

Erratum: Landau analysis of the symmetry of the magnetic structure and magnetoelectric interaction in multiferroics
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A. B. Harris

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This erratum corrects an error in the analysis of Sec. III C concerning the allowed Fourier components of the magnetization distribution of TbMn_2O_5 for wave vector $\mathbf{q}=(\frac{1}{2}, 0, q_z)$. The analysis relies on the form of the inverse susceptibility matrix which was given in Eq. (95). In constructing the symmetry properties of this matrix, the transformation under inversion was incorrectly calculated, as is apparent from Eq. (22). Thus, in Table XV the entries for inversion \mathcal{I} and $m_x m_y \mathcal{I}$ for sublattices 9–12 all were missing a factor of $\Lambda \equiv \exp(2\pi i q_z)$, where q_z is in rlu's. The consequences of this omission are the following. In the matrix of Eq. (95), all the elements $M_{ij}^{(xx)}$ for $i < 9$ and $j > 8$ should be multiplied by $\Lambda^{1/2}$ and all the elements of $M_{ij}^{(xx)}$ with $i > 8$ and $j < 9$ should be multiplied by $\Lambda^{-1/2} = \Lambda^{*1/2}$. However, as before, the Roman letters are real and the Greek ones are complex. Typically this form of the matrix arises as follows. Using the glide operations m_x and m_y one can show that $M_{1,9}^{xx} = M_{2,10}^{xx} = M_{3,11}^{xx} = M_{4,12}^{xx} = \alpha$, where α is an unspecified complex number. Then inversion symmetry indicates that $\alpha = M_{1,9}^{xx} = \Lambda [M_{10,2}^{xx}]^* = \Lambda \alpha^*$. Therefore $\alpha = \Lambda^{1/2} a$, where a is a real number.

Following this, all the symmetry adapted coordinates of Eqs. (101) and (104) which pertain to sublattices 9–12 should be multiplied by $\Lambda^{-1/2}$. The argument starting with Eq. (112) and ending with Eq. (118) has missing factors of Λ , but these cancel when Eq. (118) is reached, so that Eq. (119) remains true, but the \mathbf{O} 's involve Eqs. (101) and (104), modified as mentioned above. Then the result for the allowed spin Fourier transforms of Table XVI is modified by having all the entries for sublattices 9–12 multiplied by $\Lambda^{-1/2}$. Finally, the explicit results for the wave functions of sublattices 9–12 in Eq. (123) should be corrected by replacing (only for these sublattices) σ_n by $\sigma_n \Lambda^{-1/2} \equiv \sigma_n \exp(-i\phi_n - i\pi q_z)$. In Eq. (123) this is most easily done by replacing ϕ_n by $\phi_n + \pi q_z$ for sublattices 9–12. The details of the argument below Eq. (126) are changed but the conclusion is still that inversion fixes the phases in the wave function, although now in a way involving q_z , which is reasonable.

The reader might well ask, "What makes you think the results are now correct, since the calculation was complicated?" The answer is that this error was found by a calculation¹ which shows how the eigenfunctions of the two one-dimensional irreducible representations (for $q_x \neq 1/2$) connect (as $q_x \rightarrow 1/2$) to these of the two-dimensional irreducible representation for $q_x = 1/2$. The equations which give the wave functions and order parameter of the latter phase in terms of those of the former phase are highly overdetermined and the fact that they had no solution using the incorrect wave function (before this erratum) led me to find the error noted in this erratum. Using the corrected information of Table XVI not only led to equations which had a solution, but that solution preserved the phase relations expected from the symmetry analysis leading to Table XVI. This result provides strong evidence that indeed this erratum is conclusive. Note that this erratum does not affect the wave functions for the Mn sublattices 1–8, so the results for YMn_2O_5 do not need correction. (The nonmagnetic Y ions occupy sublattices 9–12.) Indeed my results give an excellent reproduction of the spin structure of the commensurate phase $\mathbf{q}=(1/2, 0, 1/4)$ of YMn_2O_5 , as determined by neutron diffraction.²

¹A. B. Harris, M. Kenzelman, A. Aharony, and O. Entin-Wohlman (unpublished).

²H. Kimura, S. Kobayashi, Y. Fukuda, T. Osawa, Y. Kamada, Y. Noda, I. Kagomiya, and K. Kohn, J. Phys. Soc. Jpn. **76**, 074706 (2007).