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Multivariate Data Fusion Based on Fixed-Geometry Confidence Sets

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Abstract

The successful design and operation of autonomous or partially autonomous vehicles which are capable of traversing uncertain terrains requires the application of multiple sensors for tasks such as: local navigation, terrain evaluation, and feature recognition. In applications which include a teleoperation mode, there remains a serious need for local data reduction and decision-making to avoid the costly or impractical transmission of vast quantities of sensory data to a remote operator. There are several reasons to include multi-sensor fusion in a system design: (i) it allows the designer to combine intrinsically dissimilar data from several sensors to infer some property or properties of the environment, which no single sensor could otherwise obtain; and (ii) it allows the system designer to build a robust system by using partially redundant sources of noisy or otherwise uncertain information.

At present, the epistemology of multi-sensor fusion is incomplete. Basic research topics include the following task-related issues: (i) the value of a sensor suite; (ii) the layout, positioning, and control of sensors (as agents); (iii) the marginal value of sensor information; the value of sensing-time versus some measure of error reduction, e.g., statistical efficiency; (iv) the role of sensor models, as well as *a priori* models of the environment; and (v) the calculus or calculi by which consistent sensor data are determined and combined.

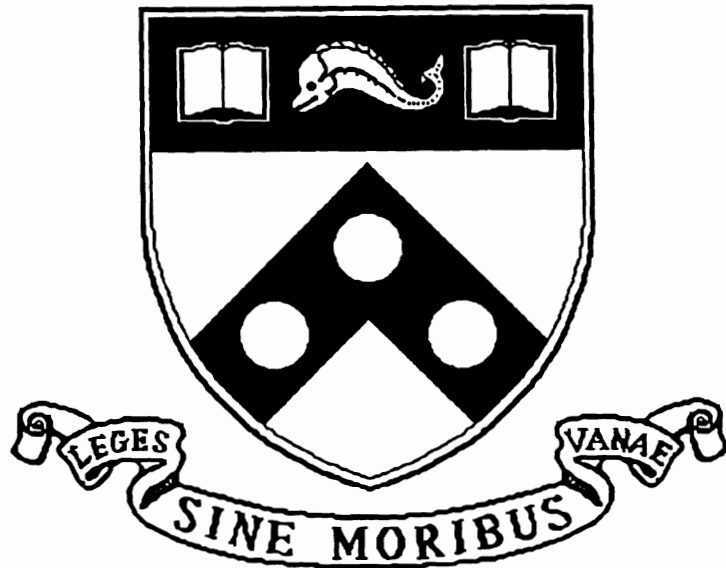
Comments

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**Multivariate Data Fusion
Based On Fixed-Geometry Confidence Sets**

**MS-CIS-92-31
GRASP LAB 312**

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Multivariate Data Fusion

Based on Fixed-Geometry Confidence Sets

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1 Introduction and Summary

The successful design and operation of autonomous or partially autonomous vehicles which are capable of traversing uncertain terrains requires the application of multiple sensors for tasks such as: local navigation, terrain evaluation, and feature recognition. In applications which include a teleoperation mode, there remains a serious need for local data reduction and decision-making to avoid the costly or impractical transmission of vast quantities of sensory data to a remote operator. There are several reasons to include multi-sensor fusion in a system design: (i) it allows the designer to combine intrinsically dissimilar data from several sensors to infer some property or properties of the environment, which no single sensor could otherwise obtain; and (ii) it allows the system designer to build a robust system by using partially redundant sources of noisy or otherwise uncertain information.

At present, the epistemology of multi-sensor fusion is incomplete. Basic research topics include the following task-related issues: (i) the value of a sensor suite; (ii) the layout, positioning, and control of sensors (as agents); (iii) the marginal value of sensor information; the value of sensing-time versus some measure of error reduction, e.g., statistical efficiency; (iv) the role of sensor models, as well as a priori models of the environment; and (v) the calculus or calculi by which consistent sensor data are determined and combined.

In our research on multi-sensor fusion, we have focused our attention on several of these issues. Specifically, we have studied the theory and application of robust fixed-size confidence intervals as a methodology for robust multi-sensor fusion. This work has been delineated and summarized in Kamberova and Mintz (1990) and McKendall and Mintz (1990a, 1990b). As we noted, this previous research focused on confidence intervals as opposed to the more general paradigm of confidence sets. The basic distinction here is between fusing data characterized by an uncertain scalar parameter versus fusing data characterized by an uncertain vector parameter, of known dimension. While the confidence set paradigm is more widely applicable, we initially chose to address the confidence interval paradigm, since we were simultaneously interested in addressing the issues of: (i) robustness to nonparametric uncertainty in the sampling distribution; and (ii) decision procedures for small sample sizes.

Recently, we have begun to investigate the multivariate (confidence set) paradigm. The delineation of optimal confidence sets with fixed geometry is a very challenging problem when: (i) the a priori knowledge of the uncertain parameter vector is *not* modeled by a Cartesian product of intervals (a hyper-rectangle); and/or (ii) the noise components in the multivariate observations are *not* statistically independent. Although it may be difficult to obtain optimal fixed-geometry confidence sets, we have obtained some very promising approximation techniques. These approximation techniques provide: (i) statistically efficient fixed-size hyper-rectangular confidence sets for decision models with hyper-ellipsoidal parameter sets; and (ii) tight upper and lower bounds to the optimal confidence coefficients in the presence of both Gaussian and non-Gaussian sampling distributions.

In both the univariate and multivariate paradigms, it is assumed that the a priori uncertainty in the parameter value can be delineated by a fixed set in an n -dimensional Euclidean space. It is further assumed, that while the sampling distribution is uncertain, the uncertainty class description for this distribution can be delineated by a given class of neighborhoods in the space of all n -dimensional probability distributions.

The following sections of this paper: (i) present a paradigm for multi-sensor fusion based on position data; (ii) introduce statistical and set-valued models for sensor errors and a priori environmental uncer-

tainty; (iii) explain the role of confidence sets in statistical decision theory and sensor fusion; (iv) relate fixed-size confidence intervals to fixed-geometry confidence sets; and (v) examine the performance of fixed-size hyper-cubic confidence sets for decision models with spherical parameter sets in the presence of both Gaussian and non-Gaussian sampling distributions.

2 Multi-Sensor Fusion of Position Data

In this section we present a paradigm for multi-sensor fusion based on position data. Figure 1 depicts three spatial position sensors S_i , $i = 1, 2, 3$, which measure the two-dimensional spatial position of a potential target (shaded object) with reference to the indicated Cartesian coordinate system. The small rectangles denote the given set-valued descriptions of the a priori uncertainty in the spatial position of the sensors. The data sets $\underline{Z}_i = (Z_{i1}, Z_{i2}, \dots, Z_{iN_i})$, $i = 1, 2, 3$, denote noisy position measurements of one or more potential targets (objects of interest). The measurement noise of each sensor is in addition to, and generally independent of, the position uncertainty of the sensor.

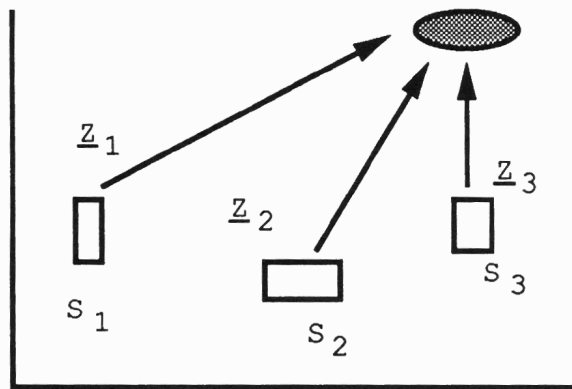


Figure 1: Multi-Sensor Fusion of Position Data

In this context, the problem of multi-sensor fusion becomes: (i) test for consistency between data sets $(\underline{Z}_1, \underline{Z}_2, \underline{Z}_3)$; and (ii) combine the data (if any) which are consistent. For example, given three sensors with known positions, and a single measurement per sensor $Z_i = \theta_i + V_i$, $i = 1, 2, 3$, the fusion problem becomes:

- (Test for Consistency:) Does $\theta_i = \theta_j$, $1 \leq i, j \leq 3$, $i \neq j$?
- (Data Fusion:) If $\theta = \theta_1 = \theta_2 = \theta_3$, how do we combine Z_1, Z_2 , and Z_3 to estimate the common value of the position parameter θ ?

In a very practical sense, this fusion problem is inherently multivariate, since: (i) spatial position is usually characterized by a two- or three-dimensional parameter vector; and (ii) there is usually stochastic dependence between the components of a position sensor noise vector when it is transformed into Cartesian coordinates.

3 Sensor and Environmental Models

In this section we introduce statistical and set-valued models for sensor errors and a priori environmental uncertainty. We focus our attention on location parameter models. Here, the term location denotes a

sensor observation relation of the form $Z = \theta + V$, where θ is an uncertain parameter, and V denotes observation noise whose characteristics are independent of θ . We assume that $\theta \in \Omega$, where Ω is a known subset of E^n , e.g., a given hyper-rectangle, or hyper-ellipsoid. Let F denote the joint CDF of the sensor noise V . We allow for uncertainty in our knowledge of F by modeling F as an unknown element of a known class of CDFs, \mathcal{F} . In certain applications, the given uncertainty class \mathcal{F} is a neighborhood in the space of all CDFs.

These set-valued uncertainty models of the environment and the sensors are valuable in applications where there is relatively limited probabilistic information available. In particular, the uncertainty in the environmental parameter θ is modeled entirely by a set of possible values, $\Omega \subset E^n$. We make *no* probabilistic assumptions about θ . Further, we do *not* require a complete, or even a parametric specification, of F . The uncertainty classes \mathcal{F} allow:

- non-Gaussian sampling distributions, i.e., non-Gaussian sensor noise;
- nonparametric uncertainty descriptions; and
- the inclusion of sporadic sensor behavior, e.g., ϵ -contamination models.

4 Confidence Sets, Statistical Decision Theory, and Sensor Fusion

Our approach to robust multi-sensor fusion makes use of robust fixed-geometry confidence sets. In this section we address this methodology and illustrate it with an example based on fixed-size confidence intervals.

Let $\Omega = [-d, d] \subset E^1$, $Z = \theta + V$, and \mathcal{F} denote a given uncertainty class for F . Let $\delta(Z)$ denote a decision rule which solves the following max-min problem:

$$\max_{\delta} \min_{\theta \in \Omega, F \in \mathcal{F}} P[\delta(Z) - e \leq \theta \leq \delta(Z) + e],$$

where $e > 0$ is given. An interval $[\delta(Z) - e, \delta(Z) + e]$ which solves this max-min problem is called a **Robust Fixed-Size Confidence Interval** of size $2e$ for θ . Here, δ is robust with respect to the distributional uncertainty modeled by \mathcal{F} . Research on robust fixed-size confidence intervals appears in Zeytinoglu and Mintz (1988). These ideas extend immediately to (convex) sets in E^n . We refer to these extensions as robust fixed-geometry confidence sets.

Based on robust fixed-geometry confidence sets, we obtain a methodology for multi-sensor fusion by constructing a robust test of hypothesis for the equality of location parameters. We illustrate the basic idea with a univariate example.

Let $Z_i = \theta_i + V_i$, $i = 1, 2$. Assume the V_i are i.i.d. with common CDF $F \in \mathcal{F}$, where \mathcal{F} is a given uncertainty class. Further, assume that $\theta_i \in [-d, d]$, $i = 1, 2$. Define: $\hat{Z} = Z_1 - Z_2$, $\hat{\theta} = \theta_1 - \theta_2$, and $\hat{V} = V_1 - V_2$, with CDF \hat{F} . Let $\hat{\mathcal{F}}$ denote the uncertainty class defined by the random variables \hat{V} where F ranges over all of \mathcal{F} . Observe that: $\hat{Z} = \hat{\theta} + \hat{V}$, where $\hat{\theta} \in [-2d, 2d]$, and $\hat{F} \in \hat{\mathcal{F}}$.

We construct a robust test of hypothesis for the equality of θ_1 and θ_2 by obtaining a confidence interval of width $2e$. We reject the hypothesis: $\theta_1 = \theta_2$, if $0 \notin [\delta(\hat{Z}) - e, \delta(\hat{Z}) + e]$. The value of the parameter e is used to select the size of the test.

In obtaining the numerical results (performance bounds) displayed in the sequel, we make use of the specific structure of the confidence procedures delineated in Zeytinoglu and Mintz (1988), Kamberova and Mintz (1990) and McKendall and Mintz (1990b). We refer the reader to these papers for the details.

5 Cartesian Products of Fixed-Size Confidence Intervals

If $Z = \theta + V \in E^n$, Ω is a hyper-rectangle, and the components of the noise vector V are independent random variables, then we can construct fixed-size hyper-rectangular confidence sets for θ . We illustrate these ideas with a two-dimensional example which is depicted in Figure 2:

- $Z = \theta + V \in E^2$;
- $|\theta_i| \leq d_i, i = 1, 2$;
- $V_i, i = 1, 2$ — independent random variables;
- A rectangular confidence set of size $2e_1 \times 2e_2$;
- The two-dimensional confidence set is based on the Cartesian product of the (independent) confidence intervals described previously.

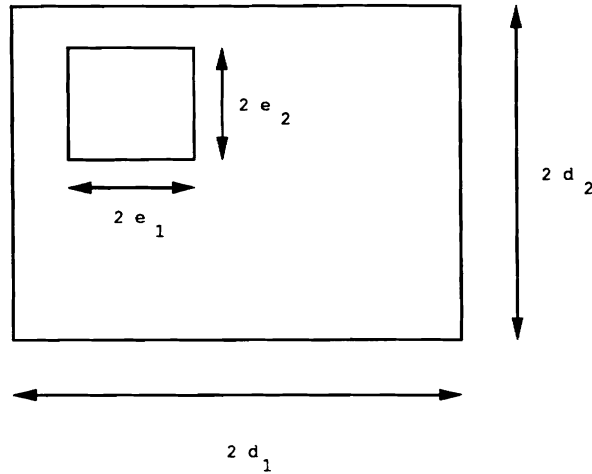


Figure 2: A Cartesian Product Paradigm

6 A Non-Cartesian Product Paradigm

In this section we consider a “small” modification to the problem statement which lead to a Cartesian product solution in the last example. The small modification entails the replacement of the rectangular Ω set in E^2 with a circular parameter space. In particular, if:

- $Z = \theta + V \in E^2$;
- $|\theta| \leq r$;
- $V_i, i = 1, 2$ — independent independent random variables;
- A circular confidence set of radius r_e ;
- A rectangular confidence set of size $2e_1 \times 2e_2$;

then: the appropriate two-dimensional confidence sets (circular, and rectangular) for the circular parameter space with independent noise components are still **open questions** — even when the V_i are i.i.d. $N(0, \sigma^2)$. The geometric components of this example appear in Figure 3.

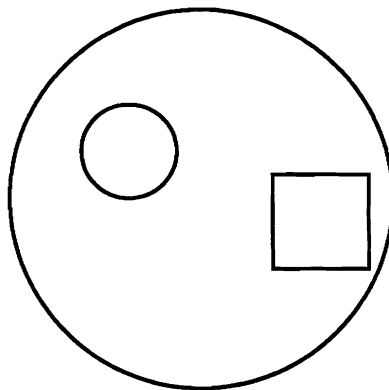


Figure 3: A Non-Cartesian Product Paradigm

Although the problem of determining exact spherical confidence sets in conjunction with spherical parameter spaces is still open, there is a special class of noise distributions for which a partial answer can be obtained, namely the spherically symmetric distributions. In this instance, the decision rules must exhibit rotational invariance. To illustrate this point, we return to the previous example and assume that the V_i , $i = 1, 2$, are i.i.d. $N(0, \sigma^2)$. In this case, the decision rule for locating the center of the circular (spherical) confidence region depends on the unit vector determined by the observation data (direction), and the modulus of the observation vector. This underlying spherical symmetry is a consequence of the intrinsic symmetry in the problem formulation. However, the form of the “retraction”, i.e., the function that depends on the modulus of the observation vector is still an **open question**. The geometric components of this example appear in Figure 4.

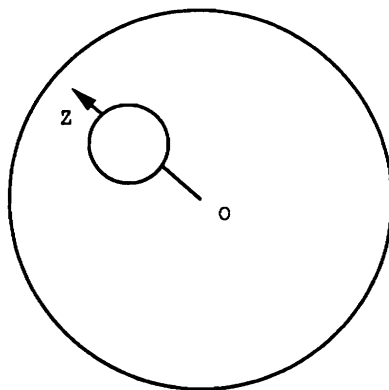


Figure 4: A Spherically Symmetric Paradigm

7 Approximate Solutions for Non-Cartesian Decision Models

We return to the problem stated in Section 6 for the case where the desired confidence set is a square. As noted previously, solution to this non-Cartesian problem is still open. In order to obtain an approximate solution, we replace the circular parameter set with its minimum bounding square and compute the optimal fixed-size confidence set with confidence coefficient $1 - \alpha$. This computation determines the size of the confidence set ($2e \times 2e$). We next compute the performance (confidence coefficient) for this procedure against the parameter space defined by the square which is inscribed in the original circular parameter set. The optimal confidence coefficient for the original problem must lie between these two

values, since the circle is contained between the two bounding squares. This inner-outer approximation technique easily extends to E^n . The related sets are depicted in Figure 5.

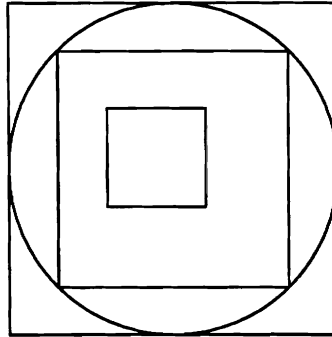


Figure 5: The Inner-Outer Approximation Technique

Table 1 presents the results of the following computations: (i) We determined the percentage difference between the upper and lower bounds to the optimal confidence coefficient based on the stated inner-outer approximation technique; (ii) We examined the 2-D and 3-D cases for values of $1 - \alpha$: 0.90 and 0.95; (iii) We considered integer values of d/e in the range 3 to 10, where $2d$ is the linear dimension of the outer hyper-cube; and (iv) We considered observation noise distributions: Gaussian $N(0,1)$, Laplacian $L(0,1)$, and Cauchy $C(0,1)$.

2-Dimensional Case						
d/e	N(0,1)		L(0,1)		C(0,1)	
	Conf. Coeff. 0.90	Conf. Coeff. 0.95	Conf. Coeff. 0.90	Conf. Coeff. 0.95	Conf. Coeff. 0.90	Conf. Coeff. 0.95
3	0	0	0	0	0	0
4	1.36	0.65	1.37	0.66	1.32	0.63
5	0.72	0.35	0.71	0.34	0.61	0.29
6	0.43	0.21	0.42	0.20	0.33	0.16
7	0.70	0.34	0.68	0.33	0.52	0.25
8	0.47	0.22	0.45	0.21	0.33	0.16
9	0.33	0.16	0.31	0.15	0.22	0.10
10	0.24	0.11	0.22	0.11	0.15	0.07

3-Dimensional Case						
d/e	N(0,1)		L(0,1)		C(0,1)	
	Conf. Coeff. 0.90	Conf. Coeff. 0.95	Conf. Coeff. 0.90	Conf. Coeff. 0.95	Conf. Coeff. 0.90	Conf. Coeff. 0.95
3	3.10	1.49	3.17	1.52	3.62	1.73
4	1.35	0.65	1.36	0.65	1.31	0.63
5	1.98	0.95	1.98	0.95	1.84	0.89
6	1.12	0.54	1.10	0.53	0.92	0.44
7	0.70	0.33	0.67	0.32	0.52	0.25
8	0.87	0.42	0.84	0.41	0.64	0.31
9	0.59	0.28	0.57	0.27	0.41	0.20
10	0.69	0.33	0.66	0.32	0.47	0.23

Table 1: Percentage difference between the upper and lower bounds to the optimal confidence coefficient.

Remarks

It is evident from the table that: (i) The percentage difference is at most weakly dependent on the tail behavior of the noise distribution; and (ii) The percentage differences between the 0.95 and 0.90 ($1 - \alpha$) baseline cases is approximately double.

Acknowledgements

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References

- Kamberova, G. and Mintz, M., 1990.** Robust Multi-Sensor Fusion: A Decision-Theoretic Approach, Proc. of the DARPA Image Understanding Workshop, September 1990, pp. 867-873.
- McKendall, R. and Mintz, M., 1990a.** Non-Monotonic Decision Rules for Sensor Fusion, Proc. of the DARPA Image Understanding Workshop, September 1990, pp. 874-880.
- McKendall, R. and Mintz, M., 1990b.** Using Robust Statistics for Sensor Fusion, SPIE Proc. of the International Symposium on Advances in Intelligent Systems, Session on Sensor Fusion, November 1990.
- Zeytinoglu, M. and Mintz, M., 1988.** Robust Fixed-Size Confidence Procedures for a Restricted Parameter Space, The Annals of Statistics, Vol. 16, No. 3, pp. 1241-1253, September 1988.