2-12-2010

Non-Abelian Black Holes in *D* = 5 maximal gauged supergravity

Mirjam Cvetič
University of Pennsylvania, cvetic@cvetic.hep.upenn.edu

H. Lü
Texas A & M University - College Station

C. N. Pope
Texas A & M University - College Station

Suggested Citation:
Cvetič, M., H. Lü, and C.N. Pope. (2010). "Non-Abelian black holes in *D* = 5 maximal gauged supergravity." *Physical Review D* 81, 044023.

http://dx.doi.org/10.1103/PhysRevD.81.044023

Non-Abelian Black Holes in $D = 5$ maximal gauged supergravity

Abstract
We investigate static non-Abelian black hole solutions of anti–de Sitter (AdS) Einstein-Yang-Millsdilaton gravity, which is obtained as a consistent truncation of five-dimensional maximal gauged supergravity. If the dilaton is (consistently) set to zero, the remaining equations of motion, with a spherically-symmetric ansatz, may be derived from a superpotential. The associated first-order equations admit an explicit solution supported by a non-Abelian $SU(2)$ gauge potential, which has a logarithmically growing mass term. In an extremal limit the horizon geometry becomes AdS_2 X S^3. If the dilaton is also excited, the equations of motion cannot easily be solved explicitly, but we obtain the asymptotic form of the more general non-Abelian black holes in this case. An alternative consistent truncation, in which the Yang-Mills fields are set to zero, also admits a description in terms of a superpotential. This allows us to construct explicit wormhole solutions (neutral spherically-symmetric domain walls). These solutions may be generalized to dimensions other than five.

Disciplines
Physical Sciences and Mathematics | Physics

Comments
Suggested Citation:
Cvetič, M., H. Lü, and C.N. Pope. (2010). "Non-Abelian black holes in $D = 5$ maximal gauged supergravity." *Physical Review D* 81, 044023.

http://dx.doi.org/10.1103/PhysRevD.81.044023

Non-Abelian black holes in $D = 5$ maximal gauged supergravity

M. Cvetič,[1] H. Lü,[2,3] and C. N. Pope[2,4]

[1]*Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA*
[2]*George and Cynthia Woods Mitchell Institute for Fundamental Physics and Astronomy, Texas A&M University, College Station, Texas 77843-4242, USA*
[3]*Division of Applied Mathematics and Theoretical Physics, China Institute for Advanced Study, Central University of Finance and Economics, Beijing, 100081, China*
[4]*DAMTP, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge CB3 OWA, UK*
(Received 31 October 2009; published 12 February 2010)

We investigate static non-Abelian black hole solutions of anti–de Sitter (AdS) Einstein-Yang-Mills-dilaton gravity, which is obtained as a consistent truncation of five-dimensional maximal gauged supergravity. If the dilaton is (consistently) set to zero, the remaining equations of motion, with a spherically-symmetric ansatz, may be derived from a superpotential. The associated first-order equations admit an explicit solution supported by a non-Abelian $SU(2)$ gauge potential, which has a logarithmically growing mass term. In an extremal limit the horizon geometry becomes $\mathrm{AdS}_2 \times S^3$. If the dilaton is also excited, the equations of motion cannot easily be solved explicitly, but we obtain the asymptotic form of the more general non-Abelian black holes in this case. An alternative consistent truncation, in which the Yang-Mills fields are set to zero, also admits a description in terms of a superpotential. This allows us to construct explicit wormhole solutions (neutral spherically-symmetric domain walls). These solutions may be generalized to dimensions other than five.

DOI: 10.1103/PhysRevD.81.044023 PACS numbers: 04.70.Bw, 04.65.+e

Explicit analytic solutions for non-Abelian black hole or soliton solutions of (gauged) supergravity theories are rare. The first such example was that of the Chamseddine-Volkov BPS monopole [1] of four-dimensional $\mathcal{N} = 4$ gauged supergravity. Its type IIB embedding can be interpreted as $D5$-branes wrapped on S^2, and the $D = 4$, $\mathcal{N} = 1$ dual field theory interpretation was given in [2]. This gravitating Bogomol'nyi-Prasad-Sommerfield (BPS) soliton is supported both by the $SU(2)$ gauge field and a scalar field. The ground state of this supergravity truncation does not have a constant scalar and so there is no AdS_4 vacuum. (For work on non-Abelian BPS black holes and Dirac-'t Hooft type monopoles in $D = 4$, $\mathcal{N} = 2$ ungauged supergravity, see [3] and references therein.)

Numerical results, which provide evidence for the existence of regular BPS monopole solutions in $D = 5$, $\mathcal{N} = 4$ gauged supergravity, were presented in [4]. The lift to type IIB superstring theory was interpreted as $D5$-branes wrapped on S^3, and the dual $D = 3$, $\mathcal{N} = 1$ field theory interpretation was given in [2]. (These latter results can also be interpreted as those of $D = 7$, $\mathcal{N} = 2$ gauged supergravity and its lifts to type IIB and M-theory, analyzed earlier in [5]. See also the review [6] and references therein.) Again these gravitating BPS solitons are supported both by the $SU(2)$ gauge field and a scalar field, and thus are not asymptotic to AdS_5. (For further numerical analysis of black hole and soliton solutions of $D = 5$, $\mathcal{N} = 4$ gauged supergravity, see [7] and references therein. Non-Abelian BPS solutions in five-dimensional $\mathcal{N} = 2$ gauged supergravity have been discussed recently in [8].)

It is believed that maximal ($\mathcal{N} = 8$) gauged supergravity in $D = 5$ can be obtained from a Kaluza-Klein reduction of ten-dimensional type IIB supergravity on S^5. The only complete demonstrations so far are for the consistency of the maximal Abelian $U(1)^3$ truncation [9], the $\mathcal{N} = 4$ gauged $SU(2) \times U(1)$ truncation [10], the scalar truncation in [11,12], and the $SO(6)$ truncation [13]. The form of the full metric reduction ansatz was conjectured in [14].

Non-Abelian solutions in any of the five-dimensional gauged supergravities that have a known embedding in type IIB supergravity are of particular interest because they can be given a ten-dimensional interpretation within string theory. We can find an exact solution in the $SU(2) \times U(1)$ gauged theory, whose type IIB embedding is given in [10], in which the $SU(2)$ Yang-Mills fields carry a magnetic charge. Unfortunately, however, the BPS condition implies that the five-dimensional metric has the wrong signature.

In this paper we consider static non-Abelian black hole solutions of five-dimensional maximal ($\mathcal{N} = 8$) gauged supergravity. We present a consistent truncation of this theory whose bosonic sector comprises gravity, $SU(2) \times SU(2)$ gauge fields, and a scalar field whose potential has an AdS_5 minimum. A further consistent truncation to just an $SU(2)$ gauge symmetry, with a fixed cosmological constant (related to the gauge coupling), results in anti–de Sitter (AdS) Einstein-Yang-Mills gravity. We show that with the assumption of a spherically-symmetric ansatz for the metric and $SU(2)$ Yang-Mills potentials, the field equations for this truncated system may be derived from a superpotential, and hence we can obtain first-order equations of motion. These give rise to an explicit $SU(2)$ black hole solution which is asymptotic to AdS_5, but which has a logarithmically divergent mass term as a consequence of the nonvanishing (constant) $SU(2)$ gauge potential. This solution was in fact obtained previously in [15].[1] We find that it admits an extremal limit, for which the $SU(2)$ gauge potential remains nonvanishing, and the horizon has the geometry $AdS_2 \times S^2$. These results are intriguing, since the solutions have an embedding into $D = 5$ maximal gauged supergravity, and hence admit a further lift to type IIB string theory.

Although one may expect that with a spherically-symmetric ansatz the more general system with $SU(2) \times SU(2)$ gauge fields and a dilatonic scalar should also admit a description in terms of a superpotential, we have not succeeded in finding it in this case. We can, nevertheless, directly study the second-order equations of motion, and investigate the asymptotic form for the more general solutions with the additional "scalar charge." We find evidence that these non-Abelian Yang-Mills solutions again describe black holes, albeit again with logarithmically divergent mass.

We also find wormhole solutions of both $D = 4$ and $D = 5$ maximal gauged supergravities. These are static neutral domain-wall solutions, which are asympotic to AdS_4 and AdS_5 respectively. However, in the interior a scalar field diverges. We obtain these solutions by finding a superpotential, and then solving the associated first-order equations of motion. However, these solutions do not have supersymmetric limits.

We start with the $SO(6)$ truncation of $D = 5$, $\mathcal{N} = 8$ gauged supergravity. It can be obtained from the S^5 reduction of the $SL(2, \mathbb{R})$-singlet sector of type IIB supergravity, for which the only bosonic fields in ten dimensions are the metric and the self-dual 5-form. The full nonlinear ansatz was given in [13]. The five-dimensional theory consists of the metric, twenty scalars, which are in the $20'$ representation of $SO(6)$ and are represented by the symmetric unimodular tensor T_{ij}, with i being a 6 of $SO(6)$, together with 15 $SO(6)$ Yang-Mills gauge fields, represented by the 1-form potentials A^{ij}, antisymmetric in i and j. The five-dimensional Lagrangian is given by [13]

$$\mathcal{L}_5 = R * \mathbb{1} - \frac{1}{4} T_{ij}^{-1} * DT_{jk} \wedge T_{k\ell}^{-1} DT_{\ell i} - \frac{1}{4} T_{ik}^{-1} T_{j\ell}^{-1} * F^{ij} \wedge F^{k\ell} - V * \mathbb{1} - \frac{1}{48} \epsilon_{i_1 \cdots i_6} \Big(F^{i_1 i_2} F^{i_3 i_4} A^{i_5 i_6} - g F^{i_1 i_2} A^{i_3 i_4} A^{i_5 j} A^{j i_6} + \frac{2}{5} g^2 A^{i_1 i_2} A^{i_3 j} A^{j i_4} A^{i_5 k} A^{k i_6} \Big), \quad (1)$$

where the potential V is given by

$$V = \frac{1}{2} g^2 (2 T_{ij} T_{ij} - (T_{ii})^2). \quad (2)$$

The Yang-Mills field strength F^{ij} and covariant derivative DT_{ij} are defined by

$$F^{ij} = dA^{ij} + g A^{ik} \wedge A^{kj}, \qquad DT_{ij} = dT_{ij} + g A^{ik} T_{kj} + g A^{jk} T_{ik}. \quad (3)$$

We now perform a further truncation of $SO(6)$ to $SU(2) \times SU(2)$, by setting

$$A^{12} = A^3, \quad A^{23} = A^1, \quad A^{31} = A^2, \quad A^{45} = \tilde{A}^3, \quad A^{56} = \tilde{A}^1, \quad A^{64} = \tilde{A}^2, \quad T_{11} = T_{22} = T_{33} = X, \quad T_{44} = T_{55} = T_{66} = X^{-1}, \quad (4)$$

with the remaining fields vanishing. This truncation is consistent provided that the additional constraint

$$F^i \wedge \tilde{F}^j = 0 \quad (5)$$

is imposed, where

$$F^i = dA^i + \frac{1}{2} g \epsilon^{ijk} A^j \wedge A^k, \qquad \tilde{F}^i = d\tilde{A}^i + \frac{1}{2} g \epsilon^{ijk} \tilde{A}^j \wedge \tilde{A}^k. \quad (6)$$

The fields satisfy equations of motion that can be derived from the Lagrangian

$$\mathcal{L} = R * \mathbb{1} - \frac{3}{2} X^{-2} * dX \wedge dX - \frac{1}{2} X^{-2} * F^i \wedge F^i - \frac{1}{2} X^2 * \tilde{F}^i \wedge \tilde{F}^i + \frac{3}{2} g^2 (X^2 + X^{-2} + 6), \quad (7)$$

together with the constraint (5).

We may now look for spherically-symmetric static solutions, by making the ansatz

$$ds_5^2 = -\alpha^2 dt^2 + d\rho^2 + \frac{1}{4} \beta^2 (\sigma_1^2 + \sigma_2^2 + \sigma_3^2), \qquad A^i = g^{-1} \gamma \sigma_i, \qquad \tilde{A}^i = g^{-1} \tilde{\gamma} \sigma_i, \quad (8)$$

where the functions α, β, γ, $\tilde{\gamma}$, and the scalar X are taken to depend only on the radial coordinate ρ. The σ_i are $SU(2)$ left-invariant 1-forms, satisfying $d\sigma_i = -\frac{1}{2} \epsilon^{ijk} \sigma_j \wedge \sigma_k$. Note that the metric ansatz is invariant under $SO(4) \sim SU(2)_L \times SU(2)_R$ rotations of the S^3 level surfaces. The Yang-Mills potentials, and field strengths, are invariant under $SU(2)_L$, while they rotate covariantly under $SU(2)_R$. The energy-momentum tensor is therefore invariant under the full $SO(4)$ action.

[1] Other recent works on non-Abelian solutions in various other Einstein-Yang-Mills systems can be found in [16–20].

The Yang-Mills equations imply that

$$(\alpha\beta\dot\gamma e^{-\phi})^{\cdot} - \frac{4\alpha}{\beta}\gamma(\gamma-1)(2\gamma-1)e^{-\phi}=0,$$
$$(\alpha\beta\dot{\tilde\gamma} e^{\phi})^{\cdot} - \frac{4\alpha}{\beta}\tilde\gamma(\tilde\gamma-1)(2\tilde\gamma-1)e^{\phi}=0, \tag{9}$$

where we have defined

$$X \equiv e^{(1/2)\phi}, \tag{10}$$

and a dot denotes a derivative with respect to ρ. The scalar equation of motion is

$$\frac{(\alpha\beta^3\dot\phi)^{\cdot}}{2\alpha\beta^3} = e^{-\phi}\Big(g^2 - \frac{4\dot\gamma^2}{g^2\beta^2} - \frac{16\gamma^2(\gamma-1)^2}{g^2\beta^4}\Big) - e^{\phi}\Big(g^2 - \frac{4\dot{\tilde\gamma}^2}{g^2\beta^2} - \frac{16\tilde\gamma^2(\tilde\gamma-1)^2}{g^2\beta^4}\Big), \tag{11}$$

and the Einstein equations are given by

$$\frac{\ddot\alpha}{\alpha} + \frac{3\dot\alpha\,\dot\beta}{\alpha\beta} = \Big(\frac{2\dot\gamma^2}{g^2\beta^2} + \frac{8\gamma^2(\gamma-1)^2}{g^2\beta^4}\Big)e^{-\phi} + \Big(\frac{2\dot{\tilde\gamma}^2}{g^2\beta^2} + \frac{8\tilde\gamma^2(\tilde\gamma-1)^2}{g^2\beta^4}\Big)e^{\phi} + g^2(\cosh\phi+3),$$
$$\frac{\ddot\alpha}{\alpha} + \frac{3\ddot\beta}{\beta} = -\Big(\frac{4\dot\gamma^2}{g^2\beta^2} - \frac{8\gamma^2(\gamma-1)^2}{g^2\beta^4}\Big)e^{-\phi} - \Big(\frac{4\dot{\tilde\gamma}^2}{g^2\beta^2} - \frac{8\tilde\gamma^2(\tilde\gamma-1)^2}{g^2\beta^4}\Big)e^{\phi} + g^2(\cosh\phi+3) + \frac{3}{8}\dot\phi^2, \tag{12}$$
$$\frac{2}{\beta^2} - \frac{\dot\alpha\,\dot\beta}{\alpha\beta} - \frac{2\dot\beta^2}{\beta^2} - \frac{\ddot\beta}{\beta} = \frac{8\gamma^2(\gamma-1)^2}{g^2\beta^4}e^{-\phi} + \frac{8\tilde\gamma^2(\tilde\gamma-1)^2}{g^2\beta^4}e^{\phi} - g^2(\cosh\phi+3).$$

The constraint (5) implies that

$$\dot\gamma\,\tilde\gamma(\tilde\gamma-1) + \dot{\tilde\gamma}\gamma(\gamma-1) = 0. \tag{13}$$

From this, it follows that

$$\gamma\tilde\gamma = c(\gamma-1)(\tilde\gamma-1), \tag{14}$$

where c is an integration constant. Combining (14) and (9), we obtain the first-order constraint

$$\beta^2[c-(c-1)\gamma]\dot\phi\,\dot\gamma = (c-1)[4\gamma^2(\gamma-1)^2 - \beta^2\dot\gamma^2]. \tag{15}$$

Finding the general solution to the equations of motion is likely to be very difficult. We can, however, obtain explicit exact solutions in some special cases.

We first consider the case where the Yang-Mills fields are nonvanishing, and the integration constant c in (14) is chosen to be $c=1$. Equation (15) then implies that either $\dot\gamma=0$ or $\dot\phi=0$. For $\dot\gamma=0$, it follows from (14) that $\tilde\gamma = 1-\gamma$, and from (9) that the only nontrivial solution is $\gamma=\frac{1}{2}=\tilde\gamma$, in which case the scalar equation (11) implies that we can also set $\phi=0$. The reduced equations of motion can now be derived from an effective Lagrangian $L=T-U$ with

$$T = \frac{6\alpha'\beta'}{\alpha\beta} + \frac{6\beta'^2}{\beta^2},$$
$$U = -\frac{3}{32}\alpha^2\beta^2\Big(-\frac{1}{2g^2} + \beta^2 + 2g^2\beta^4\Big), \tag{16}$$

where a prime denotes a derivative with respect to η, defined by $d\rho = \frac{1}{8}\alpha\beta^3 d\eta$.

Expressing the kinetic terms T as

$$T = \frac{1}{2}g_{ij}\frac{dX^i}{d\eta}\frac{dX^j}{d\eta}, \tag{17}$$

where $X^i=(\alpha,\beta)$, we find that the potential U can be written in terms of a "superpotential" W as

$$U = -\frac{1}{2}g^{ij}\frac{\partial W}{\partial X^i}\frac{\partial W}{\partial X^j}, \tag{18}$$

where

$$W = \frac{3}{4}\alpha\beta\sqrt{-M+\beta^2+g^2\beta^4 - \frac{\log(g\beta)}{g^2}}. \tag{19}$$

The parameter M is an integration constant, which can be chosen arbitrarily. The existence of the superpotential implies that the second-order equations of motion are satisfied if the first-order equations $dX^i/d\eta = g^{ij}\partial W/(\partial X^j)$ hold. Thus we obtain the equations

$$\dot\alpha = \frac{\alpha(-1+2g^2M+2g^4\beta^4+2\log(g\beta))}{2g^2\beta^2\sqrt{-M+\beta^2+g^2\beta^4-g^{-2}\log(g\beta)}}$$
$$\dot\beta = \beta^{-1}\sqrt{-M+\beta^2+g^2\beta^4-g^{-2}\log(g\beta)}. \tag{20}$$

These can be solved, giving

$$ds^2 = -fdt^2 + \frac{dr^2}{f} + \frac{1}{4}r^2(\sigma_1^2+\sigma_2^2+\sigma_3^2),$$
$$A^i = \frac{1}{2g}\sigma_i = \tilde A^i, \qquad f = 1 + g^2r^2 - \frac{M+g^{-2}\log(gr)}{r^2}. \tag{21}$$

Note that because the superpotential W itself has the arbitrary constant of integration M, the solution (21) of the first-order equations is in fact the most general solution also of the original second-order field equations. The solution describes an $SU(2)$ Yang-Mills black hole with logarithmically divergent mass. The horizon is located at the largest root of the function f.

The solution has an extremal limit, for which $f(r)$ and $f'(r)$ vanish simultaneously at $r=r_0$, when

$$M = \frac{1+\sqrt{5}+\log(8(7+3\sqrt{5}))}{8g^2}, \tag{22}$$

and

$$g^2 r_0^2 = \frac{1}{4}(\sqrt{5} - 1). \tag{23}$$

The near-horizon geometry is then a direct product of $AdS_2 \times S^3$, with the metric given by

$$ds^2 = \alpha^2 ds_2^2 + \beta^2 d\Omega_3^2, \quad \alpha^2 = \frac{5-\sqrt{5}}{40g^2}, \quad \beta^2 = \frac{\sqrt{5}-1}{4g^2}. \tag{24}$$

It is worth pointing out that it is the Yang-Mills fields that are responsible for the occurrence of the logarithmic term in the metric (21). If the Yang-Mills fields are instead set to zero, the logarithmic term disappears and the solution becomes the Schwarzschild-AdS black hole. That nonsupersymmetric solutions such as the Schwarzschild black hole can be obtained from first-order equations derived from a superpotential was previously observed in [21].

The solution can be lifted back to $D = 10$, by using the reduction ansatz given in [13]. The ten-dimensional metric is given by

$$\begin{aligned} d\hat{s}_{10}^2 = ds_5^2 + \frac{1}{g^2}\Big\{&\Big[d\mu_1 + \frac{1}{2}(\mu_2\sigma_3 - \mu_3\sigma_2)\Big]^2 \\ &+ \Big[d\mu_2 + \frac{1}{2}(\mu_3\sigma_1 - \mu_1\sigma_3)\Big]^2 + \Big[d\mu_3 + \frac{1}{2}(\mu_1\sigma_2 \\ &- \mu_2\sigma_1)\Big]^2 + \Big[d\mu_4 + \frac{1}{2}(\mu_5\sigma_3 - \mu_6\sigma_2)\Big]^2 \\ &+ \Big[d\mu_5 + \frac{1}{2}(\mu_6\sigma_1 - \mu_4\sigma_3)\Big]^2 \\ &+ \Big[d\mu_6 + \frac{1}{2}(\mu_4\sigma_2 - \mu_5\sigma_1)\Big]^2\Big\}, \end{aligned} \tag{25}$$

where $\mu_i\mu_i = 1$. The Ramond-Ramond (RR) 5-form field takes a somewhat involved structure, supported by the five-sphere coordinates; it can be readily derived from the expressions given in [13]. Thus the type IIB solution is supported by the RR 5-form with a specific non-Abelian deformation of the five-sphere coordinates, resulting in an asymptotically AdS_5 space-time supported by $D3$-brane fluxes. This structure is different from those of [6,22] where a five-brane wraps a 2-cycle in the internal Ricci-flat space, resulting in a $D = 4$ $\mathcal{N} = 1$ dual field theory on the world-volume of the five-brane.

We may also consider the more general case of non-Abelian solutions where the dilatonic scalar is also excited. For simplicity, we consider only the case for $\gamma = \tilde{\gamma} = \frac{1}{2}$, so that the Yang-Mills equations are trivially satisfied. The equations of motion for the spherically-symmetric ansatz can then be derived from the Lagrangian $L = T - U$, with

$$\begin{aligned} T &= \frac{6\alpha'\beta'}{\alpha\beta} + \frac{6\beta'^2}{\beta^2} - \frac{3}{8}\phi'^2, \\ U &= -\frac{3}{32}\alpha^2\beta^2\Big(-\frac{1}{2g^2}\cosh\phi + \frac{1}{2}g^2\beta^4\cosh\phi \\ &\quad + \beta^2 + \frac{3}{2}g^2\beta^4\Big). \end{aligned} \tag{26}$$

Reading off g_{ij} using (17), and then expressing U in terms of a superpotential W as in (18), we find that W is determined by the equation

$$\beta\frac{\partial \hat{W}^2}{\partial\beta} - 8\Big(\frac{\partial\hat{W}}{\partial\phi}\Big)^2 = 3g^2\beta^4 + 2\beta^2 + \Big(g^2\beta^4 - \frac{1}{g^2}\Big) \times \cosh\phi, \tag{27}$$

where

$$W(\alpha, \beta, \phi) = \frac{3\alpha\beta}{4}\hat{W}(\beta, \phi). \tag{28}$$

We have not been able to solve this equation explicitly.

Although we are unable to obtain the exact solution, we may nevertheless consider its large r expansion, which we find to be given by

$$\begin{aligned} ds^2 &= -N^2dt^2 + \frac{dr^2}{f} + \frac{1}{4}r^2(\sigma_1^2 + \sigma_2^2 + \sigma_3^2), \qquad A^i = \frac{1}{2g}\sigma_i = \tilde{A}^i, \\ N^2 &= 1 + g^2r^2 - \frac{\xi - 1}{g^2r^2} + \frac{q^2(8\xi - 3)}{32g^2r^6} - \frac{g^2(40\xi - 1)}{800g^4r^8} + \frac{q^2(1200\xi^2 - 60(13 + 20g^4q^2)\xi - 13 + 550g^4q^2)}{9600g^6r^{10}} \\ &\quad - \frac{q^2(4\,492\,320\xi^2 - 56(56\,713 + 25\,200g^4q^2)\xi + 3(63\,461 + 64\,400g^4q^2))}{39\,513\,600g^8r^{12}} + \cdots, \\ \frac{f}{N^2} &= 1 + \frac{q^2}{r^4} + \frac{q^2(24\xi - 1)}{32g^4r^8} - \frac{q^2(120\xi - 13)}{225g^6r^{10}} + \frac{q^2(2016\xi^2 - 48(3 + 8g^4q^2)\xi - 1 + 160g^4q^2)}{3072g^8r^{12}} + \cdots, \\ \sinh\frac{1}{2}\phi &= \frac{q}{r^2}\Big[1 + \frac{\xi}{4g^4r^4} - \frac{12\xi - 1}{72g^6r^6} + \frac{288\xi^2 - 128g^4q^2\xi - 1 + 32g^4q^2}{2048g^8} \\ &\quad \times \frac{631\,200\xi^2 - 40(8053 + 3600g^4q^2)\xi + 9(4303 + 4400g^4q^2)}{2\,880\,000g^{10}r^{10}} \frac{1}{82\,944\,000g^{12}r^{12}}(8\,100\,000\xi^3 + 28\,800(256 \\ &\quad - 285g^4q^2)\xi^2 + 15(-508\,631 + 141\,600g^4q^2 + 172\,800g^8q^4)\xi 1\,093\,091 + 18\,000g^4q^2 - 756\,000g^8q^4) + \cdots\Big], \end{aligned} \tag{29}$$

where ξ is given by

$$\xi = 1 + g^2 M + \log(gr). \tag{30}$$

Although we have not obtained the exact solution, the form of the large-r expansion indicates that it describes a black hole, at least provided that q is sufficiently small or that M is sufficiently large. To see this, we note that for the case $q = 0$, the solution reduces to the exact one that we discussed earlier. The horizon is located at r_+, where $M = r_+^2 + r_+^4 - \log r_+$. (Here we set $g = 1$ for simplicity.) With $q \neq 0$, the horizon is shifted to r'_+ where r'_+ is defined by $N(r'_+) = 0$. It is straightforward to see that

$$r'_+ - r_+ = \mathcal{O}\left(\frac{q^2}{r_+^3}\right). \tag{31}$$

We may alternatively consider a special case where the Yang-Mills fields are set to zero, so that the solution then involves only the metric and a scalar field. The scalar potential fits into the general pattern discussed in the appendix. Following the discussion in the appendix, we find that there exists a domain wall solution

$$ds_5^2 = -(k + g^2 r^2)dt^2 + \frac{dr^2}{(k + g^2 r^2)(1 + \frac{q^2}{r^4})} + r^2 d\Omega_{3,k}^2,$$
$$\sinh\frac{1}{2}\phi = \frac{q}{r^2}, \tag{32}$$

where $k = 1, 0$ or $= -1$ for spherical, flat, or hyperbolic spatial sections. This solution can also be lifted back to $D = 10$, giving rise to a solution of type IIB supergravity, using the results of [13]:

$$d\hat{s}_{10}^2 = \left(Xc^2 + \frac{s^2}{X}\right)^2 \left\{ ds_5^2 + g^{-2}\left(d\theta^2 + \frac{c^2}{X^2 c^2 + s^2} d\Omega_2^2 + \frac{s^2}{s^2 X^{-2} + c^2} d\tilde{\Omega}_2^2\right)\right\},$$
$$\hat{G}_{(5)} = g(X^2 c^2 + X^{-2} s^2 + 3)\epsilon_{(5)} - \frac{2sc}{g} X^{-1} * dX \wedge d\theta,$$
$$X = \frac{q}{r^2} + \sqrt{1 + \frac{q^2}{r^4}}, \qquad c = \cos\theta, \qquad s = \sin\theta, \tag{33}$$

where the self-dual 5-form is given by $\hat{F}_{(5)} = \hat{G}_{(5)} + \hat{*}\hat{G}_{(5)}$.

The domain wall solution (32) is massless and has a naked singularity at $r = 0$. As we have shown in the appendix for a general class of such domain walls in arbitrary dimensions, we can also add a mass term. The exact form of the solution is unknown, but the large r expansion can be obtained straightforwardly. Here we present the solution in higher orders:

$$N^2 = k + g^2 r^2 - \frac{M}{r^2} + \frac{Mq^2}{4r^6} - \frac{kMq^2}{20 g^2 r^8} + \frac{Mq^2(4k^2 + 15 g^2 M - 15 g^4 q^2)}{120 g^4 r^{10}} + \cdots,$$
$$\frac{f}{N^2} = 1 + \frac{q^2}{r^4} + \frac{3Mq^2}{4g^2 r^8} - \frac{8kMq^2}{15 g^4 r^{10}} + \cdots,$$
$$\sinh\frac{1}{2}\phi = 1 + \frac{M}{4g^2 r^4} - \frac{kM}{6 g^4 r^6} + \frac{M(8k^2 + 9g^2 M - 4g^4 q^2)}{64 g^6 r^8} - \frac{kM(120 + 263 g^2 M - 60 g^4 q^2)}{1200 g^8 r^{10}} + \cdots. \tag{34}$$

As in our previous discussion of the non-Abelian black holes, since the $q = 0$ solution here describes the Schwarzschild-AdS black hole, and the effect of the q parameter is to modify terms at higher orders in $1/r$, it follows that the solution with $q \neq 0$ will still describe a black hole, at least if q is sufficiently small.

In summary, we have in this paper studied the system of equations that arises from a consistent truncation of five-dimensional maximal gauged supergravity, in which just the metric, a dilatonic scalar, and the gauge fields of $SU(2) \times SU(2)$ are retained. Consistency requires that the gauge fields satisfy the constraint (5). Our focus has been on seeking spherically-symmetric solutions to this system. In the two special cases where either the dilaton is set to zero and a further truncation to $SU(2)$ gauge symmetry is performed, or where the dilaton is retained but the gauge fields are set to zero, we have been able to describe the system in terms of a superpotential. This allows us to obtain first-order equations of motion, which can be straightforwardly solved analytically. The first of these cases leads to a non-Abelian black hole solution, with logarithmically-diverging mass, which was first obtained in [15]. The second case gives rise to a spherically-symmetric domain wall solution. In the more general situation where both the Yang-Mills fields and the dilaton are excited, we have not succeeded in describing the system in terms of a superpotential. Nevertheless, we have studied the asymptotic behavior of spherically-symmetric solutions, and found that more general non-Abelian black holes arise here also. This discussion is extended, in an appendix, to gravity plus dilaton systems in arbitrary dimensions.

M. C., H. L. and C. N. P. are grateful to Sheridan Lorenz and the Mitchell Family Foundation for hospitality at Cook's Branch Conservancy, and H. L. and C. N. P. are grateful to the Department of Physics and Astromony at the University of Pennsylvania for hospitality, during the course of this work. The work of M. C. is supported by DOE grant DE-FG05- 95ER40893-A020, NSF RTG grant DMS-0636606 and the Fay R. and Eugene L. Langberg

Chair. The work of C. N. P. is supported in part by DOE grant DE-FG03-95ER40917.

APPENDIX: A SPHERICALLY-SYMMETRIC DOMAIN WALL SOLUTION

In this appendix, we consider a general class of D-dimensional Lagrangians, given by

$$\mathcal{L}_D = e\Big(R - \frac{1}{2}(\partial\phi)^2 - V\Big),$$
$$V = -(D-2)g^2\Big[D - 2 + \cosh\Big(\sqrt{\frac{2(D-3)}{D-2}}\phi\Big)\Big]. \quad \text{(A1)}$$

The scalar potential can be expressed in terms of a superpotential w as

$$V = \Big(\frac{dw}{d\phi}\Big)^2 - \frac{D-1}{2(D-2)}w^2,$$
$$w = \sqrt{2}(D-2)g\cosh\Big(\sqrt{\frac{D-3}{2(D-2)}}\phi\Big). \quad \text{(A2)}$$

Consider the spherically-symmetric ansatz

$$ds^2 = -\alpha^2 dt^2 + d\rho^2 + \beta^2 d\Omega^2_{n,k}, \quad \text{(A3)}$$

where $n = D - 2$, and $k = -1, 1$, and 0, corresponding to hyperbolic, flat, and sphere. The scalar and Einstein equations of motion are given by

$$\frac{(\alpha\beta^n\dot\phi)^{\cdot}}{\alpha\beta^n} - \frac{dV}{d\phi} = 0, \qquad \frac{\ddot\alpha}{\alpha} + \frac{n\dot\alpha\dot\beta}{\alpha\beta} + \frac{V}{n} = 0,$$
$$-\frac{\ddot\alpha}{\alpha} - \frac{n\ddot\beta}{\beta} - \frac{1}{2}\dot\phi^2 - \frac{V}{n} = 0, \quad \text{(A4)}$$
$$\frac{(n-1)k}{\beta^2} - \frac{\dot\alpha\dot\beta}{\alpha\beta} - \frac{(n-1)\dot\beta^2}{\beta^2} - \frac{\ddot\beta}{\beta} - \frac{V}{n} = 0.$$

These can be derived from the Lagrangian $\mathcal{L} = T - U$, where the kinetic and potential terms are given by

$$T = \frac{2(D-2)\alpha'\beta'}{\alpha\beta} + \frac{(D-2)(D-3)\beta'^2}{\beta^2} - \frac{1}{2}\phi'^2,$$
$$U = \alpha^2\beta^{2(D-3)}(\beta^2 V - (D-2)(D-3)k). \quad \text{(A5)}$$

Here, a prime denotes a derivative with respect to a new radial coordinate η, which is defined by $d\eta = \alpha\beta^n d\rho$. We find that with U given by (A5), there exists a superpotential W, *à la* (18), given by

$$W = 2(D-2)\alpha\beta^{D-3}\sqrt{k + g^2\beta^2}\cosh\Big(\sqrt{\frac{D-3}{2(D-2)}}\phi\Big). \quad \text{(A6)}$$

The resulting first-order equations of motion are

$$\dot\alpha = \frac{g^2\alpha\beta\cosh(\sqrt{\frac{D-3}{2(D-2)}}\phi)}{\sqrt{k + g^2\beta^2}},$$
$$\dot\beta = \sqrt{k + g^2\beta^2}\cosh\Big(\sqrt{\frac{D-3}{2(D-2)}}\phi\Big), \quad \text{(A7)}$$
$$\dot\phi = -\frac{\sqrt{2(D-3)(D-2)}}{\beta} \times\sqrt{k + g^2\beta^2}\sinh\Big(\sqrt{\frac{D-3}{2(D-2)}}\phi\Big).$$

Their solution gives

$$ds^2_D = -(k + g^2r^2)dt^2 + \frac{dr^2}{(k + g^2r^2)(1 + \frac{q^2}{r^{2(D-3)}})} + r^2 d\Omega^2_{D-2,\epsilon},$$
$$\sinh\Big(\sqrt{\frac{D-3}{2(D-2)}}\phi\Big) = \frac{q}{r^{D-3}}. \quad \text{(A8)}$$

The form of the scalar potential (A2) appears rather frequently in gauged supergravities. The corresponding supersymmetry transformation rules for such a truncated system are typically given by

$$\delta\psi_M = D_M\varepsilon - \frac{w}{2\sqrt{2}(D-2)}\Gamma_M\varepsilon,$$
$$\delta\lambda = \frac{1}{2\sqrt{2}}\partial_M\phi\Gamma^M\varepsilon + \frac{1}{2}\frac{dw}{d\phi}\varepsilon. \quad \text{(A9)}$$

It follows from $\delta\lambda = 0$ that the existence of supersymmetry would imply that

$$\dot\phi = \pm\sqrt{2(D-3)(D-2)}g\sinh\Big(\sqrt{\frac{D-3}{2(D-2)}}\phi\Big). \quad \text{(A10)}$$

Comparing this to the last equation in (A7), the only solution with supersymmetry is when $k = 0$. In fact it is easy to see that (A9) cannot be supersymmetric for $k \neq 0$. If it were, there would be a smooth limit when $g = 0$, and this would lead to a solution supported by the metric and a free scalar only, which could not possibly be supersymmetric.

The scalar potential for $D = 4$ occurs in four-dimensional $\mathcal{N} = 4$, $SO(4)$ gauged supergravity. The explicit reduction ansatz that gives this theory from $D = 11$ supergravity was found in [23]. We can use the ansatz to lift the ($k = 1$) solution, given by (A8) with $D = 4$, back to $D = 11$, giving

$$d\hat{s}_{11}^2 = \Delta^{1/3}\Big\{-(1+g^2r^2)dt^2 + \frac{dr^2}{(1+g^2r^2)(1+\frac{q^2}{r^2})} + r^2 d\Omega_2^2 + \frac{4}{g^2}\Big(d\xi^2 + \frac{c^2}{c^2X^2+s^2}d\Omega_3^2 + \frac{s^2}{s^2X^{-2}+c^2}d\tilde{\Omega}_3^2\Big)\Big\},$$
$$\hat{F}_{(4)} = -g(2+X^2c^2+X^{-2}s^2)\epsilon_{(4)} - \frac{4sc}{g}X^{-1} * dX \wedge d\xi,$$
$$\Delta = (c^2X^2+s^2)(s^2X^{-2}+c^2), \qquad X = \frac{q}{r} + \sqrt{1+\frac{q^2}{r^2}}, \tag{A11}$$

where $c \equiv \cos\xi$ and $s \equiv \sin\xi$.

The domain wall solutions (A8) we obtained so far have zero mass, and a naked singularity. It is possible for the solution to have nonvanishing mass, such that the second-order equations of motion [but no longer the first-order equations (A7) following from the superpotential (A6)] are still satisfied. The solution then develops a horizon. We are unable to find the exact solution for this case. However, the large r expansion of the solution can be obtained, and is given by

$$ds_D^2 = -N^2dt^2 + \frac{dr^2}{f} + r^2 d\Omega_{D-2,k}^2,$$
$$N^2 = k + g^2r^2 - \frac{M}{r^{D-3}} + \frac{(D-1)Mq^2}{2(3D-7)r^{3(D-3)}} + \cdots,$$
$$\frac{f}{N^2} = 1 + \frac{q^2}{r^{2(D-3)}} + \frac{4(D-2)(D-3)Mq^2}{(D-1)(3D-7)r^{3D-7}} + \cdots,$$
$$\sinh\Big(\sqrt{\frac{D-3}{2(D-2)}}\phi\Big) = \frac{q}{r^{D-3}}\Big(1 + \frac{(D-3)M}{2(D-1)g^2r^{D-1}} + \cdots\Big). \tag{A12}$$

For vanishing q, the solution becomes the Schwarzschild-AdS black hole, while for vanishing M, it reduces to the singular domain wall described earlier. Since the effect of introducing q is to modify the behavior only at higher inverse powers of r, it is clear that for large enough M or small enough nonzero q, the solution still describes a black hole.

[1] A. H. Chamseddine and M. S. Volkov, Phys. Rev. Lett. **79**, 3343 (1997).
[2] J. M. Maldacena and C. Nunez, Phys. Rev. Lett. **86**, 588 (2001).
[3] M. Hubscher, P. Meessen, T. Ortin, and S. Vaula, Fortschr. Phys. **57**, 600 (2009).
[4] A. H. Chamseddine and M. S. Volkov, J. High Energy Phys. 04 (2001) 023.
[5] B. S. Acharya, J. P. Gauntlett, and N. Kim, Phys. Rev. D **63**, 106003 (2001).
[6] J. P. Gauntlett, arXiv:hep-th/0305074.
[7] E. Radu, Classical Quantum Gravity **23**, 4369 (2006).
[8] J. Bellorin, Classical Quantum Gravity **26**, 195012 (2009).
[9] M. Cvetič, M. J. Duff, P. Hoxha, James T. Liu, H. Lü, J. X. Lu, R. Martinez-Acosta, C. N. Pope, H. Sati, and Tuan A. Tran, Nucl. Phys. **B558**, 96 (1999).
[10] H. Lü, C. N. Pope, and T. A. Tran, Phys. Lett. B **475**, 261 (2000).
[11] M. Cvetič, S. S. Gubser, H. Lü, and C. N. Pope, Phys. Rev. D **62**, 086003 (2000).
[12] M. Cvetič, H. Lü, C. N. Pope, and A. Sadrzadeh, Phys. Rev. D **62**, 046005 (2000).
[13] M. Cvetič, H. Lü, C. N. Pope, A. Sadrzadeh, and T. A. Tran, Nucl. Phys. **B586**, 275 (2000).
[14] A. Khavaev, K. Pilch, and N. P. Warner, Phys. Lett. B **487**, 14 (2000).
[15] N. Okuyama and K. i. Maeda, Phys. Rev. D **67**, 104012 (2003).
[16] E. Winstanley, Classical Quantum Gravity **16**, 1963 (1999).
[17] E. Radu and E. Winstanley, Phys. Rev. D **70**, 084023 (2004).
[18] G. W. Gibbons and P. K. Townsend, Classical Quantum Gravity **23**, 4873 (2006).
[19] R. B. Mann, E. Radu, and D. H. Tchrakian, Phys. Rev. D **74**, 064015 (2006).
[20] Y. Brihaye, E. Radu, and D. H. Tchrakian, Phys. Rev. D **75**, 024022 (2007).
[21] H. Lü, C. N. Pope, and J. F. Vazquez-Poritz, Nucl. Phys. **B709**, 47 (2005).
[22] J. M. Maldacena and C. Nunez, Int. J. Mod. Phys. A **16**, 822 (2001).
[23] M. Cvetič, H. Lü, and C. N. Pope, Nucl. Phys. **B574**, 761 (2000).