

ESSAYS ON INTERNATIONAL FINANCE AND RISK  
SHARING

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## ABSTRACT

### ESSAYS ON INTERNATIONAL FINANCE AND RISK SHARING

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The potential for economic agents to minimize risk through diversification is central to the study of finance. This dissertation analyzes the ability to diversify risks in an international context by studying risk sharing opportunities on two dimensions, consumption growth and portfolio wealth. The first part of this dissertation examines the ability for US based investors to diversify their portfolio risks by incorporating international corporate bonds. Based on a mean variance framework, I study the potential portfolio gains to US investors of holding foreign corporate bonds. Further, I ask if the current observed portfolio holdings match the computed optimal holdings. Using statistical analysis both with and without estimation risk, I find that comparable to the well documented phenomenon in equity markets, a similar home bias can also be found in corporate bond markets. I also investigate the hypothesis that investors can substitute direct investment in foreign corporate bond markets by holding US corporate bonds issued by foreign firms. Contrary to emerging market equity markets, I find that corporate bonds issued by foreign firms that trade in the US are insufficient to capture the gains of directly investment in the foreign corporate bond markets. The second part of this dissertation shifts focus and studies the potential welfare gains for countries to share consumption risks, when there is a small but persistent shock in the consumption growth rate. I find that a long run risk to their consumption growth has the ability to generate large welfare gains from international diversification. Further, when I estimate the model parameters using simulate method of moments, I find a large

disparity in model parameter estimates across my sample of countries, which would lead to large departures in welfare gains away from a equal consumption share.

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# Chapter 1

## Introduction

As markets and economies become increasingly connected, the idea that risks facing economic agents need to be analyzed in a open economy, global framework has become essential. The current financial crisis has highlighted the significant risks that still plague even the most advanced economies, both in terms of financial markets and real consumption. And the ability to potentially reduce these risks by pooling across different countries seems particularly valuable. Therefore, this dissertation focuses on the study of international risk diversification opportunities and gains via two channel: consumption growth and portfolio wealth. In particular, this dissertation analyzes the ability for agents to reduce the volatility by diversifying internationally within a mean variance statistical framework as well as a consumption based model framework. Exploiting data on various financial markets and across different countries, I show that there are significant risk sharing and diversification opportunities for both real consumption and portfolio returns.

The second chapter of this dissertation studies the potential portfolio diversification gains to US investors of holding foreign corporate bonds. In contrast to the vast literature on foreign equity markets, foreign corporate bond markets tend to be less developed and rarely studied. Partly, this lack of research focus has been

caused by data availability. Given the decentralized nature of bond markets, it is a recent phenomenon to have publicly available bond quotes and return data. Using a newly constructed dataset of foreign corporate bond returns and the Markowitz [33] mean variance framework, this chapter analyzes three distinct questions about US investments in foreign corporate bonds: what are the measured portfolio gains of investing in foreign corporate bond markets, does the observed US investor portfolio holdings in foreign corporate bonds match that of the model implied optimal holdings, and lastly, can the gains of direct investment in foreign bond markets be achieved with holding corporate bonds issued by foreign firms that trade in the US secondary market.

With these questions in mind, I show that there are large diversification or risk reduction benefit for US investors to holding for corporate bonds of foreign markets. In addition, contrary to what the literature finds in the emerging economy equity markets, almost none of the benefits can be captured by holding bonds issued by foreign firms in the US bond market. So direct investment in the foreign corporate bond market is necessary. Lastly, I find that despite these apparent benefits, US investors are observed holding much less quantities of foreign corporate bonds than what would be implied by mean variance efficiency. Even accounting for parameter uncertainty, the model predicts portfolio weights in foreign corporate bonds in excess of 25%, whereas, I observe a holding of 6.1% in the data. Therefore, there is strong evidence that a similar phenomenon to the equity home bias also exists, and may potentially be much stronger, in bond markets.

The third chapter departs from the empirical methodology of the second chapter, and studies the implications of consumption based asset pricing model in an open economy risk sharing framework. In particular, I analyze the welfare effects of

international risk sharing when agents are faced with a small but persistent component in their consumption growth. Here I generalize from the closed economy framework developed by Bansal and Yaron [3] and characterize the open economy equilibrium for both prices on consumption claims and consumption allocation. By introducing a small but persistent component in consumption growth or long run risk, Bansal and Yaron [3] showed that they can replicate salient features of the observed macroeconomic data and asset returns in closed economy for the US. In comparison, little is known about how this long run risk would affect the potential gains from international risk sharing.

I find that the inclusion of a small but persistent risk component to consumption growth has the potential to substantially increase the implied welfare gains to international sharing. However, this result depends crucially on the ability for agents to risk share or diversify on the long run risk component. In contrast, if all countries were driven by a common long run risk, and countries can only diversify risk on the idiosyncratic component of consumption growth volatility, then the achievable welfare gains to international diversifying would be much smaller. In addition, I find that even though both preference parameters, risk aversion and intertemporal elasticity of substitution, matter for the degree of risk sharing gains, however, the largest effects to welfare gains come from the magnitude of the long run risk component that can be diversified.

Given the importance of preference parameters and the magnitude of long run risk component, I structurally estimate the model parameters with international consumption data and asset pricing returns. Using simulated method of moments, I first show that in comparison to features of the US consumption and asset pricing data, there is a large heterogeneity across countries. In particular, when fitting the

long run risk model to the data, the large differences across the various countries leads to vastly different parameter estimates.

# Chapter 2

## Diversifying Portfolio Risk with International Corporate Bonds

### 2.1 Introduction

The US corporate bond market serves as a large capital raising market valued at \$11 trillion. And unlike with equities, insurance companies and other financial institutions hold investment grade corporate bonds not merely for return generating purposes, but also for regulatory collateral. While historically these assets have been regarded as low risk, and the question of diversification in this market has been largely ignored by the academic literature. However, given the recent turbulence in the credit markets and dramatic volatility increases in the US corporate bond market, the degree to which investors are subject to either systematic risk or diversifiable risk in this market is of both practical and academic interest.

The importance of US corporate bonds as an asset class and the recent volatility

in the bond market necessitates a better understanding of the types of international diversification opportunities available to US investors and institutions who are exposed to this market. This paper explores the potential benefits of investing in foreign investment grade corporate bonds by addressing three specific questions: What are the potential portfolio gains to investing in foreign corporate bonds for a US investor with exposure to the US corporate bond market? How does the model implied holdings compare with the observed holdings of the US investor? And can US investors use corporate bonds issued by foreign firms that trade in the US to capture the same gains of investing directly in foreign corporate bond markets<sup>1</sup>?

There are potentially many ways to analyze the benefits of holding foreign corporate bonds, I focus on the risk adjusted portfolio gains to a US investor who uses a mean variance framework to optimize over a portfolio of foreign and domestic assets. In this set up, an investor can potentially achieve portfolio gains by holding foreign corporate bonds in two ways, efficiency and diversification. Efficiency gains measure the increases portfolio risk adjusted returns; while diversification gains isolate the mean and focus purely on the reduction in portfolio volatility. Of course, any measure of gains will depend crucially on the US investor's set of benchmark assets. While the international finance literature has traditionally used the US equity market as the benchmark, I want to target the gains of holding foreign corporate bonds beyond what can be achieved in the US bond markets. As such, I assume that the US investor holds three equity portfolios represented by the Fama French portfolios of the US market (mktrf), small minus big (smb), and high minus low (hml), as well as, two US bond market portfolios represented by the excess return on the US 30 year treasury (TERM) and the excess return on US investment grade bonds (DEF).

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<sup>1</sup>The bonds under consideration are known as Yankee bonds, which are US dollar denominated, SEC regulated, and trade on the US secondary bond market.

Using this set of benchmark US equity and bond portfolios, I measure the portfolio efficiency and diversification gains of adding foreign corporate bonds.

In addition to efficiency and diversification gains, the investor's portfolio allocation problem implies a set of mean variance optimal portfolio weights. To investigate the degree to which US investors are capturing these gains, I compare the estimated portfolio weights in foreign corporate bonds against the observed US holdings of 6.1% from the Flow of Funds level tables.<sup>2</sup> However, as argued by Britten-Jones [8], estimates of portfolio weights must be analyzed in the context of the sampling distribution, and can often be statistically insignificant from zero even if the point estimate of the portfolio weight is large. When the estimated weight in the foreign corporate bonds is positive but statistically insignificant, the comparison between observed and implied holdings becomes difficult since it is optimal for the investor to choose any weight between zero and the point estimate.

For estimated weights that are positive but statistically insignificant, one way to pin down the investor's optimal allocation is to analyze the portfolio problem from a Bayesian perspective. In the Bayesian portfolio allocation problem, the investor holds the prior belief that foreign corporate bonds will contribute zero efficiency gains, but holds some uncertainty around the prior belief. Then as the investor's prior uncertainty grows, the investor is less confident that the statistical insignificance is all due to sampling variation, and the positive point estimate for the gain pushes him to put more weight on the foreign corporate bond portfolios. Therefore, as prior uncertainty increases, the implied Bayesian portfolio holdings increase continuously between zero and the mean variance point estimate. Following the methodology outlined in Pastor [36], I assess the degree to which a Bayesian

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<sup>2</sup>For detailed computation see Appendix.

investor must be confident in the prior belief that the US benchmark portfolio is fully efficient to find the observed bond holdings to be optimal.

While the majority of this paper focuses on foreign corporate bond markets, as argued by Errunza et al. [20], investors may be able to capture the gains of investing directly in foreign markets by holding foreign comparable assets that trade in the US. In order to capture this idea of lower cost "home-made" diversification, I extend the previous analysis to test if Yankee bonds<sup>3</sup> can capture any of the gains offered by directly holding foreign corporate bonds. Adding Yankee bond portfolios to the US benchmark assets, I test if there are still efficiency gains to be achieved by investing directly in foreign corporate bond markets. To better understand why Yankee corporate bonds may or may not provide the same benefits as investing directly in the home markets, I test if Yankee bond returns can be spanned by US benchmark assets and analyze the sensitivity of Yankee bonds to the US corporate bond market versus their home corporate bond market.

To implement the analysis described above, I construct a new dataset of monthly firm level corporate bond quotes for the available markets of Australia, Canada, Europe, Japan, UK, and the US. Based on the index constituent list of Merrill Lynch corporate bond indices for Jan 1997 - Dec 2008, I construct clean country bond indices aggregated from the firm level, with only senior unsecured corporate bonds issued by firms that are domiciled in the given market. Further, to limit the effects of foreign exchange return dynamics and focus primarily on corporate credit risk diversification, I hedge portfolio returns using one month forward rates and analyze hedged monthly holding period returns for each country index. This hedging strategy replicates the methodology implemented by the original unfiltered

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<sup>3</sup>Similar to cross-listed equities (or ADRs), Yankee bonds are US dollar denominated, registered with the SEC with full disclosure, and trade in the US secondary bond market.



Merrill Lynch indices<sup>4</sup>. Lastly, since all gains are from the perspective of the US investor, I compute excess returns over the US risk free rate.

The main findings of this paper can be summarized as follows. First, I find that when all the foreign corporate bonds are pooled together, they provide statistically significant risk adjusted gains to the US investor. On the other hand, when country corporate bond portfolios are tested one at a time against the US benchmark, only Japan provides statistically significant efficiency gains of 1.8% per year. This result, however, does not preclude the US investor from wanting to hold a large portion of their portfolio in foreign corporate bonds. When I account for the estimation risk faced by the US investor using a Bayesian framework, the implied weight in the foreign corporate bond portfolio is always in excess of 25% with a 1% prior uncertainty. Second, for pure risk reduction and portfolio diversification gains, I find that foreign corporate bonds have the potential to provide economically large and statistically significant gains. Computed as the variance reduction to the minimum variance portfolio, portfolio diversification gains can be as large as 77% in sample. Moreover, the out of sample risk reduction for the minimum variance portfolio is always positive relative to the US benchmark, and would have decreased portfolio volatility by 41% in the most recent crisis episode. Third, I show that including Yankee bonds in the US benchmark portfolio does not alleviate the need to invest directly in the foreign assets to capture diversification gains. It also does not materially lower the implied holdings in foreign corporate bonds. The reason why Yankee bonds do not provide more gains is that their returns follow closely the dynamics of the US corporate bond market and are much less sensitive to their home corporate bond indices.

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<sup>4</sup>For a recent analysis of investment opportunities in foreign currency markets see Campbell, De Medeiros, Vicera [14].

This paper is closely related to the literature on international equity portfolio diversification and leverages the methodology from the domestic finance literature on the efficiency of the market portfolio. The methodology used in this paper most closely resembles that of the Huberman and Kandel [24] paper analyzing the efficiency of the market portfolio relative to size portfolios in the US market. Using this methodology, the international finance literature has produced a long line of research examining the efficiency and diversification benefits of investing in both advanced economy and emerging market equities markets. Papers such as Jorion [24], DeSantis [17], Bekeart and Urias [7] showed that emerging market equities consistently provide efficiency gains to the US investor. Looking at advanced economies, Britten-Jones [8] showed that even for large implied portfolio weights on foreign equities, weights are not statistically different from zero when the sampling distribution is considered. Further, as demonstrated by Errunza et al. [20], a combination of ADRs, Multinationals, and Country Funds, can span emerging market returns, allowing the investor to capture mean variance efficiency gains at lower transaction costs. More recently, Rowland and Tesar [37] find that multinationals firms do provide significant diversification benefits, but do not exhaust all the gains from holding the international market index. However, the international finance literature that considers diversification benefits to sovereign or corporate bonds has been fairly thin. It is only recently that the literature has extended into the credit markets. The closest study to my own is the working paper by Longstaff, Peddersen, Pan, and Singleton [32], which examines portfolio efficiency gains to investing in emerging market sovereign credit default swaps. In contrast, I focus on the corporate bond markets and explore different types of gains as well as portfolio holdings with and without estimation risk.

## 2.2 The Portfolio Problem

Consider a one period portfolio allocation problem where the investor must choose an allocation between a risk free asset and (N+K) risky assets. The universe of (N+K) investable assets can be partitioned into K benchmark assets, referred to as the US Benchmark assets, and N foreign test assets. Given the investor's initial wealth,  $W_0$ , and the returns on the risky assets, the investor will choose the weights that maximize his period 1 expected utility. The investor's problem can be written as:

$$\text{Max}_{\{w_N, w_K\}} E[u(W_1)] \quad (2.1)$$

subject to the budget constraint:

$$W_1 = W_0 * (1 + \tilde{r}_p) \quad (2.2)$$

and

$$\tilde{r}_p = r_f + w'_N * \tilde{r}_N + w'_K * \tilde{r}_K \quad (2.3)$$

where  $r_f$  is the risk free rate,  $\tilde{r}_p$  is the portfolio return,  $w_N$  and  $w_K$  are column vectors of portfolio weights on the benchmark and test assets respectively, and  $\tilde{r}_N, \tilde{r}_K$  are column vectors of excess return on the benchmark assets and test assets respectively.<sup>5</sup>

In general the solution of portfolio choice problem will depend on higher order moments of the asset return distribution. However, if the risky assets are assumed to have normally distributed rates of return, the the portfolio return will also be normally distributed, which can be summarized in the first two moments of the

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<sup>5</sup>Weights and returns will be vectors if there are multiple benchmark or test assets

distribution.<sup>6</sup> Then, for any arbitrary utility function that exhibits monotonicity and strict concavity, the investor will always choose a portfolio such that he can achieve a higher mean and a lower variance.

It is important to point out that in this economy, the investor is faced with no additional constraints other than his initial wealth constraint. Therefore, it is assumed that the markets are frictionless and the investor can take limitless short-sale positions. Further, the investor is not faced with any additional costs such as transaction costs or taxes.

To take this portfolio allocation problem to the data, I must make an assumption on the universe of investable assets available to a US investor. As the goal of this paper is to test the gains from investing in foreign corporate bonds, the N test assets will be the foreign corporate bond portfolios, to be described in detail in the next section. However, one can imagine many possible sets of assets that could serve as benchmark assets for the US investor. A natural starting point is to include the US equity market portfolio (mktrf). Furthermore, motivated by the works of Fama and French [21], I also include the zero cost portfolios of small minus big (smb) and high minus low (hml). In addition to the US equity market portfolios, any gains to holding foreign corporate bond should be in excess of what can be achieved simply by holding the US corporate bond market. Therefore, I also include two US bond market assets in the benchmark assets, which are the excess return on the 30 year US treasury (TERM) and excess return on the US investment grade corporate bond index (DEF). All together, I assume that the US investor holds as benchmark assets that include the three Fama French equity portfolios and two US bond market assets.

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<sup>6</sup>Multivariate normality is sufficient, not necessary, for investors to choose mean variance efficient portfolios. For details and more general conditions, see Huang and Litzenberger [22].

## 2.3 Data Summary

To analyze the benefits of including foreign corporate bonds in the US benchmark portfolio, time series of foreign corporate bond market returns are required. Using the data from Merrill Lynch investment grade corporate bond indices as the base data<sup>7</sup>, I collect monthly constituent list of bond indices from the following markets: Australia, Canada, Europe, Japan, UK and the US<sup>8</sup>. The monthly data spans the period of Jan 1997 - Dec 2008 for the US, Canada, and UK, and Jan 1999 - Dec 2008 for Europe, and Jan 2000 - Dec 2008 for Australia. From the total pool of bonds, I eliminate any bond that is not considered Senior and Unsecured debt, or issued by a quasi-government institution.

Then, for each country index, I eliminate any bond that is issued by a firm that is domiciled outside of that country. This filtering method eliminates the effects of cross-listings, which may obscure the true investment opportunities of holding the country's bonds. For example, if there were many non-Japanese companies listed on the Japanese bond market, then the Merrill Japanese Bond index would not properly reflect the credit risk of Japanese companies. Further, in order to analyze the ability of Japanese Yankee bonds to substitute for direct investment in Japanese corporate bonds, it is essential that the bonds are comparable in the domicile of the firm. Given these reasons, the country indices that I construct contain only firms domiciled in the market. This specification is also consistent with the MSCI index for equities. Therefore, rather than using the Merrill Lynch corporate bond indices directly from Bloomberg, I use the country corporate bond indices constructed with

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<sup>7</sup>For inclusion in the indices, all bonds must be investment grade bonds, have a minimum par requirement, one year or more left to maturity, and a fixed coupon. See Merrill Lynch Rules 2000 for details.

<sup>8</sup>Europe includes Belgium, France, Germany, Italy, Netherlands, Switzerland

the above filters.

Table 1 summarizes the corresponding clean observations for each country bond portfolio. The number of observations is the total number of bond quotes for the entire sample period. The US corporate bond index has the most observations for the 1997 - 2008 sample period at 352,552 monthly bond quotes. In addition, the US market also has the largest number of bonds and issuing firms at 9224 bonds issued by 1251 firms. In comparison, Japan has 2153 bonds, but issued by only 164 firms. In general, each Japanese firm issues more bonds and at shorter maturity so that the bond turnover is large. At the opposite extreme with few bonds per firm, the UK corporate bond market has a total of 535 bonds issued by 189 firms. In addition to the total number of clean observations, Table 1 also reports the number of observations in sub-categories by rating and industry. By ratings, the majority of bonds are rated A or BBB, and accounts for over 60% of bonds in every markets. Not surprisingly, across the industry breakdown, financial firms are the heaviest issuers of corporate bonds across all markets and make up anywhere from 41% to 71% of the investment grade bond markets.

Using the constructed set of firm level bond quotes, I re-weight the local currency bond returns using the Merrill Lynch index weights<sup>9</sup>, and form clean country corporate bond index returns denominated in the local currency. Since all portfolio gains will be from the perspective of a US investor, I translate all currency bond returns into US dollars returns using foreign exchange rates from Datastream. Unhedged returns are converted using the month end spot rate, while hedged returns are computed using a 1 month forward rate on the current bond value and expected accrued interest, and spot rate on any bond value price changes.<sup>10</sup> Since

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<sup>9</sup>Merrill Lynch index weights are based on par, so these will be value weighted portfolios

<sup>10</sup>This leaves some basis risk on the realized changes in bond value. But these changes are

the focus of this paper is on the investment and diversification opportunities in the credit markets, I want to isolate the core credit returns from the foreign exchange dynamics. Therefore, going forward, all returns referenced in this paper are hedged returns, which limits the effect from currency exposure. In the robustness section, I will present the results of the diversification gains using unhedged returns, which combines the effect of foreign currency exposure and corporate credit risk.

The remainder of the data will come from the standard sources. For foreign equity index returns, I use MSCI total country equity index returns in local currency available on Datastream, and convert it into dollar hedge and unhedged returns in the same way as described earlier for the foreign corporate bond returns. Further, for the US benchmark factors, I use the Fama French portfolio returns available from WRDS, and the risk free rate from and the return on the fixed term 30 year Treasury bond from CRSP. All data is sampled for the period of Jan 1997 - Dec 2008, which corresponds to the data period for the corporate bond portfolios.<sup>11</sup>

### 2.3.1 Sample Statistics

I begin with a brief examination of some time series properties of the newly constructed corporate bond dataset. Because the primary empirical methodology is confined to a mean variance framework, I focus on the mean and standard deviation of the corporate bond returns as well as the correlation of the returns across countries. In addition, I compare the differences in hedged versus unhedged returns, as well as, equity versus corporate bond returns.

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generally small and therefore limits the foreign exchange exposure. I follow the hedging calculation used by Merrill Lynch, and use the same calculation with MSCI local equity returns to get hedged equity returns. For the detailed calculation, see the Appendix

<sup>11</sup>The available longer sample for the US benchmark assets can be exploited as detailed in Stambaugh [38], and will be analyzed in detail with further research.

Table 2 compares the summary statistics for both hedged and unhedged returns across the different asset markets. While equity hedged and unhedged returns are comparable in terms of the mean and volatility of the returns, there is a much more noticeable difference between hedged and unhedged returns for bonds. The inclusion of the foreign exchange risk dramatically increases the volatility of bond returns. In particular, unhedged bond returns often have double the volatility of their hedged counterpart.<sup>12</sup> Looking across the hedged returns for the different bond markets, mean return differences are small, while variation in return volatility is much larger. In particular, the US corporate bond portfolio has the highest annualized standard deviation at 5.42% per year as compared to the other advanced economy corporate bond markets whose return volatility ranges from 2.01% per year for Japan to 4.47% per year for the UK. In addition to the first two moments of the return distribution, Table 2 also reports the first order autocorrelation of returns. While large estimates of first order autocorrelation might imply stale data, I show that the first order autocorrelation for the constructed bond returns is comparable to the equity returns autocorrelation from the MSCI indices, which has been well studied and used in the international finance literature.

In addition to the all investment corporate bond portfolios, I subdivide country bond portfolios into groupings with the following characteristics: long maturity corporate (10+ years to maturity), intermediate maturity corporate (6-10 year maturity), and short maturity corporate (3-5 year maturity), industrial sector issues and financial sector issues<sup>13</sup>. Table 3 shows the annualized mean and standard deviations for hedged dollar returns for the country bond portfolios and sub-portfolios

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<sup>12</sup>This is similar to the finding in Berger and Warnock [9]

<sup>13</sup>There are generally not enough bonds to partition by rating and maturity, and out of the two, maturity tends to be a more dominant factor



partitioned by maturity and industry. The top panel of Table 3 repeats the hedged returns shown in Table 2 for the portfolio with all investment grade bonds. The second panel of Table 3 reports the return statistics of the portfolios across different maturity horizons. Not surprising, for every country, the volatility of the long maturity bonds are higher. Particularly, in the case of the US, the annualized standard deviation of the short term corporate bonds is 3.68%, while the long maturity bonds have a annualize volatility of 9.40%. The third panel of Table 3 outlines the returns for industry breakdowns, where differences across countries seem to be minimal for the first two moments of the return series.

While the individual asset means and variances are important for the mean variance analysis that is to follow, the portfolio variance is also heavily influenced by the correlation of across assets. Table 4 reports the correlation of hedged returns for the country level all corporate bond indices and equity indices. Comparing the top and bottom panels of Table 4, the correlation for these developed economies is some times much lower for the corporate bond markets than for the equity markets. The pairwise correlation for equity markets is always greater than 50%, while correlation for corporate bond returns can be as low as 7%. As an example, the Australian corporate bond portfolio has a 36% correlations with the US corporate bond market, whereas the Australian equity market returns are correlated with the US equity market at 69%. Since both equity and bond returns are converted to hedged dollar returns in the same way, the lower correlation are driven by the dynamics of the underlying market.

## 2.4 Mean Variance Efficiency Gains

This section explores the portfolio gains to including foreign corporate bond with the return series described above. As motivated earlier by the mean variance investor portfolio problem, the investor will choose a combination of risky assets such that it maximizes his portfolio Sharpe ratio<sup>14</sup>. In other words, the test asks if adding foreign corporate bonds can significantly increase the portfolio Sharpe ratio, or the mean variance efficiency of a portfolio with the 5 US benchmark assets.

Another way to analyze the Sharpe ratio contribution of the foreign corporate bonds to the US portfolio is to ask if the maximal portfolio Sharpe ratio with and without the foreign bonds are equivalent. To test the null hypothesis that the maximal portfolio Sharpe ratio is the same with and without a test asset, I can equivalently run a regression with the test asset as the dependent variable and the benchmark assets as the independent variables and test for a zero intercept condition.<sup>15</sup> Here the test assets will be the set of foreign corporate bond indices and the benchmark assets will be the Fama-French 5 US assets discussed in Section 2.3.

To start, I test the efficiency gain of the univariate case and test each foreign corporate bond index one at a time against the US benchmark. To test for the zero intercept restriction, I analyze the coefficient and t-statistic on the intercept term, or alpha, in the below regression equation:

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<sup>14</sup>Sharpe ratio is defined as  $\mu_p/\sigma_p$ , where  $\mu_p$  is the mean portfolio excess return and  $\sigma_p$  is the standard deviation of the portfolio excess return

<sup>15</sup>For detailed discussion of the equivalence, see Jobson [26], Jobson and Korkie [27]

$$r_t^{For} = \alpha + \beta_1 * mktrf_t + \beta_2 * smb_t + \beta_3 * hml_t + \beta_4 * TERM_t + \beta_5 * DEF_t + e_t \quad (2.4)$$

where  $r_t^{For}$  is the excess return on the foreign bond index. In addition,  $mktrf_t$  is the excess return on US equity market,  $smb_t$  is the return on small minus big portfolio,  $hml_t$  is the return on high minus low portfolio,  $TERM_t$  is the excess return on the 30 year treasury,  $DEF_t$  is the excess return on the US corporate bond index, and  $e_t$  is an error term.

Table 5 shows the result for the above excess return regressions for each of the foreign investment grade indices individually with the corresponding t-statistic for each coefficient. The top row labeled "Alpha" reports the intercept coefficient,  $\alpha$ , which measures the gain in portfolio Sharpe ratio of including foreign corporate bonds. When I include each country separately into the US benchmark of Fama-French 5 factors, most of the  $\alpha$  estimates are statistically insignificant at the 10% level with the exception of Japan, which is highly significant with a t-statistic of 3.25. Economically, however, the point estimates of the intercepts suggest that including foreign corporate bonds can increase portfolio Sharpe ratios anywhere from -0.04% to 0.15% per month, or an annualized rate of -0.48% to 1.8% per year. In particular, Japan provides the highest Sharpe ratio increase at 1.8% per year and is highly statistically significant. Moreover, Canada is narrowly rejected with a potential increase to the US benchmark Sharpe ratio of almost 1% per year.

Table 5 also reports the adjusted  $R^2$  from the regression in Equation 2.4, which varies quite dramatically depending on the country test portfolios. The lowest adjusted  $R^2$  is on the Japan corporate bond portfolio where the US benchmark asset

returns can only explain about 4% of the variation. Interestingly, Canada has an adjusted  $R^2$  of 62%, which implies that the US benchmark portfolio explains a large portion of the Canadian corporate bond return dynamics, and yet positive potential efficiency gains are narrowly rejected.

In terms of loadings on the US benchmark assets in Equation 2.4, Table 5 illustrates that most countries' corporate bond portfolios load statistically significantly on the US bond market factors of TERM and DEF. Recall, DEF is just the excess return on the US corporate bond portfolio. The one anomaly is Japan, which seem to have insignificant loading on all the US factors. There does not seem to be a consistent pattern on how foreign corporate bonds load on the US equity factors of mktrf, smb, and hml. In particular, Canada and UK corporate bond portfolios move together with the US equity market, while Japan, Europe and Australia have negative co-movements with the US equity market.

### **2.4.1 Mean Variance Efficient Portfolio Holdings**

The above section outlines the potential gains that could have been achieved by including each country corporate bond portfolio individually in the US benchmark. However, as established in the portfolio choice problem earlier, the investor must choose the weights in the portfolio to achieve any potential gains. This section will explore the tangency portfolio weights in the foreign corporate bonds that are implied by the potential gains found in the previous section. Using the methodology from Britten-Jones [8] to derive a sampling distribution for the portfolio weights, I also test if the foreign weights are statistically different from zero.

The weights of the mean variance efficient tangency portfolio, for  $N$  test asset and  $K$  number of US benchmark assets, can be constructed with the following equation:

$$w = \frac{\Sigma^{-1} * \mu}{i'_{K+N} * \Sigma^{-1} * \mu} \quad (2.5)$$

where  $w$  is a  $(K+N) \times 1$  vector of portfolio weights,  $i_{K+N}$  is a  $(K+N) \times 1$  vector of ones,  $\mu$  is  $(K+N) \times 1$  mean excess return vector, and  $\Sigma$  is an  $(K+N) \times (K+N)$  covariance matrix.

Table 6 reports the mean variance portfolio weights of the tangency portfolio that includes the US benchmark assets and the specified country corporate bond portfolio. For example, the tangency portfolio with the US benchmark and the Australian corporate bond portfolio implies an allocation of 46% in the Australian bond portfolio, 5% in the US mktrf, 17% in smb, and so on. Looking across the different country corporate bond portfolios in the top panel of Table 6, the point estimates for the implied portfolio weight in the foreign corporate bonds can be large, ranging from -26% for the UK to 81% for Japan.

While the implied portfolio weights in the foreign corporate bond portfolios seem large, as argued by Britten-Jones [8], the point estimate must be taken in context of the sampling distribution. In particular, if there is a lot of sampling variation in the estimate of the tangency portfolio weights, the implied weight might not be statistically different from zero. Table 6 also reports the corresponding t-statistics of the weight in the foreign corporate bond portfolio under the point estimate of portfolio weights. Of the five country corporate bond portfolios, only Japan has a statistically significant weight with a t-statistic of 3.25. The implied tangency portfolio holding in this case is to allocate 81% of the portfolio weight in the Japanese foreign corporate bonds.

The fact that only Japan has a statistically significant weight is not particularly surprising, since from the excess return regression results in Table 5, Japan is the

only country corporate bond portfolio that provides statistical significance Sharpe ratio gains. However, for the other countries the statistically insignificant weights in the country corporate bond portfolios makes it difficult to compare to observed foreign corporate bond holdings of the US investor. In particular, if the weight in the foreign corporate bond portfolio is statistically insignificant, then in a classical hypothesis testing framework the US investor can interpret it as optimal to hold any weight between zero and the point estimate. One way to get around the difficulty in interpreting the insignificant weights is to use a Bayesian portfolio allocation framework.

## 2.4.2 Estimation Risk and Bayesian Portfolio Holdings

In the classical estimation framework, the investor is assumed to know the true parameter values in Equation 2.4 and the statistical insignificance of the intercept from zero is driven all by sampling variation. However, in reality, the investor may be uncertain about the true parameter values in Equation 2.4, and will need to make portfolio allocation decisions taking into account estimation risk. In the Bayesian framework, the investor holds a prior belief about the true underlying parameter. But this prior is not just one parameter value, but rather a distribution where the variance of the distribution represents the perceived uncertainty of the investor.

The null hypothesis used in Section 4.1 test for the hypothesis that  $\alpha$  in Equation 2.4 is equal to zero. This does not however take into account parameter uncertainty or estimation risk. In the Bayesian framework, the investor's prior belief will be centered around that idea that the true parameter of  $\alpha$  is zero, but that there some uncertainty about the true parameter value of zero represented by the prior variance,  $\sigma_\alpha^2$ . Economically, this has the interpretation that the investor has a prior that is

centered around the belief that the US benchmark is fully efficient, and there is no Sharpe ratio gain to including foreign corporate bonds. However, the investor holds some uncertainty around this belief as represented by  $\sigma_\alpha^2$ . I will compute the implied portfolio holdings, varying this degree of uncertainty, and compare the implied holdings to the observed 6% portfolio holdings of foreign corporate bonds.

Following the methodology outlined in Pastor [36], let  $\theta = (\alpha, B_2, \sigma^2)$  be the parameter vector of Equation 2.4, where  $\alpha$  is the regression intercept,  $B_2$  is the vector of loadings on the US benchmark portfolios, and  $\sigma^2$  is the variance of the regression residuals. As motivated earlier, the prior on  $\alpha$  will have mean zero, and variance  $\sigma_\alpha^2$ . For the other parameters,  $B_2$  and  $\sigma^2$ , the prior will be the estimated from a prior estimation period of Jan 1997 - Dec 1997, and have arbitrarily large prior variances to capture the idea that the investor stands uninformed about the other parameters. Therefore, the likelihood of the parameters given the data,  $L(\theta|\Phi)$ , will be formed over the period Jan 1998 - Dec 2008. The posterior distribution of the parameters is then:

$$p(\theta|\Phi) \propto p(\theta)L(\theta|\Phi)$$

where  $p(\theta)$  is the prior distribution over the parameter estimates. The above equation simply says that the posterior distribution  $p(\theta|\Phi)$  is a combination of the prior belief and the likelihood estimates from the data.

To sample from the posterior distribution,  $p(\theta|\Phi)$ , I use the Gibbs sampler and exploit the ease of sampling from the conditional distribution of  $p(B|\Phi, \sigma^2)$  and  $p(\sigma^2|\Phi, B)$ , where  $B = (\alpha, B_2)$ . I initiate the Gibbs sampler using an estimate of  $\sigma^2$  from the prior estimation period of Jan 1997 - Dec 1997. With the initial estimate of  $\sigma^2$ ,  $s^2$ , I draw a vector  $\hat{B}$  from the distribution of  $p(B|\Phi, s^2)$ . Then given the

draw of  $\hat{B}$ , I draw a new  $s^2$  from the marginal distribution  $p(\sigma^2|\Phi, \hat{B})$ . By sampling continuously in this way, the draws converge and are then made as if they were from the joint posterior distribution of  $p(B, \sigma^2|\Phi)$ . To eliminate the effects of the initialization of the Gibbs sampler, I discard the first 1,000 draws. So in total, the Gibbs sampler produces 300,000 draws from the joint posterior distribution, less the first 1,000.

Using the posterior means of the parameters and draws from the predictive density of benchmark returns<sup>16</sup>, I use Equation 2.4 to draw from the predictive distribution of foreign corporate bond return,  $r_{t+1}^{FOR}$ . Given draws of the predictive returns,  $(r_{t+1}^{For}, mktrf_{t+1}, smb_{t+1}, hml_{t+1}, TERM_{t+1}, DEF_{t+1})$ , I compute the mean and variance of the predictive distribution,  $\tilde{E}$  and  $\tilde{V}$ . Then the mean variance optimal weights are, where  $i$  is a column vector of ones:

$$w = \frac{\tilde{V}^{-1} * \tilde{E}}{i' * \tilde{V}^{-1} * \tilde{E}} \quad (2.6)$$

Intuitively, if  $\sigma_\alpha = 0$ , the Bayesian investor is perfectly confident that the true parameter of  $\alpha = 0$  and is dogmatic that the US benchmark portfolio is fully efficient. In other words, no additional Sharpe ratio gains can be achieved by adding foreign corporate bonds into the US benchmark portfolio. In this case, the investor would choose to hold no foreign bonds. However, as the prior uncertainty on the true parameter value of  $\alpha$  grows, the Bayesian investor entertains the idea that the true parameter of  $\alpha$  may not be zero, and considers the positive point estimate as capturing both sampling variation and estimation risk. And, as the uncertainty on the true parameter value of  $\alpha$  increases, the investor considers more the positive

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<sup>16</sup>Detailed methodology is outlined in Appendix



intercept estimate from the likelihood and put increasing weight on the foreign corporate bond index. Finally, when  $\sigma_\alpha = \infty$  and the investor has a completely diffuse prior, the Bayesian portfolio allocation will rely solely on the likelihood estimate, which is the point estimate implied by the mean variance portfolio weight.

Table 7 presents the Bayesian portfolio weights on the foreign corporate bond portfolio as the prior variance on  $\alpha$  increases from zero to 10% per annum. At low 1% prior uncertainty on the true parameter of  $\alpha$ , the Bayesian investor would already choose to hold 37% of the portfolio in the Australian corporate bond. The remainder of the portfolio is allocated among the US benchmark portfolios, which is suppressed from the table for clarity. For all three countries (Australia, Canada, and Europe) that had positive but statistically insignificant intercept estimates, at 1% prior variance, the Bayesian portfolio allocation already implies a foreign holding in the range of 25% to 57%<sup>17</sup>. This implies that even with a small amount of estimation risk, a Bayesian investor would invest a reasonably large portion of his portfolio in foreign corporate bonds.

To put these weights in context with existing literature on foreign equity holdings, I report in the last panel of Table 7 the implied Bayesian portfolio weights from Pastor (2000) of foreign equity markets<sup>18</sup>. He finds that at 1% prior variance, the implied holding of foreign equity is only 7% of the portfolio. The conclusion is that in order to match the observed 8% holdings in foreign equities of US investors, the implied prior variance must be tight around 1% per annum<sup>19</sup>. In comparison, the

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<sup>17</sup>While 1% prior uncertainty was described as very tight by Pastor, the variance for bond portfolios are significantly less than equity portfolios. A more comparable "tight" prior on bonds will certainly be a much smaller prior uncertainty and proportional to the expected variance of the bond portfolio.

<sup>18</sup>The US benchmark used in Pastor 2000 is the VW NYSE and the foreign equity asset is the MSCI Morgan Stanley World-Except US portfolio (WXUS)

<sup>19</sup>8% holding of foreign equities can be found in Lewis (1999), and referenced in Pastor (2000)

weight in the foreign corporate bond indices is at 25% portfolio holding in foreign corporate bonds at the 1% prior variance level. This is because, the equity mean variance efficiency result in Pastor (2000) had a much weaker statistical significance than the analysis with foreign corporate bonds shown earlier. As an example, recall that the intercept on Canadian corporate bonds from Table 5 was narrowly rejected with a t-statistic is 1.50, whereas, Pastor (2000) reports a t-statistic on the intercept of foreign equities on the US equity market of 0.64. Therefore, a Bayesian investor must hold a much stronger confidence in the US benchmark portfolio to justify a similar percentage in the foreign corporate bonds.

### 2.4.3 Pooling all countries together

The previous sections have tested the mean variance efficiency gains and holdings when individual country corporate bond portfolios are added to the US benchmark assets. As I have demonstrated, individual country corporate bond portfolios do not provide statistically insignificant efficiency gains to the tangency portfolio, except in the case of Japan. However, countries can individually provide statistically insignificant gains, but still provide efficiency gains to the US investor jointly. This hypothesis can be tested by asking if all the intercept terms in Equation 2.4 are jointly statistically different from zero. Under the null hypothesis that all  $\alpha$ s are jointly equal to zero, the following J-statistic is unconditionally distributed with central F-distribution with N and T-N-K degrees of freedom<sup>20</sup>, where N is the number of test assets, K is the number of benchmark assets, and T is the length of time observations:

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<sup>20</sup>See Equation (6.2.12) of Campbell, Lo, and MacKinlay [13]

$$J_1 = (T - N - K)/N[1 + \hat{\mu}_k * \hat{\Omega}^{-1} * \hat{\mu}_k]^{-1}(\hat{a} * \hat{\Sigma}^{-1} * \hat{a}) \sim F(N, T - N - K) \quad (2.7)$$

where  $\hat{a}$  is the vector of estimated alphas,  $\hat{\mu}_k$  is the mean estimate of the benchmark assets,  $\hat{\Sigma}$  is the variance covariance matrix of the regression residuals, and  $\hat{\Omega}$  is the maximum likelihood estimate of variance covariance matrix for the benchmark asset portfolio returns.

Table 8 shows the results of the above J-statistic, with the corresponding p-values for the F distribution. The first column labeled "All" is the J-statistic for the joint test that includes all the country portfolios. The p-value for all the country corporate bond portfolios is 2%, which implies that the test rejects the hypothesis that all alphas are zero at the 5% confidence level. However, at the 1% confidence level, the hypothesis that all intercepts are jointly zero can not be rejected.

The remaining columns of Table 8 test for the joint statistical significance of alphas using the sub-portfolios for each country. As mentioned earlier for each country, I formed sub-indices based on the maturity and industry of the corporate bonds. The second column is the J-statistic that tests for the joint significance of all short term corporate bonds against the US benchmark assets. From the resulting p-values, all foreign corporate bond portfolios, except for the long term maturity portfolios, do jointly provide statistically significant gains to the US benchmark portfolios.

It is an interesting comparison that in pairwise tests only Japan is statistically significant, and yet jointly the corporate bond portfolios do bring higher risk adjusted returns to the US benchmark. However, it might be all driven by the Japanese

corporate bond portfolio, in which case, we don't need the remaining portfolios. To test this conjecture, I compute the mean variance weights on the tangency portfolio that includes all foreign corporate bond portfolios to see if any other countries have statistically significant weights. Table 9 reports the tangency portfolio weights when all the foreign corporate bond portfolios are pooled together with the US benchmark assets. As demonstrated by Table 9, the tangency portfolio weights imply a statistically significant long position in Japan of 69% and a short position of -22% in the UK, and the portfolio weights in the other countries are not statistically different from zero. Supposing a US investor puts zero weight in the statistically insignificant foreign corporate bond portfolios and 69% in Japan and -22% in UK, this would imply a net position of 47% in foreign corporate bonds, which in economic terms is a substantial part of the portfolio.

## 2.5 Gains from Risk Reduction

The mean variance analysis in the previous sections measured gains in terms of increasing Sharpe ratio of the tangency portfolio. To achieve mean variance efficiency, the portfolio optimization trades off the asset's contribution to the portfolio mean return, with its effect on portfolio variance. In order to decouple to the two effects, this section isolates the means and measures the gain from a pure risk reduction perspective. This can be done by analyzing the minimum variance portfolio, and asking how much pure portfolio risk reduction can be achieved with the minimum variance portfolio of including foreign corporate bonds.

The minimum variance portfolio weights are the solution to the following optimization problem:

$$\min w'_{gmv} \Sigma w_{gmv} \quad s.t \quad w'_{gmv} * i_{K+N} = 1 \quad (2.8)$$

The solution for the portfolio weight in each of the K benchmark assets plus N test assets follows the below formula:

$$w_{gmv} = \frac{\Sigma^{-1} * i_{K+N}}{i'_{K+N} * \Sigma^{-1} * i_{K+N}} \quad (2.9)$$

where  $w_{gmv}$  is a (K+N) x 1 vector of portfolio weights for the global minimum variance portfolio,  $i_{K+N}$  is a (K+N) x 1 vector of ones, and  $\Sigma$  is an (K+N) x (K+N) covariance matrix.

Comparing Equation 2.9 against Equation 2.5, it is clear that the global minimum variance portfolio excludes the effects of expected returns and creates the portfolio that has the lowest possible portfolio risk. In generating the portfolio with the smallest variance, the portfolio weights will tend to favor assets that have lower variance since potentially lower expected returns that associate with lower asset variance is not considered. From this perspective, it is unsurprising that the US equity portfolios will always have lower weights than the bond portfolios. The more interesting dynamic is the tradeoff between foreign corporate bond portfolios and the US bond portfolios represented by TERM and DEF factors.

Table 10 provides estimates of the minimum variance portfolio weights when foreign corporate bond portfolios are included one at a time. The minimum variance portfolio weights demonstrate the investor should short the US 30 Year treasury and hold a mix of US and foreign corporate bonds to achieve the lowest possible portfolio

variance. The US equity market portfolio also contribute to risk reduction but in general command a smaller portion of the portfolio. The lower panel of Table 10 shows the portfolio volatility gains that can be achieved when including each foreign corporate bond to the US benchmark. The minimum variance portfolio formed only with the US benchmark has an average annualized portfolio standard deviation of 4%, while the minimum variance portfolio with both the US benchmark and the foreign corporate bonds have a standard deviation ranging from 1.7% to 3.6%. So on average, just including one foreign corporate bond portfolio to the US benchmark could potentially decrease the global minimum variance portfolio standard deviation by anywhere from 10% to 58%.

However, these gains are computed in sample and therefore are the upper bound to potential diversification gains to a US investor over the entire estimated sample period. For out of sample portfolio variance reduction to the global minimum variance portfolio, I use the period Jan 1997 - Dec 1998, to estimate the minimum variance portfolio weights. Keeping the weights from the estimation period, I compute the minimum variance portfolio returns for each month for Jan 1999 - Dec 2008, and plot the realized return standard deviation of a rolling 12 month window. Therefore, Jan 2000 plots the annualized standard deviation of the minimum variance portfolio returns from Jan 1999 - Jan 2000, Feb 2000 plots the annualized standard deviation of the portfolio returns from Feb 1999 - Feb 2000, and so forth. Figure 1 shows the annualized portfolio standard deviation of the US benchmark minimum variance portfolio versus the US benchmark plus Canada, Japan, and UK corporate bond minimum variance portfolio. I choose to use only Canada, Japan, and the UK, because the data for Australia and Europe are shorten and begin in 1999. As Figure 1 shows, if the investor would have held the global minimum variance portfolio with

the weights estimated in 1997, they would have consistently experienced positive diversification gains, even in this current crisis episode. Further, the magnitude of risk reduction out of sample can be as large as 65%.

Of course, constant weights throughout a ten year holding period is an extreme measure of buy and hold gains. To measure the diversification gains with some investor portfolio re-balancing, I estimate the minimum variance portfolio weights of the US benchmark portfolio and Canada, Japan, and UK, with a past 24 months window and a holding period of 6 month. For example, I estimate the minimum variance portfolio weights for the period Jan 1997 - Dec 1998 and use the weights to compute the realized minimum variance portfolio returns for Jan, Feb, Mar, April, May, and Jun of 1999. Then in Jun 1999, the minimum variance portfolio weights will be re-estimated using the 24 month sample period of Jun 1997 - Jun 1999, and those weights will be used to compute the next 6 months of portfolio returns. Given the time series returns, I plot the annualized standard deviation of the realized minimum variance portfolio returns with a 12 month window. So the estimated out of sample portfolio standard deviation for Jan 2000 is an estimate of the return standard deviation for Jan 1999 - Dec 1999. Figure 2 shows the out of sample performance of the minimum variance portfolio that is re-balanced and held for 6 months. In comparison to the constant weight strategy used for Figure 1, re-balancing brings the portfolio standard deviation of the US benchmark portfolio down from a max of 10% per year to a max of 6% per year. Further, by using a re-balancing strategy with foreign corporate bonds added to the US benchmark portfolios, the standard deviation of the minimum variance portfolio drops from 6.5% per year to about 3% per year. The out of sample diversification gain to holding foreign corporate bonds in the last crisis would have been a 54% reduction

in portfolio risk.

## 2.6 Capturing foreign gains with Yankee Bonds

In the previous sections, I have established that there are potentially some efficiency gains to investing in foreign corporate bond markets directly as a US investor. There are potentially much larger and significant portfolio diversification benefits. This suggests a secondary question: can these gains can be achieved at lower costs by holding foreign corporate bonds that are issued in the US? As argued by Errunza et al. [20], a combination of ADRs, multinational corporations, and country funds provide US investors with the same gains as investing in the emerging market equities directly, but at lower transaction cost, better information, and easier access. Motivated by this argument, this section analyzes the extent to which foreign corporate bonds that trade in the US, known as Yankee bonds, can capture the gains of investing directly in the foreign corporate bond market. To explore the ability for Yankee bonds to capture gains from direct investment in foreign corporate bond markets, I re-evaluate the results of the Sharpe ratio analysis and Bayesian portfolio weight analysis with Yankee bonds added to the US benchmark.

To test the efficiency gains of the tangency portfolio, I run the following excess return regression of each foreign corporate bond index against the US benchmark plus the Yankee portfolio, where the difference from Equation 2.4 is the extra  $\beta_6$  term as a part of the benchmark:

$$r_t^{For} = \alpha + \beta_1 * mkt r f_t + \beta_2 * smb_t + \beta_3 * hml_t + \beta_4 * TERM_t + \beta_5 * DEF_t + \beta_6 * Yankee_t + u_t \quad (2.10)$$



where  $r_t^{For}$  is the excess return on the foreign bond index. In addition,  $mktrf_t$  is the excess return on US equity market,  $smb_t$  is the return on small minus big portfolio,  $hml_t$  is the return on high minus low portfolio,  $TERM_t$  is the excess return on the 30 year treasury,  $DEF_t$  is the excess return on the US corporate bond index,  $Yankee_t$  is the excess return on the Yankee bond portfolio that corresponds to the foreign bond index, and  $u_t$  is an error term.

For each country in Table 11, I compare the results of the above regression with and without Yankee bonds in the US benchmark. The first column of each country is taken directly from Table 5, and is the result of the excess return regression specified in Equation 2.4. The side by side comparison of the US benchmark portfolio with and without Yankee bonds shows that including Yankee bonds does not make a material difference in either the point estimate or statistical significance of the intercept. For example, as outlined in Table 11 the estimated Sharpe ratio gain of including Australia corporate bond portfolio to the US benchmark is .06% per month when the US benchmark portfolio does not include Yankee bonds, with a t-statistic of 1.02. In comparison, when I add Yankee bonds to the US benchmark, the implied Sharpe ratio gain to the US benchmark portfolio that contains Yankee bonds is still .06% per month, with a t-statistic of 0.98. In fact, none of the intercept estimates change when I add each country's Yankee bond portfolio to the US benchmark. As I demonstrated earlier, the only country that provides statistically significant gains to the US benchmark portfolio in Table 5 is Japan. Again, in the case of Japan, the inclusion of Yankee bonds in the US benchmark does not change the statistical significance of the implied 1.8% portfolio Share ratio gain to the US investor.

As argued earlier, the insignificance of the portfolio Sharpe ratio gains necessarily imply that the investor will allocate zero weight on the foreign corporate bond

portfolios. Particularly, when the investor is faced with estimation risk in a Bayesian framework, the implied portfolio holdings in the foreign corporate bond portfolios are above 25%, even when the investor holds only a little uncertainty that the true efficiency gain is zero. To test if the inclusion of Yankee bonds significantly decreases the Bayesian portfolio holdings analyzed earlier, I include Yankee bonds in the US benchmark assets and re-examine the implied Bayesian portfolio holdings on the foreign corporate bonds.

Table 12 compares the implied Bayesian tangency portfolio weights with and without Yankee bonds, while varying the parameter uncertainty of the true value of the intercept. For the tangency portfolio with Australian foreign corporate bonds, Yankee bonds, and US benchmark asset, at 1% annual prior variance, the implied Bayesian portfolio weight in the Australian foreign corporate bond is still fairly large at 31% of the portfolio. To facilitate the comparison, earlier results presented in Table 7 are shown underneath the results with Yankee bonds. At 1% prior variance, the implied Bayesian portfolio holdings does not change much despite the inclusion of Yankee bonds. For Australia, the implied Bayesian holding in Australian bonds decreases from 37% to 31%, which is largely driven by the slight reduction in t-statistic of the intercept estimate in Table 11. The effect of including Yankee bonds in the US benchmark portfolio have little effect on the implied holdings of the foreign corporate bond portfolio across all the countries, particularly for the lower prior variances. In fact, across all the countries, even at a fairly tight prior of 1% prior variance per annum, the holding of foreign corporate bonds across all countries is still at 23%.

### 2.6.1 Why don't Yankees capture gains?

The inability for Yankee bonds to capture the gains of investing abroad seem puzzling in light of the equities analysis by Errunza et al. [20]. While exploring all the reasons why this is the case is beyond the scope of this paper, this section tests the hypothesis that Yankee bond returns follow closely the dynamics of US corporate bond and have fewer similarities to the bond indices of the foreign markets. To test this hypothesis, I first use mean variance spanning to test which Yankee bond portfolios have investment opportunity sets that can be traced out with a combination of US benchmark assets. Then, I use a regression analysis of Yankee bonds on US benchmark assets to show that Yankee bond returns are much less statistically sensitive to the foreign corporate bond market than to the US corporate bond market.

As described earlier in section 4, a formal test of spanning involves two conditions on the regression in Equation ??, the intercept equals zero and the slope coefficients adds to one. I test to see if each Yankee bond portfolio can be spanned by the the US benchmark of equity and bond portfolios. The top panel of Table 13 shows the F-statistic and corresponding p-values of the spanning test when only DEF and TERM are used as the right hand side variable of Equation ?. Using just the two US benchmark bond variables of DEF and TERM, Europe and UK have p-values above the 5% level at 48.5% and 5.2% respectively. This means that the test can not reject the hypothesis that DEF and TERM spans the European and UK corporate bond portfolios at the 5% level. Further, when I add the US equity portfolios in the second panel of Table 13, Australia has a p-value of 6.6% which implies that at the 5% level, the test cannot reject the hypothesis that the Australian Yankee bonds is

spanned by a combination of US equity and bond portfolios.

Spanning tests places a stringent requirement on the benchmark assets in that they must trace out exactly the same investment opportunity set as the Yankee bond portfolios. However, the fact that Yankee bonds do not capture the gains from the foreign market may be because they are more correlated with the US benchmark assets and less with their home markets. Since Yankee bonds are traded in the US secondary bond market, I use the two US bond market variables, TERM and DEF, as controls, and test for the sensitivity of Yankee bond returns to their home corporate bond returns. Recall that TERM is the excess return on the 30 year US treasury and DEF is the excess return on the US corporate bond portfolio. As shown by Diebold, Li, and Yue [18], there is evidence of global factors that move all bond markets. To account for any global dynamics that affect both the foreign and the US bond markets, I also control for the interaction effect between US corporate bond returns and the foreign corporate bond returns. Therefore, to explore the sensitivity of Yankee bonds to their foreign corporate bond market, I run the following regression analysis:

$$r_t^{Yankee} = c + \gamma_1 * TERM_t + \gamma_2 * DEF_t + \gamma_3 * r_t^{For} + \gamma_4 * r_t^{For} * DEF_t + \eta_t \quad (2.11)$$

where  $r_t^{Yankee}$  is the excess return on a country specific Yankee bond portfolio. On the right hand side,  $r_t^{For}$  is the excess return on the foreign corporate bond index trading in the local market,  $TERM_t$  is the excess return on the 30 year treasury,  $DEF_t$  is the excess return on the US corporate bond index, and  $\eta_t$  is an error term.

Table 11 shows the results of the above regression of Yankee bond returns on US

bond returns and foreign corporate bond returns. First, the intercept coefficients are all statistically insignificant from zero, with t-statistics ranging from -0.13 to 0.71. This implies that adding Yankee bonds to a portfolio of 30 year US treasury, US corporate bonds, and foreign corporate bonds do not bring any significant efficiency gains. Further, Table 11 indicates that the loading of Yankee bonds on the DEF factor, which is the excess return on the US corporate bond portfolio, ranges between 0.74 to 1.06 and highly statistically significant.

In contrast, the third panel of Table 11 reveals the estimates and t-statistic of  $\gamma_3$  in Equation 2.11. The sensitivity of Yankee bond returns to their foreign market returns ranges between -0.12 to 0.26 and all have t-statistics that are statistically insignificant at the 5% level. In comparison, the sensitivity of Yankee bond returns to the US corporate bond returns, or DEF, were all above 0.74 and statistically significant.

From the regression analysis, I find that Yankee bond returns are much more sensitive to US corporate bond returns than their home corporate bond returns. This supports the earlier finding that Yankee bonds are significantly different from their home market corporate bonds as to not capture the gains from direct investment. And in the case of Australia, Europe and the US, the result of the spanning test show that some combination of the US benchmark assets can replicate the entire investment opportunity of the Yankee bonds.

## 2.7 Robustness

Sections 5 and 6 of this chapter underscores the substantial diversification gains to holding hedged foreign corporate bonds, gains that can not be mimicked by

holding Yankee bonds. For a US investor holding a benchmark portfolio of equity and bond assets, these gains seem to be particularly large both in sample and out of sample for the most recent crisis period. This section extends the previous risk reduction analysis with three alternative specifications. First, I explore the time variation in diversification gains with a rolling window estimation of the minimum variance portfolio. Second, I analyze the effect of the foreign exchange hedge on risk reduction by computing the diversification gains with unhedged foreign corporate bond portfolio returns. And last, I examine the importance of using both equity and bond portfolios in the US benchmark assets by measuring the diversification gains when only the US corporate bond portfolio is used as the benchmark.

### **2.7.1 Time Variation in Diversification Gains**

There is a large body of empirical evidence that documents the time variation in the co-movement of assets.<sup>21</sup> Since the minimum variance portfolio depends solely on the variance covariance structure of asset return, any time variation in asset return co-movements may have large effects on the diversification gains measure earlier. To analyze the time dynamics, I estimate the in-sample portfolio risk reduction to the minimum variance portfolio using a rolling estimation window of 24 months. Further, to analyze the potential of foreign equities to capture the diversification gains, I compare the risk reduction gains of the minimum variance portfolio with foreign equities versus foreign bonds.

Specifically, starting with Jan 1999, I estimate the minimum variance portfolio weights using the past 24 month window and compute the in sample reduction to annualized portfolio standard deviation. Rolling the window over month by month,

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<sup>21</sup>Most recently, Bekeart, Hodrick, and Zhang [6]

Figure 3 graphs the in sample gains to the global minimum variance portfolio of the US benchmark versus the US benchmark plus Canada, Japan, and UK corporate bonds.<sup>22</sup> The plot shows that there is substantial time variation in the diversification gains from foreign corporate bonds. In particular, during periods of heightened volatility for the US benchmark portfolio, the inclusion of foreign corporate bonds seem to greatly reduce the volatility on the global minimum variance portfolio.

While the diversification gains in Figure 3 are quite striking, it might be that much of the gains can be captured using foreign equities instead. Figure 4 graphs the time varying risk reduction of including foreign equities to the US benchmark asset versus including foreign corporate bonds<sup>23</sup>. Using the same 24 month rolling window methodology as Figure 3, Figure 4 plots the portfolio standard deviation of the US benchmark, of the US benchmark plus foreign equities, and of the US benchmark plus foreign bonds. The graph shows that the inclusion of foreign equities does provide some diversification benefits, particularly in the earlier periods. However, the diversification benefits have become more muted over time, and in the most recent credit crisis, foreign equities do not seem to provide much diversification benefits when compared against the diversification gains from foreign bonds.

## 2.7.2 Diversification Gains with Unhedged Returns

As typically done in the international finance literature on equities, diversification gains are measured with unhedged foreign returns, or returns that are inclusive of foreign exchange exposure. As previously discussed, unhedged returns from foreign corporate bond portfolios are far more volatile than their hedged counterparts. This

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<sup>22</sup>Australia and Europe portfolios do not span the full sample period.

<sup>23</sup>Foreign equities corresponds to the three countries included in foreign corporate bonds namely Canada, Japan, and UK

is a reflection of the fact that the unhedged corporate bond portfolio combines both the credit market risk as well as the foreign exchange risk. Therefore, to explore the effects of foreign exchange on previously measured diversification gains<sup>24</sup>, this section analyzes the risk reduction properties of unhedged corporate bond returns against the US benchmark.

Using the same in sample rolling methodology as described in section 7.1, Figure 5 shows the variance reduction of including unhedged foreign corporate bond portfolios into the US benchmark assets. Since there is no foreign exchange exposure on the US benchmark assets, the minimum variance portfolio of the US benchmark in Figure 5 is the same as Figure 3. The portfolio variance of the US benchmark plus the unhedged foreign corporate bonds shows much smaller diversification gains than earlier when foreign bond portfolios were hedged. In contrast to the diversification gains of up to 75% with hedged returns, Figure 5 shows that the in sample risk reduction of including unhedged foreign corporate bonds are at best 25% in the most recent crisis. In particular, because the foreign corporate bonds are much more volatile due to the foreign exchange risk, the minimum variance portfolio weights are skewed more towards the US benchmark assets. Therefore, the portfolio variance with foreign corporate bonds trails closely with the portfolio variance with just the benchmark assets.

### **2.7.3 US Corporate Bond Portfolio as the Benchmark**

Up to this point, all analysis has been conducted from the perspective of a US investor, who is exposed to the US corporate bond market, but holds a well-diversified

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<sup>24</sup>Note that the earlier "hedged" returns do include some basis risk, as only the current value of the bond and the expected accrued interest is hedged with a 1 month forward. Any price changes are still subject to foreign exchange risk. However, the bond value changes are small, which limits the exposure to foreign exchange risk.



portfolio of the US equity and bond portfolios. However, in the case of some financial institutions that hold a majority of their portfolio in corporate bonds for regulatory requirements, the exposure to the corporate bond market may be much larger than what is implied in the mean variance efficient allocation of the US benchmark bond and equity portfolios. It is standard in the literature to use the US equity markets to analyze the diversification benefits of foreign equities, so comparably, US corporate bond market may be the appropriate benchmark for foreign corporate bonds. Therefore, this section uses the US corporate bond portfolio as the sole benchmark asset and analyze the effect on diversification of adding foreign corporate bonds.

Again using the in sample rolling window estimates of variance reduction to the minimum variance portfolio, Figure 6 shows the in sample risk reduction of including foreign corporate bonds to a benchmark of the US corporate bond market. The time plot shows that including foreign corporate bonds diversifies the risks of just holding the US corporate bond market by 50% or more, and is often much greater during crisis periods. In addition, the diversification gains to the US corporate bond market of adding foreign equities was particularly pronounced during the recent crisis period where holding the global minimum variance portfolio with foreign bonds would have brought the portfolio volatility down from 7% per year to about 1% per year.

Finally, to better understand the achievable gains rather than the potential gains, I compute the out of sample portfolio standard deviation reductions as measured by re-balancing and holding the estimated minimum variance portfolio weights for 6 months. Using the same estimation method as used in Figure 2, I estimate the minimum variance portfolio weights using a sample period of the past 24 months and then compute the realized return on the minimum variance portfolio holding the estimated weights for the next 6 months. Then at the end of the 6 months,

I again use the past 24 month return to estimate a new set of minimum variance portfolio weights, and carry it forward for 6 months. In this way, I generate a time series of out of sample minimum variance portfolio returns. Then, Figure 7 graphs the 12 month standard deviation of the minimum variance portfolio returns for the US Corporate bond plus the foreign corporate bonds. In comparison, to the out of sample analysis when the benchmark was the US equity and bond portfolios, an investor holding just the US corporate bond would experience a dramatic risk reduction if he were to add corporate bonds from Canada, Japan, and the UK. In particular, for the most recent crisis period, the out of sample risk reduction to the US corporate bond market of holding the minimum variance portfolio with foreign corporate bonds would have brought down the annualized portfolio standard deviation from over 9% per year to a little under 2% per year, which is over a 75% risk reduction.

## 2.8 Conclusion

This paper has demonstrated the gains to US investor of holding foreign corporate bonds. Jointly, the inclusion of corporate bonds from Australia, Canada, Europe, Japan, and UK provides a statistically significant increase to the portfolio Sharpe ratio of the US benchmark equity and bond portfolios. Moreover, the mean variance efficient portfolio weights implied by both classical estimation and Bayesian analysis suggests that US investors who have a benchmark of US equity and bond portfolios should invest over 25% of their portfolios in foreign corporate bonds.

In addition to mean variance efficiency gains, I measure pure risk reduction

gains as variance reduction on the minimum variance portfolio. I find that including foreign corporate bonds provides economically large and statistically significant diversification benefits to benchmark of US equity and bond portfolios. While there is time variation with in diversification gains, I find that holding foreign corporate bonds has the potential to decrease the in sample portfolio volatility by as large as 77%. Further, out of sample performance for the most simplistic constant weight buy and hold strategy would imply consistently positive risk reduction for the minimum variance portfolio. And if the investor were to re-balance every six months, the inclusion of foreign corporate bonds would decrease the out of sample annualized portfolio standard deviation from 7% per year to 3% per year.

Finally, motivated by the idea that foreign issued corporate bonds that trade in the US may provide easier access and lower cost way of achieving these potentially large diversification gains of investing in foreign markets, I test the ability for Yankee bonds to capture the same benefit. I find that the addition of Yankee bonds to the US benchmark assets does not change the implied mean variance efficiency gains of investing directly in the foreign corporate bond markets. In addition, the Bayesian portfolio weight in foreign corporate bonds does not dramatically decrease when Yankee bonds are included. Using both a spanning test and a sensitivity analysis of Yankees on US benchmark assets, I find that Yankee bonds do not capture gains from foreign corporate bond markets because their return dynamics are more closely related to US corporate bonds than to their home corporate bond markets.

This analysis does raise several open questions of why these gains are not being exploited more aggressively. In particular, how large are the transaction costs and liquidity risks that may potentially alter the gains to holding foreign corporate bonds? What types of market frictions and legal barriers might be hindering the

US investor to invest directly in these foreign bond markets? Moreover, while this paper provides some preliminary evidence that these diversification gains are time-varying, a more in-depth analysis of the changing covariance structure both within the corporate bond market and in relation to the equity markets would be insightful. I leave these topics open for future research.

# Chapter 3

## Long Run Risk and International Risk Sharing

### 3.1 Introduction

An extensive literature has examined the potential risk-sharing gains from international diversification by focusing on models and data based upon consumption relationships across countries. These consumption-based studies have largely ignored the implications of the models for asset pricing moments, leading to counterfactual asset pricing relationships such as low equity premia, high risk free rates, and low volatility of asset returns. In particular, standard models assume constant relative risk aversion utility with risk aversion parameters less than 10, even though Mehra and Prescott [34] have shown that the equity premium cannot be explained with parameters in this range. Even if the utility function is generalized to recursive utility, the standard assumptions about utility parameters generate risk-free rates that are too high as shown by Weil [44] and others. Finally, the combination of

assumptions about state variables and preferences typically imply low or even constant variability in risk-free rates.<sup>1</sup> These counterfactual predictions in asset returns cast doubt on the ability of the consumption-based literature to accurately measure gains from international risk-sharing.<sup>2</sup>

Despite these implications of standard consumption-based models, new approaches that reconsider consumption behavior have achieved better success in matching basic asset pricing moments in US returns. Bansal and Yaron [3] show that the US equity premium, risk free rate, and variability of returns can be explained by a small, but persistent, component in consumption growth they term "long run risk." Campbell and Cochrane [12] show that the asset return behavior can be explained by an independently and identically distributed consumption growth process, when a slow-moving external habit to the standard power utility function is added. Lettau and Ludvigson [28] [30] [29] show that US cross-sectional and time series equity returns can be explained by a consumption-based model using consumption-wealth ratios as a proxy for the stochastic discount factor. Despite these successes, the empirical literature has focused upon the closed economy setting.

In this paper, I begin to bridge the disconnect between international risk-sharing and the empirical implications from consumption-based asset pricing. I develop an risk-sharing model which incorporates a long run risk to consumption growth in a general decentralized economy in which countries can decide to remain in autarky or enter the risk-sharing arrangement. Using this framework, I show how welfare gains from international risk-sharing in the decentralized economy can be formulated from the social-planners problem, which depends upon the utility parameters and

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<sup>1</sup>See for example, the discussion in Campbell and Cochrane [12] and Abel [1].

<sup>2</sup>For a survey of these counterfactual predictions and the implications for risk-sharing gains, see Lewis [31].

the state process distributions.

To consider the implications of international asset return behavior on consumption risk-sharing, I begin with the framework used in Bansal and Yaron [3] to target the asset returns of the US in a close economy equilibrium. As in that framework, I develop an international consumption-based asset pricing model assuming a small, but persistent component in consumption growth. With the assumption that all countries share the same preference parameters, I derive an analytical formulation of the pricing kernel for a world representative agent in the decentralized open economy. In a full risk sharing equilibrium where agents have perfect commitment, countries will pool all consumption and buy shares of "world" consumption based on the open economy pricing.

While I derive the analytical solution to the open economy equilibrium with an arbitrary number of countries, the effects of the long run risk on welfare gains in international risk sharing can be analyzed most clearly in a two country framework. I show that while preference parameters matter in the degree of welfare gains achieved, the most significant increase to welfare gains from risk sharing comes from the ability to diversify on the long run risk component of consumption growth. In this framework, if all countries shared a common long run risk which can not be diversified internationally then the implied welfare gains will be much smaller. With a better understanding of the model dynamics, I begin to pin down the magnitude of preference parameters and potential long run risk for each country using the analytical solution from the closed economy and asset pricing data.

This chapter also matches the basic moments of an international consumption-based asset pricing model.<sup>3</sup> Using consumption and asset return data from seven

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<sup>3</sup>In a complementary research agenda, Colce and Colacito [15] [16] examine the effects of long run risk on real exchange rates. However, our paper focuses upon an asset return behavior and a

countries, I use Simulated Method of Moments (SMM) to match the parameters of the close economy consumption-based model with moments from asset returns. In the base model, returns are driven by a common persistent consumption risk component across countries. However, I also estimate the model allowing these components to differ by country. As I show, there exists a great deal of heterogeneity across countries in consumption processes and asset returns. This heterogeneity has implications for parameter estimates to vary dramatically across different countries. In continuing research, I will use these parameter estimates that are disciplined by our asset return data to re-examine international risk-sharing. In this way, the estimates of international consumption risk sharing gains are consistent with the implications of asset return behavior.

## 3.2 Asset Returns and Consumption-Based Models

A large literature has considered the implications of consumption behavior on international risk-sharing. Backus, Kehoe, and Kydland [2] observed that consumption correlations are lower than output correlations which clearly violates the implications of perfect risk-sharing arising from complete markets. To understand this behavior, a large literature has considered the effects of deviations from the standard model. These deviations include incomplete markets, transactions costs, and country-specific non-tradeable risks such as immobile labor and non-tradeable goods.<sup>4</sup>

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common consumption good.

<sup>4</sup>For a very partial list, see Baxter and Crucini [4] on incomplete markets, Baxter and Jermann [5] and Heathcote and Perri [23] on labor risk, Stockman and Tesar [40] on non-tradeable goods,



A natural, but important question that arises when assessing this large literature is: What is the economic cost of rejecting perfect international risk-sharing? If these costs are minor, then even though the low correlation of consumption technically implies a failure of perfect risk-sharing, this failure is economically insignificant. On the other hand, large foregone gains to risk-sharing would imply the contrary. Unfortunately, the literature has reported a wide range of foregone gains to risk-sharing. Some studies have found the gains to be exceedingly small and on the order of less than one-thousands of a percent of permanent consumption while others have found these gains to be in excess of 100% of permanent consumption.<sup>5</sup>

While all of these studies are based upon international consumption behavior, they differ in how well they relate this behavior to asset returns. A well known feature of standard consumption models is that the implied variability of consumption cannot explain asset returns behavior. In particular, as Mehra and Prescott [34] pointed out, the standard consumption-based model generates an equity premium puzzle since the model predicts a lower premium than observed in the data. Therefore, some international risk-sharing models have attempted to match this feature of the data by using a risk-aversion coefficient that is sufficiently high to match the equity premium.<sup>6</sup> While the equity premium can be matched with the correct choice of risk aversion coefficient, other features of asset returns are not. For example, Weil [44] showed that the standard consumption-based model continues to imply a risk-free rate puzzle because the model generates a higher risk-free rate than

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and Tesar and Werner [41] and Warnock [42] on transactions costs. For a more detailed but dated survey, see Lewis [31].

<sup>5</sup>While the assumptions underlying these gain calculations differ dramatically, these numbers were taken to emphasize the wide range in the literature. For a study implying tiny gains from international risk sharing, see Cole and Obstfeld (1991) and for large gains see Obstfeld (1994b). Tesar (1995) and van Wincoop (1994) provide surveys that consider the impact of various effects such as habit persistence and the presence of non-traded goods.

<sup>6</sup>See for example Obstfeld [35] and the discussion in Lewis [31].

the data. Moreover, high risk aversion can not resolve the high volatility of asset returns in the data, compared to the low, sometimes zero, volatility in the model.<sup>7</sup>

While the behavior of asset returns is only one way to discipline an international model of consumption, for questions concerning risk-sharing gains, asset return behavior is arguably the most important. Trade in international capital markets is often viewed as the most efficient or even primary mechanism in which risks can be shared globally. As such, the prices of assets in these markets reflect equilibrium views toward inter-temporal and intra-temporal risks. For this reason, I take these asset returns as the standard on which to discipline our models of international consumption and implied risk parameters. To provide a general framework, I first review the standard consumption model and the new insights gained from "long run risks." Below, I will imbed this model into an international context to begin to match returns with consumption data.

### **3.2.1 Closed Economy Consumption Processes With and Without Long Run Risk**

I begin by examining a standard consumption model in the closed economy using a standard Mehra-Prescott approach as well as the Bansal-Yaron "long run risk" model. Since the closed economy can also be viewed as representing an autarkic equilibrium, this model will provide an important benchmark for our gains from risk-sharing. Thus, I describe the model in terms of a representative agent in each country.

Each country  $j$  has a continuum of identical consumer-investors. Under standard iid consumption growth, the log consumption growth rate processes of each of these

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<sup>7</sup>See the discussion in Campbell and Cochrane [12] and Lewis [31].

agents  $g_{c,t}^j$  is determined by a mean growth rate  $\mu^j$ , and variance to the innovation given by  $\sigma^j$ .

$$g_{c,t+1}^j = \mu^j + \sigma^j \eta_{t+1}^j$$

where  $\eta_{t+1}^j \sim N.i.i.d.(0, 1)$ . If further the agent in country  $j$  views his consumption profile as subject to long run risks as in Bansal and Yaron (BY) [3] he will have a persistent stochastic component in the conditional mean as given by  $x_t^j$  in the following equation.

$$\begin{aligned} g_{c,t+1}^j &= \mu^j + x_t^j + \sigma^j \eta_{t+1}^j \\ x_{t+1}^j &= \rho^j x_t^j + \sigma \varphi_e^j e_{t+1}^j \end{aligned} \tag{3.1}$$

where  $e_{t+1}^j \sim N.i.i.d.(0, 1)$ . Thus, the "long run risk" process,  $x^j$  induces a persistent deviation in the conditional mean of consumption away from its long run growth rate,  $\mu^j$ . Bansal and Yaron [3] argue that this deviation is difficult to detect because the difference in variance between the temporary deviation from the growth rate,  $\eta_{t+1}^j$ , and the variance of the persistent component,  $e_{t+1}^j$ , is large. In other words,  $\varphi_e^j$  is very tiny and close to 0.0003 in US data. BY also consider the effects of stochastic volatility such that  $\sigma^j$  is time-varying. In this chapter, I do not include stochastic volatility, but will include these results in future research.

In order to match asset return behavior, BY fit the behavior of dividends and consumption growth rates to the implied estimates of asset return moments. As such, they use moments of equity returns and the risk-free rate to estimate the parameters in equation 3.1 and the parameters in growth rate of dividend process

$g_{d,t}^j$  given by:

$$g_{d,t+1}^j = \mu_d^j + \phi^j \bar{x}_t + \varphi_d^j \sigma^j u_{t+1}^j \quad (3.2)$$

where  $u_{t+1}^j \sim N.i.i.d.(0, 1)$ ,  $\mu_d^j$  is the growth rate of dividends,  $\phi^j$ , is the loading of long run risk on the growth rate of dividends, and  $\varphi_d^j$  is the ratio of conditional variance in dividend growth to the transitory variance in consumption.

### 3.2.2 Closed Economy Asset Returns and Utility

I require a utility function to understand the relationship between consumption/dividend processes and asset returns and, ultimately, the welfare gains on risk-sharing. I further need a utility function that allows different risk aversion and intertemporal substitution for two reasons. First, as demonstrated by Obstfeld [35], the effects of gains from sharing differing growth rates and from reducing variability around these growth rates are confounded if relative risk aversion and intertemporal substitution are governed by the same parameter as in the constant relative risk aversion utility. Second, as the asset pricing literature has shown, constant relative risk aversion utility cannot jointly match the moments of equity and the risk-free rate.

For both of these reasons, I assume agents in each closed economy country has recursive preferences following Epstein and Zin [19] and Weil [43]. Further, in our open economy model below, I will assume that all countries have the same utility function parameters. Specifically, using the index  $j$  to refer to each country, utility at time  $t$  can be written:

$$U^j(C^j(S_t), E_t[U^j(C^j(S_{t+1}))]) = \left\{ (1 - \delta)C^j(S_t)^{\frac{1-\gamma}{\theta}} + \delta \left( E_t [U^j(C^j(S_{t+1}), E_t[U^j(C^j(S_{t+1}))])^{1-\gamma}] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \quad (3.3)$$

where  $0 < \delta < 1$  is the time discount rate, so that  $(\frac{1}{\delta} - 1)$  is the rate of time preference, where  $\gamma \geq 0$  is the risk-aversion parameter, where  $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$  for  $\psi \geq 0$ , the intertemporal elasticity of substitution, and where  $E_t(\cdot)$  is the expectation operator conditional on the information set at time  $t$   $I_t \equiv \{S_t, S_{t-1}, \dots\}$  for  $S_t$ , the realization of the state process, at time  $t$ . As described by Epstein and Zin [19], this utility function specializes to standard time-additive constant-relative risk aversion preferences when  $\gamma = \frac{1}{\psi}$ . In this case, the utility function becomes:

$$U^j(C^j(S_t), E_t[U^j(C^j(S_{t+1}))]) = \left\{ (1 - \delta) E_t \sum_{\tau=0}^{\infty} \delta^\tau C^j(S_{t+\tau})^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}$$

Since the risk aversion coefficient and the inverse of the intertemporal elasticity of substitution are no longer constrained to move together under Epstein-Zin-Weil preferences, lifetime utility can be unbounded for some combinations of the utility parameters,  $\delta$ ,  $\gamma$ , and  $\psi$ , and the growth rate of consumption.<sup>8</sup> Intuitively, the time discount rate, governed by  $\delta$  and the intertemporal elasticity of substitution in consumption, measured by  $\psi$ , parameterize the sensitivity of utility to future consumption. If these parameters are sufficiently high, then certainty-equivalent consumption growth rates as measured by the growth rate of consumption adjusted by risk-aversion can induce current utility to become unbounded. For this reason, I will also require the condition that utility is bounded:

$$U^j(C^j(S_t), E_t[U^j(C^j(S_{t+1}))]) = \left\{ (1 - \delta) C^j(S_t)^{\frac{1-\gamma}{\theta}} + \delta (E_t [U^j(C^j(S_{t+1}), E[U^j(C^j(S_{t+1})|I_{t+1}])^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} < \infty$$

The unboundedness in utility becomes more likely the higher is the time discount

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<sup>8</sup>Among others, this point has been described in Lewis (2000).

rate and the intertemporal elasticity of substitution in consumption.

Epstein and Zin [19] derive the first-order condition in this environment as:

$$E_t \left\{ \delta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{jP})^{(\theta-1)} R_{t+1}^\ell \right\} = 1 \quad (3.4)$$

where  $R_{t+1}^{jP}$  is the gross return on the market portfolio of agent  $j$  and  $R_{t+1}^\ell$  is the gross return on any asset  $\ell$  available in country  $j$ . As I show in the appendix, these first order conditions can be used to derive solutions for the asset returns in terms of the utility parameters and the parameters in the processes of consumption and dividends. To show how the effects of long run risk compare to iid consumption in asset returns, I consider results based upon the US alone in the next section.

### 3.2.3 Asset Returns with and without Long Run Risk in US Data

In this section, I estimate the parameters for US consumption processes by matching the asset pricing moments in US data. Below I present our results using international data for seven countries that can trade claims on a common consumption good. I begin by examining the US data alone for two main reasons.

First, I follow much of the risk-sharing literature based on a common consumption good, by analyzing consumption data adjusted for deviations in purchasing power parity from the Penn World Tables. By contrast, the domestic asset pricing literature has used US real consumption data. Moreover the US data has been analyzed over a longer time period than I have available for the other countries. Therefore, I first present our results for the US alone to verify whether our data provide estimates that are similar to those in the domestic literature.

Second, I provide the US results to consider whether long run risk is needed to explain asset returns. If iid consumption is sufficient, I do not need to examine other models to allow asset returns to guide us in choosing the appropriate parameters for international risk-sharing gains.

According to Bansal and Yaron [3], consumption decisions are made at a higher frequency than annual data. If the model is specified at a monthly level, then all decision parameters from the implicit income and dividend processes defined at the monthly frequency.<sup>9</sup> As such, all estimates of model parameters must be time-aggregated to match annual data.

To extract estimates of the parameters in the consumption and dividend process, I rely on the Campbell-Shiller decomposition that expresses returns as functions of the price-to-asset-payout ratio:

$$r_{t+1}^\ell = k_0^\ell + k_1^\ell z_{t+1}^\ell - z_t^\ell + g_{t+1}^\ell \quad (3.5)$$

where  $r_t^\ell$  is the net return on asset  $\ell$ ,  $z_t^\ell$  is the logarithm of the price-payout ratio, and  $g_t^\ell$  is the growth rate of the payouts, either dividends in the case of equity or consumption in the case of the market portfolio. Finally,  $k_0^\ell$  and  $k_1^\ell$  are approximating constants that capture the long run return mean and the price-payout ratio, respectively.<sup>10</sup>

Using this approximation along with the Euler equations in equation 3.4 and the utility parameters from Bansal and Yaron [3] I estimate the monthly parameters of  $\mu^j$ ,  $\sigma^j$ ,  $\rho^j$ ,  $\varphi_e^j$ ,  $\mu_d^j$ ,  $\phi^j$ , and  $\varphi_d^j$  to match annual consumption, dividends, and asset

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<sup>9</sup>See Bansal and Yaron (2004) for a more complete articulation of this argument.

<sup>10</sup>Approximation constants are defined to be  $k_1^j = \frac{\exp(\bar{z}^j)}{1+\exp(\bar{z}^j)}$  and  $k_0^j = \log(1 + \exp(\bar{z}^j)) - k_1^j \bar{z}^j$ , where  $\bar{z}^j$  is the steady state log price to consumption ratio in the close economy.

pricing moments. In particular, the moments I match are the standard deviation of log consumption growth, the first order auto-correlation of consumption growth, the standard deviation of log dividend growth, the mean equity premium, the mean risk free rate, the standard deviation of the market return, and standard deviation of risk free rate. (The appendix and Section 3.5 below describe this procedure in more detail.)

To examine the US alone, Table 15 examine three different sets of consumption and asset return data. As the first data set, Mehra and Prescott [34] use the Kuznet-Kendrik-USNIA measure of per capita real consumption of non-durables and services. For asset returns, they use the S&P composite stock price and dividend series. Both series span the period from 1889 to 1978. Second, I obtained the consumption data from NIPA following BY from 1929 - 1998, as well as dividend and return series from the value-weighted CRSP data. Third, for our study that will be applied in an international context, our sample period was necessarily shorter. In particular, I constructed consumption from the Penn World Tables spanning only 1950-2000. I then used the internationally consistent dividend growth rate data from Campbell (2003) for the US.

The first two columns of Table 15 report the first and second moments of returns for the Mehra and Prescott model assuming iid consumption growth. Using constant-relative risk aversion utility and a risk aversion coefficient of 10, Mehra and Prescott find that with an annualized consumption growth rate of 1.7% and the standard deviation of 3.6%, the model can generate an equity premium of only 1.42%, even though the equity premium in the data is 6.18%. Moreover, the risk-free rate from the model is too high at 12.71% while it is less than 1% in the data. Mehra and Prescott also assume that equity pays off the consumption growth rate.



The third and fourth columns of Table 15 show the effects of two main differences used in literature since Mehra and Prescott's seminal paper. First, using Epstein-Zin-Weil utility, the risk aversion and intertemporal elasticity of substitution parameters are allowed to differ. In particular, I use the estimates obtained by BY of 10 for risk aversion and 1.5 for the intertemporal elasticity of substitution. Second, following a number of papers, I treat equity as a payment on dividends instead of consumption. As the table shows, with slightly lower variability of consumption at 2.93%, the model generally generates the standard deviation and first-order autocorrelation of consumption and dividend growth. It also produces some variability in equity returns but at around 12%, it is lower than the 19% in the data. However, the model generates constant risk-free rates such that the variance is counterfactually equal to zero. Moreover, the iid model fails on the means of asset returns. The implied equity premium is negative at around -0.9% and the risk free rate is still too high at 2%.

The fifth column of Table 1 shows the effects of including the long run risk term. For this analysis, I use Bansal-Yaron NIPA data and time period to obtain the estimates of:  $\mu^j = .15\%$ ,  $\sigma^j = .78\%$ ,  $\rho^j = .979$ ,  $\varphi_e^j = .044$ ,  $\mu_d^j = .15\%$ ,  $\phi^j = 3$ , and  $\varphi_d^j = 4.5$ . Not surprisingly, these numbers are similar to those found by Bansal and Yaron. When the model is combined with these parameters, the equity premium becomes positive and around 4.4%, the variability of equity returns are 16.7%, the risk-free rate declines to 1.7%, and the variability of the risk-free rate increases closer to the data.

The last three columns show the implications for asset pricing moments using our PWT consumption data and model estimates. Since I have a shorter time period, our consumption data exhibit lower variability reflecting the "Great Moderation"

in the US.<sup>11</sup> In addition, the variability of dividend growth and first order autocorrelations in annual consumption and dividend growth are all lower over this time period. Despite these differences, our model generates a similar pattern to those obtained by Bansal and Yaron. Our parameter estimates imply:  $\mu^j = .19\%$ ,  $\sigma^j = .79\%$ ,  $\rho^j = .976$ ,  $\varphi_e^j = .044$ ,  $\mu_d^j = .12\%$ ,  $\phi^j = 3.95$ , and  $\varphi_d^j = 1.4$ . When I assume i.i.d. consumption, the equity risk premium is negative, the variability of the market returns is too low and that of the risk-free rate is zero. The risk-free rate is too high, albeit only slightly so since the risk free rate over our sample is higher.

When I add long run risk in the last column, our model matches the returns better. The equity premium rises to 3.5%, the risk-free rate declines closer to the data and the variability of the risk-free rate and the equity premium increase. Once I include the effects of stochastic volatility in the next version of our paper, I anticipate improving the fit of volatility even more.

Overall, then, the estimates in Table 1 show that iid consumption cannot generate plausible asset pricing implications, particularly concerning the variability in returns. Moreover, it is important to allow for differences between risk aversion and IES in utility. In the next section, I consider the impact of these features in assessing international risk-sharing gains.

### 3.3 The International Consumption-Based Economy

I now develop a standard international consumption-based model that can nest autarky within an optimal risk-sharing arrangement. My goal is to provide a general

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<sup>11</sup>See the discussion in Stock and Watson [39].

framework that encompasses many of the existing international asset pricing models as surveyed in Lewis [31]. These models vary along several dimensions. First, they make different assumptions about their underlying state processes. Some models assume temporary deviations from a long run mean. Other models explicitly assume growth rates in country outputs<sup>12</sup> As demonstrated by Obstfeld [35], the effects of gains from sharing growth and from reducing variability are confounded unless recursive utility is assumed that allows the separation between relative risk aversion and intertemporal substitution in consumption. For this reason, I continue to assume recursive utility below. Second, models differ in whether they allow for capital accumulation or assume an endowment output process. On one hand, the international real business cycle literature pioneered by Backus, Kehoe, and Kydland [2] allows for capital accumulation. The asset pricing effects of capital accumulation can potentially be important. For example, Jermann [25] shows that capital accumulation together with habit persistence can explain the equity premium and the risk-free rate in the US economy. On the other hand, much of the domestic consumption-based asset pricing literature has abstracted from production and taken the consumption process as given in analyzing returns. Moreover, the international asset pricing literature that focuses upon risk-sharing gains has often taken consumption as de facto exogenous.<sup>13</sup> In order to be consistent with the consumption data-based literature, I follow the tradition of taking the consumption process as given while staying agnostic about the production process to the greatest degree possible, and specify places where I must assume an endowment economy. Given these considerations and the importance of features of asset pricing moments

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<sup>12</sup>See in particular Obstfeld (1994). As described in Lewis (2000), many of the asset-returns based studies assume growth in dividends and/or output.

<sup>13</sup>See for example, Obstfeld (1994b) and the literature cited in Lewis (1999).

found above, I consider a canonical international economy model that can include these features. I describe this model next.

There are representative consumer-investors in  $J$  countries, indexed by  $j$ . Each country produces an output  $Y^j$  that depends upon a state process  $S_t$ , at time  $t$ . The state process spans the space of all  $J$  country production processes. The agent in each country has recursive preferences following Epstein and Zin [19] and Weil [44] given in equation 3.3. Above, I considered the closed economy version of this framework which I can be viewed as a solution to the economy under autarky.. I now consider the implications of this framework when agents consider an equilibrium full integration. I begin with the social planner's optimal allocation of resources before examining the decentralized closed and open economy equilibria. Later, I consider the welfare gains of moving from the autarky equilibrium to the full integration equilibrium.

### 3.3.1 Social Planner's Problem

I now consider the social planner's problem faced with  $J$  output processes and agent's preferences given above. The social planner maximizes an objective function that values lifetime utility across each country's representative agent with weights,  $\lambda^j$ . At time 0, the planner maximizes utility over all states and dates given the output processes in each state.

$$\begin{aligned}
 & \underset{\{C^j(S_t)\}}{\text{Max}} && \sum_{j=1}^J \lambda^j U^j(C^j(S_t), E_t[U^j(C^j(S_{t+1}))]) && (3.6) \\
 & \forall j = \{1, \dots, J\} \\
 & \forall S_t \in S \\
 & \forall t \in \mathbb{N}^+
 \end{aligned}$$

$$\text{s.t. } \sum_{j=1}^J C^j(S_t) \leq \sum_{j=1}^J Y^j(S_t), \forall S_t \in S, \forall t \in \mathbb{N}^+ \quad (3.7)$$

Note that the planner maximizes a weighted average of the utility of the representative agent in each country in every state and date subject to the constraint that aggregate consumption does not exceed aggregate output. This constraint arises directly when production is given by an endowment process. Alternatively, optimization can be seen as the allocation of consumption after production decisions have been made. Rewriting the planner's problem using the recursive utility formulation above implies:

$$\begin{aligned} & \underset{\{C^j(S_t)\}}{\text{Max}} \sum_{j=1}^J \lambda^j U^j(C(S_t), E[U^j(C^j(S_{t+1})|I_t)]) \text{ s.t. } \sum_{j=1}^J C^j(S_t) \leq \sum_{j=1}^J Y^j(S_t) \\ & \forall j = \{1, \dots, J\} \\ & \forall S_t \in S \\ & \forall t \in \mathbb{N}^+ \end{aligned} \quad (3.8)$$

Assuming that the consumption good is non-durable, the resource constraint will hold with equality. In this case, the social planner's first order conditions give the familiar condition that marginal utilities are equalized across all states. I describe these first order conditions in more detail in the appendix.

### 3.3.2 Decentralized Closed Economy

I now examine the decentralized economy by focusing first on an international economy in which all countries are in autarky. The representative agent in each country

$j$  is originally endowed with the ownership rights on the productivity stream of output from his country:  $Y^j(S_t) \forall S_t \in S$ . I further restrict this output to be generated by an exogenous endowment stream and, where there is no possibility of confusion, I adopt the convention that  $X_t \equiv X(S_t)$  for any variable  $X$  that is a function of the state. Given dividend payments from the representative agent's endowment,  $Y_t^j$ , he consumes and then buys claims on the endowment process for the following period at price  $P_t^j$ . Defining the claims on country  $\ell$ 's endowment process held by country  $j$  as  $\varpi_t^{j\ell}$ , the agent's optimization problem in autarky is given by:

$$\begin{aligned} \underset{\{C_t, \varpi_t^{jj}\}}{\text{Max}} U_t^j \quad \text{where } U_t^j &= \left\{ (1 - \delta)C_t^j \left(\frac{1-\gamma}{\theta}\right) + \delta \left( E_t [U_t^j]^{1-\gamma} \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\ \text{s.t. } C_t^j + P_t^j \varpi_t^{jj} &\leq W_t^j \\ (Y_{t+1}^j + P_{t+1}^j) \varpi_t^{jj} &= W_{t+1}^j \end{aligned} \quad (3.9)$$

In autarky, the agent in country  $j$  consumes his own output and shares on this process are not sold internationally. Since the number of shares is time invariant, I normalize the number of outstanding shares to one so that the agent in country  $j$  holds his own country's shares  $\varpi_t^{jj} = 1$  and is restricted from holding any shares in the other countries,  $\varpi_t^{ji} = 0, \forall i \neq j$ . Therefore, country  $j$  agent's problem can be written more succinctly as the Bellman equation:

$$V_i(C_t^j, W_t^j) = \underset{\{C_t^j\}}{\text{Max}} \left[ (1 - \delta)C_t^j \left(\frac{1-\gamma}{\theta}\right) + \delta E_t \left[ V_{t+1}^j(C_{t+1}^j, W_{t+1}^j)^{1-\gamma} \right]^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (3.10)$$

$$\text{s.t. } W_{t+1}^j = (W_t^j - C_t^j) * R_{t+1}^j \quad (3.11)$$

Where  $R_{t+1}^j = \frac{P_{t+1}^j + Y_{t+1}^j}{P_t^j}$  is the gross return on the claim on the home output and equation (3.11) rewrites the budget constraint using this definition and the restriction that  $\varpi_t^{jj} = 1$ .

As shown by Campbell [11] and Obstfeld [35], the solution to this Bellman equation is :<sup>14</sup>

$$V_t(C_t^j, W_t^j) = (1 - \delta)^{-\left(\frac{\psi}{1-\psi}\right)} (C_t^j)^{\left(\frac{1}{1-\psi}\right)} (W_t^j)^{-\left(\frac{\psi}{1-\psi}\right)} \quad (3.12)$$

Applying the first-order condition in equation 3.4 to the return on the endowment process in autarky, the condition becomes:

$$E_t \left\{ \delta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^j)^\theta \right\} = 1 \quad (3.13)$$

Below I use this Euler equation to determine the equilibrium price of equity in home markets under closed economies,  $P_t^j$ . In this case, the home equity is priced only by its own representative agent. Since agent in country j consumes his own output alone, then his marginal utility uniquely determines the price of the asset that pays dividends  $Y_t^j$ . Moreover, the optimal consumption path depends only upon the home output process, a restriction which clearly violates the social planner's first order conditions given above. I will use this equilibrium as a benchmark for evaluating welfare gains below.

Another way to see the relationship between the decentralized closed economy and the social planner's problem is to directly use the solution to the value function

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<sup>14</sup>Bansal and Yaron [3] and Lewis [31] use the value function solutions in Campbell [10] and Obstfeld [35], respectively, to analyze welfare gains of risk reduction.

above. Using the fact that by the envelope theorem,  $(\partial V_t / \partial Y_t) = (\partial U^j(C_t, I_t) / \partial C_t)$  along the optimal consumption path, I show in the appendix that substituting the solution from the decentralized economy into the planner's first order conditions implies the following requirement for optimality:

$$\ln(\lambda^\ell) + \left( \frac{\psi}{1 - \psi} \right) \{ \ln[C_t^\ell / W_t^\ell] \} = \ln(\lambda^j) + \left( \frac{\psi}{1 - \psi} \right) \{ \ln[C_t^j / W_t^j] \} \quad (3.14)$$

Given that country  $j$  agent wants to smooth consumption over time according to his consumption-wealth ratio,  $\ln[C_t^\ell / W(Y_t^\ell)]$ , depending upon his intertemporal elasticity of substitution in consumption,  $\psi$ , the social planner would choose to allocate consumption across countries in proportion to their consumption-wealth ratio. This relationship is consistent with the approach taken in Lettau and Ludvigson [28] who specify the stochastic discount rate to depend upon this ratio.

If I further assume that the planner's invariant weights are equal across countries so that  $\lambda^\ell = \lambda^j$  then the optimality condition can be further be simplified to:

$$\ln[C_t^\ell / W_t^\ell] = \ln[C_t^j / W_t^j] \quad (3.15)$$

Along the social planner's optimal allocation of consumption across countries consumption-wealth ratios would be equalized. Since the two economies have different output processes,  $\ln[C_t^\ell / W_t^\ell] \neq \ln[C_t^j / W_t^j]$  for all  $\ell \neq j$  with probability one, under autarky the social planner's first order condition cannot hold with probability one.



### 3.3.3 Decentralized Open Economy

I now consider the decentralized open economy in which the representative agents in each country sell off the rights to their own output streams. I define the price of claims for country  $j$  output payouts in world markets at time  $t$  as  $P_t^{*j}$ . In this case, the agent's optimization problem becomes:

$$\underset{\{C_t, \underline{\varpi}_t^j\}}{\text{Max}} U_t^j \quad \text{where } U_t^j = \left\{ (1 - \delta)C_t^j \left(\frac{1-\gamma}{\theta}\right) + \delta \left( E_t [U_{t+1}^j]^{1-\gamma} \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \quad (3.16)$$

$$\begin{aligned} \text{s.t. } C_t^j + \underline{P}_t^{*j} \underline{\varpi}_t^j &\leq W_t^{j*} \\ (\underline{Y}_{t+1} + \underline{P}_{t+1}^{*j}) \underline{\varpi}_t^j &= W_{t+1}^{j*} \end{aligned}$$

where  $\underline{\varpi}_t^j = \{\varpi_t^{j1}, \varpi_t^{j2}, \dots, \varpi_t^{jJ}\}$  is the vector of claims held by country  $j$  investors on each of the country outputs,  $\underline{Y}_t$  is the  $J \times 1$  vector of the output realizations,  $\underline{P}_t^{*j}$  is the price vector of these claims, and  $W_t^{j*}$  is the wealth of country  $j$  at world prices. Since the utility function is homogeneous in consumption and wealth, all agents will hold the same portfolio shares in a world mutual fund. If I define the portfolio share of country  $\ell$  in country  $j$ 's wealth as  $h_t^{j\ell} = (P_t^{*\ell} \varpi_t^{j\ell} / W_t^{j*})$ , the mutual fund theorem implies that the vector of portfolio shares in the wealth portfolio, i.e.,  $\underline{h}_t^j = \{h_t^{j1}, h_t^{j2}, \dots, h_t^{jJ}\}$ , is equalized for each element across countries:  $\underline{h}_t^j = \underline{h}_t^\ell, \forall j, \ell$ . The state of the economy is driven by the endowment processes of all  $J$  countries,

$\underline{Y}_t$ , so that country j agent's problem can then be rewritten as:

$$V_t(W_t^{j*}(\underline{Y}_t), \underline{Y}_t) = \underset{\{C_t, \underline{h}_t^j\}}{\text{Max}} \left[ (1 - \delta) C_t^{j(\frac{1-\gamma}{\theta})} + \delta (E_t [V_{t+1}(W_{t+1}^{j*}(\underline{Y}_{t+1}), \underline{Y}_{t+1})^{1-\gamma}])^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (3.17)$$

$$\text{s.t. } W_{t+1}^{j*} = (W_t^{j*} - C_t^j) \underline{h}_t^{j'} \underline{R}_{t+1}^* \quad (3.18)$$

where  $\underline{R}_{t+1}^*$  is the  $J \times 1$  return vector whose  $j$ -th component is  $R_{t+1}^{*j} = (Y_{t+1}^j + P_{t+1}^{*j})/P_t^{*j}$ .

Defining the return on country j's wealth portfolio as  $R_{t+1}^{jP} \equiv \underline{h}_t^{j'} \underline{R}_{t+1}^*$ , the first-order intertemporal optimization problem for the agent in country j must then satisfy the Euler equation:<sup>15</sup>

$$E_t \left\{ \delta^\theta (C_{t+1}^j/C_t^j)^{(-\frac{\theta}{\psi})} (R_{t+1}^{jP})^{(\theta-1)} R_{t+1}^{*\ell} \right\} = E_t \left\{ \delta^\theta (G_{c,t+1}^j)^{(-\frac{\theta}{\psi})} (R_{t+1}^{jP})^{(\theta-1)} R_{t+1}^{*\ell} \right\} = 1 \quad (3.19)$$

where  $G_{c,t+1}^j \equiv C_{t+1}^j/C_t^j$ . While this Euler equation holds for each individual country's agent, all countries face the same asset market and thereby view the same return vector. Moreover, since all countries have the same utility function and this function is iso-elastic, I show in the appendix that all countries will choose to hold identical shares in a world mutual fund of claims on the output of all participating countries. Therefore,  $\underline{h}_t^{j'} = \underline{h}_t' \forall j$ . Furthermore, in equilibrium, the number of shares in each country are normalized to one so that the world mutual fund returns are given by:  $R_{t+1}^{w*} \equiv \{(\underline{Y}_{t+1} + \underline{P}_{t+1}^*)\nu\} / \{(\underline{P}_t^*)\nu\}$  for  $\nu$  a  $J$ -dimensional unit vector. Defining  $\varpi^j$  as the claims on the world mutual fund held by country j I can rewrite

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<sup>15</sup>See Epstein and Zin (1991).

the value function more succinctly as:

$$V_t(W_t^{j*}(\underline{Y}_t), \underline{Y}_t) = \underset{\{C_t^j, \varpi_t^j\}}{\text{Max}} \left[ (1 - \delta) C_t^j \left( \frac{1 - \gamma}{\theta} \right) + \delta \left( E_t [V_{t+1}(W_{t+1}^{j*}(\underline{Y}_{t+1}), \underline{Y}_{t+1})^{1-\gamma}] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (3.20)$$

$$\text{s.t. } W_{t+1}^{j*} = (W_t^{j*} - C_t^j) R_{t+1}^{w*} \quad (3.21)$$

In this case, the Euler equation of all participants in world markets will be given by:

$$E_t \left\{ \delta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^{(\theta-1)} R_{t+1}^{j*} \right\} = E_t \left\{ \delta^\theta (G_{c,t+1}^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^{(\theta-1)} R_{t+1}^{j*} \right\} = 1 \quad (3.22)$$

This relationship holds as long as all countries hold a constant share of the same mutual fund.<sup>16</sup> In this case, the aggregate resource constraint together with the budget constraint for each country implies further that:  $C_t^j = \varpi^j(\underline{Y}_t \hat{\iota})$  or  $(C_{t+1}^j / C_t^j) = (C_{t+1}^\ell / C_t^\ell), \forall j, \ell$ . Then, the return on this common world consumption growth rate prices all countries and can also be priced itself by the Euler equation:

$$E_t \left\{ \delta^\theta (C_{t+1}^{rw} / C_t^{rw})^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^{(\theta-1)} R_{t+1}^{j*} \right\} = E_t \left\{ \delta^\theta (G_{c,t+1}^{rw})^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^{(\theta-1)} R_{t+1}^{j*} \right\} = 1 \quad (3.23)$$

$$E_t \left\{ \delta^\theta (C_{t+1}^{rw} / C_t^{rw})^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^\theta \right\} = E_t \left\{ \delta^\theta (G_{c,t+1}^{rw})^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^\theta \right\} = 1 \quad (3.24)$$

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<sup>16</sup>I present a justification for this assumption below

Below, I will use these Euler equations to solve for the equity prices in world markets. Since the number of shares in each country is normalized to one, the price of the share of the mutual fund is  $\underline{P}_t^* i = P_t^{w*}$ . I also solve for the individual equities on world markets using to the first order condition for individual securities above.

Before proceeding, I relate this decentralized decision-making to the planner's problem using the value function for each country:

$$V_t(W_t^{j*}(S_t), S_t) = (1 - \delta)^{-\left(\frac{\psi}{1-\psi}\right)} (C_t^j)^{\left(\frac{1}{1-\psi}\right)} (W_t^{j*}(S_t))^{-\left(\frac{\psi}{1-\psi}\right)} \quad (3.25)$$

As above, I use the envelope theorem result that  $\partial V / \partial W = \partial U / \partial C$  and substitute the result into the planner's first order condition. Taking logs implies that

$$\ln(\lambda^\ell) + \left(\frac{\psi}{1-\psi}\right) \{\ln[C_t^\ell / W_t^{\ell*}]\} = \ln(\lambda^j) + \left(\frac{\psi}{1-\psi}\right) \{\ln[C_t^j / W_t^{j*}]\}$$

A necessary condition for the planner's problem to hold is that wealth depends upon the total state vector and that consumption-wealth ratios are equalized. In the appendix, I show that these conditions hold. Note, however, that given the initial consumption weights, the planner's problem only requires the consumption-wealth ratios to be equalized across countries in each state. If welfare gains are to be generated by opening up markets, each country's agent will have to decide whether to participate. These participation constraints can lead to different allocations of the welfare gains as I describe next.

### 3.3.4 The Decision to Open Markets

Above I described an open economy equilibrium when all countries are open and willing to participate. However, I have not shown that all countries would want to participate by selling off their claims on their home output and holding diversified

shares of the world economy instead. This equilibrium requires that no country would prefer to deviate and close their markets. While this deviation could potentially dominate an open market in any date, I consider an equilibrium in which there are complete contingent claims in all future periods. For this equilibriums, I require only that no country would prefer autarky ex ante.<sup>17</sup>

To see the possibility for autarky to dominate, consider the timing of markets within the initial period. Agents enter the period with the perpetual claim on their home output and receive the initial endowment on this claim. They then sell off this claim in world markets and in turn buy shares in the world mutual fund. Thus, in equilibrium, an investor in country j faces the constraint:

$$C_0^j + P_0^{w*} \varpi_0^{jw} \leq (Y_0^j + P_0^{j*}) \quad (3.26)$$

With this timing, the agent consumes his endowment in the first period which implies:  $\varpi_0^{jw} = (P_0^{j*}/P_0^{w*})$ . Thereafter, the portfolio constraint tracks the evolution of wealth of country j as:

$$C_t^j + P_t^{w*} \varpi_t^{jw} \leq (Y_t^w + P_t^{w*}) \varpi_{t-1}^{jw} \quad (3.27)$$

where  $Y_t^w$  is the per capita endowment of the world,  $\underline{Y}_t$ , and  $P_t^{w*}$  is the price of a claim that pays off this world endowment.

Does an agent at time 0 choose to integrate with the rest of the world in all future periods? In this period the agent decides whether to sell claims in the open economy or whether to stay in autarky. In making this decision, I assume that agents can fully commit to staying in the integrated world market once they have agreed to

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<sup>17</sup>Other possible deviations are explained below.

participate. The ex-ante participation constraint requires that for all countries  $j$  in the risk sharing equilibrium the expected lifetime utility is higher than in autarky. In other words, each country will participate in open markets only if:

$$V_0^j(C_0^{j*}, W_0^{j*}) > V_0^j(C_0^{jA}, W_0^{jA}) \quad (3.28)$$

where  $C_0^{j*}$  and  $C_0^{jA}$  are the initial consumption levels of country  $j$  agents under the open economy and autarky, respectively. In particular, the initial consumption levels of the country  $j$  agents implied by the decentralized economy above would imply the constraint is:

$$V_0^j(\varpi_0^j \underline{Y}, W_0^{j*}) > V_0^j(Y_0^j, W_0^j) \quad (3.29)$$

or alternatively,

$$(\varpi_0^j Y_0^w)^{\frac{1}{1-\psi}} (W_0^{j*})^{-\frac{\psi}{1-\psi}} > (Y_0^j)^{\frac{1}{1-\psi}} (W_0^j)^{-\frac{\psi}{1-\psi}} \quad (3.30)$$

Thus an agent will find it optimal to commit to engage in the integrated world market only if his utility is not higher under autarky. Therefore, his value function is the higher of the two expected utility paths. Using A to indicate "autarky", the decision can be written as:

$$V_0^j(C_0^j, W_0^j) = \text{Max}\{V_0^j(C_0^{jA}, W_0^{jA}), V_0^j(C_0^{j*}, W_0^{j*})\} \quad (3.31)$$

This decision implies a further restriction on the social planner's problem above. Define the set of countries that choose risk-sharing as  $\hat{J}^*$  and the complementary set as  $\hat{J}$  so that  $\hat{J}^* \cup \hat{J} = J$ . Then the set  $\hat{J}^*$  is defined as:

$$\hat{J}^* = \{j : V_0^j(C_0^{jA}, W_0^{jA}) < V_0^j(C_0^{j*}(\hat{J}^*), W_0^{j*}(\hat{J}^*))\}. \quad (3.32)$$

Note that since wealth and consumption under risk-sharing depend upon the set of countries choosing to be open, pinning down these countries requires solving for the fixed point set of countries with higher utility under open markets and the countries who indeed choose to be open. This set restricts the planner's problem above to a smaller set of open markets as follows:

$$\begin{aligned} & \underset{\{C^j(S_t)\}}{\text{Max}} \quad \sum_{j=1}^J \lambda^j U^j(C(S_t), E[U^j(C^j(S_{t+1})|I_t)]) & (3.33) \\ & \forall j = \{1, \dots, J\} \\ & \forall S_t \in \mathcal{S} \\ & \forall t \in \mathbb{N}^+ \end{aligned}$$

$$\text{s.t.} \quad \sum_{j \in \hat{J}^*} C^j(S_t) \leq \sum_{j \in \hat{J}^*} Y^j(S_t), \quad (3.34)$$

$$C^j(S_t) \leq Y^j(S_t) \forall j \in \hat{J} \quad (3.35)$$

The presence of the participation constraints implies that some countries may choose to stay out of the integrated market which will in turn affect the wealth of other countries. If participation constraints are met, countries enter into an agreement where each country  $j$  forgoes  $Y_t^j$  in exchange for  $h_0^j \underline{Y}_t^w$  in consumption. By market clearing condition for the Social Planner problem in equation (3.7), then feasibility requires that  $\sum_{j \in \hat{J}^*} \varpi^j = 1$ .

As described above, I assume that countries can fully commit to share the realization of their output each period once they have sold their equity shares. Alternatively, there may be some periods when individual countries have a large realization of their own output and would prefer to revert to autarky for the period rather than share in the world output. If countries were to effectively default on dividend payments, the risk of this default would affect asset pricing relationships and is beyond the scope of this chapter. Given my assumption that countries can fully commit, countries initially sell off all rights to their own output and hence claims on world output, so that  $\varpi_t^{jw}$  are time-invariant. As a result, all countries with open markets share the same stochastic discount factor in pricing relationships. I use this property in our analysis below.

### 3.4 Evaluating International Risk-Sharing Gains

There are at least two measures for these welfare gains that have been calculated in the literature. First, the welfare gains can be calculated as the percentage increase in initial permanent consumption under autarky that would make the country indifferent between opening markets or leaving them closed. This approach followed by Obstfeld [35] and Lewis [31] requires solving for  $\Delta$  and equating the value functions under autarky and integration. Therefore, subsuming the country superscript and the dependence of wealth on the set of risk-sharing countries  $\hat{J}$  for clarity, calculating welfare gains in this case requires solving for  $\Delta$  in the following equation:

$$V_0((1 + \Delta)C_0^A, W_0^A) = V_0(C_0^*, W_0^*) \quad (3.36)$$



Where  $(C_0^A, W_0^A)$  and  $(C_0^*, W_0^*)$  are respectively the autarky and open economy consumption and wealth. Using the solution for the value function, this welfare gain has the form:

$$(1 + \Delta) = \left\{ \frac{V_0(C_0^*, W_0^*)}{V_0(C_0^A, W_0^A)} \right\}^{(1-\psi)} = \left\{ \frac{C_0^*/W_0^*}{C_0^A/W_0^A} \right\}^\psi \left( \frac{C_0^*}{C_0^A} \right)^{(1-\psi)} \quad (3.37)$$

Welfare gains depend upon both current consumption and the consumption-wealth ratio, reflecting future certainty-equivalent consumption. Welfare gains increase directly with higher current period consumption under risk sharing relative to autarky consumption,  $C_0^*/C_0^A$ , depending on whether the intertemporal elasticity of consumption is greater or less than one. If  $\psi < 1$ , intertemporal substitution is inelastic and countries prefer substitution into current period consumption and welfare gains increase with relatively higher current consumption under risk-sharing. On the other hand, welfare gains also depend upon the consumption-wealth ratio reflecting the expected stochastic discount rate in future periods. If  $\psi < 1$ , a higher consumption-wealth ratio under risk-sharing will have a smaller effect on welfare gains.

The second measure of welfare gains calculates the gains from simultaneously increasing permanent consumption and wealth. This approach is followed by Bansal and Yaron [3]. Since the value function is homogeneous of degree one in consumption and wealth, I can alternatively define welfare gains as the proportion of current consumption and wealth, that would make agents indifferent between autarky and risk sharing. In other words, the gains  $\Delta$  are defined such that:

$$V_0((1 + \Delta)C_0^A, (1 + \Delta)W_0^A) = (1 + \Delta)V_0(C_0^A, W_0^A) = V_0(C_0^*, W_0^*) \quad (3.38)$$

So I can re-write welfare gains as

$$(1 + \Delta) = \frac{V_0(C_0^*, W_0^*)}{V_0(C_0^A, W_0^A)} = \left\{ \frac{C_0^*/W_0^*}{C_0^A/W_0^A} \right\}^{\left(\frac{-\psi}{\psi-1}\right)} \left( \frac{C_0^*}{C_0^A} \right) \quad (3.39)$$

Note that this interpretation of welfare gains affects consumption and wealth in the same proportion and leave the consumption-wealth ratio undistorted.

### 3.4.1 Measuring Gains with Asset Pricing Moments

In order to calculate welfare gains, I now use the Euler equations to calculate the prices under closed and open economies. In closed economy, I use the identity  $W_0^A = C_0^A + P_0^A$ , to rewrite the value function in terms of the price to consumption ratio<sup>18</sup>. When markets open, I allow the countries to sell shares of their own contingent claim for a share of the world contingent claim. Defining  $Z_{c,t}^A$  as the price-consumption ratio under closed markets,  $Z_{c,t}^A = P_t^A/C_t^A$ , and  $Z_{c,t}^*$  as the price-consumption ratio under open markets,  $Z_{c,t}^* = P_t^*/C_t^*$ , and using the budget constraint  $W_t^* = C_t^* + P_t^*$ , I can rewrite the value function at time 0 as:

$$V_0(C_0^A, W_0^A) = (1 - \delta)^{\frac{-\psi}{1-\psi}} (1 + Z_{c,0}^A)^{\frac{\psi}{\psi-1}} C_0^A \quad (3.40)$$

$$V_0^*(C_0^*, W_0^*) = (1 - \delta)^{\frac{-\psi}{1-\psi}} (1 + Z_{c,0}^*)^{\frac{\psi}{\psi-1}} C_0^* \quad (3.41)$$

Using the above definition for the value function and solving for the definition of welfare gain, I have:

$$(1 + \Delta) = \frac{V_0(C_0^*, W_0^*)}{V_0(C_0^A, W_0^A)} = \left( \frac{1 + Z_{c,0}^*}{1 + Z_{c,0}^A} \right)^{\frac{\psi}{\psi-1}} \left( \frac{C_0^*}{C_0^A} \right) \quad (3.42)$$

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<sup>18</sup>See Appendix A

I use these autarky and open economy consumption claim price measures to calculate the potential gains from consumption risk-sharing based upon three different allocation weights of initial consumption,  $\left(\frac{C_0^*}{C_0^A}\right)$ . In the first version, the "equally weighted" allocation, I simply set  $C_0^{j*} = C_0^{jA}$ . Although I show that in many cases this allocation cannot be an equilibrium, many studies ignore the reallocation of resources required to induce countries with more productive and better hedged endowment streams to participate. Our second version is the "price weighted" allocation based upon our decentralized economy above. In this case,  $C_0^{j*} = \varpi_0^j Y_0^w$  where  $\varpi_0^j = (P_0^{j*}/P_0^{w*})$  or the share of world per capita endowment that country  $j$  can buy when selling of its endowment on world markets. Similarly, in this equilibrium,  $C_0^{jA} = Y_0^{jA}$ , country  $j$ 's initial endowment. Thus, in this allocation, the gains can be measured according to:

$$(1 + \Delta) = \frac{V_0(C_0^*, W_0^*)}{V_0(C_0^A, W_0^A)} = \left(\frac{1 + Z_{c,0}^*}{1 + Z_{c,0}^A}\right)^{\frac{\psi}{\psi-1}} \left(\frac{\varpi_0^w Y_0^w}{Y_0^A}\right) \quad (3.43)$$

As I noted above, there may be some countries for whom the autarky path dominates sharing all productivity payments with the rest of the world. For this reason, I also consider a third version of allocation weights I term the "reservation" allocation. In this allocation, I ask what the original consumption level must be under open markets in order to make each country indifferent between opening or not. This allocation is defined by setting  $\Delta$  equal to zero in equation (42), setting  $C_0^{jA} = Y_0^{jA}$ , and then solving for  $C_0^*$ . In other word, the "reservation allocation" is determined by:

$$C_0^{*R} = \left(\frac{1 + Z_{c,0}^A}{1 + Z_{c,0}^*}\right)^{\frac{\psi}{\psi-1}} Y_0^A \quad (3.44)$$

## 3.5 Matching the International Asset Return and Consumption Moments

In this section, I use our international framework to analyze the implications of an international data set of consumption, dividends, and asset returns. As with the US data, I match the implied consumption processes to asset return means and variances. I then use these parameter estimates that are disciplined by asset returns to calculate the implied risk-sharing gains in a two country framework.

I begin by describing our data set in more detail. Then I detail our methodology for estimating the model parameters. This methodology uses a simulated method of moments (SMM) procedure to find a set of country parameters that best replicates the data moments. I then describe the implied risk-sharing gains based upon these parameters.

### 3.5.1 Data Description

This data set is comprised of two main data sources broken down by asset returns and consumption. For consumption, I use PPP-adjusted per capita consumption measures from Penn World Tables National Accounts. As I described in Table 16, I also checked our results for the US against the existing literature based upon the National Product and Income Accounts (NIPA). Note that our analytical framework above is based upon growth rates except for the initial consumption allocations. Therefore, I use the US as a numeraire real consumption measure in growth rates. However, our results are not sensitive to this assumption. Details about constructing the consumption series are provided in the appendix.

For dividend and return data, I use the data from Campbell (2003). To be consistent with the annual consumption data in PWT, I aggregate the quarterly data in Campbell using the same deflator series from the Penn World Tables to form real annual equity returns, risk free rates, and dividend growth rates. The equity return data span the sample period 1970-1999. To maintain consistency, I use the real risk free rate for the same period. Details about the aggregation of the asset return and dividend series are provided in the appendix.

The set of countries I examine are restricted to a group for which I have a consistent set of asset returns and consumption. These countries are Australia, Canada, France, Germany, Japan, UK, and the US. All the data moments are presented in Table 16. The top panel shows the mean and standard deviations of equity returns and risk-free rates for the seven countries since 1970. The mean equity premium ranges from a low of about 1.5% for Australia to a high of 6.6% for the UK. The risk free rate shows a tighter range of 1.2% for Japan to 2.7% for Canada. The standard deviation of equity is high for all these countries. Moreover, the standard deviation of the risk-free rate is comparable to the size of the mean risk-free rate ranging from 1.7% to 3% per annum. As noted above, typical models based upon i.i.d. consumption imply that the variance of the risk-free rate is equal to zero.

The middle panel shows basic statistics for consumption. The mean growth rates are reported first along with their standard errors. Canada has the lowest growth rate at 1.9%. But the highest growth rate by far is Japan at 4.9%. Another outlier is Australia which exhibits a small but negative first order auto-correlation coefficient. The standard deviations show a significant amount of variability in annual consumption growth.

The bottom panel reports summary moments for dividend growth. The mean growth rate in the US is positive, generating higher anticipated pay-offs in the dividend-paying asset. However, for Canada and France, the dividend growth rate is marginally negative and for Japan it is significantly negative. However, the standard errors on these mean growth rates show that the mean dividend growth rates are typically insignificantly different from one. Fortunately, as equations 3.1 and 3.2 show, the key variable from the dividend process needed to determine the consumption process depends on the variance and not the mean of dividend growth.<sup>19</sup>

Given these summary statistics, I next discuss the methodology used to solve for the parameters in the model.

### 3.5.2 Solution Method

To discipline our model, I require the parameter values for the processes of consumption and dividends to generate the asset return moments I observe in the data. To generate the parameters values, I first use the annual means of consumption growth and dividend growth to calibrate the monthly rates  $\mu$  and  $\mu_d$ . For this purpose, I calculate the mean annual growth rates from the data and divide by 12. In trial runs of the SMM procedure described below, I find that this change makes little difference in the estimation of the remaining parameters and greatly decreases the computation time.

Next I use a reduced (first-pass) SMM to estimate the parameters for each country. Implementing this procedure involves the following step. For every set of

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<sup>19</sup>The mean of dividend growth will be important for measuring the welfare gains when I consider the incomplete markets version of the model, however, I am currently investigating longer sources of dividends that may provide a better estimate of long term dividend growth.

parameter values, I first solve the model using the analytical solutions for returns in the closed economy. I then compute a weighted difference between a targeted set of model generated moments and the data moments using a weighting matrix.<sup>20</sup> The set of parameter values that minimizes this difference is the SMM estimate.

I choose the following set of data moments to target for each country: the standard deviation of log consumption growth ( $\sigma(g_c)$ ), the first order auto-correlation of log consumption growth ( $\rho_1(g_c)$ ), the standard deviation of log dividend growth ( $\sigma(g_d)$ ), the mean equity premium ( $E(r_m - r_f)$ ), the mean risk free rate ( $E(r_f)$ ), the standard deviation of the market return ( $\sigma(r_m)$ ), and standard deviation of risk free rate ( $\sigma(r_f)$ ). Using these seven moments per country, I estimate the 5 parameters in the model for each country given in equations 3.1 and 3.2. These are: the variance of the transitory component of consumption,  $\sigma^j$ , the ratio of this variance to the long run risk variance,  $\varphi_e^j$ , and to the dividend variance,  $\varphi_d^j$ , the autocorrelation of the long run risk component,  $\rho^j$ , and the sensitivity of dividends to long run risk,  $\phi^j$ . The set of targeted moments were chosen to best represent both consumption and asset pricing data<sup>21</sup>.

Below I also consider two restricted cases. In one case, I assume all countries have the same autocorrelation coefficient on long run risk;  $\rho^j = \rho$  for all j. In this case, the autocorrelation is set equal to the mean across countries since the parameter estimates are relatively close to each other. In the other restricted case, I assume that all countries have a common long run risk component. This long run risk component is assumed to be the same as the estimate from the US.

Our estimation requires a set of preference parameters. For this purpose, I use

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<sup>20</sup>In weighting the target moments, I implement the reduced SMM procedure using both the identity matrix and a diagonal matrix with typical components equal to the sample variance.

<sup>21</sup>All the usual criticisms of moment selection apply, see Gallant and Tauchen for discussion on efficient method of moments

parameter estimates that have been found to fit asset returns best in the US. I therefore take the parameters from Bansal and Yaron [3] of  $IES = 1.5$ ,  $\gamma = 10$ , and  $\beta = .998$ . As is standard in the literature and required from our model, these parameters are the same across all countries.

As stated earlier, the model is written and estimated at the monthly level and therefore the simulated data from the model must be time-aggregated to match the annual data moments<sup>22</sup>. Therefore to match our annual consumption, dividend growth and asset return moments, I time-aggregate the model-generated data from monthly to annual frequency. Parameter estimates and simulated model moments are the averages of 500 simulations, each with 840 time-aggregated monthly observations.

Table 17 shows the resulting SMM generate parameters of  $(\rho^j, \sigma^j, \phi^j, \varphi_e^j, \varphi_d^j)$  for each country. Panel A shows the results of the monthly calibrated means of consumption and dividends. Panel B reports the estimation results assuming a common long run risk component. With this constraint, the autocorrelations  $\rho$ , and the variance of the long run component  $\sigma\varphi_e$  are the same across countries. In this case, only the idiosyncratic variance  $\sigma^j$ , the effect of long run risk on dividends,  $\phi^j$ , and the ratio of dividend variance to long run risk,  $\varphi_d^j$ , differ across countries. Panel C shows the same parameter estimates but without requiring the long run risk to be common across countries. As the estimates show, the variances on the idiosyncratic and long run components are comparable across both versions of the model. The main difference is that the long run risk variance is 2 to 3 times higher for the US, UK, and Japan in the individual long run risk case compared to the

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<sup>22</sup>By time-aggregate, I compute the growth between the levels at  $t+12$  and  $t$ , given the realizations of 12 monthly growth rates. In comparison, by annualize, I mean monthly growth rate times 12



common long run risk case.

Table 18 compares the model moments from simulated data with actual data moments for both versions of the model. For convenience, Panel A repeats the targeted data moments from Table 16. These are the standard deviation and first order autocorrelation of consumption growth, the standard deviation of dividend growth, the mean of the equity premium, the standard deviation of equity returns, the mean of the risk-free rate, and the standard deviation of the risk free rate. Panels B and C show the counterparts generated by the model for the case in which there is a common long run risk and individual long run risk component, respectively. Generally, the model does a reasonable job at matching the standard deviation patterns of consumption and dividend growth, although it has a harder time matching the first order autocorrelations across countries. The model also matches the general pattern of mean equity premia, the risk-free rate and the variance of equity returns. However, the standard deviation of the risk free rate in the model is lower than the data. This feature is likely to improve once I include stochastic volatility.

Overall, therefore, my consumption processes match the basic features of the asset return model. I next use these estimates to reconsider the implications for international risk-sharing gains in a two country framework.

### **3.5.3 Welfare Gains with Long Run Risk: A Two Country Example**

To understand the effects of embedding a long run risk component to consumption growth on international welfare gains, I show the implications for a two country example. For this example, I use the US data described above and analyze it in the context of another country that can be either similar or very different from the US.

Note that welfare gains for all three measures of initial consumption allocations depend upon the ratio of the price-to-consumption ratio,  $Z_{c,0}$ . Moreover in the equally weighted allocation, these prices determine the gains uniquely. To develop intuition about these prices, consider how these prices depend upon the autarky and world endowment processes.

For this purpose, consider the pricing of the equilibrium consumption process. First, using the Campbell-Shiller decomposition to express returns in terms of the price-to-consumption ratio, with approximating constants  $k_0^j$  and  $k_1^j$  I have:

$$r_{c,t+1}^j = k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{c,t+1}^j \quad (3.45)$$

I then solve for the equilibrium log price to consumption ratio by conjecturing that the log price to consumption ratio is linear in the source of long run risk,  $z_t^j = A_0^j + A_1^j \bar{x}_t + A_2^j x_t^j$ . Using the Euler equation and applying the properties of log normality for consumption asset return,  $r_{c,t+1}^j$ , and consumption growth,  $g_{c,t+1}^j$ , implies:

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] = 0 \quad (3.46)$$

Into this Euler equation I then substitute the stochastic processes for log consumption growth,  $g_{c,t+1}^j$ , log price to consumption ratios,  $z_{t+1}^j$ , and the long run risk terms,  $x_{t+1}$ . Taking conditional expectations and conditional variances<sup>23</sup>, the left hand side becomes the sum of a constant term and  $x_t$  terms<sup>24</sup>. In the appendix, I

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<sup>23</sup>Note conditional expectation of shocks are zero and conditional variance of shocks are equal to one

<sup>24</sup>For a detailed derivations of prices in autarky and the open economy see the appendix.

show that these steps imply the following analytical form for the coefficients of the log price to consumption ratio when long run risk is shared across countries<sup>25</sup>:

$$A_1^j = \frac{1 - \frac{1}{\psi}}{1 - k_1^j \rho} \quad (3.47)$$

$$A_0^j = \frac{\ln \delta + k_0^j + (1 - \frac{1}{\psi})\mu^j + \frac{1}{2}\theta((1 - \frac{1}{\psi})^2(\sigma^j)^2 + (k_1^j A_1^j \sigma^j \varphi_e^j)^2)}{1 - k_1^j} \quad (3.48)$$

Thus, the value function depends upon  $1 + Z_{c,0}^j = 1 + \exp(z_{c,0}^j) = 1 + \exp(A_0^j)$ . But what is  $A_0^j$ ? It measures in certainty equivalent terms the long run effect of the consumption processes on the price-to-consumption ratio. In particular, when long run risk is absent,  $A_0^j$  depends upon approximating constants, the time preference parameter,  $\delta$ , and  $(1 - \frac{1}{\psi})(\mu^j + \frac{1}{2}(1 - \gamma)(\sigma^j)^2)$

Figure A-1 illustrates the trade-offs in certainty equivalent consumption implied by these parameters under autarky and the open economy for a base case of two countries parameterized to look like the US. In this example under autarky, the US mean growth rate  $\mu^j$  and the standard deviation  $\sigma^j$  are given by our monthly parameter estimates of .19% and .79%, respectively. I consider two different assumption about risk aversion:  $\gamma = 2, 10$ . I then consider the effects upon the consumption growth path if two identical countries agree to share risk and growth rates so that  $\mu^{j*} = \sum_i \frac{1}{2}\mu^{iA}$  and  $\sigma^{j*} = \iota'\Omega\iota$  where  $\Omega$  is the variance-covariance matrix across countries. For this figure, I assume that the correlation across countries is 0.5.

The figure depicts the trade-offs in growth rates and transitory variability over time. When risk aversion is equal to 10, each country faces a flatter consumption

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<sup>25</sup>When long run risk differs across countries there is an additional effect arising from idiosyncratic long run risk. The solution is detailed in the appendix.

profile given by the solid blue line than under the world pink line. Since both countries face the same mean growth rates, the increase in consumption profile arises from the reduction in variability alone. Alternatively, when the risk aversion parameter is only 2, the improvement in consumption profile moving from the dotted green line to the world line is very minor. Finally, the figure shows the effects on long run risk when it is assumed that all countries face the same long run risk. First, note that since the variance of long run risk is small in the data (e.g., with a standard deviation of only 0.0003), the effects of this risk on the unconditional consumption profile is small. Second, for the case depicted in this figure, all countries share the same long run risk effects so there is no opportunity to share the risk. As such, the effects of long run risk on welfare have been shut down.

Figure A-2 depicts a similar example of two countries except that now country 2 is assumed to have a standard deviation in monthly consumption that is three times as variable as country 1. For this picture, I subsume the long run risk effect and assume that risk aversion is equal to 10. I also continue to assume that the initial consumption allocation is the same for both countries and therefore would correspond to an "equal weight" allocation. However, the picture demonstrates that the effect upon welfare improvement would not be the same for both countries. In particular, country 2 has much higher variability resulting in a downward sloping consumption profile. By contrast, country 1 has an increasing consumption profile. While the reduction in variance still leads to an increase in the consumption profile in the world, country 1 obviously does not benefit as much as country 2.

Figure A-3 shows an example in which the two countries have the same variance, but differing growth rates. In this case, country 2 has an autarky growth rate that is 3% higher than country 1. Now country 1 benefits more than country 2 by pooling

claims to productivity.

While this example has illustrated the effects of mean growth rates and variance on consumption profiles and thereby  $Z_{c,0}^j$ , it does not address the effects of the intertemporal elasticity of substitution in consumption,  $\psi$ . To understand this effect, it is first useful to consider the effect of the welfare ratio  $\left(\frac{1+Z_{c,0}^*}{1+Z_{c,0}^A}\right)^{\frac{\psi}{\psi-1}}$  holding constant the effects of  $\psi$  on  $Z_{c,0}^j$ . Differentiation implies that  $\partial\Delta/\partial\psi = \frac{1}{(1-\psi)^2} \left(\frac{1+Z_{c,0}^*}{1+Z_{c,0}^A}\right)^{\frac{\psi}{1-\psi}} \log\left(\frac{1+Z_{c,0}^*}{1+Z_{c,0}^A}\right)$ . Thus in the extreme when the autarky price of consumption stream is close to the open economy price of consumption stream, the log of the ratios of these price-consumption streams is zero and welfare gains are not affected by the intertemporal elasticity of consumption (IES). Alternatively when the price of the world to consumption ratio is significantly higher, the welfare gains will tend to increase with IES although here the effects of IES on  $Z$  become relevant.

I now use our two country example to consider the impacts on welfare gains assuming our three different sets of initial consumption allocations. I begin in Table 20 by assuming the two countries are symmetric but have a correlation of 0.5. Since the countries are symmetric, their price shares are equal and the equally weighted and price weighted gains are identical. I report the gains when risk aversion is equal to 2 and 10 and also when IES is equal to 0.5 and also 1.5, as BY argue. These gains are reported for three different assumptions about long run risk across countries. Panel A assumes that the countries all have the same long run risk  $\bar{x}_t$  so that  $\sigma^j \varphi_e^j = \sigma_e, \forall j$ . In this case, the gains to risk-sharing are lower because countries are not able to pool their long run risk, but can only pool their idiosyncratic risk. Panels B and C assume that the two countries have different long run risk processes,  $x_t^j$  components. Panel B attributes the correlation between countries to arise solely from the correlation between the idiosyncratic risk. Thus, in Panel B,

the correlation between monthly consumption growth rates would be determined as:  $\text{Cov}(g_{c,t+1}^1, g_{c,t+1}^2) = \text{Cov}(\eta_{t+1}^1, \eta_{t+1}^2)$ . Moreover, the long run risk components are assumed uncorrelated. In this case, risk-sharing allows the countries to diversify both the idiosyncratic risk and the long run risk component. As a result, the gains increase for given preference parameters. For example, when  $\text{IES} = 1.5$ , the gains increase from about 0.9% to 8.5% when risk aversion is 2 and from 5.4% to 66.7% when risk aversion is 10. Panel C shows the results assuming that the correlation between monthly consumption growth rates are solely determined by the long run risk components or that  $\text{Cov}(g_{c,t+1}^1, g_{c,t+1}^2) = \frac{1}{1-\rho^2} \text{Cov}(e_{t+1}^1, e_{t+1}^2)$ . In this case, since the correlation is one-half on long run risk and the idiosyncratic risk is uncorrelated, the gains from long run risk are smaller than in Panel B. For example, when  $\text{IES} = 1.5$ , the gains decline from about 8.5% to 5.5% when risk aversion is 2 and from 67% to 39% when risk aversion is 10. However, these gains are all larger than when all countries share the same long run risk as in Panel A.

Table 21 examines the gains for the individual long run risk case when correlations are based upon the transitory component. In this table, I show how gains are affected by the use of naive initial "Equal Consumption" allocations and the "Price Weighted" allocations based upon the decentralized economy. Panel A begins with the results for the symmetric case. These gains mirror the results in Table 20. The welfare gains increase in the risk aversion coefficient for a given IES. Moreover, higher IES leads to greater welfare gains for a given risk aversion. Panel A also shows the effects on gains when the correlation is negative. In this case, the two countries are better able to reduce their risk. As a result welfare gains increase as the correlations across countries decrease.

Panel B of Table 21 demonstrates the effects on gains when one country has

higher variance than the other. In particular, I assume that country 2 has 1.1 times higher variance than country 1. In this case, the equally weighted gains show that country 2 would benefit more than country 1. The intuition behind this result is clear since country 2 had a higher variance in autarky. However, the table also shows the price effects. Since country 1 has a more valuable endowment claim in world markets, it commands a higher price and therefore a higher initial consumption share. As a result, the price-weighted gains show that country 1 captures more of the welfare gains in the equilibrium.

Panel C of Table 21 shows the same exercise except that now country 2 has a mean growth rate that is 1.1 times that of country 1. In this case, equally weighted "gains" are negative for country 2 when risk aversion is 2. The reason is that the country 2 agent gives up a higher growth rate in return for a lower variability in consumption that with low risk aversion, he doesn't care as much about reducing volatility. Therefore, he clearly would prefer autarky to an equally weighted agreement. However, this example only illustrates that the equally weighted gains are not equilibria. The price weighted gains show once more that the high growth country 2 is compensated with a higher price in world markets and, hence, a greater welfare gain. In the case of risk aversion of 2 and IES of 0.5, country 2 goes from a welfare cost of 0.5% under equal weights to a gain of 7.8% under priced weights. Similarly, when risk aversion is 2 and IES is 1.5, the "gains" go from -0.7% to 31%.

Table 22 considers an alternative consumption allocation to the decentralized economy. I examine the allocation implied when the country with the highest reservation consumption level commands an "all-or-nothing" offer to the other countries. In particular, the country with the highest reservation consumption, defined as  $j = H$  for "High",  $C_0^{H,R}$  gives other countries the offer to set  $\{C_0^{1,*}, C_0^{2,*}, \dots, C_0^{j,*}, \dots, C_0^{J,*}\} =$

$\{C_0^{1,R}, C_0^{2,R}, \dots, C_0^{j,R}, \dots, C_0^{J,R}\}, j \neq H$  subject to the feasibility constraint that  $\sum_{j=1}^J C_0^{j,R} \leq \sum_{j=1}^J Y_0^j$  when populations are equal.<sup>26</sup> I also show the other extreme if the low country is able to extract the rents.

Considering once again the individual long run risk on the transitory consumption component case, Table 22 reports the allocation weights and gains for the set of BY parameters given in the table. Panel A shows the allocations for equal weighted gains equal to 1 by construction. For these parameter estimates, the gains are 67%. In the columns with headings "Reservation 1," I report the allocation for country 1 implied by his agents reservation allocation. The allocation of 0.6 implies that he would be willing to participate in risk-sharing as long as his allocation is at least 60% of his autarky endowment. If he gives the other country a take it or leave it offer of the other country's reservation allocation, he offers only 0.6 by symmetry. The table reports that country 1's gains are then 233.4%. The information under "Reservation 2" reports the symmetric version of these results. Similarly, the price allocation gains in the symmetric case imply that both countries get the same gains since the prices of their equity are the same on world markets.

Panel B of Table 22 shows the results of these allocations when country 2 has a higher variance than country 1. In this case, the equally weighted gains will again show that the higher autarky variance country 2 receives more gains. The reservation allocations show that country 2 values opening markets more. In particular, the reservation allocation is 0.52 which is lower than country 1's reservation allocation of 0.64. If country one can extract all the gains from risk-sharing then it will enjoy a gain of almost 160%. The "Price-Weighted" allocations show how the value of the endowment stream on world markets affects the allocations. Since

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<sup>26</sup>In the appendix, I describe the analysis when population weights differ.



country 1 has a lower variance, the value of its equity is higher in world markets and it gets a higher initial allocation consumption allocation of 1.14. The extra 14% is paid for by country 2 which has to reduce its initial allocation to 86%.

Finally, Panel C of Table 22 shows the initial allocation effects on risk-sharing gains when mean growth rates differ. Again, country 2 has a higher growth rate so that country 1 clearly benefits more in the equally weighted allocation. Similarly, the reservation allocations are now switched between the two countries. Country 1 is now will to give up more initial consumption to 0.56 in order to participate in the higher growth rate, which country 2 agent's reservation allocation now increases to 0.63. Once again, the impact of prices on world markets tilt the initial allocations toward the high price country. Country 2 gets 8% more of the initial consumption while country 1 get 8% less.

### 3.6 Conclusion

In this paper I have examined the implications of asset return moments for consumption-based models of international risk-sharing. Relative to standard consumption models, recent asset pricing models have achieved better success at matching these moments by either assuming a small but persistent "long run risk" component in consumption (Bansal and Yaron [3]) or by assuming a persistence to "habit" in utility (Campbell and Cochrane [12]).<sup>27</sup> I developed a framework that nests many models of international financial models and provides a benchmark for calculating welfare gains. By incorporating "long run risk" into the framework, I showed key features of the model with a two country example. In particular, I showed that risk-sharing

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<sup>27</sup>So far, I have focused upon the former explanation, but intend to include the latter in the next version of the paper.

gains depend upon the degree to which this risk component is common or idiosyncratic across countries.

I then used asset pricing and consumption data for seven countries to estimate key distributional assumptions. The estimation is disciplined with the model by requiring the parameters to match standard moments of asset returns with consumption data. The results also indicate where domestic-based estimation of asset pricing models fall short in the international arena.

Then to examine the welfare effects of including a long run risk component to consumption growth, I analyzed a two country framework. The analysis shows a range of gains that depend critically upon the degree of common or world long run risk. In fact, the existence of a long run risk component may even in some cases create the incentive necessary for agents to want to risk share. One potential way to determine the extent to which long run risks are shared among countries in the data is to utilize the correlation structure of both consumption growth and equity returns. For this purpose, I am developing an incomplete markets version of the market in which claims on equity are traded, but claims on consumption are not tradeable. This assumption will allow us to identify the degree to which long run risk co-moves across countries.

Despite these caveats, I believe that the analysis in this chapter provides an important step forward as a first attempt to use asset returns to provide insights into the consumer's views toward risk across countries.

# Appendices

## Appendix A Chapter 2

### Appendix A.1 Tables and figures

Table 1: Summary of Data

	AUS	CAN	EMU	JPN	UK	US	Yankee
Obs	8,446	49,878	48,426	84,997	26,845	352,552	46,782
Bonds	277	1047	1438	2153	535	9224	1274
Firms	97	212	318	164	189	1251	390
As percent of Bonds							
AAA	12.46	1.7	13.55	4.43	3.33	2.99	33.57
AA	24.89	15.38	28.91	25.86	17.46	12.06	6.55
A	45.18	56.28	35.63	41.79	45.09	39.79	33.57
BBB	17.48	26.64	21.91	27.92	34.12	45.15	58.25
As percent of Bonds							
Industrials	38.93	18.56	44.26	45.06	17.87	22.54	11.46
Financials	54.62	70.39	44.43	41.15	56.49	71.31	80.2
Utility	6.45	11.05	11.32	13.79	25.64	6.15	8.35

Table 2: Asset Returns: Jan 1997 - Dec 2008 (in Annualized Percent)

This table shows the mean, standard deviation, and first order autocorrelation of monthly corporate bond and equity return series. Both hedged versus unhedged dollar returns series are reported. Means and standard deviations are expressed in annualized percentages. For example, the hedged return for the Australian corporate bond portfolio is 4.62% annualized with a 2.49% standard deviation. In comparison, the un-hedged dollar return for the same Australian corporate bond portfolio is 10.85% with a 12.27% standard deviation.

	AUS	CAN	EMU	GER	FRA	JPN	UK	US
<b>Hedged</b>								
Corp Inv Grade	4.62	6.29	5.02			5.58	5.05	6.17
Stdev	2.49	3.73	2.96			2.01	4.47	5.42
AC1	0.12	0.01	0.17			0.21	0.03	0.10
Equity: MSCI	6.94	7.97		1.86	2.23	1.47	3.18	5.07
Stdev	13.59	17.66		23.49	18.77	18.64	14.42	16.42
AC1	0.12	0.24		0.11	0.15	0.23	0.07	0.16
<b>UN-Hedged</b>								
Corp Inv Grade	10.85	7.60	6.78			0.47	6.58	6.17
Stdev	12.27	9.41	11.18			11.27	9.68	5.42
AC1	0.14	0.07	0.21			0.01	0.14	0.10
Equity: MSCI	9.98	10.03		3.65	4.01	2.47	4.71	5.07
Stdev	18.50	16.64		25.13	20.66	36.72	18.53	16.42
AC1	0.21	0.23		0.05	0.04	0.22	0.06	0.16

Table 3: Returns of Corporate Bond Portfolios (in Annualized Percent)

This table reports the return statistics of corporate bond sub-portfolios. The portfolios are divided by years left to maturity and by industry. The table shows both annualized means and standard deviation of the return series in percent. For Australia, not enough long term bonds are available for the sample period and therefore is not reported.

	AUS	CAN	EMU	JPN	UK	US
<b>All Corporates</b>	4.62	6.29	5.02	5.58	5.05	6.17
<b>Stdev</b>	2.49	3.73	2.96	2.01	4.47	5.42
<b>By Maturity:</b>						
Short (3-5 Years)	4.79	6.03	4.89	5.60	4.61	5.69
Stdev	3.04	2.83	2.51	2.09	2.70	3.68
Intermediate (5-10 Years)	4.87	6.36	5.32	6.76	5.02	6.14
Stdev	4.53	3.97	3.79	3.68	4.13	5.90
Long (10+ Years)	N/A	6.91	6.13	8.18	5.82	7.11
Stdev	N/A	6.54	5.68	6.63	6.75	9.40
<b>By Industry:</b>						
Industrials	4.55	6.24	4.88	6.12	5.21	6.17
Stdev	2.76	4.11	3.19	2.67	4.66	5.74
Financials	4.52	5.94	4.94	5.26	4.55	6.00
Stdev	2.29	2.88	2.96	1.75	3.72	5.07

Table 4: Monthly Return Correlation

This table reports the pairwise monthly correlation across the investment grade bond portfolios. In comparison, the table shows the equity correlations, which are consistently higher than the bond correlations. Since equity returns is not available for pan Europe, I use the MSCI Germany equity series as a proxy. These correlations are computed using the hedged return series for the two asset markets.

	AUS	CAN	EUR	JPN	UK	US
Investment Grade Corporate Bonds						
AUS	1.00	0.43	0.59	0.20	0.42	0.36
CAN		1.00	0.72	0.11	0.60	0.78
EUR			1.00	0.11	0.66	0.70
JPN				1.00	0.07	0.16
UK					1.00	0.63
Equity						
AUS	1.00	0.65	0.66	0.61	0.72	0.69
CAN		1.00	0.68	0.57	0.69	0.80
GER*			1.00	0.52	0.79	0.79
JPN				1.00	0.54	0.54
UK					1.00	0.79

Table 5: Pairwise Portfolio Efficiency Gain: Foreign Corporate Bonds on US Benchmark

This table shows the results of the monthly excess return regression in Equation 2.4. Individual country's corporate bond portfolio is tested individually against the US benchmark of equity and bond portfolios. "Alpha" is the estimate of the regression intercept, which represents the gain in portfolio Sharpe ratio of including the foreign bond portfolio. For ease, I also report the annualized Sharpe ratio gain in percent. The symbol \* indicates I can reject the null of parameter constancy at 10%, and \*\*, at 5%. Further, the sample period for AUS (2000-2008), EMU (1999-2008) are shortened due to data availability.

	AUS	CAN	EMU	JPN	UK
Alpha	0.0006	0.0009	0.0003	0.0015	-0.0004
t-Stat	1.02	1.50	0.58	3.25**	-0.42
Gain in Annualized %	0.72	0.98	0.36	1.80	-0.48
Mktrf	-0.0500	0.0309	-0.0107	-0.0142	0.0559
t-Stat	-3.24**	2.00**	-0.71	-1.14	2.46**
SMB	0.0258	0.0352	0.0082	0.0112	0.0282
t-Stat	1.65	2.26**	0.55	0.90	1.23
HML	-0.0311	0.0182	0.0238	0.0245	0.0714
t-Stat	-1.66*	0.96	1.34	1.61	2.56**
TERM	0.0733	0.0670	0.0715	0.0076	0.0846
t-Stat	2.74**	2.30**	2.70**	0.33	1.98**
DEF	0.0507	0.4057	0.2631	0.0472	0.3458
t-Stat	0.87	6.29**	4.52**	0.91	3.65**
Adj $R^2$	0.36	0.62	0.55	0.04	0.42

Table 6: Tangency Portfolio Weights: Pairwise with US Benchmark

Corresponding to the portfolio Sharpe ratio gains are mean variance efficient portfolio weights. This table reports the implied efficient portfolio holdings in the foreign corporate bond portfolio and the US benchmark portfolios. Following Britten-Jones(1999), the table also shows the t-statistics of the estimated weight in foreign corporate bonds. The t-statistic tests if the estimated weight is statistically different from zero. The symbol \* indicates I can reject the null of parameter constancy at 10%, and \*\*, at 5%. Further, the sample period for AUS (2000-2008), EMU (1999-2008) are shortened due to data availability.

	AUS	CAN	EMU	JPN	UK
Foreign Corp Bond	0.46	0.79	0.42	0.81**	-0.26
t-Stat	1.02	1.50	0.58	3.25**	-0.42
Mktrf	0.05	0.11	0.04	0.06	0.24
smb	0.17**	0.12	0.26**	0.05	0.27
hml	0.28**	0.25**	0.30**	0.08*	0.47**
TERM	0.15	0.23	0.17	0.10	0.51
DEF	-0.11	-0.50	-0.19	-0.11	-0.22

Table 7: Bayesian Portfolio Weights: varying prior variance  $\sigma_\alpha^2$

This table reports the Bayesian portfolio weights that accounts for estimation risk. The prior variance represents the parameter uncertainty around the alpha parameter in the regression Equation 2.4. The prior mean on alpha is centered around zero with a varying prior uncertainty. With 1% prior uncertainty on alpha being different from zero, the optimal posterior portfolio weight in the Australian corporate bond index is 37%, when considering an optimal portfolio of 5 US benchmark assets and the Australian corporate bond index. Only countries with a positive and statistically insignificant alpha from Table 5 are considered for this analysis. Portfolio weights in the US benchmark portfolio are suppressed from the table.

	Prior Variance: in Annual Percent					MV
	0%	1%	2%	3%	10%	
AUS Corp Bonds	0	0.37	0.42	0.43	0.44	0.46
CAN Corp Bonds	0	0.57	0.67	0.70	0.73	0.79
EMU Corp Bonds	0	0.25	0.34	0.37	0.40	0.42
Pastor (2000)						
MSCI WXUS (70-96)	0	0.07	0.21	0.30	0.45	0.47

Table 8: Joint Test by Maturity and Industry Portfolios

This table reports the F-statistic and corresponding p-value from the hypothesis test that all intercept coefficients in Equation 2.4 are jointly zero. A p-value of greater than 0.05 implies that the corporate bond portfolios are jointly significant at the 5% level. In addition, this table shows the joint test of sub-portfolios that are partitioned by time left to maturity and industry.

	All	Short	Long	Ind	Fin
F-Stat	3.63	4.10	1.21	2.97	4.22
pvalue	0.02	0.01	0.37	0.04	0.01



Table 9: MV Portfolio Weights: All Countries with US Benchmark

This table shows the portfolio weights implied by the tangency portfolio when all foreign corporate bond portfolios are added to the US benchmark portfolios. The table shows that when all countries are included with the 5 US benchmark assets, an optimal portfolio weight in Japan is 69% and in UK is -22%. The symbol \* indicates a rejection of the null at the 10% level, and \*\*, at 5%. Further, the sample period for AUS (2000-2008), EMU (1999-2008) are shorten due to data availability.

	AUS	CAN	EMU	JPN	UK
ALL Country Portfolio against US Benchmark					
Corp IG	0.13	0.14	0.14	0.69**	-0.22**
t-Stat	0.94	1.16	0.87	3.29**	-2.66**
Britten-Jones: Equities					
MSCI (77-96)	0.13	-0.45	-0.18	0.06	0.33
t-Stat	0.54	-1.16	-0.51	0.24	1.01

Table 10: Minimum Variance Portfolio Weights: Pairwise with US Benchmark

This table shows the portfolio weights implied by the minimum variance portfolio when each foreign bond portfolio is added individually to the US benchmark portfolios. The table also reports the in sample annualized standard deviation of the minimum variance portfolio of the US benchmark versus of the US benchmark plus the foreign corporate bond portfolio. The symbol \* indicates I can reject the null of parameter constancy at 10%, and \*\*, at 5%. Further, the sample period for AUS (2000-2008), EMU (1999-2008) are shorten due to data availability.

	AUS	CAN	EMU	JPN	UK
MV Portfolio Weight: in %					
Foreign	87.1	86.7	102.0	87.5	48.7
tStat	20.90**	9.28**	14.71**	24.10**	5.91**
mktrf	5.4	0.0	2.9	2.1	0.7
smb	0.8	5.4**	4.2**	1.7	9.6**
hml	7.1**	11.0**	4.7**	1.8	12.8**
TERM	-7.4**	-12.7**	-10.6**	-2.8	-13.1**
DEF	7.1	9.7	-3.2	9.8**	41.2**
MV Portfolio Stdev: in Annualized %					
	AUS	CAN	EMU	JPN	UK
US Benchmark	4.05	4.05	4.05	4.05	4.05
US Benchmark plus foreign	1.87	3.18	2.50	1.77	3.62

Table 11: Excess Return Regression: All Investment Grade (Jan 1997 - Dec 2008)

This table compares the results of the excess return regression for the US benchmark with and without Yankee bonds. The first and second column of each country reports the result for Equation 2.4 and Equation 2.10 respectively. The inclusion of Yankee bonds into the US benchmark portfolio does not materially change the estimated portfolio Sharpe ratio gains of including any of the foreign corporate bond portfolios. In particular, Japan still provides statistically significant Sharpe ratio increase of .15% per month or 1.8% per year. The symbol \* indicates I can reject the null of parameter constancy at 10%, and \*\*, at 5%. Further, the sample period for AUS (2000-2008), EMU (1999-2008) are shorten due to data availability.

	AUS	CAN	EMU	JPN	UK
Alpha	0.0006	0.0009	0.0003	0.0015	-0.0004
t-Stat	1.02	1.50	0.58	3.25**	-0.42
Yankee	-0.045	0.049	0.045	-0.020	0.050
t-Stat	-0.76	0.57	0.71	-0.61	0.33
Mktrf	-0.050	0.031	-0.011	-0.014	0.056
t-Stat	-3.24**	2.00**	-0.71	-1.14	2.46**
SMB	0.026	0.035	0.008	0.011	0.028
t-Stat	1.65	2.26**	0.55	0.90	1.23
HML	-0.031	0.018	0.024	0.025	0.071
t-Stat	-1.66*	0.96	1.34	1.61	2.56**
TERM	0.073	0.067	0.072	0.008	0.085
t-Stat	2.74**	2.30**	2.70**	0.33	1.98**
DEF	0.051	0.406	0.263	0.047	0.346
t-Stat	0.87	6.29**	4.52**	0.91	3.65**
Adj $R^2$	0.36	0.62	0.55	0.04	0.42
					0.41

Table 12: Bayesian MV Portfolio Weights with Yankees in US Benchmark

This table compares the results of the Bayesian portfolio holding of foreign corporate bonds for the US benchmark with and without Yankee bonds. The second row of each country portfolio is taken out of Table 7 for ease of comparison. Similar to Table 7, this table shows the implied Bayesian weights in the foreign corporate bond portfolio for a given degree of parameter uncertainty on the alpha parameter in Equation 2.4. The degree of parameter uncertainty is captured by the prior variance in annual percent. Portfolio weights in the US benchmark portfolio are not shown.

	Prior Variance in Annual Percent				
	0%	1%	2%	3%	10%
AUS w/Yankee	0	0.31	0.34	0.35	0.35
AUS	0	0.37	0.42	0.43	0.44
CAN w/Yankee	0	0.63	0.66	0.67	0.68
CAN	0	0.57	0.67	0.70	0.73
EMU w/Yankee	0	0.23	0.30	0.33	0.34
EMU	0	0.25	0.34	0.37	0.40

Table 13: Yankee Bonds - Spanning Test, F-statistic (Jan 1997 - Dec 2008)

This table test the ability for US benchmark portfolios to span Yankee bond portfolios. This is equivalent to testing the joint hypothesis that the intercept equals zero and the slope coefficients add to one, in the excess return regression of Equation 2.4 where Yankee bond portfolio is the dependent variable and the US benchmark portfolios are the independent variables. Both the F-statistics and corresponding pvalues are reported. The first set of results use only the US bond portfolios (TERM and DEF) as the spanning variables, while the lower panel uses all 5 US benchmark assets to span the Yankee bond portfolios.

	AUS	CAN	EMU	JPN	UK
TERM, DEF	7.81	4.70	0.72	8.01	2.97
pvalue	0.001	0.010	0.485	0.000	0.052
Adj $R^2$	0.65	0.87	0.79	0.33	0.88
US Benchmark	2.73	4.93	0.42	4.23	3.64
pvalue	0.066	0.008	0.655	0.016	0.027
Adj $R^2$	0.65	0.88	0.80	0.35	0.88

## Appendix A.2 US Holdings of Foreign Bonds

Figure 1: Min Var Portfolio SD, Out of Sample with Constant Weights

The figure graphs the out of sample minimum variance portfolio standard deviation when portfolio weights are estimated once and held over 10 years. MV portfolio weights estimated Jan 1997 - Dec 1999, and held for the period Jan 2000 - Dec 2008. The estimated portfolio weights for the minimum portfolio of both US benchmark and foreign corporate bonds used are -4.4% in mktf, -3.27% in smb, 2.5% in hml, -3.6% in TERM, 68.3% in DEF, 5.6% in Canada, 35.6% in Japan, and 29.2% in UK corporate bonds. In comparison, the portfolio weights for the global minimum portfolio for only the US benchmark are -4.3% in mktf, 3.7% in smb, 15.6% in hml, -40.8% in TERM, and 125.8% in DEF. With the realized return series, the graph plots the annualized past 12 month portfolio volatility.

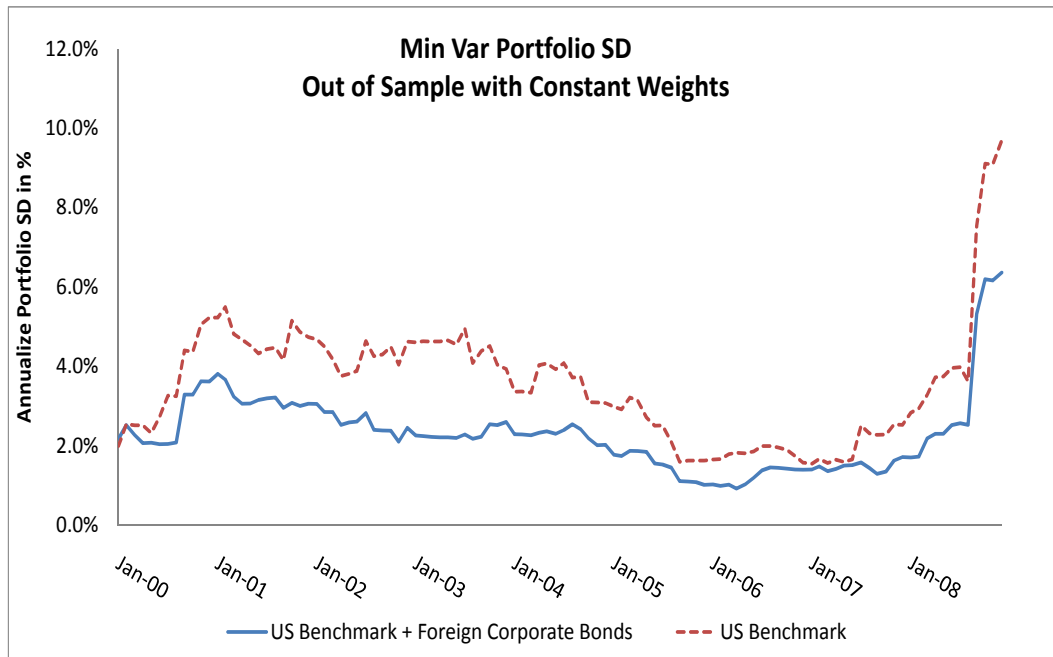


Figure 2: Rebalanced Min Var Portfolio: Out of Sample with 6 month holding period

The figure graphs the out of sample minimum variance portfolio standard deviation when portfolio weights are estimated once and held over 10 years. MV portfolio weights are estimated using the past 24 months, and held for the next 6 months. Then the portfolio weights are re-estimated with the past 24 months, and again held for the next 6 months. This provides a monthly time series of the re-balanced returns, the graph plots the annualized past 12 month portfolio volatility.

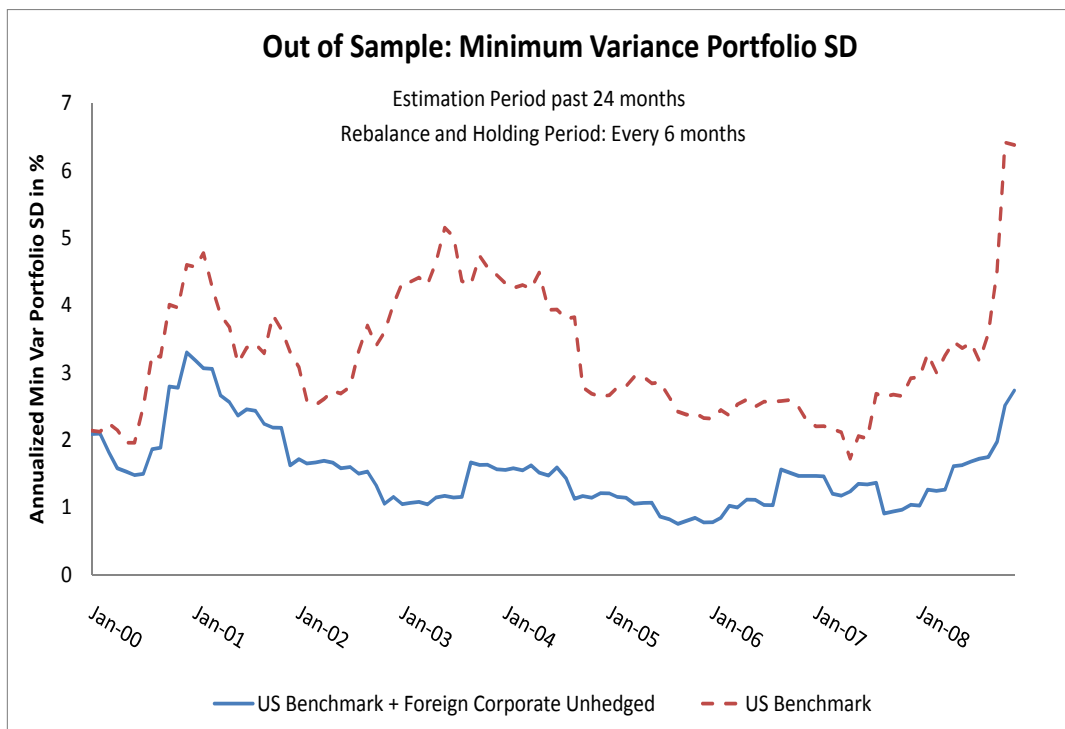


Figure 3: In Sample Rolling Window Min Var Portfolio

This graph shows a rolling estimate of the in sample portfolio standard deviation for the minimum variance portfolio. For every month, I estimate the past 24 month minimum variance portfolio of the US benchmark and the US benchmark plus foreign corporate bonds from Canada, Japan, and the UK. Then this graph plots the annualized volatility of the two estimated minimum variance portfolios. Therefore, unlike the out of sample analysis shown earlier, this figure computes standard deviation of minimum variance portfolio returns from the past 24 months, as opposed to 12 months.

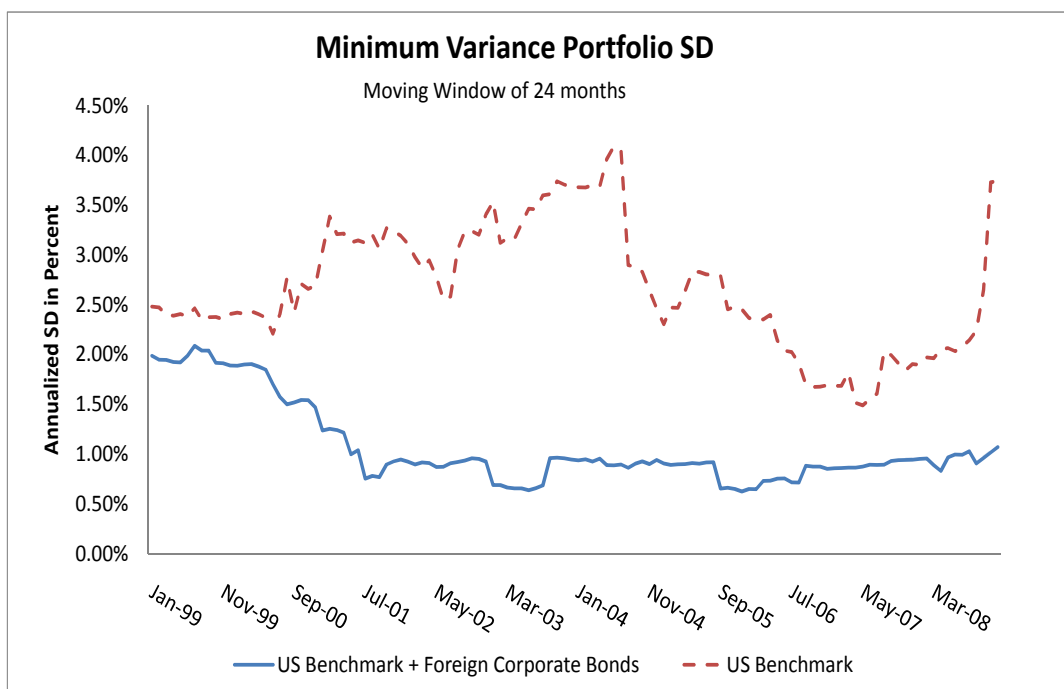


Figure 4: In Sample Rolling Window Min Var Portfolio: Equity v. Corp Bonds

This figure compares the in sample volatility of the minimum variance portfolio with the US benchmark plus equities versus the US benchmark plus bonds. Constructed in the same way as Figure 3, I estimate the past 24 month minimum variance portfolio with the specified group of assets, and plot the annualized volatility. To be consistent with the 3 foreign corporate bond portfolios, I use equity indices for only Canada, Japan, and the UK.

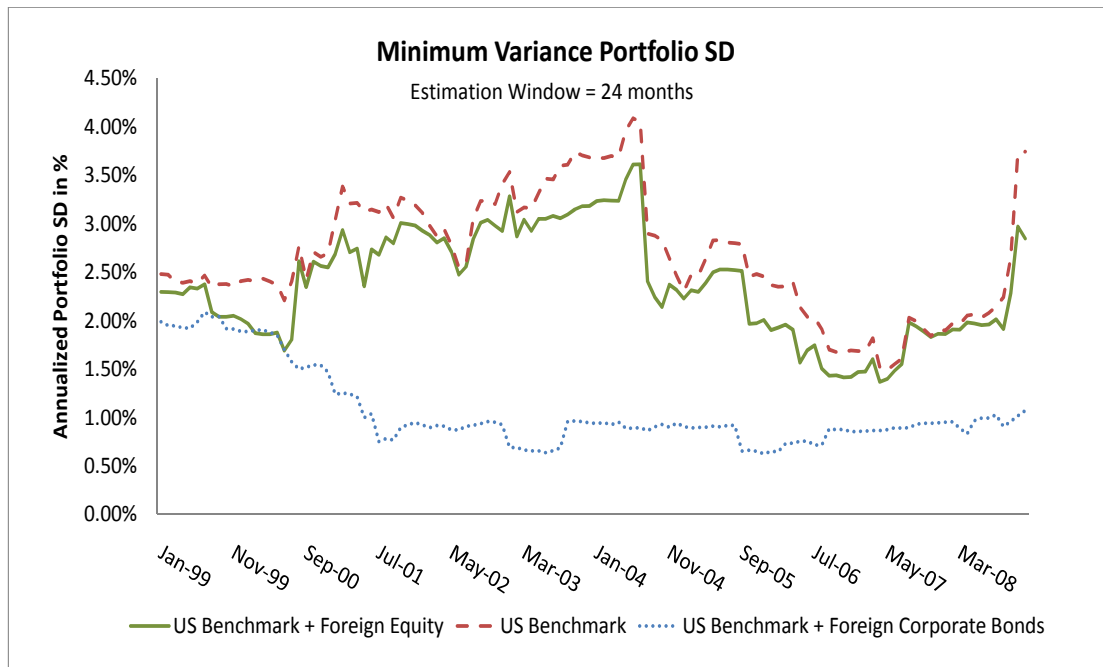




Figure 5: In Sample Rolling Window Min Var Portfolio SD: UnHedged

This figure shows the in sample volatility of the minimum variance portfolio of Unhedged foreign corporate bond returns. Constructed in the same way as Figure 3, I estimate the past 24 month minimum variance portfolio of the US benchmark and the US benchmark with Unhedged corporate bond returns from Canada, Japan, and UK. Unhedged returns are much more volatile as they also incorporate foreign exchange risks. The inclusion of foreign exchange risk greatly reduces the diversification benefits of holding foreign corporate bonds.

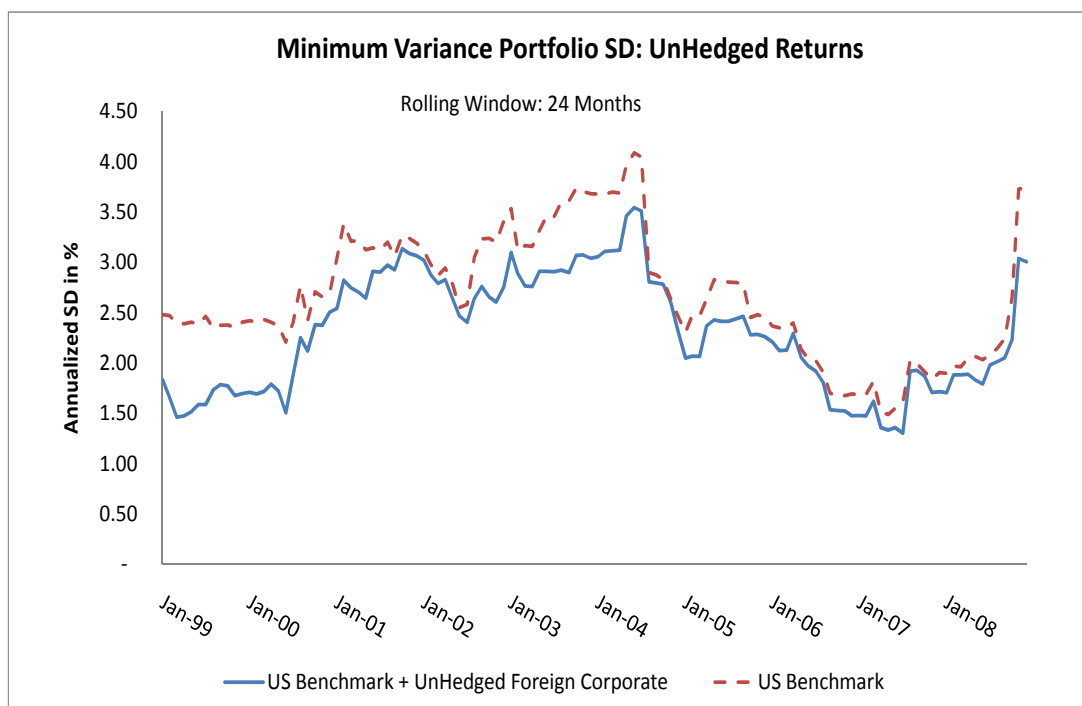


Figure 6: In Sample Min Var Portfolio: US Corporate Bond Benchmark

This figure shows the in sample volatility of the minimum variance portfolio if US corporate bonds are used as the benchmark portfolio. I estimate the past 24 month return volatility of US corporate bonds versus the minimum variance portfolio of the US corporate bonds plus hedged corporate bond returns from Canada, Japan, and UK.

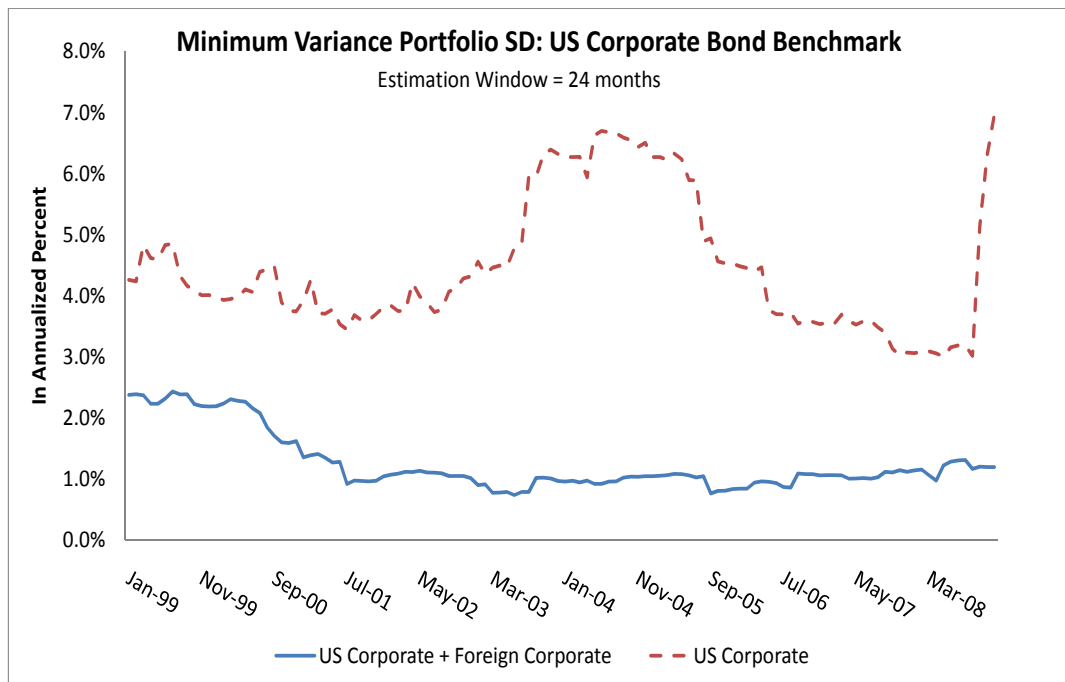


Figure 7: Out of Sample Min Var Portfolio: US Corporate Bond Benchmark

This figure shows the out of sample variance reduction of including foreign corporate bonds into a benchmark of just US corporate bonds. For the US corporate bond, the graph plots the realized volatility from the past 12 months. With foreign corporate bonds, I estimate the weights in the minimum variance portfolio of US and foreign corporate bonds from the past 24 months, then holding those weights, the realized portfolio returns for the next 6 months. And then at the end of the 6 month holding period, I re-estimate the weights and hold it for another 6 months. From the return series, I graph the annualized 12 month standard deviation. Again, the foreign corporate bond portfolios include only Canada, Japan, and the UK.

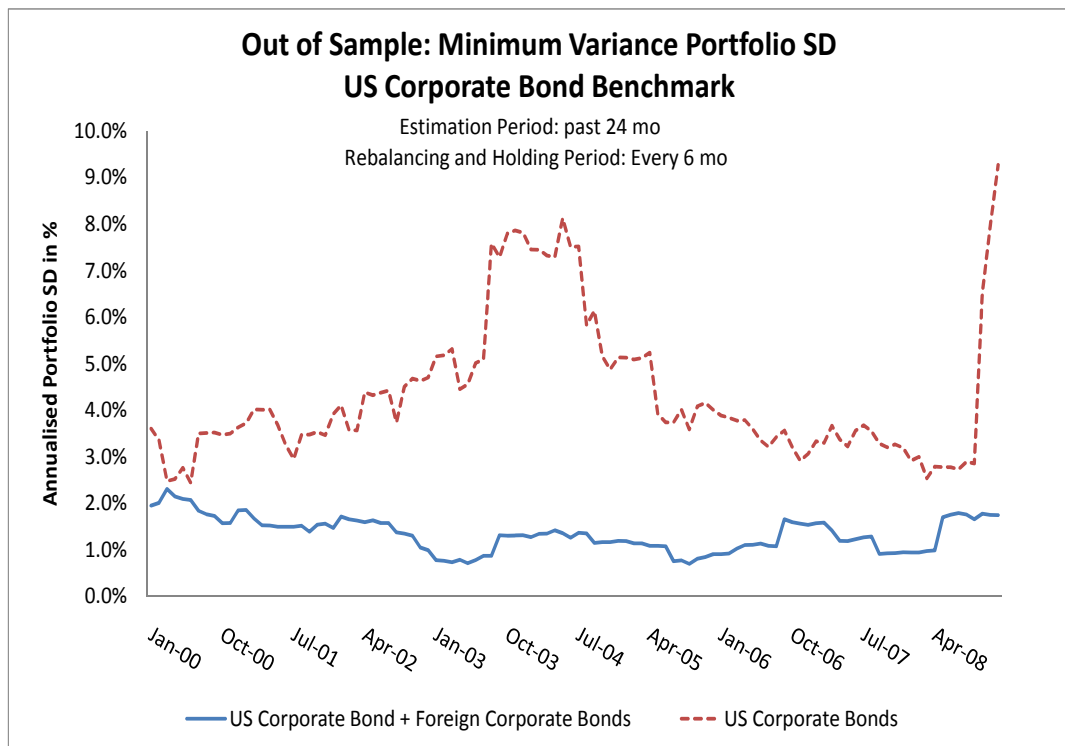


Figure 8: US Holdings of Foreign Bonds and Equities

The calculations are made based on the Flow of Funds levels tables L.212 and L.213, where I use the "Rest of the World" holdings under "Liabilities" as the holdings of foreign bonds or equities by US residents. For the total net asset position for the US, I use the total value of US corporate bonds and equities markets net of what is owed to the foreigners. Therefore, I assume that the wealth of the US residents is the value of US equity and bond market less the portion of the market that is owned by foreigners. Berger and Warnock (2007) using a much finer measure computes the weight of the US investors in local currency foreign bonds to be at 1.2% for developed countries. Using data from the Flow of Funds level tables, Figure 8 graphs foreign bonds and equities holdings by US residents. While there is an upward trend from 2003 - 2008 and the figure shows that most recently foreign bonds makes up about 6.1% of the total portfolio wealth.

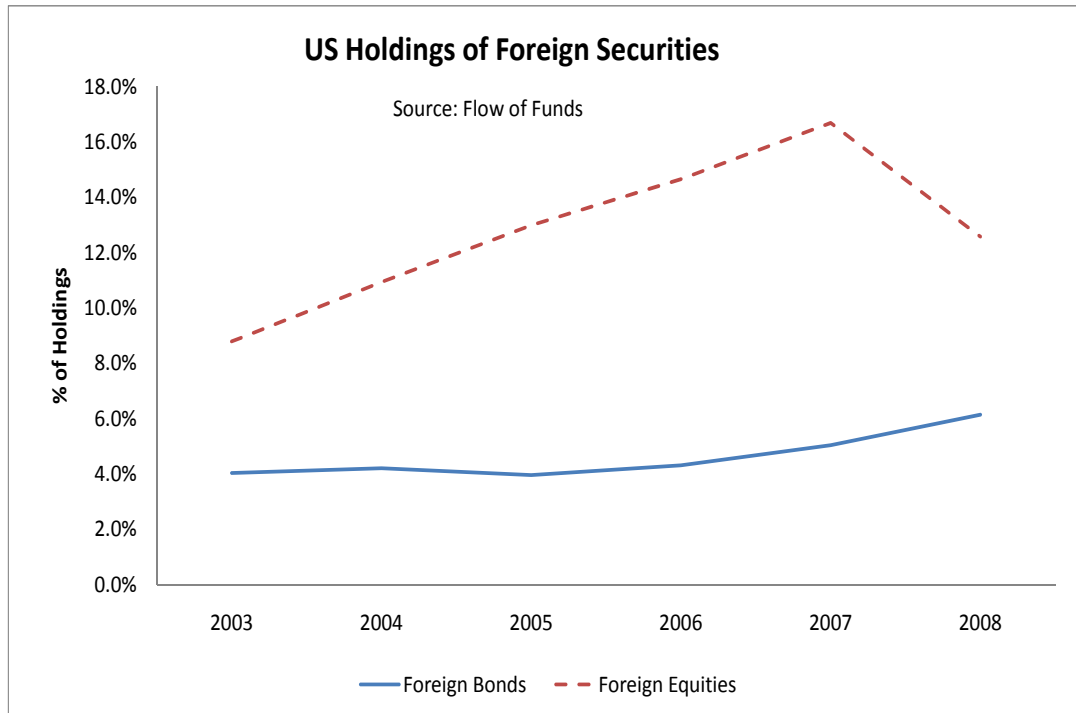


Table 14: Yankee Bond Return Sensitivity (Jan 1997 - Dec 2008)

This table shows the result of the regression of Yankee bonds on US corporate bond portfolio and the home market corporate bond portfolio, as specified in Equation 2.11. This tests the sensitivity of Yankee bond portfolios to US bond market returns versus Foreign corporate bond market returns, and controlling for the interaction term between the US market and Foreign market. The symbol \* indicates a 10% statistical significance, and \*\*, denotes 5% statistical significance. The sample period for AUS (2000-2008), EMU (1999-2008) are shorten due to data availability.

	AUS	CAN	EMU	JPN	UK
Alpha	0.0007	0.0004	-0.0001	0.0002	0.0003
tStat	0.71	0.71	-0.13	0.13	0.67
TERM	0.012	0.037	-0.034	-0.036	0.038
tStat	0.28	1.52	-0.90	-0.62	1.77*
US Corp	0.811	1.006	1.065	0.744	0.864
tStat	9.06**	15.28**	12.67**	6.23**	16.65**
US x FOR	-9.943	-10.258	9.768	-20.710	-2.797
tStat	-1.72*	-4.24**	1.87*	-1.11	-1.82*
FOR Corp	-0.121	0.126	0.026	0.017	0.013
tStat	-0.76	1.60	0.18	0.08	0.29
Adj R2	0.62	0.89	0.78	0.33	0.88

## Appendix B Chapter 3

### Appendix B.1 Tables and Figures

Table 15: Model Comparison for US Consumption and Asset Pricing Moments (In Annual Percent)

	Mehra/Prescott <sup>a</sup>		Bansal/Yaron <sup>b</sup>			Lewis/Liu <sup>c</sup>		
	Data <sup>d</sup>	Model	Data <sup>e</sup>	IID	LRR	Data <sup>f</sup>	IID	LRR
$\sigma(g_c)$	3.6	n/a	2.93	2.19	2.85	1.89	2.71	2.89
$\rho 1(g_c)$	-0.14	n/a	0.49	0.23	0.47	0.19	0.24	0.30
$\sigma(g_d)$	n/a	n/a	11.49	9.89	11.28	5.47	4.61	5.94
$\rho 1(g_d)$	n/a	n/a	0.21	0.22	0.37	0.08	0.22	0.44
$E(r_m - r_f)$	6.18	1.42	6.33	-0.89	4.37	5.47	-0.17	3.50
$\sigma(r_m)$	n/a	n/a	19.42	12.17	16.67	17.56	5.66	10.77
$E(r_f)$	0.80	12.71	0.86	2.18	1.74	1.46	1.88	1.78
$\sigma(r_f)$	n/a	n/a	0.97	0.00	0.63	1.53	0.00	0.42

<sup>a</sup>CRRA Utility with Parameters:  $\beta = 0.99$ ,  $\gamma = 10$

<sup>b</sup>Epstein-Zin-Weil Utility with Parameters:  $\beta = 0.987$ ,  $\psi = 1.5$ ,  $\gamma = 10$ ,  $\mu = 0.15$ ,  $\sigma = 0.78$ ,  $\phi_e = 0.044$ ,  $\rho = 0.979$ ,  $\phi = 3.0$ ,  $\phi_d = 4.5$

<sup>c</sup>Epstein-Zin-Weil Utility with Parameters:  $\beta = 0.987$ ,  $\psi = 1.5$ ,  $\gamma = 10$ ,  $\mu = 0.19$ ,  $\sigma = 0.64$ ,  $\phi_e = 0.044$ ,  $\rho = 0.979$ ,  $\phi = 3.4$ ,  $\phi_d = 1.7$

<sup>d</sup>Consumption: Kuznet-Kendrik-USNIA Non-durable and Services for 1889-1978, Asset Data: S&P Composite

<sup>e</sup>Consumption: NIPA Non-durable and Services for 1929-1998, Asset Data: CRSP 1929-1998

<sup>f</sup>Consumption: PWT Total Consumption for 1950-2000, Asset Data: Campbell Handbook for 1970-1998

Table 16: Summary Statistics: in Annual percent

	AUS	CAN	FRA	GER	JAP	UK	US
<b>Asset Pricing Data<sup>a</sup>:</b>							
$E(r_m)$	3.55	5.43	8.73	7.73	4.96	7.92	6.93
$\sigma(r_m)$	22.60	17.28	22.51	19.81	21.77	21.14	17.56
$E(r_f)$	2.06	2.69	2.42	2.61	1.24	1.28	1.46
$\sigma(r_f)$	2.49	1.77	1.69	1.32	2.17	2.92	1.53
$E(r_m - r_f)$	1.49	2.74	6.31	5.12	3.72	6.65	5.47
$E(g_d)$	0.637	-0.416	-0.429	0.250	-2.29	0.739	1.49
$\sigma(g_d)$	13.68	8.16	11.43	9.37	5.43	8.28	5.47
$\rho_1(g_d)$	0.181	0.397	0.514	0.490	0.545	0.137	0.076
	AUS	CAN	FRA	GER	JAP	UK	US
<b>Log Consumption Growth Data<sup>b</sup>:</b>							
$E(g_c)$	2.17	1.90	3.12	2.85	4.90	2.17	2.29
$\sigma(g_c)$	3.51	2.05	3.28	3.86	3.35	1.86	1.89
$\rho_1(g_c)$	-0.074	0.236	0.110	0.164	0.552	0.323	0.188

<sup>a</sup>Sthece: Campbell (1970-1999)

<sup>b</sup>Sthece: Penn World Tables (1950-2000)

Table 17: Estimated Monthly Parameters by Country - W/O Stochastic Volatility

	AUS	CAN	FRA	GER	JAP	UK	US	BY-US
<b>A: Calibrated</b>								
$\mu$	0.181	0.158	0.26	0.238	0.408	0.181	0.191	0.150
$\mu_g$	0.053	-0.035	-0.036	0.021	-0.191	0.062	0.124	0.150
<b>B: Common LRR (w/BY <math>\sigma = 0.78</math>, <math>\phi_e = 0.044</math>, <math>\rho = 0.979</math>)</b>								
$\sigma_j$	0.860	0.260	0.930	1.330	1.710	0.770	0.640	
$\rho$	0.979	0.979	0.979	0.979	0.979	0.979	0.979	
$\phi_e$	0.044	0.044	0.044	0.044	0.044	0.044	0.044	
$\phi_e * \sigma$	0.034	0.034	0.034	0.034	0.034	0.034	0.034	
$\phi$	3.14	3.11	3.82	3.29	3.43	3.86	3.42	
$\phi_d$	7.2	7.7	3.6	2.2	1.0	2.5	1.7	
<b>C: Individual LRR</b>								
$\sigma_j$	1.050	0.570	0.970	1.430	1.600	0.720	0.490	0.780
$\rho$	0.979	0.979	0.979	0.979	0.979	0.979	0.979	0.979
$\phi_e$	0.034	0.051	0.051	0.021	0.045	0.119	0.200	0.044
$\phi_e * \sigma_j$	0.035	0.029	0.049	0.030	0.073	0.086	0.098	0.034
$\phi$	3.43	3.49	3.50	3.50	3.50	3.50	3.50	3.00
$\phi_d$	5.0	5.8	4.5	2.9	1.6	4.5	3.8	4.5

<sup>a</sup>Preference parameters:  $\beta = 0.987$ ,  $\psi = 1.5$ ,  $\gamma = 10$

<sup>b</sup>All  $\mu$ 's,  $\sigma_j$ 's, and  $(\phi_e * \sigma_j)$ 's are in percent



Table 18: **Data Moments and Simulated Model Moments**

	AUS	CAN	FRA	GER	JAP	UK	US
<b>A. Data Moments:</b>							
$\sigma(g_c)$	3.51	2.05	3.28	3.86	3.35	1.86	1.12
$\sigma(g_d)$	13.68	8.16	11.43	9.37	5.43	8.28	5.47
$E(r_m - r_f)$	1.49	2.74	6.31	5.12	3.72	6.65	5.47
$E(r_f)$	2.06	2.69	2.42	2.61	1.24	1.28	1.46
$\sigma(r_m)$	22.60	17.28	22.51	19.81	21.77	21.14	17.56
$\sigma(r_f)$	2.48	1.77	1.69	1.32	2.17	2.92	1.53
<b>B. Model Moments: Common LRR</b>							
$\sigma(g_c)$	3.08	1.98	3.18	4.17	5.16	2.83	2.60
$\sigma(g_d)$	18.97	8.85	11.65	10.27	7.72	8.71	6.42
$E(r_m - r_f)$	1.95	4.07	5.34	4.66	4.99	5.82	5.38
$E(r_f)$	1.73	2.29	2.34	1.28	1.54	1.94	2.18
$\sigma(r_m)$	25.22	14.02	17.99	16.10	13.06	15.76	12.91
$\sigma(r_f)$	0.62	0.62	0.62	0.62	0.62	0.62	0.62
<b>C. Model Moments: Individual LRR</b>							
$\sigma(g_c)$	3.35	2.11	3.43	4.31	4.90	2.88	2.21
$\sigma(g_d)$	15.35	10.07	13.77	12.38	10.14	11.14	7.83
$E(r_m - r_f)$	2.14	3.48	6.77	4.64	7.50	7.23	6.40
$E(r_f)$	1.61	2.17	1.98	0.90	1.45	1.58	2.18
$\sigma(r_m)$	21.22	15.53	20.31	19.05	16.58	18.30	14.77
$\sigma(r_f)$	0.49	0.47	0.71	0.53	0.73	0.68	0.56

<sup>a</sup>Data moments as previously shown in Table 16 and SMM procedure described in Solution Method

Table 19: Log Consumption Growth Correlations

	AUS	CAN	FRA	GER	JAP	UK	US	World Eq <sup>a</sup>	World Pop <sup>b</sup>
AUS	1.000	0.165	-0.027	-0.111	0.055	0.089	0.046	0.388	0.102
CAN	0.165	1.000	0.147	-0.256	0.007	0.530	0.608	0.484	0.536
FRA	-0.027	0.147	1.000	0.050	0.159	0.252	0.216	0.642	0.532
GER	-0.111	-0.256	0.050	1.000	0.058	-0.259	-0.188	0.285	0.171
JPN	0.055	0.007	0.159	0.058	1.000	0.117	-0.145	0.415	0.316
UK	0.089	0.530	0.252	-0.259	0.117	1.000	0.593	0.530	0.620
US	0.046	0.608	0.216	-0.188	-0.145	0.593	1.000	0.460	0.790

<sup>a</sup>World is Equally weighted

<sup>b</sup>World is Population weighted, where Population data is from PWT

<sup>c</sup>Sthecc: PWT 1950-2000

Table 20: Symmetric Two Country Welfare Gains (in Annual percent)

	$\psi = 0.5$		$\psi = 1.5$	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
<b>A. Common LRR:</b> $corr(\eta_t^i, \eta_t^j) = 0.5$				
Gain	0.281	5.013	0.895	5.434
<b>B. Individual LRR:</b> $corr(\eta_t^i, \eta_t^j) = 0.5, corr(e_t^i, e_t^j) = 0$				
Gain	2.193	41.001	8.478	66.706
<b>C. Individual LRR:</b> $corr(\eta_t^i, \eta_t^j) = 0, corr(e_t^i, e_t^j) = 0.5$				
Gain	1.523	28.345	5.523	39.031

<sup>a</sup>Model parameters:  $\beta = 0.987, \mu_1 = \mu_2 = 0.15, \sigma_1 = \sigma_2 = 0.78, \phi_e = 0.044, \rho = 0.979$

Table 21: Two Country Welfare Gains with Individual LRR (in Annual percent)

		$\psi = 0.5$						$\psi = 1.5$					
		$\gamma = 2$			$\gamma = 10$			$\gamma = 2$			$\gamma = 10$		
		Eq Wtd	Pr Wtd	Eq Wtd	Pr Wtd	Eq Wtd	Pr Wtd	Eq Wtd	Pr Wtd	Eq Wtd	Pr Wtd	Eq Wtd	Pr Wtd
<b>A. Symmetric:</b> $\mu_1 = \mu_2, \sigma_1 = \sigma_2, \text{corr}(\eta_t^i, \eta_t^j) = 0.5, \text{corr}(e_t^i, e_t^j) = 0$													
Country 1		2.19	2.19	41.00	41.00	8.48	8.48	66.71	66.71	66.71	66.71	66.71	66.71
Country 2		2.19	2.19	41.00	41.00	8.48	8.48	66.71	66.71	66.71	66.71	66.71	66.71
<b>A. Symmetric:</b> $\mu_1 = \mu_2, \sigma_1 = \sigma_2, \text{corr}(\eta_t^i, \eta_t^j) = -0.5, \text{corr}(e_t^i, e_t^j) = 0$													
Country 1		2.75	2.75	50.37	50.37	10.46	10.46	89.75	89.75	89.75	89.75	89.75	89.75
Country 2		2.75	2.75	50.37	50.37	10.46	10.46	89.75	89.75	89.75	89.75	89.75	89.75
<b>B. Different <math>\sigma</math>:</b> $\sigma_2 = 1.10 * \sigma_1, \text{corr}(\eta_t^i, \eta_t^j) = 0.5, \text{corr}(e_t^i, e_t^j) = 0$													
Country 1		1.91	2.46	35.95	45.85	7.36	9.67	55.22	77.18	55.22	77.18	55.22	77.18
Country 2		2.99	1.35	72.01	26.04	11.42	5.04	90.93	33.26	90.93	33.26	90.93	33.26
<b>C. Different <math>\mu</math>:</b> $\mu_2 = 1.10 * \mu_1, \text{corr}(\eta_t^i, \eta_t^j) = 0.5, \text{corr}(e_t^i, e_t^j) = 0$													
Country 1		4.93	2.03	46.14	40.68	18.80	6.61	78.83	64.99	78.83	64.99	78.83	64.99
Country 2		-0.54	7.83	31.70	51.59	-0.65	30.99	59.08	92.67	59.08	92.67	59.08	92.67

<sup>a</sup>Model parameters:  $\beta = 0.987, \mu_1 = 0.15, \sigma_1 = 0.78, \phi_e = 0.044, \rho = 0.979$

Table 22: Two Country Welfare Gains with Individual LRR (in Annual percent)

		Equal Wgt		Reserve 1		Reserve 2		Price Wgt	
		Gain	Alloc	Gain	Alloc	Gain	Alloc	Gain	Alloc
<b>A: Symmetric:</b> $corr(\eta_t^i, \eta_t^j) = 0.5, corr(e_t^i, e_t^j) = 0$									
Country 1	66.7%	1.00	0.60	0.0%	0.60	233.4%	1.40	66.7%	1.00
Country 2	66.7%	1.00	1.40	233.4%	1.40	0.0%	0.60	66.7%	1.00
<b>B: Different</b> $\sigma_2 = 1.10 * \sigma_1, corr(\eta_t^i, \eta_t^j) = 0.5, corr(e_t^i, e_t^j) = 0$									
Country 1	55.2%	1.00	0.64	0%	0.64	129.1%	1.48	77.2%	1.14
Country 2	90.9%	1.00	1.36	158.9%	1.36	0%	0.52	33.3%	0.86
<b>C: Different</b> $\mu_2 = 1.10 * \mu_1, corr(\eta_t^i, \eta_t^j) = 0.5, corr(e_t^i, e_t^j) = 0$									
Country 1	78.8%	1.00	0.56	0%	0.56	145.2%	1.37	65.0%	0.92
Country 2	59.1%	1.00	1.44	129.2%	1.44	0%	0.63	92.7%	1.08

<sup>a</sup>Model parameters:  $\beta = 0.987, \psi = 1.5, \gamma = 10, \mu_1 = 0.15, \sigma_1 = 0.78, \phi_e = 0.044, \rho = 0.979$

Figure A-1: Certainty Equivalent Consumption - Symmetric

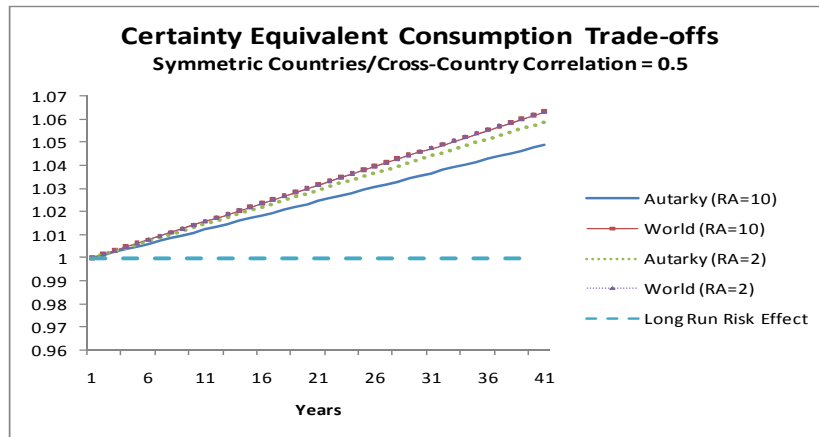


Figure A-2: Certainty Equivalent Consumption - Different Sigma

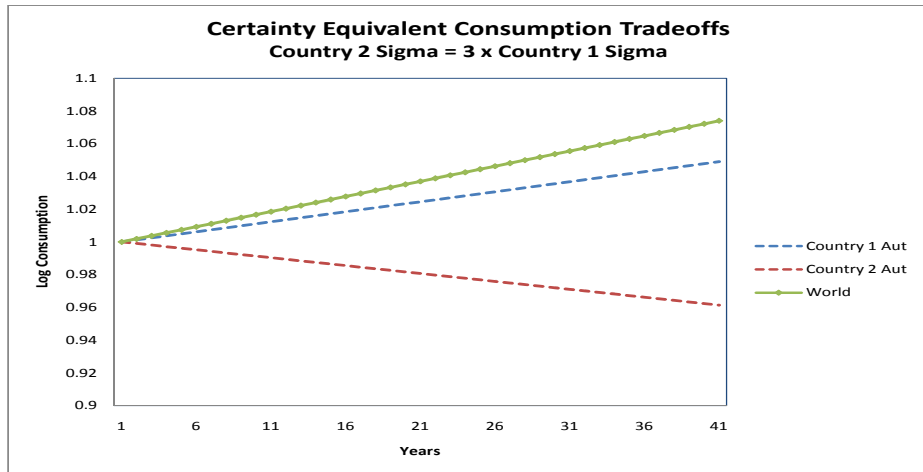
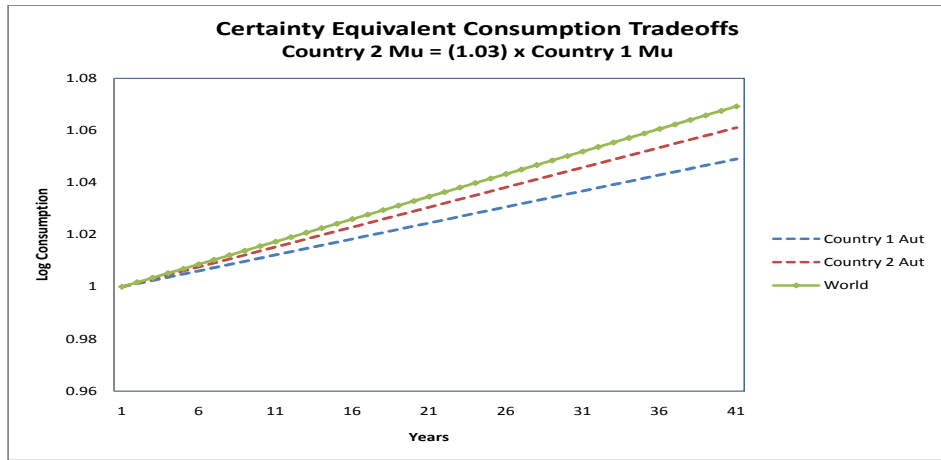


Figure A-3: Certainty Equivalent Consumption - Different Mu





## Appendix B.2 The Social Planner's Problem

Given the Epstein-Zin-Weil utility, the first order conditions of the planners problem are:

$$\begin{aligned} & \lambda^\ell \left\{ U_1^\ell(C^\ell(S_t), E[U^\ell(C^\ell(S_{t+1})|I_t)]) + U_2^\ell(C^\ell(S_t), E[U^\ell(C^\ell(S_{t+1})|I_t)]) \frac{\partial E[U^\ell(C^\ell(S_{t+1})|I_t)]}{\partial C^\ell(S_t)} \right\} \\ = & \lambda^i \left\{ U_1^i(C^i(S_t), E[U^i(C^i(S_{t+1})|I_t)]) + U_2^i(C^i(S_t), E[U^i(C^i(S_{t+1})|I_t)]) \frac{\partial E[U^i(C^i(S_{t+1})|I_t)]}{\partial C^i(S_t)} \right\} \end{aligned}$$

where  $U_n$  is the partial derivative of  $U$  with respect to the  $n$ th argument. Thus, the social planner equalizes marginal utilities across states including the effects of consumption on future expected utility.

Under CRRA preferences, utility is time separable. In this case,  $U_2 = 0$  and the first order conditions become:

$$\lambda^\ell U_1^\ell(C^\ell(S_t), E[U^\ell(C^\ell(S_{t+1})|I_t)]) = \lambda^i U_1^i(C^i(S_t), E[U^i(C^i(S_{t+1})|I_t)]) \quad (49)$$

Or,

$$\lambda^\ell C^\ell(S_t)^{-\gamma} = \lambda^i C^i(S_t)^{-\gamma} \quad (50)$$

which does not depend upon the recursive next period utility. Thus, under CRRA, marginal period utility is equalized across states whereas under Epstein Zin Weil preferences, marginal period utility relative to marginal recursive next period utility is equalized across countries and states.

Note that if the country is in autarky,  $C_t^j(S_t) = Y_t^j(S_t)$  for all countries  $j$ . In this case with CRRA preferences consumption clearly does not solve the social planner's problem unless  $Y_t^i(S_t) = Y_t^j(S_t) \forall S_t, t$ , which occurs with probability zero.

## Appendix B.3 Wealth to Consumption

By the identity  $W_t = C_t + P_t$ , I can write the wealth to consumption ratio in terms of the price to consumption ratio,  $\frac{W_t}{C_t} = 1 + \frac{P_t}{C_t} = 1 + Z_{c,t}$  (note in the Campbell-Shiller approximation, I approximate for  $z_{c,t} = \log(Z_{c,t})$ ). Then I can re-write the value

function in terms of price to consumption ratio,  $\frac{C_t}{W_t} = (1 + Z_{c,t})^{-1}$ :

$$\begin{aligned}
V_t(C_t, W_t) &= [(1 - \delta)^{-\psi} (\frac{C_t}{W_t})]^{\frac{1}{1-\psi}} W_t \\
&= (1 - \delta)^{\frac{-\psi}{1-\psi}} (C_t)^{\frac{1}{1-\psi}} (\frac{1}{W_t})^{\frac{\psi}{1-\psi}} \\
&= (1 - \delta)^{\frac{-\psi}{1-\psi}} (C_t)^{\frac{1}{1-\psi}} (\frac{1}{W_t})^{\frac{\psi}{1-\psi}} C_t^{-1} C_t \\
&= (1 - \delta)^{\frac{-\psi}{1-\psi}} (\frac{C_t}{W_t})^{\frac{\psi}{1-\psi}} C_t \\
&= (1 - \delta)^{\frac{-\psi}{1-\psi}} (1 + Z_{c,t})^{\frac{\psi}{\psi-1}} C_t
\end{aligned}$$

## Appendix B.4 Closed Economy Asset Prices with Long Run Risk

In the closed economy, each country  $j$  has a representative agent and is endowed with the stochastic consumption growth processes in equation (1). In this appendix, I detail how I calculate the price of consumption in the closed economy. For simplicity, I here consider the case when long run risk is common across countries and is given by  $\bar{x}$ .

First, I use the Campbell-Shiller decomposition to express returns in terms of price to consumption ratio, with approximating constants  $k_0^j$  and  $k_1^j$ <sup>28</sup>:

$$r_{c,t+1}^j = k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{c,t+1}^j \quad (51)$$

Following Bansal and Yaron 2004, I solve for equilibrium log price to consumption ratio by conjecturing that the log price to consumption ratio is linear in the sthce of long run risk,  $z_t^j = A_0^j + A_1^j \bar{x}_t$ . Given the Campbell-Shiller decomposition, it is sufficient solve for the coefficients  $A_0^j$  and  $A_1^j$  in order to solve for the equilibrium returns.

Using the first order condition derived earlier in equation (3.13) and applying the properties of log normality of return,  $r_{c,t+1}^j$ , and consumption growth,  $g_{c,t+1}^j$ , I have:

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] = 0 \quad (52)$$

Now I substitute in the return decomposition and the stochastic processes for

---

<sup>28</sup>Approximation constants are defined to be  $k_1^j = \frac{\exp(\bar{z}^j)}{1 + \exp(\bar{z}^j)}$  and  $k_0^j = \log(1 + \exp(\bar{z}^j)) - k_1^j \bar{z}^j$ , where  $\bar{z}^j$  is the steady state log price to consumption ratio in the close economy.

log consumption growth,  $g_{c,t+1}^j$ , log price to consumption ratios,  $z_{t+1}^j$ , and the long run risk,  $\bar{x}_{t+1}$ , into the above Euler condition. Then taking conditional expectation and conditional variances<sup>29</sup>, the left hand side of the equation (52) becomes the sum of a constant term and an  $\bar{x}_t$  term<sup>30</sup>.

$$[\theta(\ln \delta + (1 - \frac{1}{\psi})\mu_c^j + k_0^j + (k_1^j - 1)A_0^j + \frac{1}{2}\theta((k_1 A_1 \varphi_e)^2 + (1 - \frac{1}{\psi})^2\sigma^2))] + [\theta(k_1^j A_1^j \rho - A_1^j + 1 - \frac{1}{\psi})\bar{x}_t] = 0 \quad (53)$$

As I can see from equation (53), the left hand side is the sum of a constant term and an  $\bar{x}_t$  term. Since the sum of the two terms must be zero, each of the terms should be zero. This requirement gives us a system of two equations and two unknowns. Solving the system of equations gives us the following analytical form for the coefficients of the log price to consumption ratio:

$$A_1^j = \frac{1 - \frac{1}{\psi}}{1 - k_1^j \rho} \quad (54)$$

$$A_0^j = \frac{\ln \delta + k_0^j + (1 - \frac{1}{\psi})\mu^j + \frac{1}{2}\theta((1 - \frac{1}{\psi})^2(\sigma^j)^2 + (k_1^j A_1^j \sigma \varphi_e)^2)}{1 - k_1^j} \quad (55)$$

I now detail the steps required to get these solutions. For this purpose, I rewrite the dynamics of the long run risk,  $\bar{x}_{t+1}$ , and country j's log consumption growth process,  $g_{c,t+1}^j$  from equations (1) as follows:

$$\begin{aligned} g_{c,t+1}^j &= \mu^j + \bar{x}_t + \sigma^j \eta_{t+1}^j \\ \bar{x}_{t+1} &= \rho \bar{x}_t + \sigma \varphi_e e_{t+1} \end{aligned}$$

To solve for prices, I first conjecture that the closed economy log price to consumption ratio of the country j's consumption claim asset is linear in the common long run risk:  $z_t^j = A_0^j + A_1^j \bar{x}_t$

Next, using the Campbell & Shiller approximation and substituting in the stochastic processes for  $z_t^j$ ,  $z_{t+1}^j$ , and  $\bar{x}_{t+1}$  I have:

$$\begin{aligned} r_{t+1}^j &= k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{c,t+1}^j \\ &= k_0^j + k_1^j (A_0^j + A_1^j \bar{x}_{t+1}) - (A_0^j + A_1^j \bar{x}_t) + g_{c,t+1}^j \\ &= k_0^j + k_1^j A_0^j + k_1^j A_1^j (\rho \bar{x}_t + \sigma \varphi_e e_{t+1}) - A_0^j - A_1^j \bar{x}_t + g_{c,t+1}^j \\ &= [k_0^j + k_1^j A_0^j - A_0^j] + (k_1^j A_1^j \rho - A_1^j) \bar{x}_t + k_1^j A_1^j \sigma \varphi_e e_{t+1} + g_{c,t+1}^j \end{aligned}$$

<sup>29</sup>Note conditional expectation of shocks are zero and conditional variance of shocks are equal to one

<sup>30</sup>For a detailed derivations of closed economy prices see Appendix

From the Euler equation and exploiting the log-normality of the pricing kernel and the returns, I can have:

$$\exp[E_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j) + \frac{1}{2} Var_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] = 1$$

or equivalently,

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j)] = 0 \quad (56)$$

Now substituting the return decomposition into equation (56), I have that:

$$\begin{aligned} & E_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j] \\ &= E_t[\theta \ln \delta + (\theta - \frac{\theta}{\psi}) g_{c,t+1}^j + \theta[k_0^j + k_1^j A_0^j - A_0^j] + \theta(k_1^j A_1^j \rho - A_1^j) \bar{x}_t + \theta k_1^j A_1^j \sigma \varphi_e e_{t+1}] \\ &= \theta \ln \delta + (\theta - \frac{\theta}{\psi}) E_t[g_{c,t+1}^j] + \theta[k_0^j + k_1^j A_0^j - A_0^j] + \theta(k_1^j A_1^j \rho - A_1^j) \bar{x}_t \\ &= \theta[\ln \delta + k_0^j + k_1^j A_0^j - A_0^j + (1 - \frac{1}{\psi}) \mu^j] + \theta[A_1^j(k_1^j \rho - 1) + (1 - \frac{1}{\psi})] \bar{x}_t \end{aligned}$$

Where I used the fact that  $E_t[e_{t+1}] = E_t[\eta_{t+1}^j] = 0$

And,

$$\begin{aligned} & \frac{1}{2} Var_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j] = \\ &= \frac{1}{2} Var_t[\theta(1 - \frac{1}{\psi}) g_{c,t+1}^j + \theta k_1^j A_1^j \sigma \varphi_e e_{t+1}] = \\ &= \frac{1}{2} Var_t[\theta(1 - \frac{1}{\psi}) \sigma^j \eta_{t+1}^j] + \frac{1}{2} Var_t[\theta k_1^j A_1^j \sigma \varphi_e e_{t+1}] = \\ &= \frac{1}{2} (\theta(1 - \frac{1}{\psi}) \sigma^j)^2 Var_t[\eta_{t+1}^j] + \frac{1}{2} (\theta k_1^j A_1^j \sigma \varphi_e)^2 Var_t[e_{t+1}] = \\ &= \frac{1}{2} [(\theta(1 - \frac{1}{\psi}) \sigma^j)^2 + (\theta k_1^j A_1^j \sigma \varphi_e)^2] = \\ &= \frac{1}{2} \theta^2 [(1 - \frac{1}{\psi})^2 (\sigma^j)^2 + (k_1^j A_1^j \sigma \varphi_e)^2] = \end{aligned}$$

Note, I used the fact that  $Var_t(e_{t+1}) = Var_t(\eta_{t+1}^j) = 1$

Combining the above derivations, I can re-write the left hand side of the equation (56) in terms of a constant term and an  $\bar{x}_t$  term.

$$\begin{aligned} & E_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j] + \frac{1}{2} Var_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^j + \theta r_{c,t+1}^j] = \\ &= \theta[\ln \delta + k_0^j + k_1^j A_0^j - A_0^j + (1 - \frac{1}{\psi}) \mu^j + \frac{1}{2} \theta ((1 - \frac{1}{\psi})^2 (\sigma^j)^2 + (k_1^j A_1^j \sigma \varphi_e)^2)] \\ &+ \theta[A_1^j(k_1^j \rho - 1) + (1 - \frac{1}{\psi})] \bar{x}_t \\ &= 0 \end{aligned}$$

Since the sum of the constant term and the  $\bar{x}_t$  is zero, then both terms must be zero. This gives us a system of two equations and two unknowns,  $A_0^j$  and  $A_1^j$ .

1. To solve for  $A_1^j$ , I set  $\bar{x}_t$  term to zero:

$$\begin{aligned} & \theta[A_1^j(k_1^j \rho - 1) + (1 - \frac{1}{\psi})] \bar{x}_t = 0 \\ & A_1^j = \frac{(1 - \frac{1}{\psi})}{1 - k_1^j \rho} \end{aligned}$$

2. From the constant term, I can solve for  $A_0^j$ :

$$\theta[\ln \delta + k_0^j + k_1^j A_0^j - A_0^j + (1 - \frac{1}{\psi})\mu^j + \frac{1}{2}\theta((1 - \frac{1}{\psi})^2(\sigma^j)^2 + (k_1^j A_1^j \sigma \varphi_e)^2)] = 0$$

$$A_0^j = \frac{\ln \delta + k_0^j + (1 - \frac{1}{\psi})\mu^j + \frac{1}{2}\theta((1 - \frac{1}{\psi})^2(\sigma^j)^2 + (k_1^j A_1^j \sigma \varphi_e)^2)}{1 - k_1^j}$$

## Appendix B.5 Open Economy Equilibrium with Long Run Risk

In this appendix, I detail the open economy equilibrium. When markets open, all assets are priced with a common pricing kernel. This stochastic discount factor is determined by the intertemporal optimization problem of the Representative Agent and is a function of the world log consumption growth,  $g_t^w$ . I define the world consumption growth as a weighted average of the individual consumption growths with weights,  $a^j$ , for each country.

$$g_{c,t}^w = \sum_{j=1}^J a_j * g_{c,t}^j$$

$$= \bar{\mu} + \bar{x}_t + \bar{u}_{t+1}$$

where  $\bar{\mu}_{t+1} = \sum_{j=1}^J a_j \mu^j$ ,  $\bar{u}_{t+1} = \sum_{j=1}^J a_j \sigma^j \eta_{t+1}^j$ ,  $\sum_{j=1}^J a_j = 1$ . While  $a_j$  should adjust for the population size of country  $j$ 's, for now, I will assume that all countries are equally weighted.

Similar to the steps used in the closed economy solution, I can approximate open economy returns in terms of open economy price to consumption ratio using the Campbell-Shiller decomposition. But now the approximating constants are defined by  $k_1^w = \frac{\exp(\bar{z}^w)}{1 + \exp(\bar{z}^w)}$  and  $k_0^w = \log(1 + \exp(\bar{z}^w)) - k_1^w \bar{z}^w$ , where  $\bar{z}^w$  is the steady state open economy log price to world consumption ratio.

$$r_{c,t+1}^w = k_0^w + k_1^w z_{t+1}^w - z_t^w + g_{c,t+1}^w \quad (57)$$

As shown in the previous sections, in the open economy returns for each country  $j$ 's consumption asset will be priced with the first order condition in equation (3.23), and applying the normality of log returns and log consumption growth I have:

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j)] = 0 \quad (58)$$

In particular, the above equation holds for the return on the world portfolio in the open economy,  $r_{c,t+1}^w$ , which is needed for the pricing of all contingent claims. To price the return on the world portfolio, (58) becomes:

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w)] = 0 \quad (59)$$

Following similar steps as in the closed economy, I again conjecture that the world log price to consumption ratio is linear in the long run risk,  $z_t^w = A_0^w + A_1^w \bar{x}_t$ , and solve for the coefficients  $A_0^w$  and  $A_1^w$ . Like the closed economy solutions, after substituting the stochastic processes for  $g_{c,t+1}^w$ ,  $\bar{x}_{t+1}$ , and  $r_{t+1}^w$ , the left hand side of equation (59) will have a constant term and an  $\bar{x}_t$  term. I again set the constant term to zero and the  $\bar{x}_t$  term to zero, and solve the system of two equations<sup>31</sup>:

$$A_1^w = \frac{1 - \frac{1}{\psi}}{1 - k_1^w \rho} \quad (60)$$

$$A_0^w = \frac{\ln \delta + k_0^w + (1 - \frac{1}{\psi})\bar{\mu} + \frac{1}{2}\theta(1 - \frac{1}{\psi})^2 Var_t[\bar{u}_{t+1}] + \frac{1}{2}\theta(k_1^w A_1^w \sigma \varphi_e)^2}{1 - k_1^w} \quad (61)$$

Having solved the return on the world portfolio, I can price the open economy return on country j's consumption claim. I approximate open economy returns with the price to consumption ratio,  $\bar{r}_{c,t+1}^j$ , using the Campbell-Shiller decomposition. But now the approximating constants,  $\bar{k}_0^j$  and  $\bar{k}_1^j$ , are defined by the open economy steady state log price to consumption ratio,  $\bar{z}^j$ .<sup>32</sup>

$$\bar{r}_{c,t+1}^j = \bar{k}_0^j + \bar{k}_1^j \bar{z}_{t+1}^j - \bar{z}_t^j + g_{c,t+1}^j \quad (62)$$

I assume that country j's open economy log price to consumption ratio,  $\bar{z}_t^j$ , is linear in the long run risk  $\bar{z}_t^j = \bar{A}_0^j + \bar{A}_1^j \bar{x}_t$ . To solve for the coefficients,  $\bar{A}_0^j$  and  $\bar{A}_1^j$ , I substitute the defined processes for  $g_{c,t+1}^w$ ,  $\bar{x}_{t+1}$ , and  $g_{c,t+1}^j$  into equation (58). Then I simplify until the left hand side of (58) has only a constant term and an  $\bar{x}_t$  term, which again by earlier reasoning gives us a system of two equations for the two unknowns. As shown more completely in Appendix C, the coefficients  $\bar{A}_0^j$  and  $\bar{A}_1^j$  are as follows:

$$\bar{A}_1^j = \frac{1 - \frac{1}{\psi}}{1 - \bar{k}_1^j \rho} \quad (63)$$

$$\bar{A}_0^j = \frac{(\dots) + \frac{1}{2}[(\sigma^j)^2 + ((\theta - 1)k_1^w A_1^w + \bar{k}_1^j \bar{A}_1^j)^2 \sigma^2 \varphi_e^2 + (\theta - 1 - \frac{\theta}{\psi})^2 Var_t[\bar{u}_{t+1}]]}{1 - \bar{k}_1^j} \quad (64)$$

where  $(\dots) = \theta \ln \delta + (\theta - 1)(k_0^w + k_1^w A_0^w - A_0^w) + \bar{k}_0^j + (\theta - 1 - \frac{\theta}{\psi})\bar{\mu} + \mu^j$  and  $Var_t[\bar{u}_{t+1}] = Var_t[\sum_{i=1}^N a_i \sigma^i \eta_{t+1}^i]$ , the variance-covariance matrix of the idiosyncratic component of log.

I now detail how the solutions to the  $\bar{A}_0^j, \bar{A}_1^j$  are obtained.

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<sup>31</sup>See detailed solution in appendix

<sup>32</sup>Approximation constants are defined to be  $\bar{k}_1^j = \frac{\exp(\bar{z}^j)}{1 + \exp(\bar{z}^j)}$  and  $\bar{k}_0^j = \log(1 + \exp(\bar{z}^j)) - \bar{k}_1^j \bar{z}^j$

### Appendix B.5.1 Price of World Consumption Claims In Open Market

To price return in the open economy, I begin with the pricing of the "world" consumption claim. I conjecture that the log price to consumption ratio of the world consumption claim asset is linear in the common long run risk:  $z_t^w = A_0^w + A_1^w \bar{x}_t$

Again I approximate returns using Campbell-Shiller decomposition and substitute in the stochastic processes for  $z_t^w$ ,  $z_{t+1}^w$ , and  $\bar{x}_{t+1}$ :

$$\begin{aligned}
 r_{t+1}^w &= k_0^w + k_1^w z_{t+1}^w - z_t^w + g_{c,t+1}^w \\
 &= k_0^w + k_1^w (A_0^w + A_1^w \bar{x}_{t+1}) - (A_0^w + A_1^w \bar{x}_t) + g_{c,t+1}^w \\
 &= k_0^w + k_1^w A_0^w + k_1^w A_1^w \bar{x}_{t+1} - A_0^w - A_1^w \bar{x}_t + g_{c,t+1}^w \\
 &= (k_0^w + k_1^w A_0^w - A_0^w) + k_1^w A_1^w (\rho \bar{x}_t + \sigma \varphi_e e_{t+1}) - A_1^w \bar{x}_t + g_{c,t+1}^w \\
 &= (k_0^w + k_1^w A_0^w - A_0^w) + k_1^w A_1^w \rho \bar{x}_t - A_1^w \bar{x}_t + k_1^w A_1^w \sigma \varphi_e e_{t+1} + g_{c,t+1}^w
 \end{aligned}$$

From the Euler equation and exploiting the log-normality of the pricing kernel and the returns, I have:

$$\exp[E_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w) + \frac{1}{2} Var_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w)] = 1$$

or equivalently,

$$E_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w)] + \frac{1}{2} Var_t[(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w)] = 0 \quad (65)$$

Now substituting the return decomposition into equation (65), I have that:

$$\begin{aligned}
 E_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w] &= \\
 &= \theta \ln \delta + (\theta - \frac{\theta}{\psi}) E_t(g_{c,t+1}^w) + \theta (k_0^w + k_1^w A_0^w - A_0^w) + \theta A_1^w (k_1^w \rho - 1) \bar{x}_t + \theta k_1^w A_1^w \sigma \varphi_e E_t(e_{t+1}) \\
 &= \theta (\ln \delta + k_0^w + k_1^w A_0^w - A_0^w + (1 - \frac{1}{\psi}) \bar{\mu}) + \theta (1 - \frac{1}{\psi} + A_1^w (k_1^w \rho - 1)) \bar{x}_t
 \end{aligned}$$

And,

$$\begin{aligned}
 \frac{1}{2} Var_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w] &= \\
 &= \frac{1}{2} Var_t[-\frac{\theta}{\psi} g_{c,t+1}^w + \theta ((k_0^w + k_1^w A_0^w - A_0^w) + k_1^w A_1^w \rho \bar{x}_t - A_1^w \bar{x}_t + k_1^w A_1^w \sigma \varphi_e e_{t+1}) + \\
 &g_{c,t+1}^w] = \\
 &= \frac{1}{2} Var_t[\theta (1 - \frac{1}{\psi}) g_{c,t+1}^w + \theta k_1^w A_1^w \sigma \varphi_e e_{t+1}] = \\
 &= \frac{1}{2} Var_t[\theta (1 - \frac{1}{\psi}) (\bar{\mu} + \bar{x}_t + \bar{u}_{t+1}) + \theta k_1^w A_1^w \sigma \varphi_e e_{t+1}] = \\
 &= \frac{1}{2} \theta^2 (1 - \frac{1}{\psi})^2 Var_t[\bar{u}_{t+1}] + \frac{1}{2} \theta^2 (k_1^w A_1^w \sigma \varphi_e)^2
 \end{aligned}$$

Note:  $Var_t(e_{t+1}) = Var_t(\eta_{t+1}^i) = 1$  and  $Var_t[\bar{u}_{t+1}] = Var_t[\sum_{i=1}^N a_i \sigma^i \eta_{t+1}^i]$

Combining the above derivations, I re-write the left hand side of the equation (65) in terms of a constant term and an  $\bar{x}_t$  term.

$$E_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w] + \frac{1}{2} Var_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + \theta r_{c,t+1}^w] =$$

$$\begin{aligned}
&= [\theta(\ln \delta + k_0^w + k_1^w A_0^w - A_0^w + (1 - \frac{1}{\psi})\bar{\mu}) + \frac{1}{2}\theta^2(1 - \frac{1}{\psi})^2 Var_t(\bar{u}_{t+1}) + \frac{1}{2}\theta^2(k_1^w A_1^w \sigma \varphi_e)^2] \\
&+ [\theta(1 - \frac{1}{\psi} + A_1^w(k_1^w \rho - 1))]\bar{x}_t \\
&= 0
\end{aligned}$$

Since the sum of the constant term and the  $\bar{x}_t$  is zero, then both terms must be zero. This gives us a system of two equations and two unknowns,  $A_0^w$  and  $A_1^w$ .

1. To solve for  $A_1^w$ , I set  $\bar{x}_t$  term to zero:

$$\begin{aligned}
[1 - \frac{1}{\psi} + A_1^w(k_1^w \rho - 1)]\bar{x}_t &= 0 \\
A_1^w &= \frac{1 - \frac{1}{\psi}}{1 - k_1^w \rho}
\end{aligned}$$

2. From the constant term, I can solve for  $A_0^w$ :

$$\begin{aligned}
\ln \delta + k_0^w + k_1^w A_0^w - A_0^w + (1 - \frac{1}{\psi})\bar{\mu} + \frac{1}{2}\theta(1 - \frac{1}{\psi})^2 Var_t[\bar{u}_{t+1}] + \frac{1}{2}\theta(k_1^w A_1^w \sigma \varphi_e)^2 &= 0 \\
\ln \delta + k_0^w + (1 - \frac{1}{\psi})\bar{\mu} + \frac{1}{2}\theta(1 - \frac{1}{\psi})^2 Var_t[\bar{u}_{t+1}] + \frac{1}{2}\theta(k_1^w A_1^w \sigma \varphi_e)^2 &= (1 - k_1^w)A_0^w \\
A_0^w &= \frac{\ln \delta + k_0^w + (1 - \frac{1}{\psi})\bar{\mu} + \frac{1}{2}\theta(1 - \frac{1}{\psi})^2 Var_t[\bar{u}_{t+1}] + \frac{1}{2}\theta(k_1^w A_1^w \sigma \varphi_e)^2}{1 - k_1^w}
\end{aligned}$$

## Appendix B.5.2 Price of Country Contingent Claims In Open Market

Again, I conjecture that open economy log price to consumption ratio for country  $j$  is linear in the long run risk:  $\bar{z}_t^j = \bar{A}_0^j + \bar{A}_1^j \bar{x}_t$

Then by Campbell & Shiller approximation:

$$\begin{aligned}
\bar{r}_{c,t+1}^j &= \bar{k}_0^j + \bar{k}_1^j \bar{z}_{t+1}^j - \bar{z}_t^j + g_{c,t+1}^j \\
&= \bar{k}_0^j + \bar{k}_1^j \bar{A}_0^j + \bar{k}_1^j \bar{A}_1^j \bar{x}_{t+1} - \bar{A}_0^j - \bar{A}_1^j \bar{x}_t + g_{c,t+1}^j \\
&= (\bar{k}_0^j + \bar{k}_1^j \bar{A}_0^j - \bar{A}_0^j) + g_{c,t+1}^j + \bar{A}_1^j (\bar{k}_1^j \rho - 1) \bar{x}_t + \bar{k}_1^j \bar{A}_1^j \sigma \varphi_e e_{t+1}
\end{aligned}$$

Using the Euler equation and exploiting the log-normality of the pricing kernel and the returns, I have:

$$\exp[E_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j) + \frac{1}{2} Var_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j)] = 1$$

or equivalently,

$$E_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j) + \frac{1}{2} Var_t(\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j) = 0 \tag{66}$$

Now substituting the return decomposition into equation (66), I have that:

$$\begin{aligned}
&E_t[\theta \ln \delta - \frac{\theta}{\psi} g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j] \\
&= [\theta \ln \delta + (\theta - 1)(k_0^w + k_1^w A_0^w - A_0^w) + (\bar{k}_0^j + \bar{k}_1^j \bar{A}_0^j - \bar{A}_0^j) + (\theta - 1 - \frac{\theta}{\psi})\bar{\mu} + \mu^j] + [(\theta - \frac{\theta}{\psi}) \\
&+ (\theta - 1)A_1^w(k_1^w \rho - 1) + \bar{A}_1^j(\bar{k}_1^j \rho - 1)]\bar{x}_t
\end{aligned}$$



And:

$$\begin{aligned} \frac{1}{2}Var_t(\theta \ln \delta - \frac{\theta}{\psi}g_{c,t+1}^w + (\theta - 1)r_{c,t+1}^w + \bar{r}_{c,t+1}^j) = \\ = \frac{1}{2}Var_t[\sigma^j \eta_{t+1}^j] + \frac{1}{2}Var_t[(\theta - 1 - \frac{\theta}{\psi})\bar{u}_{t+1}] + \frac{1}{2}Var_t\{[(\theta - 1)k_1^w A_1^w + k_1^j \bar{A}_1^j]\sigma \varphi_e e_{t+1}\} = \\ = \frac{1}{2}(\sigma^j)^2 + \frac{1}{2}[(\theta - 1)k_1^w A_1^w + \bar{k}_1^j \bar{A}_1^j]^2 \sigma^2 \varphi_e^2 + \frac{1}{2}(\theta - 1 - \frac{\theta}{\psi})^2 Var_t[\bar{u}_{t+1}] \end{aligned}$$

Following the same logic as before, I solve for the coefficients  $\bar{A}_0^j$  and  $\bar{A}_1^j$  by setting the  $\bar{x}_t$  term to zero and the constant term to zero

1. I solve  $\bar{A}_1^j$  by setting  $\bar{x}_t$  term to zero:

$$\begin{aligned} [(\theta - \frac{\theta}{\psi}) + (\theta - 1)A_1^w(k_1^w \rho - 1) + \bar{A}_1^j(\bar{k}_1^j \rho - 1)]\bar{x}_t = 0 \\ (\theta - \frac{\theta}{\psi}) + (\theta - 1)A_1^w(k_1^w \rho - 1) = \bar{A}_1^j(1 - \bar{k}_1^j \rho) \end{aligned}$$

But from above I have  $(1 - k_1^w \rho)A_1^w = 1 - \frac{1}{\psi}$ .

$$\bar{A}_1^j = \frac{1 - \frac{1}{\psi}}{1 - \bar{k}_1^j \rho}$$

2. Solve for  $\bar{A}_0^j$  by setting the constant term to zero:

$$\begin{aligned} \theta \ln \delta + (\theta - 1)(k_0^w + k_1^w A_0^w - A_0^w) + (\bar{k}_0^j + \bar{k}_1^j \bar{A}_0^j - \bar{A}_0^j) + (\theta - 1 - \frac{\theta}{\psi})\bar{\mu} + \mu^j + \frac{1}{2}(\sigma^j)^2 + \\ \frac{1}{2}[(\theta - 1)k_1^w A_1^w + \bar{k}_1^j \bar{A}_1^j]^2 \sigma^2 \varphi_e^2 + \frac{1}{2}(\theta - 1 - \frac{\theta}{\psi})^2 Var_t[\bar{u}_{t+1}] = 0 \\ \bar{A}_0^j = \\ \frac{\theta \ln \delta + (\theta - 1)(k_0^w + k_1^w A_0^w - A_0^w) + \bar{k}_0^j + (\theta - 1 - \frac{\theta}{\psi})\bar{\mu} + \mu^j + \frac{1}{2}(\sigma^j)^2 + \frac{1}{2}[(\theta - 1)k_1^w A_1^w + \bar{k}_1^j \bar{A}_1^j]^2 \sigma^2 \varphi_e^2 + \frac{1}{2}(\theta - 1 - \frac{\theta}{\psi})^2 Var_t[\bar{u}_{t+1}]}{1 - \bar{k}_1^j} \end{aligned}$$

Where  $A_0^w$  and  $A_1^w$  are defined above.

## Appendix B.6 Data Description

This section describes the data source and filtering used in constructing the observed consumption and asset pricing moments.

### Appendix B.6.1 Consumption Series

As described in the text, I am interested in matching the US PWT consumption data implications with those of Bansal-Yaron (2004) based upon the National Income and Product Account (NIPA) data from the Bureau of Economic Analysis. Therefore, for every country except the US, I construct real per-capita consumption by taking total consumption at 1996 constant prices (CKON) and dividing it by the population (POP). For the US, I compare the PWT estimates to those reported by the NIPA estimates. Since the NIPA data give a finer breakdown of personal expenditures, I construct real per-capita consumption by using personal consumption on Non-Durables and Services chained to 2000 dollars from NIPA table 7.1 for the sample period 1950-2000.

### **Appendix B.6.2 Dividends**

I obtain the data described in Campbell (2003) from John Campbell's website. In order to make real dividend growth consistent with real consumption growth, I use the same deflator series from the Penn World Tables to adjust nominal dividends into real terms. Because the Penn World Table only gives annual series, I summed the quarterly dividends from the Campbell data to construct annual nominal dividends, then deflate by the PWT annual consumption deflator. Note that there is a difference between this annualized moments compared to Table 3 of Campbell (2003). To be consistent with time-aggregation as emphasized in long run risk, I annualize by summing the quarterly dividends. By contrast, Campbell (2003) takes average quarterly moments and annualizes by multiplying means by 400 and standard deviations by 200.

### **Appendix B.6.3 Asset Returns**

As in the case of dividends, I obtain the data from John Campbell's website. To form consistent real annual equity returns and risk free rates, I aggregate from quarterly to annual returns. All quarterly nominal rates of return are adjusted using quarterly CPI included in the Campbell data. For annual real returns in percentages, I follow the convention in Campbell (2003) and multiply means by 400 and standard deviations by 200. This numbers closely match Campbell 1999, with only slight variations due to increased sample size. I compute quarterly equity premium and annualize in the same way as the other rates of return. Similarly I aggregate quarterly dividends and use the equity price for the fourth quarter of the year to construct annual P/D ratios.

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