Relevant Consequence and Empirical Inquiry

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Abstract

A criterion of adequacy is proposed for theories of relevant consequence. According to the criterion, scientists whose deductive reasoning is limited to some proposed subset of the standard consequence relation must not thereby suffer a reduction in scientific competence. A simple theory of relevant consequence is introduced and shown to satisfy the criterion with respect to a formally defined paradigm of empirical inquiry.

1 Introduction

Deductive implication in a first-order framework has been the pivotal concept in at least two attempts to analyze aspects of scientific reasoning. On the one hand, the inductive confirmation of a given theory has been linked to the empirical verification of its logical consequences (see Hempel, 1965). On the other hand, the idea that one theory is closer to the truth than another has been construed in terms of the theories' respective sets of true and false consequences (see Popper, 1959). It is well known that both analyses founder on the richness of the class of deductive consequences of a given theory. Thus, one consequence of the axiom A is $A \vee S$ for arbitrary sentence S; yet verification of S (hence of $A \vee S$) need not confirm A. Similarly, Tichý (1974) proved that given any two theories A, B, if A is false then either A has a false consequence not implied by B, or B has a true consequence not implied by A. It follows that no false theory is closer to the truth than any other theory in the sense intended by Popper. As observed by Schurz & Weingartner (1987), Tichý's proof requires the same, valid inference from A to $A \vee S$.

Now this latter inference has an odd character inasmuch as it does not depend on any particular relation between A and S. In this sense, the form of S is "irrelevant" to the deduction. Thus is born the idea that by considering only the relevant consequences of a theory it may be possible to resurrect the two analyses of scientific reasoning cited above.

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Such is the programme defended and elaborated by Weingartner and Schurz (Weingartner, 1988; Weingartner & Schurz, 1986; Schurz & Weingartner, 1987). It is not the purpose of the present note to summarize the substantial results achieved by these investigators. Rather, we wish to suggest and illustrate a convergent criterion of adequacy for proposed definitions of relevant consequence, namely:

(1) CRITERION: Scientists whose deductive reasoning is limited to some proposed subset of the standard consequence relation must not thereby suffer a reduction in scientific competence (compared to scientists not so limited in the deductions they draw).

The justification of (1) should be apparent. An illuminating analysis of scientific practice cannot rest upon a system of reasoning that — if actually employed by scientists — would deny them empirical insights otherwise attainable.

Criterion (1) can be applied to a definition of relevant consequence only in the presence of a precise conception of scientific practice and scientific success. Such a conception amounts to formally specifying a paradigm of empirical inquiry. Many paradigms have been advanced and analyzed in recent years, mainly in the context of theoretical studies of machine learning and inductive inference (see Rivest, Haussler & Warmuth, 1989; Case & Fulk, 1990; and references cited there). For purposes of illustration we shall focus on the simplest paradigm that seems to preserve essential ingredients of scientific reasoning in a first-order framework; it was first introduced in Osherson, Stob & Weinstein (1991a). Similarly, our illustration bears on an elementary conception of relevant consequence. This conception will be shown to satisfy Criterion (1) with respect to the paradigm we describe.

Our exposition is organized as follows. A definition of relevant consequence is provided in the next section. Section 3 introduces a paradigm of empirical inquiry that frames the remaining analysis. In Section 4 we define a (formal) scientist who reasons in a relevant manner and whose scientific competence is demonstrably maximal. Concluding remarks occupy Section 5.

For the sequel we fix a countable, first-order language $\mathcal{L}$ with identity. The set of $\mathcal{L}$-formulas and $\mathcal{L}$-sentences are denoted by $\mathcal{L}_{form}$ and $\mathcal{L}_{sen}$, respectively. Unless otherwise noted, structures (or "models") are assumed to interpret $\mathcal{L}$. By "predicate" we mean a predicate letter other than $=$. We rely on the standard account of consequence for open formulas, namely: Given sets Γ, Σ of formulas, we write $\Gamma \models \Sigma$ just in case for all structures $\mathcal{S}$ and assignments g to $\mathcal{S}$, if $\mathcal{S} \models \Gamma[g]$ then $\mathcal{S} \models \Sigma[g]$.

2 Relevant consequence

The following definition is based on ideas that may be attributed to Korner (1959, 1979) (cited in Weingartner, 1988). As a preliminary, for each arity $n \geq 0$ we fix an n-ary predicate Q^n that does not occur in $\mathcal{L}$. Given $\theta \in \mathcal{L}_{form}$ and n-ary predicate R, the result of uniformly replacing every occurrence of R in θ by Q^n is denoted $\theta[R \mid Q^n]$.

(2) DEFINITION: Let $\Gamma \subseteq \mathcal{L}_{form}$ and $\theta \in \mathcal{L}_{form}$ be given. Γ *implies θ relevantly* just in case:

(a) $\Gamma \models \theta$;

(b) there is no n-ary predicate symbol R occurring in θ such that $\Gamma \models \theta[R \mid Q^n]$.

In this case we write $\Gamma \models_{\mathrm{r}} \theta$; otherwise $\Gamma \not\models_{\mathrm{r}} \theta$. If (a) is true but (b) false, then Γ *implies* θ *irrelevantly.*

(3) EXAMPLES:

(a) Suppose that $\Gamma = \{\forall x \neg Fx\}$ and $\theta = \forall x(Fx \rightarrow Gx)$. Then $\Gamma \models \theta$ but $\Gamma \not\models_{\mathrm{r}} \theta$. This is because $\Gamma \models \theta[G \mid Q^1]$, i.e., $\Gamma \models \forall x(Fx \rightarrow Q^1 x)$.

(b) For every $A, S \in \mathcal{L}_{form}$ if A and S share no predicates then $A \not\models_{\mathrm{r}} A \vee S$.

(c) $\{\forall x(Fx \vee Gx), \forall x \neg Fx\} \models_{\mathrm{r}} \forall x Gx$.

(d) $Fa \wedge Gb \models_{\mathrm{r}} (Fa \wedge Gb) \vee Fb$.

Examples (a) and (b) suggest that valid inferences labelled irrelevant by Definition (2) are defective from the point of view of empirical inquiry. Thus, a scientist who wishes to determine the truth of $\forall x \neg Fx$ would not normally attempt to verify $\forall x(Fx \rightarrow Gx)$; similarly for A and $A \vee S$. In contrast, the relevant inference of Example (c) seems to represent legitimate scientific practice; determining the truth of $\{\forall x(Fx \vee Gx), \forall x \neg Fx\}$ might well involve an attempt to verify $\forall x Gx$.

Example (d) shows that Definition (2) does not, however, cover all cases of irrelevant inference. The definition may be strengthened to label (d) irrelevant (as in Schurz & Weingartner, 1987). We shall nonetheless maintain the present version since it facilitates illustration of Criterion (1).

We note that $\models_{\mathrm{r}}$ is not proposed as a new analysis of validity, alternative to $\models$. To the contrary, we assume that in a first-order context, the (standard) consequences of a theory T exhaust the set of sentences whose truth is guaranteed by that of T. Relevant consequence should rather be construed as a model of the deductive component of scientific reasoning. As such, $\models_{\mathrm{r}}$ cannot be expected to possess all the pleasing properties of $\models$ (monotonicity, deduction theorem, etc.). In fact, contrary to standard consequence, we have the following.

(4) PROPOSITION: Suppose that $\mathcal{L}$ contains one 2-ary predicate and the 0-ary predicate P. Then the subset $X = \{(\alpha, \beta) \mid \alpha \models_{\mathrm{r}} \beta\}$ of pairs of $\mathcal{L}$-formulas is not effectively enumerable.

Proof: If X were effectively enumerable, then (contrary to fact) so would be the set of noncontradictions in the sublanguage $\mathcal{L}'$ of $\mathcal{L}$ without P. For, $\theta \in \mathcal{L}'$ is noncontradictory iff $P \wedge \theta \models_{\mathrm{r}} P$. ■

Finally, it will be convenient at this juncture to consider a kind of irrelevance that can infect individual sentences. Let $\theta = \exists x(Gx \wedge (Fx \vee \neg Fx))$, and suppose that structure $\mathcal{S}$ satisfies θ. Then every expansion $\mathcal{S}'$ of $\mathcal{S}$ to Q^1 satisfies $\theta[F \mid Q^1]$. The predicate F thus appears "inessentially" in θ, and scientific reasoning about θ is bound to have an odd character. The following definition allows us to exclude such cases from our discussion of empirical inquiry.

(5) DEFINITION: Let $\theta \in \mathcal{L}_{form}$ be given. θ is *normal* just in case for every structure $\mathcal{S}$ and every n-ary predicate F appearing in θ, if $\mathcal{S} \models \theta$ then some expansion of $\mathcal{S}$ to Q^n satisfies $\neg\theta[F \mid Q^n]$.

3 A paradigm of empirical inquiry

The present section introduces a formal paradigm of empirical inquiry.

3.1 Overview

Our fixed language $\mathcal{L}$ is assumed to be suitable for expressing scientific data and theories within some given field of inquiry. Prior research in the field is conceived as verifying a set $T \subset \mathcal{L}$ of axioms already known to be true. Each structure satisfying T thus represents a possible world consistent with background knowledge. For simplicity, attention is limited to countable structures; "structure" and related expressions should henceforth be understood in this sense. We assume that Nature has chosen some (countable) model $\mathcal{S}$ of T to be actual; her choice is unknown to us.

A given scientist ψ is conceived as attempting to divine the truth-value in $\mathcal{S}$ of some specific sentence θ that is not decided by T. At the start of inquiry the deductive consequences of T exhaust ψ's knowledge of $\mathcal{S}$. As inquiry proceeds, the following kind of information becomes available to ψ. It is assumed that ψ is able to determine, for each atomic formula $\varphi(\bar{x})$ of $\mathcal{L}$ and any $\bar{a} \in |\mathcal{S}|$ whether or not $\bar{a}$ satisfies $\varphi(\bar{x})$ in $\mathcal{S}$. ψ receives all of $|\mathcal{S}|$ in piecemeal fashion and bases its conjecture at a given moment on the finite subset of $|\mathcal{S}|$ examined by that time. Upon receiving each new datum, ψ emits a fresh conjecture about the truth of θ in $\mathcal{S}$, announcing either "true" or "false." To be counted as successful, ψ's successive conjectures must stabilize to "true" if $\mathcal{S} \models \theta$ and to "false" otherwise.

Further motivating remarks may be found in Osherson & Weinstein (1989). We turn now to the definitions that formalize the paradigm.

3.2 Definitions

$\varphi \in \mathcal{L}_{form}$ is called "basic" just in case φ is an atomic formula or the negation of an atomic formula. The basic subset of $\mathcal{L}_{form}$ is denoted BAS. In the context of an assignment of variables to the unknown structure $\mathcal{S}$, members of BAS may be conceived as encoding facts of the form $\bar{a} \in P^{\mathcal{S}}$ or $\bar{a} \notin P^{\mathcal{S}}$, for predicate $P \in \mathcal{L}$, as suggested in the overview. The set of all finite sequences over BAS is denoted SEQ. Given $\sigma \in$ SEQ, the set of formulas appearing in σ is denoted by $range(\sigma)$. Members of SEQ of length n may be conceived as potential "evidential positions" of a scientist at moment n of his inquiry. By a *complete assignment* to a structure $\mathcal{S}$ is meant a mapping of the variables of $\mathcal{L}$ onto $|\mathcal{S}|$. Given $T \subseteq \mathcal{L}_{sen}$, the class of all (countable) models of T is denoted MOD(T).

We now consider the information available to a scientist working in an unknown structure. An *environment* is any ω-sequence over BAS. Given an environment e, the set of formulas appearing in e is denoted by *range*(e) and the initial finite sequence of length n in e is denoted $\bar{e}_n$. The following definition specifies the sense in which an environment can provide information about a structure.

(6) DEFINITION: Let environment e, structure $\mathcal{S}$, and assignment g to $\mathcal{S}$ be given. e is *for $\mathcal{S}$ via g* just in case *range*$(e) = \{\beta \in \text{BAS} \mid \mathcal{S} \models \beta[g]\}$.

Thus, when g is complete, an environment e for $\mathcal{S}$ via g provides basic information about every element of $|\mathcal{S}|$, using variables as codes for elements. It is easy to see that structures sharing an environment are isomorphic.

We take a scientist to be any function (partial or total) from SEQ to $\{t, f\}$. A scientist may be conceived as a special purpose device devoted to discovering the truth-value of a single sentence θ. Given input data $\sigma \in \text{SEQ}$, the scientist conjectures a truth-value for θ in whatever structure $\mathcal{S}$ has given rise to σ. To be successful on θ in $\mathcal{S}$, ψ must "detect" the truth-value of θ in $\mathcal{S}$, as specified by the following definition.

(7) DEFINITION: Let $\theta \in \mathcal{L}_{sen}$, structure $\mathcal{S}$ and scientist ψ be given. *ψ detects θ in $\mathcal{S}$* just in case for every complete assignment g to $\mathcal{S}$, and every environment e for $\mathcal{S}$ via g, if $\mathcal{S} \models \theta$ then $\psi(\bar{e}_n) = t$ for cofinitely many $n \in N$, and if $\mathcal{S} \models \neg\theta$ then $\psi(\bar{e}_n) = f$ for cofinitely many $n \in N$.

Thus, we credit ψ with detecting θ in $\mathcal{S}$ just in case ψ's successive conjectures about the truth-value of θ in $\mathcal{S}$ eventually stabilize to the correct one in response to increasingly complete information about $\mathcal{S}$.

(8) DEFINITION: Let class $\mathcal{K}$ of structures, $\theta \in \mathcal{L}_{sen}$ and scientist ψ be given. *ψ detects θ in $\mathcal{K}$* just in case for all $\mathcal{S} \in \mathcal{K}$, ψ detects θ in $\mathcal{S}$. In this case θ is *detectable in $\mathcal{K}$*.

Pursuant to the conception of scientific inquiry described in the overview, we shall be particularly concerned with detectability in elementary classes of structures. Given $\theta \in \mathcal{L}_{sen}$ and $T \subseteq \mathcal{L}_{sen}$ we ask whether θ is detectable in MOD(T). Examples of detectability and nondetectability in the foregoing sense are provided in Osherson, Stob & Weinstein (1991a). The following theorem is also proved there.

(9) THEOREM: Let $\theta \in \mathcal{L}_{sen}$ and $T \subseteq \mathcal{L}_{sen}$ be given. θ is detectable in MOD(T) iff both θ and $\neg\theta$ are equivalent over T to existential-universal sentences.

In the sequel we shall consider general-purpose scientists who are parameterized by a background theory T and target sentence θ. Formally, such a scientist Ψ is a function with range $\{t, f\}$ and defined in the set of triples of form (T, θ, σ), where $T \subseteq \mathcal{L}_{sen}$, $\theta \in \mathcal{L}_{sen}$, and $\sigma \in \text{SEQ}$. Ψ may be thought of as converting arbitrary T and θ into a scientist $\lambda\sigma.\Psi(T, \theta, \sigma)$ to be evaluated for its ability to detect θ in MOD(T).

4 A scientific method based on relevant consequence

In the present section we define a general-purpose scientist Ψ with the following properties.

(10) (a) Ψ embodies a method of inquiry that rests squarely on relevant consequence, with no role for irrelevant consequence.

(b) Ψ enjoys maximal scientific competence in the following sense. Ψ can detect any normal sentence in the models of any given theory, provided only that this is possible in principle.

We take the existence of such a Ψ to demonstrate that scientists who reason in conformity with Definition (2) of relevant consequence need suffer thereby no reduction in scientific competence — at least, not within the framework of the present paradigm of empirical inquiry. In this sense, Definition (2) satisifes Criterion (1).

Towards the specification of Ψ we fix an enumeration $\Pi = \{\pi_j \mid j \geq 0\}$ of all universal formulas in $\mathcal{L}_{form}$. The following definitions will also be helpful.

(11) DEFINITION: Let $\pi_j \in \Pi$ and $\sigma \in$ SEQ be given. σ *cancels* π_j just in case there is $\beta \in$ BAS such that:

(a) $\pi_j \models_{\rm r} \beta$

(b) $range(\sigma) \models_{\rm r} \neg\beta$.

(12) DEFINITION: The functions TRUE($\cdot$) and FALSE($\cdot$) are defined as follows. Let $T \subseteq \mathcal{L}_{sen}$, $\theta \in \mathcal{L}_{sen}$, and $\sigma \in$ SEQ be given. TRUE(T, θ, σ) is the least $j \geq 0$ such that:

(a) σ does not cancel π_j;

(b) there is $\alpha \in \mathcal{L}_{sen}$ with $T \cup \{\pi_j\} \models_{\rm r} \alpha \wedge \theta$ and $\alpha \wedge \theta \models_{\rm r} \theta$.

If no such j exists, TRUE$(T, \theta, \sigma) = \omega$. FALSE$(T, \theta, \sigma)$ is the least $j \geq 0$ such that:

(a) σ does not cancel π_j;

(b) there is $\alpha \in \mathcal{L}_{sen}$ with $T \cup \{\pi_j\} \models_{\rm r} \alpha \wedge \neg\theta$ and $\alpha \wedge \neg\theta \models_{\rm r} \neg\theta$.

If no such j exists, FALSE$(T, \theta, \sigma) = \omega$.

Finally, given $T \subseteq \mathcal{L}_{sen}$ and $\sigma \in$ SEQ, define $\Psi(T, \theta, \sigma) =$

$$\begin{cases} t & \text{if TRUE}(T,\theta,\sigma) < \text{FALSE}(T,\theta,\sigma) \\ f & \text{if FALSE}(T,\theta,\sigma) < \text{TRUE}(T,\theta,\sigma) \\ \text{undefined} & \text{otherwise.} \end{cases}$$

That Ψ satisfies (10)b is shown by the following.

(13) THEOREM: Let $T \subseteq \mathcal{L}_{sen}$ and normal $\theta \in \mathcal{L}_{sen}$ be given. If θ is detectable in MOD(T) then $\lambda\sigma.\Psi(T, \theta, \sigma)$ detects θ in MOD(T).

Proof: See the Appendix. ■

Regarding (10)a, it is clear that the core of Ψ's reasoning rests on two kinds of relevant deduction: one from $T \cup \{\pi_j\}$ to $\alpha \wedge \pm\theta$, the other from $\alpha \wedge \pm\theta$ to $\pm\theta$. To see that the combination of such steps is not a disguised form of standard consequence, it suffices to note the following. There exist $A, B \in \mathcal{L}_{form}$ such that (a) $A \models B$, but (b) there is no $C \in \mathcal{L}_{form}$ with $A \models_r C \wedge B$ and $C \wedge B \models_r B$. An example is $A = Fa$, $B = Fa \vee Gb$. We conclude that Ψ satisfies (10)a.

5 Concluding remarks

Although Definition (2) meets Criterion (1), we saw that it does not correctly label all cases of irrelevant deduction. The definition thus cannot serve to rehabilitate the idea that verified consequences of a theory are confirmatory (and similarly for verisimilitude). Alternative conceptions of relevant consequence must therefore be considered, for example, those reviewed in Weingarten (1988).[1] Within the perspective of the present paper, each such conception may be evaluated from two points of view. On the one hand, it should help make sense of confirmation, verisimilitude, and other aspects of empirical inquiry. On other hand, it should help explain why scientific reasoning sometimes succeeds, by serving as the sole deductive resource in a successful method of inductive inference.

Criterion (1) has here been interpreted in terms of a particular paradigm of empirical inquiry, but several other models would have served equally well (for alternative paraidigms, see Osherson, Stob & Weinstein, 1989, 1991b,c 1992). The most satisfying analysis would characterize the class of paradigms in which scientists who rely on relevant consequence have maximal competence. Achieving such a characterization will no doubt depend on progress in the theory of inductive inference and scientific discovery.

Appendix: Proof of Theorem (13)

Let $T \subseteq \mathcal{L}_{sen}$, normal $\theta \in \mathcal{L}_{sen}$, and $\mathcal{S} \in \text{MOD}(T)$ be given, and suppose that θ is detectable in $\text{MOD}(T)$. (If $\text{MOD}(T) = \emptyset$ nothing needs to be proved.) Suppose that $\mathcal{S} \models \theta$ (the case $\mathcal{S} \models \neg\theta$ is parallel). Let environment e for $\mathcal{S}$ via complete assignment g to $\mathcal{S}$ be given. We must show:

(14) $\Psi(T, \theta, \bar{e}_m) = t$ for cofinitely many $m \geq 0$.

Since $\mathcal{S} \models \theta$, for every $\pi_j \in \Pi$ such that $T \cup \{\pi_j\} \models \neg\theta$ there is $m \geq 0$ such that $\bar{e}_m$ cancels π_j (because g maps the variables of $\mathcal{L}$ onto $|\mathcal{S}|$). Consequently, by the definition of Ψ, (14) follows from:

[1] In contrast, relevance logics within the tradition established by Anderson & Belnap (1975; Dunn, 1986) do not seem suited to the present context. Virtually all such logics allow the inference from A to $A \vee B$, and rule out the inference from $\{A \vee B, \neg B\}$ to A.

(15) There exists $\pi_j \in \Pi$ and $\alpha \in \mathcal{L}$ such that:

(a) for all $m \geq 0$, $\bar{e}_m$ does not cancel π_j;

(b) $T \cup \{\pi_j\} \models_{\mathrm{r}} \alpha \wedge \theta$;

(c) $\alpha \wedge \theta \models_{\mathrm{r}} \theta$.

By Theorem (9), let $\pi_k \in \Pi$ be such that:

(16) (a) $\mathcal{S} \models \pi_k$

(b) $T \cup \{\pi_k\} \models \theta$.

For each n-ary predicate F appearing in θ, let $\hat{F} =$

$$\begin{cases} \neg\exists\bar{x}F\bar{x} & \text{if } \mathcal{S} \models \neg\exists\bar{x}F\bar{x} \\ F\bar{x} & \text{otherwise.} \end{cases}$$

where in the latter case $\bar{x}$ is the lexicographically first set of n variables such that $\mathcal{S} \models F\bar{x}[g]$.

Let $\beta = \bigwedge\{\hat{F} \mid$ the predicate F appears in $\theta\}$. We take α to be the existential closure of β. We take π_j to be $\pi_k \wedge \beta$. It is easy to verify that $\pi_j \in \Pi$. By (16)a it follows that π_j satisfies (15)a, and that:

(17) (a) $\mathcal{S} \models \pi_j[g]$

(b) $\mathcal{S} \models \alpha \wedge \theta$

In view of (16)b we have:

(18) (a) $T \cup \{\pi_j\} \models \alpha \wedge \theta$

(b) $\alpha \wedge \theta \models \theta$

Thus, it remains to show that the latter implications are relevant.

For (18)a suppose that n-ary predicate F appears in θ. It is easy to see that some extension $\mathcal{S}'$ of $\mathcal{S}$ to Q^n satisfies $\neg\hat{F}[F|Q^n]$. By (17)a, $\mathcal{S}' \models T \cup \{\pi_j\}$. However, $\mathcal{S}' \not\models (\alpha \wedge \theta)[F \mid Q^n]$. It follows that $T \cup \{\pi_j\} \models_{\mathrm{r}} \alpha \wedge \theta$.

As for (18)b, let n-ary predicate F appear in θ. By Definition (5) and the normality of θ there is an expansion $\mathcal{S}'$ of $\mathcal{S}$ to Q^n such that $\mathcal{S}' \not\models \theta[F \mid Q^n]$. By (17)b, $\mathcal{S}' \models \alpha \wedge \theta$. It follows that $\alpha \wedge \theta \models_{\mathrm{r}} \theta$. ■

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