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# Extremal Black Hole/CFT Correspondence in (Guaged) Supergravities

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**Abstract**

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**Disciplines**

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# Extremal black hole/CFT correspondence in (gauged) supergravities

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We extend the investigation of the recently proposed Kerr/conformal field theory correspondence to large classes of rotating black hole solutions in gauged and ungauged supergravities. The correspondence, proposed originally for four-dimensional Kerr black holes, asserts that the quantum states in the near-horizon region of an extremal rotating black hole are holographically dual to a two-dimensional chiral theory whose Virasoro algebra arises as an asymptotic symmetry of the near-horizon geometry. In fact, in dimension $D$ there are $[(D-1)/2]$ commuting Virasoro algebras. We consider a general canonical class of near-horizon geometries in arbitrary dimension $D$, and show that in any such metric the $[(D-1)/2]$ central charges each imply, via the Cardy formula, a microscopic entropy that agrees with the Bekenstein-Hawking entropy of the associated extremal black hole. In the remainder of the paper we show for most of the known rotating black hole solutions of gauged supergravity, and for the ungauged supergravity solutions with four charges in $D=4$ and three charges in $D=5$, that their extremal near-horizon geometries indeed lie within the canonical form. This establishes that, in all these examples, the microscopic entropies of the dual conformal field theories agree with the Bekenstein-Hawking entropies of the extremal rotating black holes.

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## I. INTRODUCTION

A recent paper [1] proposed a new holographic duality symmetry in quantum gravity, in which the quantum states in the near-horizon region of a four-dimensional extremal Kerr black hole are identified with a certain two-dimensional chiral conformal field theory (CFT). This CFT arises by examining the asymptotic symmetry generators associated with a class of diffeomorphisms of the near-horizon Kerr geometry that obey suitably chosen boundary conditions at infinity. The Lie brackets of the infinitesimal diffeomorphism transformations close on a centerless Virasoro algebra. By defining charges associated with the transformations, and evaluating the Dirac brackets of the charges, one obtains a Virasoro algebra with a central charge that is related to the angular momentum of the black hole. By using the Cardy formula, the microscopic entropy of the chiral CFT can be computed. This calculation requires that one invoke the ideas of Frolov and Thorne [2] in order to define a quantum theory in the extremal black hole geometry, and to associate a nonzero temperature $T_{\rm FT}$ with the vacuum state. It was shown in [1] that the microscopic entropy so calculated agrees precisely with the Bekenstein-Hawking entropy of the extremal Kerr black hole. (See [3–9] for some earlier related work, and [10–15] for recent follow-ups.)

The proposed Kerr/CFT correspondence was extended to a wider class of rotating black hole backgrounds in [11]. It was shown that the microscopic entropy of the dual CFT again agrees with the Bekenstein-Hawking entropy in the case of extremal Kerr-anti-de Sitter (AdS) black holes, both in four dimensions and also in all higher dimensions. A new feature that arises in more than four dimensions is that there is a Virasoro algebra, and a corresponding chiral CFT, associated with each of the orthogonal 2-planes in which the black hole is rotating. Curiously, although the central charges are different for the different CFTs, their Frolov-Thorne temperatures differ too, in precisely such a way that the Cardy formula leads to an identical microscopic entropy for each of the CFTs. Furthermore, each one of these entropies agrees precisely with the Bekenstein-Hawking entropy of the extremal rotating Kerr-AdS black hole [11].

It is perhaps useful at this point to elaborate a little on the rôle of the Frolov-Thorne temperature in the calculation of microscopic entropy via the Cardy formula. The Cardy formula gives the entropy of the two-dimensional CFT as

$$S = 2\pi\sqrt{\frac{cL}{6}}, \tag{1.1}$$

where $c$ is the central charge and $L$ is the energy. The temperature of the CFT is then given by $dL = TdS$, and so from (1.1) we have $dS = \pi\sqrt{c/(6L)}TdS$ and hence

$$\sqrt{L} = \pi\sqrt{\frac{c}{6}}T. \tag{1.2}$$

 

Substituting back into (1.1) gives

$$S = \frac{\pi^2}{3} c T. \tag{1.3}$$

It is in this form, with $c$ being the central charge of the Virasoro algebra, and $T$ being the Frolov-Thorne expression for the temperature of the near-horizon metric, that the Cardy formula delivers an expression for the microscopic entropy of the CFT that can be compared with the Bekenstein-Hawking entropy of the extremal black hole.

Another extension of the original proposal in [1] has also recently been given, in which it was shown that the microscopic entropy of the dual CFT agrees with the Bekenstein-Hawking entropy in the case of the Kerr-Newman-(A)dS charged rotating extremal black hole in four dimensions [13]. It was also noted in [13] that if one makes an assumption about the Frolov-Thorne temperature for black hole solutions to a class of four-dimensional theories involving the coupling of gravity to electromagnetic and scalar fields, one could establish an equality of the microscopic CFT entropy and the Bekenstein-Hawking entropy for a wide class of higher-dimensional extremal black holes that are related by dimensional reduction.

In this paper, we shall probe the Kerr/CFT correspondence for a large class of extremal higher-dimensional rotating charged black holes. Our strategy will be first to establish, for a general ansatz for near-horizon geometries, a result that demonstrates the equality of the microscopic entropy derived via the Cardy formula and the Bekenstein-Hawking entropy. Then, for any specific black hole solution it only remains to construct its extremal near-horizon limit, and to show that it is contained within the general ansatz mentioned above, in order to establish the equality of the microscopic and the Bekenstein-Hawking entropies for that case.

The charged rotating black hole examples that we shall consider in this paper include: the solution in four-dimensional $\mathcal{N} = 2$ (Einstein-Maxwell) gauged supergravity [16]; five-dimensional minimal gauged supergravity [17]; four-dimensional ungauged supergravity with 4 unequal charges [18]; four-dimensional gauged supergravity with 2 sets of pairwise equal charges [19]; five-dimensional ungauged supergravity with 3 unequal charges [20]; five-dimensional gauged supergravity with 3 charges, of which 2 are equal [21–23]; five-dimensional gauged supergravity with both angular momenta equal and 3 charges [24]; six-dimensional gauged supergravity [25]; seven-dimensional gauged supergravity with two equal charges [26]; the higher-dimensional Kerr-AdS solution [27,28]; and a general class of black holes in arbitrary dimension with two equal charges [26,29].

## II. GENERAL EXTREMAL ROTATING BLACK HOLES AND CFT DUALS

It was argued in [13] from the general structure of four-dimensional extremal rotating black holes that the entropy of the black hole can be obtained from the Cardy formula of the two-dimensional conformal field theory in the boundary of the black hole near-horizon geometry. Here, we shall present a general argument for higher-dimensional black holes.

We consider first $D = 5$ black holes that are asymptotic to flat or AdS spacetimes, with the asymptotic metric given by

$$ds^2 = -(1 + \hat{r}^2\ell^{-2})d\hat{t}^2 + \frac{d\hat{r}^2}{1 + \hat{r}^2\ell^{-2}} + \hat{r}^2(d\theta^2 + \cos^2\theta d\hat{\phi}_1^2 + \sin^2\theta d\hat{\phi}_2^2). \tag{2.1}$$

The discussion that follows is applicable for both vanishing and nonvanishing cosmological constant $\ell^{-2}$. In the extremal limit, it is possible to extract the near-horizon geometry as an exact solution in its own right, by first making the coordinate transformations

$$\hat{r} = r_0(1 + \lambda\rho), \qquad \hat{\phi}_1 = \phi_1 + \Omega_1^0 \hat{t}, \qquad \hat{\phi}_2 = \phi_2 + \Omega_2^0 \hat{t}, \qquad \hat{t} = \frac{t}{2\pi T_{\rm H}'^0 r_0 \lambda}. \tag{2.2}$$

Here $r_0$ is defined to be the horizon radius in the extremal limit. The quantities $\Omega_i^0$ are the angular velocities on the horizon for the two azimuthal angles $\hat{\phi}_i$, with the superscript 0 indicating that they are evaluated in the extremal limit. Let $r_+$ be the outer horizon radius of the general nonextremal black hole, which we regard as one of the parameters of the general nonextremal family of solutions, and $T_{\rm H}(r_+)$ be the corresponding Hawking temperature. The quantity $T_{\rm H}'^0$ is defined to be

$$T_{\rm H}'^0 := \frac{\partial T_{\rm H}}{\partial r_+}\bigg|_{r_+ = r_0}. \tag{2.3}$$

For later purposes, we also define

$$\Omega_i'^0 := \frac{\partial \Omega_i}{\partial r_+}\bigg|_{r_+ = r_0}, \tag{2.4}$$

where $\Omega_i(r_+)$ are the angular velocities for the general nonextremal black hole.

Taking the scaling parameter $\lambda$ to zero, we obtain the near-horizon geometry of the extremal black hole, whose metric has the form

$$ds_5^2 = A(\theta)\left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2}\right) + F(\theta)d\theta^2 + B_1(\theta)\tilde{e}_1^2 + B_2(\theta)(\tilde{e}_2 + C(\theta)\tilde{e}_1)^2, \tag{2.5}$$
$$\tilde{e}_1 = d\phi_1 + k_1\rho dt, \qquad \tilde{e}_2 = d\phi_2 + k_2\rho dt,$$

where $A$, $B_i$, $C$ and $F$ are functions of the latitude coordinate $\theta$. The metric can be viewed as an $S^3$ bundle over $AdS_2$. The $AdS_2$ base of the metric, written here in Poincaré coordinates $(t, \rho)$, can be recast in global coordinates $(\tau, r)$ by means of the transformations

$$\rho = r + \sqrt{1+r^2}\cos\tau, \qquad t = \frac{\sqrt{1+r^2}\sin\tau}{r + \sqrt{1+r^2}\cos\tau}. \tag{2.6}$$

Since this implies that $\rho dt = r d\tau + d\gamma$, where

$$\gamma := \log\left(\frac{1+\sqrt{1+r^2}\sin\tau}{\cos\tau + r\sin\tau}\right), \tag{2.7}$$

it follows that if in addition we send $\phi_i \to \phi_i - k_i\gamma$, then the metric (2.5) becomes

$$\begin{aligned} ds_5^2 &= A(\theta)\left(-(1+r^2)dt^2 + \frac{dr^2}{1+r^2}\right) + F(\theta)d\theta^2 \\ &\quad + B_1(\theta)\tilde{e}_1^2 + B_2(\theta)(\tilde{e}_2 + C(\theta)\tilde{e}_1)^2, \\ \tilde{e}_1 &= d\phi_1 + k_1 r dt, \qquad \tilde{e}_2 = d\phi_2 + k_2 r dt. \end{aligned} \tag{2.8}$$

In either form, the constants $k_1$ and $k_2$ are given by

$$k_i = \frac{1}{2\pi T_i}, \tag{2.9}$$

with

$$T_i = \lim_{r_+ \to r_0} \frac{T_H}{\Omega_i^0 - \Omega_i} = -\frac{T_H'^0}{\Omega_i'^0}. \tag{2.10}$$

The quantities $T_i$, defined first for higher-dimensional Kerr-AdS black holes in [11], can be interpreted as the Frolov-Thorne temperatures [1,2] associated with the CFTs for each azimuthal angle $\phi_i$. The Bekenstein-Hawking entropy for the extremal black hole is given by

$$S_{BH} = \frac{1}{4}\int d\theta\sqrt{B_1 B_2 F}\int d\phi_1 d\phi_2. \tag{2.11}$$

The five-dimensional near-horizon geometry (2.8) has a pair of commuting diffeomorphisms that generate two commuting Virasoro algebras:

$$\begin{aligned} \zeta_{(n)}^{(1)} &= -e^{-in\phi_1}\frac{\partial}{\partial\phi_1} - inre^{-in\phi_1}\frac{\partial}{\partial r}, \\ \zeta_{(n)}^{(2)} &= -e^{-in\phi_2}\frac{\partial}{\partial\phi_2} - inre^{-in\phi_2}\frac{\partial}{\partial r}. \end{aligned} \tag{2.12}$$

The central charges $c_i$ in these Virasoro algebras, at the level of Dirac brackets of the associated charges $Q_{(n)}^i = 1/(8\pi)\int_{\partial\Sigma} k_{(n)}^i$, can be calculated in the manner described in [30,31] and applied in [1], namely, from the $m^3$ terms in the expressions

$$\frac{1}{8\pi}\int_{\partial\Sigma} k_{\zeta_{(m)}^i}[\mathcal{L}_{\zeta_{(-m)}^i} g, g] = -\frac{i}{12}(m^3 + \alpha m)c_i, \tag{2.13}$$

where

$$\begin{aligned} k_\zeta[h,g] &= \tfrac{1}{2}[\zeta_\nu\nabla_\mu h - \zeta_\nu\nabla_\sigma h_\mu{}^\sigma + \zeta_\sigma\nabla_\nu h_\mu{}^\sigma \\ &\quad + \tfrac{1}{2}h\nabla_\nu\zeta_\mu - h_\nu{}^\sigma\nabla_\sigma\zeta_\mu \\ &\quad + \tfrac{1}{2}h_{\nu\sigma}(\nabla_\mu\zeta^\sigma + \nabla^\sigma\zeta_\mu)] * (dx^\mu \wedge dx^\nu), \end{aligned} \tag{2.14}$$

Taking $g_{\mu\nu}$ to be given by (2.8), we find that the central charges are

$$c_i = \frac{3}{2\pi}k_i\int d\theta\sqrt{B_1B_2F}\int d\phi_1 d\phi_2 = \frac{6k_iS_{BH}}{\pi}, \tag{2.15}$$

for $i=1$ and $i=2$. Thus we have

$$S_{BH} = \frac{\pi^2}{3}c_1T_1 = \frac{\pi^2}{3}c_2T_2, \tag{2.16}$$

in precise agreement with the microscopic entropy given by the Cardy formula (1.3).

The argument above can be straightforwardly generalized to higher dimensions. The near-horizon geometry of extremal rotating black holes in $D = 2n + \epsilon$ dimensions, with $\epsilon = 0, 1$, can be written, using Poincaré $AdS_2$ coordinates, as

$$\begin{aligned} ds^2 &= A\left(-\rho^2dt^2 + \frac{d\rho^2}{\rho^2}\right) + \sum_{\alpha=1}^{n-1}F_\alpha dy_\alpha^2 + \sum_{i,j=1}^{n-1+\epsilon}\tilde{g}_{ij}\tilde{e}_i\tilde{e}_j, \\ \tilde{e}_i &= d\phi_i + k_i\rho dt, \qquad k_i = \frac{1}{2\pi T_i}, \qquad T_i = -\frac{T_H'^0}{\Omega_i'^0}, \end{aligned} \tag{2.17}$$

or alternatively, using global $AdS_2$ coordinates, as

$$\begin{aligned} ds^2 &= A\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2}\right) + \sum_{\alpha=1}^{n-1}F_\alpha dy_\alpha^2 \\ &\quad + \sum_{i,j=1}^{n-1+\epsilon}\tilde{g}_{ij}\tilde{e}_i\tilde{e}_j, \\ \tilde{e}_i &= d\phi_i + k_i r d\tau, \qquad k_i = \frac{1}{2\pi T_i}, \qquad T_i = -\frac{T_H'^0}{\Omega_i'^0}. \end{aligned} \tag{2.18}$$

Here we follow [32] and use a set of unconstrained latitudinal coordinates $y_\alpha$, rather than the direction cosines $\mu_a$ subject to $\sum_{a=1}^n \mu_a^2 = 1$ that were used in the original formulation of the higher-dimensional Ricci-flat [33] or asymptotically AdS [27,28] rotating black holes. The functions $A$, $F_\alpha$ and $\tilde{g}_{ij}$ depend only on these latitudinal coordinates. The metric has $n - 1 + \epsilon$ copies of the Virasoro algebra. It has been shown that near-horizon geometries are generally of this form for classes of theories that are of interest in four and five dimensions [34], and also for cohomogeneity-1 horizons in arbitrary dimension [35]. We have verified for dimensions $D \le 7$ that the central charges are given by

$$c_i = \frac{3}{2\pi} k_i \int d^{n-1} y_\alpha \Big(\det \tilde{g}_{ij} \prod_{\alpha=1}^{n-1} F_\alpha\Big)^{1/2} \times \int d\phi_1 \ldots d\phi_{n-1+\epsilon} = \frac{6k_i S_{\rm BH}}{\pi}. \tag{2.19}$$

Since this relation does not have any features relying on a particular dimension, it is very likely to hold in arbitrary dimension. It follows that

$$S_{\rm BH} = \tfrac{1}{3}\pi^2 c_i T_i, \qquad \text{for each } i, \tag{2.20}$$

holds in general, in complete agreement with the microscopic entropy given by the Cardy formula (1.3).

In the next few sections, we shall examine a large class of charged rotating black holes in diverse dimensions. We obtain the near-horizon geometries of these black holes in the extremal limit. We demonstrate that the metrics can all be cast into the form (2.17), and hence that the Cardy formulae are all satisfied.

## III. EINSTEIN-MAXWELL ADS SUPERGRAVITIES IN FOUR AND FIVE DIMENSIONS

We shall start our main discussion with two relatively simple examples, namely, the charged rotating black holes in Einstein-Maxwell AdS supergravities in four and five dimensions.

### A. Four-dimensional Einstein-Maxwell AdS supergravity

This example, the Kerr-Newman-AdS solution, was discussed in detail in [13]; we include it here for completeness. The metric is given by

$$\begin{aligned} ds^2 &= \rho^2\Big(\frac{d\hat{r}^2}{\Delta} + \frac{d\theta^2}{\Delta_\theta}\Big) + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\Big(a d\hat{t} - \frac{\hat{r}^2 + a^2}{\Xi} d\hat{\phi}\Big)^2 - \frac{\Delta}{\rho^2}\Big(d\hat{t} - \frac{a \sin^2\theta}{\Xi} d\hat{\phi}\Big)^2, \\ \rho^2 &= \hat{r}^2 + a^2\cos^2\theta, \\ \Delta &= (\hat{r}^2 + a^2)(1 + \hat{r}^2\ell^{-2}) - 2M\hat{r} + Q^2, \\ \Delta_\theta &= 1 - a^2\ell^{-2}\cos^2\theta, \qquad \Xi = 1 - a^2\ell^2. \end{aligned} \tag{3.1}$$

Here $Q^2 = p^2 + q^2$, with $(q, p)$ being the electric and magnetic charges. The solution describes a charged black hole with the outer horizon at $\hat{r} = r_+$, where $r_+$ is the largest root of the function $\Delta(\hat{r})$. The metric is asymptotically $AdS_4$ in global coordinates, but with nonvanishing angular velocity $\Omega_\infty = -a^2\ell^{-2}$. The Hawking temperature, entropy and angular velocity on the horizon are given by

$$\begin{aligned} T_{\rm H} &= \frac{r_+^2 - a^2 - Q^2 + r_+^2\ell^{-2}(3r_+^2 + a^2)}{4\pi r_+(r_+^2 + a^2)}, \\ \Omega_\phi &= \frac{\Xi a}{r_+^2 + a^2}, \qquad S = \frac{\pi(r_+^2 + a^2)}{\Xi}. \end{aligned} \tag{3.2}$$

The extremal limit is achieved when the parameters $M$ and $Q$ take the following values:

$$\begin{aligned} M &= r_0 + r_0(2r_0^2 + a^2)\ell^{-2}, \\ Q^2 &= r_0^2 - a^2 + r_0^2(3r_0^2 + a^2)\ell^{-2}. \end{aligned} \tag{3.3}$$

The horizon of the metric is at $\hat{r} = r_0$, with the function $\Delta$ near the horizon given by

$$\begin{aligned} &\Delta = V(\hat{r} - r_0)^2 + {\rm O}(\hat{r} - r_0)^3, \quad \text{with} \\ &V = 1 + (6r_0^2 + a^2)\ell^{-2}. \end{aligned} \tag{3.4}$$

To obtain the near-horizon geometry, we make the coordinate transformation

$$\hat{r} = r_0(1 + \lambda\rho), \qquad \hat{\phi} = \phi + \Omega_\phi^0 \hat{t}, \tag{3.5}$$

where $\Omega_\phi^0 = \Omega_\phi|_{r_+ = r_0}$. We then scale the time coordinate $\hat{t}$ by

$$\hat{t} = \frac{r_0^2 + a^2}{r_0 V \lambda} t, \tag{3.6}$$

and send $\lambda \to 0$. We obtain the metric

$$\begin{aligned} ds^2 &= \frac{\rho_0^2}{V}\Big(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + \frac{V d\theta^2}{\Delta_\theta}\Big) + \frac{(r_0^2 + a^2)^2 \sin^2\theta \Delta_\theta}{\Xi^2 \rho_0^2} \times \Big(d\phi + \frac{1}{2\pi T_\phi}\rho dt\Big)^2, \\ \rho_0^2 &= r_0^2 + a^2\cos^2\theta, \end{aligned} \tag{3.7}$$

where the Frolov-Thorne temperature $T_\phi$ is given by

$$T_\phi = -\frac{\partial_{r_+} T_{\rm H}}{\partial_{r_+} \Omega_\phi}\bigg|_{r_+ = r_0} = \frac{V(r_0^2 + a^2)}{4\pi \Xi a r_0}. \tag{3.8}$$

The entropy in the extremal limit is

$$S = \frac{\pi(r_0^2 + a^2)}{\Xi}. \tag{3.9}$$

The central charge can be easily obtained, given by

$$c = \frac{12 a r_0}{V}. \tag{3.10}$$

### B. Five-dimensional minimal gauged supergravity

The general nonextremal rotating black hole in five-dimensional minimal gauged supergravity with two arbitrary angular momenta was obtained in [17]. Here we shall adopt the notation given in [36]. The metric is given by

$$ds^2 = -e^0 e^0 + \sum_{i=1}^{4} e^i e^i, \tag{3.11}$$

where

$$e^0 = \sqrt{\frac{R}{\hat{r}^2 + y^2}}\mathcal{A}, \qquad e^1 = \sqrt{\frac{\hat{r}^2 + y^2}{R}}dr,$$
$$e^2 = \sqrt{\frac{Y}{\hat{r}^2 + y^2}}(dt' - \hat{r}^2 d\psi_1), \qquad e^3 = \sqrt{\frac{\hat{r}^2 + y^2}{Y}}dy,$$
$$e^4 = \frac{ab}{\hat{r}y}\left(dt' + (y^2 - \hat{r}^2)d\psi_1 - \hat{r}^2 y^2 d\psi_2 + \frac{qy^2}{ab(\hat{r}^2 + y^2)}\mathcal{A}\right), \tag{3.12}$$

and

$$R = \frac{(1 + \hat{r}^2\ell^{-2})(\hat{r}^2 + a^2)(\hat{r}^2 + b^2) + 2abq + q^2}{\hat{r}^2} - 2M,$$
$$Y = -\frac{(1 - y^2\ell^{-2})(a^2 - y^2)(b^2 - y^2)}{y^2}, \tag{3.13}$$
$$\mathcal{A} = dt' + y^2 d\psi_1.$$

The coordinates $t'$, $\psi_1$ and $\psi_2$ are not proper canonical time and azimuthal coordinates. The proper coordinates $(\hat{t}, \hat{\phi}_1, \hat{\phi}_2)$ are given by

$$t' = \hat{t} - (a^2 + b^2)\psi_1 - a^2 b^2 \psi_2,$$
$$\psi_1 = \frac{a\hat{\phi}_1}{\Xi_a(a^2 - b^2)} + \frac{b\hat{\phi}_2}{\Xi_b(b^2 - a^2)}, \tag{3.14}$$
$$\psi_2 = \frac{\hat{\phi}_1}{a(b^2 - a^2)\Xi_a} + \frac{\hat{\phi}_2}{b(a^2 - b^2)\Xi_b},$$

where $\Xi_a = 1 - a^2\ell^{-2}$ and $\Xi_b = 1 - b^2\ell^{-2}$. Then the coordinates $\phi_1$ and $\phi_2$ have period $2\pi$. The metric is $AdS_5$ asymptotically, but in a rotating coordinate frame with angular velocities $\Omega_1 = -a\ell^{-2}$ and $\Omega_2 = -b\ell^{-2}$. The thermodynamic quantities for this black hole were obtained in [17]. Here we shall present the temperature, entropy and the angular velocities of the horizon. These are given by

$$T_{\rm H} = \frac{r_+^2}{4\pi[r_+^4 + (a^2 + b^2)r_+^2 + ab(ab + q)]}\left(\frac{\partial R}{\partial \hat{r}}\right)\bigg|_{\hat{r}=r_+},$$
$$S = \frac{\pi^2[r_+^4 + (a^2 + b^2)r_+^2 + ab(ab + q)]}{2r_+\Xi_a\Xi_b},$$
$$\Omega_1 = \frac{\Xi_a(ar_+^2 + ab^2 + qb)}{(r_+^2 + a^2)(r_+^2 + b^2) + qab}, \tag{3.15}$$
$$\Omega_2 = \frac{\Xi_b(br_+^2 + a^2 b + qa)}{(r_+^2 + a^2)(r_+^2 + b^2) + qab}.$$

We now consider the extremal limit, given by the following conditions:

$$M = \frac{(1 + r_0^2\ell^{-2})(r_0^2 + a^2)(r_0^2 + b^2) + q^2 + 2abq}{2r_0^2},$$
$$\ell^{-2} = \frac{(ab + q)^2 - r_0^4}{r_0^4(a^2 + b^2 + 2r_0^2)}. \tag{3.16}$$

Near the horizon, we have

$$R = V(\hat{r} - r_0)^2 + O(\hat{r} - r_0)^3, \qquad V = \tfrac{1}{2}R''(r_0). \tag{3.17}$$

To extract the near-horizon geometry, we make the following coordinate transformation:

$$\hat{r} = r_0(1 + \lambda\rho), \qquad \hat{\phi}_1 = \phi_1 + \Omega_1^0\hat{t},$$
$$\hat{\phi}_2 = \phi_2 + \Omega_2^0\hat{t}, \tag{3.18}$$

where $\Omega_i^0 = \Omega_i|_{r_+=r_0}$. We then scale the time coordinate $\hat{t}$ as

$$\hat{t} = \beta t,$$
$$\beta = \frac{1}{2\pi r_0 T_{\rm H}'^0\lambda} = \frac{r_0^4 + (a^2 + b^2)r_0^2 + ab(ab + q)}{Vr_0^3\lambda}. \tag{3.19}$$

Taking the limit of $\lambda \to 0$, the vielbeins become

$$e^0 = \sqrt{\frac{r_0^2 + y^2}{V}}\rho dt, \qquad e^1 = \sqrt{\frac{r_0^2 + y^2}{V}}\frac{d\rho}{\rho},$$
$$e^3 = \sqrt{\frac{r_0^2 + y^2}{Y}}dy,$$
$$e^2 = \sqrt{\frac{Y}{r_0^2 + y^2}}\left(\frac{a(a^2 + r_0^2)\tilde{e}_1}{\Xi_a(a^2 - b^2)} + \frac{b(b^2 + r_0^2)\tilde{e}_2}{\Xi_b(b^2 - a^2)}\right),$$
$$e^4 = \frac{ab}{r_0 y}\left(\frac{(a^2 - y^2)(aqy^2 + b(a^2 + r_0^2)(r_0^2 + y^2))}{ab(a^2 - b^2)\Xi_a(r_0^2 + y^2)}\tilde{e}_1 + \frac{(b^2 - y^2)(bqy^2 + a(b^2 + r_0^2)(r_0^2 + y^2))}{ab(b^2 - a^2)\Xi_b(r_0^2 + y^2)}\tilde{e}_2\right), \tag{3.20}$$

where

$$\tilde{e}_i = d\phi_i + k_i\rho dt, \qquad k_i = \frac{1}{2\pi T_i}, \tag{3.21}$$

and $T_i$'s are the Frolov-Thorne temperatures defined in (2.10). Thus we see that the near-horizon geometry of the extremal black hole can be put in the general form (2.17) discussed in Sec. II, and hence the Cardy formulas (2.20) are satisfied. To be specific, we have

$$T_1 = \frac{r_0 V[(r_0^2 + a^2)(r_0^2 + b^2) + qab]}{4\pi\Xi_a[a(r_0^2 + b^2)^2 + qb(b^2 + 2r_0^2)]},$$
$$T_2 = \frac{r_0 V[(r_0^2 + a^2)(r_0^2 + b^2) + qab]}{4\pi\Xi_b[b(r_0^2 + a^2)^2 + qa(a^2 + 2r_0^2)]}. \tag{3.22}$$

The corresponding central charges are given by

$$c_1 = \frac{6\pi[a(r_0^2+b^2)^2 + qb(b^2+2r_0^2)]}{r_0^2 \Xi_b V},$$
$$c_2 = \frac{6\pi[b(r_0^2+a^2)^2 + qa(a^2+2r_0^2)]}{r_0^2 \Xi_a V}. \tag{3.23}$$

## IV. FOUR DIMENSIONS

In this and the following sections, we consider a variety of rotating black holes involving multiple charges in various dimensions. We start here with four dimensions, and then later proceed to increase the dimensionality.

### A. Ungauged supergravity with four unequal charges

Black holes with four unequal charges arise from the bosonic sector of the four-dimensional $\mathcal{N}=2$ ungauged supergravity coupled to three vector multiplets. The metric was first obtained in [18], and the explicit form of the gauge potentials was given in [19].

The solution is specified by mass, angular momentum, and two electric and two magnetic charges. The metric takes the form

$$ds_4^2 = -\frac{\rho^2 - 2m\hat{r}}{W}(d\hat{t} + Bd\hat{\phi})^2 + W\left(\frac{d\hat{r}^2}{\Delta} + d\theta^2 + \frac{\Delta \sin^2\theta d\hat{\phi}^2}{\rho^2 - 2m\hat{r}}\right), \tag{4.1}$$

where

$$\Delta = \hat{r}^2 - 2m\hat{r} + a^2, \qquad \rho^2 = \hat{r}^2 + a^2\cos^2\theta,$$
$$B = \frac{2m(a^2-u^2)[\hat{r}c_{1234} - (\hat{r}-2m)s_{1234}]}{a(\rho^2 - 2m\hat{r})},$$
$$W^2 = r_1 r_2 r_3 r_4 + u^4 + u^2[2\hat{r}^2 + 2m\hat{r}(s_1^2+s_2^2+s_3^2+s_4^2) + 8m^4 c_{1234}s_{1234} - 4m^4(s_{123}^2 + s_{124}^2 + s_{134}^2 + s_{234}^2 + 2s_{1234}^2)],$$
$$r_i = \hat{r} + 2ms_i^2, \qquad u = a\cos\theta,$$
$$c_{i_1\ldots i_n} = \cosh\delta_{i_1}\ldots\cosh\delta_{i_n},$$
$$s_{i_1\ldots i_n} = \sinh\delta_{i_1}\ldots\sinh\delta_{i_n}. \tag{4.2}$$

The outer and inner horizons are at $\hat{r} = r_\pm$, with

$$r_\pm = m \pm \sqrt{m^2 - a^2}. \tag{4.3}$$

The entropy $S$, Hawking temperature $T_{\rm H}$ and the angular velocity $\Omega$ have the explicit form

$$S = 2\pi[m^2(c_{1234}+s_{1234}) + m\sqrt{m^2-a^2}(c_{1234}-s_{1234})],$$
$$T_{\rm H} = \frac{1}{4\pi m[c_{1234} - s_{1234} + (c_{1234}+s_{1234})m/\sqrt{m^2-a^2}]},$$
$$\Omega = 2\pi T_{\rm H}\frac{a}{\sqrt{m^2-a^2}}. \tag{4.4}$$

The extreme black hole corresponds to

$$m = a \qquad \text{and} \qquad r_+ = a. \tag{4.5}$$

The near-horizon geometry of the extreme black hole is obtained by taking

$$\hat{r} = a(1+\lambda\rho), \qquad \hat{\phi} = \phi + \Omega\hat{t}, \qquad \hat{t} = \frac{t}{\lambda}, \tag{4.6}$$

with $\lambda \to 0$. The near-horizon metric is then

$$ds_4^2 = W_0\left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + d\theta^2\right) + \frac{a^2\sin^2\theta B_0^2}{W_0}(d\phi + k\rho dt)^2, \tag{4.7}$$

where

$$B_0 = B|_{\hat{r}=a,m=a} = -2a(c_{1234} - s_{1234}),$$
$$k = \frac{1}{2\pi T_\phi} = -\frac{\partial_{r_+}\Omega}{2\pi\partial_{r_+}T_{\rm H}}\bigg|_{r_+=a} = \frac{c_{1234}-s_{1234}}{c_{1234}+s_{1234}},$$
$$W_0 = W|_{\hat{r}=a,m=a}. \tag{4.8}$$

Thus, we see that the form of the near-horizon geometry of the extremal black hole fits into the general pattern discussed in Sec. II, and hence the Cardy formula is satisfied.

### B. $U(1)^4$ gauged supergravity with pairwise equal charges

The most general charged rotating black hole solution known in four-dimensional $U(1)^4$ gauged supergravity has the four U(1) charges pairwise equal [19].

The metric is

$$ds^2 = H\left[-\frac{R}{H^2(\hat{r}^2+y^2)}\left(d\hat{t} - \frac{a^2-y^2}{\Xi a}d\hat{\phi}\right)^2 + \frac{\hat{r}^2+y^2}{R}d\hat{r}^2 + \frac{\hat{r}^2+y^2}{Y}dy^2 + \frac{Y}{H^2(\hat{r}^2+y^2)} \times\left(d\hat{t} - \frac{(\hat{r}+q_1)(\hat{r}+q_2)+a^2}{\Xi a}d\hat{\phi}\right)^2\right], \tag{4.9}$$

where

$$R = \hat{r}^2 + a^2 + g^2(\hat{r}+q_1)(\hat{r}+q_2)[(\hat{r}+q_1)(\hat{r}+q_2)+a^2] - 2m\hat{r},$$
$$Y = (1-g^2y^2)(a^2-y^2), \qquad \Xi = 1 - a^2g^2,$$
$$H = \frac{(\hat{r}+q_1)(\hat{r}+q_2)+y^2}{\hat{r}^2+y^2}, \qquad q_I = 2ms_I^2,$$
$$s_I = \sinh\delta_I. \tag{4.10}$$

Note that, as is standard in the gauged supergravity literature, we are using $g$ to denote the gauge-coupling constant, which is related to the AdS length scale $\ell$ by $g = \ell^{-1}$. We have used a shifted azimuthal coordinate $\hat{\phi}$ that gives an asymptotically rotating coordinate frame; the coordinate change $\hat{\phi} \to \hat{\phi} - ag^2\hat{t}$ would give an asymptotically non-rotating coordinate frame. This shifted azimuthal coordinate is used merely to make the metrics more convenient to write, and is not otherwise significant. The Hawking temperature and entropy are

$$\begin{aligned} T_{\mathrm{H}} &= \frac{R'|_{\hat{r}=r_+}}{4\pi[(r_+ + q_1)(r_+ + q_2) + a^2]} \\ &= \frac{r_+^2 - a^2 + a^2g^2(r_+^2 - q_1q_2) + g^2(r_+ + q_1)(r_+ + q_2)(3r_+^2 + q_1r_+ + q_2r_+ - q_1q_2)}{4\pi r_+[(r_+ + q_1)(r_+ + q_2) + a^2]}, \\ S &= \frac{\pi[(r_+ + q_1)(r_+ + q_2) + a^2]}{\Xi}. \end{aligned} \tag{4.11}$$

In our asymptotically rotating coordinate frame, the angular velocity of the horizon is

$$\hat{\Omega} = \frac{\Xi a}{(r_+ + q_1)(r_+ + q_2) + a^2}. \tag{4.12}$$

For an extremal solution, with a horizon at $\hat{r} = r_0$, we have $R|_{\hat{r}=r_0} = 0$ and $R'|_{\hat{r}=r_0} = 0$, and so

$$r_0^2 - a^2 + a^2g^2(r_0^2 - q_1q_2) + g^2(r_0 + q_1)(r_0 + q_2) \times (3r_0^2 + q_1r_0 + q_2r_0 - q_1q_2) = 0. \tag{4.13}$$

Then we have the near-horizon expansion

$$R = V(\hat{r} - r_0)^2 + \mathrm{O}(\hat{r} - r_0)^3, \tag{4.14}$$

where

$$V = 1 + g^2(6r_0^2 + 6q_1r_0 + 6q_2r_0 + a^2 + q_1^2 + q_2^2 + 4q_1q_2). \tag{4.15}$$

To obtain the near-horizon geometry, we make the coordinate changes

$$\hat{r} = r_0(1 + \lambda\rho), \qquad \hat{\phi} = \phi + \Omega^0\hat{t}, \qquad \hat{t} = \frac{t}{2\pi T_{\mathrm{H}}'^0 r_0\lambda}, \tag{4.16}$$

and then take the limit $\lambda \to 0$. The near-horizon geometry is

$$\begin{aligned} ds^2 = H_0\bigg[&(r_0^2 + y^2)\left(-\rho^2dt^2 + \frac{d\rho^2}{\rho^2}\right) + \frac{r_0^2 + y^2}{Y}dy^2 \\ &+ \frac{Y}{H_0^2(r_0^2 + y^2)}\left(\frac{2r_0 + q_1 + q_2}{V}\rho dt \right. \\ &\left. + \frac{(r_0 + q_1)(r_0 + q_2) + a^2}{\Xi a}d\phi\right)^2\bigg], \end{aligned} \tag{4.17}$$

where $H_0 = H|_{\hat{r}=r_0}$. This can be cast in the form of (2.17), so the Cardy formulas are satisfied.

For the extremal solution, the Frolov-Thorne temperatures are

$$T_0 = 0, \qquad T_1 = \frac{V[(r_0 + q_1)(r_0 + q_2) + a^2]}{2\pi\Xi a(2r_0 + q_1 + q_2)}. \tag{4.18}$$

The central charge is

$$c_1 = \frac{6a(2r_0 + q_1 + q_2)}{V}. \tag{4.19}$$

## V. FIVE DIMENSIONS

### A. Ungauged supergravity with three unequal charges

The $U(1)^3$ charged black hole in $D = 5$ ungauged supergravity was obtained in [20]. The solution was expressed in a simpler form in [22], in which the metric is given by

$$\begin{aligned} ds_5^2 &= (H_1H_2H_3)^{1/3}(x + y) \\ &\quad \times \left(-\frac{G}{(x + y)^3H_1H_2H_3}(d\hat{t} + \mathcal{A})^2 + ds_4^2\right), \end{aligned} \tag{5.1}$$
$$ds_4^2 = \left(\frac{dx^2}{4X} + \frac{dy^2}{4Y}\right) + \frac{U}{G}\left(d\chi - \frac{Z}{U}d\sigma\right)^2 + \frac{XY}{U}d\sigma^2,$$

where

$$\begin{aligned} X &= (x + a^2)(x + b^2) - 2Mx, \\ Y &= -(a^2 - y)(b^2 - y), \\ G &= (x + y)(x + y - 2M), \\ U &= yX - xY, \qquad Z = ab(X + Y), \\ \mathcal{A} &= \frac{2Mc_1c_2c_3}{x + y - 2M}[(a^2 + b^2 - y)d\sigma - abd\chi] \\ &\quad - \frac{2Ms_1s_2s_3}{x + y}(abd\sigma - yd\chi), \\ H_i &= 1 + \frac{2Ms_i^2}{x + y}, \qquad s_i = \sinh\delta_i, \qquad c_i = \cosh\delta_i. \end{aligned} \tag{5.2}$$

Here, $x$ is the radial coordinate with the asymptotic flat region at $x = \infty$. The horizon is at $x = x_+$, where $x_+$ is the largest root of $X$. The latitude coordinate $y$ runs from $a$ to $b$. The U(1) coordinates $\sigma$ and $\chi$ are related to the canonical azimuthal coordinates as follows:

$$\sigma = \frac{a\hat{\phi}_1 - b\hat{\phi}_2}{a^2 - b^2}, \qquad \chi = \frac{b\hat{\phi}_1 - a\hat{\phi}_2}{a^2 - b^2}. \tag{5.3}$$

The entropy, Hawking temperature, and angular velocities on the horizon are given by

$$\begin{aligned} S &= \frac{\pi^2(x_+ + a^2)(x_+ + b^2)(c_1c_2c_3x_+ + s_1s_2s_3ab)}{2x_+^{3/2}}, \\ T_{\rm H} &= \frac{\sqrt{x_+}(x_+^2 - a^2b^2)}{2\pi(x_+ + a^2)(x_+ + b^2)(c_1c_2c_3x_+ + s_1s_2s_3ab)}, \\ \Omega_1 &= \frac{ax_+}{(x_+ + a^2)(c_1c_2c_3x_+ + s_1s_2s_3ab)}, \\ \Omega_2 &= \frac{bx_+}{(x_+ + b^2)(c_1c_2c_3x_+ + s_1s_2s_3ab)}. \end{aligned} \tag{5.4}$$

The extremal limit of the solution is achieved with the condition $M = \frac{1}{2}(a+b)^2$, in which case the horizon is at $x = x_0$, where $x_0 = ab$. As in the previous case, the extremal limit can be extracted by the following coordinate transformation:

$$x = x_0(1 + \lambda\rho), \qquad \hat{\phi}_i = \phi + \Omega_i^0 \hat{t}, \qquad \hat{t} = \frac{t}{2\pi x_0 T'_{\rm H}(x_0)\lambda}. \tag{5.5}$$

We then take the limit $\lambda \to 0$. The near-horizon geometry then has the following form:

$$\begin{aligned} ds_5^2 = \frac{ab + y}{4}\prod_{i=1}^{3}\left(1 + \frac{(a+b)^2 s_i^2}{ab + y}\right)^{1/3} &\times \left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} + \frac{dy^2}{(a^2 - y)(b^2 - y)}\right. \\ &\left. + \sum_{i,j=1}^{2} \tilde{g}_{ij}(y)\tilde{e}_i\tilde{e}_j\right), \end{aligned} \tag{5.6}$$

where $\tilde{e}_i = d\phi_i + k_i\rho dt$, with $k_i = 1/(2\pi T_i)$. This is precisely the same form as in (2.17), and so the Cardy formulas are satisfied. Here we present the entropy, the Frolov-Thorne temperatures and central charges:

$$\begin{aligned} S &= \frac{1}{2}\pi^2(a+b)^2\sqrt{ab}(c_1c_2c_3 + s_1s_2s_3), \\ T_1 &= \frac{\sqrt{ab}(c_1c_2c_3 + s_1s_2s_3)}{\pi(bc_1c_2c_3 - as_1s_2s_3)}, \\ T_2 &= \frac{\sqrt{ab}(c_1c_2c_3 + s_1s_2s_3)}{\pi(ac_1c_2c_3 - bs_1s_2s_3)}, \\ c_{\phi_1} &= \frac{3}{2}\pi(a+b)^2(bc_1c_2c_3 - as_1s_2s_3), \\ c_{\phi_2} &= \frac{3}{2}\pi(a+b)^2(ac_1c_2c_3 - bs_1s_2s_3). \end{aligned} \tag{5.7}$$

### B. U(1)$^3$ gauged supergravity

The maximal five-dimensional gauged supergravity has gauge group SO(6), which has Cartan subgroup U(1)$^3$. We have already considered black holes in minimal gauged supergravity, which corresponds to all three Abelian charges being equal. Here, we consider some further black hole solutions of the theory.

#### *1. Charge parameters* $\delta_1 = \delta_2$ *and* $\delta_3 = 0$

Another particularly simple charged and rotating black hole in five-dimensional U(1)$^3$ gauged supergravity has three charge parameters $\delta_I$ that satisfy $\delta_1 = \delta_2 =: \delta$ and $\delta_3 = 0$, as well as both angular momenta independent [21].

The metric, using the vielbeins presented in [26] but here with Boyer-Lindquist azimuthal coordinates, is

$$\begin{aligned} ds^2 = H^{2/3}\Bigg[ &-\frac{R}{H^2(\hat{r}^2 + y^2)}\mathcal{A}^2 + \frac{\hat{r}^2 + y^2}{R}d\hat{r}^2 \\ &+ \frac{\hat{r}^2 + y^2}{Y}dy^2 + \frac{Y}{\hat{r}^2 + y^2}\bigg(d\hat{t} - \frac{a(\hat{r}^2 + a^2)}{\Xi_a(a^2 - b^2)}d\hat{\phi}_1 \\ &- \frac{b(\hat{r}^2 + b^2)}{\Xi_b(b^2 - a^2)}d\hat{\phi}_2 - \frac{q}{H(\hat{r}^2 + y^2)}\mathcal{A}\bigg)^2 \\ &+ \frac{a^2b^2}{\hat{r}^2y^2}\bigg(d\hat{t} - \frac{(\hat{r}^2 + a^2)(a^2 - y^2)}{\Xi_a a(a^2 - b^2)}d\hat{\phi}_1 \\ &- \frac{(\hat{r}^2 + b^2)(b^2 - y^2)}{\Xi_b b(b^2 - a^2)}d\hat{\phi}_2 - \frac{q}{H(\hat{r}^2 + y^2)}\mathcal{A}\bigg)^2\Bigg], \end{aligned} \tag{5.8}$$

where

$$\begin{aligned} R &= \frac{(\hat{r}^2 + a^2)(\hat{r}^2 + b^2)}{\hat{r}^2} + g^2(\hat{r}^2 + a^2 + q)(\hat{r}^2 + b^2 + q) - 2m, \\ Y &= -\frac{(1 - g^2y^2)(a^2 - y^2)(b^2 - y^2)}{y^2}, \\ \Xi_a &= 1 - a^2g^2, \qquad \Xi_b = 1 - b^2g^2, \\ H &= 1 + \frac{q}{\hat{r}^2 + y^2}, \qquad q = 2ms^2, \qquad s = \sinh\delta, \\ \mathcal{A} &= d\hat{t} - \frac{a(a^2 - y^2)}{\Xi_a(a^2 - b^2)}d\hat{\phi}_1 - \frac{b(b^2 - y^2)}{\Xi_b(b^2 - a^2)}d\hat{\phi}_2. \end{aligned} \tag{5.9}$$

We have used shifted azimuthal coordinates $\hat{\phi}_i$ that give an asymptotically rotating coordinate frame; the coordinate changes $\hat{\phi}_1 \to \hat{\phi}_1 - ag^2\hat{t}$ and $\hat{\phi}_2 \to \hat{\phi}_2 - bg^2\hat{t}$ would give an asymptotically nonrotating coordinate frame. The Hawking temperature and entropy are

$$T_{\rm H}=\frac{(\hat{r}^2R)'|_{\hat{r}=r_+}}{4\pi[(r_+^2+a^2)(r_+^2+b^2)+qr_+^2]}=\frac{r_+^4-a^2b^2+g^2r_+^4(2r_+^2+a^2+b^2+2q)}{2\pi r_+[(r_+^2+a^2)(r_+^2+b^2)+qr_+^2]},\qquad S=\frac{\pi^2[(r_+^2+a^2)(r_+^2+b^2)+qr_+^2]}{2\Xi_a\Xi_b r_+}. \tag{5.10}$$

In our asymptotically rotating coordinate frame, the angular velocities of the horizon are

$$\hat{\Omega}_a=\frac{\Xi_a a(r_+^2+b^2)}{(r_+^2+a^2)(r_+^2+b^2)+qr_+^2},\qquad \hat{\Omega}_b=\frac{\Xi_b b(r_+^2+a^2)}{(r_+^2+a^2)(r_+^2+b^2)+qr_+^2}. \tag{5.11}$$

For an extremal solution, with a horizon at $\hat{r}=r_0$, we have $R'|_{\hat{r}=r_0}=0$, and so

$$r_0^4-a^2b^2+g^2r_0^4(2r_0^2+a^2+b^2+2q)=0. \tag{5.12}$$

Then we have the near-horizon expansion

$$R=V(\hat{r}-r_0)^2+{\rm O}(\hat{r}-r_0)^3, \tag{5.13}$$

where

$$V=1-\frac{3a^2b^2}{r_0^4}+g^2(6r_0^2+a^2+b^2+2q). \tag{5.14}$$

To obtain the near-horizon geometry, we make the coordinate changes

$$\hat{r}=r_0(1+\lambda\rho),\qquad \hat{\phi}_1=\phi_1+\hat{\Omega}_a^0\hat{t},\qquad \hat{\phi}_2=\phi_2+\hat{\Omega}_b^0\hat{t},\qquad \hat{t}=\frac{t}{2\pi T_{\rm H}'^0 r_0\lambda}, \tag{5.15}$$

and then take the limit $\lambda\to 0$. The near-horizon geometry is

$$\begin{aligned}ds^2=H_0^{2/3}\Big\{&\frac{r_0^2+y^2}{V}\Big(-\rho^2dt^2+\frac{d\rho^2}{\rho^2}\Big)+\frac{r_0^2+y^2}{Y}dy^2+\frac{Y}{r_0^2+y^2}\Big[\frac{2r_0}{H_0V}\rho dt+\frac{a(r_0^2+a^2+q)}{H_0\Xi_a(a^2-b^2)}d\phi_1+\frac{b(r_0^2+b^2+q)}{H_0\Xi_b(b^2-a^2)}d\phi_2\Big]^2\\&+\frac{a^2b^2}{r_0^2y^2}\Big[\frac{2}{H_0r_0V}\Big(r_0^2+y^2+\frac{qy^2}{r_0^2+y^2}\Big)\rho dt+\frac{[(r_0^2+a^2)(r_0^2+y^2)+qr_0^2](a^2-y^2)}{H_0(r_0^2+y^2)\Xi_a a(a^2-b^2)}d\phi_1\\&+\frac{[(r_0^2+b^2)(r_0^2+y^2)+qr_0^2](b^2-y^2)}{H_0(r_0^2+y^2)\Xi_b b(b^2-a^2)}d\phi_2\Big]^2\Big\},\end{aligned} \tag{5.16}$$

where $H_0=H|_{\hat{r}=r_0}$. This can be cast in the form of (2.17), so the Cardy formulas are satisfied.

For the extremal solution, the Frolov-Thorne temperatures are

$$T_0=0,\qquad T_1=\frac{Vr_0[(r_0^2+a^2)(r_0^2+b^2)+qr_0^2]}{4\pi\Xi_a a[(r_0^2+b^2)^2+qb^2]},\qquad T_2=\frac{Vr_0[(r_0^2+a^2)(r_0^2+b^2)+qr_0^2]}{4\pi\Xi_b b[(r_0^2+a^2)^2+qa^2]}. \tag{5.17}$$

The central charges are

$$c_1=\frac{6\pi a[(r_0^2+b^2)^2+qb^2]}{V\Xi_b r_0^2},\qquad c_2=\frac{6\pi b[(r_0^2+a^2)^2+qa^2]}{V\Xi_a r_0^2}. \tag{5.18}$$

### 2. Equal angular momenta

Charged rotating black holes with both angular momenta equal and three arbitrary U(1) charges in $D=5$ gauged supergravity were obtained in [24]. Owing to the equality of the angular momenta, the solution is of cohomogeneity 1. The metric has the form [24]

$$ds_5^2=-\frac{RY}{f_1}d\hat{t}^2+\frac{\hat{r}^2R}{Y}d\hat{r}^2+\frac{1}{4}R(d\theta^2+\sin^2\theta d\phi^2)+\frac{f_1}{4R^2}\Big(d\psi+\cos\theta d\phi-2\frac{f_2}{f_1}d\hat{t}\Big)^2, \tag{5.19}$$

where $Y$, $R$, $f_1$ and $f_2$ are functions of the radial variable $\hat{r}$ only, and were presented in detail in [24]. The angular coordinates $\phi$ and $\psi$ are related to the standard $2\pi$-period azimuthal coordinates $\hat{\phi}_1$ and $\hat{\phi}_2$ as follows:

$$\phi=\hat{\phi}_1-\hat{\phi}_2,\qquad \psi=\hat{\phi}_1+\hat{\phi}_2. \tag{5.20}$$

The extremal limit is achieved when $Y$ has a double root at $\hat{r}=r_0$. As in the previous cases, we make the following coordinate transformation:

$$\hat{r}=r_0(1+\lambda\rho),\qquad \hat{\phi}_i=\phi_i+\frac{f_2(r_0)}{f_1(r_0)}\hat{t},\qquad \hat{t}=\frac{\sqrt{f_1(r_0)}}{V\lambda}t, \tag{5.21}$$

where $V=\frac{1}{2}Y''(r_0)$. Taking the $\lambda\to 0$ limit, it is straightforward to obtain the near-horizon geometry, given by

$$ds_5^2 = \frac{r_0^2 R(r_0)}{V}\Big(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2}\Big) + \frac{1}{4}R(r_0)[d\theta^2 + \sin^2\theta(\tilde{e}_1 - \tilde{e}_2)^2] + \frac{f_1(r_0)}{4R(r_0)^2}(\tilde{e}_1 + \tilde{e}_2 + \cos\theta(\tilde{e}_1 - \tilde{e}_2))^2, \quad (5.22)$$

where $\tilde{e}_i = d\phi_i + k_i\rho dt$. This is exactly the same form discussed in Sec. II, and hence the Cardy formula is satisfied.

## VI. SIX AND SEVEN DIMENSIONS

### A. Six-dimensional gauged supergravity

We consider here the black hole solution of six-dimensional SU(2) gauged supergravity [25], which has two independent angular momenta and a single U(1) charge in the Cartan subgroup of the gauge group.

The metric is

$$ds^2 = H^{1/2}\Big[-\frac{R}{H^2U}\mathcal{A}^2 + \frac{U}{R}d\hat{r}^2 + \frac{(\hat{r}^2+y^2)(y^2-z^2)}{Y}dy^2 + \frac{(\hat{r}^2+z^2)(z^2-y^2)}{Z}dz^2 + \frac{Y}{(\hat{r}^2+y^2)(y^2-z^2)}\Big(d\hat{t} - (\hat{r}^2+a^2)(a^2-z^2)\frac{d\hat{\phi}_1}{\epsilon_1} - (\hat{r}^2+b^2)(b^2-z^2)\frac{d\hat{\phi}_2}{\epsilon_2} - \frac{q\hat{r}\mathcal{A}}{HU}\Big)^2 + \frac{Z}{(\hat{r}^2+z^2)(z^2-y^2)}\Big(d\hat{t} - (\hat{r}^2+a^2)(a^2-y^2)\frac{d\hat{\phi}_1}{\epsilon_1} - (\hat{r}^2+b^2)(b^2-y^2)\frac{d\hat{\phi}_2}{\epsilon_2} - \frac{q\hat{r}\mathcal{A}}{HU}\Big)^2\Big], \quad (6.1)$$

where

$$R = (\hat{r}^2+a^2)(\hat{r}^2+b^2) + g^2[\hat{r}(\hat{r}^2+a^2)+q][\hat{r}(\hat{r}^2+b^2)+q] - 2m\hat{r}, \qquad Y = -(1-g^2y^2)(a^2-y^2)(b^2-y^2),$$
$$Z = -(1-g^2z^2)(a^2-z^2)(b^2-z^2), \qquad U = (\hat{r}^2+y^2)(\hat{r}^2+z^2), \qquad \epsilon_1 = \Xi_a a(a^2-b^2), \qquad \epsilon_2 = \Xi_b b(b^2-a^2),$$
$$\Xi_a = 1-a^2g^2, \qquad \Xi_b = 1-b^2g^2, \qquad H = 1+\frac{q\hat{r}}{U}, \qquad q = 2ms^2, \qquad s = \sinh\delta, \quad (6.2)$$
$$\mathcal{A} = d\hat{t} - (a^2-y^2)(a^2-z^2)\frac{d\hat{\phi}_1}{\epsilon_1} - (b^2-y^2)(b^2-z^2)\frac{d\hat{\phi}_2}{\epsilon_2}.$$

The coordinate changes $\hat{\phi}_1 \to \hat{\phi}_1 - ag^2\hat{t}$ and $\hat{\phi}_2 \to \hat{\phi}_2 - bg^2\hat{t}$ would give an asymptotically nonrotating coordinate frame. The Hawking temperature and entropy are

$$T_{\rm H} = \frac{R'|_{\hat{r}=r_+}}{4\pi[(r_+^2+a^2)(r_+^2+b^2)+qr_+]} = \frac{2(1+g^2r_+^2)r_+^2(2r_+^2+a^2+b^2) - (1-g^2r_+^2)(r_+^2+a^2)(r_+^2+b^2) + 4qg^2r_+^3 - q^2g^2}{4\pi r_+[(r_+^2+a^2)(r_+^2+b^2)+qr_+]},$$
$$S = \frac{2\pi^2[(r_+^2+a^2)(r_+^2+b^2)+qr_+]}{3\Xi_a\Xi_b}. \quad (6.3)$$

In our asymptotically rotating coordinate frame, the angular velocities of the horizon are

$$\hat{\Omega}_a = \frac{\Xi_a a(r_+^2+b^2)}{(r_+^2+a^2)(r_+^2+b^2)+qr_+}, \qquad \hat{\Omega}_b = \frac{\Xi_b b(r_+^2+a^2)}{(r_+^2+a^2)(r_+^2+b^2)+qr_+}. \quad (6.4)$$

For an extremal solution, with a horizon at $\hat{r} = r_0$, we have $R|_{\hat{r}=r_0} = 0$ and $R'|_{\hat{r}=r_0} = 0$, and so

$$3r_0^4 + (a^2+b^2)r_0^2 - a^2b^2 + g^2r_0^2[5r_0^4 + 3(a^2+b^2)r_0^2 + a^2b^2] + 4qg^2r_0^3 - q^2g^2 = 0. \quad (6.5)$$

Then we have the near-horizon expansion

$$R = V(\hat{r}-r_0)^2 + O(\hat{r}-r_0)^3, \quad (6.6)$$

where

$$V = 6r_0^2 + a^2 + b^2 + g^2[15r_0^4 + 6(a^2+b^2)r_0^2 + 6qr_0 + a^2b^2]. \quad (6.7)$$

To obtain the near-horizon geometry, we make the coordinate changes

$$\hat{r} = r_0(1+\lambda\rho), \qquad \hat{\phi}_1 = \phi_1 + \hat{\Omega}_a^0\hat{t}, \qquad \hat{\phi}_2 = \phi_2 + \hat{\Omega}_b^0\hat{t}, \qquad \hat{t} = \frac{t}{2\pi T_{\rm H}'^0 r_0\lambda}, \quad (6.8)$$

and then take the limit $\lambda \to 0$. The near-horizon geometry is

$$ds^2 = H_0^{1/2}\Big[\frac{\tilde{U}}{V}\Big(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2}\Big) + \frac{(r_0^2+y^2)(y^2-z^2)}{Y}dy^2 + \frac{(r_0^2+z^2)(z^2-y^2)}{Z}dz^2 + \frac{Y}{(r_0^2+y^2)(y^2-z^2)}\Big(\frac{2r_0(r_0^2+z^2)}{V}\rho dt + (r_0^2+a^2)(a^2-z^2)\frac{d\phi_1}{\epsilon_1} + (r_0^2+b^2)(b^2-z^2)\frac{d\phi_2}{\epsilon_2} + \frac{qr_0}{H_0\tilde{U}}\tilde{\mathcal{A}}\Big)^2 + \frac{Z}{(r_0^2+z^2)(z^2-y^2)}\Big(\frac{2r_0(r_0^2+y^2)}{V}\rho dt + (r_0^2+a^2)(a^2-y^2)\frac{d\phi_1}{\epsilon_1} + (r_0^2+b^2)(b^2-y^2)\frac{d\phi_2}{\epsilon_2} + \frac{qr_0}{H_0\tilde{U}}\tilde{\mathcal{A}}\Big)^2\Big], \quad (6.9)$$

where $\tilde{U} = U|_{\hat{r}=r_0}$, $H_0 = H|_{\hat{r}=r_0}$, and

$$\tilde{\mathcal{A}} = \frac{-3r_0^4 - r_0^2(y^2+z^2) + y^2z^2}{Vr_0}\rho dt - (a^2-y^2)(a^2-z^2)\frac{d\phi_1}{\epsilon_1} - (b^2-y^2)(b^2-z^2)\frac{d\phi_2}{\epsilon_2}. \quad (6.10)$$

This can be cast in the form of (2.17), so the Cardy formulas are satisfied.

For an extremal solution, the Frolov-Thorne temperatures are

$$T_1 = \frac{V[(r_0^2+a^2)(r_0^2+b^2)+qr_0]}{2\pi\Xi_a a[2r_0(r_0^2+b^2)^2 + q(b^2-r_0^2)]}, \qquad T_2 = \frac{V[(r_0^2+a^2)(r_0^2+b^2)+qr_0]}{2\pi\Xi_b b[2r_0(r_0^2+a^2)^2 + q(a^2-r_0^2)]}, \quad (6.11)$$

and $T_0 = 0$. The central charges are

$$c_1 = \frac{4\pi a[2r_0(r_0^2+b^2)^2 + q(b^2-r_0^2)]}{V\Xi_b}, \qquad c_2 = \frac{4\pi b[2r_0(r_0^2+a^2)^2 + q(a^2-r_0^2)]}{V\Xi_a}. \quad (6.12)$$

## B. Seven-dimensional gauged supergravity

We consider here the black hole solution of [26] in seven-dimensional SO(5) gauged supergravity. It possesses three independent angular momenta and a single charge parameter, corresponding to two equal U(1) charges in the $U(1)^2$ Cartan subgroup of the full gauge group.

The metric is

$$ds^2 = H^{2/5}\Big\{-\frac{R}{H^2U}\mathcal{A}^2 + \frac{U}{R}d\hat{r}^2 + \frac{(\hat{r}^2+y^2)(y^2-z^2)}{Y}dy^2 + \frac{(\hat{r}^2+z^2)(z^2-y^2)}{Z}dz^2 + \frac{Y}{(\hat{r}^2+y^2)(y^2-z^2)}\Big(d\hat{t} - \sum_{i=1}^3 \frac{(\hat{r}^2+a_i^2)\gamma_i}{a_i^2-y^2}\frac{d\hat{\phi}_i}{\epsilon_i} - \frac{q}{HU}\mathcal{A}\Big)^2 + \frac{Z}{(\hat{r}^2+z^2)(z^2-y^2)}\Big(d\hat{t} - \sum_{i=1}^3 \frac{(\hat{r}^2+a_i^2)\gamma_i}{a_i^2-z^2}\frac{d\hat{\phi}_i}{\epsilon_i} - \frac{q}{HU}\mathcal{A}\Big)^2 + \frac{a_1^2a_2^2a_3^2}{\hat{r}^2y^2z^2}\Big[d\hat{t} - \sum_{i=1}^3 \frac{(\hat{r}^2+a_i^2)\gamma_i}{a_i^2}\frac{d\hat{\phi}_i}{\epsilon_i} - \frac{q}{HU}\Big(1 + \frac{gy^2z^2}{a_1a_2a_3}\Big)\mathcal{A}\Big]^2\Big\}, \quad (6.13)$$

where

$$R = \frac{1+g^2\hat{r}^2}{\hat{r}^2}\prod_{i=1}^3(\hat{r}^2+a_i^2) + qg^2(2\hat{r}^2+a_1^2+a_2^2+a_3^2) - \frac{2qga_1a_2a_3}{\hat{r}^2} + \frac{q^2g^2}{\hat{r}^2} - 2m, \qquad Y = \frac{1-g^2y^2}{y^2}\prod_{i=1}^3(a_i^2-y^2),$$

$$Z = \frac{1-g^2z^2}{z^2}\prod_{i=1}^3(a_i^2-z^2), \qquad U = (\hat{r}^2+y^2)(\hat{r}^2+z^2), \qquad \gamma_i = a_i^2(a_i^2-y^2)(a_i^2-z^2), \qquad \epsilon_i = \Xi_i a_i\prod_{j\neq i}(a_i^2-a_j^2),$$

$$\Xi_i = 1 - a_i^2g^2, \qquad H = 1 + \frac{q}{(\hat{r}^2+y^2)(\hat{r}^2+z^2)}, \qquad q = 2ms^2, \qquad s = \sinh\delta, \qquad \mathcal{A} = d\hat{t} - \sum_{i=1}^3\gamma_i\frac{d\hat{\phi}_i}{\epsilon_i}. \quad (6.14)$$

The coordinate changes $\hat{\phi}_i \to \hat{\phi}_i - a_ig^2\hat{t}$ would give an asymptotically nonrotating coordinate frame. The Hawking temperature and entropy are

$$T_{\rm H} = \frac{(\hat{r}^2 R)'|_{\hat{r}=r_+}}{4\pi[(r_+^2 + a_1^2)(r_+^2 + a_2^2)(r_+^2 + a_3^2) + q(r_+^2 - a_1 a_2 a_3 g)]}$$
$$= \frac{(1 + g^2 r_+^2) r_+^2 \sum_i \prod_{j\neq i}(r_+^2 + a_j^2) - \prod_i (r_+^2 + a_i^2) + 2q(g^2 r_+^4 + g a_1 a_2 a_3) - q^2 g^2}{2\pi r_+[(r_+^2 + a_1^2)(r_+^2 + a_2^2)(r_+^2 + a_3^2) + q(r_+^2 - a_1 a_2 a_3 g)]}, \quad (6.15)$$
$$S = \frac{\pi^3[(r_+^2 + a_1^2)(r_+^2 + a_2^2)(r_+^2 + a_3^2) + q(r_+^2 - a_1 a_2 a_3 g)]}{4\Xi_1 \Xi_2 \Xi_3 r_+}.$$

In our asymptotically rotating coordinate frame, the angular velocities of the horizon are

$$\hat{\Omega}_i = \frac{\Xi_i[a_i \prod_{j\neq i}(r_+^2 + a_j^2) - q\prod_{j\neq i} a_j g]}{(r_+^2 + a_1^2)(r_+^2 + a_2^2)(r_+^2 + a_3^2) + q(r_+^2 - a_1 a_2 a_3 g)}. \quad (6.16)$$

For an extremal solution, with a horizon at $\hat{r} = r_0$, we have $R'|_{\hat{r}=r_0} = 0$, and so

$$2r_0^6 + (a_1^2 + a_2^2 + a_3^2) r_0^4 - a_1^2 a_2^2 a_3^2 + g^2[3r_0^8 + 2(a_1^2 + a_2^2 + a_3^2) r_0^6 + (a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2 + 2q) r_0^4 - q^2] + 2qg a_1 a_2 a_3 = 0. \quad (6.17)$$

Then we have the near-horizon expansion

$$R = V(\hat{r} - r_0)^2 + O(\hat{r} - r_0)^3, \quad (6.18)$$

where

$$V = 6r_0^2 + a_1^2 + a_2^2 + a_3^2 + \frac{3(a_1 a_2 a_3 - qg)^2}{r_0^4} + g^2[15 r_0^4 + 6(a_1^2 + a_2^2 + a_3^2) r_0^2 + a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2 + 2q]. \quad (6.19)$$

To obtain the near-horizon geometry, we make the coordinate changes

$$\hat{r} = r_0(1 + \lambda\rho), \qquad \hat{\phi}_i = \phi_i + \Omega_i^0 \hat{t}, \qquad \hat{t} = \frac{t}{2\pi T_{\rm H}'^0 r_0 \lambda}, \quad (6.20)$$

and then take the limit $\lambda \to 0$. The near-horizon geometry is

$$ds^2 = H_0^{2/5}\bigg\{\frac{\tilde{U}}{V}\bigg(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2}\bigg) + \frac{(r_0^2 + y^2)(y^2 - z^2)}{Y} dy^2 + \frac{(r_0^2 + z^2)(z^2 - y^2)}{Z} dz^2$$
$$+ \frac{Y}{(r_0^2 + y^2)(y^2 - z^2)}\bigg(\frac{2r_0(r_0^2 + z^2)}{V}\rho dt + \sum_{i=1}^3 \frac{(r_0^2 + a_i^2)\gamma_i}{a_i^2 - y^2}\frac{d\phi_i}{\epsilon_i} + \frac{q}{H_0\tilde{U}}\tilde{\mathcal{A}}\bigg)^2$$
$$+ \frac{Z}{(r_0^2 + z^2)(z^2 - y^2)}\bigg(\frac{2r_0(r_0^2 + y^2)}{V}\rho dt + \sum_{i=1}^3 \frac{(r_0^2 + a_i^2)\gamma_i}{a_i^2 - z^2}\frac{d\phi_i}{\epsilon_i} + \frac{q}{H_0\tilde{U}}\tilde{\mathcal{A}}\bigg)^2$$
$$+ \frac{a_1^2 a_2^2 a_3^2}{r_0^2 y^2 z^2}\bigg[\frac{2}{V r_0}\bigg(\tilde{U} - \frac{qgy^2z^2}{a_1 a_2 a_3}\bigg)\rho dt + \sum_{i=1}^3 \frac{(r_0^2 + a_i^2)\gamma_i}{a_i^2}\frac{d\phi_i}{\epsilon_i} + \frac{q}{H_0\tilde{U}}\bigg(1 + \frac{gy^2z^2}{a_1 a_2 a_3}\bigg)\tilde{\mathcal{A}}\bigg]^2\bigg\}, \quad (6.21)$$

where $\tilde{U} = U|_{\hat{r}=r_0}$, $H_0 = H|_{\hat{r}=r_0}$, and

$$\tilde{\mathcal{A}} = -\frac{2r_0(2r_0^2 + y^2 + z^2)}{V}\rho dt - \sum_{i=1}^3 \gamma_i \frac{d\phi_i}{\epsilon_i}. \quad (6.22)$$

This can be cast in the form of (2.17), and so the Cardy formulas are satisfied.

For an extremal solution, the Frolov-Thorne temperatures are

$$T_1 = \frac{V r_0[(r_0^2 + a_1^2)(r_0^2 + a_2^2)(r_0^2 + a_3^2) + q(r_0^2 - a_1 a_2 a_3 g)]}{4\pi\Xi_1} \times [a_1(r_0^2 + a_2^2)^2(r_0^2 + a_3^2)^2 + q a_1(a_2^2 a_3^2 - r_0^4) - qg a_2 a_3(3r_0^4 + 2a_2^2 r_0^2 + 2a_3^2 r_0^2 + a_2^2 a_3^2) - q^2 g a_2 a_3]^{-1}, \quad (6.23)$$

temperatures $T_2$ and $T_3$, obtained by cyclic permutation of $a_i$, $i = 1, 2, 3$, and also $T_0 = 0$. The central charges are

$$c_1 = \frac{3\pi^2}{\Xi_2\Xi_3 V r_0^2}[a_1(r_0^2+a_2^2)^2(r_0^2+a_3^2)^2 + qa_1(a_2^2a_3^2 - r_0^4) - qga_2a_3(3r_0^4 + 2a_2^2r_0^2 + 2a_3^2r_0^2 + a_2^2a_3^2) - q^2ga_2a_3], \tag{6.24}$$

and also $c_2$ and $c_3$, obtained by cyclic permutation of $a_i$, $i = 1, 2, 3$.

## VII. ARBITRARY DIMENSIONS

### A. Higher-dimensional Kerr-AdS

The extremal black hole/CFT correspondence for the higher-dimensional Kerr-AdS solution [27,28] was previously considered in [11], where it was shown that the Cardy formulas are satisfied. We return to this example, showing directly that the near-horizon geometry of its extremal limit can be cast in the form of (2.17). (Note that the near-horizon geometry of the extremal Kerr-AdS black hole in $D = 5$ was obtained in [37], and that of the Myers-Perry solution in [35].)

#### *1. Even dimensions* $D = 2n$

The Kerr-AdS metric in even dimensions $D = 2n$ is

$$ds^2 = -\frac{R}{U}\mathcal{A}^2 + \frac{U}{R}d\hat{r}^2 + \sum_{\alpha=1}^{n-1}\frac{U_\alpha}{X_\alpha}dy_\alpha^2 + \sum_{\alpha=1}^{n-1}\frac{X_\alpha}{U_\alpha}\left(d\hat{t} - \sum_{i=1}^{n-1}\frac{(\hat{r}^2+a_i^2)\gamma_i}{a_i^2 - y_\alpha^2}\frac{d\hat{\phi}_i}{\epsilon_i}\right)^2, \tag{7.1}$$

where

$$R = \prod_{k=1}^{n-1}(\hat{r}^2 + a_k^2) - 2m\hat{r}, \qquad X_\alpha = -\prod_{k=1}^{n-1}(a_k^2 - y_\alpha^2),$$
$$U = \prod_{\alpha=1}^{n-1}(\hat{r}^2 + y_\alpha^2), \qquad U_\alpha = -(\hat{r}^2 + y_\alpha^2)\prod_{\beta=1}^{n-1}{}'(y_\beta^2 - y_\alpha^2),$$
$$\gamma_i = \prod_{\alpha=1}^{n-1}(a_i^2 - y_\alpha^2), \qquad \epsilon_i = \Xi_i a_i \prod_{k=1}^{n-1}{}'(a_i^2 - a_k^2),$$
$$\Xi_i = 1 - a_i^2g^2, \qquad \mathcal{A} = d\hat{t} - \sum_{i=1}^{n-1}\gamma_i\frac{d\hat{\phi}_i}{\epsilon_i}. \tag{7.2}$$

The notation $\prod'$ means that we omit the factor that vanishes from a product.

For the extremal solution, the Frolov-Thorne temperatures are

$$T_i = \frac{V(r_0^2 + a_i^2)}{4\pi\Xi_i a_i r_0 \prod_{j\neq i}(r_0^2 + a_j^2)}, \tag{7.3}$$

where $V = \frac{1}{2}R''|_{\hat{r}=r_0}$, and also $T_0 = 0$. The near-horizon geometry is [11]

$$ds^2 = \frac{\tilde{U}}{V}\left(-\rho^2dt^2 + \frac{d\rho^2}{\rho^2}\right) + \sum_{\alpha=1}^{n-1}\frac{\tilde{U}_\alpha}{X_\alpha}dy_\alpha^2 + \sum_{\alpha=1}^{n-1}\frac{X_\alpha}{U_\alpha}\times\left(\frac{2r_0\tilde{U}}{V(r_0^2+y_\alpha^2)}\rho dt + \sum_{i=1}^{n-1}\frac{(r_0^2+a_i^2)\gamma_i}{a_i^2 - y_\alpha^2}\frac{d\phi_i}{\epsilon_i}\right)^2, \tag{7.4}$$

where $\tilde{U} = U|_{\hat{r}=r_0}$ and $\tilde{U}_\alpha = U_\alpha|_{\hat{r}=r_0}$.

We can cast this near-horizon geometry in the form of (2.17), reading off $\tilde{g}_{ij}$ from $g_{\phi_i\phi_j}$. To explicitly see how for this example, we need to account for the correct coefficients of $dt$ within the vielbeins. From the partial fraction decomposition

$$\frac{\tilde{U}}{(r_0^2+y_\alpha^2)\prod_{k=1}^{n-1}(r_0^2+a_k^2)} = \sum_{i=1}^{n-1}\frac{\Xi_i a_i\gamma_i}{\epsilon_i(a_i^2 - y_\alpha^2)(r_0^2+a_i^2)}, \tag{7.5}$$

we see that this is indeed the case. It follows that the Cardy formulas are satisfied.

#### *2. Odd dimensions* $D = 2n + 1$

The Kerr-AdS metric in odd dimensions $D = 2n + 1$ is

$$ds^2 = -\frac{R}{U}\mathcal{A}^2 + \frac{U}{R}d\hat{r}^2 + \sum_{\alpha=1}^{n-1}\frac{U_\alpha}{X_\alpha}dy_\alpha^2 + \sum_{\alpha=1}^{n-1}\frac{X_\alpha}{U_\alpha}\left(d\hat{t} - \sum_{i=1}^{n}\frac{(\hat{r}^2+a_i^2)\gamma_i}{a_i^2 - y_\alpha^2}\frac{d\hat{\phi}_i}{\epsilon_i}\right)^2 + \frac{\prod_{k=1}^n a_k^2}{r^2\prod_{\alpha=1}^{n-1}y_\alpha^2}\left(d\hat{t} - \sum_{i=1}^{n}\frac{(\hat{r}^2+a_i^2)\gamma_i}{a_i^2}\frac{d\hat{\phi}_i}{\epsilon_i}\right)^2, \tag{7.6}$$

where

$$R = \frac{1}{\hat{r}^2}\prod_{k=1}^{n}(\hat{r}^2 + a_k^2) - 2m, \qquad X_\alpha = \frac{1}{y_\alpha^2}\prod_{k=1}^{n}(a_k^2 - y_\alpha^2),$$
$$U = \prod_{\alpha=1}^{n-1}(\hat{r}^2 + y_\alpha^2), \qquad U_\alpha = -(\hat{r}^2 + y_\alpha^2)\prod_{\beta=1}^{n-1}{}'(y_\beta^2 - y_\alpha^2),$$
$$\gamma_i = a_i^2\prod_{\alpha=1}^{n-1}(a_i^2 - y_\alpha^2), \qquad \epsilon_i = \Xi_i a_i \prod_{k=1}^{n}{}'(a_i^2 - a_k^2),$$
$$\Xi_i = 1 - a_i^2g^2, \qquad \mathcal{A} = d\hat{t} - \sum_{i=1}^{n}\gamma_i\frac{d\hat{\phi}_i}{\epsilon_i}. \tag{7.7}$$

For the extremal solution, the Frolov-Thorne temperatures are

$$T_i = \frac{Vr_0(r_0^2 + a_i^2)}{4\pi\Xi_i a_i \prod_{j\neq i}(r_0^2 + a_j^2)}, \tag{7.8}$$

where $V = \frac{1}{2}R''|_{\hat{r}=r_0}$, and also $T_0 = 0$. The near-horizon geometry is [11]

$$ds^2 = \frac{\tilde{U}}{V}\left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2}\right) + \sum_{\alpha=1}^{n-1}\frac{\tilde{U}_\alpha}{X_\alpha}dy_\alpha^2 + \sum_{\alpha=1}^{n-1}\frac{X_\alpha}{U_\alpha}\left(\frac{2r_0\tilde{U}}{V(r_0^2+y_\alpha^2)}\rho dt + \sum_{i=1}^{n}\frac{(r_0^2+a_i^2)\gamma_i}{a_i^2-y_\alpha^2}\frac{d\phi_i}{\epsilon_i}\right)^2 + \frac{\prod_{k=1}^n a_k^2}{r^2\prod_{\alpha=1}^{n-1}y_\alpha^2}\left(\frac{2\tilde{U}}{Vr_0}\rho dt + \sum_{i=1}^{n}\frac{(r_0^2+a_i^2)\gamma_i}{a_i^2}\frac{d\phi_i}{\epsilon_i}\right)^2, \tag{7.9}$$

where $\tilde{U} = U|_{\hat{r}=r_0}$ and $\tilde{U}_\alpha = U_\alpha|_{\hat{r}=r_0}$.

Analogously to the even-dimensional case, the near-horizon geometry can be cast in the form of (2.17). The analogous partial fraction decomposition used is

$$\frac{r_0^2\tilde{U}}{(r_0^2+y_\alpha^2)\prod_{k=1}^n(r_0^2+a_k^2)} = \sum_{i=1}^{n}\frac{\Xi_i a_i\gamma_i}{\epsilon_i(a_i^2-y_\alpha^2)(r_0^2+a_i^2)}. \tag{7.10}$$

It again follows that the Cardy formulas are satisfied.

### B. Charged rotating black holes in ungauged supergravity

The solution considered here is the two-charge Cvetič-Youm solution [29], with the simplification of [26] that both charges are equal. It can be regarded as a solution of toroidally compactified heterotic supergravity in dimension $4 \le D \le 9$, although the construction generalizes to arbitrary dimension as a solution of a low-energy effective action of bosonic strings. This solution underlies the ungauged limit of some of the gauged black hole solutions that we have considered above. We use the form of the metric in [26].

#### *1. Even dimensions $D = 2n$*

In even dimensions $D = 2n$, the metric is

$$ds^2 = H^{2/(D-2)}\left[-\frac{R}{H^2U}\mathcal{A}^2 + \frac{U}{R}d\hat{r}^2 + \sum_{\alpha=1}^{n-1}\frac{U_\alpha}{X_\alpha}dy_\alpha^2 + \sum_{\alpha=1}^{n-1}\frac{X_\alpha}{U_\alpha}\left(d\hat{t} - \sum_{i=1}^{n-1}\frac{(\hat{r}^2+a_i^2)\gamma_i}{a_i^2-y_\alpha^2}\frac{d\hat{\phi}_i}{\epsilon_i} - \frac{q\hat{r}}{HU}\mathcal{A}\right)^2\right], \tag{7.11}$$

where

$$R = \prod_{k=1}^{n-1}(\hat{r}^2+a_k^2) - 2m\hat{r}, \qquad X_\alpha = -\prod_{k=1}^{n-1}(a_k^2-y_\alpha^2),$$
$$U = \prod_{\alpha=1}^{n-1}(\hat{r}^2+y_\alpha^2), \qquad U_\alpha = -(\hat{r}^2+y_\alpha^2)\prod_{\beta=1}^{n-1}{}'(y_\beta^2-y_\alpha^2),$$
$$\gamma_i = \prod_{\alpha=1}^{n-1}(a_i^2-y_\alpha^2), \qquad \epsilon_i = a_i\prod_{k=1}^{n-1}{}'(a_i^2-a_k^2),$$
$$H = 1 + \frac{q\hat{r}}{U}, \qquad q = 2ms^2,$$
$$s = \sinh\delta, \qquad \mathcal{A} = d\hat{t} - \sum_{i=1}^{n-1}\gamma_i\frac{d\hat{\phi}_i}{\epsilon_i}. \tag{7.12}$$

The Hawking temperature and entropy are

$$T_{\rm H} = \frac{R'|_{\hat{r}=r_+}}{4\pi[\prod_{k=1}^{n-1}(r_+^2+a_k^2)+qr_+]}, \qquad S = \frac{\mathcal{A}_{D-2}[\prod_{k=1}^{n-1}(r_+^2+a_k^2)+qr_+]}{4r_+}, \tag{7.13}$$

where $\mathcal{A}_{D-2} = 2\pi^{(D-1)/2}/\Gamma[(D-1)/2]$ is the volume of a unit $(D-2)$-sphere, so for example $\mathcal{A}_2 = 4\pi$ and $\mathcal{A}_4 = \frac{8}{3}\pi^2$. The angular velocities of the horizon are

$$\Omega_i = \frac{a_i\prod_{j\neq i}(r_+^2+a_j^2)}{\prod_{k=1}^{n-1}(r_+^2+a_k^2)+qr_+}. \tag{7.14}$$

For an extremal solution, with a horizon at $\hat{r} = r_0$, we have $R|_{\hat{r}=r_0} = 0$ and $R'|_{\hat{r}=r_0} = 0$, and so

$$\sum_{i=1}^{n-1}\frac{1}{r_0^2+a_i^2} = \frac{1}{2r_0^2}. \tag{7.15}$$

Then we have the near-horizon expansion

$$R = V(\hat{r}-r_0)^2 + {\rm O}(\hat{r}-r_0)^3, \qquad V = \tfrac{1}{2}R''|_{\hat{r}=r_0}. \tag{7.16}$$

Since $R'|_{\hat{r}=r_0} = 0$ for an extremal solution, we have

$$T_{\rm H}'^0 = \frac{V}{2\pi[\prod_{k=1}^{n-1}(r_0^2+a_k^2)+qr_0]}, \tag{7.17}$$

and, using (7.15), we obtain

$$\Omega_i'^0 = -\frac{2a_ir_0\prod_{j\neq i}(r_0^2+a_j^2)}{(r_0^2+a_i^2)[\prod_{k=1}^{n-1}(r_0^2+a_k^2)+qr_0]}. \tag{7.18}$$

Therefore the Frolov-Thorne temperatures are

$$T_0 = 0, \qquad T_i = \frac{V(r_0^2+a_i^2)}{4\pi a_ir_0\prod_{j\neq i}(r_0^2+a_j^2)}. \tag{7.19}$$

To obtain the near-horizon geometry, we make the coordinate changes

$$\hat{r} = r_0(1+\lambda\rho), \qquad \hat{\phi}_i = \phi_i + \Omega_i^0 \hat{t}, \qquad \hat{t} = \frac{t}{2\pi T_{\rm H}'^0 r_0 \lambda}, \tag{7.20}$$

and then take the limit $\lambda \to 0$. The near-horizon geometry is

$$ds^2 = H_0^{2/(D-2)}\left[\frac{\tilde{U}}{V}\left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2}\right) + \sum_{\alpha=1}^{n-1}\frac{\tilde{U}_\alpha}{X_\alpha}dy_\alpha^2 + \sum_{\alpha=1}^{n-1}\frac{X_\alpha}{U_\alpha}\left(\frac{2r_0\tilde{U}}{V(r_0^2+y_\alpha^2)}\rho dt + \sum_{i=1}^{n-1}\frac{(r_0^2+a_i^2)\gamma_i}{a_i^2-y_\alpha^2}\frac{d\phi_i}{\epsilon_i} + \frac{qr_0}{H_0\tilde{U}}\tilde{\mathcal{A}}\right)^2\right], \tag{7.21}$$

where $\tilde{U} = U|_{\hat{r}=r_0}$, $\tilde{U}_\alpha = U_\alpha|_{\hat{r}=r_0}$, $H_0 = H|_{\hat{r}=r_0}$, and

$$H_0' = \frac{\partial H_0}{\partial r_0} = \frac{qr_0}{\tilde{U}}\left(\frac{1}{r_0} - \sum_{\alpha=1}^{n-1}\frac{2r_0}{r_0^2+y_\alpha^2}\right), \qquad \tilde{\mathcal{A}} = \frac{H_0'\tilde{U}^2}{Vqr_0}\rho dt - \sum_{i=1}^{n-1}\gamma_i\frac{d\phi_i}{\epsilon_i}. \tag{7.22}$$

By checking $dt$ coefficients within the vielbeins, we can directly see that this near-horizon geometry may be cast in the form of (2.17). Some terms follow in the same way as for the higher-dimensional Kerr-AdS solution. There are also extra terms when charge is included; these extra terms are within $\tilde{\mathcal{A}}$. To check these extra terms, we use the identity

$$\frac{1}{\tilde{U}}\sum_{i=1}^{n-1}\frac{a_i\gamma_i\prod_{j\neq i}(r_0^2+a_j^2)}{\epsilon_i(r_0^2+a_i^2)} = \sum_{\alpha=1}^{n-1}\frac{1}{r_0^2+y_\alpha^2} - \sum_{i=1}^{n-1}\frac{1}{r_0^2+a_i^2}, \tag{7.23}$$

which is seen to hold by a partial fraction decomposition of the entire left-hand side. On the right-hand side, the coefficients of $1/(r_0^2+a_i^2)$ are trivial, and the coefficients of $1/(r_0^2+y_\alpha^2)$ in turn follow from the partial fraction decomposition

$$\frac{U_\alpha}{(r_0^2+y_\alpha^2)X_\alpha} = \sum_{i=1}^{n-1}\frac{a_i\gamma_i}{\epsilon_i(a_i^2-y_\alpha^2)}. \tag{7.24}$$

Now using the extremality condition (7.15), we see that

$$\frac{2r_0}{\tilde{U}}\sum_{i=1}^{n-1}\frac{a_i\gamma_i\prod_{j\neq i}(r_0^2+a_j^2)}{\epsilon_i(r_0^2+a_i^2)} = -\left(\frac{1}{r_0} - \sum_{\alpha=1}^{n-1}\frac{2r_0}{r_0^2+y_\alpha^2}\right) = -\frac{H_0'\tilde{U}}{qr_0}, \tag{7.25}$$

completing the verification. It follows that the Cardy formulas are satisfied. The central charges are

$$c_i = \frac{3\mathcal{A}_{D-2}a_ir_0\prod_{j\neq i}(r_0^2+a_j^2)[\prod_{k=1}^{n-1}(r_0^2+a_k^2)+qr_0]}{\pi V(r_0^2+a_i^2)}. \tag{7.26}$$

### 2. *Odd dimensions* $D = 2n+1$

In odd dimensions $D = 2n+1$, the metric is

$$ds^2 = H^{2/(D-2)}\left[-\frac{R}{H^2U}\mathcal{A}^2 + \frac{U}{R}d\hat{r}^2 + \sum_{\alpha=1}^{n-1}\frac{U_\alpha}{X_\alpha}dy_\alpha^2 + \sum_{\alpha=1}^{n-1}\frac{X_\alpha}{U_\alpha}\left(d\hat{t} - \sum_{i=1}^{n}\frac{(\hat{r}^2+a_i^2)\gamma_i}{a_i^2-y_\alpha^2}\frac{d\hat{\phi}_i}{\epsilon_i} - \frac{q}{HU}\mathcal{A}\right)^2 + \frac{\prod_{k=1}^{n}a_k^2}{r^2\prod_{\alpha=1}^{n-1}y_\alpha^2}\times\left(d\hat{t} - \sum_{i=1}^{n}\frac{(\hat{r}^2+a_i^2)\gamma_i}{a_i^2}\frac{d\hat{\phi}_i}{\epsilon_i} - \frac{q}{HU}\mathcal{A}\right)^2\right], \tag{7.27}$$

where

$$R = \frac{1}{\hat{r}^2}\prod_{k=1}^{n}(\hat{r}^2+a_k^2) - 2m, \qquad X_\alpha = \frac{1}{y_\alpha^2}\prod_{k=1}^{n}(a_k^2-y_\alpha^2),$$
$$U = \prod_{\alpha=1}^{n-1}(\hat{r}^2+y_\alpha^2), \qquad U_\alpha = -(\hat{r}^2+y_\alpha^2)\prod_{\beta=1}^{n-1}{}'(y_\beta^2-y_\alpha^2),$$
$$\gamma_i = a_i^2\prod_{\alpha=1}^{n-1}(a_i^2-y_\alpha^2), \qquad \epsilon_i = a_i\prod_{k=1}^{n}{}'(a_i^2-a_k^2),$$
$$H = 1+\frac{q}{U}, \qquad q = 2ms^2, \qquad s = \sinh\delta,$$
$$\mathcal{A} = d\hat{t} - \sum_{i=1}^{n}\gamma_i\frac{d\hat{\phi}_i}{\epsilon_i}. \tag{7.28}$$

The Hawking temperature and entropy are

$$T_{\rm H} = \frac{(\hat{r}^2R)'|_{\hat{r}=r_+}}{4\pi[\prod_{k=1}^{n}(r_+^2+a_k^2)+qr_+^2]}, \qquad S = \frac{\mathcal{A}_{D-2}[\prod_{k=1}^{n}(r_+^2+a_k^2)+qr_+^2]}{4r_+}, \tag{7.29}$$

where $\mathcal{A}_{D-2} = 2\pi^{(D-1)/2}/\Gamma[(D-1)/2]$ is the volume of a unit $(D-2)$-sphere, so for example $\mathcal{A}_3 = 2\pi^2$ and $\mathcal{A}_5 = \pi^3$. The angular velocities of the horizon are

$$\Omega_i = \frac{a_i\prod_{j\neq i}(r_+^2+a_j^2)}{\prod_{k=1}^{n}(r_+^2+a_k^2)+qr_+^2}. \tag{7.30}$$

For an extremal solution, with a horizon at $\hat{r} = r_0$, we have $R'|_{\hat{r}=r_0} = 0$, and so

$$\sum_{i=1}^{n}\frac{1}{r_0^2+a_i^2} = \frac{1}{r_0^2}. \tag{7.31}$$

Then we have the near-horizon expansion

$$R = V(\hat{r}-r_0)^2 + O(\hat{r}-r_0)^3, \qquad V = \tfrac{1}{2}R''|_{\hat{r}=r_0}. \tag{7.32}$$

Since $R'|_{\hat{r}=r_0} = 0$ for an extremal solution, we have

$$T'^0_{\rm H} = \frac{r_0^2 V}{2\pi[\prod_{k=1}^n (r_0^2 + a_k^2) + q r_0^2]}, \tag{7.33}$$

and, using (7.31), we obtain

$$\Omega'^0_i = -\frac{2a_i r_0 \prod_{j\neq i}(r_0^2 + a_j^2)}{(r_0^2 + a_i^2)[\prod_{k=1}^n (r_0^2 + a_k^2) + q r_0^2]}. \tag{7.34}$$

Therefore the Frolov-Thorne temperatures are

$$T_0 = 0, \qquad T_i = \frac{V r_0 (r_0^2 + a_i^2)}{4\pi a_i \prod_{j\neq i}(r_0^2 + a_j^2)}. \tag{7.35}$$

To obtain the near-horizon geometry, we make the coordinate changes

$$\hat{r} = r_0(1 + \lambda\rho), \qquad \hat{\phi}_i = \phi_i + \Omega^0_i \hat{t}, \qquad \hat{t} = \frac{t}{2\pi T'^0_{\rm H} r_0 \lambda}, \tag{7.36}$$

and then take the limit $\lambda \longrightarrow 0$. The near-horizon geometry is

$$\begin{aligned} ds^2 = H_0^{2/(D-2)}\bigg[&\frac{\tilde{U}}{V}\left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2}\right) + \sum_{\alpha=1}^{n-1}\frac{\tilde{U}_\alpha}{X_\alpha}dy_\alpha^2 \\ &+ \sum_{\alpha=1}^{n-1}\frac{X_\alpha}{U_\alpha}\left(\frac{2r_0\tilde{U}}{V(r_0^2+y_\alpha^2)}\rho dt + \sum_{i=1}^n \frac{(r_0^2+a_i^2)\gamma_i}{a_i^2 - y_\alpha^2}\frac{d\phi_i}{\epsilon_i}\right. \\ &\left.+ \frac{q}{H_0\tilde{U}}\tilde{\mathcal{A}}\right)^2 + \frac{\prod_{k=1}^n a_k^2}{r^2\prod_{\alpha=1}^{n-1}y_\alpha^2} \\ &\times\left(\frac{2\tilde{U}}{V r_0}\rho dt + \sum_{i=1}^n \frac{(r_0^2+a_i^2)\gamma_i}{a_i^2}\frac{d\phi_i}{\epsilon_i} + \frac{q}{H_0\tilde{U}}\tilde{\mathcal{A}}\right)^2\bigg], \end{aligned} \tag{7.37}$$

where $\tilde{U} = U|_{\hat{r}=r_0}$, $\tilde{U}_\alpha = U_\alpha|_{\hat{r}=r_0}$, $H_0 = H|_{\hat{r}=r_0}$, and

$$\begin{aligned} H'_0 &= \frac{\partial H_0}{\partial r_0} = -\frac{q}{\tilde{U}}\sum_{\alpha=1}^{n-1}\frac{2r_0}{r_0^2 + y_\alpha^2}, \\ \tilde{\mathcal{A}} &= \frac{H'_0\tilde{U}^2}{Vq}\rho dt - \sum_{i=1}^n \gamma_i \frac{d\phi_i}{\epsilon_i}. \end{aligned} \tag{7.38}$$

As in the even-dimensional case, we can directly see that this near-horizon geometry can be cast in the form of (2.17) by checking $dt$ coefficients. The analogous identities needed are

$$\frac{1}{r_0^2\tilde{U}}\sum_{i=1}^n \frac{a_i\gamma_i\prod_{j\neq i}(r_0^2+a_j^2)}{\epsilon_i(r_0^2+a_i^2)} = \sum_{\alpha=1}^{n-1}\frac{1}{r_0^2+y_\alpha^2} - \sum_{i=1}^n \frac{1}{r_0^2+a_i^2} + \frac{1}{r_0^2}, \tag{7.39}$$

$$\frac{U_\alpha}{(r_0^2+y_\alpha^2)X_\alpha} = \sum_{i=1}^n \frac{\gamma_i}{a_i\epsilon_i(a_i^2 - y_\alpha^2)}. \tag{7.40}$$

The $1/r_0^2$ coefficient on the right-hand side of (7.39) follows from the identity

$$\sum_{i=1}^n \frac{\gamma_i}{a_i^3\epsilon_i} = \frac{\prod_{\alpha=1}^{n-1} y_\alpha^2}{\prod_{k=1}^n a_k^2}, \tag{7.41}$$

as seen by a partial fraction decomposition of one of the terms on the left-hand side. Now using the extremality condition (7.31), we see that

$$\frac{2}{r_0\tilde{U}}\sum_{i=1}^n \frac{a_i\gamma_i\prod_{j\neq i}(r_0^2+a_j^2)}{\epsilon_i(r_0^2+a_i^2)} = \sum_{\alpha=1}^{n-1}\frac{2r_0}{r_0^2+y_\alpha^2} = -\frac{H'_0\tilde{U}}{q}. \tag{7.42}$$

For the $dt$ coefficient of the unpaired vielbein, we also need to use the partial fraction decomposition

$$\frac{\tilde{U}}{\prod_{k=1}^n(r_0^2+a_k^2)} = \sum_{i=1}^n \frac{a_i\gamma_i}{\epsilon_i(r_0^2+a_i^2)}, \tag{7.43}$$

hence completing the verification. It again follows that the Cardy formulas are satisfied. The central charges are

$$c_i = \frac{3\mathcal{A}_{D-2}a_i\prod_{j\neq i}(r_0^2+a_j^2)[\prod_{k=1}^{n-1}(r_0^2+a_k^2) + q r_0^2]}{\pi V r_0(r_0^2+a_i^2)}. \tag{7.44}$$

## VIII. CONCLUSIONS

In this paper, we have generalized the recently proposed extremal black hole/CFT correspondence to large classes of charged rotating black holes in a variety of dimensions. For extremal black holes, the near-horizon geometry can be obtained by a limiting (or decoupling) procedure that implies that the near-horizon geometry is a solution in its own right. We started with a general argument that the near-horizon geometry of extremal rotating black holes is of the form of a sphere bundle over $AdS_2$, with the connection potentials proportional to the inverse of the Frolov-Thorne temperatures. It is then straightforward to demonstrate that the Cardy formulas for these near-horizon geometries are satisfied, which we have verified in low dimensions. Since the formulas do not rely on any special features of a particular dimension, they are very likely to be satisfied in arbitrary dimension. With this general argument, to show that the Cardy formulas are satisfied for a particular black hole solution, it suffices to show that its near-horizon geometry may be cast in a canonical form.

We then obtained the near-horizon geometries for a variety of charged rotating black holes in gauged and ungauged supergravities in a variety of dimensions, and in gravity theories that are low-energy effective actions of bosonic strings in arbitrary dimension. In all of these examples, the near-horizon geometry has the form established in the general argument. Consequently the Cardy formulas are satisfied and the microscopic entropies of the dual CFTs agree with the Bekenstein-Hawking entropies of the extremal rotating black holes.

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