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# Metamaterial monolayers and bilayers for enhanced transmission through a sub-wavelength aperture in a flat perfectly conducting screen

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# Metamaterial monolayers and bilayers for enhanced transmission through a sub-wavelength aperture in a flat perfectly conducting screen

## **Abstract**

In this paper, we will provide an overview of our theoretical work on the role of metamaterial covers in dramatically enhancing the wave transmission through a subwavelength aperture over a perfectly conducting flat screen. It is well known that a low-permittivity or a low-permeability grounded slab may support surface polaritons. In our problem these natural modes are exploited to collect and redirect the impinging radiation into the tiny hole and to reshape the radiation pattern at the exit side of the screen towards an observer. The sum of these two effects may potentially lead to a dramatic increase in the total power transmission through the aperture. Moreover, we show how this effect may be further increased and optimized by employing bilayer covers with "conjugate" materials, i.e., materials with oppositely signed constitutive parameters. In some earlier works, we have indeed utilized such a coupling to induce a compact "interface" resonance, suggesting several microwave and optical applications. Here, the same resonant phenomenon can lead to the transmission enhancement together with a reduction of the required cover thickness. We provide some insights into the physical basis of this effect, and we speculate some potential applications.

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# Metamaterial monolayers and bilayers for enhanced transmission through a sub-wavelength aperture in a flat perfectly conducting screen

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*SUMMARY. – In this paper, we will provide an overview of our theoretical work on the role of metamaterial covers in dramatically enhancing the wave transmission through a subwavelength aperture over a perfectly conducting flat screen. It is well known that a low-permittivity or a low-permeability grounded slab may support surface polaritons. In our problem these natural modes are exploited to collect and redirect the impinging radiation into the tiny hole and to reshape the radiation pattern at the exit side of the screen towards an observer. The sum of these two effects may potentially lead to a dramatic increase in the total power transmission through the aperture. Moreover, we show how this effect may be further increased and optimized by employing bilayer covers with “conjugate” materials, i.e., materials with oppositely signed constitutive parameters. In some earlier works, we have indeed utilized such a coupling to induce a compact “interface” resonance, suggesting several microwave and optical applications. Here, the same resonant phenomenon can lead to the transmission enhancement together with a reduction of the required cover thickness. We provide some insights into the physical basis of this effect, and we speculate some potential applications.*

## 1. Introduction

The challenge of transporting optical information in a structure with lateral dimensions below the diffraction limit is currently an exciting topic in the scientific community, due to the increased attention in science and technology of nanoscales, and optical computation (see, e.g., [1]). One such

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problem has been the study of techniques to enhance wave transmission through sub-wavelength apertures, which are being investigated by several groups worldwide [2-4].

In this work, we report and summarize our recent theoretical work on the role of metamaterial covers in dramatically enhancing the wave transmission through a subwavelength aperture over a perfectly conducting flat screen. It is well known that a low-permittivity or a low-permeability grounded slab may support surface polaritons, which are commonly used in leaky-wave antenna applications [5]. In our problem these natural modes are exploited to collect and redirect the impinging radiation into the tiny hole and to reshape the radiation pattern at the exit side of the screen towards an observer. The combination of these two effects may potentially lead to a dramatic increase in the total power transmission through the aperture.

Moreover, we show here how this effect may be further increased and optimized by employing bilayer covers with “conjugate” materials, i.e., materials with oppositely signed constitutive parameters. In some earlier works (see, e.g., [6-7]), some of us have indeed utilized such a coupling to induce a compact “interface” resonance, suggesting several microwave and optical potential applications. Here, the same resonant phenomenon may lead to the transmission enhancement and a reduction of the required cover thickness.

The huge enhancement effect in the wave transmission through tiny holes may offer several interesting potential applications. It has been suggested in the literature that such enhancement may be used to spatially filter the electromagnetic radiation, avoiding the diffraction limitations [8], for tunable optical filters [9], for photolithography, for near-field microscopy or to extract light from LED [3]. In the following, we provide some insights into the physical basis of this effect. We also address the sensitivity of our designs to losses and to a few other realistic limitations.

## 2. Theoretical analysis

The geometry we refer to in the following is depicted in Figure 1, and consists of a flat perfectly conducting and infinitely thin metallic screen embedded in a suitable Cartesian reference system. The screen is placed on the plane  $y=0$  and an electrically tiny aperture, with arbitrary cross section, is positioned at the origin. Let the region that the hole occupies on the screen be denoted by  $A$ . An  $e^{i\omega t}$ -monochromatic source is placed somewhere in the region  $y<0$ , far from the hole, illuminating the structure. The screen is covered on both sides by layers of isotropic material, with complex constitutive parameters  $\epsilon$  and  $\mu$  at the operating frequency  $f=\omega/2\pi$ , and with thicknesses  $d_{in}$  and  $d_{out}$ , respectively for the  $y<0$  and  $y>0$  sides. The covered screen is surrounded by vacuum, with permittivity and permeability  $\epsilon_0$  and  $\mu_0$ , respectively.

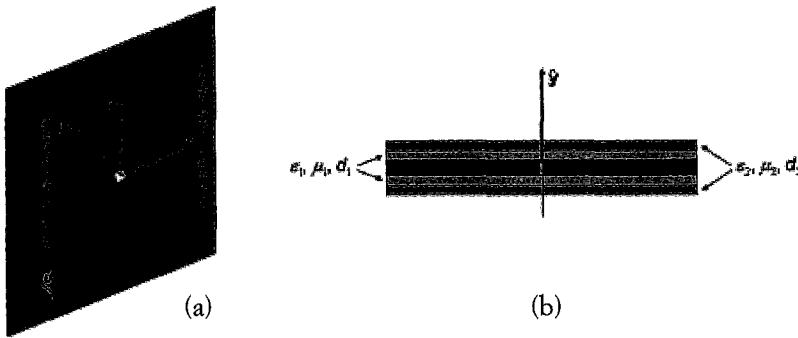


FIG. 1

Geometry of the problem: a) a sub-wavelength hole in a perfectly electric conducting (PEC) flat screen covered by a metamaterial slab (entrance face), b) cross-sectional view of the setup with bilayer covers in the entrance and exit faces

According to Bethe's theory [10], we should first calculate the field induced by the excitation in the region  $A$ , neglecting the presence of the hole, since its small dimensions should not sensibly affect the field on the entrance face. Then, we may calculate the equivalent magnetic sources to be put on the other side of the screen, again in the region  $A$ , in order to solve the radiation problem on the exit face. As shown by Bethe, these magnetic sources are directly proportional to the amplitude of the field in  $A$  when the hole is closed, and for an observer far from the hole on the exit side they are represented by an electric and a magnetic dipole as follows:

$$[1] \quad \begin{aligned} \mathbf{p} &= \alpha_1 \mathbf{E}_0 \\ \mathbf{m} &= -\alpha_2 \cdot \mathbf{H}_0 \end{aligned}$$

where the electric polarizability  $\alpha_1$  and the magnetic polarizability tensor  $\alpha_2$  depend on the shape of the hole and, due to the small dimensions of  $A$ , they may be evaluated using the static limits. The fields  $\mathbf{E}_0$  and  $\mathbf{H}_0$  represent, respectively, the uniform electric and magnetic fields present in region  $A$  when the hole is closed, which implies the normal component of the total electric field and the tangential component of the total magnetic field in formula (1).

It may be shown that an efficient way to dramatically increase the electromagnetic field on the hole, which corresponds to a proportional increase of the equivalent radiating dipoles from the hole, as justified by formula (1), is to use at the entrance face of the screen a cover with:

$$[2] \quad |\mu| \leq \mu_0, \quad d_{in} = \frac{(2N-1)\pi}{2\sqrt{k^2 - \bar{k}_t^2}},$$

where  $N$  is a positive integer,  $k = \omega\sqrt{\epsilon\mu}$  and  $\bar{k}_t$  is the transverse wave number of the plane wave impinging at the angle from which most part of the radiation is expected to come. This condition, in fact, implies that the grounded slab at the entrance face supports a natural mode (leaky wave) with transverse wave number  $k_t \approx \bar{k}_t$ , with a very small imaginary part taking into account the typical radiation losses of such modes, which couples very well with the incoming radiation. This polariton easily collects power from the entrance side into the tiny hole, drastically increasing the total power transmission.

Correspondingly by reciprocity, placing a similar cover at the exit side of the hole, induces a parallel increase of transmission towards the observer, similar to the effects shown in another setup by Enoch *et al.* [5].

### 3. Numerical results

Figure 2 shows the normalized radiation patterns of a setup with a monolayer cover, designed following Eq. 1. The main drawback of this setup is that the required cover may easily become relatively thick, since the wavelength in the cover material is larger than in the free space (in the example here shown  $d \leq 8\lambda_0$ ). A way to overcome this problem is to utilize cover bilayers constituted by conjugate materials, in the sense reported in [6-7]. In this case, a compact interface resonance may be induced in the cover, which allows designing a thin cover still supporting a material polariton. The optimized design gives a total power enhancement of about 90 dB with a cover thick less than a wavelength in free space.

Clearly, the results here reported are ideal numerical simulations, whereas in practice we should take into account several intrinsic limitations: material losses, sensitivity to a variation in the material parameters, cover and screen finiteness in the transverse plane, finiteness of the screen conductivity and longitudinal thickness. We have considered also these realistic limitations in our work, even though the results are not reported here for lack of space. However, it is interesting to underline that still promising results are obtained also in these cases, which in the optimized setups lead to major enhancements of power transmission through tiny subwavelength apertures.

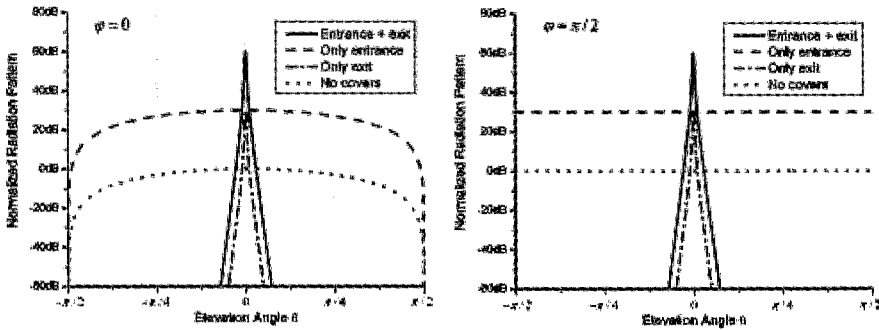


FIG. 2

Radiation patterns in the E and H planes for covers with  $\epsilon = \epsilon_0$ ,  $\mu = 10^{-3}\mu_0$ ,  $d_{in} = d_{out} = \pi/2k$ , normalized to the peak of the pattern with no covers, compared to those with only the entrance slab, only the exit one, and the one with no covers.

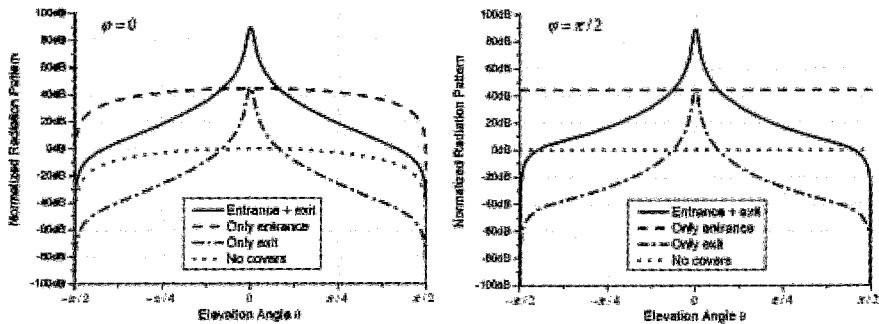


FIG. 3

Radiation patterns in two planes  $\phi = 0^\circ$  and  $\phi = 90^\circ$ , normalized to the pattern with no covers. Cover bilayers are employed with  $\epsilon_1 = 9.957\epsilon_0$ ,  $\mu_1 = 0.034\mu_0$ ,  $\epsilon_2 = 5.747\epsilon_0$ ,  $\mu_2 = -0.283\mu_0$ ,  $d_1 = d_2 = 0.36\lambda_0$

## REFERENCES

- (1) S.A. MAIER, P.G. KIK, H.A. ATWATER, S. MELTZER, E. HAREL, B.E. KOEL, A.A.G. REQUICHA, *Local detection of electromagnetic energy transport below the diffraction limit in metal nanoparticle plasmon waveguides*, Nature Materials, **2**, 229-232, 2003.
- (2) T.W. EBBSEN, H.J. LEZEC, H. F. GHAEMI, T. THIO, P.A. WOLFF, *Extraordinary optical transmission through sub-wavelength hole arrays*, Nature, **391**, 667-669, 1998.

- (3) D.E. GRUPP, H.J. LEZEC, T. THIO, T.W. EBBSEN, *Beyond the Bethe limit: tunable enhanced light transmission through a single sub-wavelength aperture*, *Advanced Materials*, **11**, 860-862, 1999.
- (4) A.A. OLINER, D.R. JACKSON, *Leaky surface-plasmon theory for dramatically enhanced transmission through a sub-wavelength aperture, Part I: Basic features*, *Proceedings of IEEE Antennas and Propagation Society Symposium*, Columbus, OH, **2**, 1091-1094, 2003.
- (5) S. ENOCH, G. TAYEB, P. SABORNOUX, N. GUERIN, P. VINCENT, *A metamaterial for directive emission*, *Physical Review Letters*, **89**, 213902, 2002.
- (6) A. ALÙ, N. ENGHETA, *Pairing an epsilon-negative slab with a mu-negative slab: resonance, tunneling and transparency*, *IEEE Transactions on Antennas and Propagation*, Special Issue on Metamaterials, **51**, 2558-2571, 2003.
- (7) A. ALÙ, N. ENGHETA, *Peculiar radar cross section properties of double-negative and single-negative metamaterials*, *Proceedings of IEEE Radar Conference*, Philadelphia, PA, 2004, to appear.
- (8) R. SAMBLES, *More than transparent*, *Nature*, **391**, 641-642, 1998.
- (9) T.J. KIM, T. THIO, T.W. EBBSEN, D.E. GRUPP, H.J. LEZEC, *Control of optical transmission through metals perforated with subwavelength hole arrays*, *Optics Express*, **24**, 256-258, 1999.
- (10) H.A. BETHE, *Theory of diffraction by small holes*, *Physical Review*, **66**, 163-182, 1944.