

FIG. 1. (Color) (a, b) Multiple exposures of tracer particles in a  $10\text{ cm} \times 10\text{ cm}$  region of the flow. (c, d) Corresponding velocity (arrows) and vorticity (color) fields. Left column (a, c)  $\text{Re}=32$ . Right column (b, d)  $\text{Re}=245$ .

### Detecting topological features of chaotic fluid flow

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Traditionally, fluid flows are characterized by studying their velocity fields. New experimental techniques based on following the motion of tracer particles give us more information and allow us to study the geometric and topological properties of the flow. In particular, we can use the curvature of particle trajectories to help us locate the primary topological feature of the flow.

By driving electric current across a layer of conducting fluid positioned above an array of permanent magnets, we

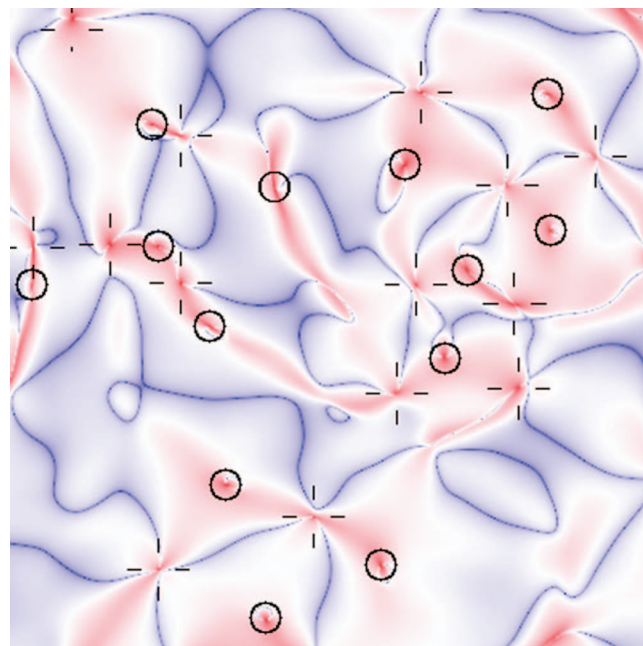


FIG. 2. (Color) Curvature field (red: high curvature; blue: low curvature) with hyperbolic (crosses) and elliptic (circles) points marked.

produce quasi-two-dimensional flow.<sup>1,2</sup> A square lattice of alternating north and south poles produces a square vortex lattice at low Reynolds number ( $\text{Re}$ ) that becomes disordered as  $\text{Re}$  increases, as shown in Fig. 1.

We measure the instantaneous curvature for each of our thousands of simultaneous particle trajectories and thereby construct a *curvature field*, as shown in Fig. 2. The local maxima of the curvature field indicate the topological singularities of the flow.<sup>1,2</sup> In two dimensions, these points are either hyperbolic (saddle points) or elliptic (vortex cores).

As  $\text{Re}$  increases, the hyperbolic and elliptic points make larger excursions from their forced locations, until they break free from the forced lattice. They also appear and disappear in pairs beyond the onset of spatiotemporal chaos.<sup>1,2</sup>

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<sup>1</sup>N. T. Ouellette and J. P. Gollub, *Phys. Rev. Lett.* **99**, 194502 (2007).

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