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EFFECT OF RANDOMNESS ON CRITICAL BEHAVIOR OF SPIN MODELS*

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ABSTRACT

Renormalization group methods are used to analyze the critical behavior of random Ising models. The Wilson-Fischer ϵ-expansion for the recursion relations for n-component continuous spin models are developed for randomly inhomogeneous systems. In addition to the usual variables for a homogeneous system there appears a variable which in essence describes local fluctuations in T_c. From the structure and stability of the fixed points we conclude that critical exponents are unaffected by randomness for $n > 4$ but are renormalized by randomness for $1 < n < 4$. In both cases $\alpha < 0$, as expected from a simple physical argument.

It is well known that uniform magnetic systems undergo sharp phase transitions with divergent susceptibilities. If, however, the system is randomly diluted, or if the interactions between spins are randomized, the situation is less clear. Is the transition sharp or smeared? If the transition is sharp, are the exponents the same as for the homogeneous system or are they renormalized? High temperature expansions[1] seem unable to answer these questions. An exact solution[2] for a special two-dimensional random Ising model predicts a smeared transition. However, in view of the long range correlations in the randomness of this special model, it is not clear whether the results represent behavior typical of local randomness. In view of these uncertainties it is natural to try to clarify the situation using renormalization group techniques which have been so successful in calculating critical properties of homogeneous systems.[3] Two formulations of the renormalization group suitable for this purpose are the cluster expansion for discrete spins given by Niemeyer and van Leeuwen[4] and the ϵ-expansion for continuous spins of Wilson and Fisher.[3] Previously[5] we outlined the general scheme for applying the renormalization group to systems with random potentials. Since most of that discussion described the discrete-spin method, we will confine the present discussion to the continuous spin technique. Results to first order in ϵ will be given here; higher order results will be presented elsewhere.

We begin with the reduced Hamiltonian

$$\mathcal{H} = \tfrac{1}{2} \int V_2(\vec{q}_1,\vec{q}_2)\, \vec{S}_1\cdot\vec{S}_2\, dq_1\, dq_2 + \int V_4\, \vec{S}_1\cdot\vec{S}_2\, \vec{S}_3\cdot\vec{S}_4\, dq_1\, dq_2\, dq_3\, dq_4, \qquad (1)$$

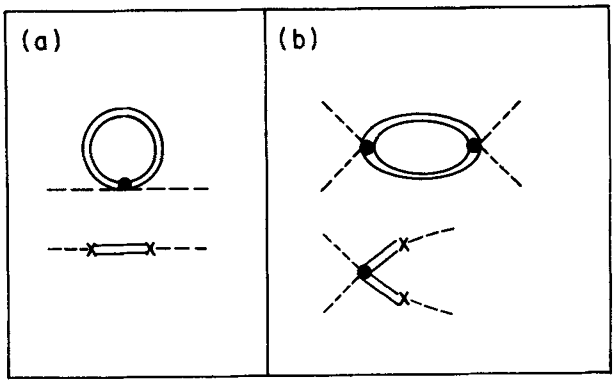
where $\vec{S}_n = \vec{S}(\vec{q}_n)$ is the Fourier transform of an n-component vector field, $\int dq \equiv (2\pi)^{-d} \int d^d\vec{q}$, where the integration is over a sphere of radius Λ, and V_2 and $V_4 \equiv V_4(q_1,q_2,q_3,q_4)$ are arbitrary random potentials for an inhomogeneous system governed by a probability distribution P. We then develop recursion relations for the inhomogeneous potentials in the standard way:[6]

$$\{V'_\ell\} = R\{V_\ell\} \equiv R_s\, R_b\, \{V_\ell\}, \qquad (2)$$

where R_b represents the removal of all spin degrees of freedom with $b^{-1}\Lambda < |\vec{q}| < \Lambda$ and R_s represents a scale change $\vec{q} \to b\vec{q}$ and a spin renormalization $\vec{s} \to \zeta\vec{s}$. As shown in Fig. 1, R_b can be developed diagrammatically as in the homogeneous case. As discussed in Ref. 4, Eq. (2) gives rise to recursion relations for the probability distribution:

$$P'(\{V'_\ell\}) = \int \delta(\{V'_\ell\} - R\{V_\ell\}) P(\{V_\ell\}) d\{V_\ell\} \qquad (3)$$

where the integral is over all degrees of freedom in $\{V_\ell\}$. Thus in the random problem, one seeks a fixed point for the probability distribution $P(\{V_\ell\})$ rather than for the potentials.



1. Diagrams for V_2' (a) and V_4' (b) to leading order in V_4 and δV_2. The double line represents the Gaussian propagator for the inhomogeneous system $[V_2(\vec{q},\vec{q}')]^{-1}$. To obtain Eqs. (6) use $V_2^{-1} = \langle V_2\rangle^{-1} - \langle V_2\rangle^{-1} \delta V_2 \langle V_2\rangle^{-1} \ldots$ in these diagrams and average the resulting equations using $\langle \delta V_2 \delta V_2\rangle = \Delta(q_1 + q_2 + q_3 + q_4)$.

It is obvious that $P(\{V_n\}) = \delta(\{V_n\} - \{V_n^*\})$ is a fixed point of Eq. (3) if $\{V_n{}^*\}$ is the fixed point value of $\{V_n\}$ for the homogeneous system. To study systems with narrow probability distributions, we develop recursion relations for the cumulants, $\langle V_\ell\rangle$ $\langle V_\ell V_m\rangle - \langle V_\ell\rangle\langle V_m\rangle$ etc. of P. The averaging process restores translational invariance, so we can write $V_2(\vec{q}, \vec{q}') = \langle V_2(\vec{q}, \vec{q}')\rangle + \delta V_2(\vec{q}, \vec{q}')$, where

$$\langle V_2(\vec{q}, \vec{q}')\rangle = (r + q^2)\, \delta^d(\vec{q} + \vec{q}'). \qquad (4)$$

The spin renormalization coefficient ζ is then chosen so that the coefficient of q^2 in Eq. (4) remains unity after each iteration (i.e. $\zeta = b^{1 + \frac{1}{2}d - \frac{1}{2}\eta}$). In the long wavelength limit, we can also write

$$\langle V_4(\vec{q}_1,\vec{q}_2,\vec{q}_3,\vec{q}_4)\rangle = u\, \delta^d(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4) \qquad (5a)$$

$$\langle \delta V_2(\vec{q}_1,\vec{q}_2)\, \delta V_2(\vec{q}_3,\vec{q}_4)\rangle = \Delta\delta^d(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4). \qquad (5b)$$

If there are no long range correlations in the random potentials, Δ will be a constant in the long wavelength limit. Thus Δ behaves like a four-spin potential and must be treated on the same level as u in the recursion relation. All other cumulants and momentum dependences are irrelevant variables near four dimension for the same reason that u_6 and q-dependent corrections to u are irrelevant in the homogeneous case.[6]

To first order in $\epsilon = 4-d$ the recursion relations are

$$r' = b^{2-\eta}\{r - A(r)[4(n+2)u - \Delta]\} \qquad (6a)$$

$$u' = b^{\epsilon-2\eta}\{u - K \ln b[4(n+8)u^2 - 6u\Delta]\} \qquad (6b)$$

$$\Delta' = b^{\epsilon-2\eta}\{\Delta - K \ln b[8(n+2)u\Delta - 4\Delta^2]\}, \qquad (6c)$$

where $K^{-1} = 2^{d-1}\pi^{\frac{1}{2}d}\,\Gamma(\tfrac{1}{2}d)$ and $A(r) = \int_{\Lambda/b}^{\Lambda} (q^2 + r)^{-1} d^d\vec{q}$. The analysis of these equations proceeds exactly as for a spin system with a hypercubic potential.[7] There are four fixed points to first order in ϵ, and the flow diagram showing their stability is given in Fig. 2. They are

1) a Gaussian fixed point with $u^* = \Delta^* = 0$;
2) a Heisenberg fixed point with $u^* = \epsilon/[4K(n+8)]$, $\Delta^* = 0$, $\lambda_\Delta = \epsilon(4-n)/(n+8)$, $\alpha = \frac{1}{2}\epsilon(4-n)/(n+8)$;
3) an unphysical fixed point with $u^* = 0$, $\Delta^* = -\frac{1}{4}K^{-1}\epsilon$, $\lambda_\Delta = -\epsilon$, $\lambda_u = -\frac{1}{2}\epsilon$, $2\nu = 1 + \epsilon/8$;

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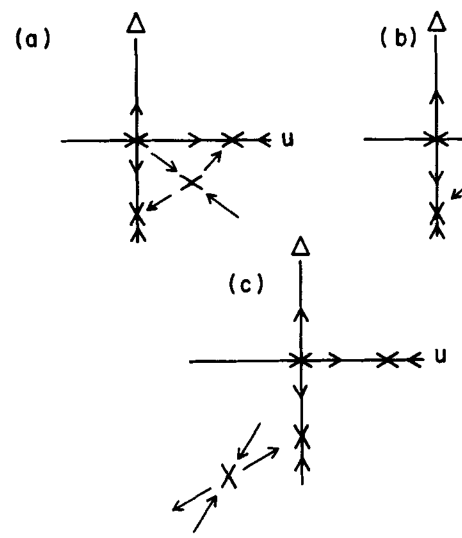

2. Fixed point flow diagrams for a) $n > 4$, b) $1<n<4$, and c) $n < 1$.

4) a randomness dominated fixed point with $u^* = \epsilon/[16K(n-1)]$, $\Delta^* = \epsilon(4-n)/[8K(n-1)]$, $\lambda_1 = -\epsilon$, $\lambda_2 = \frac{1}{4}(n-4)\epsilon/(n-1)$, $2\nu = 1+3n\epsilon/[16(n-1)]$, $\alpha = \epsilon(n-4)/[8(n-1)]$.

Our conclusions are therefore: A) the third fixed point is always stable but can never be reached, since physically Δ must be positive. B) If $n > 4$, the Heisenberg fixed point is stable, in particular, with respect to turning on a small amount of randomness. We interpret this to mean that for $n > 4$, there is a sharp phase transition in the random system with the same exponents as in the homogeneous system. C) For $1 < n < 4$, the random fixed point is stable. At this fixed point Δ is non zero and the exponents differ from those of the homogeneous system. D) for $n < 1$, there is no stable fixed point with u and Δ positive. This presumably corresponds to a transition which is different from the usual second order one. The behavior for n near unity is not well understood yet.

A heuristic argument by one of us[8] predicts that there can be a sharp transition only if the specific heat exponent α is negative. Note that conclusions B and C are in accord with this argument inasmuch as α is negative in both cases. Intuitively, making n large decreases the effect of randomness because the number of degrees of freedom is increased.

A second order (in ϵ) calculation of the stability of the Heisenberg fixed point gives

$$\lambda_\Delta = [(4-n)/(n+8)]\epsilon - [(n+2)(13n+44)/(n+8)^3]\epsilon^2 \quad (7)$$

and to order ϵ^2 we may write this as $\lambda_\Delta = \alpha/\nu$.[9] Thus the Heisenberg fixed point is never stable with respect to randomness when α is positive in agreement with the heuristic argument.

The above results can also be obtained by a formulation due to Emery.[10] In his method one studies the free energy, F_R, of the random model with a Hamiltonian

$$\mathcal{H}_R = \Sigma_{rs} J_{rs} \Sigma_\alpha S_{r\alpha} S_{s\alpha} + v\Sigma_r \Sigma_{\alpha,\beta} S_{r\alpha}^2 S_{r\beta}^2 - \Sigma_r \psi_r\, g(S_{r1}, S_{r2}, \ldots S_{rn}), \quad (8)$$

where r and s are spatial indices, α and β are component labels and are summed from 1 to n, and ψ_r is a random variable governed by the distribution function $P(\psi_r)$. Emery [10] shows that F_R is the same as the free energy F_e associated with the Hamiltonian

$$\mathcal{H}_e = \Sigma_{rs}\Sigma_\alpha\Sigma_k J_{rs}\, x_{r\alpha k}\, x_{s\alpha k} + v\Sigma_r\Sigma_{\alpha,\beta}\Sigma_k\, x_{r\alpha k}\, x_{r\alpha k}\, x_{r\beta k}\, x_{r\beta k} + \Sigma_r\, f[i\Sigma_m\, g(x_{r1k}, x_{r2k}, \ldots x_{rnk})], \quad (9)$$

where $\vec{x}_r$ is a vector variable with components $x_{r\alpha k}$, $f(s) = -\ln[\int P(z)e^{-isz}\,dz]$, the component label k is summed from 1 to m, and the limit $m \to 0$ is taken. We take $g(S_{r1}, S_{r2}, \ldots S_{rn}) = \sum_\alpha (S_{r\alpha})^2$. We now develop recursion relations for Hamiltonians of the form $\mathcal{H}_e$. Since terms of sixth order in x are irrelevant[7], we replace f(s) by its expansion up to order s^2: $f(s) = \frac{1}{2}\overline{\psi^2} s^2$, where $\overline{\psi^2} > 0$ is the average value of ψ^2. Thus the last term in Eq. (9) is of the form $w\Sigma_r(\Sigma_{k\alpha} x_{r\alpha k}^2)^2$. Then the recursion relations for the generalized hypercubic model[11] follow:

$$r' = b^{2-\eta}\{r - A(r)[(4n+8)v + (4mn + 8)w]\} \quad (10a)$$

$$v' = b^{\epsilon-2\eta}\{v - K\ln b[4(n+8)v^2 + 48\, vw]\} \quad (10b)$$

$$w' = b^{\epsilon-2\eta}\{w - K\ln b[8(n+2)vw + 4(nm+8)w^2]\}. \quad (10c)$$

In the limit $m\to 0$, these relations reproduce Eq. (6), if the identifications $v = u$ and $w = -\Delta/8$ are made.[12]

If $V_2(x,x')$, where x is a position coordinate, is constrained to be constant within a p-dimensional subspace, then Δ in Eq. (5b) will be proportional to $\delta^p(\vec{k}_1 + \vec{k}_2)$, where $\vec{k}_1$ is the projection of $\vec{q}_1$ onto the p-dimensional subspace. In this case the recursion relations yield $\Delta' \sim b^{\epsilon+p}\Delta$ and all fixed points are unstable with respect to randomness within the ϵ expansion. This may explain why the "striped" randomness treated in Ref. 2 leads to a broadened transition, whereas the renormalization group treatment given elsewhere[5] suggests a sharp transition. This result also suggests that the transition for $n < 1$ (see D above) may be a broadened one.

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