A Proposal for Example Induced Programming

Rochelle Fleischmann

University of Pennsylvania

Follow this and additional works at: https://repository.upenn.edu/cis_reports

Recommended Citation


This paper is posted at ScholarlyCommons. https://repository.upenn.edu/cis_reports/741
For more information, please contact repository@pobox.upenn.edu.
A Proposal for Example Induced Programming

Abstract
The user, programmer-analyst, programmable device chain of communication is analyzed as a type of authoritarian hierarchy. The elimination of the function of the programmer-analyst is presented as a desirable goal. The modus operandi of the programmer-analyst is discussed and analogs suitable for implementation on a programmable device are presented. These analogs are embedded in a computer simulation and tested. Problem areas are discussed and further areas of investigation are suggested.

Comments
A PROPOSAL FOR EXAMPLE INDUCED PROGRAMMING

A thesis submitted to the Faculty of the Moore School of Electrical Engineering in partial fulfillment of the requirements for the degree of Master of Science in Engineering (Computer and Information Science)

Rochelle Fleischmann

1975
University of Pennsylvania

THE MOORE SCHOOL OF ELECTRICAL ENGINEERING

A PROPOSAL FOR EXAMPLE INDUCED PROGRAMMING

Rochelle Fleischmann

A thesis submitted to the Faculty of the MOORE SCHOOL OF ELECTRICAL ENGINEERING in partial fulfillment of the requirements of the degree of Master of Science in Engineering for graduate work in Computer and Information Sciences

April, 1975
Title of thesis: A PROPOSAL FOR EXAMPLE INDUCED PROGRAMMING

Abstract

The user, programmer-analyst, programmable device chain of communication is analyzed as a type of authoritarian hierarchy. The elimination of the function of the programmer-analyst is presented as a desirable goal. The modus operandi of the programmer-analyst is discussed and analogs suitable for implementation on a programmable device are presented. These analogs are embedded in a computer simulation and tested. Problem areas are discussed and further areas of investigation are suggested.

A thesis submitted to the Faculty of the MOORE SCHOOL OF ELECTRICAL ENGINEERING in partial fulfillment of the requirements for the degree of Master of Science in Engineering for graduate work in Computer and Information Sciences.

April, 1975
Acknowledgements

My sincere thanks to Dr. John W. Carr III for initially suggesting the problem investigated in this thesis and for his constant support and encouragement. Special appreciation is extended to Phyllis Wharton and to Edward Treister for their patience and helpful suggestions. My thanks go also to I. P. Sharp Associates Limited for providing me most generously with the use of their facilities.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Analysis of the Problem</td>
<td>3</td>
</tr>
<tr>
<td>Communication Between the User and the Programmer-Analyst</td>
<td>4</td>
</tr>
<tr>
<td>Construction of the Program</td>
<td>7</td>
</tr>
<tr>
<td>Description of the Growth Algorithm</td>
<td>9</td>
</tr>
<tr>
<td>Conclusions</td>
<td>15</td>
</tr>
<tr>
<td>Appendices</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>I  Program Listing</td>
<td>21</td>
</tr>
<tr>
<td>II Program Flowcharts</td>
<td>36</td>
</tr>
<tr>
<td>III File Layouts</td>
<td>60</td>
</tr>
<tr>
<td>IV File Descriptions</td>
<td>63</td>
</tr>
<tr>
<td>V  Input to the Model</td>
<td>67</td>
</tr>
<tr>
<td>VI Program Messages and their Meanings</td>
<td>71</td>
</tr>
<tr>
<td>VII Sample Runs</td>
<td>76</td>
</tr>
<tr>
<td>Hero's Algorithm for the Area of a Triangle</td>
<td>77</td>
</tr>
<tr>
<td>Calculation of the Final Temperature of a Mixture of Two Known Quantities of Ice and Water</td>
<td>82</td>
</tr>
<tr>
<td>VIII Other Approaches to the Problem</td>
<td>96</td>
</tr>
<tr>
<td>IX Bibliography</td>
<td>100</td>
</tr>
</tbody>
</table>
INTRODUCTION

Since the introduction of programmable devices, their availability has been severely limited to:

- those who can program or are willing and able to learn how;
- those who can pay someone to do the programming for them;
- those who can take advantage of preprogrammed packages.

Although historically access to programmable devices has been simplified considerably by the introduction of higher level languages, operating systems, etc., there still exists an elite of users determined by aptitude, extensive training and money. Declining costs of the devices themselves have made them available to an ever widening segment of society; however, true democratization rests on the elimination of the programmer.

The programmer-analyst acts as an intermediary between user and machine. In many respects his function resembles that of the high priest in a cultic religion. As long as the uninitiated must turn to him to intercede on their behalf, programmable devices will remain a mysterious and vaguely threatening force in society. This work therefore
has as its ultimate aim the democratization and demystification of the device.
ANALYSIS OF THE PROBLEM

Let us continue the cultic analogy. It is clear that in many such religions, it is not the priest himself who is central to the man god relationship but rather the knowledge that the priest possesses. Assumedly, man does not communicate directly with his god because he doesn't know how. His god demands a specific form of address which only the priest has mastered. The priest provides him with the formalized mode of behaviour appropriate to the message which he wishes to convey. This behaviour, of whatever form, reflects his needs, be they penitential, supplicational, or of some other nature. In a similar vein, the user approaches the programmer-analyst with a "message" which he wishes to have conveyed. The user would communicate directly with the computer if he could, but he doesn't know how. He therefore asks the programmer-analyst to provide him with a formalization of his problem--one which is acceptable to the computer. One must note, however, that the programmer-analyst's knowledge is not simply a catalogue of syntactically correct statements; he also often undertakes the crucial task of problem analysis. The importance of this point will become clear as this paper progresses.

We can now see that the program is a formal embodiment
of a user's message, or let us say, concept. We must now look more closely at two things:

1) how the concept is communicated from the user to the programmer-analyst.
2) how the programmer-analyst uses this concept to construct a program.

**Communication Between the User and the Programmer-Analyst**

The process which is to take place is the teaching of a concept to a student by a non-professional teacher. In this study the teacher is the user and the student is the programmer-analyst. By assuming a non-professional teacher (i.e., a non-programmer) we infer that the user knows what he wants done but has never formalized the process in the sense of a systematic analysis of the problem. He is capable of demonstrating what he wants to have done given an appropriate facility but he may not be skilled in analysis or communication. As a result he is likely to change his procedure from example to example without specifically explaining why. The effectiveness of this process depends, therefore, on the ability of the student to take an active role. He must be able to identify the point at which he fails to continue understanding--i.e., the point at which the teacher has made some logical leap from the student's point of view. He must also be able to ask questions which
will extract missing information merely implied in the statement of the problem.

Since it is our intention to eliminate the programmer-analyst as intermediary, his function must be taken over by the programmable device. The device must be able to accept information directly from the user in a form natural to him and must be able to elicit missing information.

We have said that the user is at least capable of demonstrating his concept. The programmer-analyst often takes advantage of this ability by asking the user to give some examples of what he wishes to have accomplished. This procedure obviates the necessity of an analytic approach on the part of the user and generally reduces communication problems. The programmer-analyst takes note of the examples and compares one to another. If he notices at any time that one example deviates from another one, he asks the user why the difference exists. Thus although the user has failed initially to point out the reason for a crucial change in procedure, the programmer-analyst has been able to account for it. Basically, the active participation of the programmer-analyst reduces to making three statements:

1) Show me what you want to do.

2) That's not what you did last time.
3) Why did you change procedure?

The model described herein uses analogs of these three statements. It contains an input section which accepts examples from the user in a form natural to him; it contains a comparison section which searches for changes between examples; and it contains an interrogatory section which actively requests information to account for unexplained inconsistencies. Note that underlying this approach is the assumption that the user will be able to explain any apparent inconsistencies which are discovered and that ultimately no such anomalies will remain.

There seems to exist a similarity between this approach and the TOTE unit (test, operate, test, exit) which Miller, Galanter, and Pribram use as a model of most human problem solving. The first test corresponds to statement two above. The operate phase corresponds to statement three. The comparison of examples is now carried out again to determine whether the contradiction still exists; if it does not, the intake portion of the method is concluded corresponding to the exit element of the TOTE unit.

At the conclusion of the intake phase the proposed method uses the information it has gleaned to construct a program embodying the user's concept.

**Construction of the Program**

In analyzing the nature of the given problem, a graphic representation of programs has proven useful. Of the various equivalent forms available, I have found the tree structure to be the most appropriate. In this structure, the nodes of the tree correspond to control statements and the branches correspond to computational statements. When only one branch extends downward from a node, the control statement is unconditional; otherwise, the flow passes through one of the possible branches based on a decision made at the node. We will ignore unconditional nodes in this study and instead focus in on the investigation of conditional ones. We will also limit ourselves to the investigation of binary trees in which exactly two branches extend downward from each node.

Since a program is the embodiment of a concept, a concept may be graphically represented as a tree. Each endpoint of that tree represents a subclass of the concept. The endpoints define a partition of the concept into mutually exclusive subconcepts. Each example presented by
the user falls into one of these subconcepts. Therefore in order to reconstruct the complete concept, the user must provide at least one example for each endpoint of the tree. It is then a matter of taking the union of these branches to arrive at the fully grown tree. A description of the algorithm to achieve this union follows in the next chapter.
DESCRIPTION OF TREE GROWTH ALGORITHM

There are two individuals involved in the communication process, the teacher and the student (the user and the programmable device). The first example given by the user becomes the first approximation of the tree. Its structure is trivial; it has no nodes and only one endpoint. Its only branch represents a list of computational statements which are always to be performed on the same constant data. The student remains passive because there can be no contradictions at this point.

As the second example is entered, the student compares each statement to its counterpart in the first example. Comparison is made on two dimensions, operand and operator. A difference in either dimension is sufficient to initiate the operate phase of the TOTE unit. Let us assume a new operand has been entered in the second example. Since, as we noted above, all operands in the first example are assumed to be constants, the use of a new constant must be accounted for. There are two possibilities which the student will examine:

1) The conflicting operands are not really constants but rather they are variables.
2) The conflicting operands are indeed constants and the user has neglected to explain the basis upon which the two different constants were chosen.

To determine which hypothesis is true, the student need only ask the question, "constant or variable?" The method of posing this question depends on the facilities available. A programmer-analyst will ask the question verbally; the computer simulation used in this thesis asks the question in print; a programmable calculator could ask it by locking its keyboard except for the two keys labelled constant and variable.

If the answer to the question is "variable", the student takes note that operands in this position in this example and all future ones are variables and need no longer agree. The second example can then be continued. If the answer to the question is "constant", the student can assume that a node in the tree has been reached and that he must grow a new branch from this node. The old branch emanating from the node will represent subconcepts of the same type as the first example, and the new branch will represent those similar to the second example. As we noted earlier, nodes correspond to conditional statements in a
program. Thus "growing the tree" means inserting a conditional control statement immediately preceding the noticed change in procedure.

We will assume that the condition which is tested to determine flow of control is of the form

\[ A \text{ op } B \]

where \( A \) is a significant variable whose value is being tested, \( \text{op} \) is a relational operator such as =, and \( B \) is a pivot value which splits the universe of possible values of \( A \) into two mutually exclusive partitions. For example,

\[ \text{SPEED} = 30 \]

is a condition in which \( \text{SPEED} \) is the significant variable, \( = \) is the relational operator, and 30 is the pivot value. We will also assume that the significant variable is one which has already made its appearance earlier in the tree.

The student must now identify the three constituents of the conditional part of the control statements. To do this he will begin to question the user about each variable which appeared earlier on the tree, including the intermediate results obtained from the computational statements. Of each one he will ask, "Is this the one that made you change procedure?" The programmer-analyst will pose the question verbally; the present simulation displays a variable in
print with a question mark following it and waits for a yes or no answer before proceeding to the next one; a programm-
able calculator could again lock its keyboard and display the variables on its screen in the same manner as the simu-
lation.

When the student has identified the significant vari-
able he can proceed to ask for the pivot value. If the pivot value is a constant, it can be entered directly by the user; if, however, the pivot value is another variable, the student can continue presenting variables from the point at which he left off. Again, the programmer-analyst will pose the question directly; the simulation prints the state-
ment "ENTER PIVOT VALUE" and waits for a response; the programmable calculator could lock all operator keys except the pivot value entry key and require the user to enter his pivot value.

The last piece of information required is the rela-
tional operator. The programmer-analyst asks for it direct-
ly; the simulation prints the statement "ENTER RELATION" and waits for a response; and the programmable calculator could again use the strategy of locking appropriate keys. The student can now compose the condition which should result in having the constant from the first example and the con-
stant from the second example on different branches.

The student would now enter the third phase of the TOTE unit--the retesting phase. Does the condition described by the user actually place his two examples into two different subcategories? If not, the student can only say something equivalent to "Sorry, but you're not doing what you told me you're doing. Try another explanation." If, however, all goes well, the fourth phase of the TOTE unit is entered, the exit phase. The student now realizes he is on a new branch of the tree and has no comparing to do. He accepts whatever is described to him by the user.

We have discussed what takes place if the student notices a difference in operands which are both determined to be constants. If instead the difference is in operators, the procedure is identical to what takes place following the "constant or variable?" question: the student assumes that he has reached a node and asks the necessary questions to produce the conditional control statement at the node.

We now see that although we spoke earlier of constructing the tree by the union of its separate branches, we are doing something slightly different. Instead of merging the identical beginnings of the examples, we are actually
growing the tree as the examples are presented. The tree grows cotemporaneously with example presentation. In other words, the student expands his knowledge of the user's concept as it is presented to him.

All succeeding examples are handled in the same way as the second. When the current example reaches an already established node, the branch to be taken is determined from the appropriate, already entered variables in the current example. Then, when the correct branch is determined, comparisons are made with the data on it.

The resulting tree can correctly handle all examples entered by the user if the user has entered examples of each subconcept. The tree can be said to be a faithful reproduction of the user's concept.
CONCLUSIONS

As the accompanying examples show, the present model can successfully learn a user's concept on the basis of examples given by him. However, it has at least two distinct problem areas.

A major area for continued investigation must be the problem of learning that a loop is present in the user's concept. The problem has at least two facets to it.

In the first place, the user may not know that his concept contains a loop. In fact, he may not know what a loop is. We might think of this possibility as the student looking for an entity or pattern for which no word exists in the user's language. The student and user might be speaking the equivalent of two different languages called analytic and Gestalt. Or we could account for this missing knowledge in another way. E. B. Hunt\(^2\) talks about Piaget's theories of mental capabilities being a function of maturational. He says that the availability of concepts (e.g., looping) to individuals probably profoundly effects their learning strategies. I would think that it effects their

concept teaching strategies as well. A frustrating breakdown in communication is often observed between teacher and student when one of them is explaining or asking about something in terms which the other simply cannot relate to. It is not a matter of simplicity or complexity but rather a missing link, and in this case it is not necessarily a matter of a logical connection which has been skipped. Asking about a loop would work with professionals in the elite. But what about everyone else?

Perhaps a very simple example would make this clearer. Suppose that the user wishes to convey the concept "take an average". He might begin by entering three numbers, totaling them and dividing the sum by three. As a second example he might wish to enter four numbers, total them and divide by four. He will trigger the operate phase of the TOTE unit as he enters his fourth number, and he won't be able to answer the questions posed because they are irrelevant to what he is doing. What is his significant variable? Even if he had entered as his first variable the number of numbers to be averaged, and thus had a significant variable, he would still have no pivot value. In order to have one, he would have to have a counter for the number of numbers he has entered so far, a most unnatural thing to have—unless of course you happen to be a programmer. The user might
recognize that he is looping, but then again, he would be
more likely to say that he is entering four numbers. One
cannot assume that he will understand what the programmer
means by entering a variable number four times. The unini-
tiated user will see four as an adjective, the professional
will see four as an adverb. This syntactic distinction
might turn out to be the key to the problem.

The second facet of this problem is in the area of
pattern recognition. Given that we have a tree which con-
tains an unwound loop in it, how do we go about recognizing
it? Let us take the simplest case, the trivial tree with no
nodes in it. The one branch represents a series of computa-
tional statements. Let us represent them by the string

abcabcabcabcabcabcab

We now have several options for the pattern. It could be

abc
bca
cab
abcabc
bcabca
cabcab
abcabcabc
bcababcabca
bcabcabca
cabcabcab
How do we determine which is the correct one? In addition, how do we determine the basis on which the loop continues to cycle? Given the first facet of the problem, can we ask the user in a way which is intelligible and yet does not presuppose knowledge of a professional nature? At what point would we ask the user, assuming that we know what to ask? Cotemporaneously with the presentation of the examples as we did for the conditional statements? At the end of each example? Or perhaps at the end of all example presentation?

Although I have given this first problem considerable thought, I have not come up with a viable solution. My guess is that the best approach will be from a learning theoretic and human engineering angle. Determine what questions would elicit the information needed without mentioning loops and then find a way of asking those questions.

The second problem area is the simple tedium of entering a large number of examples necessary for complex concepts. While this drawback does no damage to the theoretic basis of the model, it is a severe limitation to practical implementation. The model attempts to alleviate this problem by prompting the user and thus relieving him of some of the tedium; however, improved prompting algorithms should be considered.
The range of applications which the model proposed herein can handle is wide but still severely limited by the dual problems of looping and tedium; however, it can demonstrably produce a valid working program based on examples provided by a user.
APPENDIX I

PROGRAM LISTING
CR

NUM
1234567890. ω

OPRS
VARCREATE[] = -23-
VAR CREATE N

[1] TIE NUMBER N IN USE?
[2] +L1X ;N IN NUMS
[3] TIE IN LIB BUT NOT TIED?
[4] +L2X o = +L(LIB29)[11+10] = 10+EFFECT
[5] TIE
[6] FN IN TIE N
[7] ERASE FN
[8] L1:FN ERASE N
[9] CREATE FN
[10] L2:FN CREATE N

V
DELETE[] V
V DELETE I;K;L;S;N

[1] DELETE I TH EXAMPLE
[2] PACK=10
[3] N+1
[4] LEAF?
[5] 0:=(1=1+K+;READ 1,1+K)+0
[6] BRANCH?
[7] =(2=x K)+L1
[8] DETEFT WHICH BRANCH WAS TAKEN.
[9] CANNOT USE POINTS SINCE PIVOT VALUE MAY HAVE BEEN ERASED.
[10] N+READ 1,1+K
[12] =i=READ 2,K[3])+L2

[16] L IS LOGICAL VECTOR OF NON I TH EXAMPLE.
[17] L1:=(L/L+I=S+;READ 2, K[2])+0
[18] DELETE I TH EXAMPLE FROM INDEX COMPONENT.
[19] (S I $) REPLACE 2, K[2]
[20] DELETE I TH OPERAND FROM PROGRAM COMPONENT.
[21] DELETE I TH RESULT FROM PROGRAM COMPONENT.
[22] =i=s+;READ 1, K[1]+L3
[23] L+(~L)/pL+1+L
[24] L=(S;,')/pS
[25] K+((S")'+0 1)/L
[26] (S+(K[3]+5)+K[3]) REPLACE 1,1+K
[27] INDEX THROUGH EXAMPLE.
[31] =L0,K+K[1]+1
[32] =i=READ 2,K[3])+L2
[33] =L0,K+K[2]

V
\textbf{DISPLAY[\_\_]}\textasciitilde
\textbf{R=DISPLAY;}\textasciitilde

[1] \(\rightarrow(0=\text{STACK})+L1, R+0\)
[2] \(\rightarrow0, \text{R}+1+\text{STACK}\)
[3] \(L1:'Q'\)

\textbf{DUMP[\_\_]}\textasciitilde
\textbf{DUMP I;J;K;L;R;S}\textasciitilde

[1] \(\text{DISPLAYS ITH EXAMPLE.}\)
[2] \(\rightarrow((I<1)\lor I>N)+\text{ERR1}\)
[3] \(\text{'EXAMPLE ', 'I2' EQMT I}\)
[4] \(K+0, 0\)
[5] \(L2: K+\text{POINTER K[1]}\)
[6] \(\text{ALEAF REACHED?}\)
[7] \(\rightarrow(K[1]=-'1')+0\)
[8] \(\text{THIS EXAMPLE ABORTED HERE?}\)
[9] \(\rightarrow((aS)<L+(S+\text{READ } 2, K[2]))+I)+0\)
[10] \(\text{CHECK FOR NULLARY OPERATOR.}\)
[11] \(\rightarrow(1=\text{READ } 1, I+K)+L3\)
[12] \(J+2, (S='\prime')/I\text{R}\)
[13] \(L+2\times L-1\)
[14] \(\prime; J[\text{L-1}]+-I+J[\text{L}]+S; 1+\text{S}\)
[15] \(J[\text{L}]+-I+J[\text{L+1}]+\text{S}\)
[16] \(L4: \rightarrow L4\)
[17] \(L3: ?\)
[18] \(\rightarrow(S='0')+L4\)
[19] \(\rightarrow L4, R+0\)
[20] \(\text{ERR1: 'INVALID EXAMPLE NUMBER.'}\)
\[\text{ENTER}[]\]
\[\text{R=ENTER EC;}i;N\]

1. Returns a zero if input string contains an EPROP. Otherwise it returns 1: the operation of the operator.

2. OP is vector of recognized operators.

3. AI is index of operator in input string.

4. OS is set to one if operand comes from stack.

5. \((1+i+\text{EG}+\text{OPPS})/\text{EG}+\text{ERR}1,\text{P}+\text{OS}+0\)

6. The operator should follow an operand.

7. UNARY is the vector of operators which need an operand.

\[\text{ERR2}\]

8. \(\text{NUM} \text{ is vector of valid numeric characters.}\)

9. \(\text{AN} \text{ is the operand in literal form.}\)

10. \(\text{ERR3} \text{ is the position of operand in input string.}\)

11. \(\text{ERR4} \text{ is the operator not last symbol. Re-enter.}\)

12. \(\text{FIRST} \text{ is the first element of EPROP.}\)

13. \(\text{PIEST} \text{ is the second element of EPROP.}\)

14. \(\text{ACC} \text{ is the accumulator.}\)

15. \(\text{MAKE} \text{ new program and index files.}\)

16. \(\text{PROC} \text{ ASCRIPT 1}\)

17. \(\text{INDEX} \text{ ASCRIPT 2}\)

18. \(\text{INITIALIZE} \text{ first component of each.}\)

19. \((2,1) \text{ APPEND 1}\)

20. \((0,1) \text{ APPEND 2}\)

21. \(\text{ERR} \text{ returns a zero scalar when EPROP detected.}\)

22. \(\text{ERR} \text{ is the position of the operator in the literal input string.}\)

23. \(\text{APPEND} \text{ this expression to the program tree.}\)

24. \(\text{NEXT}\)

25. \(+L1\)

26. \(\text{APPEND} \text{ the end of branch symbol.}\)

27. \(\text{L2}:(^1,(\text{SIZE} 2)(2)-1) \text{ REPLACE 1,(\text{SIZE} 1)[2]-1}\)
\[ \forall \text{GETL}(\square) \forall \]
\[ \forall \text{R+GETL } X \]
\[ [1] \quad R = (-\rho X) + (\rho X) + \mathfrak{m}, \mathfrak{m} + X + X \]
\[ \forall \]
\[ \forall \text{LAST}(\square) \forall \]
\[ \forall \text{R+LAST } Y; Z; 0 \]
\[ [1] \quad Z = \text{READ } 1, \mid Y \]
\[ [2] \quad Y+2, (', /=Z) / \rho Z \]
\[ [3] \quad \rightarrow (Y > 0) \rightarrow L1 \]
\[ [4] \quad R = (1 + 2 + Y) + 1 + Z \]
\[ [5] \quad \rightarrow 0 \]
\[ [6] \quad L1: R = (1 - 3 + Y) + 1 + (1 - 2 + Y) + Z \]
\[ \forall \]
\[ \forall \text{MAIN}(\square) \forall \]
\[ \forall \text{MAIN}; ; ; S; OSW; P; PROG; STACK; PIVOTS \]
\[ [1] \quad PIVOTS + 1 0 \]
\[ [2] \quad N + 1 \]
\[ [3] \quad \text{FIRSTEG} \]
\[ [4] \quad L0: = (', 1) = 1 + \text{GETL} 'ANY MORE EXAMPLES (Y/N)?') + L2 \]
\[ [5] \quad L4: = \text{EXT3G} \]
\[ [6] \quad \rightarrow L0 \]
\[ [7] \quad L2: = (', 1 + \text{GETL} 'DO YOU WANT TO TRY IT OUT?(Y/N)') + L3 \]
\[ [8] \quad \text{PRINT} \text{PROG} \]
\[ [9] \quad F \]
\[ [10] \quad P = 3 \quad \text{FD } F \]
\[ [11] \quad \text{PROG} \]
\[ [12] \quad L2 \]
\[ [13] \quad L3: = (', 1 + \text{GETL} 'ANY MORE EXAMPLES (Y/N)?') + L4 \]
\[ NEXT([\_]) \]
\[ NEXT R; TEMP; \_ \]

[1] 
\[ +P IS THE POSITION OF THE OPERATOR IN THE LITERAL INPUT STRING. \]

[2] 
\[ OSN=1 IF OPERAND COMES FROM STACK. \]

[3] 
\[ \rightarrow(L2, L3, L4, L5, L6, L1)['1**o-'1EC[R]] \]

[4] 
\[ \rightarrow PUSH ACC ONTO STACK \]

[5] 
\[ L2 := L7, STACK + (\_FI ACC), STACK \]

[6] 
\[ \rightarrow DISPLAY TOP POSITION OF STACK \]

[7] 
\[ L3 := 0, \_DISPLAY \]

[8] 
\[ \rightarrow DROP TOP POSITION OF STACK \]

[9] 
\[ L4 := L7, STACK + 1 \_STACK \]

[10] 
\[ \rightarrow CLEAR ACC \]

[11] 
\[ L5 := L7, \_ACC+ '0' \]

[12] 
\[ \rightarrow ROTATE STACK ONE POSITION \]

[13] 
\[ L6 := L7, STACK + 1 \_STACK \]

[14] 
\[ \rightarrow TEMP IS LITERAL STRING ARITHMETIC EXPRESSION. \]

[15] 
\[ L1 := OSWAL10 \]

[16] 
\[ \rightarrow L11, \_TEMP+ACC, EC[P], (R-1)+EC \]

[17] 
\[ L10 := TP-ACC, EC[R], Z+, 'E15.10' \_FMT 1 + STACK \]

[18] 
\[ \rightarrow EXECUTE TEMP AND DISPLAY. \]

[19] 
\[ L11 := ACC+XEQ TEMP \]

[20] 
\[ \rightarrow OSWAL12 \]

[21] 
\[ \rightarrow APPEND OPERATOR, OPERAND TYPE, OPERAND, '\_RESULT, ', ' \]

[22] 
\[ (EC[P], 'O', (R-1)+EC), '\_RESULT, ' \]

[23] 
\[ \rightarrow SET UP NEW POINTERS. \]

[24] 
\[ L9 := ((\_SIZE 1)[2]+1), (\_SIZE 2)[2] \_APPEND 1 \]

[25] 
\[ (0, ') \_APPEND 2 \]

[26] 
\[ \rightarrow \]

[27] 
\[ \rightarrow NULLARY: APPEND OPERATOR ONLY \]

[28] 
\[ L7 := EC[R] \_APPEND 1 \]

[29] 
\[ \rightarrow L8 \]

[30] 
\[ L12 := EC[R], '2', Z, ' ', (ACC+, 'E15.10' \_FMT ACC), ', ') \]

[31] 
\[ \_APPEND 1 \]

\[ \rightarrow L8 \]
\texttt{\textbackslash V} \texttt{NEXTEG}[]\texttt{V} \\
\texttt{\textbackslash V} \texttt{NEXTEG;}J;\texttt{ACC;;P;}X;X;\texttt{TEMP;}EG;B
[1] \texttt{\_M IS EXAMPLE NUMBER (GLOBAL)}
[2] \texttt{'ENTERING EXAMPLE ';'M+1}
[3] \texttt{STACK+10}
[4] \texttt{ACC+0'}
[5] \texttt{J+20}
[6] \texttt{L0;}J+\texttt{POINTER 1+J}
[7] \texttt{L1;}+(L0,L1+1)[\texttt{PROMPT}]
[8] \texttt{+("END'\_M=3\_EG)+L14}
[9] \texttt{+("CANCEL'\_M=6+\_EG)+L8}
[10] \texttt{ENTER RETURNS A ZERO SCALAR IF AN ERROR IS DETECTED IN THE INPUT STRING.}
[11] \texttt{+(0=M\_ENTER FC)+L1}
[12] \texttt{IF THIS IS A DISPLAY REQUEST, DO SO AND THEN GET NEXT INPUT LINE.}
[14] \texttt{+L1,,DISPLAY}
[15] \texttt{J[1]} \texttt{IS COMPONENT NUMBER IN FILE 1 OF NEXT OPERAND IN THIS EXAMPLE.}
[16] \texttt{J[2]} \texttt{IS RELATED COMPONENT NUMBER IN FILE 2.}
[17] \texttt{IF J[1]=-1, PREVIOUS EXAMPLES ENDED HERE. CREATE A NEW BRANCH.}
[18] \texttt{L21;}+("1=1+J)+L4
[19] \texttt{IF NULLARY OPERATOR EXPECTED,}
[20] \texttt{SKIP OPERAND CHECKS}
[21] \texttt{+(1=x\_READ 1,1+J)+L9}
[22] \texttt{IF OPERAND IS A VARIABLE CHECK NO FURTHER.}
[23] \texttt{+(\_OSY)A'1'=}X[2])+L2
[24] \texttt{CHECK WHETHER THIS IS A STACK VARIABLE}
[25] \texttt{+("2'=}X[2])+L20
[26] \texttt{+(L4,L2)\{OSY+1}]
[27] \texttt{IF NEXT OPERAND IS A CONSTANT AND IS THE SAME AS ALL OF THE PRECEDING CONSTANTS, CHECK NO FURTHER.}
[28] \texttt{L20;}+(\_FI LAST 1+J)=\_FI(R-1)+\_EG)+L2
[29] \texttt{+("C'}1\_GETL \_CONSTANT OR VARIABLE (C/V)?')+L4
[30] \texttt{CHANGE SWITCH TO VARIABLE INDICATOR.}
[31] \texttt{X[2]}="1'
[32] \texttt{L2;}X+y,((R-1)+\_EG),'\','
[33] \texttt{("READ 2,J[2]),X') \_REPLACE 2,J[2]}
[34] \texttt{CHECK OPERATOR NEXT.}
[35] \texttt{L9;}+(\_EG[P]=1+x)+L10
[36] \texttt{+(L22,L23,L24,L25,L26)\{'+0='\_EG[P]}
[37] \texttt{L22;}\_LO,\_STACK+(\_PT ACC),\_STACK
[38] \texttt{L23;}\_LO,\_STACK+1+\_STACK
[39] \texttt{L24;}\_LO,\_ACC+0'
[40] \texttt{L25;}\_LO,\_STACK+1+\_STACK
[41] a CHECK FOR STACK OPERAND
[42] L26:=(~OSW)+L28
[43] TEMP+ACC,EG[R],,'E15.10' □FMT 1+STACK
[44] →L27
[45] □TEMP IS ARITHMETIC EXPRESSION IN LITERAL STRING FORM.
[46] L28:TEMP+ACC,EG[R],(R-1)+EC
[47] □EXECUTE AND DISPLAY TEMP.
[48] L27:□+ACC+XEQ TEMP
[49] □SAVE RESULT
[50] L30:(X+X,(ACC+,,'E15.10' □FMT ACC),'') □REPLACE 1,1+J
[51] →L
[52] □REMOVE OPERAND INFO PREVIOUSLY STORED BECAUSE A BRANCH
   Point has been reached.
[53] L10:X=□READ 1,1+J
[54] □CREATE A NEW BRANCH.
[55] L4:=(0=SIG J)+L7
[56] 'CONTINUING'
[57] →(~'END'∧,=3+EC)+L5
[58] P=□READ 1,X=(□SIZE 1)[2]-1
[59] TEMP=□X+1
[60] P[TEMP]=''1
[61] P □REPLACE 1,X
[62] →Q
[63] L5:□TEXT R
[64] L5:=("END"∧,=3+EC+GETL ' ')+L11
[65] →("CANCEL"∧,=6+EC)+L8
[66] →(L5,L5)[1+0*R+ENTER EC]
[67] □ERROR IN SETTING UP OF CONDITION.
[68] L7:"DOES NOT TALLY WITH PAST EXAMPLES.'
[69] □DISPLAY MTH EXAMPLE.
[70] DUMP M
[71] □DELETE MTH EXAMPLE.
[72] L8:DELETE M
[73] →0
[74] L11:=("1,(□SIZE 2)[2]-1) □REPLACE 1,(□SIZE 1)[2]-1
[75] →0
[76] L14:=(~1=1+J)+L4
\( \text{7POINTER[]} \)
\( \text{R+POINTER I; J; X; X1} \)

[1] \text{RETURN} \text{S A VECTOR OF LENGTH TWO.}

[2] \text{X[1] POINTS TO COMPONENT OF FILE 1 CONTAINING THE NEXT OPERAND.}

[3] \text{X[2] POINTS TO COMPONENT OF FILE 2 RELATED TO THE NEXT OPERAND.}

[4] \text{\( +((2=0)^{-1} = (7+1+1, 1, 2+2) \{1\} + 0 \)}

[5] \text{\( +((2=0)^{+1} + 0 \)}

[6] \text{A BRANCH POINT IN THE PROGRAM HAS BEEN REACHED.}

[7] \text{THE CONDITIONAL IS EXECUTED TO DETERMINE WHICH BRANCH TO TAKE.}

[8] \text{RETURN NEXT OPERAND ALONG THE APPROPRIATE BRANCH.}

[9] \text{L1: \( = \{1 = Y + 1 + 1, 0 \} \) READ 1, P+1; L2}

[10] \text{Z + LAST Y}

[11] \text{L3}

[12] \text{L2: \( = \{1 = 15, 0 \} \) END X[7]}

[13] \text{L3: \( X1 + 2 + \), READ 1, P; LAST X[6]}

[14] \text{J = 2 \times X1 X1}

[15] \text{B = P + 1}

[16] \text{R+X[J, J+2]}

[17] \text{TEST FOR ANOTHER BRANCH}

[18] \text{\( = (0=X[2]) + 0 \)}

[19] \text{\( = L1, P + R[1] \)}
VPRIJ'ITPROGC~~V

PRINTPROG;Z;STACK;K;LIM;P:T;Y;P;X;A1;A2;A3;A4

[1] CREATE STACK OF BRANCH POINTS.
[2] STACK=P+0
[3] Z+(-Z'=')/Z+,,'Z';O];I3' □FMT PIVOTS
[4] F+ 2 50 (50+R=PROG;T;S',Z),50+'S+1T+0'
[5] K+1
[6] Z+''
[7] LIM+([SIZE 1][2]
[8] L0:=(LIM<K)>0
[9] □BRANCH POINT OR LEAF?
[10] +('1=(R+,□READ 1,K)[1]+LEAP
[12] →(2=R,P)+L1
[13] □SAVE LEFT BRANCH ON STACK.
[14] L16:STACK-STACK,(T+□READ 1,P+1)[1 3],P
[15] □SET UP CONDITIONAL STATEMENT.
[16] →((Q=□T[3])+L7)
[17] Z+Z',('1=(Q+,□READ K)[1]+L8
[18] I3' □FMT T[1]
[20] R2:=(T[1]="1")+L14
[21] □DELETE BLANKS.
[22] R+P,[1] 50+(-Z'=')/Z
[23] Z+''
[24] □TAKE RIGHT BRANCH.
[25] P+T[2 4],P
[26] L1:=(R[1]="1")+LEAP
[27] →((P[2]=0)+L15
[28] P+P,P
[29] A1+''
[30] A2+□R+''
[31] □PIVOTS)+L17
[32] A1+"O'",,'I3' □FMT R[1]),,'+'
[33] L17:+(-P[1]+1□PIVOTS)+L18
[34] A2+"O'",,'I3' □FMT R[1]+1
[37] L19:+(L9,L10,L11,L12,L13)[++O+11+Y+□READ 1,P[1]]
[38] L9:+L4,0Z+Z,'S+',A3,'S'
[39] L10:Z+Z,'S+1S'
[40] →L4
[41] L11:Z+2,A2,'+0'
[42] →L3
[43] L12:Z+2,'S+1PS'
[44] →L4
[45] #IS OPERAND A CONSTANT, VARIABLE, OP FROM THE STACK?
[46] L13:→(L2,L5,L6)[1'012:\Y[2]]
[47] #VARIABLE ASSIGNMENT STATEMENT
[48] L5:Z+2,A2,'+',A3,Y[1],A1,'1'
[50] #DELETE BLANKS AND PRINT
[51] L4:F+F,[1] 50+(~Z=' ')/Z
[52] Z=' '
[53] →L0,K+R[1]+1
[54] LEAF:A4='T'
[55] →(-P<PIVOTS)+L20
[56] A4='O',',I3' #FMT P
[57] L20:Z+'0',R+1',A4
[58] P+P,[1] 50+(~Z=' ')/Z
[59] #POP STACK.
[60] →(O=STACK)+0
[61] P='3+STACK
[62] STACK='3+STACK
[63] Z='L',(X+,'I3' [FMT R[1]],' ')
[64] →L1
[65] #CONSTANT ASSIGNMENT STATEMENT.
[66] L2:Z+2,A2,'+',A3,Y[1],2+((Y[1]',')-1)+Y
[67] →L3
[68] #STACK ASSIGNMENT STATEMENT
[69] L6:Z+2,A2,'+',A3,Y[1],1'S'
[70] →L3
[71] L14:A4='T'
[72] →(-P<PIVOTS)+L21
[73] A4='O',',I3' #FMT P
[74] L21:Z+(4+Z),'0',R+1',A4
[75] STACK='3+STACK
[76] →L3
[77] L15:P+P[1]
[78] →L16
[79] →L17
\[ \text{\textbackslash{}PROMPT}[\text{\textbackslash{}\textbackslash{}}] \text{\textbackslash{}V} \]
\[ \text{\textbackslash{}PROMPT;X;Y;I} \]
\[ \text{\textbackslash{}R} + \text{\textbackslash{}PROMPT;X;Y;I} \]
\[ (\text{\textbackslash{}1\text{\textbackslash{}=}1\text{\textbackslash{}+J}) + \text{\textbackslash{}L1} \]
\[ \text{\textbackslash{}(CR\text{\textbackslash{}=}1\text{\textbackslash{}+EG}\text{\textbackslash{}+,GETL \text{"END"}} \text{\textbackslash{}+0,\text{\textbackslash{}R}+2} \]
\[ \text{\textbackslash{}L7,\text{\textbackslash{}oEG\text{\textbackslash{}+,\text{"END"}}} \]
\[ \text{\textbackslash{}L1;Y\text{"\textbackslash{}READ} 1,1+J \text{"\textbackslash{}I} \]
\[ \text{\textbackslash{}IS THIS A NULLARY OPERATOR?} \]
\[ (1\text{\textbackslash{}<\text{\textbackslash{}p\text{\textbackslash{}Y}} + \text{\textbackslash{}L2} \]
\[ \text{\textbackslash{}(CR\text{\textbackslash{}=}1\text{\textbackslash{}+EG}\text{\textbackslash{}+,GETL \text{"Y"}, \text{\textbackslash{}+0,\text{\textbackslash{}R}+2} \]
\[ \text{\textbackslash{}(L8,L9,L10,L11)[\text{"+0+" \text{"Y}} \]
\[ \text{\textbackslash{}L8;=L6,\text{\textbackslash{}STACK}+(\text{\textbackslash{}\textbackslash{}Fi ACC}),\text{\textbackslash{}STACK} \]
\[ \text{\textbackslash{}L9;=L6,\text{\textbackslash{}STACK}+1\text{\textbackslash{}STACK} \]
\[ \text{\textbackslash{}L10;=L6,\text{\textbackslash{}oACC\text{\textbackslash{}+,\text{"0"}}} \]
\[ \text{\textbackslash{}L11;=L6,\text{\textbackslash{}STACK}+1\text{\textbackslash{}STACK} \]
\[ \text{\textbackslash{}IS A NULLARY OPERATOR. CHECK OPERAND} \]
\[ \text{\textbackslash{}L2;=(L3,L4,L5)[\text{"012" \text{"Y}[2]} \]
\[ \text{\textbackslash{}OPEPAND IS A CONSTANT} \]
\[ \text{\textbackslash{}L3;=(CP\text{\textbackslash{}=}1\text{\textbackslash{}+EG\text{\textbackslash{}+,GETL(X+LAST 1+J)),Y[1]}, \text{"+0,\text{\textbackslash{}R}+2} \]
\[ \text{\textbackslash{}L41} \]
\[ \text{\textbackslash{}OPEPAND IS FROM STACK} \]
\[ \text{\textbackslash{}L5;=(CP\text{\textbackslash{}=}1\text{\textbackslashes{}+EG\text{\textbackslashes{}+,GETL \text{"Y[1]}}, \text{"+0,\text{\textbackslashes{}R}+2} \]
\[ \text{\textbackslash{}L1;=L5,\text{\textbackslashes{}X}+,\text{"15.10" \text{"\textbackslashes{}Fi 1\text{\textbackslashes{}STACK} \]
\[ \text{\textbackslash{}OPEPAND IS A VARIABLE} \]
\[ \text{\textbackslash{}L6;=L5,\text{\textbackslashes{}GETL \text{"Y"},1+Y} \]
\[ \text{\textbackslash{}OPEPAND IS A VARIABLE} \]
\[ \text{\textbackslash{}L7;=(CP\text{\textbackslashes{}=}1\text{\textbackslashes{}+EG\text{\textbackslashes{}+,GETL \text{"0"},\text{"Y[1]}, \text{"+0,\text{\textbackslashes{}R}+2} \]
\[ \text{\textbackslash{}L8\text{"\textbackslashes{}EQ\text{\textbackslashes{}+,ACC}X,ACC,Y[1]},X \]
\[ \text{\textbackslash{}ACC\text{"\textbackslashes{}+,\text{"15.10" \text{"\textbackslashes{}FiT ACC} \]
\[ \text{\textbackslash{}REPLACE:(Y,X,\text{"ACC,\text{"Y[1]} \text{"\textbackslashes{}REPLACE 1,J[1]} \]
\[ \text{\textbackslash{}L6;=1} \]
\[ \text{\textbackslash{}L7;=(\text{\textbackslashes{}\textbackslashes{}READ 2,J[2]},\text{")),\text{"\textbackslashes{}REPLACE 2,J[2]} \]
\[ \text{\textbackslashes{}} \]
\begin{verbatim}

VSIC[]

\text{\%} R+SIG J;K;Y;X;S;T;P;Q;E;O;E;TEMP;T1;S1
1 K+2P+0
2 TEMP+B
3 J[1] IS COMPONENT NUMBER OF OPERAND AT BRANCH.
4 K[1] WILL HAVE VALUE OF COMPONENT NUMBER OF CONSECUTIVE OPERANDS.
5 L1:=(J[1]-(K+POINTER 1+K)[1])+0
6 SKIP NULLARY OPERATORS.
7 +(1=p?+\text{READ 1,}1+K)+L1
8 DISPLAY OPERAND AND ASK IF IT IS SIGNIFICANT.
9 L5:=('Y'=1+GETL(0+LAST 1+K),' (Y/N)'):L2,T1+ 1 0
10 DISPLAY INTERMEDIATE RESULT AND ASK IF IT IS SIGNIFICANT.
11 +(="'=1+GETL(0+LAST(-1+K)), (Y/N)):L1,T1+ 0 1
12 L2:PIVOTS+PIVOTS,K+T1[1]=0
13 +(=J[1])+L7,B+TEMP
14 +(O=P)+L6
15 aP IS THE POINTER VECTOR WHICH PRECEDES THE OPERAND.
16 P+\text{READ 1,J[1]-1}
17 aPLACE IT BY A BRANCH POINTER.
18 ((\%SIZE 1)[2]) \text{\%READ 1,J[1]-1}
19 aY IS LITERAL STRING REPRESENTATION OF PIVOT VALUE.
20 L3:=GETL 'ENTER PIVOT VALUE.'
21 \text{\%COMPARING TWO OPERANDS?}
22 +(=''=1+Y)+L10
23 -(\text{\%YEU?})+ERR3
24 +(1=L+\text{E+Y})+ERR5
25 T+Y,0
26 aT IS PIVOT VALUE IN NUMERIC FORM.
27 L4:=X+1+GETL 'ENTER RELATION.'
28 +(=E+POE)=ERR6,S+0
29 aZ IS CONDITIONAL EXPRESSION IN LITERAL STRING FORM.
30 \text{\%CHECK THAT CONDITION IS EITHER TRUE FOR LAST EXAMPLE ONLY OR FALSE FOR LAST EXAMPLE ONLY.}
31 z[1]=('C','')/1+p":"+'1+2+0]'+
32 z+(p?+T1)+TFI 0
33 z+('(','E,X,0',',E15,10') ?PIT=(((1+\text{READ 2,K[2])}/0),'),',',',Y,X,0
34 aZ IS USED AS SWITCH TO INDICATE WHICH BRANCH IS TRUE AND WHICH IS FALSE.
35 S+S+2x(X/XEO 2)
36 z+('~'=E,X,0,')',',Y,X,0
37 +(0=S+S+X/XEO 2)+0
38 aSET UP BRANCH INFORMATION.
40 X \text{\%APPEND 1}
\end{verbatim}
((S+JC1Iy(QSIZE 1[2]+1),(S+JC2Iy(QSIZE 2[2]),(1+P),
K[1],T) □ APPEND 1
(0,M) □ APPEND 2
-0,R+1
L6: P+□ READ 1,B
CHECK WHICH BRANCH HAS TAKEN
Y+(P[1,2],J[1])-1
J+P[Y+1,3]
P[Y+1,3]+(QSIZE 1[2],0
P □ REPLACE 1,B
L3
ERR3: □ L3,ρ □ 'INVALID NUMERIC CHARACTER.'
ERR5: □ L3,ρ □ 'ONE VALUE ONLY.'
ERR6: □ L4,ρ □ 'INVALID OPERATOR.'
L10: "+",X[1]
+(T1[1]=1)+L12
L11: □ (J[1]=(N-POINTER 1+N)[1])+0
□ SKIP NULLARY OPERATORS
-(1=N-□ READ 1,N[1])+L11
□ DISPLAY OPERAND AND ASK IF IT IS SIGNIFICANT.
L14: -(Y=1+GETL(Y+LAST M[1]),' (Y/N)?')+L13,S1+ 1 0
□ DISPLAY INTERMEDIATE RESULT AND ASK IF IT IS SIGNIFICANT.
L12: -(N=1+GETL(Y+LAST(-M[1]),' (Y/N)?')+L11,S1+ 0 1
□ MAKE T A TWO ELEMENT VECTOR POINTING TO SECOND SIGNIFICANT OPERAND.
L13: □ Y
PIVOTS+PIVOTS,M[1]+S1[1]=0
L7: P+□ READ 1,B
L8: □ (QSIZE 1[2]) □ REPLACE 1,B
L3
APPENDIX II

PROGRAM FLOWCHARTS
ENM ABSENCE N
ENTER

INITIALIZE

READ NEXT SEGMENT OF TREE

IS THIS A LEAF?

Y

EXIT

N

IS THIS A NODE?

Y

DETERMINE WHICH BRANCH WAS TAKEN

N

EXAMPLE # IN INDEX COMPONENT?

Y

DELETE EXAMPLE NUMBER FROM INDEX COMPONENT

N

DELETE CIFRANO AND OPERATE FROM PROGRAM COMPONENT

DELETE I
DISPLAY

ENTER

IS STACK EMPTY?

DISPLAY TOP ELEMENT OF STACK

DISPLAY 'X'

EXIT

EXIT

DISPLAY
INITIALIZE FILES, STACK AND ACCUMULATORS

ACCEPT INPUT STRING FROM USER

'END'? Y

CANCEL'?

Y

ZS INPUT STRING VALID?

Y

APPEND STRING TO PROGRAM TREE

N

APPEND STRING TO PROGRAM TREE

N

EXIT

EXIT

PRINT MESSAGE
```
EVENT

PRINT LITERAL STRING 'X'

ACCEPT LITERAL STRING 'X' FROM USER

EXIT
```

GETL X
READ COMPONENT Y1 FROM PROGRAM TREE

IS Y NEGATIVE?

PICK UP LAST RESULT

EXIT

PICK UP LAST OPERAND

LAST Y
ENTRY

DISPLAY TOP ELEMENT OF STACK

ENTER

OPERATOR

STACK OPERAND?

EXECUTE USER'S INSTRUCTIONS AND DISPLAY RESULT

SET UP NEW POINTERS

EXIT

FIND TOP POSITION OF STACK

PUSH ACCUMULATOR ONTO STACK

SET ACCUMULATOR TO ZERO

ADD TOP OPERAND, OPERATOR, AND RESULT TO THE PROGRAM TREE

ROTATE STACK ONE POSITION

NEXT
null
DRAW A PROGRAM STATEMENT TO
APPEND "TRUE" BRANCH TO LEAF
Y

REWRITE GATE LABEL PART OF
STATEMENT

APPEND RESULT ASSIGNMENT
AFTER LABEL

DROP "FALSE" BRANCH = CON
SPACE

PINTIKOLL (CONT'D)
A

OPERAND?

DISPLAY CONSTANT OPERAND AND OPERATOR

DISPLAY 'x'

USER RESPONSE OR 'y'?

EXECUTE CALCULATION AND DISPLAY RESULT

ATTEND OPERAND AND RESULT TO PROGRAM FILE

EXIT

EXIT

VALID NUMERIC STRING?

EXIT

EXIT

EXIT (CONT'D)
INITIALIZE VARIABLES

PICK UP NEXT SEGMENT OF THE PROGRAM TREE

IS THE NODE ACTIVE INVESTIGATED?

IS THE NULLARY OPERATOR?

DISPLAY OPERAND

X

SO IT THE SIGNIFICANT VARIABLE?

DISPLAY INTERMEDIATE RESULT

X

IS IT THE SIGNIFICANT VARIABLE?
NOTE: POSITION OF SIGNIFICANT VARIABLE

YES
RESTART PROGRAM TREE TO CREATE A NODE AT THIS LEAF

NO
IS THE PRESENT NODE PREVIOUSLY A LEAF?

YES
Determine on which branch the new node belongs

NO
RESTART INCOMPLETE TREE TO CREATE A NODE ON THIS BRANCH

ACCEPT PIVOT VALUE FROM USER

YES

NO

IS IT A CONSTANT?

YES

NO

IS IT A VALID NUMBER?

YES
PRINT ERROR MESSAGE

NO

NOT SIG J (CONT'D)

C

N

WAS SIGNIFICANT VARIABLE AN OPERAND?

YES

NO

PRINT ERRORS MESSAGE

HOLE THAN ONE NUMBER?
LAYOUT OF PROGRAM AND INDEX FILES AND THEIR RELATIONSHIP

COMPONENT NUMBER

PROGRAM FILE (TIED TO 1)

INDEX FILE (TIED TO 2)

1 (NUMERIC)

| 1 |

2 (ALPHA)

3 (NUMERIC)

4 (ALPHA)

5 (NUMERIC)

6 (ALPHA)

INDEX FILE

| 1  | 2  | N |

| 1  | 2  | N |

| 1  | 2  | N |

| 1  | 2  | N |

ON INDEX CODE:

0 - CONSTANT
1 - VARIABLE
2 - STACK
APPENDIX IV

FILE DESCRIPTIONS
The file PROG is tied to 1 and is composed of four types of components:

I Pointers
II Operation Traces
III Relations
IV Branch Pointers

**TYPE I: Pointers (numeric)**
The first element of the component is a pointer whose value is the number of the component in file PROG which contains the next segment of the trace. It is generally the next component in the file. A value of minus one for this pointer indicates that the end of the trace has been reached.
The second element of the component, if it exists, is a pointer whose value is the number of the component in file INDEX which contains significant information relating to the next segment of the trace (see the description of the INDEX file). If the second element is missing, the first element indicates a branch point at the next point in the trace.

**TYPE II: Operation Traces (alphabetical)**
The first element is an operator. If the operator is null-ary, there are no more elements in this component.
The second element, if it exists, indicates the type of operand used with the operator. A value of zero indicates a
constant operand; a value of one indicates a variable operand; a value of two indicates a stack operand. The remainder of the component is composed of pairs of strings of indefinite length separated by commas. The first string of each pair is the literal representation of an operand used with this operator; the second string is the literal representation of the result obtained from the operator and the aforementioned operand. There are as many operand-result pairs as there were examples which passed through this trace.

**TYPE III: Relations (alphabetic)**
The only element of this component is the relational operator involved at a branch point.

**TYPE IV: Branch Pointers (numeric)**
The first four elements of this component correspond to type I components discussed earlier. The first pair of elements are the pointers of the true and false paths of a trace. That is, if the value of the conditional statement is true, the trace continues at the component number whose value is contained in the first element of the pair; if the value of the conditional statement is false, the trace continues at the component number whose value is contained in the second element of the pair. A value of minus one
indicates that the end of a trace has been reached. The fifth element of this component points to the closest preceding nonbranch point in the trace. The next two elements of this component correspond to the second element of type I data when true and false values are respectively obtained for the conditional statement. A value of zero is equivalent to the non-existence of this element.

The sixth element of the component points to the first operand which participates in the conditional. If the value of the pointer is positive, the operand is indicated; if it is negative, the result is indicated.

The seventh and eighth elements of the component point to the second operand participating in the conditional. If the eighth element is a zero, then the value of the second operand is given in the seventh element; if the eighth element is non-zero, it points to an operand within PROG. As in element six, if it is positive, the pointer indicates an operand; if it is negative, it indicates a result.

The file INDEX is tied to 2 and is composed of only one type of component.

Each component consists of a list of numbers. These numbers correspond to the example numbers represented in the operation trace components of PROG.
APPENDIX V

INPUT TO THE MODEL
<table>
<thead>
<tr>
<th>OPERANDS</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
<td><strong>Description</strong></td>
</tr>
<tr>
<td>constant</td>
<td>any type acceptable to APL</td>
</tr>
<tr>
<td>ω</td>
<td>indicates that the top element of the stack is to be used</td>
</tr>
<tr>
<td>α</td>
<td>indicates that another program variable is to be used</td>
</tr>
<tr>
<td>END</td>
<td>indicates the example is complete</td>
</tr>
<tr>
<td>CANCEL</td>
<td>indicates the user wishes to erase the current example</td>
</tr>
</tbody>
</table>
OPERATORS

Nullary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td>Push the accumulator onto the stack</td>
</tr>
<tr>
<td>↓</td>
<td>Pop the stack--i.e., allow the top element of the stack to disappear</td>
</tr>
<tr>
<td>→</td>
<td>Rotate the stack by one position; the top element goes to the bottom</td>
</tr>
<tr>
<td>c</td>
<td>Clear the accumulator</td>
</tr>
<tr>
<td>□</td>
<td>Display the top element of the stack</td>
</tr>
</tbody>
</table>

Unary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>Add the operand into the accumulator and store the result in the accumulator</td>
</tr>
<tr>
<td>-</td>
<td>Subtract the operand from the accumulator and store the result in the accumulator</td>
</tr>
<tr>
<td>x</td>
<td>Multiply the operand by the accumulator and store the result in the accumulator</td>
</tr>
<tr>
<td>÷</td>
<td>Divide the accumulator by the operand and store the result in the accumulator</td>
</tr>
<tr>
<td>*</td>
<td>Raise the accumulator to the power of the operand and store the result in the accumulator</td>
</tr>
</tbody>
</table>
Binary:

<  Test whether the significant variable is less than the pivot value

≤  Test whether the significant variable is less than or equal to the pivot value

=  Test whether the significant variable is equal to the pivot value

>  Test whether the significant variable is greater than the pivot value

≥  Test whether the significant variable is greater than or equal to the pivot value

≠  Test whether the significant variable is unequal to the pivot value

**NOTE:** If the model prompts the user by printing a nullary operator or an operand and unary operator, the user may respond with a carriage return which is equivalent to accepting the prompt as the next step in the example, or may override the prompt by entering some other valid computational step.
APPENDIX VI

PROGRAM MESSAGES AND THEIR MEANINGS
BEGIN ENTERING FIRST EXAMPLE
Initialization of the files is complete. The user may begin entering his example.

TOO MANY OPERANDS. RE-ENTER
A computational step has too many operands in it. This may result from a blank or comma embedded in the operand.

WRONG NUMBER OF OPERATORS. RE-ENTER
There must be exactly one operator.

NO OPERAND. RE-ENTER
A unary operator has been entered without an operand.

INVALID NEXERIC CHARACTER. RE-ENTER
The operand is an invalid APL constant.

OPERATOR NOT LAST SYMBOL. RE-ENTER
The computational step is syntactically incorrect.

ANY MORE EXAMPLES(Y/N)?
Asks if the user wishes to enter any more examples. Entering the letter N indicates no; any other response will cause the model to expect another example.
ENTERING EXAMPLE xx
Indicates that the model is ready to accept the next example and tells the user which example is about to be accepted.

?  
Indicates the operator which the model expects next takes a variable operand. Waits for an operand or an override.

CONSTANT OR VARIABLE (C/V)?  
Asks whether the operand which was just entered is a constant or a variable. Entering the letter C indicates a response of constant. Any other response will be interpreted as a variable.

CONTINUING  
The user may continue entering his example from the point at which he left off.

DOES NOT TALLY WITH PAST EXAMPLES  
Information supplied by the user is not consistent with past examples. The current example will be displayed then deleted.

(Y/N)?  
The model is displaying the value of a variable operand used
earlier in the example and is asking whether it is the significant variable. The user may respond with the letter Y for yes or N for no.

ENTER PIVOT VALUE
The user may enter either a valid APL constant or the letter alpha to indicate that the pivot value is another variable operand.

ENTER RELATION
The user is requested to enter the suitable binary operator.

INVALID NUMERIC CHARACTER
Re-enter the pivot value.

ONE VALUE ONLY
Re-enter the pivot value.

INVALID OPERATOR
Re-enter the relational operator.

DO YOU WANT TO TRY IT OUT?(Y/N)
The program has been created. The user may respond with a Y for yes or an N for no.
EXAMPLE xx

Precedes the example being displayed.

INVALID EXAMPLE NUMBER

A request was made to display an example which doesn't exist.
APPENDIX VII

SAMPLE RUNS
Hero's Algorithm for the Area of a Triangle

\[ A = \sqrt{s(s-a)(s-b)(s-c)} \]

where \( s = \frac{1}{2}(a+b+c) \)

\( a, b, c \) are the lengths of the sides of the triangle.

```
BEGIN ENTERING FIRST EXAMPLE.

3
+ 3  
+ 3

5
+ 0
+ 5

5
+ 0
+ 5

7
+ 7
+ 7

12
+ 12
+ 12

3
+ 3
+ 3

15
+ 15
+ 15

5
+ 5
+ 5

12
+ 12
+ 12

3
+ 3
+ 3

5
+ 5
+ 5

7
+ 7
+ 7

5
+ 5
+ 5

7
```

**Note:**

User's input is indented or underlined. Members in left margin show value of accumulator after a unary operation has been performed, or show value of an element of stack after a question.
0.5

4.5

84.37

42.1875

6.49533329

END INDICATES END OF EXAMPLE

ANY MORE EXAMPLES (Y/N)? Y

ENTERING EXAMPLE 2

3+

5+

CONSTANT OR VARIABLE (C/V)?

5+

12+

CONSTANT OR VARIABLE (C/V)?

12+

0

7+

13+

CONSTANT OR VARIABLE (C/V)?
ANY MORE EXAMPLES (Y/N)?
DO YOU WANT TO TRY IT OUT? (Y/N)?
R+PROG;T;S
S+1;?0
T+T+?0
S+T;S
T+0
T+T+?0
S+T;S
T+0
T+T+?0
S+T;S
T+1;0S
T+T+1+S
S+1;0S
T+T+1+S
S+T;S
T+T;2
S+1;5S
T+T-1+S
S+1;5S
S+T;S
S+1;5S
S+1;5S
T+0
T+T+1;S
S+1;5S
S+1;5S
T+T-1;S
S+1;5S
S+T;S
S+1;5S
S+1;5S
T+0
T+T-1+S
S+1;5S
S+1;5S
S+1;5S
T+T-1+S
S+1;5S
T+T-1+S
S+1;5S
S+1;5S
T+T-1+S
T+1;5S
T+T-1+S
S+1;5S
S+T;S
T+T-1+S
S+1;5S
T+T-1+S
S+1;5S
T+T-1+S
T+T*;S
T+T*;S
+0,R+T
□: 3
REQUESTS INPUT
&
Do you want to try it out? (Y/N)
Any more examples (Y/N)?

More examples may be entered after testing. If additions are made, the simulation recycles for more testing.
Calculation of the Final Temperature of a Mixture of Two Known Quantities of Ice and Water. Temperature and Mass of Container are Ignored.

**MAIN**
BEGIN ENTERING FIRST EXAMPLE.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 ozs. of ice at 100°F</td>
<td>15 ozs. of ice at 200°F</td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>CALCIULATE AND SAVE TOTAL NUMBER OF CALORIES PRESENT</td>
</tr>
<tr>
<td>2000</td>
<td>C = (10 x 100) + (15 x 200)</td>
</tr>
<tr>
<td>4000</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>CALCIULATE TOTAL MASS OF ICE</td>
</tr>
<tr>
<td>15</td>
<td>M = 10 + 15</td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>T = 4000 / 25</td>
</tr>
<tr>
<td>150</td>
<td>END</td>
</tr>
</tbody>
</table>

ANY MORE EXAMPLES (Y/N)? Y
ENTERING EXAMPLE 2

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>10+</td>
<td>15+</td>
</tr>
<tr>
<td>CONSTANT OR VARIABLE (C/V)? V</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>100x</td>
<td>75x</td>
</tr>
</tbody>
</table>
CONSTANT OR VARIABLE (C/V)? Y
1125
+
0
15 + 10 +

CONSTANT OR VARIABLE (C/V)? Y
10
+
200x 350x

CONSTANT OR VARIABLE (C/V)? Y
3500
+

ω+
4625
+
+
+
0
15 (Y/N)?
1.500000000000 (Y/N)?
75 (Y/N)?

1.120000000000 (Y/N)?
10 (Y/N)?

1.000000000000 (Y/N)?
350 (Y/N)?

ENTER PROXIMATE VALUE. 273
ENTER RELATION. >

CONTINUE.
+
+
+
10
30x

5425
+
+
+
+
+
+ 0

15 +
ω+
25 +
+
+
0

15 CMS. OF ICE AT 75° K.
10 CMS. OF WATER AT 350° K.

THE USER IS PRESENTED WITH ALL OPERANDS AND INTERMEDIATE RESULTS USED IN THE EXAMPLE TO THIS POINT. THE USER MUST INDICATE WHICH OF THE VALUES HAS CAUSED HIM TO CHANGE PROCEDURE. PROGROUTIATION ENDS WHEN THE USER INDICATES A PARTICULAR VALUE.

SINCE THE TEMPERATURE OF THE SECOND QUANTITY IS GREATER THAN 273° K, WE MUST INCLUDE THE NUMBER OF CALORIES USED FOR THE CONVERSION OF ICE TO WATER.

C = (15 x 75) + (10 x 350) + (10 x 85)
54 25
→
ω+ 217

END
ANY MORE EXAMPLES (Y/N)? Y
ENTERING EXAMPLE 3

? +10
10 gms. of Ice at 75°K
+ 10
15
?×75
750 + 0
? - 15
15 + 0
?×250
5250 +

ω+ 6000 + + 0 + + + +
ω+ 15 80× 1200 +
ω+ 7200 + + + + + 0 0 + 10 ω+ 25 + + + + + + ω+
7200
+
$\omega$
288
END 0
10 (Y/N)? N
1.00000000E0
75 (Y/N)? N
7.50000000E2
15 (Y/N)? N
1.50000000E1
350 (Y/N)? N
5.25000000E3
7.50000000E2
6.00000000E3
1.50000000E1
1.50000000E1
80 (Y/N)? N
1.25000000E3
6.00000000E3
7.20000000E3
1.00000000E1
1.00000000E1
1.50000000E1
2.50000000E1
7.20000000E3
7.20000000E3
2.50000000E3
2.88000000E2
ENTER SECOND VALUE 273
ENTER RELATION >
CONTINUING

273
END

ANY 00 SAMPLE EXAMPLES (Y/N)? Y
ENTERING EXAMPLE 4
$\omega$
10
?x75
750
+$\omega$
30
?x400
12000
+

SINCE THE FINAL TEMPERATURE IS GREATER THAN 273°K AND LESS THAN 353°K, WE MUST SET IT TO 273°K.
(SLUSH)
$$\omega^+$$

12750

10

0

$$\omega^+$$

30

80x

2400

$$\omega^+$$

15150

+ 

+ 

+ 

0

$$\omega^+$$

10

+ 

$$\omega^+$$

40

+ 

+ 

0

$$\omega^+$$

15150

+ 

$$\omega^+$$

378.75

0

20

10 (Y/N)? Y

1.00000000E1 (Y/N)? Y

75 (Y/N)? N

7.50000000E2 (Y/N)? N

30 (Y/N)? Y

3.00000000E1 (Y/N)? Y

400 (Y/N)? Y

1.20000000E4 (Y/N)? Y

7.50000000E2 (Y/N)? Y

1.27500000E4 (Y/N)? Y

3.00000000E1 (Y/N)? Y

3.00000000E1 (Y/N)? Y

80 (Y/N)? N

2.40000000E3 (Y/N)? Y

1.27500000E4 (Y/N)? Y

Since the final temperature is greater than 300°K, we assume it as 300° to account for the conversion of ice to water.
1.515000000E4  (Y/N)?N
1.000000000E1  (Y/N)?N
1.000000000E1  (Y/N)?N
3.000000000E1  (Y/N)?N
4.000000000E1  (Y/N)?N
1.515000000E4  (Y/N)?N
1.515000000E4  (Y/N)?N
4.000000000E1  (Y/N)?N
3.787500000E2  (Y/N)?Y
ENTER PIVOT VALUE. 353
ENTER RELATION. >
CONTINUING
298.75
END
ANY MORE EXAMPLES (Y/N)?Y
ENTERING EXAMPLE 5
?+15
15 6MS. OF WATER AT 350° K
?x35
9250
+ 0
?+10
10
+ ?x75
750
+ w
6000
+
15 (Y/N)?N
1.500000000E1  (Y/N)?N
350 (Y/N)?N
ENTER PIVOT VALUE. 273
ENTER RELATION. >
CONTINUING
+ w
15
80x
1200
+
7200

THIS TIME THE FIRST
TEMPERATURE IS GREATER
THAN 273° K
ANY MORE EXAMPLES (Y/N)?Y
ENTERING EXAMPLE 6
?+30
30
?x400
12000
?+10
10
?x75
750
?+12750
12750
?+1

30 ENTS. OF WATER AT 4000 K.
10 ENTS. OF ICE AT 750 K.
Example 4

SAME AS EXAMPLE 4

\[
\begin{array}{cccc}
\omega + \\
15150 \\
+ \\
+ \\
+ \\
\omega + \\
30 \\
+ \\
\omega + \\
40 \\
+ \\
+ \\
\omega + \\
15150 \\
+ \\
\omega + \\
378.73 \\
\end{array}
\]

\[
\begin{array}{cccc}
\frac{30-}{30} \\
\frac{(Y/N)?N}{(Y/N)?N} \\
\frac{3.0000000081}{(Y/N)?N} \\
\frac{400}{(Y/N)?N} \\
\frac{1.2000000064}{(Y/N)?N} \\
\frac{10}{(Y/N)?N} \\
\frac{1.0000000061}{(Y/N)?N} \\
\frac{75}{(Y/N)?N} \\
\frac{7.5000000022}{(Y/N)?N} \\
\frac{1.2000000054}{(Y/N)?N} \\
\frac{1.2750000014}{(Y/N)?N} \\
\frac{3.0000000081}{(Y/N)?N} \\
\frac{3.0000000061}{(Y/N)?N} \\
\frac{80}{(Y/N)?N} \\
\frac{2.4000000083}{(Y/N)?N} \\
\frac{1.2750000014}{(Y/N)?N} \\
\frac{1.5150000004}{(Y/N)?N} \\
\frac{3.0000000081}{(Y/N)?N} \\
\frac{3.0000000061}{(Y/N)?N} \\
\frac{1.0000000061}{(Y/N)?N} \\
\frac{4.0000000001}{(Y/N)?N} \\
\frac{1.5150000004}{(Y/N)?N} \\
\frac{1.5150000004}{(Y/N)?N} \\
\frac{4.0000000001}{(Y/N)?N} \\
\frac{3.7875000002}{(Y/N)?N} \\
\end{array}
\]

ENTER PIVOT VALUE 353
ENTER RELATION >
CONTINUING
298.75

END
ANY MORE EXAMPLES (Y/N)? Y
ENTERING EXAMPLE 7

10

10 6 MS. OF WATER AT 2,900 K.

?x290

2900

10 6 MS. OF WATER AT 300 K.

+ 0

?+10

10

?x300

3000

+ w+

5900

+ o

+ w+

10

80x

800

+ w+

6700

+ w+

10

33x

10 (Y/N)? Y

1.0000000031

290 (Y/N)? Y

ENTER PIVOT VALUE .273

ENTER RELATION .2
CONTINUING

500

+ w+

7500

+ w+

+ w+
ANY MORE EXAMPLES (Y/N)? Y
ENTERING EXAMPLE 8

10 +
0 +
20 +
0 +
7500 +
0 +
375 +
295 +
END

10 CMS. OF WATER AT 350° K.

15 CMS. OF ICE AT 75° K.

1

10 +
0 +
3500 +
15 +
75 +
1125 +
0 +
4625 +
0 +
0 +
5425 +
THE FINAL TEMPERATURE IS LESS THAN 273 K.

END

0
10 (Y/N)?N
1.00000000E1 (Y/N)?N
350 (Y/N)?N
3.50000000E3 (Y/N)?N
15 (Y/N)?N
1.50000000E1 (Y/N)?N
75 (Y/N)?N
1.12500000E3 (Y/N)?N
3.50000000E3 (Y/N)?N
4.62500000E3 (Y/N)?N
1.00000000E1 (Y/N)?N
1.00000000E1 (Y/N)?N
80 (Y/N)?N
8.00000000E2 (Y/N)?N
4.62500000E3 (Y/N)?N
5.42500000E3 (Y/N)?N
1.00000000E1 (Y/N)?N
1.00000000E1 (Y/N)?N
1.50000000E1 (Y/N)?N
2.50000000E1 (Y/N)?N
5.42500000E3 (Y/N)?N
5.42500000E3 (Y/N)?N
2.50000000E1 (Y/N)?N
2.17000000E2 (Y/N)?N
ENTER PILOT VALUE: 273
ENTER RELATION: >
CONTINUING
ANY MORE EXAMPLES (Y/N)?N
DO YOU WANT TO TRY IT OUT? (Y/N)?Y
R+PROC; T; S; 016; 08; 093; 047; 093; 047; 06; 03; 0147; 074; 0
S+1; T+0
O3+T+[]
S+O3; S
T+O3×06+[]
S+T; S
T+0
T+T+[]
S+T; S
T+T×016+[]
S+1φS
T+T+1+S
S+1+S
S+T; S
+(2.73000000E2×016)+L104
T+0
S+10S
S+15S
T+T+1+S
T+T×30
S+16S
T+T+1-S
S+14S
S+T; S
S+1φS
T+0
T+T+1+S
+(2.73000000E2×06)+L76
T+T×80
S+1φS
S+1φS
T+T+1+S
S+1+S
S+T; S
S+1φS
T+0
T+T+1+S
S+1+S
T+T+1+S
S+1+S
S+T; S
S+10S
T+0
T+T+1+S
S+1φS
T+T+1+S
T+T-80
→0, R+T
L76; S+1+S
$S+1+S$
$S+T+S$
$S+1\phi S$
$T+0$
$T+T+1+S$
$S+1\phi S$
$T+T+1+S$
$\rightarrow 0, T+T$

\[\begin{array}{c}
\text{\(\square\):} \\
\text{\(\square\):} \\
\text{\(\square\):} \\
\text{\(\square\):} \\
\end{array}\]

\[\begin{array}{c}
20 \\
194 \\
10 \\
210 \\
193.3333333 \\
\end{array}\]

DO YOU WANT TO TRY IT OUT? (Y/N)?
ANY MORE EXAMPLES? (Y/N)?
APPENDIX VIII

OTHER APPROACHES TO THE PROBLEM
Many disciplines other than computer and information science are attempting to grapple with the process of communication and with its underlying structure. The various approaches used in these disciplines may help us come to a better understanding of the problem.

The subject of this thesis was suggested to me by Dr. John W. Carr III. He described the problem as the induction of an incompletely specified finite state machine from traces of its output function (data or test operations) and inputs \( T, P \). The finite state machine when induced was to be able to accept (generate) the language of all traces. In this thesis the program output by the simulation is the finite state machine which has been inferred from the traces left by the user on the programmable device.

Although it may not be immediately apparent, modern mathematical linguistics is deeply engaged in investigation of the same problem. The notion of an acceptor or generator of valid strings of a language has been used by Noam Chomsky in his theory of transformational grammars for natural language. Each native speaker of a language constructs a "grammar" which allows him to recognize and produce valid sentences of that language. Chomsky presented three possible models of a grammar to account for human language capabili-
ties: a finite state grammar, a phrase structure grammar, and a transformational grammar, each one more powerful than its predecessor. The finite state grammar is directly analogous to the finite state machine above. However, it is an inadequate grammar for the systematic analysis of natural language since it cannot deal with dependencies holding between non-adjacent words (e.g., He who hesitates is lost). A phrase structure grammar is context free. It is generally represented in linguistic literature by tree structures. Chomsky was dissatisfied with the power of phrase structure grammars in dealing with certain types of structural ambiguity in natural language. He claims that in a certain sense transformational grammars which are context sensitive, are not only more powerful but simpler. In *Mathematical Models in Linguistics* Maurice Gross outlines the basic mathematical concepts underlying Chomsky's theories. He discusses the relationship between various types of grammar and their counterparts in automata theory.

Another approach to this problem is through algebra and its relationship to automata theory. Any machine has
an associated semigroup (Theorem in *Binary Systems*, McNaughton Bruck). Carr has discussed in his *Notes on Languages and Semigroups* a variety of reasons for approaching the problem of grammatical inference from this angle. A detailed discussion of semigroups can be found in *Introduction to Discrete Structures* by Preparata and Yeh.

One of the earliest attempts at solving this learning problem was made by E. B. Hunt. It is his terminology which I have used extensively in this thesis. He analyzes concept learning or inductive reasoning as the discovery and utilization of a classification rule. (recognition and generation by a grammar) He points out that concepts can be represented as sequential decision rules (trees) and investigates different strategies for developing such trees under various constraints such as limited memory and under various methods of presentation of sample strings from the concept.

This discussion barely scratches the surface of the variety of approaches taken to solve essentially the same problem--induction, inference, learning. There is a short bibliography included in this thesis which can be used as an entry point into the extensive literature available.
APPENDIX IX

BIBLIOGRAPHY
Carr, John W. III

Inference of Computer Programs from their Traces
internal publication, U. of Pennsylvania, 1974

Carr, John W. III

The Moore Machine Model of a Program Machine
internal publication, U. of Pennsylvania, 1974

Carr, John W. III

Notes on Languages and Semigroups
internal publication, U. of Pennsylvania, 1973

Carr, John W. III

Notes on Machine Decomposition
internal publication, U. of Pennsylvania, 1973

Carr, John W. III

Proof that a Set of Productions Generates a Corresponding Semigroup of its Derivation Classes
internal publication, U. of Pennsylvania, 1973

Carr, John W. III

A Tentative Protocol for the Construction of a Substitutional Grammar
internal publication, U. of Pennsylvania, 1973

Epstein, R.

Notes for CIS 680
internal publication, U. of Pennsylvania, 1974

Fu, K. S.

A Survey of Grammatical Inference
-102-

Gross, Maurice and A. Lentin

Introduction to Formal Grammars
Gauthier-Villars, Paris, 1970

Gross, Maurice

Mathematical Models in Linguistics

Hockett, Ch. F.

Language, Mathematics, and Linguistics
Mouton & Co., The Hague, 1967

Hunt, Earl B.

Concept Learning an Information Processing Problem

Hunt, Earl B. and J. Marins and Philip J. Stone

Experiments in Induction

Lyons, John

Chomsky

Miller, George Arthur

Finite State Machine Induction
Master's Thesis, Moore School of Electrical Engineering
U. of Pennsylvania, 1969

Miller, G. A., E. Galanter and K. H. Pribham

Plans and the Structure of Behaviour
Holt, N.Y., 1960
Pao, Tsyh-Wen Lee

A Solution of the Syntactical Induction Inference Problem for a Non-Trivial Subset of Context-Free Language


Preparata, F. P. and R. T. Yeh

Introduction to Discrete Structures
Addison-Wesley Publishing Co., 1973

Simon, H. A. and L. Siklossy (eds.)

Representation and Meaning: Experiments with Information Processing Systems