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Essays in Financial Econometrics

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Abstract
In the first chapter, I estimate dynamic factors from the term structure of credit spreads and the term structure of equity option implied volatilities, and I provide a comprehensive characterization of the dynamic relationships among those credit spread factors and equity volatility factors. I find strong evidence that the volatility factors, especially the volatility level factor, Granger cause credit spread levels, confirming the theoretical predictions of Merton (1974) in a significantly richer and more nuanced environment than previously achieved. Simultaneously, I also find evidence of reverse Granger causality from credit spreads to equity volatility, operating through the slope factors, consistent with the market microstructure literature such as Fleming and Remolona (1999a, 1999b), which finds that price discovery often happens first in bond markets. Hence my results extend and unify both the corporate bond pricing and market microstructure literatures, deepening our understanding of stock and bond market interaction and suggesting profitable trading strategies. In the second chapter, which is a joint work with Frank X. Diebold, we study the dynamics of the U.S. Treasury yield term structure by applying the Nelson-Siegel models introduced in Diebold and Li (2006) and its arbitrage-free version developed by Christensen, Diebold, and Rudebusch (2007). We analyze in-sample by testing the risk-neutral restrictions, which establish the absence from arbitrage, on the Nelson-Siegel factors estimated under the physical measure. We thus compare the term structure modeling between the risk-neutral measure and the physical measure. Specifically, we show that the risk-neutral restrictions on the factor dynamics are well-satisfied, and those factors have the following properties: 1) the level factor is a unit-root process and does not affect the other two factors; 2) the slope and curvature factors are mean-reverting processes that revert at the same rate; 3) the curvature factor forecasts the slope factor, but not conversely. Moreover, we find that utilizing these restrictions can improve out-of-sample forecast performance.

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To Lin

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ABSTRACT

ESSAYS IN FINANCIAL ECONOMETRICS

Jian Hua

Francis X. Diebold

In the first chapter, I estimate dynamic factors from the term structure of credit spreads and the term structure of equity option implied volatilities, and I provide a comprehensive characterization of the dynamic relationships among those credit spread factors and equity volatility factors. I find strong evidence that the volatility factors, especially the volatility level factor, Granger cause credit spread levels, confirming the theoretical predictions of Merton (1974) in a significantly richer and more nuanced environment than previously achieved. Simultaneously, I also find evidence of reverse Granger causality from credit spreads to equity volatility, operating through the slope factors, consistent with the market microstructure literature such as Fleming and Remolona (1999a, 1999b), which finds that price discovery often happens first in bond markets. Hence my results extend and unify both the corporate bond pricing and market microstructure literatures, deepening our understanding of stock and bond market interaction and suggesting profitable trading strategies. In the second chapter, which is a joint work with Frank X.

Diebold, we study the dynamics of the U.S. Treasury yield term structure by applying the Nelson-Siegel models introduced in Diebold and Li (2006) and its arbitrage-free version developed by Christensen, Diebold, and Rudebusch (2007). We analyze in-sample by testing the risk-neutral restrictions, which establish the absence from arbitrage, on the Nelson-Siegel factors estimated under the physical measure. We thus compare the term structure modeling between the risk-neutral measure and the physical measure. Specifically, we show that the risk-neutral restrictions on the factor dynamics are well-satisfied, and those factors have the following properties: 1) the level factor is a unit-root process and does not affect the other two factors; 2) the slope and curvature factors are mean-reverting processes that revert at the same rate; 3) the curvature factor forecasts the slope factor, but not conversely. Moreover, we find that utilizing these restrictions can improve out-of-sample forecast performance.

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CHAPTER I

Introduction

Macroeconomists, financial economists, and market participants all have strong interests in modeling the dynamic evolutions of various financial instruments. Particularly, they have spent a great effort on interest rates and volatility. In this dissertation, I conduct a formal econometric analysis of dynamic term structure models for U.S. Treasury bond yields, corporate bond yields, and equity option implied volatilities. In the first chapter, titled "Option Implied Volatilities and Corporate Bond Yields: A Dynamic Factor Approach," I estimate dynamic factors from the term structure of credit spreads and the term structure of equity option implied volatilities, and I provide a comprehensive characterization of the dynamic relationships among those credit spread factors and equity volatility factors. I find strong evidence that the volatility factors, especially the volatility level factor, Granger cause credit spread levels, confirming the theoretical predictions of Merton (1974) in a significantly richer and more nuanced environment than previously achieved. Simultaneously, I also find evidence of reverse Granger causality from credit spreads to equity volatility, operating through the slope factors, consistent with the market microstructure literature such as Fleming and Remolona (1999a, 1999b), which finds that price discovery often happens first in bond markets. Hence my results extend and unify both the corporate bond pricing and market microstructure literatures, deepening our understanding of stock and bond market interaction and suggesting profitable trading strategies.

The second chapter, which is a joint work with Francis X. Diebold, is titled "Yield Curve Modeling in Risk-neutral vs. Physical Environments." We study the dynamics of the U.S. Treasury yield term structure by applying the Nelson-Siegel models introduced in Diebold and Li (2006) and its arbitrage-free version developed by Christensen, Diebold, and Rudebusch (2007). We analyze in-sample by testing the risk-neutral restrictions, which establish the absence from arbitrage, on the Nelson-Siegel factors estimated under the physical measure. We thus compare the term structure modeling between the risk-neutral measure and the physical measure. Specifically, we show that the risk-neutral restrictions on the factor dynamics are well-satisfied, and those factors have the following properties: 1) the level factor is a unit-root process and does not affect the other two factors; 2) the slope and curvature factors are mean-reverting processes that revert at the same rate; 3) the curvature factor forecasts the slope factor, but not conversely. Moreover, we find that utilizing these restrictions can improve out-of-sample forecast performance.

CHAPTER II

Option Implied Volatilities and Corporate Bond Yields: A Dynamic Factor Approach

In the celebrated model of Merton (1974), corporate debt and equity represent alternative claims on a firm's assets. The two securities' common dependence on the firm's asset value implies that expected credit spreads are driven (at least in part) by equity volatility. This is so because, other things equal, high equity volatility increases the likelihood of a drop in equity value large enough to trigger default. This is especially interesting and intriguing as it links conditional *means* in *bond* markets to conditional *variances* in *stock* markets. This link is displayed in Figure 2.1, which shows that ten-year spreads of Aa-rated bonds are strongly correlated with one-year implied volatility.

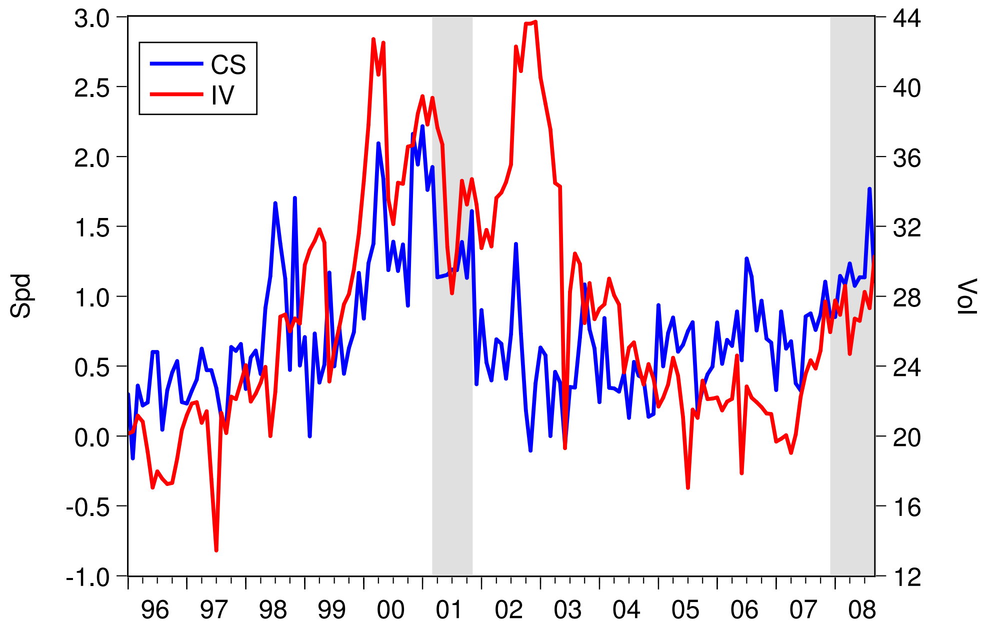
Most literature thus far has related individual credit spreads (on five-year bonds, say) to individual equity volatilities, as for example in Campbell and Taksler (2003). More generally, however, the entire term structure of credit spreads should be linked to the term structure of volatilities, a conjecture that is explored in this chapter.

Despite the fact that both the empirics and the theory of corporate bond pricing suggest that equity volatility affects credit spreads, market microstructure studies suggest that bond markets affect equity volatility. Using high-frequency intraday data, these studies document news announcements' effects on stock and bond markets and show that in terms of price discovery, bond markets respond more, and more *quickly*, than stock markets; see Fleming and Remolona (1999a, 1999b), for example. Hence, I also analyze whether the term structure of volatilities is affected by the term structure of credit spreads. In fact, I examine comprehensively the dynamic interaction between volatility and credit spread term structures.

Using portfolios of investment-grade bonds and equity options from nonfinancial firms with homogeneous credit ratings, I estimate factors from the two term structures via the Dynamic Nelson-Siegel (DNS) model (Diebold and Li, 2006) and uncover predictive relationships among those credit spread factors and equity volatility factors using a vector-autoregression (VAR) analysis. Precisely, I find strong evidence that the volatility factors, especially the volatility level factor, Granger cause credit spread levels, confirming the theoretical predictions of Merton (1974) in a significantly richer and more nuanced environment than previously achieved. Simultaneously, I also find evidence of reverse Granger causality from credit spreads to equity volatility, operating through the slope factors, consistent with the market microstructure literature such as Fleming and Remolona (1999a, 1999b), which finds that price discovery often happens first in bond markets.

This chapter relates to earlier work in both corporate bond pricing and price

Figure 2.1: Ten-year corporate spreads vs. one-year implied volatility



Note: Monthly credit spreads and implied volatility comparisons, 1996:01-2008:09. Monthly ten-year corporate spreads to the Treasury of Aa-rated zero coupon bonds are plotted against the one-year implied volatility of the same rating group. The shaded regions are the official recession periods according to the NBER.

discovery, but at least three features differentiate my analysis from previous results along important dimensions. These include the focus on the interaction between two term structures, the focus on implied volatility instead of realized volatility, and the length and breadth of the options and bond data sample. These will be discussed briefly in turn.

First, I focus on term structure interactions. Few studies thus far examine the impact between the two term structures. Most studies concentrate on a single credit spread or volatility or a few spreads or volatilities from one part of the maturity spectrum. In particular, the literature still lacks an analysis of the interaction between the whole curves. Even if studies analyze the term structure effect, the literature seems to focus on just one term structure. For example, Fleming and Remolona (1999c) study the announcements' effects on the government yield curve. Moreover, so far there have been no studies on the announcements' effects on the term structure of volatilities. This chapter is the first attempt to address the dynamic relationship between two term structures, and my analysis extend previous research into a much richer environment.

Second, I use option implied volatility as a measure of equity volatility. One reason that implied volatility is selected instead of realized volatility is that implied volatility is forward-looking, whereas realized volatility, widely used in many previous studies, including Campbell and Taksler (2003), is backward looking. Since the bond yield is forward looking, it is crucial to retain the forward-looking property for both markets. Moreover, in an efficient market the option price incorporates all available relevant information, including past returns. Therefore, implied volatility is selected for the analysis.

Third, the dataset used in this chapter spans a comparatively long sample period, including both recessions and financial crises, and uses the recently developed model-free implied volatility approach due to its ability to obtain volatility directly from a whole set of option prices. Instead of applying the Black-Scholes formula on at-the-money options, the model-free approach aggregates across all available strike prices, which resolves the skewness problem in the strike dimension of equity options. Furthermore, I establish the term structures of implied volatilities using portfolios of firms with homogeneous credit ratings. Most previous research on the term structure of implied volatilities that applies the model-free approach focuses on index options such as the S&P 500. I undertake the analysis of the term structure of option implied volatilities by going beyond the overall index options to portfolio levels grouped by firms' credit ratings.

Notwithstanding the improvements obtained through the above considerations, my results not only are consistent with existing work in both corporate bond pricing and price discovery, but also extend findings from both the corporate bond pricing and market microstructure literatures into a much richer and more nuanced environment. Hence my results unify both literatures, deepening our understanding of stock and bond market interaction. Moreover, there is a simple explanation for the direction of the predictive relations between the level factors and the slope factors in the two term structures. For the level interaction, the predictive relation is from the volatility to the spread. Because the options market focuses on the short term (less than 2 years), macroeconomic shocks can affect the entire maturity spectrum. As a result, the level movement in the options market is apparent first, and the spread level updates as the risk level has changed. For the slope interaction, however, it is the reverse, i.e. the direction is from the spread to the volatility. Long-term bonds are more sensitive to macroeconomic shocks than short-term bonds, and thus the spread slope movement is apparent first. As the bond market updates market prices of risk into the future, the options market adjust accordingly. In addition, my results also suggest profitable trading strategies to explore the predictable relationships.

The study proceeds as follows. Subsection 2.1 provides a survey on both the corporate bond pricing literature and the market microstructure and price discovery literature. Subsection 2.2 describes the bond and options data. Subsection 2.3 states the modeling approach of the two term structures and describes the properties of each estimated factor. Subsection 2.4 characterizes the dynamic relationship and explores the possible reasons for such interaction. This chapter concludes in subsection 2.5.

2.1 Research questions and related literature

Before turning to the analysis, I briefly review the literature related to the following research questions to be addressed:

a) Does the term structure of option implied volatilities impact the term structure of credit spreads?

b) Does the term structure of credit spreads impact the term structure of volatilities?

c) What is the dynamic relationship between bond and equity markets?

Research on question a) deals with corporate bond pricing studies, whereas question b) and c) are related to the market microstructure and price discovery literature.

Corporate bond pricing

The theoretical literature on the pricing of corporate bonds distinguishes between structural and reduced-form models. In structural models, a firm is assumed to default when the value of its liabilities exceeds its asset value, and then bondholders assume control of the firm in exchange for its residual value. Black and Scholes (1973) and Merton (1974) are some of the classic papers in this area. Huang and Huang (2003) analyze a wide range of structural firm value models and show that these models typically explain only 20% to 30% of observed credit spreads, which has emerged as the credit spread puzzle.

The structural models move in an equilibrium framework, where prices and default probabilities are endogenous to the models, and the market value of assets and their volatility are exogenous. Reduced-form models, by contrast, assume an exogenous stochastic process for the default probability and the recovery rate, and use prices as input. These models can allow for premia to compensate investors for illiquidity and systematic credit risk, and they can be fitted econometrically to corporate yield data (See, for example, Jarrow and Turnbull (1995), Duffie and Singleton (1997, 1999)). The flexibility of the reduced-form approach allows default risk to play a greater role in corporate bond pricing.

Recently, there have been several less structured econometric studies on corporate bond pricing. Collin-Dufresne et al. (2001) find that a single unobserved factor, common to all corporate bonds, drives most variation in credit spread changes. Kwan (1996) shows that changes in a firm's stock price are negatively correlated with contemporaneous and future changes in the yields of its bonds. Campbell and Taksler (2003) show a strong effect of realized equity volatility on individual credit spreads.

My study is closely related to that of Campbell and Taksler (2003) and undertakes a less structured econometric approach. It explores whether the entire term structure of credit spreads is affected by the entire term structure of option implied volatilities.

Market microstructure and price discovery

Market microstructure studies, using high-frequency intraday price data, are able to analyze price movements in relation to particular announcement effects. Fleming and Remolona (1999a, 1999b) analyze the intraday Treasury bond market and show that the largest price movements stem from the arrival of news announcements. They also conclude that bond markets are more responsive than stock markets.[1] Andersen, Bollerslev, Diebold, and Vega (2007) analyze stock, bond, and foreign exchange markets across multiple countries and qualify earlier work suggesting that bond markets react most strongly to macroeconomic news.

The existing literature on price discovery focuses on firm level information

[1]From a theoretical perspective, the link between asset prices and economic news tends to be more straightforward for the bond market than for the stock market. An upward revision of expected real activity, for example, raises expected cash flows for stocks while raising discount rates for both stocks and bonds. The effect on bond prices is clearly negative, while the effect on stock is ambiguous, depending on whether the cash flow effect dominates the discount rate effect.

flow between the credit default swaps (CDS), bond, equity, and equity options markets. Briefly all areas of the literature are reviewed here.

Longstaff, Mithal, and Neis (2005) show that among the CDS spread changes, corporate bond spreads, and stock returns, both the stock and CDS markets lead the corporate bond market, which supports the finding that information seems to flow first into stock and credit derivative markets and then into the corporate bond market. Norden and Weber (2004) look at the comovement of the CDS, bond, and stock markets and find that the stock market generally leads the CDS and bond markets, that the CDS market is more responsive to the stock market than to the bond market, and that the CDS market plays a more important role in price discovery than the bond market does.

For the lead-lag relationships between equity and equity options at the firm level, Chakravarty, Gulen, and Mayhew (2004) claim that option trading contributes to price discovery in the underlying stock market. Informed traders trade in both stocks and options, suggesting an important informational role for options. Easley, O'Hara, and Srinivas (1998) and Pan and Poteshman (2006) find that signed trading volume in the options market can help forecast future stock returns. However, when looking at the equity options and CDS markets together, Berndt and Ostrovnaya (2008) conclude that options prices reveal information about forthcoming adverse events at least as early as do credit spreads.

Given the monthly data used in this study, it is not possible to identify the effect of particular news announcement, but I am able to analyze the comovement between the two term structures. In addition to testing the impact of the term structure of volatilities on the term structure of spreads, I also explore the reverse impact, i.e., the dynamic impact of the spread term structure on the volatility term structure. Thus, I present a complete characterization of the dynamics between the bond and stock markets.

2.2 The data

I use data on nonfinancial firms' prices of corporate bond transactions in conjunction with their equity option prices. This section describes their source, construction methods, and summary statistics.

2.2.1 Data description

The corporate bond data are retrieved from the Fixed Income Securities Database (FISD) and National Association of Insurance Commissioners (NAIC) transactions data for the period spanning January 1996 through September 2008. The FISD contains bond-specific information such as issue date, bond features, bond ratings, coupon rate, and payment frequency. The NAIC database[2] consists of all transactions by life insurance companies, property and casualty insurance companies, and health maintenance organizations (HMOs) as distributed by Warga (2000), and hence the transaction data contain details such as trade date, volume, and bond price (stated as a percentage of the bond face value). End-of-month bond prices (i.e., the price at the last transaction of the month) are collected for all fixed-rate U.S. dollar-denominated bonds.

[2]This database is an alternative to the no longer available Warga (1998) database used by Duffee (1998), Blume, Lim, and MacKinlay (1998), Elton et al. (2000, 2001), Hecht (2000), and Collin-Dufresne et al. (2001).

A limitation of the NAIC is that it only contains data on transactions conducted by U.S. insurance companies, and thus, this constraint may reduce the generality of the results since insurance companies tend to focus on long-term and investment-grade bonds. However, three comments are warranted:

- First, according to the Flow of Funds accounts published by the Federal Reserve Board, insurance companies hold between 30% and 40% of corporate bonds.[3] Therefore, the NAIC should be adequately representative of entire credit market.

- Second, bond prices shown in the data are not believed to be biased. As long as the corporate bond market is competitive, the transaction prices are arbitrage-free and reflect all publicly available information in an unbiased manner. Therefore, the transaction prices do not reflect any bias to institutional peculiarities of the insurance industry.

- Finally, the NAIC data are essentially the only source of bond-exchange transaction data that dates back to the 1990s.[4]

Yield differences between investment- and non-investment-grade bonds are quite significant. Insurance companies are often limited or altogether prohibited from purchasing non-investment-grade debt. Furthermore, the National Association of Insurance Commissioners' Securities Valuation Office requires a modest reserve ratio of 1% for Aaa-rated bonds and 2% for Baa or better-rated bonds,

[3]Other important holders include foreign residents (15% to 20%), household (15%), pension and retirement funds (15%), mutual funds (5% to 10%), and commercial banks (5%).

[4]Introduced in 2002, TRACE consolidates transaction data for all corporate bonds. One big weakness of that dataset is that it spans a much shorter time period, since it starts only in 2002.

but the ratio jumps to 5% for Ba, which is considered a non-investment-grade debt. The spread on non-investment-grade debt thus appears unattractive to insurance companies because yields are determined by the market where no reserve requirement is in place. Therefore, because insurance companies are not attracted to this spread on non-investment-grade debt, these transactions are not likely to be representative of the general market.

The corporate bond dataset covers 5,981 issuers, rated Aa, A, Baa by Moody's[5] and divided into two sector groups: industrial and financial. Out of 5,981 issuers, 3,972 are industrial, and the rest are financial.

I am interested in explaining credit spreads, the difference between corporate yields and Treasury yields, whose data are also important. The Treasury yields used in the study are the zero-coupon yields constructed by the method described in Gürkaynak, Sack, and Wright (2006).[6] They fit a Svensson (1994)[7] model,

$$y_t(\tau) = \beta_0 + \beta_1 \left(\frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right) + \beta_2 \left(\frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right) + \beta_2 \left(\frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right),$$

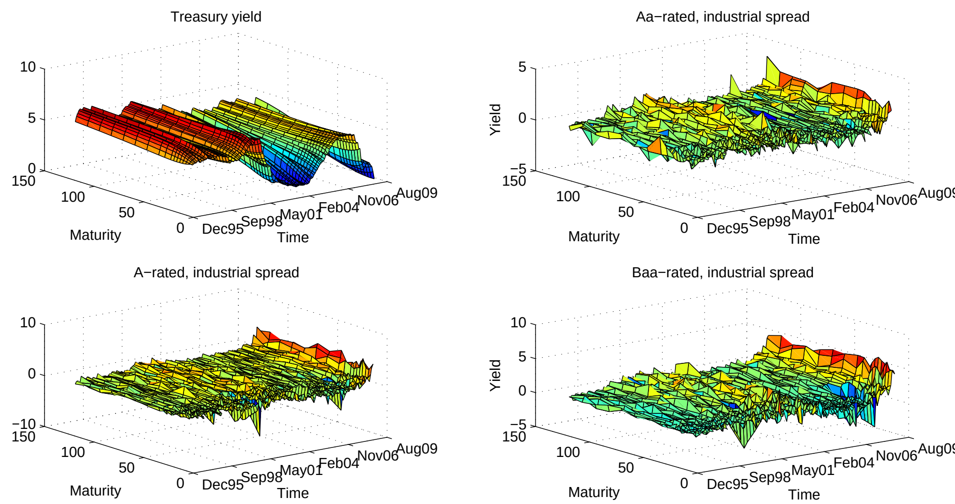
to a large pool of underlying off-the-run Treasury bonds. Thus, fitted values of the four factors $(\beta_0(t), \beta_1(t), \beta_2(t), \beta_3(t))$ and the two parameters $(\lambda_1(t), \lambda_2(t))$ are estimated each day. As demonstrated by Gürkaynak, Sack, Wright (2006), this model fits the underlying bonds extremely well, which implies that zero-coupon yields derived from these factors and parameters constitute a good approximation

[5]Notice Aaa-rated bonds are not considered in the sample. Campbell and Taksler (2003) and Elton et al. (2000, 2001) also do not include Aaa-rated bonds.

[6]The Federal Reserve Board updates the factors and parameters of this method daily, and the data can be downloaded from the Board's website.

[7]Svensson model is an extension of the Nelson and Siegel (1987) model.

Figure 2.2: Term structure of the Treasury yield curves and credit spreads

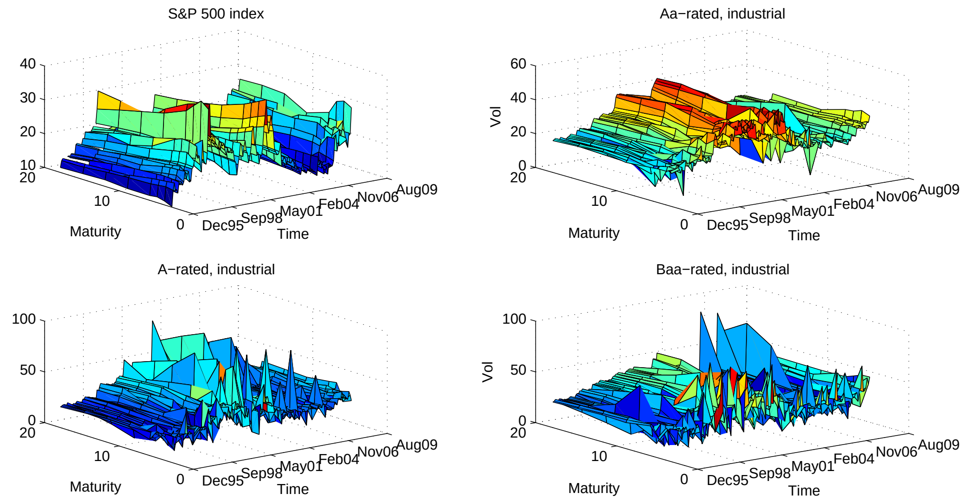


Note: Government yield curves and credit spread curves for Aa-, A-, and Baa-rated firms, 1996.01-2008.09. The sample consists of monthly yield data from January 1996 to September 2008 at maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months.

to the true Treasury zero-coupon yield curve. If this approach introduces any bias, the credit spreads across all ratings will be biased in the same direction.

The data used for option implied volatility came from OptionMetrics' IVY database. IVY is a comprehensive database of historical prices for the equity options market and includes daily data of all U.S.-listed equity options for the time period corresponding to the corporate bond data. Only those options from the firms existing in the bond dataset are downloaded. Out of 5,981 firms in the bond database, 1,122 firms have traded options, of which 896 are industrial and 226 are financial. The index option data for the S&P 500 are also obtained for illustration purposes.

Figure 2.3: Term structure of option implied volatilities of S&P 500 and Aa, A and Baa-rated firms



Note: S&P 500 index option implied volatility curves and option implied volatility curves for Aa-, A-, and Baa-rated firms, 1996.01-2008.09. The sample consists of monthly yield data from January 1996 to September 2008 at maturities of 3, 4, 5, 6, 9, 12, 15, and 18 months.

In the final dataset of corporate bonds and equity options, there are 1,122 firms that have traded options and bonds with an investment-grade rating between Aa and Baa. Since 896 of them are industrial, the study will focus on industrial firms. The results for financial firms are similar and available upon request.

2.2.2 Data construction

2.2.2.1 Yield data

I exclude corporate bonds that have special features (callable, putable, and sinking fund options). For each rating group of each month, following Fama and Bliss' (1987) treatment of government bonds, the filtered corporate bond prices are converted into unsmoothed forward rates, which, in turn, are converted into unsmoothed zero yields.[8] In order to simplify the process of obtaining credit spreads, the corporate zero yields are linearly interpolated into fixed maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months for each rating group, where a month is defined as 30.4375 days.[9] For the government yields, the four factors and two parameters of the Svensson model for the corresponding periods are obtained from the Federal Reserve Board website.[10] The zero yields of the same fixed maturities are calculated by fitting the Svensson model with the four factors, two parameters, and maturities.

2.2.2.2 Volatility data

Based on option prices, strikes, and types (call or put), the implied volatility is calculated using the same method used by the new VIX, [11] which is a ticker symbol for the Chicago Board Options Exchange Volatility Index that measures the implied volatility of S&P 500 index options over the next 30-day period. Papers by Jiang and Tian (2003) and Carr and Wu (2009) show that

$$\sigma_T^2 \equiv \frac{1}{T} E_0^Q [V(o, T)], \tag{2.1}$$

[8]See the appendix for detailed discussion of the construction method.

[9]For corporate bond transactions, there are significantly fewer trades for maturity of less than one year, and the resulting bias is minimal, since the dynamic factors are going to fit the entire term structure.

[10]The web address is http://www.federalreserve.gov/pubs/feds/2006/index.html.

[11]For a detailed discussion of VIX, see Jiang and Tian (2003) and Carr and Wu (2009). The method is shown with an example on the Chicago Board Options Exchange's (CBOE) website, http://www.cboe.com/micro/vix/vixwhite.pdf.

Table 2.1: Descriptive statistics: industrial sector credit spreads

	3m	6m	9m	12m	18m	24m	30m	36m	48m	60m	72m	84m	96m	108m	120m
Aa															
Mean	0.52	0.55	0.57	0.47	0.50	0.49	0.47	0.58	0.60	0.61	0.66	0.74	0.76	0.75	0.82
Std	0.40	0.33	0.32	0.31	0.39	0.35	0.37	0.47	0.40	0.61	0.58	0.47	0.43	0.46	0.55
Min	-0.63	-0.43	-0.18	-0.51	-1.41	-0.82	-0.69	-0.76	-1.00	-3.26	-2.24	-0.73	-0.40	-1.37	-0.11
Max	1.64	1.63	1.56	1.26	1.72	1.60	1.36	2.57	1.82	2.09	2.15	1.88	2.08	2.22	2.67
$\rho(1)$	0.20	0.21	0.28	0.22	0.44	0.28	0.45	0.47	0.50	0.22	0.26	0.46	0.32	0.41	0.64
	(0.45)	(0.34)	(0.31)	(0.31)	(0.35)	(0.34)	(0.33)	(0.40)	(0.35)	(0.60)	(0.57)	(0.42)	(0.41)	(0.42)	(0.42)
A															
Mean	0.50	0.57	0.59	0.52	0.59	0.66	0.75	0.78	0.89	0.84	0.93	0.98	0.92	0.93	1.01
Std	0.94	0.78	0.63	0.66	0.70	0.59	0.53	0.51	0.53	0.42	0.45	0.46	0.51	0.46	0.50
Min	-5.17	-4.19	-3.16	-2.16	-3.06	-2.57	-1.10	-0.93	-0.26	-0.63	-0.10	-0.03	-0.60	-0.01	0.24
Max	2.52	2.26	2.04	2.62	2.34	2.36	2.87	2.20	2.75	1.98	2.43	2.45	2.18	2.59	2.56
$\rho(1)$	0.17	0.09	0.07	0.03	0.14	0.29	0.33	0.35	0.55	0.29	0.58	0.43	0.30	0.44	0.58
	(1.01)	(0.79)	(0.63)	(0.66)	(0.70)	(0.57)	(0.51)	(0.48)	(0.45)	(0.41)	(0.37)	(0.42)	(0.49)	(0.41)	(0.41)
Baa															
Mean	0.92	0.97	0.87	0.90	1.02	1.08	1.13	1.12	1.21	1.26	1.39	1.39	1.34	1.29	1.37
Std	0.87	0.67	0.52	0.58	0.76	0.75	0.70	0.62	0.63	0.63	0.58	0.68	0.58	0.63	0.61
Min	-2.32	-1.42	-0.74	-1.31	-2.93	-3.33	-2.13	-0.87	-0.04	-0.04	-0.29	0.15	-0.48	-1.28	-0.35
Max	3.89	4.10	2.48	2.50	3.95	3.75	3.70	3.44	3.17	3.50	3.21	3.35	2.67	3.66	3.11
$\rho(1)$	0.28	0.18	0.38	0.34	0.52	0.26	0.47	0.55	0.64	0.31	0.41	0.47	0.34	0.32	0.55
	(0.90)	(0.68)	(0.49)	(0.55)	(0.66)	(0.73)	(0.62)	(0.52)	(0.49)	(0.60)	(0.54)	(0.60)	(0.54)	(0.60)	(0.51)

Note: Descriptive statistics of industrial sector monthly credit spreads at different maturities. Standard errors of the AR(1) regression appear in parenthesis. The $\rho(1)$ indicates the first-order serial correlation coefficient. The sample period is 1996:01-2008:09.

which is the risk-neutral expectation of variance over the next 30 days under risk-neutral measure Q, and the value of the variance can be replicated by all available call and put options,

$$E_0^Q[V(0,T)] = 2e^{rT}\left(\int_L^F \frac{P(K,S_0,T)}{K^2}dK + \int_F^H \frac{C(K,S_0,T)}{K^2}dK\right). \tag{2.2}$$

Such an approach encompasses all strike prices that result in two-dimensional curves rather than three-dimensional surfaces. The discretized version of equation (2) under the physical measure used to construct the volatility of various horizons is:

$$\sigma_T^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2, \tag{2.3}$$

where

- σ_T: implied volatility ($VOL/100$)
- T: time to expiration
- F: forward price level derived from option prices
- K_i: strike price of ith out-of-money option; a call if $K_i > F$ and a put if $K_i < F$
- ΔK_i: interval between strike prices
- K_0: first strike below the forward price, F

- r: risk-free interest rate to expiration[12]

- $Q(K_i)$: the midpoint of the bid-ask spread for each option with strike K_i.

The forward level, F, is based on at-the-money option prices. The strike price of the at-the-money options is the price at which the difference between the call and put prices is smallest. Then the forward level is obtained via put and call parity,

$$F = \text{Strike Price} + e^{rT} \times (\text{Call Price} - \text{Put Price}), \tag{2.4}$$

and K_0 is the strike price immediately below the forward level F.

I adopt this approach for individual equity options.[13] This approach of implied volatility construction has several advantages over applying the Black-Scholes formula directly on at-the-money options, widely used in previous studies. First, the implied volatility does not depend on any particular option pricing model but is derived directly from option prices. Second, a wider range of strike prices is used for the construction, and thus, the skewness issue is resolved. Finally, at each point in time, there is a two-dimensional volatility curve rather than a three-dimensional volatility surface to be modeled, as in Haerdle and Benko (2005) and Haerdle et al. (2005).

[12] In the construction process, the Treasury zero-coupon yields are considered to be the risk-free rate for corresponding expiration.

[13] The option exercise style could potentially negatively affect the implied volatility calculation. The VIX is based on European index options, but individual equity options are mostly American. Since Jiang and Tian (2003) develop the model-free implied volatility based on European options, potentially it might be problematic to apply it to American options. However, for non-dividend paying stock, it is never optimal to exercise American call options before expiration, so they are equivalent to European calls. Although American put options do not have this property, studies have shown that using the Black-Scholes formula to obtain implied volatility yields minimal errors. The VIX method aggregates both calls and puts across strike prices. Therefore, applying the VIX method to equity options should also result in negligible errors.

After calculating VOL for each firm on each trading day, the last observed VOL in each month is considered to be the volatility of that month. The firms are grouped by the corresponding rating of their debt; i.e., a firm that has an A-rated bond is considered to be an A-rated firm. In each month, the latest rating of the bond issue is assumed to be the rating of the issuer. So if there is a change to one of the bonds of a firm, the updated rating is the firm's overall new rating. For some firms with equity option transactions that occur before their debt issuance, they presumably receive the rating when the debt is issued. Given each rating category, the volatilities are interpolated to the fixed expirations of 3, 4, 5, 6, 9, 12, 15, 18 months, where a month is again defined as 30.4375 days. This facilitates comparisons across ratings and also across the two term structures.

2.2.3 Summary statistics

Figure 2.2 provides three-dimensional plots of the government yields and yield spreads for each rating category. For the Treasury yields, the large amount of temporal variation in the level is visually apparent. For credit spreads, the temporal variation in the level becomes more sudden and volatile when moving from the higher rating groups to the lower ones. The slope and curvature variation for the government yields is still quite apparent, but less obvious for the credit spreads. Table 2.1 presents the descriptive statistics of the credit spreads. For each rating group, the credit spreads are, on average, upward sloping, more volatile in the short end than in the long end, and less persistent for the short maturity than for the long maturity. For a given maturity, the credit spreads are, on average, higher, more volatile and less persistent for the lower rated groups than for the

Table 2.2: Descriptive statistics: industrial sector option implied volatility

	3m	4m	5m	6m	9m	12m	15m	18m
*A*a								
Mean	29.786	28.666	27.545	26.196	26.066	27.526	27.742	27.312
Stdev	7.614	7.360	7.261	7.244	7.320	7.197	7.116	6.997
Min	5.597	3.105	11.005	7.260	8.627	13.441	16.938	14.388
Max	47.373	45.217	45.575	44.446	43.933	43.713	45.651	44.842
$\rho(1)$	0.807	0.842	0.844	0.747	0.751	0.895	0.942	0.960
	(4.51)	(3.96)	(3.89)	(4.83)	(4.84)	(3.20)	(2.35)	(1.91)
A								
Mean	30.800	31.817	29.224	26.276	28.705	31.524	31.875	31.616
Stdev	8.575	14.208	10.372	9.795	11.490	12.068	10.121	10.792
Min	10.701	7.946	5.748	9.138	8.014	12.754	17.172	8.955
Max	59.650	97.540	84.748	79.458	89.982	88.270	78.523	88.120
$\rho(1)$	0.480	0.337	0.366	0.528	0.470	0.633	0.680	0.600
	(7.45)	(13.46)	(9.72)	(8.35)	(10.20)	(9.38)	(7.42)	(8.65)
*B*aa								
Mean	37.238	34.566	32.040	29.535	30.621	33.165	33.976	34.230
Stdev	13.526	14.255	10.897	13.166	9.956	8.140	7.347	10.014
Min	12.351	8.688	7.578	6.290	9.329	13.756	19.283	9.566
Max	83.127	88.282	69.519	70.898	61.869	55.654	56.010	88.882
$\rho(1)$	0.426	0.239	0.329	0.236	0.312	0.676	0.811	0.245
	(12.27)	(13.95)	(10.31)	(12.89)	(9.52)	(6.02)	(4.27)	(9.75)

Note: Descriptive statistics of industrial sector option implied volatility at different maturities. Standard errors of the AR(1) regression appear in parenthesis. The $\rho(1)$ indicates the first-order serial correlation coefficient. The sample period is 1996:01-2008:09.

higher rated ones.

Table 2.2 presents the descriptive statistics of option implied volatilities. For any given rating, the typical implied volatility curve is downward sloping or close to flat. At the short end, the volatilities are usually higher, since the options are about to expire. The long-term implied volatilities possess less fluctuation and are more persistent than the short-term ones. Figure 2.3 provides three-dimensional plots of option implied volatilities for the S&P 500 index and firms in this study. For the S&P 500 index, its option implied volatilities have a substantial amount of temporal variation in the level. For the slope, the amount is less but still apparent. Aa-rated firms seem to follow the overall shape of the index, but it is less apparent for A- and Baa-rated firms. There are spikes in those two groups, especially at the short end.

Figure 2.7 presents a time series plot of 5-year credit spreads for Baa-rated bonds. The credit spreads widen around the third quarter of 2000 and at the end of 2001 and rise sharply in 2008. It seems that the spreads are sensitive to economic contractions such as the ones in 2001 and 2008.

Unlike credit spreads, which are mostly correlated with business cycle conditions, implied volatilities are also affected by other shocks, such as corporate scandals and financial crises. In Figure 2.6, the 3-month and 6-month implied volatilities from Aa-rated firms are plotted against the VIX, which is the 30-day implied volatility of the S&P 500 index options. The firms in this study must have both bonds and options traded and are grouped into corresponding ratings, but these firms are not necessarily part of the S&P 500, so they could be more volatile than the overall index. The implied volatilities of Aa-rated firms still rise sharply during market turbulence such as the 1998 Asian financial crisis and 2002-03 corporate scandals. For firms that are rated lower than Aa, their volatilities have a more dramatic variation during those periods. The empirical observations of implied volatilities are that they react quickly to shocks but do not always move correspondingly with economic expansions and recessions, as credit spreads do.

Similar to credit spreads, implied volatilities rise as the rating moves from high to low for any given expiration. This indicates that ratings affect volatilities, which, of course, affect credit spreads; however, *WITHIN* a rating group, the linkage between volatilities and spreads can be weak. In addition, the focus of this study is on the aggregate level analysis of the bond and equity options markets. Therefore, it is appealing to analyze the dynamics by pooling across all ratings.

2.3 Modeling the term structures

In order to capture the variation of the entire term structure, the study extracts dynamic factors from each term structure. This section describes how this is achieved and illustrates the properties of those estimated factors.

2.3.1 Dynamic Nelson-Siegel (DNS) model

The Nelson and Siegel (1987) framework that Diebold and Li (2006) made dynamic,

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \varepsilon_t, \qquad (2.5)$$

is the workhorse of term structure modeling. β_1 changes all yields uniformly and it can be called the level factor (L_t); β_2 loads the short rate more heavily, its loading decays to zero as maturity lengthens, and it can be called the slope factor (S_t); β_3 loads the medium term more heavily, its loading starts at zero and decays back to zero as maturity increases, and it can be called the curvature factor (C_t). λ determines the maturity at which the medium-term (or the curvature factor) loading achieves its maximum.

I apply the Dynamic Nelson-Siegel (DNS) framework to distill the term structures of credit spreads and volatilities into sets of level, slope, and curvature factors for each rating group. The aim of a factor approach is to capture and model the dynamics of spreads and volatilities. Going back at least to Litterman and Scheinkmann (1991), it is an established fact that close to 99.9% of the variation in Treasury yields can be explained by three latent variables referred to as a level, a slope, and a curvature factor. Principal analysis of the volatility curve shows that three factors can explain 81%, 84%, and 96% for each of the three rating categories. Thus, Thus, the DNS model provides a robust yet flexible method to decompose both credit spreads and volatilities while having the compelling interpretation of level, slope and curvature factors.[14]

[14]The DNS model not only is able to fit curves of various shapes, including upward sloping, downward sloping, humped, and inverted humped, but also has a theoretical foundation for yield-curve modeling. Krippner (2009) shows that the Nelson-Siegel class of models is effectively a reduced-form representation of the affine model in Dai and Singleton (2002), which makes the DNS model both a theoretically and empirically appealing model for the yield curve.

2.3.2 Corporate bond modeling

Credit spreads, the corresponding difference between the Treasury yield ($y^G(\tau)$) and corporate yield ($y_t^j(\tau)$) at given maturity τ, where j can be Aa, A or Baa rated, are modeled following the DNS approach, i.e.

$$\begin{aligned} CS_t^j(\tau) &= y_t^j(\tau) - y_t^G(\tau) \\ &= L_{s,t}^j + S_{s,t}^j\left(\frac{1-e^{-\lambda_s\tau}}{\lambda_s\tau}\right) + C_{s,t}^j\left(\frac{1-e^{-\lambda_s\tau}}{\lambda_s\tau} - e^{-\lambda_s\tau}\right) + \varepsilon_{s,t}. \end{aligned}$$

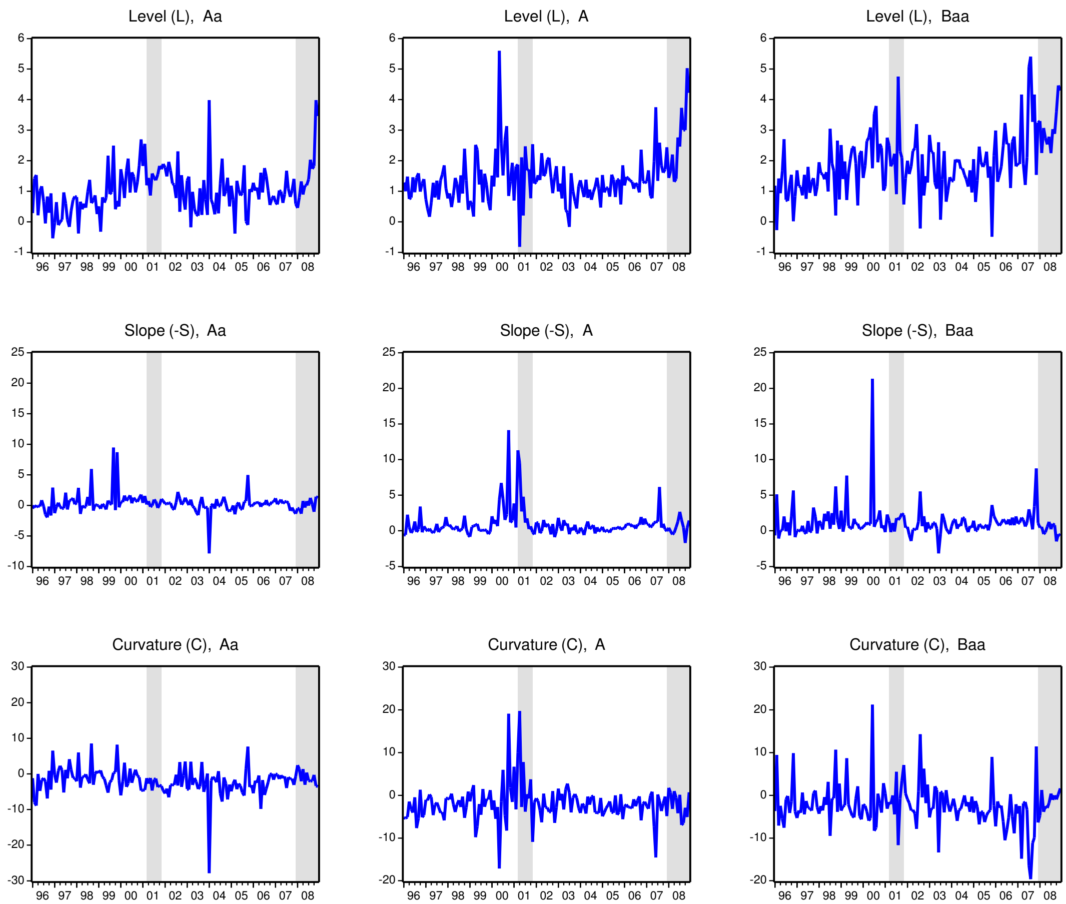
A set of three factors (L_t^j, S_t^j, and C_t^j) with a parameter λ_s is to be estimated for rating j. Following the recommendation by Diebold and Li (2006), the λ_s value is fixed at 0.0609. Two- or three-year maturities are commonly used as the maturity at which the curvature factor achieves a maximum. The average is 30 months, so the corresponding λ_s value that maximizes the curvature factor loading at 30 months is 0.0609. Therefore, the factors can be estimated via ordinary least squares (OLS).[15] Doing a two-step procedure like this enhances simplicity and numerical credibility and eliminates the potential problem of numerical optimizations.[16]

The descriptive statistics for the credit spread factors are in Table 2.3. From the autocorrelation perspective, the level factor is generally the most persistent,

[15]When λ is not preselected, a nonlinear estimation or a Kalman filter via a state space model can be performed. Moreover, λ is implicitly assumed to be the same for both Treasury and corporate bonds. Fitting DNS to the Treasury and corporate yield curves, respectively, shows that estimated λ_ts are not significantly different from each other.

[16]Another popular approach is to transform the system into a state-space representation and estimate λ and the factors via a Kalman filter. Dieobld, Rudebusch and Aruoba (2006) take that approach. However, the numerical optimization is challenging. The results using factors estimated via a Kalman filter are generally similar and available upon request.

Figure 2.4: Estimated factors of the credit spread term structure



Note: The figure plots estimated level, slope, and curvature factors of the credit spreads for Aa-, A-, and Baa-rated firms between 1996:1 and 2008:09. The shaded regions are the official recession periods according to the NBER. These factors are estimated from the DNS model,

$$
\begin{aligned}
CS_t^j(\tau) &= y_t^j(\tau) - y_t^G(\tau) \\
&= L_{s,t}^j + S_{s,t}^j\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + C_{s,t}^j\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right),
\end{aligned}
$$

The λ is fixed at 0.0609, which corresponds to maximizing the curvature factor loading at 30 months, to facilitate the estimation process. It is important to note that the slope factor plotted is the negative of the estimated $S_{s,t}$, i.e., $Slope_t = -S_{s,t}$ because in the DNS model a negative value of $S_{s,t}$ indicates an upward-sloping curve.

Table 2.3: Descriptive statistics, estimated credit spread factors

	$\hat{L}_t$			$\hat{S}_t$			$\hat{C}_t$		
	Mean	$\rho(1)$	ADF	Mean	$\rho(1)$	ADF	Mean	$\rho(1)$	ADF
Aa	1.165	0.333	-3.61	-0.383	0.270	-6.30	-2.723	0.007	-11.28
	(0.716)			(1.651)			(3.870)		
A	1.472	0.316	-9.79	-0.961	0.356	-2.26	-2.558	0.055	-4.61
	(0.717)			(1.990)			(4.191)		
Baa	1.918	0.297	-10.03	-1.202	0.006	-11.79	-2.644	-0.077	-12.28
	(0.833)			(2.243)			(4.686)		

Note: Fit the three-factor Nelson-Siegel model,

$$CS_t^j(\tau) = L_{s,t}^j + S_{s,t}^j \left(\frac{1 - e^{-\lambda_s \tau}}{\lambda_s \tau} \right) + C_{s,t}^j \left(\frac{1 - e^{-\lambda_s \tau}}{\lambda_s \tau} - e^{-\lambda_s \tau} \right) + \varepsilon_{s,t},$$

for Aa, A, and Baa ratings, respectively, using monthly yield data 1996:01-2008:09 with λ_s fixed at 0.0609. This table presents descriptive statistics for the three estimated factors ($\hat{L}_t$, $\hat{S}_t$, and $\hat{C}_t$). The ADF column contains Augmented Dickey-Fuller (ADF) unit root test statistics and the column to the left contains sample autocorrelations at a displacement of one month. The standard errors are reported in parenthesis.

except for the A-rated group where the slope is more persistent than the level. For the level factors, the degree of persistent decreases as ratings deteriorate, and the variations increase. On average, the slopes are more negative as ratings worsen. Since the negative slope factor indicates an upward-sloping curve, the actual credit spread curves are usually more upward-sloping for the lower-rated group, which compensates for the greater risk of a long-maturity lower-rated bond. Table 2.4 describes the in-sample fit. The DNS model fits remarkably well, especially on the long end of the curve. The residual sample autocorrelation indicates that the pricing errors are not persistent. Principal component analysis of the residuals reveals no common factors dominant in the fitting errors.

2.3.3 Volatility modeling

Since the DNS model has been shown to be flexible in modeling curves of various shapes and its estimated factors have an intuitive interpretation as level, slope, and curvature, the same framework is also used to fit implied volatility curves. For firms with rating j, the implied volatility [17] (vol_t) of maturity τ is modeled as,

$$VOL_t^j(\tau) = L_{v,t}^j + S_{v,t}^j \left(\frac{1 - e^{-\lambda_v \tau}}{\lambda_v \tau} \right) + C_{v,t}^j \left(\frac{1 - e^{-\lambda_v \tau}}{\lambda_v \tau} - e^{-\lambda_v \tau} \right) + \varepsilon_{v,t},$$

and the same level, slope, and curvature factor interpretations are preserved.

Estimating the parameters $\theta = \{\lambda_v, L_{v,t}, S_{v,t}, C_{v,t}\}$ by nonlinear least squares is definitely feasible. However, λ_v is also being pre-specified because OLS has

[17] One can also use other methods, such as principal components, to extract factors, and the results are similar.

Table 2.4: Descriptive statistics, credit spread curve residuals

	Aa					*A*					*Baa*				
Maturity	Mean	Std.	MAE	RMSE	$\hat{\rho}(1)$	Mean	Std.	MAE	RMSE	$\hat{\rho}(1)$	Mean	Std.	MAE	RMSE	$\hat{\rho}(1)$
3m	0.165	0.531	0.551	0.552	0.23	-0.123	1.970	1.036	1.966	0.07	-0.348	2.237	1.166	2.254	0.03
6m	-0.091	0.850	0.518	0.851	0.07	-0.255	1.394	0.761	1.413	0.02	-0.427	1.556	0.864	1.608	-0.02
9m	-0.145	0.782	0.507	0.793	-0.01	-0.302	1.141	0.700	1.177	0.09	-0.323	1.135	0.660	1.177	-0.09
12m	-0.107	0.721	0.511	0.726	-0.02	-0.253	1.005	0.722	1.033	0.04	-0.366	0.892	0.697	0.961	-0.08
15m	-0.174	0.638	0.529	0.659	0.05	-0.244	0.893	0.662	0.924	0.06	-0.362	0.755	0.698	0.835	0.13
18m	-0.236	0.546	0.497	0.594	-0.05	-0.306	0.766	0.643	0.823	0.02	-0.429	0.712	0.697	0.829	0.27
21m	-0.272	0.475	0.484	0.546	0.06	-0.321	0.699	0.647	0.767	-0.04	-0.407	0.555	0.611	0.687	0.25
24m	-0.247	0.450	0.455	0.512	0.16	-0.283	0.636	0.596	0.694	0.24	-0.424	0.603	0.643	0.735	0.20
30m	-0.252	0.399	0.451	0.471	0.04	-0.401	0.519	0.573	0.655	0.06	-0.425	0.620	0.667	0.750	0.22
36m	-0.338	0.441	0.524	0.555	0.06	-0.413	0.484	0.561	0.635	0.00	-0.417	0.474	0.602	0.630	-0.02
48m	-0.276	0.308	0.417	0.413	0.12	-0.438	0.435	0.576	0.616	0.11	-0.444	0.444	0.577	0.627	0.23
60m	-0.194	0.450	0.466	0.488	0.03	-0.265	0.409	0.483	0.486	0.05	-0.311	0.494	0.544	0.582	-0.11
72m	-0.184	0.444	0.434	0.480	0.12	-0.257	0.337	0.417	0.423	0.21	-0.323	0.481	0.535	0.578	0.20
84m	-0.189	0.373	0.441	0.417	0.25	-0.214	0.337	0.432	0.398	-0.07	-0.235	0.499	0.482	0.550	0.20
96m	-0.146	0.357	0.406	0.385	0.16	-0.065	0.365	0.408	0.369	-0.01	-0.090	0.514	0.483	0.520	0.17
108m	-0.073	0.379	0.407	0.385	0.24	0.003	0.345	0.403	0.344	0.13	0.027	0.552	0.499	0.551	0.09
120m	-0.077	0.395	0.430	0.401	0.17	0.003	0.407	0.443	0.405	-0.03	0.052	0.472	0.468	0.473	0.04

Note: The study fits the three factor model,

$$CS_t^j(\tau) = L_{s,t}^j + S_{s,t}^j \left(\frac{1 - e^{-\lambda_s \tau}}{\lambda_s \tau} \right) + C_{s,t}^j \left(\frac{1 - e^{-\lambda_s \tau}}{\lambda_s \tau} - e^{-\lambda_s \tau} \right) + \varepsilon_{s,t},$$

for Aa, A, and Baa ratings, respectively, using monthly spread data 1996:01-2008:09 with λ fixed at 0.0609. The table presents descriptive statistics for the corresponding residuals at various maturities. The last column in each rating contains residual sample autocorrelations at a displacement of one month.

Table 2.5: Descriptive statistics, estimated implied volatility factors

	$\hat{L}_t$			$\hat{S}_t$			$\hat{C}_t$		
	Mean	$\rho(1)$	ADF	Mean	$\rho(1)$	ADF	Mean	$\rho(1)$	ADF
Aa	28.719	0.873	-2.13	25.709	0.065	-5.01	-37.123	0.119	-10.06
	(7.927)			(24.898)			(38.003)		
A	35.065	0.513	-2.52	32.242	0.065	-10.70	-62.990	0.169	-9.61
	(14.447)			(59.629)			(109.825)		
Baa	38.066	0.167	-9.64	58.561	0.060	-10.73	-95.828	-0.023	-11.65
	(12.467)			(84.728)			(140.806)		

Note: Fit the three-factor Nelson-Siegel model,

$$VOL_t^j(\tau) = L_{v,t}^j + S_{v,t}^j\left(\frac{1-e^{-\lambda_v\tau}}{\lambda_v\tau}\right) + C_{v,t}^j\left(\frac{1-e^{-\lambda_v\tau}}{\lambda_v\tau} - e^{-\lambda_v\tau}\right) + \varepsilon_{v,t},$$

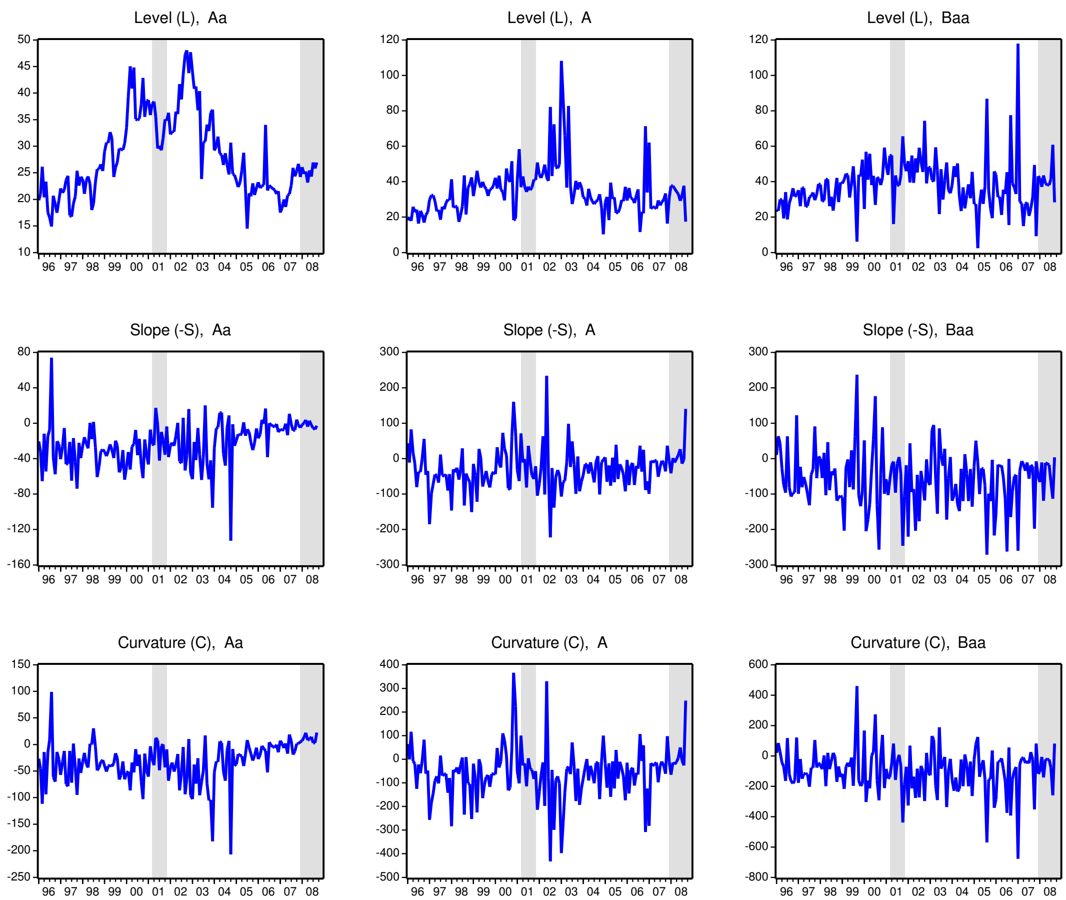
for Aa, A, and Baa ratings, respectively, using monthly volatility data for the period 1996:01-2008:09 with λ_v fixed at 0.56. This table presents descriptive statistics for the three estimated factors ($\hat{L}_t$, $\hat{S}_t$, and $\hat{C}_t$). The ADF column contains Augmented Dickey-Fuller (ADF) unit root test statistics and the column to the left contains sample autocorrelations at a displacement of one month. The standard errors are reported in parenthesis.

the edge in convenience and trustworthiness of the estimation results.[18] The prespecified value for λ_v is 0.56, which corresponds to the maximization of the loading on the medium-term factor at exactly 6 months.

Table 2.5 displays the descriptive statistics for the estimated factors of the term structure of the industrial sector's option implied volatility. The factors usually rise when moving from higher ratings to lower ones, and their variability also increases. As for the autocorrelations of the three factors, only the first factor is strongly autocorrelated, and the persistent level is greater for the higher rated

[18] Again we can transform it into a state-space model and estimate using a Kalman filter, and the estimated factors are similar to those from OLS. The results are available upon request.

Figure 2.5: Estimated factors of the option implied volatility term structure



Note: The figure plots estimated level, slope, and curvature factors of the option implied volatility for Aa-, A-, and Baa-rated firms between 1996:1 and 2008:09. The shaded regions are the official recession periods according to the NBER. These factors are estimated from the DNS model,

$$vol_t^j(\tau) = L_{v,t}^j + S_{v,t}^j \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + C_{v,t}^j \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right),$$

The λ is fixed at 0.56, which corresponds to maximizing the curvature loading at 30 months, to facilitate the estimation process. It is important to note that the slope factor plotted is the negative of the estimated $S_{v,t}$, i.e., $Slope_t = -S_{v,t}$ because in the DNS model a negative value of $S_{s,t}$ indicates an upward-sloping curve.

group. Augmented Dickey-Fuller tests suggest that $\hat{L}_v$ may have a unit root, whereas $\hat{S}_v$ and $\hat{C}_v$ do not have a unit root and are less persistent. The typical slope factors are positive, which indicate downward-sloping curves, of which the steepness is elevated as ratings deteriorate. Table 2.6 describes the in-sample fit. The fit of the DNS model on the volatility curve is good given that, for a particular maturity, the volatility is usually several times that of the spreads. The residual sample autocorrelation indicates that the pricing errors are not persistent. After checking the principal component of the residuals, there seems to be no dominant common factors affecting fitting errors.

2.3.4 Factor descriptive analysis

Principal component analysis of the data reveals that the first two factors (level and slope) can explain as much as 72% of the variation in credit spreads in the Aa-rated group and 94% of the variation in volatilities. For other rating groups, the percentages are slightly lower, but the level and slope factors can still explain significant portions of the variation in both term structures, and thus they are the focus of the intertemporal analysis. Representations of the two term structure factors are in Figures 2.4 and 2.5. Table 2.7 reports the cross-correlation between the credit spread level and the Auroba-Diebold-Scotti (ADS) index.[19]

The credit spread level factors, the most persistent ones, seem to follow the

[19]The ADS index, initiated by Auroba, Diebold and Scotti (2009), tracks real business conditions at high frequency by using the release of various economic indicators. The average value of the ADS index is zero. Bigger positive values indicate better than average conditions, whereas more negative values indicate worse than average conditions. The ADS index is updated as data on the underlying components are released and is maintained by the Federal Reserve Bank of Philadelphia.

Table 2.6: Descriptive statistics, volatility curve residuals

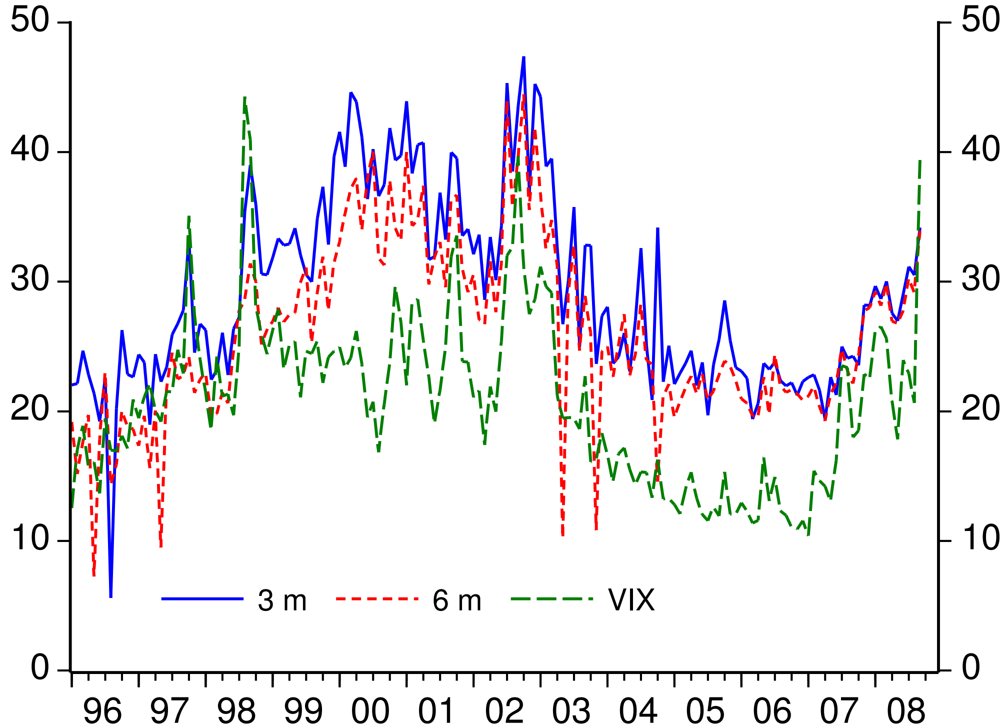
Maturity	3m	4m	5m	6m	9m	12m	15m	18m
Aa								
Mean	0.281	-0.460	-0.363	0.439	0.608	-0.405	-0.343	0.246
Std.Dev.	1.025	2.158	1.976	2.836	2.420	1.090	0.605	1.264
MAE	0.720	1.514	1.310	1.874	1.770	0.873	0.472	0.888
RMSE	1.059	2.199	2.002	2.861	2.487	1.160	0.694	1.284
$\hat{\rho}(1)$	-0.017	-0.139	-0.259	-0.266	0.125	0.375	0.397	0.220
A								
Mean	0.927	-1.899	-0.663	1.915	0.685	-0.993	-0.479	0.514
Std.Dev.	3.402	8.018	5.827	6.562	5.730	4.022	2.826	4.287
MAE	2.084	4.550	3.552	4.508	3.401	2.210	1.433	2.173
RMSE	3.516	8.214	5.846	6.815	5.752	4.130	2.858	4.303
$\hat{\rho}(1)$	0.001	-0.014	-0.124	-0.017	0.166	0.309	0.011	0.120
Baa								
Mean	0.582	-1.082	-0.573	1.078	0.775	-0.697	-0.442	0.364
Std.Dev.	4.248	9.162	6.750	8.802	6.084	3.231	3.431	5.161
MAE	3.068	6.689	5.134	6.503	4.292	2.258	1.737	2.989
RMSE	4.273	9.196	6.753	8.839	6.113	3.295	3.448	5.157
$\hat{\rho}(1)$	-0.136	-0.107	-0.163	-0.059	-0.049	0.104	-0.098	-0.112

Note: The study fits the three-factor model,

$$VOL_t^j(\tau) = L_{v,t}^j + S_{v,t}^j \left(\frac{1 - e^{-\lambda_v \tau}}{\lambda_v \tau} \right) + C_{v,t}^j \left(\frac{1 - e^{-\lambda_v \tau}}{\lambda_v \tau} - e^{-\lambda_v \tau} \right) + \varepsilon_{v,t},$$

for Aa, A, and Baa ratings, respectively, using monthly implied volatility data 1996:01-2008:09 with λ fixed at 0.56. The table presents descriptive statistics for the corresponding residuals at various expiration horizons. The last row in each rating contains residual sample autocorrelations at a displacement of one month.

Figure 2.6: Aa-rated implied volatilities vs. the VIX



Note: The three- and six-month implied volatility measures from Aa-rated firms are plotted against the VIX for the sample period 1996:01-2008:09.

Table 2.7: Cross-correlogram of ADS index and spread/volatility factors

	Spread Level			Spread Slope			Vol Level			Vol Slope		
lag	Aa	A	Baa	Aa	A	Baa	Aa	A	Baa	Aa	A	Baa
-12	-0.27	-0.20	-0.25	-0.19	-0.26	-0.19	-0.43	-0.22	-0.20	-0.01	-0.13	0.00
-11	-0.25	-0.20	-0.24	**-0.29**	-0.31	-0.23	-0.47	-0.23	-0.25	0.06	-0.16	0.06
-10	-0.28	-0.20	-0.23	-0.20	-0.37	-0.27	-0.51	-0.22	-0.30	0.03	-0.15	0.05
-9	-0.28	-0.23	-0.20	-0.22	-0.36	-0.26	-0.53	-0.21	-0.27	0.05	-0.17	0.06
-8	-0.32	-0.25	-0.25	-0.20	-0.41	-0.32	-0.52	-0.22	-0.34	0.07	-0.16	0.10
-7	-0.31	-0.25	-0.24	-0.16	-0.45	-0.35	-0.52	-0.25	-0.29	0.07	-0.17	0.12
-6	-0.29	-0.16	-0.27	-0.10	**-0.49**	**-0.37**	-0.52	-0.25	-0.28	0.05	-0.19	0.02
-5	-0.28	-0.21	-0.25	-0.08	-0.47	-0.35	-0.55	-0.27	-0.24	-0.03	-0.13	-0.02
-4	-0.27	-0.22	-0.27	-0.10	-0.43	-0.29	-0.59	-0.28	-0.28	-0.13	-0.19	0.10
-3	-0.25	-0.23	-0.33	-0.07	-0.48	-0.33	**-0.61**	-0.36	-0.24	-0.05	-0.20	0.07
-2	-0.29	-0.25	-0.35	0.01	-0.42	-0.32	-0.54	**-0.38**	-0.34	-0.10	-0.17	0.07
-1	-0.36	-0.33	-0.31	0.08	-0.41	-0.30	-0.50	-0.34	**-0.39**	-0.17	-0.14	0.05
0	**-0.38**	**-0.34**	**-0.35**	-0.01	-0.41	-0.27	-0.47	-0.31	-0.35	**-0.19**	**-0.22**	-0.02
1	-0.34	-0.29	-0.29	0.01	-0.29	-0.13	-0.46	-0.35	-0.30	-0.15	-0.21	0.06
2	-0.29	-0.16	-0.24	-0.07	-0.27	-0.13	-0.44	-0.35	-0.24	-0.10	-0.16	0.08
3	-0.22	-0.07	-0.12	-0.12	-0.23	-0.14	-0.40	-0.30	-0.29	-0.05	-0.15	0.04
4	-0.21	-0.12	-0.11	0.02	-0.20	-0.13	-0.37	-0.27	-0.27	-0.01	-0.15	0.04
5	-0.26	-0.07	-0.12	0.04	-0.13	-0.08	-0.34	-0.34	-0.31	-0.06	-0.08	0.05
6	-0.24	0.00	-0.09	-0.09	-0.07	-0.02	-0.31	-0.33	-0.26	0.03	-0.07	0.03
7	-0.16	-0.02	-0.04	0.04	0.02	0.04	-0.30	-0.29	-0.25	-0.05	-0.05	0.14
8	-0.16	-0.02	0.00	0.03	-0.02	0.01	-0.35	-0.33	-0.22	-0.06	-0.02	0.17
9	-0.14	-0.04	-0.04	0.09	-0.04	0.01	-0.37	-0.30	-0.25	-0.01	-0.04	0.08
10	-0.19	-0.06	-0.03	0.15	0.03	0.05	-0.36	-0.28	-0.22	-0.02	-0.07	0.08
11	-0.20	-0.05	-0.03	0.12	0.10	0.11	-0.36	-0.26	-0.27	-0.10	-0.08	0.08
12	-0.15	-0.07	-0.03	0.09	0.17	0.16	-0.36	-0.31	-0.29	-0.11	0.04	0.11

Note: Sample period: 1996:01-2008:09. The cross correlogram of Aruoba-Diebold-Scotti (ADS) business conditions index with up to 12-month lags and leads of the level and slope factors for credit spreads and implied volatility.

same general pattern for all three rating categories. There is significant upward movement before recessions. As the recession in 2001 looms, the level of the A-rated spreads widens dramatically, and the levels of Aa and Baa also increase. Since the ADS index represents real economic conditions, a negative correlation between the ADS index and the credit spread level is expected from theory and is also confirmed here. That is, during recessions there are higher probabilities of a credit downgrade or default, so the spreads should rise to adjust for more risk.

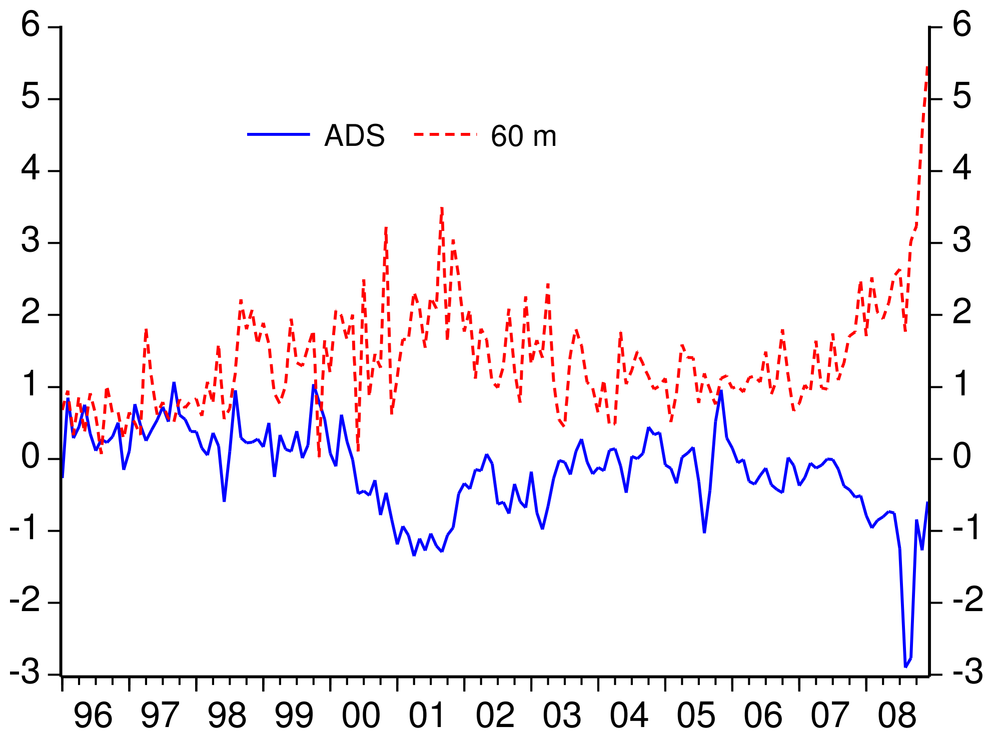
The slope factors of the credit spreads have stronger business cycle correlations. Specifically, as the economy is about to enter a recession in 2001, the slopes of A- and Baa-rated spreads steepen substantially, but Aa-rated firms seem to be less affected. This interesting observation indicates that the market is expecting a bigger increase in the default risk for the A- and Baa-rated long-term bonds than the short-term ones. This is consistent with Gilchrist, Yankov, Zakrajšek (2009), who find that the long-term yield spreads of medium risk (i.e., A-, Baa-, Ba-rated) bonds contain substantial predictive power for economic activity and outperform standard indicators. The cross correlogram also indicates that a strong correlation with the ADS index arrives as early as six months ahead of the tough. Table 2.7 shows that both the A- and the Baa-rated groups have cross-correlations of -0.49 and -0.37 six months ahead of the trough.

The level of the option implied volatility of higher-rated firms, such as Aa-rated firms, exhibits a pattern similar to that of an overall market indicator such as the VIX, but the lower-rated firms (Baa-rated) have quite a different time series plot. The variability of the level increases as ratings decrease. Comparing them with business conditions, the Aa-rated group has a cross-correlation of -0.61, the strongest of all three rating categories, three months ahead of the trough. The other two grades have a significant correlation one or two months ahead of the trough.

The slope of implied volatilities, which captures the difference between the short-run and long-run volatilities, are mostly downward sloping for all ratings groups. The reason for the downward-sloping volatility across maturities could be the following: movements in stock price will affect short-term options more and trigger a bigger increase in short-term volatility, while the effect on long-term contracts are smaller and long-term volatility remains less affected, thus resulting in a downward-sloping curve. The downward-sloping shape of the volatility curve is consistent with the established empirical fact that stocks are usually more volatile in the short-run and more stable in the long-run. However, during recessions or financial crisis, the slope might be upward sloping because the market worries about the economic conditions in the future (say one year from now). The variation of the slope factors is highest for Baa-rated firms and lowest for A-rated firms. Once again, as the economy expands or contracts, the slope factor does not seem to react much. There are fairly weak cross-correlations between the slope of the credit spreads and the ADS index.

In summary, the level and slope factors capture a significant portion of the variation and they are the focus of the rest of the analysis. Similar to previous findings, the slope of the credit spread curve possesses strong predictive power for the deterioration of real business conditions, whereas the volatility slope does not seem to have that property. However, the volatility level does widen before business cycles. Therefore, the volatility level and spread slope seem to be more

Figure 2.7: 60-month Baa-rated credit spreads vs. the ADS index



Note: The 60-month credit spreads of the Baa-rated group are plotted against the Auroba-Diebold-Scotti (ADS) index for the sample period 1996:01-2008:09. The ADS index tracks real business conditions at high frequency by using the release of various economic indicators. See Auroba, Diebold, and Scotti (2009) or the website of the Federal Reserve Bank of Philadelphia for more details.

responsive to new information.

2.4 Intertemporal analysis between the two markets

The previous section estimates dynamic factors for each section. This section analyzes the dynamic interaction among the factors and the relationship between the bond and equity markets.[20] The dynamic relation is examined through a series of different analyses to ensure the consistency and robustness of the results. They are (i) a panel VAR, (ii) Granger causality analysis, and (iii) impulse response functions.

2.4.1 The panel VAR estimation

The model for the intertemporal analysis uses a VAR framework. More specifically, a VAR model that combines spread and volatility factors is compared with models using only the credit spread factors or only the volatility factors. A VAR approach is appropriate for this purpose because it has been developed precisely to capture lead-lag relationships within or between variables. Moreover, it represents a simultaneous equation estimation. As a result, it is unnecessary to estimate single-equation distributed models that include lags and leads, since a VAR model captures all intertemporal relationships simultaneously. There are three ratings categories in the sample and the specific VAR set-up is a panel VAR[21]

[20]The procedure used in this chapter is a two-step approach. The first step extract factors from the data, and the second step analyzes the intertemporal relations of the factors. A two-step approach is easy to implement, whereas a one-step procedure would be numerically challenging and require more parameters to be estimated.

[21]This approach was initiated by Holtz-Eakin, Newey, Rosen (1988).

structure so that rating-specific fixed effects are controlled for and standard estimation techniques can be applied. For all model specifications, a lag order of one is used,[22] which seems reasonable for monthly data, and the Durbin-Waston statistics reported subsequently also confirm that.

The model specification for the combined spread and volatility system is the following:

$$\begin{bmatrix} \text{Spread} \\ \text{Vol} \end{bmatrix} = \begin{pmatrix} a_{ss} & a_{vs} \\ a_{sv} & a_{vv} \end{pmatrix} \begin{bmatrix} \text{Spread}(-1) \\ \text{Vol}(-1) \end{bmatrix} + C_1 D_{i=Aa} + C_2 D_{i=A} + C_3 D_{i=Baa} + \begin{bmatrix} \nu_{st} \\ \nu_{vt} \end{bmatrix},$$

where *Spread* is the set of three factors of the credit spreads (L_s, S_s, C_s), *Vol* is the set of three factors of the implied volatilities (L_v, S_v, C_v), and D is a dummy variable corresponding to each rating category. This joint panel VAR is called a *Volatility-Spread* system. The individual market panel VAR is also estimated for comparison purpose. The individual market model is the *Volatility* system and the *Spread* system, which uses factors from the bond market or the equity options market alone. The specification for the Spread system is

$$\begin{bmatrix} L_s \\ S_s \\ C_s \end{bmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{bmatrix} L_s(-1) \\ S_s(-1) \\ C_s(-1) \end{bmatrix} + C_4 D_{i=Aa} + C_5 D_{i=A} + C_6 D_{i=Baa} + \epsilon_{st},$$

[22]When including one more lag, almost all coefficients on the second lag are not significant.

Table 2.8: Spread-volatility system panel VAR(1)

	L_s	S_s	C_s	L_v	S_v	C_v
$L_s(-1)$	0.282**	-0.490	1.180	5.341***	5.037	-9.947
	(0.03)	(0.15)	(0.12)	(0.00)	(0.65)	(0.60)
$S_s(-1)$	0.03	0.067	0.146	1.558***	7.342**	-13.671**
	(0.47)	(0.54)	(0.54)	(0.01)	(0.04)	(0.02)
$C_s(-1)$	0.043	0.003	0.161	0.934**	1.573	-2.508
	(0.13)	(0.97)	(0.34)	(0.02)	(0.52)	(0.55)
$L_v(-1)$	0.018***	-0.045***	0.052	0.681***	0.145	-0.365
	(0.01)	(0.01)	(0.16)	(0.00)	(0.79)	(0.69)
$S_v(-1)$	0.007***	-0.018***	0.010	0.105***	0.176	0.078
	(0.00)	(0.00)	(0.48)	(0.00)	(0.37)	(0.82)
$C_v(-1)$	0.005***	-0.014***	0.012	0.097***	0.093	0.050
	(0.01)	(0.00)	(0.20)	(0.00)	(0.50)	(0.83)
Fixed Effect						
Aa	0.452	1.436	-4.906	6.993	21.708	-27.289
A	0.636	1.077	-5.090	10.001	31.520	-55.337
Baa	0.886	1.212	-5.671	9.422	55.415	-86.460
Adj R^2	0.276	0.108	0.026	0.384	0.079	0.071
SER	0.722	1.883	4.209	9.825	60.925	104.07
DW	2.027	2.054	1.971	2.225	2.004	2.009
F Test						
Test statistics	3.962	3.622	1.756	4.225	3.850	5.030
P-value	0.013	0.009	0.155	0.006	0.010	0.002

Note: The results of the spread-volatility system panel regression are reported for the sample period 1996:01-2008:09. The p-values are reported in parenthesis. ***, **, and * indicate significance at 1%, 5%, and 10%, respectively. SER is the standard error of the regression. DW is the Durbin-Watson statistics. The F test is implemented to check if the coefficients on the volatility factors are jointly significant in explaining the spread factors, i.e., the null is $a_{vs} = 0$; the same test is carried out for the reverse direction as well, i.e., the null is $a_{sv} = 0$.
The spread volatility system specification is

$$\begin{bmatrix} \text{Spread} \\ \text{Vol} \end{bmatrix} = \begin{pmatrix} a_{ss} & a_{vs} \\ a_{sv} & a_{vv} \end{pmatrix} \begin{bmatrix} \text{Spread}(-1) \\ \text{Vol}(-1) \end{bmatrix} + C_1 D_{i=Aa} + C_2 D_{i=A} + C_3 D_{i=Baa} + \begin{bmatrix} \nu_{st} \\ \nu_{vt} \end{bmatrix}.$$

and the Volatility system is,

$$\begin{bmatrix} L_v \\ S_v \\ C_v \end{bmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{bmatrix} L_v(-1) \\ S_v(-1) \\ C_v(-1) \end{bmatrix} + C_7 D_{i=Aa} + C_8 D_{i=A} + C_9 D_{i=Baa} + \epsilon_{vt}.$$

Comparisons of the individual market with the two joint markets indicates how much additional explanatory or predictive powers one market can bring to the other. Table 2.8 reports the results for the joint Spread-Volatility system panel VAR, and Table 2.9 reports results for the individual system panel VAR. Based on the results in those tables, most of the interactions are between the level and slope factors, and the corresponding results are discussed below.

First, I examine the impact of volatilities on credit spreads. The level of the credit spread is the most persistent factor among all three credit spread factors. When the volatility factors are added, the persistent level decreases. A positive coefficient between the two level factors is consistent with theory; i.e., when the level of the volatility rises, the credit spread level increases. When the slope of the volatility steepens, the credit spread level also widens, but to a much lesser degree than the impact from the volatility levels. When the volatility factors are added to the VAR, the adjusted R^2 of the credit spread level increases by about 7% from around 20% to 27.6%. The F-test confirms the joint significance of the volatility factors. However, there is a much smaller improvement in explaining the slope of the credit spreads by adding volatility factors, although all coefficients on the volatility factors are statistically significant, and the F-test also confirms the

Table 2.9: Individual system panel VAR(1)

Spread System				Volatility System			
	L_s	S_s	C_s		L_v	S_v	C_v
$L_s(-1)$	0.539***	-1.096***	1.792***	$L_v(-1)$	0.828***	0.042	-0.159
	(0.00)	(0.00)	(0.00)		(0.00)	(0.92)	(0.82)
$S_s(-1)$	0.093**	-0.061	0.245	$S_v(-1)$	0.151***	0.187	0.056
	(0.01)	(0.52)	(0.25)		(0.00)	(0.27)	(0.85)
$C_s(-1)$	0.097***	-0.117*	0.277**	$C_v(-1)$	0.130***	0.080	0.072
	(0.00)	(0.05)	(0.04)		(0.00)	(0.48)	(0.71)
Fixed effect							
Aa	0.842	0.546	-3.972	Aa	5.919	22.713	-31.375
A	1.015	0.283	-4.227	A	9.466	30.392	-55.646
Baa	1.256	0.503	-5.043	Baa	10.268	54.247	-86.991
Adj R^2	0.204	0.098	0.020	Adj R^2	0.368	0.045	0.042
SER	0.729	1.904	4.221	SER	9.947	61.594	105.68
DW	2.072	2.039	1.982	DW	2.266	2.014	2.010

Note: The results of the spread volatility system and volatility system panel regression are reported for the sample period 1996:01-2008:09. The p-values are reported in parenthesis. ***, **, and * indicate significance at 1%, 5%, and 10%, respectively. SER is the standard error of the regression. DW is the Durbin-Watson statistics.
The spread system specification is

$$\left[\text{Spread}\right] = b\left[\text{Spread}(-1)\right] + C_1 D_{i=Aa} + C_2 D_{i=A} + C_3 D_{i=Baa} + \epsilon_{st}.$$

The volatility system specification is

$$\left[\text{Vol}\right] = b\left[\text{Vol}(-1)\right] + C_1 D_{i=Aa} + C_2 D_{i=A} + C_3 D_{i=Baa} + \epsilon_{vt}.$$

joint significance.

Second, I explore the impact of credit spreads on volatilities. The level factor of the volatility is highly persistent. When the volatility level is regressed with either volatility factors alone or both volatility and spread factors, all coefficients are positive and statistically significant, and the F-test supports the joint significance of the spread factors. But the improvement from the adjusted R^2 perspective to explain the volatility level is small, around 1.6%. Therefore, comparing these results with the results from the direction of volatilities to credit spreads, it seems that the impact from volatility to spreads is stronger than the impact in the other direction.

However, from the perspective of volatility slope, there is a significant impact from the spread factors to the volatility factors and the improvement from the adjusted R^2 is more than 3% when adding spread factors to the regression. Moreover, the only statistically significant coefficient is that of the credit spread slope factor, and the F-test supports the importance of the spread factors.

In summary, the panel VAR results suggest that there are bi-directional impacts among the credit spread and volatility factors, and the results also indicate that the effects from the volatility level to the spread level and from the spread slope to the volatility slope are stronger than others.

2.4.2 Granger causality analysis

For each ratings group, a Granger causality test is carried out for direct comparisons between the two level factors and the two slope factors. The aim of the test is to confirm the dynamic relationship, especially the predictive potential,

Table 2.10: Granger causality test for the level and slope factors

Null	Aa	A	Baa
Level factors			
Vol Level NOT Granger cause Spread Level	14.396	4.994	3.763
	(0.00)	(0.03)	(0.06)
Spread Level NOT Granger cause Vol Level	0.527	1.011	6.269
	(0.47)	(0.32)	(0.01)
Slope factors			
Vol Slope NOT Granger cause Spread Slope	0.879	0.017	0.002
	(0.35)	(0.90)	(0.96)
Spread Slope NOT Granger cause Vol Slope	4.871	8.647	5.528
	(0.03)	(0.00)	(0.02)

Note: Sample period: 1996:01-2008:09. The one-lag Granger causality test statistics and the corresponding P-values are reported. The P-values are in parenthesis.

between the factors for each ratings group.

Table 2.10 reports the Granger causality test between the credit spread factors (level and slope) and the implied volatility factors (level and slope). The level of the volatility Granger causes the level of the credit spread, whereas the slope of the credit spread consistently Granger causes the slope of the implied volatility. These results imply that the volatility level has predictive power for the level of corporate yield spreads. In addition, the shape of the credit spread curve does not correspond with the movement in the shape of the volatility curve; instead, the predictive power is the opposite, i.e., the spread slope has predictive power for the volatility slope. One interesting result of the Granger causality test is that for Baa-rated firms, the Granger causality test rejects the null hypothesis for both directions.

These results are generally consistent with the results from the panel VAR model, which indicates that the direction of the impact is consistent among all ratings.

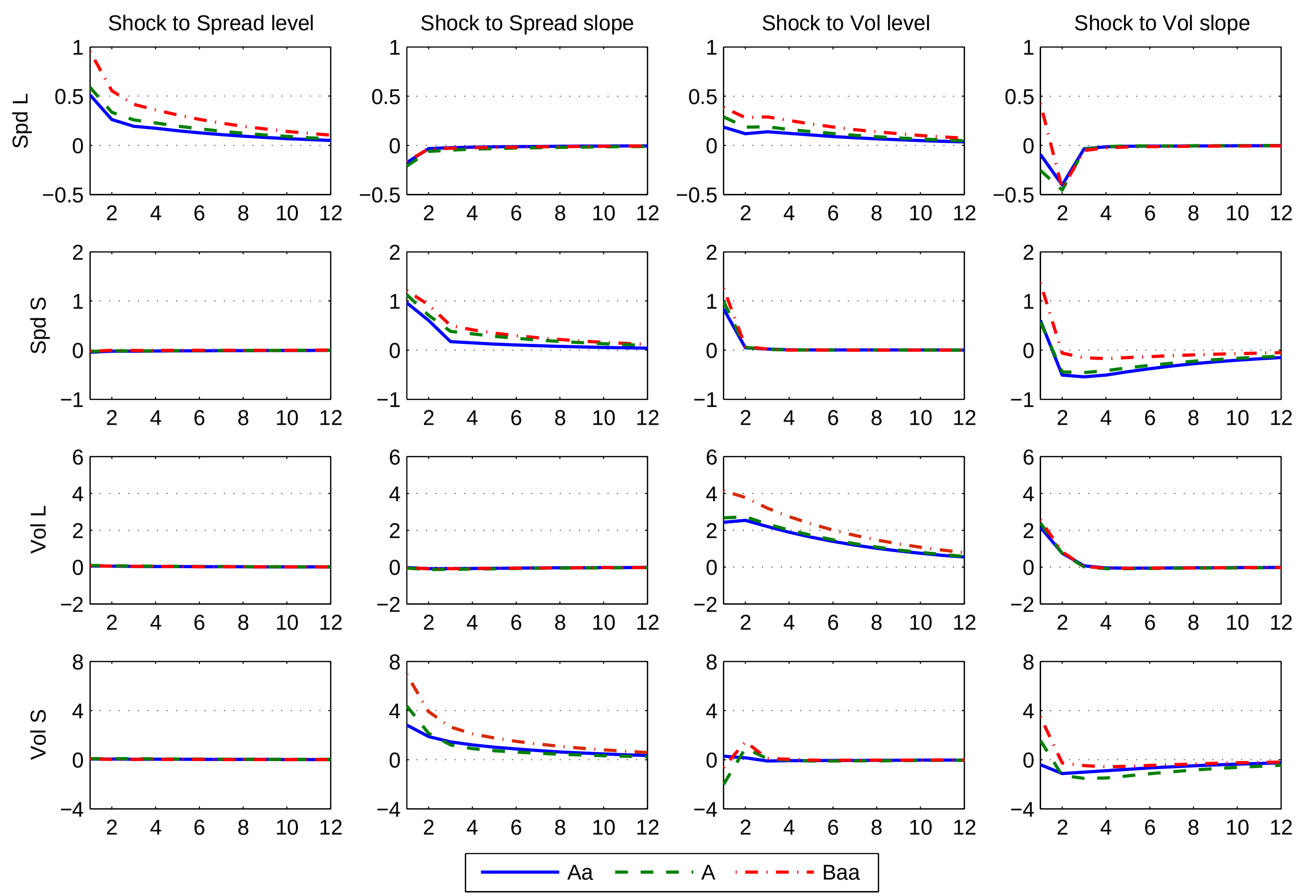
2.4.3 Impulse response analysis

Another way to examine the dynamics is to use impulse response functions of the Volatility-Spread system. Figure 2.8 presents the responses of one-standard-deviation shocks using a Cholesky decomposition. I consider four groups of impulse responses: spread responses to spread shocks, spread responses to volatility shocks, volatility responses to spread shocks, and volatility responses to volatility shocks.

The volatility components add an interesting element to the credit spread responses: an increase in the volatility level shock is followed by a persistent and dramatic rise in the credit spread level. That is, there is a close connection between the level of the volatility and the level of the credit spread. Thus, it is consistent with corporate bond pricing theory when focusing on the level of the two term structures. An increase in the shock to the volatility slope is not persistent and is followed by a significant upward shift of the credit spread slope, but the shift does not last longer than 3 months.

Now consider the responses of the volatility curve to the credit spread factor shocks. While the volatility level factor shows very little response, the volatility slope factor responds directly to positive shocks in the credit spread factors. This implies that when the credit spread curve changes its slope, the volatility curve also changes its slope. This is consistent with the Granger causality test results

Figure 2.8: Impulse responses of the spread-volatility VAR(1) system



Note: Impulse responses for the VAR(1) of the spread-volatility system given one-standard-deviation changes to the shocks. The solid lines are for Aa-rated firms, the dashed lines are for A-rated, and the dashed-pointed lines are for Baa-rated. The responses are up to 12 months ahead.

reported earlier.

Let us consider the block of own-dynamics of the two term structures. The spread level is more persistent than the spread slope, the volatility level factor exhibits significant persistence, and most of the off-diagonal responses are insignificant.

In summary, impulse response results suggest the following dynamics between the bond and equity options markets. The level shocks from volatility are associated with persistent responses from the credit spread levels, whereas the slope shocks from volatility do not appear to have persistent effect on other factors. Shocks from the bond market have a different dynamic. The slope shocks from the credit spread are associated with persistent responses from the volatility slopes, while the level shocks from the credit spread do not trigger significant responses from other factors. These results are consistent with the findings in the panel VAR estimation and the Granger causality test.

2.4.4 Summary of results and explanation

The analysis of the panel VAR estimation, Granger causality tests, and impulse response functions points to the same empirical results. That is, the options market leads the bond market in terms of level shifts, while the bond market leads the options market in slope changes. Another way to state the results is that the volatility level affects all maturity of the credit spreads, but the shape of the volatility curve does not affect the shape of the credit spread. In fact it is the reverse, i.e., the bond market slope impacts the next period's volatility curve slope.

Fundamentally the bond market and the equity options market represent the same underlying process of determining a firm's value and both should reflect changes in that process. Thus it is reassuring that the empirical results support the theory that both markets react to the arrival of information. However, from the movement of the two term structures, it seems that the two markets adjust to the new information *differently*. The volatility level and the spread slope respond to news earlier than the other two factors, consistent with results from the cross-correlation analysis with the ADS index in Table 2.7. The following discussion provides a possible explanation of the leading role of the credit spread slope and the volatility level.

For the bond market, the more responsive maturity spectrum is toward the long end. The risk of default and the associated market price of such risk are greater, since most of the coupon payments for those bonds have not been made, and the further one has to look into the future, the harder it is to predict business conditions. Unlike long-term bonds, which have a longer duration, short-term bonds carry much smaller associated risks, since most of their coupon payments have been made, and from the firm's accounting statements, the risk of default can be assessed with a greater degree of certainty. Therefore, when new information is released, the prices of long-term bonds are more likely to change. Since the long end of the curve is more sensitive to shocks than the short end, the slope movement should be apparent first. Similar results are found in Fleming and Remolona (1999c). They analyze the effects of announcements on the term structure for maturities from 3 months to 30 years and find that the effects are weak for the short term and strong for the intermediate term, 5 to 10 years, which corresponds to the long end of the spread curve in this analysis. In addition, Gilchrist, Yankov, Zakrajšek (2008) conclude that the 10-year Baa- or A-rated corporate bond has strong predictive power for business cycles, and thus they are the more responsive maturities of the credit spread curve.

On the contrary, the level in the volatility term structure moves first because the equity option prices across all maturities are affected by the new information. Since the options market consists mostly of short-term contracts (less than 2 years), aggregate shocks can have an impact for the entire maturity spectrum, and thus the level change in that term structure is apparent. Previous studies have documented volatility's response to aggregate shocks. Some papers report a connection between equity volatility and macroeconomic conditions. Hamilton and Susmel (1994) and Sinha (1996) estimate GARCH models of monthly U.S. equity returns with high and low volatility regimes depending on economic conditions. They conclude that macro conditions significantly affect equity returns in the sense that equity volatility is more likely to become (remain) high during a recession. Moreover, recent studies have started to analyze the impact of particular macro announcements on equity market return volatility, e.g. Flannery and Protopapadakis (2002). They estimate a GARCH model of equity returns and show that conditional volatility varies with macroeconomic series' announcements. However, research studies on the impact of the entire volatility term structure seem non-existent. I fill that gap and show that the entire volatility term structure moves when aggregate information arrives.

After movement in the spread slope and volatility level, the volatility slope and spread level will adjust accordingly. As the level of volatility rises, the credit risk increases, so the spread level should widen. The volatility slope indicates the direction in which the market's expected future volatility is heading. As the spread slope steepens, the associated future risk also rises, and thus the volatility slope needs to adjust and reflect the increasing expected future volatility.

These findings suggests the potential of segmentation between the corporate bond market and equity options market. The investors for the corporate bonds tend to focus on the status of longer horizon investments, whereas options market participants are more concerned about the short-term hedging. Moreover, these results reconcile results from corporate bond pricing and market microstructure and suggest that both bond and equity markets are responsive and they affect each other. Bond markets have a slope movement first, whereas equity options markets have a parallel movement first. This simple explanation could be the reason why Berndt and Ostrovnaya (2008) conclude that there is no clear leader in terms of news announcements between spreads and equity options. Moreover, my results also enhance understanding of the stock and bond relationship from a term structure perspective, which has never been studied previously.

2.5 Concluding remarks

This study has characterized the dynamic relationship between the bond market and the equity options market by analyzing the portfolios of investment-grade firms that have both bonds and equity options. From the two term structures, I found that the level of the option implied volatility Granger causes the level of the credit spread, while the slope of the credit spread Granger causes the slope of the implied volatility.

My results are especially interesting and intriguing since they consolidate and enhance findings from the corporate bond pricing and the market microstructure literatures. The theoretical predictions of Merton (1974) are confirmed through the level of the two term structures. Simultaneously, the impact from credit spreads to equity volatility, as concluded from the market microstructure literature, operates through the slope factor interactions. The term structure representation presents a significantly richer and more nuanced environment than achieved previously, and the intertemporal relationships between the two term structures extend and unify the two literatures. Therefore, my analysis deepens our understanding of stock and bond market interactions in a new perspective.

In addition to contributing to the academic literature, this chapter also has implications for practitioners. It suggests that trading strategies can possibly be formed by taking advantage of the predictive powers one market has on the other. Specifically, one could use the level factor in the options market as a signal to buy or sell corporate bonds, or could use information on the slope of the credit spreads to exploit the relative price of long- and short-term options.

CHAPTER III

Yield Curve Modeling in Risk-neutral vs. Physical Environments

Modeling of the dynamic evolution of the yield curve has long been a very important subject of research. Such models should not only allow for accurate pricing of financial derivatives, but also produce accurate forecasts of yields. Litterman and Scheinkman (1991) and Litterman, Scheinkman, and Weiss (1991) initiate the tradition of level, slope and curvature factor analysis of the yield curve modeling. Most studies suggest the level factor is highly correlated with inflation, whereas the slope factor is highly correlated with real activity. However, despite the significant correlations of the level and slope factors, the curvature factor appears unrelated to any of the main macroeconomic variables and remains poorly understood. Christensen, Diebold, and Rudebusch (2007, 2008) derive a dynamic affine term structure that rules out opportunities for riskless arbitrage. They work under the risk-neutral measure and impose restrictions on the transition dynamics. Specifically, under such a measure there is an impact of curvature factor on other factors, which operates through the slope factor. Many literature also proceed their work under the Q-measure, such as prediction markets, options densities, forward rates, etc. However, there is risk-premium associated with the risk-neutral measure and the physical measure. Our goal is to find a dynamic term structure model that fits the observed yields in-sample while predicts future yields. Moreover, such a model also reveals the dynamics of the three factors that drive the Treasury yield curve and, more importantly, the associated risk-premium between the two measures.

In this chapter, we estimate Nelson-Siegel models with or without freedom from arbitrage, as done in Diebold and Li (2006) and Christensen, Diebold, and Rudebusch (2007), respectively. In the process of making dynamic Nelson-Siegel model without arbitrage opportunities, Christensen, Diebold, and Rudebusch (2007) impose restrictions on the transition dynamics of the three factors under the risk-neutral measure. We test these risk-neutral restrictions on the factors estimated from Nelson-Siegel models with or without absence of arbitrage imposed. Since these factors are obtained in the physical measure, by testing these factors in-sample we shred some light on the difference between the risk-neutral measure and the physical measures of the term structure modeling. In addition, we also utilize these restrictions for out-of-sample forecasting. Therefore, we provide a comprehensive analysis of the dynamic term structure model for the Treasury yield curve.

Our test results indicate the risk-neutral restrictions are well satisfied under the physical measure, so the Treasury bond market is close to be risk-neutral during our sample period. Precisely, we are able to show that the three factors have the following properties. First, the level factor is a unit-root process and does not affect the other two factors. Second, the slope and curvature factors are mean-reverting processes that revert at the same rate. Third, the curvature factor forecasts the slope factor, and not conversely, which implies the curvature factor is a leading indicator of the slope factor. Moreover, imposing these properties in the forecast construction can enhance out-of-sample performance for horizons of 6-month or longer. The Nelson-Siegel model with all the risk-neutral restrictions is consistently the preferred choice among all Nelson-Siegel models.

The way we proceed is as follows. Subsection 3.1 describes the dynamic Nelson-Siegel models we use. Subsection 3.2 describes the U.S. Treasury yield data and introduces specific dynamic Nelson-Siegel models used for estimation. Subsection 3.3 analyzes the estimation results, whereas Subsection 3.4 describes the forecast exercise and its results. Subsection 3.5 introduces an alternative estimation via a two-step procedure. Subsection 3.6 analyzes the model-free factors (i.e. constructed directly from the yields). Finally, Subsection 3.7 concludes the chapter.

3.1 Dynamic Nelson-Siegel Models

The underlying workhorse of the yield curve modeling is the representation introduced by Nelson and Siegel (1987), which fits the cross section of the yield curve. The functional form is

$$y(\tau) = \beta_1 + \beta_2\Big(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\Big) + \beta_3\Big(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\Big), \tag{3.1}$$

where $y(\tau)$ is the zero coupon yield with maturity τ, and β_1, β_2, β_3, and λ are model parameters. It is the most parsimonious model to generate a level, a slope, and a curvature factor property using just a single parameter λ. Consequently, this model is popular with central bank researchers and financial market practitioners.

In order to understand the evolution of the bond returns over time, Diebold and Li (2006) develop a dynamic version of the Nelson-Siegel Model (DNS) that yields three latent factors with the same level, slope, and curvature interpretation. Essentially, they make the coefficient β's as time varying factors of level L_t, slope S_t, and curvature C_t, so

$$y_t(\tau) = L_t + S_t\Big(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\Big) + C_t\Big(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\Big). \tag{3.2}$$

An autoregressive structure is assumed for each of these three factors,

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = A \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix}, \tag{3.3}$$

so it is a fully dynamic specification. Due to the DNS model's simplicity, it is easy to estimate, forecasts remarkably well, and delivers very robust results across different data sets.[1]

Despite its good performance, this model is incompatible with desirable theoretical restrictions that rule out opportunities for riskless arbitrage. Chris-

[1]Diebold, Li, and Yue (2008) extend dynamic Nelson-Siegel to a global context, modeling a large set of country yield curves in a framework that allows for both global and country-specific factors.

tensen, Diebold, and Rudebusch (2007) resolve this problem, by deriving the affine arbitrage-free class of dynamics Nelson-Siegel (AFNS) term structure models. The AFNS model is based on standard continuous time affine arbitrage-free structure as in Duffie and Kan (1996), resulting the following affine arbitrage-free model that satisfies a set of ordinary differential equations.

The instantaneous risk-free rate is defined by

$$r_t = X_t^1 + X_t^2. \tag{3.4}$$

The state variables $X_t = (X_t^1, X_t^2, X_t^3)$ are described by a system of stochastic differential equations (SDEs) under the risk-neutral measure

$$\begin{pmatrix} dX_t^1 \\ dX_t^2 \\ dX_t^3 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{pmatrix}}_{K^Q} \begin{bmatrix} \theta_1^Q - X_t^1 \\ \theta_2^Q - X_t^2 \\ \theta_3^Q - X_t^3 \end{bmatrix} dt + \Sigma \begin{pmatrix} dW_t^{1,Q} \\ dW_t^{2,Q} \\ dW_t^{3,Q} \end{pmatrix}, \quad \lambda > 0. \tag{3.5}$$

The matrix K^Q restricts cross interactions between X_t^1, X_t^2, and X_t^3. This cross restriction matrix is the key element to establish the freedom from arbitrage. Then, the zero coupon bond prices are given by

$$P(t,T) = E_t^Q[exp(-\int_t^T r_u du)] = exp\big(B^1(t,T)X_t^1 + B^2(t,T)X_t^2 + B^3(t,T)X_t^3 + C(t,T)\big), \tag{3.6}$$

where $B^1(t,T)$,$B^2(t,T)$,$B^3(t,T)$ and $C(t,T)$ are the unique solutions to the following system of ordinary differential equations (ODEs):

$$\begin{pmatrix} \frac{dB^1(t,T)}{dt} \\ \frac{dB^2(t,T)}{dt} \\ \frac{dB^3(t,T)}{dt} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{pmatrix} \begin{pmatrix} B^1(t,T) \\ B^2(t,T) \\ B^3(t,T) \end{pmatrix}, \tag{3.7}$$

and

$$\frac{dC(t,T)}{dt} = -B(t,T)'K^Q\theta^Q - \frac{1}{2}\sum_{j=1}^{3}(\Sigma' B(t,T)B(t,T)'\Sigma)_{j,j'} \tag{3.8}$$

with boundary conditions $B^1(T,T) = B^2(T,T) = B^3(T,T) = C(T,T) = 0$.

Finally, after solving these ODEs, zero coupon bond yields are given by

$$y(t,T) = X_t^1 + X_t^2\Big(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\Big) + X_t^3\Big(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\Big) - \frac{C(t,T)}{T-t}, \tag{3.9}$$

where

$$\begin{aligned} C(t,T) \;\; &= \;\; (K^Q\theta^Q)_2 \int_t^T B^2(s,T)ds + (K^Q\theta^Q)_3 \int_t^T B^3(s,T)ds \\ & \quad + \frac{1}{2}\sum_{j=1}^{3}\int_t^T (\Sigma' B(t,T)B(t,T)'\Sigma)_{j,j'}ds. \end{aligned} \tag{3.10}$$

The result above defines the class of AFNS models. In the AFNS model, the factor loadings exactly match the DNS ones, but there is an adjustment term $C(t,T)$ in the yield function, which only depends on maturity of the bond. Thus, the X_t's have the same level L_t, slope S_t, curvature C_t factor interpretation as those in the DNS model. Also, we note that the level factor is a unit-root process under the Q-measure, while the curvature factor has the same mean reverting rate as the slope factor under the pricing measure. Moreover, under the same pricing measure the curvature factor affects the slope factor. As we will demonstrate, these properties are preserved under the P-measure as well.

As in Christensen, Diebold, and Rudebusch (2007), the AFNS model is formulated in the continuous-time framework, and thus the relationship between the real world dynamics and the risk-neutral dynamics is given by a measure change

$$dW_t^Q = dW_t^P + \Gamma_t dt, \tag{3.11}$$

where Γ_t represents the risk premium. The stochastic differential equation for the state variables under the P-measure,

$$dX_t = K^P[\theta^P - X_t]dt + \Sigma dW_t^P, \tag{3.12}$$

remains affine. Because of the flexible specification Γ_t, any mean vector θ^P and mean reversion matrix K^P under P-measure can be chosen while the Q-dynamics is preserved. Moreover, one specific K^P that satisfies arbitrage free under the physical measure could be

$$K^P = K^Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{pmatrix}. \tag{3.13}$$

Thus from equation (3.7), we can obtain the following discrete version of the state equation under the physical measure:

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-\lambda & \lambda \\ 0 & 0 & 1-\lambda \end{pmatrix}}_{A} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix}, \quad (3.14)$$

which corresponds to imposing restrictions on autoregressive dynamics through the matrix A. There are essentially 8 restrictions.[2] 6 are obvious and can be investigated using t-type tests, and they are $a_{11} = 1$, $a_{12} = 0$, $a_{13} = 0$, $a_{21} = 0$, $a_{31} = 0$, and $a_{32} = 0$. The other two are $a_{22} + a_{23} = 1$ and $a_{23} + a_{33} = 1$, which should be tested jointly using an F type test. Moreover, combinations of these restrictions should be analyzed jointly as well. We can interpret these constraints as the following:

(1) The level factor is a random walk and does not affect the other two factors;

(2) The curvature factor and slope factor are mean reverting, and they are reverting at the same rate;

(3) The lagged curvature factor impacts the slope factor, which implies the curvature factor can forecast the slope factor;

(4) The mean reverting rate of the slope factor (or the curvature factor) and the impact of the lagged curvature factor on the slope factor sum to one.

[2] Another way to think about these restrictions is that the eigenvalues of matrix A have to be 1 and $1 - \lambda$ precisely.

The main task of the chapter is to show empirically whether these properties are preserved in-sample under the physical measure and whether we can improve out-of-sample performance by utilizing these properties.

3.2 Specific Models and Estimation

In general, the DNS and AFNS models are silent about the appropriate specifications of the P-dynamics of the state variables, so the number of possible specifications is infinite. Here, we limit our focus to affine models, that is, the dynamics in all the models considered here are unrestricted VAR(1) processes, which allow for the most amount of flexibility.

3.2.1 The Treasury Bond Yield Data

The specific Treasury bond yields we use are zero-coupon Treasury bond yields calculated based on the unsmoothed Fama-Bliss (1987) method. These yields are observed at a monthly frequency over the period of January 1987 to December 2002 with maturities covering the entire spectrum from three months up to thirty years in the Treasury bond market. Table 1 presents summary statistics for all 16 maturities.

3.2.2 The General DNS Model

Under general specification as in Diebold, Rudebusch, and Aruoba (2006), the state equation has an unconstrained VAR(1) processes. Thus, the state equation is given by

Table 3.1: Summary Statistics for the Unsmoothed Fama-Bliss Treasury Zero Coupon Bond Yields.

Maturity	Mean	St.dev.	Skewness	Kurtosis	Minimum	Maximum
3	5.09	1.74	-0.06	2.85	1.18	9.13
6	5.22	1.75	-0.14	2.82	1.20	9.32
9	5.33	1.76	-0.17	2.77	1.19	9.34
12	5.48	1.78	-0.20	2.79	1.21	9.63
18	5.70	1.74	-0.20	2.79	1.37	9.66
24	5.81	1.66	-0.18	2.77	1.58	9.53
36	6.06	1.55	-0.12	2.72	2.03	9.46
48	6.26	1.48	-0.08	2.61	2.40	9.35
60	6.36	1.44	-0.02	2.46	2.67	9.29
84	6.60	1.38	0.05	2.22	3.35	9.40
96	6.70	1.37	0.06	2.14	3.52	9.52
108	6.74	1.36	0.06	2.07	3.66	9.59
120	6.74	1.36	0.06	1.99	3.75	9.53
180	7.16	1.24	0.21	1.89	4.94	9.95
240	7.25	1.13	0.08	1.78	5.22	9.73
360	6.77	1.21	0.06	1.74	4.72	9.46

The summary statistics for the sample of monthly observed Fama-Bliss zero-coupon Treasury bond yields spanning the period from January 1987 to December 2002.

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{A} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix}. \tag{3.15}$$

Here, the innovations $\eta_t(L)$, $\eta_t(S)$, and $\eta_t(C)$ are allowed to be correlated with a conditional covariance matrix given by $Q = qq'$, where the Cholesky factor q of the covariance matrix is

$$q = \begin{pmatrix} q_{11} & 0 & 0 \\ q_{21} & q_{22} & 0 \\ q_{31} & q_{32} & q_{33} \end{pmatrix}. \tag{3.16}$$

The measurement equation takes the form

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_n) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} & \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} - e^{-\lambda\tau_n} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_n) \end{pmatrix}, \tag{3.17}$$

where the measurement errors for each maturity, $\varepsilon_t(\tau_i)$, are assumed to be white noise.

3.2.3 The General AFNS Model

The state space representation of the AFNS model is very similar to the DNS model, but it is in the continuous-time framework. The state equation also indicates unconstrained VAR(1) processes,

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \underbrace{\begin{pmatrix} \kappa_{11}^P & \kappa_{12}^P & \kappa_{13}^P \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P \\ \kappa_{31}^P & \kappa_{32}^P & \kappa_{33}^P \end{pmatrix}}_{K^P} \begin{bmatrix} \theta_L^P - L_t \\ \theta_S^P - S_t \\ \theta_C^P - C_t \end{bmatrix} dt + \begin{pmatrix} \sigma_{11} & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \begin{pmatrix} dW_t^{L,P} \\ dW_t^{S,P} \\ dW_t^{C,P} \end{pmatrix}. \tag{3.18}$$

This is the arbitrage-free equivalent of the general DNS model.

The measurement equation for the general AFNS model has an additional adjustment term,

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_n) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} & \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} - e^{-\lambda\tau_n} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_n) \end{pmatrix} - \begin{pmatrix} c(\tau_1)/\tau_1 \\ c(\tau_2)/\tau_2 \\ \vdots \\ c(\tau_n)/\tau_n \end{pmatrix}. \tag{3.19}$$

3.2.4 Estimation Methods

In the estimations, all maturities in the Treasury yield data are used throughout. Diebold and Li (2006) use a two-step procedure. They fix λ at 0.0609, which implies that the loading on the curvature factor achieves maximum at 30-month maturity, and once λ is fixed, the ordinary least squared can produce the three factors. We will show the results following such a procedure later as a robustness check. Here, we adopt a one-step procedure. We estimate them by maximizing the likelihood function in the standard Kalman filter algorithm since all the mod-

Table 3.2: Descriptive Statistics, Estimated Factors

Factor	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$
DNS factors						
$\hat{\beta}_{1t}$	7.213	1.211	5.009	9.817	0.976	0.755
$\hat{\beta}_{2t}$	-2.235	1.658	-5.547	0.802	0.976	0.376
$\hat{\beta}_{3t}$	-0.787	2.026	-7.373	3.480	0.889	0.211
AFNS factors						
$\hat{\beta}_{1t}$	7.414	1.337	4.874	10.061	0.982	0.748
$\hat{\beta}_{2t}$	-2.425	1.753	-6.127	0.618	0.980	0.414
$\hat{\beta}_{3t}$	-0.695	1.560	-5.651	2.507	0.860	0.160

We fit the three factor DNS model and AFNS model using monthly yield data 1987:01-2002:12, and we present descriptive statistics for the corresponding three estimated factors $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$, and $\hat{\beta}_{3t}$. The last two columns contain sample autocorrelations at displacements of 1 and 12 months.

els are affine Gaussian and Kalman filter algorithm is an efficient and consistent estimator in this setting.[3]

For the DNS model, we follow Diebold, Rudebusch, and Aruoba (2006), so we start the algorithm at the unconditional mean and variance of the state variables. We also impose the constraint such that the eigenvalues of A are smaller than 1 to ensure the state variables are stationary.

For the AFNS model, the conditional mean vector and conditional covariance

[3]All models considered here are Gaussian with constant volatility, so the Kalman filter is both consistent and efficient. The disadvantage is that the stochastic volatility of bond yields is not taken into consideration. However, its impact on forecast performance should be relative modest.

matrix are given by

$$E^P[X_T|F_t] = (I - exp(-K^P\Delta t))\theta^P + exp(-K\Delta t)X_t, \tag{3.20}$$

$$V^P[X_T|F_t] = \int_0^{\Delta t} e^{-K^P s}\Sigma\Sigma' e^{-(K^P)' s} ds, \tag{3.21}$$

where $\Delta t = T - t$. By discretizing the continuous dynamics under the P-measure, we obtain the state equation

$$X_t = (I - exp(-K^P\Delta t))\theta^P + exp(-K\Delta t)X_{t-1} + \eta_t, \tag{3.22}$$

The conditional covariance matrix for the shock terms is given by

$$Q = \int_0^{\Delta t} e^{-K^P s}\Sigma\Sigma' e^{-(K^P)' s} ds. \tag{3.23}$$

The real component of each eigenvalue of K^P is restricted to be positive to ensure stationarity of the system. The Kalman filter for the AFNS model also starts at the unconditional mean and covariance. The measurement equation is similar to the DNS model with the adjustment term.

For all models the error structure is

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix}\right], \tag{3.24}$$

Table 3.3: The General DNS Model Parameter Estimates

	L_{t-1}	S_{t-1}	C_{t-1}	U
L_t	0.984*	0.000	-0.004	7.093*
	(0.020)	(0.022)	(0.020)	(1.423)
S_t	0.004	0.934*	0.078*	-3.023
	(0.027)	(0.023)	(0.025)	(2.031)
C_t	0.027	0.054	0.883*	-1.345
	(0.056)	(0.047)	(0.040)	(1.379)

Note: Each row presents coefficients from the transition equation for the respective state variable. The standard errors are reported in parenthesis.The parameter values with an asterisk are significant at the 5% level.

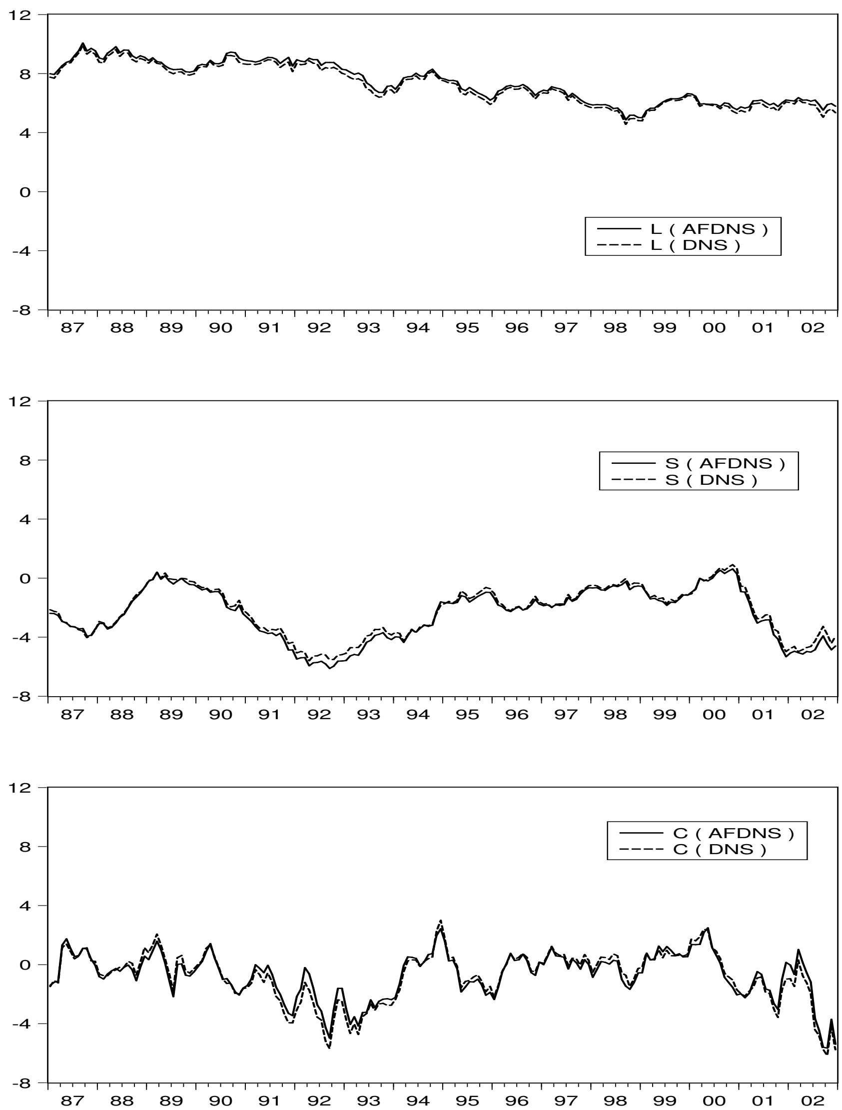
where H is a diagonal matrix

$$H = \begin{pmatrix} \sigma^2(\tau_1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma^2(\tau_N) \end{pmatrix}. \tag{3.25}$$

3.3 Estimation Results

In this section, we present the results from the Kalman filter algorithm and detail the analysis of the risk-neutral restrictions. In Figure 3.1, we plot $\{\hat{\beta}_{1t}, \hat{\beta}_{2t}, \hat{\beta}_{3t}\}$ of the DNS model along with the AFNS estimated factors. In table 3.2, we report the descriptive statistics of the estimated factors for both the DNS and the AFNS models. Despite the imposition of freedom from arbitrage, the estimated factors appear to be quite similar from both models.

Figure 3.1: Estimated DNS factor and Estimate AFNS factors.



Estimated DNS factors are plotted along with estimated AFNS factors using the Treasury yield data from January 1987 to December 2002.

3.3.1 The DNS Model

In Table 3.3 we present estimation results for the general DNS model. The estimate of the A matrix indicates highly persistent own dynamics of L_t, S_t, and C_t with estimated own-lag coefficients of 0.98, 0.93 and 0.88, respectively. Cross-factor dynamics appear mostly unimportant, with the exception of statistically significant effect of C_{t-1} on S_t.

The risk-neutral restrictions on the factor dynamics present a different set of models from the general DNS model, which is the DNS model with unconstrained factor dynamics. In order to assess the satisfaction of these restrictions, we proceed to test them via likelihood ratio tests. The restricted models have the following risk-neutral dynamics:

- Univariate AR, $A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$
- Curvature added, $A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$
- λ restriction, $A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & 1 - a_{23} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$

Table 3.4: Test for DNS Models with Risk-Neutral Restrictions

	Test statistics	P-value
Likelihood ratio of restricted models		
Univariate AR	25.696	0.000
Curvature added	1.876	0.866
λ restriction	2.664	0.850
Cross λ restrictions	2.754	0.907
All restrictions	6.148	0.631
Granger causality test		
C_{t-1} does not Granger cause S_t	23.710	0.000
S_{t-1} does not Granger cause C_t	1.580	0.210

We present likelihood ratio test statistics for DNS models that possess various combinations of the risk-neutral restrictions in the factor dynamics, and the corresponding test statistics are Chi-square distributed. In the second panel, we also present Granger casuality test between the slope and curvature factors.

- Cross λ restrictions, $A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & 1-a_{23} & a_{23} \\ 0 & 0 & 1-a_{23} \end{bmatrix}$
- All restrictions, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-a_{23} & a_{23} \\ 0 & 0 & 1-a_{23} \end{bmatrix}$.

Therefore, the test statistics are $LR = 2(l(\hat{\theta}_{unrestricted}) - l(\hat{\theta}_{restricted}))$.

Table 3.4 presents likelihood ratio test statistics and corresponding P-values under the Chi-square distribution. The LR tests support the risk-neutral restrictions overwhelmingly, and the test also suggests univariate AR dynamics are not satisfied. In the second panel of the table, the Granger casuality test suggests the DNS curvature factor can forecast the DNS slope factor. The results are summarized as the following

- The level factor is block super exogenous.
- The level factor is random walk.
- The slope and curvature factors are mean-reverting at the same rate.
- The curvature factor is block exogenous.
- There is cross factors interaction between slope and curvature factors.
- The curvature factor Granger-cause slope factor, but not conversely.

Although all of those properties are driven theoretically under the risk-neutral dynamics, empirically they are all complied in the physical measure. These results suggest the risk premium between the physical and risk-neutral measures is quite low, and thus the Treasury bond market appears to close be risk-neutral.

3.3.2 The AFNS Model

In Table 3.5 we present estimation results for the general AFNS model. The estimate of the A matrix also indicates highly persistent own dynamics of L_t, S_t, and C_t with estimated own-lag coefficients of 0.99, 0.96, and 0.91, respectively. Cross-factor dynamics appear mostly unimportant, and there is statistically significant effect of C_{t-1} on S_t as well.

Similar to the DNS model, the risk-neutral restrictions on the factor dynamics also present a different set of models from the general AFNS model, which is

Table 3.5: The General AFNS Model Parameter Estimates

	L_{t-1}	S_{t-1}	C_{t-1}	U
L_t	0.994*	0.108	-0.138	7.309*
	(0.093)	(0.065)	(0.067)	(2.892)
S_t	0.029	0.964*	0.054*	-2.560
	(0.029)	(0.074)	(0.029)	(3.109)
C_t	-0.070	-0.012	0.908*	-0.669
	(0.118)	(0.108)	(0.100)	(0.974)

Note: Each row presents coefficients from the transition equation for the respective state variable. The standard errors are reported in parenthesis. The parameter values with an asterisk are significant at the 5% level.

the AFNS model with unconstrained factor dynamics. In continuous-time frame, the arbitrage-free equivalent of the DNS models is the following set of restricted models.

- Univariate AR, $K^P = \begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{22} & 0 \\ 0 & 0 & \kappa_{33} \end{bmatrix}$

- Curvature added, $K^P = \begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{22} & \kappa_{23} \\ 0 & 0 & \kappa_{33} \end{bmatrix}$

- λ restriction, $K^P = \begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{22} & -\kappa_{22} \\ 0 & 0 & \kappa_{33} \end{bmatrix}$

- Cross λ restrictions, $K^P = \begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{22} & -\kappa_{22} \\ 0 & 0 & \kappa_{22} \end{bmatrix}$

Table 3.6: Test for AFNS Models with Risk-Neutral Restrictions

	Test statistics	P-value
Likelihood ratio of restricted models		
Univariate AR	13.420	0.037
Curvature added	8.340	0.138
λ restriction	2.380	0.882
Cross λ restrictions	10.100	0.183
All restrictions	9.000	0.342
Granger causality test		
C_{t-1} does not Granger cause S_t	15.005	0.000
S_{t-1} does not Granger cause C_t	1.956	0.164

We present likelihood ratio test statistics for AFNS models that possess various combinations of the risk-neutral restrictions in the factor dynamics, and the corresponding test statistics are Chi-square distributed. In the second panel, we also present Granger casuality test between the slope and curvature factors.

- All restrictions, $K^P = \begin{bmatrix} 10^{-7} & 0 & 0 \\ 0 & \kappa_{22} & -\kappa_{22} \\ 0 & 0 & \kappa_{22} \end{bmatrix}$.

Table 3.6 presents likelihood ratio test statistics and corresponding P-values under the Chi-square distribution. The LR tests present similar results as the DNS model does. The test supports the risk-neutral restrictions overwhelmingly, and the test also suggests univariate AR dynamics are not satisfied. In the second panel of the table, the Granger casuality test also suggests the curvature factor can forecast the slope factor under the AFNS model.

3.4 Forecast Performance

In this section, we investigate out-of-sample forecast accuracy. First, describe the recursive estimation and forecasting procedure; then results are compared and contrasted.

3.4.1 Forecast Construction

The DNS model has been proven to have great forecasting performance. Diebold and Li (2006) have done extensive comparison to show that in their data sample for 6-month ahead and above the DNS model with uncorrelated factors (only a diagonal matrix A, i.e. a univariate AR model) outperforms many competitors including the random walk. The random walk was considered by Duffee (2002) to dominate those Dai and Singleton (2000) affine model.

The AFNS model also performs well as demonstrated by Christensen, Diebold, and Rudebusch (2007). Using a slightly different data sample from Diebold and Li (2006), they find that improvements in predictive performance are achieved by the imposition of absence of arbitrage. We, therefore, not only compare the performance between the general DNS and the general AFNS models, but impose restrictions on the transition dynamics to see whether those restrictions can improve the forecasts even further.

After extensive testing in the last section, we have strong reason to believe that for both DNS and AFNS models the curvature factor impacts the slope factor, and adding curvature factor into the specification could improve the forecasting performance. We also find other restrictions like the unit-root process of the level factor are also satisfied, so adding those restrictions could be beneficial to the forecasting performance as well. As great in-sample fit does not necessarily lead to the best out-of-sample performance, we proceed by imposing various combinations of the restrictions on the matrix A to evaluate the out-of-sample forecasting performance. Therefore, we estimate and forecast recursively, using data from 1987:1 to the time that the forecast is made, beginning in 1997:1 and extending through 2002:12.

We compare h-month ahead out-of sample forecasting results from both the DNS and the AFNS models with various combinations of the restrictions for maturities 3, 12, 24, 36, 60, 120, 240, and 360 months, and forecast horizons of h=1, 6, 12, 24, and 36 months. First let us describe how various forecasts are generated. All yield forecasts are produced by the Nelson-Siegel model, but β forecasts are specified by various specifications, i.e. for the DNS model,

$$\hat{y}_{t+h/t}(\tau) = \hat{\beta}_{1,t+h/t} + \hat{\beta}_{2,t+h/t}\Big(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\Big) + \hat{\beta}_{3,t+h/t}\Big(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\Big), \quad (3.26)$$

where $\hat{\beta}$'s are the DNS factor forecasts made by VAR(1) specifications; for the AFNS model,

$$\hat{y}_{t+h/t}(\tau) = \hat{\beta}_{1,t+h/t} + \hat{\beta}_{2,t+h/t}\Big(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\Big) + \hat{\beta}_{3,t+h/t}\Big(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\Big) - \frac{C(t,T)}{T-t}, \quad (3.27)$$

where $\hat{\beta}$'s are the AFNS factor forecasts made by VAR(1) specifications. There are 6 different VAR model specifications, which correspond to various combinations of the risk-neutral restrictions on the transition dynamics.

1. *Model A*: unrestricted VAR

$$\begin{pmatrix} \hat{\beta}_{1,t+h/t} \\ \hat{\beta}_{2,t+h/t} \\ \hat{\beta}_{3,t+h/t} \end{pmatrix} = \begin{pmatrix} \hat{c}_1 \\ \hat{c}_2 \\ \hat{c}_3 \end{pmatrix} + \begin{pmatrix} \hat{\gamma}_{11} & \hat{\gamma}_{12} & \hat{\gamma}_{13} \\ \hat{\gamma}_{21} & \hat{\gamma}_{22} & \hat{\gamma}_{23} \\ \hat{\gamma}_{31} & \hat{\gamma}_{32} & \hat{\gamma}_{33} \end{pmatrix} \begin{pmatrix} \hat{\beta}_{1,t} \\ \hat{\beta}_{2,t} \\ \hat{\beta}_{3,t} \end{pmatrix} \tag{3.28}$$

2. *Model B*: univariate AR

$$\hat{\beta}_{i,t+h/t} = \hat{c}_i + \hat{\gamma}_i \hat{\beta}_{it} \qquad i = 1, 2, 3. \tag{3.29}$$

   This is a special case of the unrestricted VAR with all the off-diagonal terms to be zero. Diebold and Li (2006) conclude this is the benchmark in terms of forecast.

3. *Model C*: curvature added

$$\begin{aligned} \hat{\beta}_{1,t+h/t} &= \hat{c}_1 + \hat{\gamma}_1 \hat{\beta}_{1t} \\ \hat{\beta}_{2,t+h/t} &= \hat{c}_2 + \hat{\gamma}_2 \hat{\beta}_{2t} + \hat{\gamma}_4 \hat{\beta}_{3t} \\ \hat{\beta}_{3,t+h/t} &= \hat{c}_3 + \hat{\gamma}_3 \hat{\beta}_{3t} \end{aligned} \tag{3.30}$$

4. *Model D*: λ restriction on slope

$$\begin{aligned} \hat{\beta}_{1,t+h/t} &= \hat{c}_1 + \hat{\gamma}_1 \hat{\beta}_{1t} \\ \hat{\beta}_{2,t+h/t} &= \hat{c}_2 + (1 - \hat{\gamma}_2) \hat{\beta}_{2t} + \hat{\gamma}_2 \hat{\beta}_{3t} \\ \hat{\beta}_{3,t+h/t} &= \hat{c}_3 + \hat{\gamma}_3 \hat{\beta}_{3t} \end{aligned} \tag{3.31}$$

5. *Model E*: cross λ restrictions on slope and curvature

$$\begin{aligned} \hat{\beta}_{1,t+h/t} &= \hat{c}_1 + \hat{\gamma}_1 \hat{\beta}_{1t} \\ \hat{\beta}_{2,t+h/t} &= \hat{c}_2 + (1-\hat{\gamma}_2)\hat{\beta}_{2t} + \hat{\gamma}_2 \hat{\beta}_{3t} \\ \hat{\beta}_{3,t+h/t} &= \hat{c}_3 + (1-\hat{\gamma}_2)\hat{\beta}_{3t} \end{aligned} \tag{3.32}$$

6. *Model F*: all restrictions

$$\begin{aligned} \hat{\beta}_{1,t+h/t} &= \hat{c}_1 + \hat{\beta}_{1t} \\ \hat{\beta}_{2,t+h/t} &= \hat{c}_2 + (1-\hat{\gamma}_2)\hat{\beta}_{2t} + \hat{\gamma}_2 \hat{\beta}_{3t} \\ \hat{\beta}_{3,t+h/t} &= \hat{c}_3 + (1-\hat{\gamma}_2)\hat{\beta}_{3t} \end{aligned} \tag{3.33}$$

In addition, random walk is also included for completeness.

$$\hat{y}_{t+h,t}(\tau) = y_t(\tau), \tag{3.34}$$

so the forecast is always "no change."

3.4.2 DNS Forecasts

Table 3.7 to 3.11 present the forecasting root mean squared errors (RMSE) for 1-month, 6-month, 1-year, 2-year, and 3-year into the future for the 6 different dynamic specifications using the DNS setup. Table 3.12 to 3.16 present the results in terms of RMSE rankings. The 1-month-ahead forecasting results for the DNS model, reported in Table 3.7, appear suboptimal. In relative terms, RMSE

Table 3.7: DNS Out-of-Sample 1-Month-Ahead Forecast Errors

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	0.235	**0.259**	**0.290**	0.300	**0.288**	**0.257**	**0.211**
Unrestricted VAR	0.213	0.261	0.307	0.302	0.310	0.314	0.419
Univariate AR	0.248	0.282	0.315	0.301	0.311	0.314	0.418
Curvature added	0.200	0.260	0.303	0.299	0.304	0.308	0.417
λ restriction	0.198	0.266	0.305	0.299	0.305	0.309	0.417
Cross λ restrictions	0.189	0.265	0.303	0.300	0.302	0.307	0.417
All restrictions	**0.186**	0.267	0.302	**0.298**	0.293	0.291	0.397

Note: We present the results of out-of-sample 1-month-ahead forecast using different specifications of the DNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the root mean square errors.

Table 3.8: DNS Out-of-Sample 6-Month-Ahead Forecast Errors

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	0.910	0.904	**0.908**	**0.857**	**0.813**	**0.651**	**0.486**
Unrestricted VAR	0.909	1.050	1.078	0.999	0.949	0.796	0.717
Univariate AR	0.972	1.030	1.001	0.912	0.868	0.740	0.706
Curvature added	0.783	0.941	0.972	0.909	0.875	0.745	0.708
λ restriction	0.780	0.934	0.973	0.911	0.879	0.750	0.710
Cross λ restrictions	0.728	0.918	0.976	0.926	0.893	0.758	0.712
All restrictions	**0.694**	**0.890**	0.938	0.881	0.827	0.655	0.570

Note: We present the results of out-of-sample 6-month-ahead forecast using different specifications of the DNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the root mean square errors.

Table 3.9: DNS Out-of-Sample 1-Year-Ahead Forecast Errors

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	1.552	1.505	1.343	1.190	1.065	0.858	**0.615**
Unrestricted VAR	1.643	1.731	1.641	1.483	1.338	1.123	0.979
Univariate AR	1.510	1.533	1.394	1.237	1.124	0.974	0.900
Curvature added	1.388	1.466	1.367	1.225	1.124	0.977	0.900
λ restriction	1.457	1.497	1.392	1.244	1.144	0.997	0.910
Cross λ restrictions	1.433	1.487	1.396	1.253	1.153	1.002	0.910
All restrictions	**1.383**	**1.445**	**1.338**	**1.182**	**1.041**	**0.819**	0.622

Note: We present the results of out-of-sample 1-year-ahead forecast using different specifications of the DNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the root mean square errors.

Table 3.10: DNS Out-of-Sample 2-Year-Ahead Forecast Errors

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	2.063	2.094	1.847	1.606	1.336	1.002	0.762
Unrestricted VAR	1.914	1.843	1.604	1.389	1.197	1.055	1.059
Univariate AR	**1.737**	**1.654**	**1.403**	**1.207**	**1.010**	0.855	0.855
Curvature added	1.943	1.797	1.494	1.265	1.034	0.855	0.852
λ restriction	2.114	1.928	1.600	1.349	1.108	0.912	0.883
Cross λ restrictions	2.257	2.054	1.696	1.422	1.146	0.922	0.883
All restrictions	2.364	2.172	1.805	1.514	1.180	**0.840**	**0.654**

Note: We present the results of out-of-sample 2-year-ahead forecast using different specifications of the DNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the root mean square errors.

Table 3.11: DNS Out-of-Sample 3-Year-Ahead Forecast Errors

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	1.643	1.717	1.543	1.375	1.205	1.060	0.938
Unrestricted VAR	2.046	1.928	1.724	1.609	1.568	1.642	1.829
Univariate AR	1.713	**1.426**	1.107	0.950	0.875	0.991	1.214
Curvature added	1.777	1.462	1.120	0.953	0.870	0.986	1.213
λ restriction	**1.545**	1.367	**1.085**	**0.912**	**0.852**	0.988	1.222
Cross λ restrictions	1.710	1.482	1.166	0.983	0.887	0.995	1.222
All restrictions	2.240	1.984	1.591	1.322	0.980	**0.702**	**0.670**

Note: We present the results of out-of-sample 3-year-ahead forecast using different specifications of the DNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the root mean square errors.

comparison at various maturities reveals forecasts with all the restrictions perform similarly as the random walk model and the DNS with univariate AR setting. For maturity beyond 10-year, the random walk model clearly outperforms.

Matters improve as the forecast horizon lengthens. The 1-year-ahead forecast results, reported in Table 3.9, reveal that the DNS model with all restrictions consistently outperforms the DNS with univariate AR setup for all maturities less or equal to 10-year.

However, when we lengthen the horizon to longer than 1 year, the DNS with all restrictions does not have the lead while the DNS with univariate AR becomes the superior model. From table 3.10, the DNS model with univariate AR setup outperforms all competitors for maturities up and including 10-year, whereas the DNS models with curvature restrictions (i.e. curvature factor is added to the slope equation) are now the best for maturities of more than 10-year.

When we lengthen the horizon to 3-year, the DNS models with curvature factor imposed on the slope equation have great forecast performance. For example, the DNS model with λ restriction has a RMSE of 1.545, 1.085, 0.912, and 0.852 for 3-, 24-, 36-, and 60-month, respectively. Moreover, the DNS model with all the restrictions has a RMSE of 0.67 for 30-year maturity.

Clearly, the DNS curvature factor has substantial forecasting power, and imposing restriction on the vector auto-regression can lead to improvement. The random walk is hard to beat for near-term forecasting, but we can do much better by utilizing the risk-neutral restrictions for longer horizons. Moreover, the DNS model with all restrictions are the preferred choice among all 6 dynamic specifications.

Table 3.12: DNS Out-of-Sample 1-Month-Ahead Forecast Rankings

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	6	1	1	5	1	1	1
Unrestricted VAR	5	3	6	7	6	7	7
Univariate AR	7	7	7	6	7	6	6
Curvature added	4	2	4	3	4	4	3
λ restriction	3	5	5	2	5	5	4
Cross λ restrictions	2	4	3	4	3	3	5
All restrictions	1	6	2	1	2	2	2

Note: We present the results of out-of-sample 1-month-ahead forecast using different specifications of the DNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t + h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the rankings in terms of root mean square errors. 1 is the best, whereas 7 is the worst.

Table 3.13: DNS Out-of-Sample 6-Month-Ahead Forecast Rankings

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	6	2	1	1	1	1	1
Unrestricted VAR	5	7	7	7	7	7	7
Univariate AR	7	6	6	5	3	3	3
Curvature added	4	5	3	3	4	4	4
λ restriction	3	4	4	4	5	5	5
Cross λ restrictions	2	3	5	6	6	6	6
All restrictions	1	1	2	2	2	2	2

Note: We present the results of out-of-sample 6-month-ahead forecast using different specifications of the DNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t + h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the rankings in terms of root mean square errors. 1 is the best, whereas 7 is the worst.

Table 3.14: DNS Out-of-Sample 1-Year-Ahead Forecast Rankings

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	6	5	2	2	2	2	1
Unrestricted VAR	7	7	7	7	7	7	7
Univariate AR	5	6	5	4	4	3	4
Curvature added	2	2	3	3	3	4	3
λ restriction	4	4	4	5	5	5	5
Cross λ restrictions	3	3	6	6	6	6	6
All restrictions	1	1	1	1	1	1	2

Note: We present the results of out-of-sample 1-year-ahead forecast using different specifications of the DNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+1$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the rankings in terms of root mean square errors. 1 is the best, whereas 7 is the worst.

Table 3.15: DNS Out-of-Sample 2-Year-Ahead Forecast Rankings

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	4	6	7	7	7	6	2
Unrestricted VAR	2	3	4	4	6	7	7
Univariate AR	1	1	1	1	1	3	4
Curvature added	3	2	2	2	2	2	3
λ restriction	5	4	3	3	3	4	5
Cross λ restrictions	6	5	5	5	4	5	6
All restrictions	7	7	6	6	5	1	1

Note: We present the results of out-of-sample 2-year-ahead forecast using different specifications of the DNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+1$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the rankings in terms of root mean square errors. 1 is the best, whereas 7 is the worst.

Table 3.16: DNS Out-of-Sample 3-Year-Ahead Forecast Rankings

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	2	5	5	6	6	6	2
Unrestricted VAR	6	6	7	7	7	7	7
Univariate AR	4	2	2	2	3	4	4
Curvature added	5	3	3	3	2	2	3
λ restriction	1	1	1	1	1	3	5
Cross λ restrictions	3	4	4	4	4	5	6
All restrictions	7	7	6	5	5	1	1

Note: We present the results of out-of-sample 3-year-ahead forecast using different specifications of the DNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+1$ as $y_{t+h}(\tau)-\hat{y}_{t+h/t}(\tau)$ and report the rankings in terms of root mean square errors. 1 is the best, whereas 7 is the worst.

One very interesting observation is that the unit root restriction on the level factor plays an important role for forecasting horizon up to 1-year. When the restriction is added, we can see a large reduction in RMSE.

These results are slightly different from Dieobld and Li (2006). Diebold and Li (2006) have a great success in forecasts using a different dataset with maturities up to 10-year, whereas we have maturities up to 30-year. The original Nelson-Siegel framework might fit the long maturities suboptimally. [4]

Overall, we are able to improve the forecasting ability by imposing all the restrictions on the VAR structure compared to the DNS model with univariate AR framework. Although among various specifications we cannot see a clear winner, we can conclude imposing some restrictions definitely improves the forecast performance beyond what Diebold and Li (2006)'s suggestion of univariate AR factor dynamics.

[4]Svensson (1995) introduces a variation of the Nelson-Siegel model to allow better fit at the long maturities.

Table 3.17: AFNS Out-of-Sample 1-Month-Ahead Forecast Errors

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	**0.235**	**0.259**	**0.290**	**0.300**	**0.288**	**0.257**	**0.211**
Unrestricted VAR	1.021	1.696	1.596	1.377	1.085	0.674	0.326
Univariate AR	1.002	1.691	1.593	1.374	1.081	0.671	0.323
Curvature added	1.009	1.690	1.591	1.373	1.081	0.670	0.323
λ restriction	1.014	1.692	1.593	1.374	1.081	0.671	0.323
Cross λ restrictions	1.029	1.692	1.591	1.373	1.080	0.670	0.323
All restrictions	1.029	1.692	1.591	1.373	1.080	0.670	0.323

Note: We present the results of out-of-sample 1-month-ahead forecast using different specifications of the AFNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t + h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the root mean square errors.

3.4.3 AFNS Forecasts

For the AFNS model, we estimate and forecast recursively, using data from 1987:1 to the time that the forecast is made, beginning in 1997:1 and extending through 2000:12. We adapt the same 6 forecasting specifications (Model A to F) used in the DNS model, and their corresponding root mean square errors are reported in table 3.17 to 3.21. The forecasts are constructed using the 6 different dynamic specifications under the AFNS setup. Table 3.22 to 3.26 present the results in terms of RMSE rankings.

The results of those forecasting exercises indicate that the AFNS factors do suboptimal job across maturities for the horizons less than 1-year. It starts to outperform the random walk after 1-year.

The forecast results seem to suggest imposing restrictions on the transition dynamics also improve the forecast performance when we forecast beyond 1-year ahead. It is similar to what we see from the DNS models. However, the AFNS improves the forecasts of the long-maturity yields more than the DNS model does.

Table 3.18: AFNS Out-of-Sample 6-Month-Ahead Forecast Errors

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	**0.910**	**0.904**	**0.908**	**0.857**	**0.813**	**0.651**	**0.486**
Unrestricted VAR	1.533	1.956	1.797	1.563	1.290	0.910	0.509
Univariate AR	1.411	1.935	1.802	1.578	1.309	0.928	0.535
Curvature added	1.422	1.942	1.806	1.581	1.312	0.930	0.536
λ restriction	1.432	1.945	1.808	1.582	1.312	0.930	0.536
Cross λ restrictions	1.458	1.951	1.812	1.586	1.315	0.932	0.537
All restrictions	1.347	1.838	1.697	1.472	1.200	0.815	0.501

Note: We present the results of out-of-sample 6-month-ahead forecast using different specifications of the AFNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the root mean square errors.

Table 3.19: AFNS Out-of-Sample 1-Year-Ahead Forecast Errors

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	1.552	**1.505**	**1.343**	**1.190**	**1.065**	**0.858**	0.615
Unrestricted VAR	2.161	2.315	2.089	1.836	1.565	1.205	0.735
Univariate AR	1.791	2.137	1.990	1.767	1.521	1.174	0.713
Curvature added	1.594	2.087	1.971	1.757	1.518	1.176	0.714
λ restriction	1.777	2.136	1.992	1.769	1.524	1.177	0.714
Cross λ restrictions	1.762	2.131	1.990	1.769	1.524	1.178	0.715
All restrictions	**1.522**	1.846	1.703	1.484	1.246	0.912	**0.611**

Note: We present the results of out-of-sample 1-year-ahead forecast using different specifications of the AFNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the root mean square errors.

Table 3.20: AFNS Out-of-Sample 2-Year-Ahead Forecast Errors

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	2.063	2.094	1.847	1.606	1.336	1.002	0.762
Unrestricted VAR	2.206	2.363	2.157	1.944	1.709	1.426	1.059
Univariate AR	**1.741**	2.022	1.873	1.673	1.444	1.166	0.840
Curvature added	1.795	2.034	1.877	1.676	1.446	1.166	0.840
λ restriction	2.074	2.141	1.931	1.709	1.464	1.174	0.841
Cross λ restrictions	2.064	2.138	1.929	1.708	1.464	1.174	0.841
All restrictions	1.981	**1.895**	**1.653**	**1.414**	**1.154**	**0.856**	**0.710**

Note: We present the results of out-of-sample 2-year-ahead forecast using different specifications of the AFNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the root mean square errors.

Table 3.21: AFNS Out-of-Sample 3-Year-Ahead Forecast Errors

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	1.643	1.717	1.543	1.375	1.205	1.060	0.938
Unrestricted VAR	2.319	2.457	2.308	2.146	1.982	1.823	1.542
Univariate AR	**1.295**	1.784	1.809	1.706	1.593	1.462	1.216
Curvature added	1.307	1.784	1.809	1.706	1.594	1.462	1.216
λ restriction	1.482	1.893	1.861	1.738	1.609	1.467	1.216
Cross λ restrictions	1.458	1.894	1.862	1.738	1.609	1.467	1.216
All restrictions	1.536	**1.501**	**1.351**	**1.179**	**0.991**	**0.805**	**0.720**

Note: We present the results of out-of-sample 3-year-ahead forecast using different specifications of the AFNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the root mean square errors.

Table 3.22: AFNS Out-of-Sample 1-Month-Ahead Forecast Rankings

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	1	1	1	1	1	1	1
Unrestricted VAR	7	7	7	7	7	7	7
Univariate AR	2	3	5	5	6	6	5
Curvature added	3	2	2	2	4	3	2
λ restriction	4	5	6	6	5	5	4
Cross λ restrictions	6	6	4	4	3	4	6
All restrictions	5	4	3	3	2	2	3

Note: We present the results of out-of-sample 1-month-ahead forecast using different specifications of the AFNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the rankings in terms of root mean square errors. 1 is the best, whereas 7 is the worst.

Moreover, among different model specifications ($Model\ A-F$), the superior performance is obtained when we impose all restrictions, i.e. $Model\ F$, and thus the restricted transition matrix do help the forecast for AFNS models. Furthermore, excluding the random walk results, the *All restrictions* specification is the preferred choice among the 6 dynamic specifications for almost all horizons of 6-month or longer.

Table 3.23: AFNS Out-of-Sample 6-Month-Ahead Forecast Rankings

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	1	1	1	1	1	1	1
Unrestricted VAR	7	7	7	3	3	3	3
Univariate AR	3	3	3	4	4	4	4
Curvature added	4	4	4	5	5	5	5
λ restriction	5	5	5	6	6	6	6
Cross λ restrictions	6	6	6	7	7	7	7
All restrictions	2	2	2	2	2	2	2

Note: We present the results of out-of-sample 6-month-ahead forecast using different specifications of the AFNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t + h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the rankings in terms of root mean square errors. 1 is the best, whereas 7 is the worst.

Table 3.24: AFNS Out-of-Sample 1-Year-Ahead Forecast Rankings

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	2	1	1	1	1	1	2
Unrestricted VAR	7	7	7	7	7	7	7
Univariate AR	6	6	5	4	4	3	3
Curvature added	3	3	3	3	3	4	4
λ restriction	5	5	6	5	5	5	5
Cross λ restrictions	4	4	4	6	6	6	6
All restrictions	1	2	2	2	2	2	1

Note: We present the results of out-of-sample 1-year-ahead forecast using different specifications of the AFNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t + h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the rankings in terms of root mean square errors. 1 is the best, whereas 7 is the worst.

Table 3.25: AFNS Out-of-Sample 2-Year-Ahead Forecast Rankings

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	4	4	2	2	2	2	2
Unrestricted VAR	7	7	7	7	7	7	7
Univariate AR	1	2	3	3	3	4	4
Curvature added	2	3	4	4	4	3	3
λ restriction	6	6	6	6	5	6	5
Cross λ restrictions	5	5	5	5	6	5	6
All restrictions	3	1	1	1	1	1	1

Note: We present the results of out-of-sample 2-year-ahead forecast using different specifications of the AFNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the rankings in terms of root mean square errors. 1 is the best, whereas 7 is the worst.

Table 3.26: AFNS Out-of-Sample 3-Year-Ahead Forecast Rankings

Output-of-sample 3-year-ahead forecast rankings

	3-month	1-year	2-year	3-year	5-year	10-year	30-year
Random walk	6	2	2	2	2	2	2
Unrestricted VAR	7	7	7	7	7	7	7
Univariate AR	1	4	4	4	3	4	4
Curvature added	2	3	3	3	4	3	3
λ restriction	4	5	5	5	5	5	5
Cross λ restrictions	3	6	6	6	6	6	6
All restrictions	5	1	1	1	1	1	1

Note: We present the results of out-of-sample 3-year-ahead forecast using different specifications of the AFNS factors described in the text. We estimate all models recursively from 1987:1 to the time forecast is made, beginning in 1997:1 and extending through 2002:12. We define forecast errors at $t+h$ as $y_{t+h}(\tau) - \hat{y}_{t+h/t}(\tau)$ and report the rankings in terms of root mean square errors. 1 is the best, whereas 7 is the worst.

3.5 Alternative Estimation Method

In this section, we estimate factors via a two-step procedure as a support to the results presented earlier. We then impose a VAR on the estimated factors ($X_t = \{L_t, S_t, C_t\}$), and we test whether the risk-neutral restrictions are satisfied for these factors, which are obtained under the physical measure.

3.5.1 The DNS Model

Following Diebold and Li (2006), we also use a two-step procedure to estimate the factors. We fix λ at 0.0609, which implies that the loading on the curvature factor achieves maximum at 30-month maturity, and once λ is fixed, the ordinary least squared can produce the three factors.

A standard VAR(1) structure is imposed on these estimated DNS factors,

$$\hat{X}_t = U + A\hat{X}_{t-1} + \eta_t, \tag{3.35}$$

and the estimated parameter values are reported in Table 3.27.

The DNS level factor appears to be a unit-root process, and it is the most persistent factor. The DNS slope and curvature factors are both mean-reverting. Importantly, the only significant off diagonal element ($a_{23} = 0.072$) in the estimated A matrix is $A_{S_t,C_{t-1}}$, which is the key non-zero off-diagonal element required for arbitrage free version of the specification. In addition, $A_{S_t,C_{t-1}}$ corresponds to the λ, which is prefixed at the value of 0.0609 in Diebold and Li (2006). Given the standard error of 0.015, they are statistically indifferent from each other.

Since the restriction on the K^Q matrix is key to establish the arbitrage-free

Table 3.27: Two-Step VAR Estimation Results for L_t, S_t, C_t Factors from the DNS Model

	L_{t-1}	S_{t-1}	C_{t-1}	U
L_t	0.989*	0.005	-0.004	5.946
2	(0.017)	(0.013)	(0.011)	(11.073)
S_t	-0.016	0.937*	0.072*	-5.712
	(0.022)	(0.018)	(0.015)	(20.182)
C_t	0.075	0.060	0.895*	-3.923
	(0.059)	(0.049)	(0.040)	(19.401)

Note: VAR estimation results for L_t, S_t, C_t factors from the DNS model from 1987:1 to 2002:12. The standard errors are reported in parenthesis. The parameter values with an asterisk are significant at the 5% level.

Table 3.28: Two-Step DNS Factors Test Results

	Test statistics	P-value
F test		
Univariate AR	14.410	0.025
Curvature added	5.481	0.360
λ restriction	0.401	0.527
Cross λ restrictions	1.105	0.576
All restrictions	4.564	0.803
Granger causality test		
C_{t-1} does not Granger cause S_t	11.403	0.000
S_{t-1} does not Granger cause C_t	1.114	0.330

The DNS factors are tested via an F type test for various combinations of the restrictions on the A matrix and Granger casuality test between the slope and curvature factors. The sample period is 1987:01-2002:12. The values with asterisk indicate a reject of the null hypothesis.

Table 3.29: Two-Step VAR Estimation Results for L_t, S_t, C_t Factors from the AFNS Model

	L_{t-1}	S_{t-1}	C_{t-1}	U
L_t	0.989*	-0.001	0.002	6.688
	(0.015)	(0.014)	(0.014)	(9.026)
S_t	0.015	0.967*	0.060*	-7.416
	(0.022)	(0.018)	(0.015)	(30.616)
C_t	0.018	0.054	0.873*	-3.058
	(0.047)	(0.035)	(0.040)	(14.272)

Note: VAR estimation results for L_t, S_t, C_t factors from the AFNS model from 1987:1 to 2002:12. The standard errors are reported in parenthesis. The parameter values with an asterisk are significant at the 5% level.

dynamic Nelson-Siegel Model, we perform F test on the estimated transition matrix A. The testing of corresponding hypotheses is thus to check whether those risk-neutral restrictions are satisfied under the physical measure.

Table 3.28 illustrates the F test of the validity of the restrictions imposed on the transition dynamics. By wide margins, various combinations of the restrictions are well accepted by the data. The Granger casuality test confirms the ability of the curvature factor to predict the slope factor, and not conversely. Therefore, we can confirm the properties stated earlier are indeed satisfied for the estimated factors.

3.5.2 The AFNS Model

A standard VAR(1) structure, same as the one imposed on the DNS factors, is established for the AFNS factors and the corresponding results are reported in table 3.29.

The AFNS level factor appears closely to be an unit root process, and it is

Table 3.30: Two-Step AFNS Factors Test Results

	Test statistics	P-value
F test		
Univariate AR	11.159	0.084
Curvature added	5.227	0.389
λ restriction	2.788	0.095
Cross λ restrictions	6.918	0.032
All restrictions	10.811	0.213
Granger causality test		
C_{t-1} does not Granger cause S_t	14.674	0.000
S_{t-1} does not Granger cause C_t	1.910	0.169

The AFNS factors are tested via an F type test for various combinations of the restrictions on the A matrix and Granger casuality test between the slope and curvature factors.

the most persistent factor. The AFNS slope and curvature factors are both mean reverting. Importantly, the only significant off diagonal element[5] in the estimated A matrix is $A_{S_t,C_{t-1}}$, which is the key non-zero off-diagonal element required for arbitrage free version of the specification.

In table 3.30, all the test statistics and corresponding P-values are reported. All the null hypothesis are accepted at 5% level except for the combination restrictions on a_{22}, a_{23}, a_{33}, which requires a significance level of 1% to accept the null hypothesis. The Granger causality test also supports one direct and the other, i.e. curvature Granger causes the slope and not vice verse. Similar to the results obtained for the DNS factors, we can conclude the same set of properties are also well satisfied for the AFNS factors.

[5] Its value is 0.06, which is also statistically indifferent from 0.0609 in the DNS setup.

3.6 Model-free factors

The L_t, S_t, C_t estimated from the DNS and the AFNS models are model-based factors, while following the literature such as Frankel and Lown (1994) model-free factors of level, slope and curvature (L_t^f, S_t^f and C_t^f) can be constructed. For robustness, we also test those restrictions on the model-free factors. The model-free level factor is the long term yield $y_t(\infty)$, the model-free slope factor is difference of the long term and short term yield $y_t(\infty)) - y_t(0)$, and the model free curvature factor is twice of the medium term yield minus the sum of the long term and short term yields. Therefore, model-free factors are defined as the follows:

$$\begin{aligned} L_t^f &= y_t(360), \\ S_t^f &= y_t(360) - y_t(3), \\ C_t^f &= 2y_t(36) - y(360) - y(3). \end{aligned} \tag{3.36}$$

We impose the same VAR(1) structure on the model free factors ($X_t^f = \{L_t^f, S_t^f, C_t^f\}$) as

$$X_t = U + AX_{t-1} + \eta_t. \tag{3.37}$$

The estimation results are reported in table 3.31.

The model-free level factor also appears to be a unit root process, and it is the most persistent factor. The model-free slope and curvature factors are both mean reverting as well. Moreover, the only significant off diagonal element (the 0.113) in the estimated A matrix is $A_{S_t^f, C_{t-1}^f}$, which corresponds to λ and is the key non-zero off-diagonal element required for arbitrage-free specifications.

Table 3.31: VAR Estimation Results for Model-Free Level, Slope, Curvature Factors

	L^f_{t-1}	S^f_{t-1}	C^f_{t-1}	U^f
L^f_t	0.983*	0.003	0.012	4.689
	(0.020)	(0.017)	(0.030)	(11.519)
S^f_t	0.003	0.967*	0.113*	-5.887
	(0.021)	(0.017)	(0.033)	(20.095)
C^f_t	0.024	0.015	0.865*	-0.615
	(0.028)	(0.021)	(0.039)	(0.689)

Note: VAR estimation results for model free factors L^f_t, S^f_t, C^f_t. The standard errors are reported in parenthesis. The parameter values with an asterisk are significant at 1%.

Table 3.32: Model-free Factors Test Results

	Test statistics	P-value
F test		
Univariate AR	13.638	0.034
Curvature added	2.702	0.746
λ restriction	2.460	0.620
Cross λ restrictions	10.765	0.005*
All restrictions	23.182	0.003*
Granger causality test		
C^f_{t-1} does not Granger cause S^f_t	26.709	0.000
S^f_{t-1} does not Granger cause C^f_t	0.526	0.469

Note: The model free factors are tested via an F type test for various combinations of the restrictions on the A matrix and Granger casuality test between the slope and curvature factors.

Table 3.32 reports the results from F tests and Granger causality tests. Most of the properties for the model-based factors are also satisfied for the model free factors. The restrictions that are violated are the λ restrictions on the slope and curvature factors. Therefore, the results from the model-free factors confirm an almost identical set of properties as the model-based factors.

3.7 Concluding Remarks

In this chapter, we have comprehensively analyzed DNS and AFNS models. We found that risk-neutral restrictions are satisfied in-sample for factors estimated from both DNS and AFNS models in the physical measure, and out-of-sample performance is improved by imposing these restrictions. This leads us to conclude the risk premium for the U.S. Treasury bond market is quite low. Empirically, we show that: 1) the level factor is a unit-root process and does not affect the other two factors; 2) the slope and curvature factors are mean-reverting processes that reverts at the same rate; 3) the curvature factor forecasts the slope factor. Moreover, these properties also facilitate out-of-sample forecasting for 6-month ahead and above. Given the good performance of the Nelson-Siegel models both in-sample and out-of-sample, we consider the Nelson-Siegel class of models an excellent model for the U.S. Treasury yields.

We have also enhanced the understanding of the risk-neutral dynamics. These risk-neutral restrictions are well complied in-sample, and utilizing them also improves forecasts. Moreover, we do not actually explain the determinants of the curvature factor; rather, we explain what the curvature factor determines, which is the one-way Granger causality from the curvature factor to the slope factor.

The performance of Nelson-Siegel models are impressive, and we have established several stylized facts about this class of term structure models. In the future it is desirable to compare its performance against affine term structure models.

APPENDICES

APPENDIX A

Chapter II: "Raw" yields construction

This section describes the construction method for corporate bond yields. Let $P_t(\tau)$ be the price of a τ-period discount bond and $y_t(\tau)$ denote its continuously compounded zero-coupon nominal yield to maturity. Thus, the discount curve is,

$$P_t(\tau) = e^{-\tau y_t(\tau)}.$$

From the discount curve, the forward rate curve is,

$$f_t(\tau) = -P_t'(\tau)/P_t(\tau).$$

Therefore, the relationship between the yield to maturity and the forward rate is,

$$y_t(\tau) = \frac{1}{\tau}\int_0^{\tau} f_\tau(u)du,$$

which implies that the zero-coupon yield is an equally weighted average of forward rates. Once the yield curve or the forward curve is given, any coupon bond can be priced as the sum of the present values of future coupon and principal payments.

However, in practice, neither yield curves nor forward curves are observed, so they must be estimated from observed bond prices. Fama and Bliss (1987) establish a popular yield curve construction method. They construct yields not via an estimated discount curve, but rather via estimated forward rates at the observed maturities. Their method sequentially constructs the forward rates necessary to price successively longer-maturity bonds, often called an "unsmoothed Fama-Bliss" forward rate, and then constructs "unsmoothed Fama-Bliss yields" by averaging the appropriate unsmoothed Fama-Bliss forward rates. The unsmoothed Fama-Bliss yields price the included bonds exactly. Throughout the chapter, unsmoothed Fama-Bliss yields are adapted for corporate bonds.

APPENDIX B

Chapter II: Theory of corporate bond pricing

This section briefly discusses the theory of corporate bond pricing.[1] In Merton's model, the value of the firm's assets is assumed to obey a lognormal diffusion process with constant volatility, and the firm issues two classes of securities: debt and equity, where debt is a pure discount bond of payment D promised at time T, and the equity, E, pays no dividends. If at time T the firm's asset value exceeds the promised payment, D, the lenders are paid the promised amount and the shareholders receive the residual asset value. If the asset value is less than D, the firm defaults, and the lenders receive a payment equal to the asset value while the shareholders get nothing.

[1]See Hull, Nelken, White (2004) for more detailed reviews.

B.1 Default probability and equity value

Define A as the value of the firm's assets, let E_0 and A_0 be today's value of E and A, and let E_T and A_T be their values at time T. According to Merton's framework, the payment to shareholders at time T is given by

$$E_T = max\{A_T - D, 0\}.$$

The equity is a call option on the firm's assets with the strike price equal to the promised debt payment. Define $D^* = De^{-rT}$ as the present value of the debt and let $L = D^*/A_0$ be the leverage ratio. Hence, using these definitions the current $(t = 0)$ equity price is

$$E_0 = A_0[N(d_1) - LN(d_2)], \tag{B.1}$$

where

$$\begin{aligned} d_1 &= \frac{ln(L)}{\sigma_A\sqrt{T}} + 0.5\sigma_A\sqrt{T}, \\ d_2 &= d_1 - \sigma_A\sqrt{T}, \end{aligned}$$

σ_A is the volatility of the asset value, and r is the risk-free interest rate, both of which are assumed to be constant.

As shown by Jones et al. (1984), Ito's Lemma relates the equity volatility as a function of the asset volatility,

$$E_0\sigma_E = \frac{\partial E}{\partial A}A_0\sigma_A,$$

where σ_E is the volatility of the firm's equity at time zero. From equation (1), this leads to

$$\sigma_A = \sigma_E \frac{N(d_1) - LN(d_2)}{N(d_1)}. \tag{B.2}$$

Equations (1) and (2) allow A_0 and σ_A to be obtained from E_0, σ_E, L and T.[2] The risk-neutral probability of default is given by $P = N(-d_2)$, which depends on the leverage, L, the asset volatility, σ_A, and the time to repayment, T.

B.2 Spreads of risky debt

The credit risk premium measured by the spread over comparable maturity U.S. Treasury securities can be explained by Merton's model. Define B_0 as the market price of the debt at time zero. The firm's assets at any time equal the total value of the two financing sources, bond and equity, and thus

$$B_0 = A_0 - E_0. \tag{B.3}$$

Using equation (1), (3) becomes

$$B_0 = A_0[N(-d_1) + LN(d_2)]. \tag{B.4}$$

The yield to maturity of the debt, y, is defined by

$$B_0 = De^{-yT} = D^* e^{(r-y)T}. \tag{B.5}$$

[2]This implementation has been used by many practitioners. Moody's KMV uses it to estimate relative default probability. CreditGrades, a venture supported by major banks such as Goldman Sachs, JP Morgan, and Deutsche Bank, uses it to estimate credit default swap spreads.

Substituting this into the equation and applying $A_0 = D^*/L$ gives the credit spread, CS, as

$$CS = y - r = -ln[N(d_2) + N(-d_1)/L]T. \tag{B.6}$$

The credit spread derived from Merton's model, like the risk-neutral default probability, depends only on the leverage, L, the asset volatility, σ_A, and the time to repayment, T. Since asset volatility can be estimated from equity volatility, the credit spread is linked with the equity volatility: the higher the volatility, the higher the spread.

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