Proceedings of the 1999 IEEE/ASME
International Conference on Advanced Intelligent Mechatronics
September 19-23, 1999 • Atlanta, USA

A Hybrid Swing up Controller for a Two-link Brachiating Robot

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Abstract

In this paper, we report on a "hybrid" scheme for regulating the swing up behavior of a two degree of freedom brachiating robot. In this controller, a previous "target dynamics" controller and a mechanical energy regulator are combined. The proposed controller guarantees the boundedness of the total energy of the system. Simulations suggest that this hybrid controller achieves much better regulation of the desired swing motion than the target dynamics method by itself.

1 Introduction

This paper proposes a "hybrid" control scheme for a two degree of freedom brachiating robot depicted in Figure 1. For the last few years, we have been studying the control of this two-link brachiating robot [7, 8, 9] exploring how dynamically dexterous tasks can be achieved using the physical insight into the task and the dynamics of the system, following the initial study of robot brachiation by Saito *et al.* [11, 12]. As we have mentioned in [7], the study of brachiation is closely related to other problems involving dynamical dexterity such as legged locomotion [10, 13], dexterous manipulation [1, 3] and underactuated mechanisms [2, 5, 6, 14].

In our previous work, we have proposed a control algorithm based on what we term the "target dynamics" method. Preliminary analysis, numerical studies and experiments show that the proposed algorithm achieve brachiation on a level ladder with either uniform or irregular intervals as well as swing up from a suspended posture with one hand grip to the target bar with two hand grip [7, 8, 9]. However, a number of formal questions remain to be addressed, such as stability of the system and sensitivity to initial conditions in the swing up problem.

In this paper, we address certain issues that the original controller design ignored. We introduce a "hybrid" controller for the swing up problem, in which the target dynamics controller and a mechanical energy regulator are combined in a suitable fashion. "Swing up" is the task of swinging from the suspended posture at rest and catching the next bar as described in [7]. The problem of brachiation—swinging up to an unstabilizable handhold— requires rather different notion of task encoding than seen in the related literature of the control of underactuated systems such as joint position/tracking control [2, 5, 6] and stabilization to the vertical equilibrium position [14].

*This work was supported in part by NSF IRI-9510673

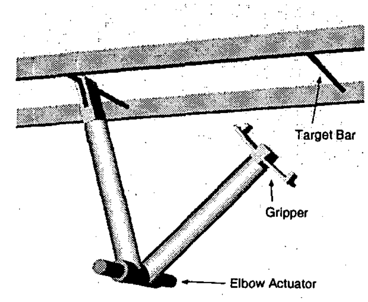

Figure 1: A Two-link Brachiating Robot

The proposed hybrid controller achieves good regulation of the desired behavior even from various initial conditions while the original target dynamics controller is quite sensitive to initial states. It also guarantees total energy boundedness, which implies that the energy of the system will not grow beyond a certain level. We consider that these features—good regulation of swing motion and mechanical energy, and a safety net—to be essential for our further investingation of robot brachiation such as the "leap" problem. The leap problem arises when the next branch is far out of reach. The task cannot be accomplished without good regulation of initial energy and a large component of free flight. Numerical studies suggest that the proposed strategy successfully improve the performance of the swing up behavior of the robot.

2 A Two-link Brachiating Robot

2.1 Model

We consider a simplified point mass lossless model of a two-link brachiating robot as depicted in Figure 2. The dynamical equation of the robot takes the form of a standard two-link planar manipulator

$$\dot{Tq} = \mathcal{L}(Tq, \tau) \quad (1)$$

where

$$\mathcal{L}(Tq,\tau) = \begin{bmatrix} \dot{q} \\ M^{-1}\left(-V - k + \begin{bmatrix} 0 \\ \tau \end{bmatrix}\right) \end{bmatrix},$$

$q = [\ \theta_1,\ \theta_2\ ]^T \in \mathcal{Q}$, $Tq = [\ q^T,\ \dot{q}^T\ ]^T \in T\mathcal{Q}$, M is the inertia matrix, V is the Coriolis/centrifugal vector,

k is the gravity vector, and τ is the joint torque. In the following, we use the dynamical parameters of the robot shown in Table 1 based on the physical two-link robot we have used in [8].

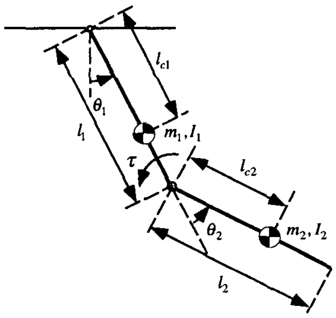

Figure 2: The mathematical model of the two-link brachiating robot used in this paper.

Description		i=1	i=2
Mass	m_i(kg)	3.499	1.232
Moment of inertia	I_i(kgm^2)	0.090	0.033
Link length	l_i(m)	0.50	0.50
Location of CG	l_{ci}(m)	0.414	0.333

Table 1: The dynamical parameters of the lossless model of the robot used in this paper. These parameters are based on the physical two-link robot described in [8].

3 A Hybrid Controller

A review of the target dynamics method [7] can be found in Appendix. The swing up task can be achieved by the modified target dynamics (17), introducing the desired limit cycle to the target variable, θ [7]. To accomplish this task, we need not only to pump up the energy, but also to control the position of the arm at the capture of the next bar.

As we have discussed in [8], the procedure for choosing the pseudo energy gain, K_e, defined in (17) is somewhat ad hoc. Some experience is helpful in determining the proper choice of K_e for a given initial condition. Since we have found that large K_e yields "chaotic" motion, we prefer to choose K_e small, which achieves the desired neutral orbit but with relatively slow energy pumping. Numerical studies suggest that some particular choices of larger K_e may result in robot trajectories which go through the next bar's position after a few of swings. Such motion allows for faster swing up times, as long as the robot catches the bar when the gripper's position coincides with that of the target bar. However, numerical simulations show that fast swing up behavior is quite sensitive to initial conditions.

In the fast swing up, "chaotic" motion in the swing behavior is observed if we let the robot keep swinging without grasping the bar at that time. We also observe that the mechanical energy of the system behaves in undesirable manner when "chaotic" motion stimulated by an overly large choice of K_e or wrong choice of ω even while the pseudo energy is well regulated. Thus, we find it useful to consider not only the pseudo energy (which has the nice property of being constant during the desired motion with respect to the target variable, θ) but also the mechanical energy which regulates the unactuated portion of the system.

In this section, we introduce a "hybrid" controller based on a new idea of combining the target dynamics and mechanical energy control in a suitable fashion. We successfully improve the performance of the swing up controller respecting insensitivity to initial conditions and mechanical energy boundedness. Numerical simulation suggests that good regulation of the desired swing motion can be achieved even when the robot starts from various initial conditions under the proposed hybrid controller. The proposed controller ensures the boundedness of the total energy. We suspect but not have yet proven that the desired orbit is also asymptotically stable—simulations to date bear out that suspicion.

3.1 Energy Regulation of Lagrangian Systems

The total mechanical energy of lagrangian mechanical systems in the form

$$M(q)\ddot{q} + B(q,\dot{q}) + k(q) = \tau \tag{2}$$

is given by

$$E = \frac{1}{2}\dot{q}^T M(q)\dot{q} + U(q) \tag{3}$$

where $U(q)$ denotes the gravitational potential. The time derivative of the mechanical energy along the motion is calculated as

$$\dot{E} = \dot{q}^T \tau \tag{4}$$

using the skew-symmetric property in the coriolis term [4].

For the particular example of our two-link brachiating robot, this relationship reduces to

$$\dot{E} = \dot{\theta}_2 \tau. \tag{5}$$

Supposing we choose the control law,

$$\tau := \tau_{E^*} = -K_{e2}(E - E^*)\dot{\theta}_2 \tag{6}$$

where K_{e2} is a positive constant and E^* is a desired mechanical energy level, then we have

$$\dot{E} = -K_{e2}(E - E^*)\dot{\theta}_2^{\,2}, \tag{7}$$

which implies that the energy regulation around the desired level can be achieved by this approach.

3.2 Hybrid Target Dynamics Controller

Consider the following hybrid controller:

$$\tau = \begin{cases} \tau_{\bar{E}^*}(q,\dot{q}) & \text{if } E < E^* \\ \tau_{\bar{E}^*}(q,\dot{q}) + \tau_{E^*}(q,\dot{q}) & \text{if } E^* \le E < E_{max1} \\ (1-\rho)[\tau_{\bar{E}^*}(q,\dot{q}) + & \text{if } E_{max1} \le E < E_{max2} \\ \quad \tau_{E^*}(q,\dot{q})] - \rho K_{e3}\dot{\theta}_2 & \\ -K_{e3}\dot{\theta}_2 & \text{if } E_{max2} \le E \end{cases} \tag{8}$$

where

$$\rho(E) = \frac{E - E_{max1}}{E_{max2} - E_{max1}}, \tag{9}$$

$\tau_{\bar{E}^*}(q, \dot{q})$ is the original swing up controller (18), τ_{E^*} is the mechanical energy regulator defined in (6) around the desired energy level, E^*, as discussed in the previous section. K_{e2} and K_{e3} are some positive gain, and E denotes the mechanical energy of the system.

The first equation regulates the swing motion of the robot through the original target dynamics controller. However, if the mechanical energy of the system exceeds the desired level E^*, then it is refined during the swing motion according to the second equation. The third equation is introduced to obtain a continuous switching from the energy refinement controller to the energy regulator. The fourth equation acts as a "safety net." Consider the time derivative of the mechanical energy of along the motion when this controller is turned on

$$\dot{E} \leq -K_{e3}\dot{\theta_2}^2. \tag{10}$$

This implies that the total energy is bounded. Note that the overall switching scheme in (8) is not smooth, but does not introduce discontinuity in the controller.

4 Simulation

In the sequel, we use the robot parameters specified in Table 1. The target bar is located at the distance of $d^* = 0.6$. For this setting, the virtual frequency, ω, and the desired pseudo energy, $\bar{E}^*$ are chosen as $\omega = 3.3649$ and $\bar{E}^* = \frac{1}{2}\omega^2\left(\frac{\pi}{2}\right)^2$ respectively for both of the original and the target dynmics and the hybrid controller. For the hybrid controller, the additional parameters are chosen as $K_{e3} = 2.0$, $E^* = U \circ c(d^*) = -12.9254$, where $U(q)$ is the potential energy of the system and $c(d)$ denotes the "ceiling" [1] parameterized by the distance between the grippers, d, as

$$c(d) = \begin{bmatrix} \arcsin\left(\frac{d}{2l}\right) \\ \pi - 2\arcsin\left(\frac{d}{2l}\right) \end{bmatrix}. \tag{11}$$

The values for E_{max1} and E_{max2} are chosen as $E_{max1} = E^* + 5.0$ and $E_{max2} = E^* + 10.0$. We use the same value for these parameters in the following simulations. The values for the other parameters are specified in each simulation. Numerical simulations illustrate the effectiveness of the proposed controller in comparison to the original swing up controller.

4.1 Insensitivity to Initial Conditions

As we have pointed out, the original swing up controller is quite sensitive to the initial condition when the robot swings up from the bottom state. In general, when the initial condition varies from its corresponding pseudo energy gain, K_e, the robot cannot always reach the target bar located at d^*.

In this section, we present a numerical study suggesting that hybrid controller can indeed achieve the task of swinging up and catching the target bar from variety of initial conditions. In order to evaluate sensitivity to initial conditions, we consider time sampled trajectories originating from various initial conditions. In the following numerical simulation, we take $17 \times 17 \times 3 \times 3 = 2601$ initial conditions on a grid in the hyper rectangular neighborhood of the origin from -0.8 to 0.8 rad in the joint angles and from -0.1 to 0.1 rad/s in the angular velocities with the interval of 0.1 respectively as depicted in Figure 3.

[1]The ceiling is defined to be the configurations where the gripper of the robot reaches the height of the ladder, $y = 0$.

Figure 4 depicts the evolution of the trajectories under the hybrid controller from the specified initial conditions above. This result suggests asymptotic convergence to the desired neutral orbit which achives the desired locomotion shown in Figures 10 and 11. In contrast, Figure 5 depicts the growth of the trajectories under the original swing up controller starting from the same initial conditions, which shows divergence from the initial conditions. In Figures 6 and 7, we show the typical movement of the robot and joint trajectories of the corresponding simulations shown above respectively. The task can be successfully achieved under the hybrid controller.

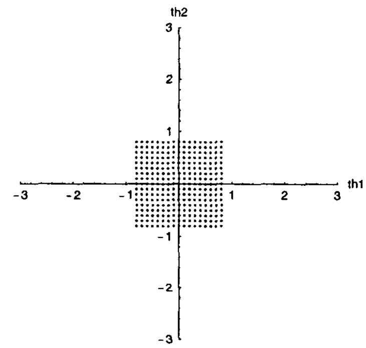

Figure 3: Initial conditions in the (θ_1, θ_2) plane. We take $17 \times 17 \times 3 \times 3 = 2601$ initial conditions on a grid in the hyper rectangular neighborhood of the origin from -0.8 to 0.8 rad in the joint angles and from -0.1 to 0.1 rad/s in the angular velocities with the interval of 0.1 respectively.

4.2 Total Energy Boundedness

We have observed that the large ω calls for unrealistically high torque and the motion of the robot sometimes becomes "wild." In this section, we illustrate the energy boundedness feature of the proposed hybrid controller.

Figures 8 and 9 show the motion of the robot and its mechanical energy when $\omega = 4.5$ instead of the correct value, $\omega^* = 3.3649$. $K_e = 0.7$ is chosen for both controllers. For the hybrid controller, the additional papmeters, $K_{e2} = 2.3$, $K_{e3} = 2.0$, $E_{max1} = E^* + 5.0 = -7.9254$ and $E_{max2} = E^* + 10.0 = -2.9254$ are chosen. The results show that the original controller yields very "wild" motion with large mechanical energy as shown in Figure 8, however, the hybrid controller indeed bounds the total energy of the system as depicted in Figure 9.

5 Conclusion

In this paper, we have introduced a hybrid controller combining the original target dynamics controller and the mechanical energy regulator. This hybrid controller

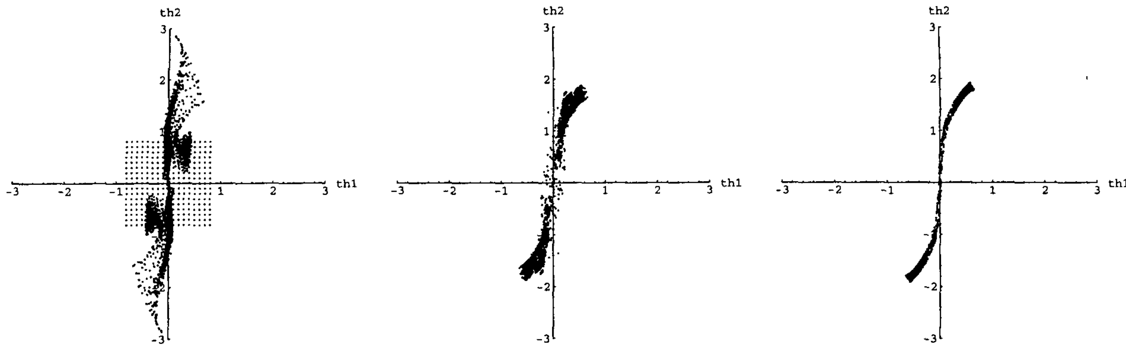

Figure 4: Time sampled trajectories in the (θ_1, θ_2) plane under the hybrid controller. Left: at $t = 0$ and $t = 4$, middle: at $t = 9$, right: at $t = 23$. These points show the evolution of the 2601 initial conditions along the motion of the system. This numerical evidence suggests convergence to a near-neutral orbit shown in Figure 11.

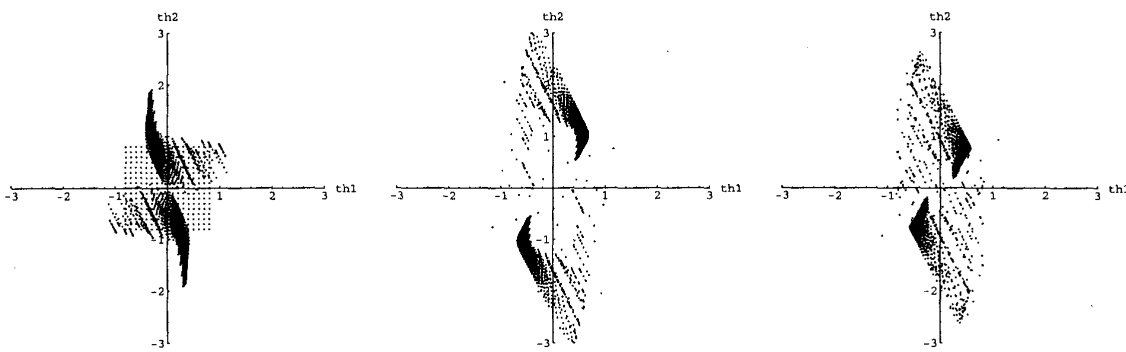

Figure 5: Time sampled trajectories in the (θ_1, θ_2) plane under the original swing up controller. Left: at $t = 0$ and $t = 4$, middle: at $t = 8$, right: $t = 21$. These points show the evolution of the 2601 initial conditions along the motion of the system. These results show that the trajectories do not converge to the desired neutral orbit.

guarantees boundedness of the total energy. Moreover, as the numerical simulations illustrate, we achieve good regulation of the swing motion of the robot, which suggests the desired orbit itself is also asymptotically stable (although we have yet to show this mathematically). Notwithstanding the favorable numerical results, it is not still clear how to choose suitable gains for the controller. Our attempts to implement this hybrid controller on the physical two-link robot [8] in our lab has not yet succeeded, largely due to discrepancies between the model and robot, and torque saturation of the elbow motor. In practice, the harmonic drive DC motor at the elbow joint bears fairly complicated nonlinear friction and seems to exhibit torque saturation over the range of operation. Preliminary numerical simulations introducing torque saturation and unmodelled nonlinear frictions in the actuator dynamics match closely observed motions in the initial experimental attempts. This suggests a future exploration of robust versions of the proposed controller to parameter uncertainty. Further mathematical analysis will be required to truly understand the properties of the proposed controller.

As we have mentioned, we consider the properties of the new controller—good regulation of the swing motion and mechanical energy from wide range of initial conditions, and energy boundedness—to be essential for our further investigation of the leap problem. In the leap problem, good control of the initial mechanical energy and angular momentum for the flight phase seems very important. Also, the next swing phase after the free flight starts from various initial conditions. In this light, we believe that the first feature plays an important role in the study of the leap problem.

Appendix

A Review of Target Dynamics Method[2]

In this section, we briefly review our control strategy for a simplified two-link brachiating robot. A detailed development of the target dynamics controller can be found in [7]. The strategy is a particular instance of input/output linearization Specifically, brachiation is encoded as the output of a target dynamical system—a

[2]Portions of this section are exerpted from [7].

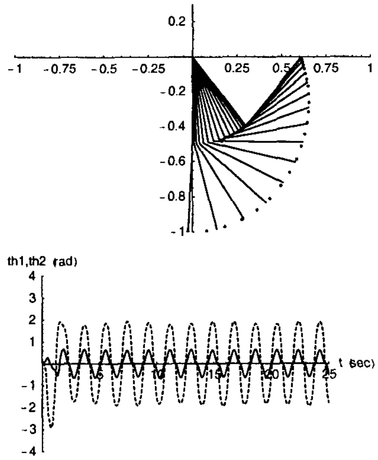

Figure 6: Typical movement of the well regulated swing motion under the hybrid controller, where initial condition is $Tq_0 = [0.1, 0, 0, 0]^T$. Top: motion of the robot at the capture of the bar at $d^* = 0.6$, when $t = 20 \sim 20.5$. Bottom: Joint trajectories (solid: θ_1, dashed: θ_2).

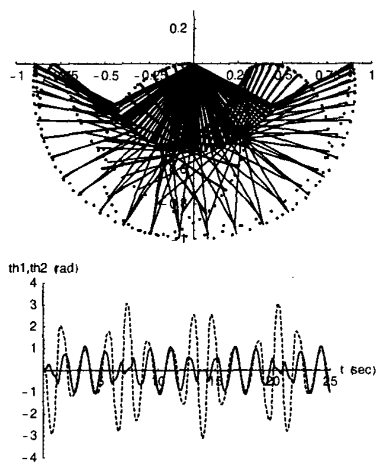

Figure 7: Typical "chaotic" motion under the original controller, where initial condition is $Tq_0 = [0.1, 0, 0, 0]^T$. Top: motion of the robot, when $t = 10 \sim 15$. Bottom: Joint trajectories (solid: θ_1, dashed: θ_2).

harmonic oscillator determined by a "virtual frequency", ω, which we will force the robot to mimic.

A.1 Task Encoding: Target Dynamics

Consider the dynamics of the two-link brachiating robot which take the form of a standard two-link planer manipulator in (1).

Motivated by the pendulum-like motion of an ape's brachiation, we choose to encode the task in terms of the even simpler linearized version,

$$y = Tx = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \; f_\omega(Tx) = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} Tx, \qquad (12)$$

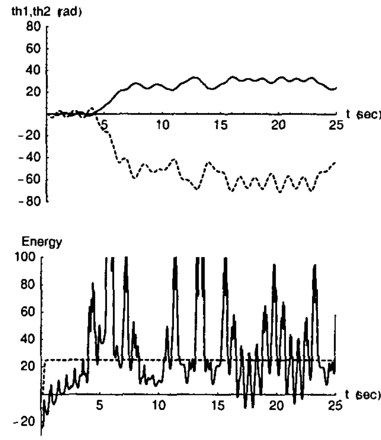

Figure 8: Top: Joint trajectories (solid: θ_1, dashed: θ_2), Bottom: pseudo energy (dashed), and mechanical energy (solid) under the original swing up controller with the "wrong" choice of $\omega = 4.5$. Note that "wild" motion is observed driven by unrealistically high torque in this case.

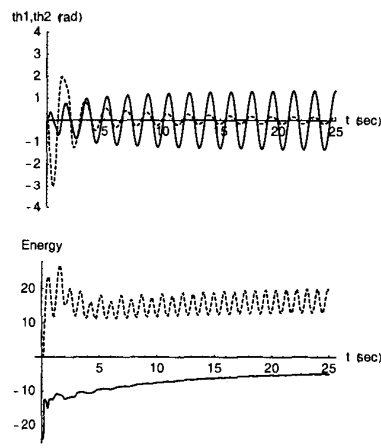

Figure 9: Joint trajectories (top, solid: θ_1, dashed: θ_2), Bottom: pseudo energy (dashed) and mechanical energy (solid) under the hybrid controller with the "wrong" choice of $\omega = 4.5$. In contrast, the total energy is bounded under the hybrid controller.

which will serve as the target dynamical system.

Now, we will find it useful to introduce a submersion arising from the change of coordinates from joint space to polar coordinates on $\mathbb{R}^2$,

$$\begin{bmatrix} r \\ \theta \end{bmatrix} = \tilde{q} = \bar{g}(q) = \begin{bmatrix} l\sqrt{2(1+\cos\theta_2)} \\ \theta_1 + \frac{1}{2}\theta_2 \end{bmatrix}. \qquad (13)$$

Specifically, we will take the second component of (13)

$$x = h(q) := \theta = [0, 1]\; \bar{g}(q) = \theta_1 + \frac{1}{2}\theta_2. \qquad (14)$$

The torque input realizing the characteristics of the target dynamical system (12) is formulated using input/output linearizing scheme

$$\begin{aligned}\tau &:= \left(D_q h \begin{bmatrix} n_{12} \\ n_{22} \end{bmatrix}\right)^{-1} \left[-\omega^2\theta - (\dot{D_q h})\dot{q} + D_q h M^{-1}(V+k)\right] \\ &= \frac{1}{n_{12}+\frac{1}{2}n_{22}} \left[-\omega^2(\theta_1 + \frac{1}{2}\theta_2) + (n_{11} + \frac{1}{2}n_{21})(V_1 + k_1)\right] \\ &\quad + V_2 + k_2 \end{aligned} \quad (15)$$

where, n_{ij} denotes each component of M^{-1}. Note that

$$D_q h \begin{bmatrix} n_{12} \\ n_{22} \end{bmatrix} = \frac{m_1 l_{c1}^2 + m_2({l_1}^2 - {l_{c2}}^2) + I_1 - I_2}{2\det(M)} \neq 0, \quad (16)$$

i.e., the invertibility condition of the first term in (15) is satisfied in the particular setting of concern.

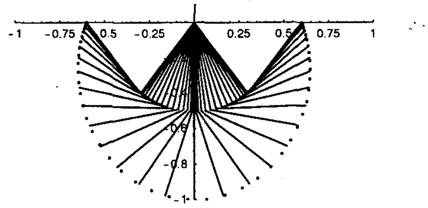

Figure 10: Motion of the robot in the ladder problem achieving a neutral orbit, where $d = 0.6, \omega = 3.3649$.

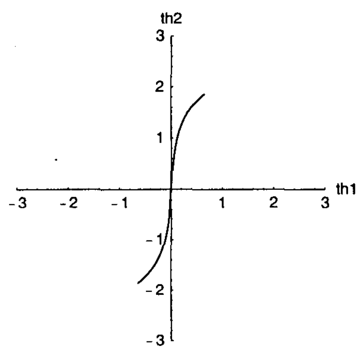

Figure 11: Trajectories of the motion of the robot shown in Figure 10 on the (θ_1, θ_2) plane achieving a neutral orbit.

A.2 Review of the Swing up Controller

Swing up requires energy pumping in a suitable fashion. The target dynamics (12) is modified to introduce a limit cycle in order to achive the task as

$$\dot{Tx} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -K_e(\bar{E} - \bar{E}^*) \end{bmatrix} Tx := f_{\bar{E}^*}(Tx) \quad (17)$$

where, $x = \theta = \theta_1 + \frac{1}{2}\theta_2$ as defined in (14)
K_e: a positive constant
$\bar{E} := \frac{1}{2}\dot{\theta}^2 + \frac{1}{2}\omega^2\theta^2$: "pseudo energy"
$\bar{E}^*$: the desired pseudo energy level

To achieve this target dynamics, the control law is formulated for the experimental system as

$$\begin{aligned}\tau_{\bar{E}^*} &:= \left(D_q h \begin{bmatrix} n_{12} \\ n_{22} \end{bmatrix}\right)^{-1} \left[-\omega^2\theta - K_e(\bar{E} - \bar{E}^*)\dot{\theta} \right. \\ &\quad \left. - (\dot{D_q h})\dot{q} + D_q h M^{-1}(V+k))\right] \\ &= \frac{1}{n_{12}+\frac{1}{2}n_{22}} \left[-\omega^2(\theta_1 + \frac{1}{2}\theta_2) \right. \\ &\quad - K_e(\bar{E} - \bar{E}^*)(\dot{\theta_1} + \frac{1}{2}\dot{\theta_2}) \\ &\quad \left. + (n_{11} + \frac{1}{2}n_{21})(V_1 + k_1)\right] + V_2 + k_2 \end{aligned} \quad (18)$$

The time derivative of the pseudo energy, $\dot{\bar{E}}^*$ along the motion suggests the convergence of $\bar{E} \to \bar{E}^*$. Therefore, it suggests that this control law achieves a stable limit cycle whose trajectory is characterized by $\frac{1}{2}\dot{\theta}^2 + \frac{1}{2}\omega^2\theta^2 = \bar{E}^*$ on the phase plane of $(\theta, \dot{\theta})$.

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