Random-anisotropy-axis magnet with infinite anisotropy

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We have studied the random-axis magnet with infinite anisotropy by three methods: Cayley-tree approximation, Migdal-Kadanoff renormalization group (MKRG), and Imry-Ma scaling. In the Cayley-tree approximation, by an examination of susceptibilities, it is shown that there exists a competition between the coordination number z and the number of components n of the spins which leads to either ferromagnetic or spin-glass order. Using the MKRG at very low temperature we map out approximately the regimes of the ferromagnetic, spin-glass, and disordered phases as a function of n and the spatial dimension, d. The Imry-Ma arguments are made as an additional method for obtaining information on the critical dimension. Comparisons of these results with the previous literature are made.

I. INTRODUCTION

In this paper we consider the random-anisotropy-axis model (RAM) in the limit of infinite anisotropy, D. The RAM is defined for a d-dimensional lattice of (classical) n-component spins, $\mathbf{s}_i$, of unit length, with (quenched) random-axis directions, $\hat{\mathbf{n}}_i$, and the Hamiltonian

$$H_{\mathrm{RAM}} = -\sum_{\langle ij \rangle} J\mathbf{s}_i \cdot \mathbf{s}_j - D\sum_i (\mathbf{s}_i \cdot \hat{\mathbf{n}}_i)^2 \ . \tag{1.1}$$

Here $\langle ij \rangle$ designates nearest-neighbor pairs and the $\hat{\mathbf{n}}_i$ are all unit vectors. This model was introduced by Harris and co-workers[1-3] in order to explain the magnetic properties of materials such as amorphous $TbFe_2$, and has since been applied to a variety of other intermetallic compounds. This is a very interesting problem because it allows both spin-glass and ferromagnetic ordering at sufficiently low temperature T. To see this, observe that in the case $n=1$, that necessarily $\hat{\mathbf{n}}_i = \hat{\mathbf{x}}$ for all i, Eq. (1.1) gives the Ising model which exhibits ferromagnetic ordering for $T < T_c \sim zJ/k_B$, where z is the coordination number of the lattice. On the other hand, for larger n, no such simplification exists, and the randomness in the $\hat{\mathbf{n}}_i$'s can be enough to induce spin-glass behavior as the Imry-Ma argument[4] shows. Thus, there is a competition between these types of ordering in which qualitatively speaking spin-glass ordering is favored by making n and/or D large and ferromagnetic ordering is favored by making z large. There is, at present, disagreement amongst different methods in describing this competition.

A variety of techniques have been applied to the investigation of the competition between D and J, including mean-field theory[5-8] (MFT), Monte Carlo[9-12] (MC), renormalization-group ϵ expansion[13-16] (ϵRG), position-space renormalization group[17] (PSRG), and $1/n$ expansion.[18,19]

The infinite-range RAM has been solved[5] for all D and J. No spin-glass phase is observed. The ferromagnetic phase transition is observed at exactly the same temperature as the pure n-component spin system. MFT is exact in this limit. On the other hand, the Imry-Ma and ϵRG arguments say that for $d \leq 4$ ferromagnetic long-range order is not possible. The MC calculations have not agreed with each other, but the most recent and detailed[12] have found that the ground states were nonmagnetic.

In the $D \to \infty$ limit of the RAM (the IRAM) the $\mathbf{s}_i$ align along $\pm\hat{\mathbf{n}}_i$ and the problem is equivalent to a modified Ising model with Hamiltonian

$$H = -\sum_{\langle ij \rangle} J_{ij}\sigma_i\sigma_j \ , \tag{1.2}$$

where $\mathbf{s}_i = \sigma_i\hat{\mathbf{n}}_i$ and $J_{ij} = J\hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_j$. We will use Eq. (1.2) to study the IRAM. In the remainder of this paper we will deal with the IRAM. Thomas[20] has solved this model exactly in one dimension for arbitrary D and J. Bray and Moore[17] have applied the large-cell PSRG method for $d=2$ with $n=2,3,10$ and obtained a zero-temperature spin-glass phase.

The degeneracy of choosing $-\hat{\mathbf{n}}_i$ instead of $\hat{\mathbf{n}}_i$ for the ith axis can be removed by specifying the "gauge." The simplest example is to choose a direction $\hat{\mathbf{a}}_i$ (independent of the random orientation) and require $\hat{\mathbf{n}}_i \cdot \hat{\mathbf{a}}_i \geq 0$. The remaining degeneracy (for those axis orientations with $\hat{\mathbf{n}}_i \cdot \hat{\mathbf{a}}_i = 0$) can be fixed in a similar way, but such orientations comprise only an infinitesimal fraction of the choices. Call the vector $\hat{\mathbf{a}}_i$ the gauge for the site i. If the vector $\hat{\mathbf{a}}_i$ is the same for all sites i then call this gauge the uniformly $\hat{\mathbf{a}}$ gauge. However, any such $\hat{\mathbf{a}}$ can be chosen. Furthermore, the overwhelming majority of available gauge choices are not at all spatially uniform. Yet *all* gauge choices are equally valid mathematical descriptions. This implies that ferromagnetic ordering must mean something other than simply a net Ising magnetization, because by randomly choosing another gauge any such magnetization would disappear. The ferromagnetic and spin-glass ordering will be distinguished by thermo-

dynamic properties which are gauge independent.

In Sec. II we will describe the IRAM on a Cayley tree. In Sec. III we will present an analysis of the IRAM by the Migdal-Kadanoff[21,22] position-space renormalization group (MKPSRG) and illustrate the detailed differences between the scaling properties of the spin-glass, ferromagnetic, and paramagnetic phases. In Sec. IV we give arguments of the Imry-Ma type and support these by explicit numerical calculations. In Sec. V we present our general conclusions.

II. THE CAYLEY-TREE APPROXIMATION

In this section we study the IRAM on a Cayley tree. We will compute the mth-order susceptibility $\chi^{(m)}$,

$$\chi^{(m)}=\sum_j \chi_{ij}^{(m)}, \quad \chi_{ij}^{(m)}=\langle [\chi_{ij}(C)]^m \rangle_C \tag{2.1}$$

for $m=1,2$. Here $\langle \ldots \rangle_C$ is an average over configurations C, $\chi_{ij}(C)$ is the susceptibility in the configuration C,

$$\chi_{ij}(C)=\frac{\mathrm{Tr}\, \mathbf{s}_i\cdot\mathbf{s}_j e^{-\beta H}}{\mathrm{Tr} e^{-\beta H}} , \tag{2.2}$$

$\beta\equiv(k_B T)^{-1}$, Tr represents a trace over the spin states, and H is given by Eq. (1.2). $\chi^{(1)}$ is the standard definition of the susceptibility for ferromagnetic ordering, whereas $\chi^{(2)}$ reflects the spin-glass correlations. By looking at the singularities of these quantities one can determine the critical temperature and determine which ordered phase (if any) occurs first as the temperature is lowered.

We apply a standard graphical series expansion technique to evaluate these quantities. After reexpressing the Boltzmann weight

$$e^{\beta J_{ij}\sigma_i\sigma_j}=\cosh(\beta J_{ij})[1+\sigma_i\sigma_j\tanh(\beta J_{ij})] \tag{2.3}$$

and then performing the trace over spin configurations, any expression of the form $\mathrm{Tr}\sigma_{i_1}\sigma_{i_2}\ldots\sigma_{i_m}e^{-\beta H}$, can be expanded graphically. Such an expansion is a sum over all graphs for which the sites i_α, $\alpha=1,\ldots,m$ have an odd number of connected lines, and all other sites have an even number. Each such graph has associated with it the weight $P\prod_{\langle ij\rangle\epsilon G}\tanh(\beta J_{ij})$, where $P\equiv\prod_{\langle ij\rangle}\cosh(\beta J_{ij})$ and G is the set of bonds in the graph. The Cayley tree is simple because there are no closed loops. Thus, only the graph without any lines contributes to $\mathrm{Tr}e^{-\beta H}$ and only the (unique) graph with a single curve connecting i and j contributes to $\mathrm{Tr}\mathbf{s}_i\cdot\mathbf{s}_j e^{-\beta H}$. So

$$\chi_{ij}(C)=\hat{\mathbf{n}}_i\cdot\hat{\mathbf{n}}_j\prod_{r=i+1}^{j}\tanh(\beta J_{r-1,r}) , \tag{2.4}$$

where r indexes the unique path connecting i and j. Let z be the coordination number of the Cayley tree and $\sigma=z-1$. The number of pairs ij separated by k bonds is $z\sigma^{k-1}$. Putting all of this information together,

$$\chi^{(1)}=1+\sum_{k=1}^{\infty} z\sigma^{k-1}C_k , \tag{2.5}$$

with

$$C_k\equiv\left\langle \hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_k\prod_{r=1}^{k}\tanh(\beta J_{r-1,r})\right\rangle_C . \tag{2.6}$$

The lack of loops allows C_k to be evaluated very directly. Define θ and $\hat{\mathbf{n}}_\perp$ by $\hat{\mathbf{n}}_k=\hat{\mathbf{n}}_{k-1}\cos\theta+\hat{\mathbf{n}}_\perp\sin\theta$, where $\hat{\mathbf{n}}_{k-1}\cdot\hat{\mathbf{n}}_\perp=0$ and $|\hat{\mathbf{n}}_\perp|=1$. Then

$$C_k=\left\langle(\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_{k-1}\cos\theta+\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_\perp\sin\theta)\times\tanh(\beta J\cos\theta)\prod_{r=1}^{k}\tanh(\beta J_{r-1,r})\right\rangle_C . \tag{2.7}$$

Because there are no closed loops in this graph the averaging over the last axis can be decoupled from the earlier ones. In addition, the averaging over $\hat{\mathbf{n}}_\perp$ decouples from the averaging over θ. The term containing $\hat{\mathbf{n}}_\perp$ vanishes by reflection symmetry, leaving

$$C_k=C_{k-1}\langle\cos\theta\tanh(\beta J\cos\theta)\rangle_C\equiv C_{k-1}C_1 . \tag{2.8}$$

Hence $C_k=(C_1)^k$ and

$$\chi^{(1)}=\frac{1+C_1}{1-\sigma C_1} , \tag{2.9}$$

which gives a critical temperature $T^{(1)}$ at

$$\sigma^{-1}=\langle\cos\theta\tanh(\beta J\cos\theta)\rangle_C \sim\langle\beta J\cos^2\theta\rangle_C=\beta J/n \tag{2.10}$$

so that $k_B T^{(1)}\sim\sigma J/n$. Of course, this expansion is only valid for $\sigma C_1<1$.

Arguments similar to those leading to Eqs. (2.4) and (2.5) yield

$$\chi^{(2)}=1+\sum_{k=1}^{\infty} z\sigma^{k-1}D_k , \tag{2.11}$$

with

$$D_k\equiv\left\langle(\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_k)^2\prod_{r=1}^{k}\tanh^2(\beta J_{r-1,r})\right\rangle_C =\left\langle(\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_{k-1}\cos\theta+\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_\perp\sin\theta)^2\times\tanh^2(\beta J\cos\theta)\prod_{r=1}^{k-1}\tanh^2(\beta J_{r-1,r})\right\rangle_C \tag{2.12}$$

in the notation of Eq. (2.7). As before, reflection symmetry causes the terms linear in $\hat{\mathbf{n}}_\perp$ to vanish. The remaining terms decouple to produce

$$D_k=\left\langle(\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_{k-1})^2\prod_{r=1}^{k-1}\tanh^2(\beta J_{r-1,r})\right\rangle_C \times\langle\cos^2\theta\tanh^2(\beta J\cos\theta)\rangle_C +\left\langle(\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_\perp)^2\prod_{r=1}^{k-1}\tanh^2(\beta J_{r-1,r})\right\rangle_C \times\langle\sin^2\theta\tanh^2(\beta J\cos\theta)\rangle_C . \tag{2.13}$$

Choose the unit vector $\hat{\mathbf{x}}$ such that $\hat{\mathbf{x}}\cdot\hat{\mathbf{n}}_{k-1}=0$ and $\hat{\mathbf{n}}_0=(\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_{k-1})\hat{\mathbf{n}}_{k-1}+a\hat{\mathbf{x}}$. Then $a^2=1-(\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_{k-1})^2$ and performing the average over $\hat{\mathbf{n}}_\perp$ (designated as $\langle\ldots\rangle_{\hat{\mathbf{n}}_\perp}$) gives

$$\langle(\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_\perp)^2\rangle_{\hat{\mathbf{n}}_\perp}=\langle a^2(\hat{\mathbf{x}}\cdot\hat{\mathbf{n}}_\perp)^2\rangle_{\hat{\mathbf{n}}_\perp}$$
$$=a^2\langle(\hat{\mathbf{x}}\cdot\hat{\mathbf{n}}_\perp)^2\rangle_{\hat{\mathbf{n}}_\perp}=[1-(\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_{k-1})^2]\frac{1}{n-1}\ . \tag{2.14}$$

Inserting these results we obtain

$$D_k=\langle\cos^2\theta\tanh^2(\beta J\cos\theta)\rangle_C D_{k-1}$$
$$+\frac{1}{n-1}\left\langle[1-(\hat{\mathbf{n}}_0\cdot\hat{\mathbf{n}}_{k-1})^2]\prod_{r=1}^{k-1}\tanh^2(\beta J\hat{\mathbf{n}}_{r-1}\cdot\hat{\mathbf{n}}_r)\right\rangle_C$$
$$\times\langle\sin^2\theta\tanh^2(\beta J\cos\theta)\rangle_C\ . \tag{2.15}$$

Setting $E\equiv\langle\tanh^2(\beta J\cos\theta)\rangle_C$ and using $D_1=\langle\cos^2\theta\times\tanh^2(\beta J\cos\theta)\rangle_C$, then Eq. (2.15) can be rewritten as

$$D_k=D_1D_{k-1}+\frac{1}{n-1}(E-D_1)(E^{k-1}-D_{k-1})\ . \tag{2.16}$$

Choosing $g_k=D_kE^{-k}-1/n$, Eq. (2.16) reduces to

$$g_k=g_{k-1}g_1\frac{n}{n-1}=\cdots=\frac{n-1}{n}\left[\frac{g_1n}{n-1}\right]^k\ . \tag{2.17}$$

Reinserting D_k gives

$$D_k=\frac{1}{n}E^k+\frac{n-1}{n}\left[\frac{nD_1-E}{n-1}\right]^k \tag{2.18}$$

and thus

$$\chi^{(2)}=1+\frac{zE/n}{1-\sigma E}+\frac{z}{n}\frac{nD_1-E}{1-\sigma(nD_1-E)/(n-1)}\ . \tag{2.19}$$

This gives the spin-glass critical temperature[23] $T^{(2)}$ at $\sigma E=1$, which yields $k_BT^{(2)}\sim J(\sigma/n)^{1/2}$, so that

$$\frac{T^{(1)}}{T^{(2)}}\sim\left[\frac{\sigma}{n}\right]^{1/2}\ . \tag{2.20}$$

We expect ferromagnetic ordering when $T^{(1)}>T^{(2)}$, (i.e., when $\sigma\gtrsim n$) and spin-glass ordering otherwise. After expressing the averages in the form

$$\langle f(\theta)\rangle_C\equiv\frac{\int_0^\pi f(\theta)\sin^{n-2}\theta d\theta}{\int_0^\pi\sin^{n-2}\theta d\theta}\ , \tag{2.21}$$

the phase boundary between the ferromagnetic and spin-glass regimes can be calculated more precisely by simultaneously solving $C_1=E=\sigma^{-1}$. The results are shown in Fig. 1, where it is seen that $\sigma=n$ is a good approximation to the phase boundary. We stress again that the above results are exact and gauge independent.

We do not expect any other types of thermodynamic singularities for the transition from the disordered phase, although quantities such as $\tilde{\chi}=\sum_j\langle\,|\chi_{ij}(C)|\,\rangle_C$ have to be considered. In the Appendix we discuss the possible relevance of $\tilde{\chi}$.

III. THE MIGDAL-KADANOFF APPROXIMATION

We have investigated IRAM by the MKPSRG method. The general MKPSRG method has been extensively described elsewhere.[24–26] We have used a version which has recently been applied to the Ising spin-glass problem.[26] The procedure has the following steps. Initially, assign axis directions $\hat{\mathbf{n}}_i$ to each site of the lattice according to the uniformly $\hat{\mathbf{x}}$ gauge and compute the nearest-neighbor pair interactions, $J_{ij}=\hat{\mathbf{n}}_i\cdot\hat{\mathbf{n}}_j$. For each MKPSRG recursion, move the interactions on every other row to the remaining rows (see Fig. 2) and then sum over the spin states of those spins which have a low enough bond connectivity (see Fig. 2). This produces a renormalized set of pair interactions $\tilde{J}_{ij}$, for the larger length scale. Repeating the recursion allows the problem to be solved. The choice of the uniformly $\hat{\mathbf{x}}$ gauge was important for our procedure because even when the bond is moved there is no question of the correct sign for the initial pair interaction J_{ij}. The procedure in Fig. 2 is for two dimensions but is easily generalized to arbitrary d. The only change is that the bond-moving step adds together 2^{d-1} distinct interactions.

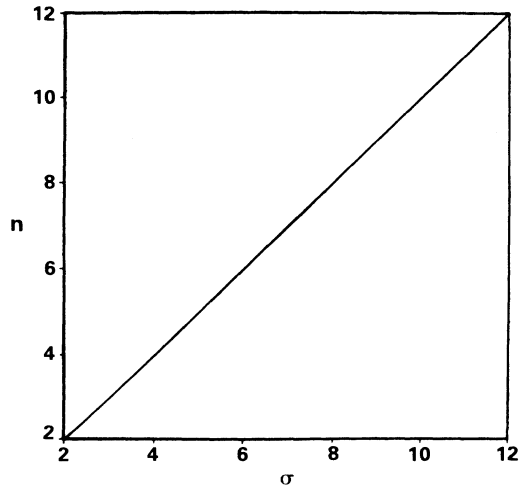

FIG. 1. The value, $\sigma_c(n)$, of σ as a function of n, for which the spin-glass and ferromagnetic critical temperatures coincide. For $\sigma>\sigma_c(n)$ there is a paramagnet to spin-glass transition. On the other hand, for $\sigma<\sigma_c(n)$ the phase transition is into a ferromagnetic phase.

The MKPSRG is applied locally to groups of bonds. Since the bonds are not all necessarily the same, the MKPSRG has to be applied to many such bond configurations to produce a renormalized distribution of bonds. Typically, we considered distributions made up of a pool of 10 000 bonds with configurations carried out over 10 000 samples.

Our main interest is in what happens at very low temperatures. Clearly at high enough temperatures the phase is disordered. However at very low temperature the phase may be (disordered) paramagnetic (PM), spin-glass ordered (SG), or ferromagnetically ordered (FM). These may be distinguished in the following way. In the Ising ferromagnetic phase, the renormalized couplings all become positive and grow as L^{d-1}. It is this low-temperature scaling behavior that yields the Widom scaling relation for the disappearance of the surface tension at

the critical point, viz., $\mu=(d-1)\nu$. The PM phase is characterized by the renormalized couplings decaying exponentially to zero. Physically, this is because distant spins are effectively uncoupled from each other. The SG phase has a striking behavior quite distinct from both the FM and PM phases. An equilibrium SG phase is characterized by increased coupling strength as in the FM phase except that the renormalized bonds are equally likely to be positive or negative. The average value of the pair interactions tends to zero, whereas the average absolute value $A(L)$ grows as the length scale increases. Previous studies of an Ising spin glass (ISG) using the MKPSRG method have shown that $A(L)\sim L^{(d-d_c)/2}$ with $d_c\sim 2.5$.

Thus in the MKPSRG method, the ISG has a lower critical dimension (LCD) of 2.5. The factor of $\frac{1}{2}$ in the exponent arises due to the frustration and randomness present in the spin glass. This distinctive dependence of the characteristic coupling on the length scale leads to a Widom-like scaling relation for spin glasses,[26] viz., $\mu=[(d-d_c)/2]\nu$. A SG phase for $d<d_c$ is characterized by the effective coupling tending to zero (as in the PM case) but only algebraically.

Our analysis of the IRAM at low temperatures by MKPSRG yielded results in accord with those found for the pure Ising SG and FM cases. The value of d_c for the SG case was found to be ~ 2.5. A phase diagram for the low-temperature phases as a function of n and d could therefore be obtained.

We looked at the case $J/(k_BT)=100\,000$, for arbitrary n and d. Our results are shown in Fig. 3. The $n=1$ results are of course those of the Ising model and agree with the facts that the LCD is 1 and that there is no SG phase. For larger n, say $n=5$, as d increases there is a continuous increase in the amount of order and thus all three phases are observed. The critical dimension for SG to FM is very sensitive to the choice of n. However the critical dimension for PM to SG is quite insensitive to the choice of n. For fixed, nonzero J/k_BT the critical dimension for PM to SG must grow, though, because the root mean square of the initial pair interactions is exactly known to be $J/\sqrt{n}$, which vanishes as $n\to\infty$ for any d. For the n's which we show, this critical dimension (PM to SG) is about 2.53, which is in rough agreement with the lower critical dimension obtained for the Ising spin glass.[19]

These results are in disagreement with the ϵRG method because of the FM phase predicted below $d=4$ for some values of n. They do however show the qualitative behavior found from the Cayley-tree approximation.

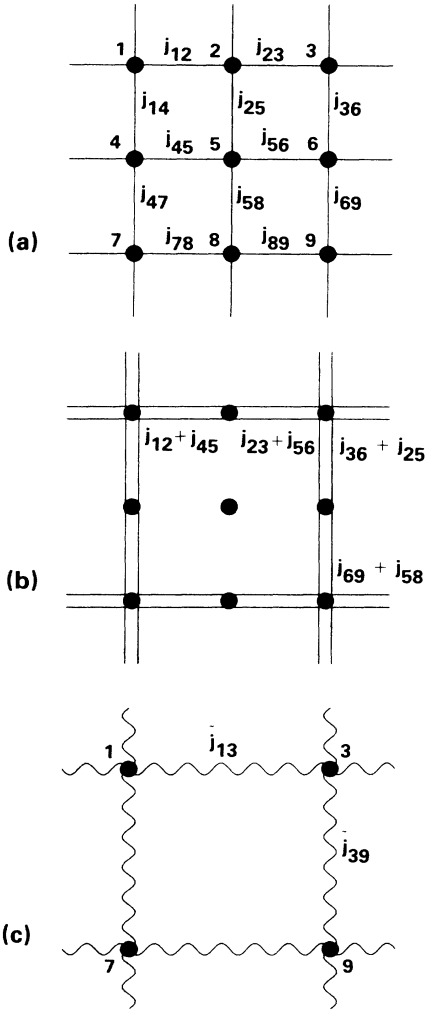

FIG. 2. The two-dimensional MKPSRG: (a) the original lattice, (b) the bond-moved lattice, and (c) the renormalized lattice.

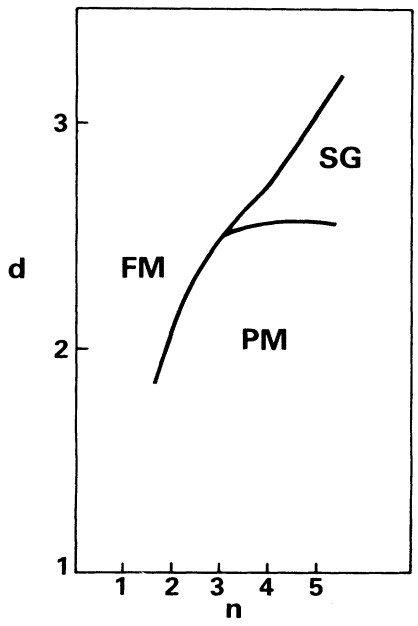

FIG. 3. The d vs n phase diagram of the IRAM in the MKPSRG approximation, at the temperature $J/(k_BT)=100\,000$. PM stands for paramagnetic, SG for spin glass ordered, and FM for ferromagnetically ordered.

IV. AN IMRY-MA ARGUMENT

The MKPSRG approximation yields a LCD less than 4. If this result is not an artifact of the approximation, one reaches a somewhat surprising conclusion of a reentrant behavior (for $d \lesssim 4$) as follows. The system is ferromagnetic in the absence of any anisotropy. On turning on a weak anisotropy, the magnetization is destroyed (since $d<4$). The MKPSRG results suggest that when the anisotropy is made infinitely strong the magnetization returns.

To verify that this result is indeed spurious, we carried out an Imry-Ma[4]–like analysis in the limit of very strong anisotropy. We assume ferromagnetic ordering and ask when such an ordering becomes unstable. The system gains energy by reorienting its magnetization to make optimal use of the random anisotropy. On the other hand, such a reorientation costs energy due to the formation of domain walls. In the weak anisotropy limit, the anisotropy energy gained per spin by having domains of length L goes like $L^{-d/2}$, whereas the domain wall energy per spin $\sim L^{-2}$. Thus in this limit the uniformly magnetized state is unstable to the formation of domains for $d<4$.

Here we adapt the original Imry-Ma argument to treat the case when $D=\infty$. To do this we must formulate an effective anisotropy energy and an effective domain wall energy since the former, in particular, appears only implicitly in the Hamiltonian when $D=\infty$. To be more specific, we considered d-dimensional regions with L lattice spacings in each direction and periodic boundary conditions. Choose an initial set, Ω, of axis orientations and a unit vector $\hat{\mathbf{a}}$. Initialize all the spins to $\sigma_i=1$ in the uniformly $\hat{\mathbf{a}}$ gauge. We then decrease the energy by making single spin flips until no single spin flip will reduce the energy any further. This produces a local minimum whose magnetization points roughly along $\hat{\mathbf{a}}$. The energy of this final configuration is called $\varepsilon(\Omega,\hat{\mathbf{a}})$. Let $\delta(\Omega)$ be the difference between the maximum and minimum values of $\varepsilon(\Omega,\hat{\mathbf{a}})$ as $\hat{\mathbf{a}}$ is varied. $B(L,d)$, the gain in anisotropy energy by having optimally oriented domains of size L in d dimensions, is defined as the average of $\delta(\Omega)$ over the configurations Ω with the size L and dimension d.

To compute the domain-wall energy S, consider the same lattice and axis orientations except that one of the lattice directions, $\hat{\mathbf{x}}$ (the "longitudinal" direction), does not have periodic boundary conditions. Define the $\hat{\mathbf{a}},\hat{\mathbf{b}}$ rotating gauge to be independent of the transverse components of the lattice position, but changing continuously from $\hat{\mathbf{a}}$ to $\hat{\mathbf{b}}$ as one crosses the sample in the longitudinal direction. Initialize the spins to $\sigma_i=1$ in the $\hat{\mathbf{a}},\hat{\mathbf{b}}$ rotating gauge. This produces a configuration whose local magnetization rotates between roughly $\hat{\mathbf{a}}$ and roughly $\hat{\mathbf{b}}$ as one moves longitudinally across the sample. The same single-spin-flip minimization is performed except that the ends in the longitudinal direction are held fixed. If we allowed these spins to flip the boundary would eventually disappear. Let $\omega(\hat{\mathbf{a}},\hat{\mathbf{b}})$ be the resulting energy and $\eta(\hat{\mathbf{a}},\delta\hat{\mathbf{a}}) \equiv \omega(\hat{\mathbf{a}},\hat{\mathbf{a}}+\delta\hat{\mathbf{a}})-\omega(\hat{\mathbf{a}},\hat{\mathbf{a}})$, with $\hat{\mathbf{a}}+\delta\hat{\mathbf{a}}$ being slightly rotated from $\hat{\mathbf{a}}$. Averaging over $\hat{\mathbf{a}}$ and Ω with the amount of rotation kept constant gives the energy $S(L,d)$.

We have performed zero-temperature Monte-Carlo simulations in $d=2,3,4$ for systems containing up to 4096 spins. The averaging was carried out over 50–200 different configurations of the anisotropy axes and over 10–50 different orientations of the magnetic gauge direction. While the $d=2$ and $d=3$ results indicate that the gain in anisotropy energy exceeds the domain wall cost, the $d=4$ simulations are consistent with the energies being equal. The systems studied are unfortunately not large enough to make any definitive statement, but are consistent with an LCD of 4 even in the strong anisotropy limit.

V. DISCUSSION AND CONCLUSIONS

The methods of this paper can be extended to treat the regime $J \ll D < \infty$. To do this one would let

$$\mathbf{s}_i = \sigma_i \hat{\mathbf{n}}_i (1-s_{i\perp}^2)^{1/2} + \mathbf{s}_{i\perp} , \tag{5.1}$$

where $\mathbf{s}_{i\perp}$ is a vector perpendicular to $\hat{\mathbf{n}}_i$. One can integrate out $\mathbf{s}_{i\perp}$ to obtain an effective Hamiltonian in terms of σ_i. This effective Hamiltonian would differ from that in Eq. (1.2) due to the contributions from terms of order $\mathbf{s}_{i\perp}^2$, etc. Thus for $D<\infty$ one has

$$J_{ij} = J_{ij}^{(0)} + J \sum_{k,l>0} a_{ij}^{(k,l)} (\beta D)^{-k} (J/D)^l . \tag{5.2}$$

Thus, within perturbation theory, the symmetry of the Hamiltonian in Eq. (1.2), would remain unchanged, but the coupling constants J_{ij} would be perturbed from their $D=\infty$ values. In other words, the results of the present paper are expected to be qualitatively correct as long as $D \gg J$ and $D \gg k_B T$ and do not rely on the strict limit $D \to \infty$. Physically, this reflects the idea that Heisenberg spins with a large anisotropy should be considered equivalent to Ising spins, as in Eq. (1.2).

We may summarize our conclusions as follows.

(1) For large D, there is a competition within mean-field theory between ferromagnetic order which occurs at low temperature when $z \gtrsim n$ and spin-glass order which occurs at low temperatures when $z \lesssim n$.

(2) The low-temperature MKPSRG shows qualitatively the behavior of mean-field theory as illustrated in Fig. 3. In particular, the spin-glass phase is in the same universality class as that of a pure Ising spin glass with randomly distributed couplings.

(3) The statement listed under (1) probably remains true as long as fluctuations do not play a decisive role in destroying long-range order. From the Imry-Ma argument, as generalized here, we therefore expect this type of competition as long as $d>4$.

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APPENDIX

It may be thought that the susceptibility, $\tilde{\chi} = \sum_j \langle\, |\chi_{ij}(C)|\, \rangle_C$, associated with the "shattered"

field, leads to a higher-temperature divergence than exists for $\chi^{(m)}$. We believe this to be true only for the Cayley tree with free boundaries and not for high-dimensional hypercubic lattices. Independent of this assertion, it is clear that we have compared the stability of two proposed orderings, ferromagnetic and spin glass, and find an interesting crossover from one type to the other as the number of spin components is varied.

On the Cayley tree, $\tilde{\chi}$ is only relevant if the signs of the bonds may be "gauged away" as one works out to the surface. However this may be avoided by a constraint which pins the surface. Because of the anomalously large surface to volume ratio [in the thermodynamic limit, this ratio is $(\sigma-1)/\sigma$] one cannot simply ignore the randomness by pushing it to the surface; the randomness remains. Because our results did not require any similar gauging procedure, they do not suffer from this difficulty. Furthermore, because of the loops, there is no analogous gauging procedure in a d-dimensional hypercubic lattice and thus $\tilde{\chi}$ should not show such a divergence for a hypercubic lattice.

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