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Mode Orthogonality in Chirowaveguides

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Abstract—In this paper, we derive the orthogonality relations for modes supported by a general cylindrical chirowaveguide. As introduced in our earlier work, a chirowaveguide is a cylindrical waveguide filled with chiral or optically active materials. As in conventional waveguides, the orthogonality relations reported here can be used to expand an arbitrary E or H field within a chirowaveguide in terms of a complete set of mutually orthogonal modes in the waveguide.

I. INTRODUCTION

THE CONCEPT of chirality, or handedness, has been a subject of interest in a variety of fields, such as chemistry [1], particle physics [2], optics [3] and mathematics [4]. The original investigations of the effect of material chirality on light polarization, known as optical activity, date back to the 19th century. Arago [5], Biot [6]-[8], Pasteur [1], and Fresnel [9] all examined optical activity in solid and liquid chiral media. For the time harmonic excitation ($e^{-i\omega t}$) and isotropic case, a chiral medium is electromagnetically characterized by the following set of constitutive relations:

$$D = \varepsilon_{r} \varepsilon_{0} E + i \frac{\varepsilon_{c}}{\varepsilon_{0}} B,$$

$$H = i \frac{\varepsilon_{c}}{\mu_{0}} E + \frac{B}{\mu_{c}},$$

where $\varepsilon_{r}$, $\mu_{c}$, and $\varepsilon_{c}$ represent, respectively, the permittivity, permeability, and chirality admittance of the chiral medium [10]. It has been shown that electromagnetic waves in these media display two unequal characteristic wavenumbers,

$$k_{\pm} = \pm \sqrt{\omega \mu_{c} \varepsilon_{c} - \omega^{2} \mu_{r} \varepsilon_{r} + \mu_{r}^{2} \varepsilon_{r}^{2}},$$

for the right and left circularly polarized (RCP, LCP) eigenmodes [11]. The set of chiral constitutive relations given in (1) and (2) is actually a subset of the more general constitutive relations used to describe bianisotropic media. These generalized relations have been studied extensively by Kong [12]-[15]. Recently, there has been renewed attention to the area of wave propagation in chiral media owing to the possibility of fabricating such materials for microwaves and millimeter waves. In the past few years, electromagnetic chirality [16] and chiral materials have been extensively investigated in a large number of applications. Among these, one should mention wave-guiding structures filled with chiral materials [17]-[19], dyadic Green's functions in chiral media [11], [20], transition radiation caused by a chiral slab [21], Doppler effects in chiral media [22], wave propagation in periodic chiral structures [23], spherical lenses made from chiral materials [24], [25], and reflection and refraction at a chiral-nonchiral interface [26]-[29].

In our previous works, we introduced the idea of a chirowaveguide, which is a cylindrical guided-wave structure filled with isotropic chiral materials, and we reported a detailed analysis of the propagation characteristics of electromagnetic waves guided through such structures [17], [18]. We also addressed the notable features of these waveguides and discussed their potential applications in microwave, millimeter-wave, and optical regimes. Here we analyze orthogonality relations for the modes of chirowaveguides. As in conventional waveguides, such orthogonality relations can be used to represent an arbitrary electric or magnetic field within a chirowaveguide in terms of the superposition of mode functions.

II. ORTHOGONALITY RELATIONS

Fig. 1 presents the geometry of the problem. A cylindrical waveguide with an arbitrary cross-sectional shape is filled with isotropic chiral materials described by (1) and (2). The axis of the waveguide is along the $z$ axis. The walls of this chirowaveguide are assumed to be perfectly conducting. The cross section of the waveguide, which is bounded by the curve $C$, and parameters of the material filling the guide are independent of $z$. We have analyzed and reported elsewhere the general characteristics of guided modes in such a guided-wave structure. Let us now consider two different modes, viz., $m$th and $n$th modes, propagating in this chirowaveguide. The electric and magnetic fields of these modes are $E_{m}$, $H_{m}$, and $E_{n}$, $H_{n}$.
respectively. $p,,$ and $p,,$ denote the wavenumbers in the guide for the $m$th and $n$th modes. The electromagnetic fields considered inside the chirowaveguide propagate along the $z$ axis. Thus we have

$$E_m = e_m e^{i p, x} = (e_m + e_{nz} \hat{z}) e^{i p, z}$$

and

$$H_m = h_m e^{i p, x} = (h_m + h_{nz} \hat{z}) e^{i p, z}$$

for the $m$th guided mode and

$$E_n = e_n e^{i p, x} = (e_n + e_{nz} \hat{z}) e^{i p, z}$$

and

$$H_n = h_n e^{i p, x} = (h_n + h_{nz} \hat{z}) e^{i p, z}$$

for the $n$th guided mode, where $\hat{z}$ is the unit vector along the $z$ axis. Here, $e_m, h_m,$ and $e_n, h_n,$ with transverse parts $e_{nz}, h_{nz},$ and the longitudinal components $e_{nz}, h_{nz}$ are functions of the transverse coordinates $x$ and $y.$ Without loss of generality, we assume that positive indices correspond to modes traveling in the positive $z$ direction and negative indices to those traveling in the negative $z$ direction. These modes satisfy the Maxwell equations and the boundary conditions on the walls of the chirowaveguide. Thus we have

$$\nabla \times E = i \omega \mu_\epsilon H + \omega \mu_\epsilon \hat{z} E$$

and

$$\nabla \times H = \omega \mu_\epsilon \hat{z} H - i \omega \epsilon_\epsilon \left(1 + \frac{\mu_\epsilon}{\epsilon_\epsilon} \hat{z}^2\right) E$$

for the $m$th and $n$th modes, respectively. Here $n$ is the unit normal to the wall of the chirowaveguide. From (7), (8), (10), (11), and vector identities, it can be easily shown that

$$\nabla \cdot (E \times H^*) = H^* \nabla \times E - E \nabla \times H^*$$

and

$$\nabla \cdot (E^* \times H) = H \nabla \times E^* - E^* \nabla \times H$$

where the asterisk denotes complex conjugation. Adding (13) and (14), we obtain

$$\nabla \cdot [(E_m \times H_n^* + E_n^* \times H_m)]$$

$$= i \omega (\mu_\epsilon - \mu_\epsilon^*) H_m \cdot H_n^*$$

$$+ i \omega \left(e_\epsilon \left(1 + \frac{\mu_\epsilon}{\epsilon_\epsilon} \hat{z}^2\right) e_\epsilon - \epsilon_\epsilon \left(1 + \frac{\mu_\epsilon^*}{\epsilon_\epsilon^*} \hat{z}^2\right) e_\epsilon^* \right) E_m \cdot E_n^*$$

and

$$+ \omega (\mu_\epsilon \hat{z} - \mu_\epsilon^* \hat{z}^*) (E_m \cdot H_n^* - H_m \cdot E_n^*) \right) \left((E_m \cdot H_n^* - H_m \cdot E_n^*) \right) \left((E_m \cdot H_n^* - H_m \cdot E_n^*) \right)$$

(15)

Now, provided that the chiral material filling the chirowaveguide is lossless, $\epsilon_\epsilon, \mu_\epsilon,$ and $\hat{z}$ are real quantities; thus from (15) we obtain

$$\nabla \cdot (E_m \times H_n^* + E_n^* \times H_m) = 0.$$  

(16)

Integrating (16) over the cross section of the waveguide and using $\nabla = \nabla + (\partial / \partial z) \hat{z},$ we obtain

$$\int_S \nabla \cdot (E_m \times H_n^* + E_n^* \times H_m) \ dS = 0$$

for the two different modes $n$ and $m.$ This implies that the above integral must be zero. That is,

$$\int_S (E_m \times H_n^* + E_n^* \times H_m) \cdot n \ dS = 0.$$  

(17)

By using the two-dimensional form of the divergence theorem, the integral having the operator $\nabla$ in (17) can be written in the form of an integral over the contour $C.$ Hence (17) can be written as

$$\int_C (E_m \times H_n^* + E_n^* \times H_m) \cdot n \ dl = 0.$$  

(18)

where $dl$ is an infinitesimal line element along the curve $C.$ Since the tangential components of the electric field on the surface of the boundary must vanish, the vector $(E_m \times H_n^* + E_n^* \times H_m)$ is tangent to the boundary; hence the line integral given in (18) is identically zero. Thus we have

$$\hat{z} \cdot \int_S \frac{\partial}{\partial z} (E_m \times H_n^* + E_n^* \times H_m) \ dS = 0.$$  

(19)

Substituting (3)–(6) into (19) yields

$$(\beta_m - \beta_n) \int_S (\epsilon_m \times h_n^* + e_n^* \times h_m) \cdot \hat{z} dS = 0.$$  

(20)

For the two different modes $n$ and $m,$ $\beta_m \neq \beta_n.$ This implies that the above integral must be zero. That is,

$$\int_S (\epsilon_m \times h_n^* + e_n^* \times h_m) \cdot \hat{z} dS = 0.$$  

(21)

This is an orthogonality relation for the modes in a lossless chirowaveguide. If the two modes are degenerate, i.e., $\beta_m = \beta_n,$ (21) does not necessarily hold. To ensure orthogonality for degenerate modes, one can construct a proper linear combination of the degenerate modes such that the new subset becomes an orthogonal set and (21) applies to them. This technique is commonly used for conventional waveguides filled with nonchiral materials [30].

1 For evanescent modes, this convention corresponds to modes decaying in the positive and negative $z$ directions, respectively.
i.e., \( n = m \), the above integral yields a nonzero value which is proportional to the power \( P_n \) carried by the mode. Thus, we can write (21) in the following form:

\[
\oint_S (e_m \times h_n^* + e_n \times h_m^*) \cdot \hat{z} dS = 4 P_n \text{sgn}(n) \delta_{mn} \quad (22)
\]

where \( \delta_{mn} \) is a Kronecker delta, \( P_n \) is the power carried by the \( n \)th mode, and \( \text{sgn}(n) \) denotes the sign of \( n \), i.e., the direction of propagation of the \( n \)th mode in the guide.

Using a similar derivation, we obtain another type of orthogonality relation for modes in chirowaveguides. That is,

\[
\oint_S (e_m \times h_n - e_n \times h_m) \cdot \hat{z} dS = 0. \quad (23)
\]

These orthogonality relations resemble those obtained for gyrotropic waveguides [31].

As in conventional waveguides, the orthogonality relation (22) has the physical meaning that the power carried by an arbitrary electromagnetic field within a chirowaveguide is the sum of the powers carried by all possible modes in that waveguide. Indeed, (22) can be used to expand an arbitrary electric or magnetic field in a chirowaveguide in terms of the mode functions. More specifically, for a given time harmonic electric or magnetic field, \( E \) or \( H \), satisfying the Maxwell equations and the boundary conditions within a chirowaveguide, one can write

\[
E = \sum_m a_m (e_m + e_m^*) e^{i\beta_m z} \quad (24)
\]

\[
H = \left( \frac{1}{i\omega \mu_e} \right) (\nabla \times E - \omega \mu_e \xi, \xi, E) = \sum_m a_m (h_m + h_m^*) e^{i\beta_m z} \quad (25)
\]

where the sum is extended over all possible modes. By using (22), the expansion coefficients \( a_m \) are obtained as

\[
a_m = \frac{e^{-i\beta_m z}}{4P_n \text{sgn}(n)} \oint_S (E \times h_n^* + e_n^* \times H) \cdot \hat{z} dS. \quad (26)
\]

The alternative orthogonality relation given in (23) does not have physical meaning and the above-mentioned interpretation of power orthogonality does not hold for that relation. It must be noted that the mode orthogonality relation expressed in (22) applies only to lossless chirowaveguides. If either \( \mu_e \), \( \epsilon_r \), or \( \xi \) is a complex quantity, relation (22) will not hold. However, (23) holds for lossy as well as lossless chirowaveguides. It is also worth noting that in deriving the two orthogonality relations we did not need to assume that the material parameters are constant over the cross section of the chirowaveguide, only that they are independent of \( z \). Therefore, these orthogonality relations also hold for cylindrical chirowaveguides partially filled with chiral materials.

For open wave-guiding structures containing chiral materials, such as dielectric chirowaveguides which have no conducting walls, the foregoing results can also be applied. However, care must be taken in using these orthogonality relations for such open structures. It is well known that open waveguides can support two types of modes: guided modes with discrete wavenumbers, and radiation modes whose wavenumbers form a continuum. The guided modes have fields that decay exponentially away from the guiding region of the structure, whereas radiation modes have fields whose distributions are not localized near the guiding region. The indices \( m \) and \( n \) used to distinguish between two different modes may indicate either guided modes or radiation modes. For guided modes, these indices are discrete quantities while for radiation modes they form a continuum. In using (22) for open chirowaveguides, the surface \( S \), over which the integral is carried out, is the entire transverse plane, normal to the longitudinal axis of the guide, extending to infinity.

If the two different modes (\( m \)th and \( n \)th) are both guided modes or one is guided and the other is a radiation mode, the Kronecker delta can still be used and (22) still holds. However, if the two modes in (22) are radiation modes, the Kronecker delta must be replaced by the Dirac delta function \( \delta(m - n) \). Furthermore, it must be noted that for open chirowaveguides, the expansions (24) and (25) should also include radiation modes for which integration must be used instead of summation. Therefore, for simplicity, the single summation symbol can be used to indicate both the sum over discrete guided modes and the integration over the continuum radiation modes.

### III. Summary

In this paper, we have obtained orthogonality relations for electromagnetic modes supported by cylindrical chirowaveguides. These wave-guiding structures are cylindrical waveguides containing chiral materials. It has been pointed out that one of these relations is valid for lossless waveguides while the other holds for the lossy as well as the lossless case. We have also demonstrated that the orthogonality relations can be used to express an arbitrary electric or magnetic field within a chirowaveguide in terms of the mode functions. The orthogonality relations obtained here resemble those used for gyrotropic waveguides.

\[\text{Indeed, in this case the surface } S \text{ is split into two portions: one is the cross section of the waveguide } S_e \text{ and the other is the rest of the transverse plane } S_u. \text{ Equation (17) is then used for each of the surfaces } S_e \text{ and } S_u. \text{ When the two-dimensional form of the divergence theorem is used in the first integral of (17) over each surface, the line integral over contour } C \text{ for the integral over surface } S_e \text{ will be canceled by the line integral over the same contour for the integral over } S_u. \text{ This is due to the continuity of tangential electric and magnetic fields at the boundary of open wave-guiding structures. Thus the only line integral in (18) is the one being carried out at infinity, which, for guided modes, vanishes.}\]
Nader Engheta (S’80–M’82–SM’89) was born in Tehran, Iran, on October 8, 1955. He received the B.S. degree (with honors) in electrical engineering from the University of Tehran in 1978 and the M.S. degree in electrical engineering and the Ph.D. degree in electrical engineering and physics from the California Institute of Technology, Pasadena, in 1979 and 1982, respectively.

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