}_j(\mathbf{U}), \dots, \hat{V}_J(\mathbf{U})), \dots\}, \quad (3.14)$$

which is proven to be a contraction, $\|\mathbf{T}(\mathbf{U}') - \mathbf{T}(\mathbf{U}'')\| < \|\mathbf{U}' - \mathbf{U}''\|$ under equations (3.12) and (3.13) as follows. $\|\mathbf{T}(\mathbf{U}') - \mathbf{T}(\mathbf{U}'')\| = \max_i |\hat{U}_i(\hat{V}_1(\mathbf{U}'), \dots, \hat{V}_j(\mathbf{U}'), \dots, \hat{V}_J(\mathbf{U}')) - \hat{U}_i(\hat{V}_1(\mathbf{U}''), \dots, \hat{V}_j(\mathbf{U}''), \dots, \hat{V}_J(\mathbf{U}''))| < \|\hat{\mathbf{V}}(\mathbf{U}') - \hat{\mathbf{V}}(\mathbf{U}'')\| = \max_j |\hat{V}_j(\mathbf{U}') - \hat{V}_j(\mathbf{U}'')| < \|\mathbf{U}' - \mathbf{U}''\|$, where two inequalities come from the facts $\sum_j \left| \frac{\partial \hat{U}_i}{\partial \hat{V}_j} \right| < 1$ and $\sum_i \left| \frac{\partial \hat{V}_j}{\partial U_i} \right| < 1$. For example, applying the Mean Value Theorem to $\hat{V}_j(\mathbf{U})$ to get $\hat{V}_j(\mathbf{U}') - \hat{V}_j(\mathbf{U}'') = \sum_i \frac{\partial \hat{V}_j(\mathbf{U}^{\theta_i})}{\partial U_i} (U_i' - U_i'')$ where \mathbf{U}^{θ_i} is between \mathbf{U}' and \mathbf{U}'' . It follows that $\sum_i \frac{\partial \hat{V}_j(\mathbf{U}^{\theta_i})}{\partial U_i} (U_i' - U_i'') \leq \sum_i \frac{\partial \hat{V}_j(\mathbf{U}^{\theta_i})}{\partial U_i} \|\mathbf{U}' - \mathbf{U}''\| < \|\mathbf{U}' - \mathbf{U}''\|$ under $\sum_i \left| \frac{\partial \hat{V}_j}{\partial U_i} \right| < 1$.

Applying Banach Fixed Point Theorem to this contraction mapping on a complete metric space, \mathbb{R}^I , a unique fixed point of \mathbf{U}^* is established. The same logic holds for finding \mathbf{V}^* , and finally $\varepsilon_{ij}^* = \hat{\varepsilon}(U_i^*, V_j^*) \forall i, j$ are uniquely determined. □

3.1.2 Proof of Proposition 2

Proposition 2 (The existence of a marriage market equilibrium $\{\varepsilon^*, (s^m, s^f)\}$) For any primitives $\{u_{ij}^m, u_{ij}^f, F(\cdot), \sigma, M(\cdot, \cdot), \mu_{ij}, \lambda, \beta, r, \delta\}$ and exogenous distributions (g^m, g^f) , the fixed point $\{\varepsilon^*, (s^m, s^f)\}$ of equations (1.13), (1.19) and (1.20) exists.

Proof. From Proposition 1, we have shown that equation (1.13) gives a solution of ε^* as a continuous function of (s^m, s^f) , denoted as $\varepsilon^*(s^m, s^f)$. Using this function, one can construct mappings by stacking equations (1.19) and (1.20).

$$s_i^m = \frac{g_i^m}{1 + \sum_l \frac{\alpha_{il}^m(s^m, s^f)[1 - F(\varepsilon_{il}^*(s^m, s^f))]}{\delta + \lambda F(\varepsilon_{il}^*(s^m, s^f))}} \quad (1.19)$$

$$s_j^f = \frac{g_j^f}{1 + \sum_k \frac{\alpha_{kj}^f(s^m, s^f)[1 - F(\varepsilon_{kj}^*(s^m, s^f))]}{\delta + \lambda F(\varepsilon_{kj}^*(s^m, s^f))}} \quad (1.20)$$

I will show that all these mappings are continuous over the compact domain $[\underline{s}, g^m], [\underline{s}, g^f]$. The lower bound for s can be found once we assume that the rate of meetings cannot ex-

ceed ($M \times \min\{S^m, S^f\}$) for some finite number M . The lower bound of s_i^m is evaluated at the lower bound of ε , $\underline{\varepsilon}$ (from step1 of 3.1.1) and the upper bound of α^m , $\bar{\alpha}^m$. Recall that $\alpha_{ij}^m = \mu_{ij} \frac{M(S^m, S^f)}{S^m} \frac{s_j^f}{S^f}$. Consider the case where there are more males than females, $G^m < G^f$. The stock of marriages is equal to $G^m - S^m$ and $G^f - S^f$, thus $S^f = G^f - G^m + S^m$. The gender ratio, $\frac{S^f}{S^m}$, is the decreasing function of S^m if $G^m < G^f$, while $\frac{M(S^m, S^f)}{S^m} = M(1, \frac{S^f}{S^m})$ is the increasing function of $\frac{S^f}{S^m}$. Under the assumption that the number of meetings at a given time is bounded above ($\sum_i \sum_j M_{ij} < M \times \min\{S^m, S^f\} = MS^m$), the number of marriages is also bounded above ($\sum_i \sum_j \eta_{ij} = \sum_i \sum_j M_{ij} \frac{1-F(\varepsilon_{ij})}{\delta+\lambda F(\varepsilon)} < MS^m \frac{1-F(\underline{\varepsilon})}{\delta+\gamma F(\underline{\varepsilon})}$), and accordingly the stock of male singles is bounded below, $\underline{S}^m = \frac{G^m}{1+M \frac{1-F(\underline{\varepsilon})}{\delta+\gamma F(\underline{\varepsilon})}} < S^m$. One can then find the upper bound of α_{ij}^m , $\bar{\alpha}^m$ which is defined as $\bar{\alpha}^m \equiv \max_{i,j} \{\mu_{ij}\} M(1, \frac{G^f - G^m - \underline{S}^m}{S^m})$. Finally, the lower bound of s_i^m is found as $\underline{s}_i^m = \frac{g_i^m}{1+J \bar{\alpha}^m \frac{1-F(\underline{\varepsilon})}{\delta+\lambda F(\underline{\varepsilon})}}$ and also, $\underline{s} = \min_{i,j} \{\underline{s}_i^m, \underline{s}_j^f\} > 0$.³ I can establish the existence of the equilibrium $\{\varepsilon^*, (s^m, s^f)\}$ by the Brouwer Fixed Point Theorem. \square

3.1.3 Derivation of value functions

This subsection derives equation (1.1) where U_i denotes the expected lifetime utility of staying single as type i male.

$$(r + \delta)U_i^m = u_i^m + \sum_l \alpha_{il}^m \mathbb{E} \max[U_{il}^m(\tilde{\varepsilon}) - U_i, 0] \quad (1.1)$$

Let $U_{i,t}^m$ be the value of staying single for male i at t . If the length of a period is given by Δ ,

$$U_{i,t}^m = \frac{1 - \delta\Delta}{1 + r\Delta} \{u_i^m \Delta + \sum_l \alpha_{il}^m \Delta \mathbb{E} \max[U_{il,t+\Delta}^m(\tilde{\varepsilon}), U_{i,t+\Delta}^m] + (1 - \sum_l \alpha_{il}^m \Delta) U_{i,t+\Delta}^m + o(\Delta)\}.$$

The right-hand side is the expected value of living to $t + \Delta$, in which the type i male gets flow utilities of $u_i^m \Delta$ (this will be normalized to 0) and receives meeting opportunities with different female types with probabilities $\alpha_{il}^m \Delta$. $o(\Delta)$ captures the payoff in the event of more than one Poisson arrival in a period, and therefore satisfies $\frac{o(\Delta)}{\Delta} \rightarrow 0$ as $\Delta \rightarrow 0$. Arrange both equations to get

$$(1 + r\Delta)U_{i,t}^m = (1 - \delta\Delta) \{u_i^m \Delta + \sum_l \alpha_{il}^m \Delta \mathbb{E} \max[U_{il,t+\Delta}^m(\tilde{\varepsilon}), U_{i,t+\Delta}^m] + (1 - \sum_l \alpha_{il}^m \Delta) U_{i,t+\Delta}^m + o(\Delta)\}.$$

³Note that $\frac{1-F(\varepsilon)}{\delta+\lambda F(\varepsilon)}$ decreases in ε and is maximized at $\underline{\varepsilon}$.

Subtract $U_{i,t}^m \equiv (1 - \delta\Delta)[U_{i,t+\Delta}^m - (U_{i,t+\Delta}^m - U_{i,t}^m)] + \delta\Delta U_{i,t}^m$ from both sides and rearrange.

$$(r\Delta + \delta\Delta)U_{i,t}^m = (1 - \delta\Delta)\{u_i^m\Delta + \sum_l \alpha_{il}^m \Delta \mathbb{E} \max[U_{il,t+\Delta}^m(\tilde{\epsilon}) - U_{i,t+\Delta}^m, 0] + o(\Delta) + (U_{i,t+\Delta}^m - U_{i,t}^m)\}$$

Divide both sides by Δ .

$$(r + \delta)U_{i,t}^m = (1 - \delta\Delta)\{u_i^m + \sum_l \alpha_{il}^m \mathbb{E} \max[U_{il,t+\Delta}^m(\tilde{\epsilon}) - U_{i,t+\Delta}^m, 0] + \frac{o(\Delta)}{\Delta} + \frac{U_{i,t+\Delta}^m - U_{i,t}^m}{\Delta}\}$$

As $\Delta \rightarrow 0$ we have

$$(r + \delta)U_{i,t}^m = u_i^m + \sum_l \alpha_{il}^m \mathbb{E} \max[U_{il,t}^m(\tilde{\epsilon}) - U_{i,t}^m, 0] + \dot{U}_{i,t}^m.$$

Note the similarity of this to asset flow value equations. The return of the asset in a small period equals the sum of the instantaneous payoff and the expected excess value of any changes in the value of states. In the stationary environment where $\dot{U}_{i,t}^m = 0$ and $U_{i,t}^m = U_i^m$ for all t , we finally obtain equation (1.1).

$$(r + \delta)U_i^m = u_i^m + \sum_l \alpha_{il}^m \mathbb{E} \max[U_{il}^m(\tilde{\epsilon}) - U_i^m, 0] \quad (1.1)$$

3.2 Data Construction

1. PSID data

The Panel Study of Income Dynamics (PSID 1968-2011) includes longitudinal information on individual characteristics and marriage histories. I combine three major data sets: 1) family data (1968-1996 recorded annually, 1997-2011 recorded biannually) that contain characteristics of households and household members (mainly a financially responsible single adult or a husband and a wife) in a given year, 2) marriage history data (2011) that record marital transitions (transition to and from the first and the most recent marriage and all marriage transitions that occurred after 1985), and 3) individual data (2011) that carry basic information on all surveyed individuals and how their information is presented in each family data.

2. Sample selection

I consider racial/ethnic types (white, black and Hispanic) of individuals and their spouses. I use information on self-reported and first mentioned race and ethnicity from family files. Regardless of their race, if individuals report themselves as Hispanic descent in response to the question on ethnicity, they are categorized as Hispanic.⁴ PSID had oversampled Hispanic households for the Latino Wave from 1990

⁴The question on race is asked several times over survey years. I choose their report from the earliest years from 1985 to 2011. I then use information from 1968-1984, considering that in those years some race identifications are automatically coded based on their parents' race. 97.5 % of samples' report of their race

to 1995. I did not use information from 241 non-Hispanics in the Latino wave, considering the possibility of oversampling non-Hispanics who marry Hispanics. Their marriages are still captured by Hispanic spouses of the Latino wave. Finally, information on 12,967 whites, 7,790 blacks, 4,972 Hispanics is used in the analysis. 641 non-Hispanic others are dropped due to small sample size.

3. Singlehood durations

I construct singlehood spells from the marriage history data. Although the unit is a year, I used information up to the month of the events (birth, marriage, divorce and data collection). Information contains all singlehood durations including first marriages and later marriages in the PSID. ‘The age at first marriage - 16’ is used for calculating the first durations. The later durations measure the period between the end of the previous marriage and the beginning of the new marriage.⁵ I restrict singlehood durations for 50 years in which people actively participate in the marriage market, the stages and durations up to age 65 are only captured, and the spells are considered as censored even though some exits occur after this period. This window (age 16-65) covers 98.5% of all exits of singlehood. Lastly, singlehood spells that began before individuals came to the U.S. are excluded (1.1%).

4. Marriage durations

Marriage spells in the PSID data, regardless of their order, are used.⁶ This observation combines histories from males and females, and thus the same marriage can be counted twice. I treated a marriage that ended with widowhood as an incomplete spell. Marriages that began outside of the U.S are excluded (0.3%).

5. Type distributions in population

Due to the small sample size of interracial marriages, I need to pool all individuals who had participated in the marriage market in different time periods. To determine type distributions, I first construct the fractions of each type in population based on PSID observations taking into account their sampling weights, which leads to $\{W, B, H\} = \{0.755, 0.118, 0.128\}$ for males and $\{W, B, H\} = \{0.735, 0.134, 0.131\}$ for females.⁷ The distributions in Table 1.4 based on census data are close to the PSID sample distributions with more precise gender ratio (0.980).

3.3 Robustness Check

3.3.1 Identification and calibration of λ

The model has $2 \times I \times J + 1$ unknowns, which are preference (ω_{ij}) and opportunity parameters (μ_{ij}), and the rate of updating match quality shocks to married couples (λ). Observed

is consistent across years. 96.1 % of individuals who are classified as white or black did not mention their second race. Results are robust to how I determine the racial identity of samples.

⁵For observations that have negative number for durations (0.5%), I adjust them to be 0.

⁶For observations that have negative number for durations (0.2%), I adjust them to be 0.

⁷Including non-Hispanic others (denoted as O), the distributions become $\{W, B, H, O\} = \{0.730, 0.114, 0.124, 0.032\}$ for males and $\{0.714, 0.131, 0.128, 0.028\}$ for females.

formation and dissolution behaviors of each marriage will be used to recover these parameters, which constitute $2 \times I \times J$ observables, hazard rate of marital formation (h_{ij}^0) and hazard rate of marital dissolution (h_{ij}^1).⁸ Since there are more unknowns than observables, restrictions on parameters are required to reduce the number of unknowns. I therefore fix the value of λ and estimate preference and opportunity parameters as shown in the baseline analysis. In this section, I will first discuss how I can estimate λ with additional restrictions or information and how these methods justify the current choice of λ . I will then show that key estimation results are robust to the choice of λ .

METHOD 1: lower bound of λ Note first that the hazard rate of divorce is the product of two parts; the rate of updating match quality shock λ and the probability of rejecting new draws $F(\varepsilon^*)$, that is $h_{ij}^1 = \lambda F(\varepsilon_{ij}^*)$. This implies that the level of λ should be bigger than the highest hazard rate of divorce observed in the data.⁹

METHOD 2: additional restrictions on type-specific arrivals By placing some restrictions on the meeting frequencies μ_{ij} , λ can be estimated. I will assume that meetings are uniformly random, that is, $\mu_{ij} = \check{\mu}$. In this case, two parameters that govern arrival of meetings and new match quality shocks, $\theta = (\check{\mu}, \lambda)$, can be simultaneously estimated based on hazard rates recovered from the first step.¹⁰

Recall the equations (1.24), (1.26) that constitute the equilibrium hazard rate of divorce for ij marriage, h_{ij}^1 , and hazard rate of marriage with a j female to a i male, h_{ij}^{0m} .

$$h_{ij}^1 = \lambda F(\varepsilon_{ij}^*) \quad (1.24)$$

$$h_{ij}^{0m} = \alpha_{ij}^m [1 - F(\varepsilon_{ij}^*)] \quad (1.26)$$

Connecting two equations and applying meeting specifications yield the following for males and females.

$$\pi(\theta, \mathbf{h}) = \begin{cases} \check{\mu} \frac{\sqrt{S^m S^f}}{S^m S^f} s_j^f \left(1 - \frac{h_{ij}^1}{\lambda}\right) - h_{ij}^{0m} & \forall i, j \\ \check{\mu} \frac{\sqrt{S^m S^f}}{S^m S^f} s_i^m \left(1 - \frac{h_{ij}^1}{\lambda}\right) - h_{ij}^{0f} & \forall i, j \end{cases} \quad (3.15)$$

Estimated parameters $\hat{\theta}$ will minimize

$$\pi(\theta, \mathbf{h})' W \pi(\theta, \mathbf{h}) \quad (3.16)$$

where W is a $2 \times I \times J$ symmetric and positive definite matrix that defines the distance

⁸Note that the hazard rates of marital formation from the male side are not independent of those from the female side.

⁹However, the exact value of $\max_{ij} \{h_{ij}^1\} = h_{BW}^1 = 0.0236$ cannot be chosen as a value of λ ; then there would be no BW marriages ($F(\varepsilon_{BW}^*) = 1$). If the value close to h_{BW}^1 is chosen, the rejection rate of BW marriages should approach 1, then to have the observed BW marriage formation, α_{BW} becomes higher. I thus choose value 0.0241 that satisfies another reasonable condition $\mu_{BW} \leq 1$ (BW meetings will be less frequent than the uniformly random process.)

¹⁰The minimum requirement is the equalization of two parameters in m , for example, the gender symmetric case $\mu_{ij} = \mu_{ji}$.

of $\pi(\theta, \mathbf{h})$ from zero.¹¹ I can finally obtain the minimum distance estimates and standard errors for λ and $\check{\mu}$ as follows: $\check{\mu} = 0.1045(0.0278)$ $\lambda = 0.0318(0.0069)$.

METHOD 3: additional information on dating behaviors If the aggregate information on meeting behaviors is available, the type-independent match quality shock generating process to couples λ can be inferred. Assume for simplicity the model where gender is the only characteristic. There is one threshold ε^* that governs formation and separation of matches. In this simple case, $\frac{1}{F(\varepsilon^*)}$ is the expected number of new draws before divorce and $\frac{1}{1-F(\varepsilon^*)}$ is the expected number of meetings(dates) before marriage.¹²

I use this last method to calibrate the value of λ for results in the main text. I approximate the average number of meetings till marriage for females to the median number of opposite-sex sexual partners, which is available information in National Health Statistics Reports 2011 (U.S. Department of Health and Human Services). The median number of opposite-sex sexual partners in a lifetime among females aged 15-44 years who are currently married (2006-2008) is 2.5. Given that 15.1% of them had already been married at least two times, the number of sexual partners till marriages is obtained as 2.172.¹³ If we set the level of λ at 0.0289, we can fit this target for the average number of meetings till marriage. However, since the observed proxy for the average number of meetings is subject to several limitations (e.g., people may underreport their number of sexual partners, not all meetings may involve sexual contacts, not all sexual partners are considered as potential partners), it is required to be assured that my main results are robust to the choice of λ at the particular value.

¹¹I use the identity matrix for W .

¹²This follows the geometric distribution. If the probability of acceptance on each meeting is p , then the probability that the k th trial is the first success is $Pr(X = k) = (1 - p)^{k-1} p$ for $k = 1, 2, \dots$. Its expectation is $\mathbb{E}(X) = \sum_{x=1}^{\infty} x p q^{x-1}$ (where $p + q = 1$) = $p \frac{d}{dq} \sum_{x=1}^{\infty} q^x = p \frac{d}{dq} (1 - q)^{-1} = \frac{1}{p}$. With the case of J possible exits, the expected number of meetings until marriage will be $\frac{\sum_{j=1}^J \alpha_{ij}^m}{\sum_{j=1}^J \alpha_{ij}^m [1 - F(\varepsilon_{ij}^*)]}$ for male i .

¹³The number of marriages is observed in PSID samples who were aged 15-44 in year 2006-2008. Solving the equation with one unknown to find the number of sexual partners till their marriages, $x, x * (0.849) + (x + x) * (0.151) = 2.5$ gives $x = 2.172$.

3.3.2 Robustness check with respect to λ

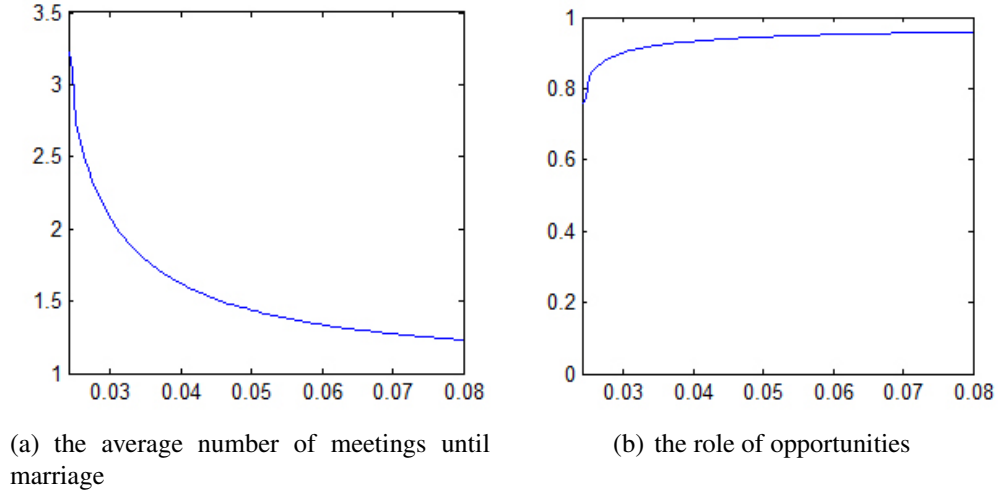


Figure 3.1: Robustness check with respect to λ (x-axis)

The robustness check regarding the choice of λ is shown in Figure 3.1. First, note that the final choice of $\lambda = 0.030$ lies between estimates based on the second and the third method. It is also above the lower bound shown in the first method. The chosen value of $\lambda = 0.030$ is related to the observed mean value of meetings until marriage, 2.08.¹⁴ As shown in panel (a) of Figure 3.1, as the value of λ becomes higher, the value for the average number of meetings until marriage decreases. Panel (b) in Figure 3.1 demonstrates that as the value λ increases, the importance of opportunities in explaining marital behaviors also increases. Since the estimated hazard rates of divorce ($h_{ij}^1 = \lambda F(\varepsilon_{ij}^*)$) is fitted by this higher value of λ , the higher λ leads to the lower rejection rates $F(\varepsilon_{ij}^*) \downarrow$, or higher acceptance rates, $[1 - F(\varepsilon_{ij}^*)] \uparrow$. Given the acceptance rules, to fit the estimated hazard rates of marriage formation ($h_{ij}^{0m} = \alpha_{ij}^m [1 - F(\varepsilon_{ij}^*)]$) with the higher acceptance rates, lower arrival rates are implied, $\alpha_{ij}^m \downarrow$. If people accept meetings easily, whom they meet becomes more significant in explaining whom they match with, which leads to the higher role of opportunities with the higher value of λ . However, in overall ranges, the significant role of opportunities compared to preferences can be shown. In addition, rankings among estimated parameters are also preserved across different values of λ .

3.4 Identification of more parameters

recovering separate flow utilities of marriages u_{ij}^m, u_{ij}^f , (σ and β) using data on transfers

¹⁴Across race and gender, the number of meetings till marriage for male whites, blacks and Hispanics is $\{2.05, 2.81, 1.44\}$ and $\{2.06, 2.62, 1.58\}$ for female counterparts.

If we have information on transfers from husbands to wives¹⁵, we can recover preferences parameters u_{ij}^m and u_{ij}^f separately, still based on the assumptions on bargaining parameter β and the dispersion of $F(\cdot)$ σ using the following likelihood function.

$$l(\tau_{0c}, \gamma_{0c}, d_c^k, \tau_{1c}, \gamma_{1c}, t_c) = \prod_c \exp\left\{-\sum_j \alpha_{ij}^m [1 - F(\varepsilon_{ij}^*)] \tau_{0c}\right\} \left[\prod_j (\alpha_{ij}^m [1 - F(\varepsilon_{ij}^*)])^{d_c^j}\right]^{1-\gamma_{0c}} \\ \prod_j [\exp\{-\lambda F(\varepsilon_{ij}^*) \tau_{1c}\} (\lambda F(\varepsilon_{ij}^*))^{1-\gamma_{1c}} f(t_c | \varepsilon \geq \varepsilon_{ij}^*)]^{d_c^j} \quad (3.17)$$

The equilibrium transfers are given as follows.

$$t_{ij}(\varepsilon) = \beta u_{ij}^m - (1 - \beta) u_{ij}^f + \sigma(\beta - \frac{1}{2}) \varepsilon - \frac{\sigma\beta(1-\beta)}{r+\delta+\lambda} \left\{ \sum_l \alpha_{il}^m \varphi(\varepsilon_{il}^*) - \sum_k \alpha_{kj}^f \varphi(\varepsilon_{kj}^*) \right\} \quad (3.18)$$

I assume $F(\cdot)$ the standard normal distribution and yield

$$f(t_c) = \frac{1}{\sigma(\beta - \frac{1}{2})} \phi \left(\frac{t_c - \beta u_{ij}^m + (1 - \beta) u_{ij}^f + \frac{\sigma\beta(1-\beta)}{r+\delta+\lambda} \left\{ \sum_l \alpha_{il}^m \varphi(\varepsilon_{il}^*) - \sum_k \alpha_{kj}^f \varphi(\varepsilon_{kj}^*) \right\}}{\sigma(\beta - \frac{1}{2})} \right) \quad (3.19)$$

$$f(t_c | \varepsilon \geq \varepsilon_{ij}^*) = \frac{f(t_c)}{1 - \Phi(\varepsilon_{ij}^*)} \quad (3.20)$$

Up to Step 3 even without the information on transfers, $\frac{u_{ij}^m + u_{ij}^f}{\sigma}$, the total flow utility from marriage i, j , and $\alpha_{ij}^m, \alpha_{ij}^f$, the arrival rate of j females to i males, can be recovered. If the data on transfers are available, one can additionally recover $\sigma(\beta - \frac{1}{2})$ and $-\beta u_{ij}^m + (1 - \beta) u_{ij}^f + \frac{\sigma\beta(1-\beta)}{r+\delta+\lambda} \left\{ \sum_k \alpha_{ik}^m \varphi(\varepsilon_{ik}^*) - \sum_l \alpha_{lj}^f \varphi(\varepsilon_{lj}^*) \right\}$ and the functional form of $F(\cdot)$ above threshold values. To recover separately u_{ij}^m and u_{ij}^f , I need to assume one of β or σ .¹⁶

¹⁵Household bargaining outcomes t_{ij} can be measured based on the survey questions on division of household chores, whether a husband or a wife had contraceptive operations, subject evaluation on the fairness in several areas, gender specific consumption.

¹⁶This discussion can be made based on population moments. Note also the followings

$$\mathbb{E}[t_{ij} | \varepsilon \geq \varepsilon_{ij}^*] = \beta u_{ij}^m - (1 - \beta) u_{ij}^f + \sigma(\beta - \frac{1}{2}) \mathbb{E}[\varepsilon | \varepsilon \geq \varepsilon_{ij}^*] - \frac{\sigma\beta(1-\beta)}{r+\delta+\lambda} \left\{ \sum_l \alpha_{il}^m \varphi(\varepsilon_{il}^*) - \sum_k \alpha_{kj}^f \varphi(\varepsilon_{kj}^*) \right\} \\ = \beta u_{ij}^m - (1 - \beta) u_{ij}^f + \sigma(\beta - \frac{1}{2}) \tilde{\lambda}(\varepsilon_{ij}^*) - \frac{\sigma\beta(1-\beta)}{r+\delta+\lambda} \left\{ \sum_l \alpha_{il}^m \varphi(\varepsilon_{il}^*) - \sum_k \alpha_{kj}^f \varphi(\varepsilon_{kj}^*) \right\} \quad (\text{eq10})$$

$$\mathbb{V}[t_{ij} | \varepsilon \geq \varepsilon_{ij}^*] = \sigma^2(\beta - \frac{1}{2})^2 \mathbb{V}[\varepsilon_{ij} | \varepsilon \geq \varepsilon_{ij}^*] = \sigma^2(\beta - \frac{1}{2})^2 (1 - \tilde{\lambda}(\varepsilon_{ij}^*)) [\tilde{\lambda}(\varepsilon_{ij}^*) - \varepsilon_{ij}^*]. \quad (\text{eq11})$$

Define the inverse Mills ratio(IMR) $\tilde{\lambda}(\varepsilon_{ij}^*) \equiv \frac{\phi(\varepsilon_{ij}^*)}{1 - \Phi(\varepsilon_{ij}^*)}$. Note that $\mathbb{E}[\varepsilon | \varepsilon \geq \varepsilon_{ij}^*] = \tilde{\lambda}(\varepsilon_{ij}^*)$.

3.5 Application to one-sided models

Assume a situation where there are i consumers and j goods. Researchers are interested in two parameters: 1) arrival rate of good j to consumer type i (α_{ij}), and 2) utility of type i from consuming good j (ω_{ij}). There are also idiosyncratic and stochastic consumer tastes ε . After purchasing an item, the consumer updates his or her taste shock according to the Poisson rate of λ . If this taste shock falls too low, the consumer may decide to stop using this good. Then the value function of type i consuming item j upon shock realization ε , $V_{ij}(\varepsilon)$, and the value function of type i who does not buy an item yet and is waiting for arrivals, V_i , are given by

$$rV_{ij}(\varepsilon) = \omega_{ij} + \varepsilon + \lambda \mathbb{E} \max[U_{ij}(\tilde{\varepsilon}) - U_{ij}(\varepsilon), U_i - U_{ij}(\varepsilon)] \quad (3.21)$$

$$rV_i = \sum_j \alpha_{ij} \mathbb{E} \max[U_{ij}(\tilde{\varepsilon}) - U_i]. \quad (3.22)$$

Threshold shocks, ε^* , will be solved from the following conditions where function $\varphi(\cdot)$ depends on the distribution of taste shock ε , $\varphi(\varepsilon^*) = \int_{\varepsilon^*}^{\infty} [1 - F(x)] dx$.

$$rV_{ij}(\varepsilon_{ij}^*) = rV_i \quad (3.23)$$

$$\omega + \varepsilon_{ij}^* + \frac{\lambda}{r + \lambda} \varphi(\varepsilon_{ij}^*) = \sum_j \frac{\alpha_{ij}}{r + \lambda} \varphi(\varepsilon_{ij}^*) \quad (3.24)$$

With observed durations before consumers choose one good, the type of good the consumers select, and durations of the consumers using the good, researchers can disentangle between α_{ij} and $F(\varepsilon_{ij})$. This decomposition is based on estimated hazard rates of transition into consuming good j (h_{ij}^0) and estimated hazard rates of transition out of consuming good j (h_{ij}^1). Given α and ε^* , preference parameters are then recovered based on equation (3.24).

$$h_{ij}^1 = \lambda F(\varepsilon_{ij}^*) \quad (3.25)$$

$$h_{ij}^0 = \alpha_{ij} [1 - F(\varepsilon_{ij}^*)] \quad (3.26)$$

The crucial identification assumption is that λ , the arrival of a new taste shock to consumers who are currently using this good, is type-independent and calibrated outside of the model based on, for example, the average number of arrivals until purchase. In other cases, type-independent λ can be assumed if information of this level is known from independent sources (rather than durations of usage data), such as technical reports or testings with the sample of consumers. It is also critical that opportunities arrive only before purchasing the good, not after purchasing. These identifying assumptions can be restrictive in analyzing some markets.

3.6 Additional tables

Table 3.1: Experiment: balanced gender ratio for blacks (non-institutional population)

	balanced black gender ratio					baseline				
	W	B	H	S	index	W	B	H	S	index
white male	0.718	0.002	0.020	0.259	0.865	0.719	0.003	0.021	0.257	0.857
white female	0.706	0.007	0.022	0.266	0.833	0.707	0.005	0.022	0.266	0.837
black male	0.039	0.534	0.023	0.405	0.884	0.036	0.567	0.021	0.377	0.900
black female	0.013	0.524	0.019	0.443	0.934	0.016	0.469	0.022	0.494	0.917
Hispanic male	0.141	0.022	0.608	0.229	0.758	0.139	0.026	0.610	0.225	0.757
Hispanic female	0.128	0.025	0.603	0.244	0.768	0.130	0.020	0.605	0.246	0.773
Total	(inter marriage=6.1%)				0.839	(inter marriage=6.0%)				0.837

Notes: Predicted type distribution in steady-state stocks. Fraction in each state: being married to column type spouses (W:white, B:black, H:Hispanic), or being single (S:single). Balanced black gender ratio: model outcomes under the increased measure of black males from the type distribution in 2000. Baseline: model outcomes under the type distribution of non-institutional population in 2000.

Chapter 4

Appendix to Chapter 2

4.1 Proofs

4.1.1 Proof of Proposition 1

Proposition 1. *Two types of pure strategy equilibria.* Under the Nash Bargaining solution given in Lemma 1, there are two types of pure strategy equilibria. If the penalty for intermarriage is high, $\tau \geq \tau^* \equiv \frac{x(1+2x)Q}{2(x+g)(x+(1-g))}$, only homogamous marriages (AA, BB) are formed in the equilibrium (the homogamous equilibrium). If $\tau < \tau^*$, all matches (AA, BB, AB, BA) are formed in the equilibrium (the all-matches equilibrium).

Proof. Under Lemma 1, there are eight potential types of pure strategy equilibria, where three indicator functions $\{\beta_{AA}, \beta_{BB}, \beta_{AB}(=\beta_{BA})\}$ take value from $\{0, 1\}$. I will first show that the only possible equilibrium types are $\{\beta_{AA}, \beta_{BB}, \beta_{AB}\} = \{1, 1, 0\}, \{1, 1, 1\}$, by excluding other 6 cases.

First, at least one type is rejecting all matches, $\{\beta_{AA}, \beta_{BB}, \beta_{AB}\} = \{0, 0, 0\}, \{1, 0, 0\}, \{0, 1, 0\}$, cannot be an equilibrium outcome, since then one of $V_i = 0, i \in \{A, B\}$ are always smaller than $V_{ii} = Q$. Second, rejecting homogamous matches and accepting heretogamous matches, $\{\beta_{AA}, \beta_{BB}, \beta_{AB}\} = \{0, 0, 1\}$, cannot be an equilibrium, since $2Q > 2(Q - \tau) = V_{AB} + V_{BA} \geq V_A + V_B \geq \min\{2V_A, 2V_B\}$ contradicts $Q < V_A$ or $Q < V_B$. Third, if they accept intermarriages, both types should accept homogamous marriages. In other words, $\{\beta_{AA}, \beta_{BB}, \beta_{AB}\} = \{1, 0, 1\}, \{0, 1, 1\}$ cannot be an equilibrium. Suppose, to the contrary, B types reject their BB marriages, while A types accept all marriages, AA, AB . This will be the case where $V_B > Q \geq V_A$ and $2(Q - \tau) \geq (V_A + V_B)$. Then the values of being single become $rV_A = \alpha g(Q - V_A) + \alpha(1 - g)\frac{2(Q - \tau) - V_A - V_B}{2}$ and $rV_B = \alpha g\frac{2(Q - \tau) - V_A - V_B}{2}$. Using these two equations, two unknowns (V_A, V_B) can be found $V_A = \frac{Q(\alpha^2 g^2 + 2\alpha r) - 2(1 - g)\alpha r \tau}{\alpha^2 g^2 + 2\alpha g r + 2r^2 + \alpha r}$ and $V_B = \frac{(Q - \tau)(\alpha^2 g^2 + 2\alpha r) - \alpha^2 g^2 \tau}{\alpha^2 g^2 + 2\alpha g r + 2r^2 + \alpha r}$, leading to $V_B < Q$, which is a contradiction. This shows that by accepting only intermarriages, members of group B cannot collect enough surpluses to reject their homogamous marriages.

Let us then consider the equilibrium where all marriages are accepted (*all-matches equilibrium*), which gives the following equations:

$$rV_A = \alpha g(Q - V_A) + \alpha(1 - g)\frac{2(Q - \tau) - V_A - V_B}{2} \quad (4.1)$$

$$rV_B = \alpha(1 - g)(Q - V_B) + \alpha g\frac{2(Q - \tau) - V_A - V_B}{2} \quad (4.2)$$

By solving two equations with two unknowns, the following matching outcomes are obtained.

$$V_A = \frac{Q(1 + 2x) - 2(1 - g)(1 - g + x)t}{1 + 3x + 2x^2} \quad (4.3)$$

$$V_B = \frac{Q(1 + 2x) - 2g(g + x)t}{1 + 3x + 2x^2} \quad (4.4)$$

Note that the conditions for acceptance of homogamous marriages ($Q \geq V_A, Q \geq V_B$) always hold. For intermarriages to be accepted, $(Q - \tau) \geq V_A + V_B$ is required, which leads to the condition:

$$\tau \leq \tau_1^* \equiv \frac{x(1+2x)Q}{2(g-g^2+x+x^2)} \quad (4.5)$$

For the equilibrium where only homogamous marriages are accepted (*homogamous equilibrium*), values functions take simple forms:

$$rV_A = \alpha g(Q - V_A) \quad (4.6)$$

$$rV_B = \alpha(1-g)(Q - V_B) \quad (4.7)$$

which lead to the following solutions:

$$V_A = \frac{gQ}{g+x} \quad (4.8)$$

$$V_B = \frac{(1-g)Q}{1-g+x} \quad (4.9)$$

The conditions for accepting homogamous marriages ($Q \geq V_A, Q \geq V_B$) are again always satisfied. For intermarriages to be rejected, $(Q - \tau) < V_A + V_B$ is required, which leads to the condition:

$$\tau > \tau_0^* \equiv \frac{x(1+2x)Q}{2(g-g^2+x+x^2)} \quad (4.10)$$

Since two conditions, shown in equations (4.5) and (4.10), depend on the same threshold, $\tau^* = \tau_0^* = \tau_1^* = \frac{x(1+2x)Q}{2(g-g^2+x+x^2)}$, there is only one pure strategy equilibrium at given parameter values. □

4.1.2 Proof of Proposition 3 and 4

Proposition 3. *The existence and uniqueness of the frictionless market equilibrium. Under symmetric frictionless settings with two types, there exists a unique marriage market equilibrium.*

Proposition 4. *Negative group size effects. Regardless of utility specifications, the group size effect is always positive. In other words, as the fraction of i increases, the marital prospects of type i decreases.*

Proof. Assume that type A (B) gets u_{AA} (u_{BB}) from homogamous marriage and u_{AB} (u_{BA}) from intermarriage.

Let μ_i be the measure of i type singles and let μ_{ij} be the measure of marriage between i husbands and j wives. Based on equations (2.18) and (2.19), the measure of each marriage

type is determined as follows.

$$\ln\mu_{AA} = \ln\mu_A + u_{AA} \quad (4.11)$$

$$\ln\mu_{BB} = \ln\mu_B + u_{BB} \quad (4.12)$$

$$\ln\mu_{AB} = \ln\mu_A + u_{AB} + t_{B \rightarrow A} \quad (4.13)$$

$$= \ln\mu_B + u_{BA} + t_{A \rightarrow B} \quad (4.14)$$

Due to gender symmetry, there is no transfer in homogenous marriages. In addition, the measure of AB marriages is equal to that of BA marriages ($\mu_{AB} = \mu_{BA}$). Transfers should be consistent with this gender symmetry assumption, and thus what type A gets from type B is the negative value of what type A gives to type B ($t_{B \rightarrow A} = -t_{A \rightarrow B}$).

Based on the market clearing condition, the measure of marriage is connected by the utility specification, as shown in equation (1.43).

$$\mu_{AB} = \exp\left\{\frac{u_{AB} + u_{BA}}{2}\right\} \sqrt{\mu_A \mu_B} \quad (4.15)$$

Moreover, the measures in each state should be consistent with the fraction of each type. The measure of type A and B single can be found at given $t_{B \rightarrow A}$.

$$\begin{aligned} g &= \mu_A + \mu_{AA} + \mu_{AB} \\ &= \mu_A [1 + e^{u_{AA}} + e^{u_{AB}} + t_{B \rightarrow A}] \end{aligned} \quad (4.16)$$

$$\begin{aligned} 1 - g &= \mu_B + \mu_{BB} + \mu_{BA} \\ &= \mu_B [1 + e^{u_{BB}} + e^{u_{BA}} - t_{B \rightarrow A}] \end{aligned} \quad (4.17)$$

By equating equations (4.13) and (4.15) given equations (4.16) and (4.17), I can solve for $t_{B \rightarrow A}$ from the following quadratic equation,

$$\mu_A e^{u_{AB}} e^{-t_{B \rightarrow A}} = e^{\frac{u_{AB} + u_{BA}}{2}} \sqrt{\mu_A \mu_B} \quad (4.18)$$

which leads to

$$\begin{aligned} t_{B \rightarrow A} &= \ln \left[\frac{1}{2(1 + e^{u_{BB}})g} e^{-u_{AB}} (e^{u_{AB} + u_{BA}} (1 - 2g) + \right. \\ &\quad \left. \sqrt{e^{u_{AB} + u_{BA}} (e^{u_{AB} + u_{BA}} (1 - 2g)^2 + 4(1 + e^{u_{AA}})(1 + e^{u_{BB}})(1 - g)g)} \right]. \end{aligned} \quad (4.19)$$

The measure of type singles can be found at given $t_{B \rightarrow A}$ by equations (4.16) and (4.17). Once μ_A is known, other measures μ_{AA}, μ_{AB} can be obtained accordingly, based on equations (4.11) and (4.13).

$$\mu_A = \frac{g}{1 + e^{u_{AA}} + \frac{e^{u_{AB} + u_{BA}}(1 - 2g) + \sqrt{e^{u_{AB} + u_{BA}}(e^{u_{AB} + u_{BA}}(1 - 2g)^2 + 4(1 + e^{u_{AA}})(1 + e^{u_{BB}})(1 - g)g)}}{2(1 + e^{u_{BB}})g}} \quad (4.20)$$

$$\mu_{AA} = \mu_A e^{u_{AA}} \quad (4.21)$$

$$\mu_{AB} = \mu_A e^{u_{AB} + t_{B \rightarrow A}} \quad (4.22)$$

Moreover, type i 's expected value of being single before he sees his realization of disturbances, $\mathbb{E}U_A$, is given by the following where c is Euler's constant ($\simeq 0.5772157$)¹

$$\mathbb{E}U_A = c + \ln\left(\frac{g}{\mu_A}\right) \quad (4.23)$$

$$= c + \ln\left[1 + e^{u_{AA}} + \frac{e^{u_{AB}+u_{BA}}(1-2g) + \sqrt{e^{u_{AB}+u_{BA}}(e^{u_{AB}+u_{BA}}(1-2g)^2 + 4(1+e^{u_{AA}})(1+e^{u_{BB}})(1-g)g)}}{2(1+e^{u_{BB}})g}\right] \quad (4.24)$$

What is noteworthy is that the utility of being a type i decreases as the fraction of type i increases. This property does not depend on utility specifications.

$$\frac{\partial \mathbb{E}U_A}{\partial g} = - \frac{e^{u_{AB}+u_{BA}}}{g \sqrt{e^{u_{AB}+u_{BA}}(e^{u_{AB}+u_{BA}}(1-2g)^2 + 4(1+e^{u_{AA}})(1+e^{u_{BB}})(1-g)g)}} \quad (4.25)$$

□

¹For the derivation, see Choo and Siow (2006b).

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