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Mamdouh M. I. Saadoun
Saudi Consolidated Electric Company

Nader Engheta
University of Pennsylvania, engheta@ee.upenn.edu

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Theoretical Study of Variation of Propagation Constant in a Cylindrical Waveguide Due to Chirality: Chiro-Phase Shifting

Mamdouh M. I. Saadoun and Nader Engheta

Abstract—We discuss a conceptual idea for a reciprocal phase shifting using chiral materials. Such phase shifters, which can be called chiro-phase shifters, may, in principle, consist of a general cylindrical waveguide in which a thin chiral rod is inserted longitudinally. A standard commonly used perturbation technique is utilized to analyze theoretically the effects of a chiral rod's parameters and in particular its chirality admittance on propagation constants of guided modes in such waveguides. A simple expression relating the relative change of propagation constants to other relevant quantities such as polarization characteristics of unperturbed guided modes, material parameters of chiral rod, and location of the rod is presented, and the case of a circular metallic chiro-phase shifter is discussed in some detail. Physical insights into the results are also provided.

I. INTRODUCTION

The similarities between chiral media, which in the optical regime are also known as media with natural optical activity [1], and magnetically biased ferrites have been addressed and studied since the early days of the introduction of the concept of chirality in electromagnetics and optics. However, to date, relatively less has been done to explore the possibility of using chiral materials instead of (or combined with) magnetically biased ferrites in the design of novel devices and components. Here, an idea for a reciprocal phase shifter whose principles of operation lie in the properties of chiral materials is discussed. Unlike magnetically biased ferrite devices, which are generally nonreciprocal [2]-[4], this proposed device can conceptually be reciprocal owing to the property of reciprocity in isotropic chiral media [5]-[7]. This may be of interest in certain applications of phase shifters where reciprocity is desired. We have proposed elsewhere [8] another conceptual idea for a reciprocal phase shifter based on a certain class of bianisotropic media, which we introduced recently and called the pseudochiral ω-medium.

An isotropic chiral material is a medium whose microstructures lack mirror symmetry leading to the property of handedness. Such a property is reflected in the constitutive relations of the medium which can take the form $D = \varepsilon_r E + i\xi\varepsilon_r B$ and $H = i\xi_r E + (1/\mu_r)B$. The additional parameter $\xi$ is the chirality admittance describing the handedness of the medium, and $\varepsilon_r$ and $\mu_r$ are the permittivity and permeability of the medium, respectively [9]. The eigenmodes of propagation in chiral media are shown to be right-circularly polarized (RCP) and left-circularly polarized (LCP) plane waves with two differing propagation constants $k_{z} = \omega \left[ \pm \xi_r \varepsilon_r + \sqrt{\varepsilon_r \mu_r + \mu_0^2 \xi_r^2} \right]$, where the plus and minus signs refer to the RCP and LCP eigenmodes, respectively [5]-[7]. It is also well known that in the case of magnetically biased ferrites, the eigenmodes of propagation along the direction of the biasing magnetic field are, also, RCP and LCP plane waves with two unequal propagation constants [10],[11]. It is in this sense that a chiral medium is similar to a magnetically biased ferrite material. However, it is well known that while the former is a reciprocal medium, the latter is not. Furthermore, in isotropic chiral media, the two wavenumbers $k_{z}$ shown above are independent of the direction of propagation, whereas magnetically biased ferrites are intrinsically anisotropic, and their wavenumbers of eigenmodes are indeed direction-dependent. Since magnetically biased ferrites have been extensively utilized in the design of a variety of microwave devices and components [2]-[4], similarities between such materials and chiral media have motivated us to investigate theoretically the possibility of utilizing chiral materials in the design of devices which would be counterparts to ferrite devices. Phase shifters, in particular, are good candidates for this study of chiral-based devices since they are not functionally required to always be nonreciprocal. As in the case of ferrite phase shifters where a thin ferrite slab inserted in a cylindrical waveguide affects the propagation constant of the guide, here we discuss theoretically similar effects when a thin rod of chiral material is to be inserted in a waveguide. Such a device may be called the chiro-phase shifter.

In the following section, we will review the variation of propagation constant due to the insertion of a thin chiral rod in a generic waveguide (metallic or open waveguide) with arbitrary cross section. In Section III, as an illustrative example, we will analyze a special device configuration for a chiro-phase shifter.

II. VARIATION OF THE GUIDE WAVE NUMBER DUE TO INSERTION OF A THIN CHIRAL ROD

Fig. 1 presents the geometry of the present problem where a cylindrical waveguide with arbitrary cross-sectional shape

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can be an open waveguide such as a dielectric waveguide loaded with a longitudinal thin rod of chiral material. The permeability insertion of the chiral rod, is filled with a material with waveguide can have a perfectly conducting boundary or it supporting a guided surface wave. The waveguide, before this loaded waveguide are obtainable only for a few special geometries, here for the general geometry and not to have sharp corners or edges. In general, the rod can be placed anywhere in the waveguide with its axis parallel to the waveguide axis. Since the exact solutions for guided modes in this loaded waveguide are obtainable only for a few special geometries, here for the general geometry of the waveguides and chiral rods, the standard perturbation technique must be used to study the effects of the thin chiral rod on the guide propagation constant. This perturbation technique has been previously used for cavity resonators wherein a small bianisotropic or bisotropic sample has been inserted. For an excellent description of this technique, the reader is referred to [12], [13]. For the perturbation theory for waveguides, the reader is referred to the work of Waldron [10] and Viitanen and Lindell [14]. In [14], Viitanen and Lindell have done an excellent analysis for perturbation theory in a corrugated waveguide with an inserted bianisotropic rod. Here, to explore the role of chiral material (instead of magnetically biased ferrite) in the possible design of a phase shifter, this use the polarizability dyadics. For small bianisotropic spheres, the polarizability dyadics have been studied extensively by Sihvola [16] and for bianisotropic ellipsoids by Sihvola [17], and they have been used in the perturbation analysis for the cavity resonators [12], [13]. Here, however, since the cross-sectional shape of the chiral rod has been assumed to be arbitrary in this section, the Born-type approximation is being used, although we realize that this approximation will provide an approximate (not exact) expression for the fields inside the rod. Since polarization and magnetization sources contribute to the displacement vector $D$ and magnetic intensity vectors $H$, respectively, it seems to be reasonable to consider that, in this approximation, the electric field intensity $E$ and the magnetic induction $B$ remain unchanged within the chiral rod, while the displacement vector $D$ and magnetic intensity vectors $H$ are perturbed. Therefore, we have $E^{(1)} \equiv E$ and $H^{(1)} \equiv H$.

As will be shown later, clearly it would not be preferred to locate the rod in regions of vanishing unperturbed fields.

$$\frac{\delta \beta}{\beta} = \frac{\omega}{\beta} \frac{d\beta}{d\omega} \frac{\int_{S_{1}} \{ E^{(1)} \cdot D^{(0)*} - E^{(0)*} \cdot D^{(1)} + H^{(1)} \cdot B^{(0)*} - H^{(0)*} \cdot B^{(1)} \} \, dS_{1}}{\int_{S_{0}} \{ E^{(0)} \cdot D^{(0)*} + H^{(0)*} \cdot B^{(0)} \} \, dS_{0}}$$  (1)
From the above equation and the constitutive relations in chiral media, the following relations for the fields' incremental perturbations are \( \Delta D(1) \equiv (\varepsilon - \varepsilon_0)E(0) + i\varepsilon_0 B(0) \) and \( \Delta B(1) \equiv (1/\mu - (1/\mu_0)B(0) + i\varepsilon E(0) \). Having obtained the unperturbed and incremental perturbations of the field, one can find \( \delta \beta / \beta \) from (1) as given below:

\[
\frac{\delta \beta}{\beta} = -f(\omega) \left\{ \frac{\varepsilon - \varepsilon_0}{4} |E(0)|^2 - \frac{1}{4} \left( \frac{1}{\mu_0} - \frac{1}{\mu} \right) |B(0)|^2 \right. \\
+ \left. \frac{\varepsilon_0}{2} \text{Im} \left( \frac{E(0)}{B(0)} \right) \right\}_{\rho \rightarrow \text{rod's center}} \tag{2}
\]

where \( f(\omega) \equiv -4(\omega S_1/\beta)(d\beta/d\omega) \int_S \left( E(0)^* \right. \cdot \left. D(0)^* + B(0)^* \right) dS \) and the symbol \( \rho \rightarrow \text{rod's center} \) means that the expressions in curly brackets are evaluated at the center of the rod. The function \( f(\omega) \) is independent of the material parameters of the perturbing rod, and depends only on the waveguide geometry, the unperturbed waveguide fields, and the cross-sectional area of the rod. The degree of accuracy of this perturbation method depends on the relative difference of material parameters of the chiral rod and the waveguide material. As mentioned earlier, such relative differences are to be small, i.e., \( |\mu_0 - \mu| \ll \mu, |\varepsilon - \varepsilon_0| \ll \varepsilon, \) and \( \varepsilon_0 \ll \sqrt{\mu} \). It must be noted, however, that as an alternative, numerical techniques such as the finite-element method can be used to obtain the propagation constants in chiral loaded waveguides [18]. Equation (2) shows that the chirality of the rod has a direct effect (to the first order) on propagation constant \( \beta \) when the imaginary part of the scalar product \( (E(0)^*, B(0)^*) \) of the unperturbed fields has a nonvanishing value. This can happen when \( B(0) \) has a component along \( E(0) \) with some nonzero phase difference. For example, if \( E(0) \) and \( B(0) \) are elliptically polarized fields, \( \text{Im} \left( E(0)^*, B(0)^* \right) \) may be nonzero depending on the orientation of the polarization ellipses of these two fields. If \( E(0) \) and \( B(0) \) are in phase and linearly polarized, \( \text{Im} \left( E(0)^*, B(0)^* \right) \) will be zero. Clearly, the perturbation of the propagation constant due to chirality is strongest for circularly polarized fields. For the case of linearly polarized fields in waveguides, and for the case of propagation of modes of corrugated waveguides due to the biisotropic rod, the reader is referred to [8] and [14], respectively. The importance of the imaginary part of \( (E(0)^*, B(0)^*) \) in the shift of resonance frequency in cavity resonators was discussed in [13].

From a microscopic point of view, one can visualize the chiral medium as a collection of small helices of certain handedness. The incident (unperturbed) electric field polarizes the stem of the helix. Since the field is time varying, a current flows in the stem, and consequently in the loop portion of the helix. This current, which has a 90° phase difference with the inducing electric field, provides a magnetic dipole moment. This magnetic moment, which is in phase with the induced current, is therefore 90° out of phase with the inducing electric field, and is mainly parallel to the stem of the helix. Hence, the incident electric field induces a magnetization parallel to the electric field and 90° out of phase with it. If the incident unperturbed wave has a magnetic field component parallel to the inducing electric field with a nonzero phase difference with it, there will be some interaction between the electrically induced magnetization and the incident magnetic field. A similar argument can be addressed for the interaction between the magnetically induced electric dipole moments and the incident electric field. The above discussion provides some intuitive insight into (2). Here, it is reviewed that the chirality of the thin rod can, in principle, affect the propagation constant \( \beta \), and consequently the phase \( \phi \) of a signal propagating through the guide. Therefore, phase shifting can be achieved in such a device. This phase shifter may be called the chiro-phase shifter. It is worth noting that in order to use chirality as a mechanism for achieving variable shift on the phase of propagating signals in a waveguide, one should find ways by which the chirality can be varied in real time through the variation of other quantities such as biasing fields. Such variation and control of chirality, particularly in optically active materials and in chiral liquid crystals, is presently under study. Combining the effect of chirality and Faraday rotation was also theoretically studied to explore possible ways to control chirality [19]. Moreover, a change of the chiral rod's location may also lead to a change in propagation constant \( \beta \), and hence in phase \( \phi \). Thus, this idea can be a basis for a device in sensing displacement and flow speed. In the following section, as an example, we apply the results of the perturbation technique to a specific case of a circular metallic chiro-phase shifter where a circular cylindrical metallic waveguide is loaded with a thin chiral rod located at an arbitrary position.

### III. An Example: A Circular Chiro-Phase Shifter

Consider a device geometry in which a circular cylindrical waveguide with metallic walls is loaded longitudinally with a thin rod of chiral material located at some arbitrary position. Apart from the chiral rod, the waveguide is assumed to be
filled with a homogeneous isotropic dielectric of permittivity \( \varepsilon \) and permeability \( \mu \). The material parameters of the rod are \( \varepsilon_r, \mu_r, \xi_r \). As mentioned earlier, it is also assumed that \( |\mu - \mu_r| \ll \mu, |\varepsilon - \varepsilon_r| \ll \varepsilon, \) and \( \xi_r \ll \sqrt{\varepsilon/\mu} \). The radius of the rod \( d \) is assumed to be much smaller than the radius of the waveguide \( R \). These assumptions are required to justify the use of perturbation techniques in analyzing this problem. The exact solutions for guided electromagnetic waves in such loaded waveguides in general (with arbitrary location of the chiral rod) appear to be untractable in closed form. For special cases, such as the case where the chiral rod is circular and located at the center of the circular waveguide, the closed form solutions are obtainable. This problem has been studied by Monte and Uslenghi [20]. Here, however, we consider the general case where the rod is located longitudinally at some arbitrary location other than the waveguide's axis, and as a result, we need to use the perturbation technique described in the previous section. It is well known that the field components of the dominant \( TE_{11} \) mode of this unperturbed waveguide (with \( E_z^{(0)} = 0 \)) can be written as [11]

\[
H_z^{(0)} = J_1 \left( \frac{x_1}{R} \right) \left[ C_1 \cos \phi + i C_2 \sin \phi \right] e^{i \beta z} e^{-i \omega t} \quad (3a)
\]

\[
H_p^{(0)} = -\frac{i \beta}{(x_1/R)^2} J_1 \left( \frac{x_1}{R} \right) \left[ -C_1 \sin \phi + i C_2 \cos \phi \right] e^{i \beta z} e^{-i \omega t} \quad (3b)
\]

\[
H_\phi^{(0)} = E_p^{(0)}/Z_{TE_{11}} \quad (3c)
\]

where \( Z_{TE_{11}} = \omega \mu/\beta \) is the transverse waveguide impedance for the \( TE_{11} \) mode, \( \beta = \sqrt{\varepsilon \mu c^2 - (x_1/R)^2} \) is the unperturbed guide propagation constant, \( x_1 \) is the first root of the derivative of the first-order Bessel function of the first kind, and \( (x_1/R) = \omega \mu/\beta \), \( (\rho, \phi, z) \) are used as cylindrical coordinate systems for the circular waveguide, and \( C_1 \) and \( C_2 \) are two arbitrary complex constants. By properly choosing the two constants \( C_1 \) and \( C_2 \), one can obtain a desired polarization for the transverse fields at the center (or any other given location) of the guide's cross section. For instance, \( C_1 \) and \( C_2 \) can be chosen to be \( C_1 = C \) and \( C_2 = ipC \) where \( C \) is an arbitrary constant and \( p \) is a real constant. Thus, one can write

\[
H_z^{(0)} = C J_1 \left( \frac{x_1}{R} \right) \left[ \cos \phi + i \sin \phi \right] e^{i \beta z} e^{-i \omega t} \quad (4a)
\]

\[
H_p^{(0)} = C \frac{i \beta}{(x_1/R)^2} J_1 \left( \frac{x_1}{R} \right) \left[ \cos \phi + i \sin \phi \right] e^{i \beta z} e^{-i \omega t} \quad (4b)
\]

\[
H_\phi^{(0)} = C \frac{i \beta}{(x_1/R)^2} J_1 \left( \frac{x_1}{R} \right) \left[ -\sin \phi + i \cos \phi \right] e^{i \beta z} e^{-i \omega t} \quad (4c)
\]

The electric field can be obtained in a similar way. From (4), we can clearly see that the transverse electric and magnetic fields at the center of the waveguide are, in general, elliptically polarized. In fact, the transverse magnetic field is given by

\[
H_{11}^{(0)} = \left( C/2 \right) (i \beta R/x_1^2) [e_x + ipe_y] e^{i \beta z} e^{-i \omega t}. \quad \text{Here, } e_x \text{ and } e_y \text{ are unit vectors along the } x \text{ and } y \text{ axes of a Cartesian coordinate system, and the parameter } p \text{ describes the axial ratio of the ellipse of polarization of the field at the center of the waveguide. For positive (negative) values of } p, \text{ the field is right (left) elliptically polarized. For } p \text{ being zero or infinity, the field is linearly polarized. For } p \text{ being } +1 \text{ or } -1, \text{ the field is right-circularly polarized RCP (left-circularly polarized LCP). When the location of the chiral rod is given in terms of } \rho_0 \text{ and } \phi_0, \text{ the unperturbed field quantities can be obtained from (3) and (4), and the n the value of } \delta \beta/\beta \text{ can be evaluated from (2). As can be seen from (2), the relative change in } \beta \text{ is due to three terms: the first and second terms are the effects of variation of the permittivity and permeability due to insertion of the unperturbed waveguide to be lossless and the unperturbed mode to be propagating), this third term can be explicitly written at the location of the chiral rod, i.e., } \rho_0 \text{ and } \phi_0, \text{ as}
\]

\[
\frac{\delta \beta}{\beta} \bigg|_{\rho_r \rightarrow \text{rod's center}} = \frac{p \xi_r Z_{TE_{11}} \mu |C|^2}{\beta^2} \left( \frac{x_1^2}{R^2} \right)^3 \rho_0 J_1 \left( \frac{x_1}{R} \right) J_1 \left( \frac{x_1}{R} \right) \rho_0.
\]

If the chiral rod is attached to the wall, \( \rho_0 = R \), and thus \( \delta \beta/\beta \) due to chirality is nil since \( J_1'(x_1) = 0 \). If the chiral rod is placed at the center of the waveguide, i.e., \( \rho_0 = 0 \), and since the rod is assumed to be very small, (5) can be evaluated at \( \rho_0 = 0 \), resulting in \( \frac{\delta \beta}{\beta} = \frac{p \xi_r Z_{TE_{11}} \mu |C|^2}{\beta^2} \left( \frac{x_1^2}{R^2} \right)^3 \rho_0 J_1 \left( \frac{x_1}{R} \right) J_1 \left( \frac{x_1}{R} \right) \rho_0 \).

Finally, we have the contribution of chirality to the relative change of the propagation constant due to insertion of the chiral rod, which can be expressed as

\[
\frac{\delta \beta}{\beta}
\]

This relation reveals several important interesting effects of the chiral rod on the propagation constant. First, it shows that the relative change in \( \beta \) is proportional to the local helicity of the unperturbed fields \( p \) and the chirality admittance of the chiral rod \( \xi_r \). If the handedness of the chiral medium (i.e., the sign of \( \xi_r \)) is reversed, the relative change in \( \beta \) is reversed. Likewise, if the helicity of the unperturbed field is reversed, a similar effect occurs. When the fields are linearly polarized (\( p = 0 \) or \( \infty \)), the relative change due to chirality is nil. However, for RCP or LCP unperturbed fields where \( p = +1 \) or \( -1 \), \( \delta \beta/\beta \) receives the maximum contribution from chirality. This suggests that a chiro-phase shifter would operate most effectively when there exists a right- (or left-) circularly polarized field at the rod's location in an otherwise unperturbed waveguide. This result was intuitively anticipated since the eigenmodes of propagation in chiral media are RCP and LCP.
and LCP waves, and chiral media interact with circularly polarized waves most effectively. It can also be observed from (5) [or (6)] that, if the direction of propagation is reversed (i.e., \( \beta \rightarrow -\beta \)), \( Z_{TE1} \) will change its sign. Thus, in order to have the same \( \delta \beta /\beta \), the sign of \( p \) should change, which means that the helicity of the wave is reverse. But since the direction of propagation is also changed, the handedness of the wave is unaltered. More explicitly, for example, unperturbed RCP fields would give rise to the same \( \delta \beta /\beta \) regardless of the direction of propagation. This is the case, however, for the magnetically biased ferrite [10]–[11]. This feature can, in principle, be used in the design of a reciprocal phase shifter using chiral materials. As shown above, the relative value of \( \delta \beta /\beta \) depends on the helicity of the local unperturbed fields. For RCP and LCP fields, maximum interaction occurs between the chiral rod and the fields, while for linearly polarized fields, the chirality of the rod has no effect (to first order) on the value of the propagation constant. In the example given above, it was shown that unperturbed transverse fields can be chosen to be circularly polarized at the center of the guide. However, on the walls, these transverse fields will always be linearly polarized since at \( \theta = a \), \( E_p^{(0)} = 0 \) and \( E_{\phi}^{(0)} = 0 \). Therefore, if the chiral rod is on the walls, chirality will not contribute to the relative change in \( \beta \). This suggests that \( \delta \beta /\beta \) is also affected by the location of the rod: when the rod is at the center of the guide, the effect of chirality on \( \delta \beta /\beta \) is maximum (if the local fields are not linearly polarized), and as the rod moves towards the guide’s walls, this effect is reduced. This feature can conceptually be used as a mechanical technique for achieving variable phase shifts. This can offer potential applications in the design of novel devices as displacement and pressure sensors for measuring mechanical displacement and pressure.

IV. SUMMARY

Using a standard perturbation technique, we have discussed the effect of a thin chiral rod inserted longitudinally in a cylindrical metallic waveguide with arbitrary cross section on the propagation constant of guided modes. It has been found that in order to have a first-order effect due to the rod’s chirality, the unperturbed fields \( E_{\phi}^{(0)} \) and \( B_{\phi}^{(0)} \) at the rod’s location must provide a nonzero value for \( \text{Im} (E_{\phi}^{(0)} . B_{\phi}^{(0)*}) \). A simple relation among the relative change in propagation constant and material parameters was obtained and given. This study has led us to suggest an idea for a novel phase shifter using chiral materials. This study has also shown that the phase shift can depend on the location of the chiral rod. This feature can potentially be used in the design of novel displacement sensors and for measuring chirality admittance.

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We wish to thank the anonymous reviewers for their helpful comments and remarks on this paper, and in particular for bringing to our attention [12], [13], [18]. The work of [14] was submitted and appeared in print after the submission of our present paper. That work was brought to our attention while we were revising our manuscript. In that reference, the perturbation of the propagation constant of guided modes in a corrugated waveguide when a thin biisotropic rod is inserted is studied. In the problem studied here in our present paper, the waveguide is not corrugated, and the variation of the propagation constant is given in terms of the polarization of guided modes in a circular metallic waveguide.

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